DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE
End – Semester Examination (Supplementary): November 2018

Branch: B. Tech (Common to all) **Semester:** I
Subject with code: Engineering Mathematics – I (MATH 101)
Date: 26/11/2018 **Marks:** 60 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1 (a)Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ by reducing it to normal form

[6 $Marks$]

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [6 $Marks$]

Q.2 (a) If $y = e^{a \sin^{-1} x}$, prove that$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

[6 $Marks$]

(b) Using Taylor's theorem, express the polynomial

$f(x) = 2x^3 + 7x^2 + x - 6$in powers of $(x-1)$. [6 $Marks$]

Q.3 **Solve any TWO:**

(a) If$v = \log(x^2+y^2+z^2)$, prove that$(x^2+y^2+z^2)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = 2$.

[6 $Marks$]

(b) If z is a homogeneous function of degree $n$in $, y$, prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$ [6 $Marks$]

(c) If $z = f(x,y)$ where $x = e^u + e^{-v}$& $y = e^{-u} - e^v$, then show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$ [6 $Marks$]

Q.4 (a) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$ and $w = \frac{xy}{z}$, show that $\frac{\partial(u\ ,\ v,\ w)}{\partial(x,\ y,\ z)} = 4$. [4 $Marks$]

(b) The focal length of a mirror is found from the formula $\frac{2}{f} = \frac{1}{v} - \frac{1}{u}$. Find the percentage error in f if u & v are both in error by 2% each. [4 $Marks$]

(c) Find the maximum value of $x^m y^n z^p$, when $x + y + z = c$. [4 $Marks$]

Q.5 (a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\ dy\, dx.$

[6 $Marks$]

(b) Change to polar co-ordinates to evaluate $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$. [6 $Marks$]

(c) Evaluate the integral $I = \int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$. [6 $Marks$]

Q.6 (a) State D' Alembert's ratio test, and hence check the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{(n^n)^2} .$$ [6 $Marks$]

(b) State Cauchy's root test, and hence check the convergence of the series:

$$\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}} .$$ [6 $Marks$]

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Regular Winter Examination – 2024

Course: B. Tech. **Branch: Common to all branches** **Semester: I**

Subject Code & Name: (24AF1000BS101) Engineering Mathematics - I

Max Marks: 60 **Date: 06/02/2025** **Duration: 3 Hr.**

Instructions to the Students:

1. Each question carries 12 marks.
2. Question No. 1 will be compulsory and include objective-type questions.
3. Candidates are required to attempt any four questions from Question No. 2 to Question No. 6.
4. The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.
5. Use of non-programmable scientific calculators is allowed.
6. Assume suitable data wherever necessary and mention it clearly.

					(CO)	Marks
Q.1	**Objective type questions. (Compulsory Question)**					**12**
1	Homogeneous system of linear equations is/has				(CO1)	1
	a. always consistent	b. always inconsistent	c. no solution	d. None		
2	The rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ is equal to				(CO1)	1
	a. 1	b. 2	c. 3	d. None		
3	If $A = [a_{ij}]$ is a square matrix of order n, then trace of matrix A is				(CO1)	1
	a. product of diagonal elements	b. sum of diagonal elements	c. sum of row elements	d. None		
4	If $u = f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to				(CO2)	1
	a. 2u	b. u	c. 0	d. None		
5	If $u = x^y$ then $\frac{\partial u}{\partial y}$ is equal to				(CO2)	1
	a. $x^y \log x$	b. 0	c. yx^{y-1}	d. None		
6	If $u = f(y-z, z-x, x-y)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$				(CO3)	1
	a. 1	b. **0**	c. -1	d. None		
7	The condition for the function $f(x,y)$ to have maximum value at (a, b) is				(CO3)	1
	a. $rt - s^2 > 0 \ and \ r < 0$	b. $rt - s^2 > 0 \ and \ r > 0$	c. $rt - s^2 < 0 \ and \ r < 0$	d. None		
8	If $x = uv, y = \frac{u}{v}$ then $\frac{\partial(x,y)}{\partial(u,v)}$ is				(CO3)	1
	a. $\frac{-2u}{v}$	b. $\frac{-2v}{u}$	c. 0	d. None		
9	The formula for $\int_0^{\frac{\pi}{2}} \sin^n\theta d\theta$ is equal to				(CO4)	1
	a. $\frac{n(n-1)(n-3)......}{n(n-2)(n-4).....} \times \frac{\pi}{2}$	b. $\frac{n(n-1)(n-3)......}{n(n-2)(n-4).....} \times 1$	c. $\frac{n(n-1)(n-3)......}{n(n-2)(n-4).....} \times 1 \ or \ \frac{\pi}{2}$	d. None		

10	The number of loops in the polar curve $r = a\sin 2\theta$ are				(CO4)	1
	a. 4	b. 2	c. 6	d. None		
11	The value of $\int_0^2 \int_1^y xy\, dxdy$ is equal to				(CO5)	1
	a. 0	b. 1	c.-1	d. None		
12	In polar co-ordinate system (r, θ) ; value of $dy\, dx$ is equal to				(CO5)	1
	a. $dr\, d\theta$	b. $rdr\, d\theta$	c. $r^2 dr\, d\theta$	d. None		
Q.2	**Solve the following.**					12
A)	Solve the equations: $x + 3y + 2z = 0;\ 2x - y + 3z = 0;\ 3x - 5y + 4z = 0.$				(CO1)	6
B)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$				(CO1)	6
Q.3	**Solve the following.**					12
A)	If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$.				(CO2)	6
B)	If z is a homogeneous function of degree n in x, y, then prove that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$				(CO2)	6
Q.4	**Solve any TWO of the following.**					12
A)	Expand $f(x, y) = e^x \sin y$ in the powers of x and y as far as the terms of third degree.				(CO3)	6
B)	Test the function $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$ for maxima, minima and saddle point				(CO3)	6
C)	Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$.				(CO3)	6
Q.5	**Solve any TWO of the following.**					12
A)	Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^8}$.				(CO4)	6
B)	Trace the curve $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ (Cycloid).				(CO4)	6
C)	Trace the curve $r = a \sin 3\theta$ (3 Leaved Rose).				(CO4)	6
Q.6	**Solve any TWO of the following.**					12
A)	Evaluate $\int_0^1 \int_0^x e^{x+y}\, dy\, dx$.				(CO5)	6
B)	Find the area of the circle $x^2 + y^2 = a^2$.				(CO5)	6
C)	Define and verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$				(CO2)	6
	*** End ***					

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2022

Course: B. Tech. (Common to all Branches) **Semester : I**

Subject Code & Name: Engineering Mathematics – I (BTBS 101)

Max Marks: 60 **Date:** **Duration: 3 Hrs.**

Instructions to the Students:

1. *All the questions are compulsory.*
2. *The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Solve Any Three of the following.**		**12**
A)	Reduce to the Normal form and find the rank of the given matrix. $A=\begin{bmatrix}1 & -2 & 0 & 1\\ 2 & -1 & 1 & 0\\ 3 & -3 & 1 & 1\\ -1 & -1 & -1 & 1\end{bmatrix}$	**Understand/ CO1**	**4**
B)	Test the consistency and solve: $3x+y+2z=3, 2x-3y-z=-3, x+2y+z=4$	**Understand/ CO1**	**4**
C)	Find the eigen value & eigen vector for least positive eigen value of the matrix: $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$	**Understand/ CO1**	**4**
D)	Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$	**Understand/ CO1**	**4**
Q.2	**Solve Any Three of the following:**		**12**
A)	If $u=\log(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$ then find the value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$	**Understand/ CO2**	**4**
B)	If $\mathrm{v}=\log(\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2)$, prove that $(\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2)\left(\frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2}+\frac{\partial^2 \mathrm{v}}{\partial \mathrm{y}^2}+\frac{\partial^2 \mathrm{v}}{\partial \mathrm{z}^2}\right)=2$	**Understand/ CO2**	**4**
C)	$u=\sin^{-1}\left(x^2+y^2\right)^{\frac{1}{5}}$ then find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$	**Understand/ CO2**	**4**
D)	Find $\frac{du}{dt}$ when $u=xy^2+x^2y$, $x=at^2$, $y=2at$	**Understand/ CO2**	
Q. 3	**Solve Any Three of the following:**		**12**
A)	If $u=x^2-2y^2$, $v=2x^2-y^2$ Where $x=r\cos\theta$, $y=r\sin\theta$ then show	**Understand/ CO3**	**4**

	that $\frac{\partial(u,v)}{\partial(r,\theta)}=6r^3\sin 2\theta$		
B)	Show that $JJ'=1$ if $x=u(1-v)$, $y=uv$	Understand/ CO3	4
C)	Discuss the maxima and minima of the function $x^2+y^2+6x+12$	Understand/ CO3	4
D)	Expand $f(x,y)=x^2y+3y-2$ in the powers of $(x-1)$ and $(y+2)$ using Taylor's theorem	Understand/ CO3	4
Q.4	**Solve Any Three of the following:**		**12**
A)	Prove that $\int_0^\infty \frac{t^4}{(1+t^2)^3}dt=\frac{3\pi}{16}$	Understand/ CO4	4
B)	Trace the Curve $a^2y^2=x^2\left(a^2-x^2\right)$	Understand/ CO4	4
C)	Trace the Curve $x=a\left(t-\sin t\right)$, $y=a\left(1-\cos t\right)$	Understand/ CO4	4
D)	Trace the Curve $r=a\cos 2\theta$	Understand/ CO4	4
Q. 5	**Solve the following:**		**12**
A)	Evaluate $\int_0^1\int_0^y xy\,dxdy$	Understand/ CO5	4
B)	Change the order of integration $\int_0^a\int_0^{\sqrt{a^2-x^2}} f(x,y)\,dydx$	Understand/ CO5	4
C)	Find the volume bounded by paraboloid $x^2+y^2=az$, the cylinder $x^2+y^2=2ay$ and the plane $z=0$	Understand/ CO5	4
	*** End ***		

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Regular & Supplementary Winter Examination-2023

Course: B. Tech **Branch: All** **Semester: I**

Course Code & Name: Engineering Mathematics-I (BTBS101)

Max Marks: 60 **Date:01-01-24** **Duration: 3 Hr**

Instructions to the Students:

1. *All the questions are compulsory.*
2. *The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Solve Any Two of the following.**		**12**
A)	Find the rank of matrix by converting it into Normal Form $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$	**Understand (CO1)**	**6**
B)	Find eigen values & eigen vector for largest eigen value for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	**Understand (CO1)**	**6**
C)	Check the consistency and solve: $2x - 3y + 5z = 1, 3x + y - z = 2, \ x + 4y - 6z = 1$	**Understand (CO1)**	**6**
Q.2	**Solve Any Two of the following.**		**12**
A)	If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$	**Understand (CO2)**	**6**
B)	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$	**Understand (CO2)**	**6**
C)	If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$	**Understand (CO2)**	**6**
Q. 3	**Solve Any Two of the following.**		**12**
A)	If $u = x + 2y^2 - z^3$, $v = 2x^2yz$, $w = 2z^2 - xy$ then evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$ 6	**Understand (CO3)**	**6**
B)	Discuss the maxima and minima for the function $x^2 + y^2 + (30 - x - y)^2$ and hence find the extreme value of the function. $(10,10) \to 300 \ f_{min}$	**Understand (CO3)**	**6**
C)	Using Lagrange's undetermined multipliers find the maximum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$	**Understand (CO3)**	**6**

Q.4	**Solve Any Two of the following.**		12
A)	Evaluate $\int_0^a x^3(a-x)^{\frac{3}{2}}\,dx$	Understand (CO4)	6
B)	Trace the curve $y^2(2a-x)=x^3$.	Understand (CO4)	6
C)	Trace the curve $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$.	Understand (CO4)	6
Q. 5	**Solve Any Two of the following.**		12
A)	Evaluate $\int_0^1\int_0^{z^2}\int_0^{z^2-x} xz\,dx\,dy\,dz$	Understand (CO5)	6
B)	Find the area bounded by $y^2=4x$ and $2x-3y=-4$.	Understand (CO5)	6
C)	Change to polar and evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$.	Understand (CO5)	6

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Regular & Supplementary Winter Examination-2023

Course: B. Tech | **Branch: All** | **Semester: I**

Course Code & Name: Engineering Mathematics-I (BTBS101)

Max Marks: 60 | **Date:01-01-24** | **Duration: 3 Hr.**

Instructions to the Students:

1. *All the questions are compulsory.*
2. *The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Solve Any Two of the following.**		**12**
A)	Find the rank of matrix by converting it into Normal Form $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$	Understand (CO1)	6
B)	Find eigen values & eigen vector for largest eigen value for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	Understand (CO1)	6
C)	Check the consistency and solve: $2x - 3y + 5z = 1$, $3x + y - z = 2$, $x + 4y - 6z = 1$	Understand (CO1)	6
Q.2	**Solve Any Two of the following.**		**12**
A)	If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$	Understand (CO2)	6
B)	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$	Understand (CO2)	6
C)	If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$	Understand (CO2)	6
Q. 3	**Solve Any Two of the following.**		**12**
A)	If $u = x + 2y^2 - z^3$, $v = 2x^2yz$, $w = 2z^2 - xy$ then evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$	Understand (CO3)	6
B)	Discuss the maxima and minima for the function $x^2 + y^2 + (30 - x - y)^2$ and hence find the extreme value of the function.	Understand (CO3)	6
C)	Using Lagrange's undetermined multipliers find the maximum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$	Understand (CO3)	6

Q.4	**Solve Any Two of the following.**		**12**
A)	Evaluate $\int_0^a x^3 (a-x)^{\frac{3}{2}}\, dx$	**Understand (CO4)**	**6**
B)	Trace the curve $y^2(2a-x) = x^3$.	**Understand (CO4)**	**6**
C)	Trace the curve $x = a(\theta - sin\theta)$, $y = a(1 - cos\theta)$.	**Understand (CO4)**	**6**
Q. 5	**Solve Any Two of the following.**		**12**
A)	Evaluate $\int_0^1 \int_0^{z^2} \int_0^{z^2-x} xz\, dx\, dy\, dz$	**Understand (CO5)**	**6**
B)	Find the area bounded by $y^2 = 4x$ and $2x - 3y = -4$.	**Understand (CO5)**	**6**
C)	Change to polar and evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}\, dx\, dy$.	**Understand (CO5)**	**6**

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,

LONERE – RAIGAD - 402 103

SEMESTER EXAMINATION: DECEMBER – 2018

Course: B. Tech (All Branches) **Semester: I**
Subject with Subject Code: Engineering Mathematics – I (BTMA101)
Date: 11/12/2018 **Marks: 60** **Time: 3 Hrs.**

Instructions to the Students:-

1. *Each question carries 12 marks.*
2. ***All** the questions are compulsory.*
3. *Illustrate your answers with neat sketches, diagram etc., wherever necessary.*
4. *If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.*

(Marks)

Q.1. Attempt any three **(12)**

(a) Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{bmatrix}$ hence find the A^{-1}

(b) For what values of $\boldsymbol{\lambda}$ and μ the equations $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have (i) no solution (ii)a unique solution or (iii) an infinite number of solutions.

(c) Use Gauss-Jordan method to find the inverse of the matrix $A=\begin{bmatrix}2 & 1 & -1\\ 0 & 2 & -1\\ 5 & 2 & -3\end{bmatrix}$

(d) Find the rank of a matrix A by reducing it to normal form, where $A=\begin{bmatrix}1 & -12 & 3\\ 4 & 1\ 0 & 2\\ 0 & 31 & 4\\ 0 & 10 & 2\end{bmatrix}$

Q.2. Attempt any three **(12)**

(a) If $z(x+y)=x^2+y^2$ show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$

(b) If u $sin^{-1}\sqrt{\frac{x^2+y^2}{x+y}}$, Show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{4}tanu[tan^2u-1]$

(c) $If u=x^2+y^2, v=2xy \wedge z=f(u,v)$ then show that $x.\frac{\partial z}{\partial x}-y.\frac{\partial z}{\partial y}=2\sqrt{u^2-v^2}.\frac{\partial z}{\partial u}$

(d) If u= x log xy, where $x^3+y^3+3xy=1, Find\ \frac{du}{dx}$

Q.3. **Attempt any two** **(12)**

(a) $x=\sqrt{vw}, y=\sqrt{uw}, z=\sqrt{uv}$ and $u=r\sin\theta\cos\emptyset, v=r\sin\theta\sin\emptyset, w=r\cos\theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$

(b) Expand $f(x,y)=e^{x}\cos y$ at $\left(1,\frac{\pi}{4}\right)$ using Taylor's theorem

(c) Divide 24 into three parts such that the continued product of the first square of the second and cube of the third may be maximum.

Q.4 **Attempt any three** **(12)**

(a) Evaluate $\int_0^\infty \frac{t^2}{(1+t^2)^{\frac{7}{2}}}dt$

(b) Trace the curve $x=a(\theta+\sin\theta), y=a(1+\cos\theta)$

(c) Trace the curve $y^2(2a-x)=x^3$

(d) Trace the curve $r=1+2\cos\theta$

Q.5. **Attempt any two** **(12)**

(a) Change the order of integration and evaluate

$$\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy.dx.dy$$

(b) Change to polar co-ordinates and evaluate

$$\int_0^4 \int_{\frac{y^2}{4}}^{y} \frac{x^2-y^2}{x^2+y^2}dx.dy$$

(c) Find the area included between the curves $y=x^2-6x+3 \wedge y=2x-9$ by double integration

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Winter Examination – Dec 2019

Course: B. Tech (All Courses) **Sem: I**

Subject Name: Engineering Mathematics-I **Subject Code: BTMA101**

Max Marks: 60M **Date:-11/12/2019** **Duration:- 3 Hrs.**

Instructions to the Students:

1. All questions are compulsory.
2. Use of non-programmable calculator is allowed.
3. Figures to right indicate full marks.
4. Illustrate your answer with neat sketches, diagram etc. whatever necessary.
5. If some part of parameter is noticed to be missing you may appropriately assume it and should mention it clearly.

		Marks
Q. 1	**Solve the following questions.**	
A)	Reduce to the Normal form and find the rank of the given matrix. $A=\begin{bmatrix}1 & 2 & 3 & -1\\ -1 & -1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$	**4**
B)	Test the consistency and solve: $2x_1+x_2-x_3+3x_4=11$, $x_1-2x_2+x_3+x_4=8$, $4x_1+7x_2+2x_3-x_4=0$, $3x_1+5x_2+4x_3+4x_4=17$	**4**
C)	Find the eigen value & eigen vector for least positive eigen value of the matrix : $A=\begin{bmatrix}4 & 2 & -2\\ -5 & 3 & 2\\ -2 & 4 & 1\end{bmatrix}$	**4**
Q.2	**Solve any *three* of the following.**	
A)	If $x^x y^y z^z = c$ show that at point $x=y=z$, $\frac{\partial^2 z}{\partial x \partial y} = -[x \log ex]^{-1}$	**4**
B)	If $u=\sin\left(x/y\right)$ & $x=e^t$, $y=t^2$ verify $\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{du}{dt}$	**4**
C)	If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u \cos 2u}{4\cos^3 u}$	**4**
D)	If $u=f(2x-3y,3y-4z,4z-2x)$ prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$	**4**
Q. 3	**Solve any *three* of the following.**	
A)	Expand $f(x,y)=e^{x+y}$ in Maclaurin's theorem up to fourth term.	**4**
B)	If $x=u(1-v)$, $y=uv$ prove that $JJ'=1$	**4**
C)	A rectangular box open at the top is to have volume of 256 cubic feet, determine the dimensions of the box required least material for the construction of the box.	**4**
D)	Examine the function x^3+y^3-3axy for maxima & minima where $a>0$	**4**

Q.4	**Solve any *three* of the following.**	
A)	Evaluate $\int_0^{2a} x\sqrt{(2ax-x^2)}\,dx$	**4**
B)	Trace the Curve $y^2(a-x)=x^2(a+x)$	**4**
C)	Trace the Curve $x = a\cos^3 t$, $y = a\sin^3 t$	**4**
D)	Trace the Curve $r = a \cos 3\theta$	**4**
Q. 5	**Solve the following questions.**	
A)	Change the order of integration $I=\int_0^a \int_x^{a^2/x} f(x,y)\,dxdy$	**4**
B)	Change to polar and evaluate $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$	**4**
C)	Find the volume bounded by the cylinders $x^2+y^2=ax$ & $z^2=ax$	**4**
	END	

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE		
End Semester Winter Examination – Dec 2019		
Course: B. Tech (All Courses)		**Sem: I**
Subject Name: Engineering Mathematics-I		**Subject Code: BTMA101**
Max Marks: 60M	**Date:-11/12/2019**	**Duration:- 3 Hrs.**

Instructions to the Students:

1. All questions are compulsory.
2. Use of non-programmable calculator is allowed.
3. Figures to right indicate full marks.
4. Illustrate your answer with neat sketches, diagram etc. whatever necessary.
5. If some part of parameter is noticed to be missing you may appropriately assume it and should mention it clearly.

		Marks
Q. 1	**Solve the following questions.**	
A)	Reduce to the Normal form and find the rank of the given matrix. $A=\begin{bmatrix}1 & 2 & 3 & -1\\ -1 & -1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$	**4**
B)	Test the consistency and solve: $2x_1+x_2-x_3+3x_4=11$, $x_1-2x_2+x_3+x_4=8$, $4x_1+7x_2+2x_3-x_4=0$, $3x_1+5x_2+4x_3+4x_4=17$	**4**
C)	Find the eigen value & eigen vector for least positive eigen value of the matrix : $A=\begin{bmatrix}4 & 2 & -2\\ -5 & 3 & 2\\ -2 & 4 & 1\end{bmatrix}$	**4**
Q.2	**Solve any *three* of the following.**	
A)	If $x^x y^y z^z = c$ show that at point $x=y=z$, $\frac{\partial^2 z}{\partial x \partial y} = -[x\log ex]^{-1}$	**4**
B)	If $u=\sin\left(x/y\right)$ & $x=e^t$, $y=t^2$ verify $\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{du}{dt}$	**4**
C)	If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}$	**4**
D)	If $u=f(2x-3y,3y-4z,4z-2x)$ prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$	**4**
Q. 3	**Solve any *three* of the following.**	
A)	Expand $f(x,y)=e^{x+y}$ in Maclaurin's theorem up to fourth term.	**4**
B)	If $x=u(1-v)$, $y=uv$ prove that $JJ^{'}=1$	**4**
C)	A rectangular box open at the top is to have volume of 256 cubic feet, determine the dimensions of the box required least material for the construction of the box.	**4**
D)	Examine the function x^3+y^3-3axy for maxima & minima where $a>0$	**4**

Q.4	**Solve any *three* of the following.**	
A)	Evaluate $\int_0^{2a} x\sqrt{(2ax-x^2)}dx$	**4**
B)	Trace the Curve $y^2(a-x)=x^2(a+x)$	**4**
C)	Trace the Curve $x = a\ cos^3 t\ ,\ y = a\ sin^3 t$	**4**
D)	Trace the Curve $r = a\ \cos 3\theta$	**4**
Q. 5	**Solve the following questions.**	
A)	Change the order of integration $I=\int_0^a \int_x^{a^2/x} f(x,y)dxdy$	**4**
B)	Change to polar and evaluate $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$	**4**
C)	Find the volume bounded by the cylinders $x^2+y^2=ax \,\&\, z^2=ax$	**4**
	END	

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE RAIGAD-402 103.

Semester Examination – May - 2019

Branch: First year B. Tech.(ALL) Semester: I

Subject with Code: Engg. Math-I (BTMA101)

Time: 3 hrs. Date: 14/05/2019 Max. Marks: 60

Instructions to Students:

(**1**) All questions are compulsory.
(2) Use of non-programmable calculator is allowed.
(3) Figures to right indicate full marks.
(4) Illustrate your answer with neat sketches, diagram etc. whatever necessary.
(5) If some part or parameter is noticed tobe missing you may appropriately assume it and should mention it clearly.

Q.1 Attempt any three from the following: 4 X 3 = 12

(a) Reduce the following matrix into normal form & find its rank

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

(b) Test the consistency & solve

$$4x - 2y + 6z = 8\ ;$$

$$x + y - 3z = -1\ ;$$

$$15x - 3y + 9z = 21.$$

(c) Find the Eigen values & Eigen vectors of the following matrix

$$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

(d) Find A^{-1} by using Cayley-Hamilton Theorem

Where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Q.2 Attempt any three from the following: 4 X 3 = 12

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}\ .$$

(b) If $f(u)$ is a homogeneous function of degree n in x & y, then show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=G(u)\,[\,G'(u)-1\,] \quad \text{where } G(u)=n\frac{f(u)}{f`(u)}.$$

(c) Verify Euler's theorem for $u=\frac{x^2+y^2}{x+y}$.

(d) If $z=f(u,v)$ where $u=lx+my \; and \; v=ly-mx$, then show that

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(l^2+m^2)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right).$$

Q.3 Attempt any three from the following: 4 X 3 = 12

(a) Show that $J.J'=1$ If $x=e^u cosv$ & $y=e^u sinv$.

(b) Expand $f(x,y)=\sin(xy)$ in the powers of $(x-1)$ & $(y-\frac{\pi}{2})$ by using Taylor's Theorem.

(c) Discus the maxima & minima of $xy(a-x-y)$.

(d) If $u=\frac{a^3}{x^2}+\frac{b^3}{y^2}+\frac{c^3}{z^2}$ where $x+y+z=1$, then find the stationary values by using Lagrange's method of multipliers.

Q.4 Attempt any three from the following: 4 X 3 = 12

(a) Evaluate $\int_0^4 x^3\sqrt{4x-x^2}\,dx$.

(b) Trace the curve $y^2(4-x)=x(x-2)^2$.

(c) Trace the curve $r=a\cos(2\theta)$.

(d) Trace the curve $x=a(t+sint)$

$y=a(1-cost)$.

Q.5 Attempt any three from the following: 4 X 3 = 12

(a) Change the order of integration & evaluate it

$$\int_0^1\int_y^1 x^2e^{xy}\,dx\,dy.$$

(b) Change to polar co-ordinates & evaluate it

$$\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)\,dx\,dy$$

(c) Evaluate $\int_{-1}^{1}\int_0^z\int_{(x-z)}^{(x+z)}(x+y+z)dx\,dy\,dz$.

(d) Find the area outside the circle $r=a$

& inside the cardioide $r=a(1+cos\theta)$.

***** END *****

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103

Semester Winter Examination – Dec.- 2019

Branch: B. Tech. (Common to all) **Semester:- I**
Subject:- Engineering Mathematics – I (MATH 101) **Marks: 60**
Date:- 11/12/2019 **Time:- 3 Hr.**

Instructions to the Students

1. Attempt **any five** questions of the following.
2. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly

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Q.1

(a) Determine the consistency of the set of equations:

$\boldsymbol{x - 2y + z = -5}$; $\boldsymbol{x + 5y - 7z = 2}$; $\boldsymbol{3x + y - 5z = 1}$. [6 *Marks*]

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [6 *Marks*]

Q.2

(a) If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$. [6 *Marks*]

(b) Using Taylor's theorem,

Prove that $\log \sin x = \log \sin a + (x-a)\cot a - \frac{1}{2}(x-a)^2 \operatorname{cosec}^2 x + \cdots$ [6 *Marks*]

Q.3 Solve any TWO:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. [6 *Marks*]

(b) If z is a homogeneous function of degree n in x *and* y , prove that

$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$. [6 *Marks*]

(c) If $z = f(x, y)$ where $x = e^u + e^{-v}$ & $y = e^{-u} - e^v$,

then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. [6 *Marks*]

Q.4

(a) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. [4 *Marks*]

(b) Find the percentage error in the measurement of the area of an ellipse when an error of **1.5 %** is made in measuring its major and minor axes. [4 $Marks$]

(c) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [4 $Marks$]

Q.5 Solve any TWO:

(a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\, dy\, dx$. [6 $Marks$]

(b) Change the order of integration and evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y}\, dx\, dy.$ [6 $Marks$]

(c) Evaluate the integral $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dx\, dy$. [6 $Marks$]

Q.6

(a) State D' Alembert's ratio test, and hence check the convergence of the series:
$\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$. [6 $Marks$]

(b) State Cauchy's root test, and hence check the convergence of the series:
$\sum \frac{[(2n+1)\,x\,]^n}{n^{n+1}}$ $(x > 0)$. [6 $Marks$]

****************************Paper End********************************

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End – Semester Examination (Supplementary): May 2019

Branch: B. Tech (Common to all) **Semester:** I

Subject with code: Engineering Mathematics – I (MATH 101) **Marks:** 60

Date: 28.05.2019 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1

(a) Solve the equations:

$x + 2y + 4z + w = 0$; $2x + 4y + 8z + 2w = 0$; $3x + 6y + 2z = 0$ [6 *Marks*]

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [6 *Marks*]

Q.2

(a) Find the n^{th} order derivative of $y = \cos^4 x$ [6 *Marks*]

(b) Using Taylor's theorem, express the polynomial $f(x) = 2x^3 + 7x^2 + x - 6$ in the powers of $(x - 1)$. [6 *Marks*]

Q.3 Solve any TWO:

(a) Evaluate $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ if $z = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$. [6 *Marks*]

(b) If z is a homogeneous function of degree n in x *and* y, prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$ [6 *Marks*]

(c) If $u = f(y - z, z - x, x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [6 *Marks*]

Q.4

(a) Expand $\boldsymbol{f(x, y) = e^{xy}}$ at $\boldsymbol{(1, 1)}$ by using Taylor's theorem. [4 $Marks$]

(b) Find the percentage error in the measurement of the area of an ellipse when an error of **1.5 %** is made in measuring its major and minor axes. [4 $Marks$]

(c) Find the maximum value of $\boldsymbol{x^m\, y^n\, z^p}$ **,** when $\boldsymbol{x + y + z = c}$ [4 $Marks$]

Q.5

(a) Evaluate the integral $\boldsymbol{I = \int_1^a \int_1^b \frac{dy\, dx}{xy}}$. [6 $Marks$]

(b) Change the order of integration and evaluate $\boldsymbol{\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y}\, dx\, dy}$. [6 $Marks$]

(c) Evaluate the integral $\boldsymbol{I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dx\, dy}$. [6 $Marks$]

Q.6

(a) State D' Alembert's ratio test, and hence check the convergence of the series:

$\boldsymbol{\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)}$. [6 $Marks$]

(b) State Cauchy's root test, and hence check the convergence of the series:

$\boldsymbol{\sum \frac{[(2n+1)\, x\,]^n}{n^{n+1}}}$ $\boldsymbol{(x > 0)}$ [6 $Marks$]

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE
End – Semester Examination (Supplementary): November 2018

Branch: B. Tech (Common to all) **Semester:** I
Subject with code: Engineering Mathematics – I (MATH 101)
Date: 26/11/2018 **Marks:** 60 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1 (a)Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ by reducing it to normal form

$[6\ Marks]$

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. $[6\ Marks]$

Q.2 (a) If $y = e^{a\sin^{-1}x}$, prove that$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

$[6\ Marks]$

(b) Using Taylor's theorem, express the polynomial

$f(x) = 2x^3 + 7x^2 + x - 6$in powers of $(x-1)$. $[6\ Marks]$

Q.3 **Solve any TWO:**

(a) If$v = \log(x^2+y^2+z^2)$, prove that$(x^2+y^2+z^2)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = 2$.

$[6\ Marks]$

(b) If z is a homogeneous function of degree $n$in $, y$, prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z. \quad [6\ Marks]$$

(c) If $z = f(x,y)$ where $x = e^u + e^{-v}$& $y = e^{-u} - e^v$, then show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}. \quad [6\ Marks]$$

Q.4 (a) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$ and $w = \frac{xy}{z}$, show that $\frac{\partial(u\ ,\ v,\ w)}{\partial(x,\ y,\ z)} = 4$. [4 $Marks$]

(b) The focal length of a mirror is found from the formula $\frac{2}{f} = \frac{1}{v} - \frac{1}{u}$. Find the percentage error in f if u & v are both in error by 2% each. [4 $Marks$]

(c) Find the maximum value of $x^m y^n z^p$, when $x + y + z = c$. [4 $Marks$]

Q.5 (a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\ dy\, dx$.

[6 $Marks$]

(b) Change to polar co-ordinates to evaluate $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$. [6 $Marks$]

(c) Evaluate the integral $I = \int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$. [6 $Marks$]

Q.6 (a) State D' Alembert's ratio test, and hence check the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{(n^n)^2} .$$ [6 $Marks$]

(b) State Cauchy's root test, and hence check the convergence of the series:

$$\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}} .$$ [6 $Marks$]

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD - 402 103
Semester Examination: December – 2017

Branch: All Courses **Semester: I**

Subject with Subject Code: Engineering Mathematics-I (MATH101) **Marks: 60**

Date: 11/12/2017 **Time: 3 Hrs.**

Instructions to the Students:-

1. Each question carries 12 marks.
2. Attempt **any five** questions of the following.
3. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

(Marks)

Q.1. (a) For what value of λ the following system of linear equations is consistent and solve it completely in each case: **(06)**

$$x+y+z=1, x+2y+4z=\lambda, x+4y+10z=\lambda^2 .$$

(b) Find the eigen values and the corresponding eigen vectors for the matrix **(06)**

$$A=\begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

Q.2. (a) If $y=\sin px+\cos px$, then prove that $y_n=p^n\left[1+(-1)^n\sin(2px)\right]^{\frac{1}{2}}$. **(04)**

(b) If $y=e^{a\cos^{-1}x}$, then prove that **(04)**

$$(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+a^2)y_n=0 .$$

(c) Expand y = $\log(\cos x)$ about the point $x=\frac{\pi}{3}$ up to third degree by using Taylor's series. **(04)**

Q.3. Attempt **Any Three:** **(12)**

(a) If $x^x y^y z^z = c$, then prove that at point $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \ln ex)^{-1}$.

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ **,** prove that $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u$.

(c) If $x^2 = au + bv, y^2 = au - bv$ and $z = f(u, v)$, then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\left(u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}\right)$.

(d) If $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t, y = t^2$, then find $\frac{du}{dt}$.

Q.4. Attempt **Any Three:** **(12)**

(a) If $ux = yz, vy = zx, wz = xy$, then prove that J J* = 1 where $J = \frac{\partial(u, v, w)}{\partial(x, y, z)}$ and J* = $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(b) If the sides and angles of a plane triangle vary in such a that its circum-radius remains constant, then prove that $\frac{da}{\cos A}$ + $\frac{db}{\cos B} + \frac{dc}{\cos C}$ = 0.

(c) A rectangular box open at the top is to have volume of 32cubic units. Find the dimensions of the box requiring the least material for its construction by Lagrange’s method of undetermined multipliers.

(d) Expand $f(x, y) = x^y$ as for as second degree in the powers of $(x - 1)$ and $(y - 1)$ using Taylor's theorem.

Q.5. Attempt **Any Three:** **(12)**

(a) Change the order of integration and evaluate I = $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y} dxdy$.

(b) Use elliptical polar form to evaluate I = $\iint_R xy\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^{\frac{n}{2}} dxdy$, where R is the region of ellipse in positive quadrant.

(c) Use spherical polar transformation to evaluate I = $\int_0^\infty dx \int_0^\infty dy \int_0^\infty \frac{dz}{(x^2 + y^2 + z^2)^2}$.

(d) Find the centroid of the positive loop of the curve $r^2 = a^2 \cos 2\theta$.

Q.6. **(a)** Test the convergence of the series $\sum_{1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2} \right)$. **(04)**

(b) Test the convergence of the series $\sum_{1}^{\infty} \frac{(n+1)^n x^n}{n^{(n+1)}}$. **(04)**

(c) Test the absolute convergence of the series $\sum_{2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$. **(04)**

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103

Semester Winter Examination – Dec.- 2019

Branch: B. Tech. (Common to all) **Semester:- I**
Subject:- Engineering Mathematics – I (MATH 101) **Marks: 60**
Date:- 11/12/2019 **Time:- 3 Hr.**

Instructions to the Students

1. Attempt **any five** questions of the following.
2. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly

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Q.1

(a) Determine the consistency of the set of equations:

$x - 2y + z = -5\,;\; x + 5y - 7z = 2\,;\; 3x + y - 5z = 1\,.$ [6 *Marks*]

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [6 *Marks*]

Q.2

(a) If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$. [6 *Marks*]

(b) Using Taylor's theorem,

Prove that $\log \sin x = \log \sin a + (x-a)\cot a - \frac{1}{2}(x-a)^2 \operatorname{cosec}^2 x + \cdots$ [6 *Marks*]

Q.3 Solve any TWO:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. [6 *Marks*]

(b) If z is a homogeneous function of degree n in x *and* y , prove that

$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$ [6 *Marks*]

(c) If $z = f(x,y)$ where $x = e^u + e^{-v}$ & $y = e^{-u} - e^v$,

then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. [6 *Marks*]

Q.4

(a) If $u = \frac{yz}{x},\; v = \frac{zx}{y},\; w = \frac{xy}{z},$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. [4 *Marks*]

(b) Find the percentage error in the measurement of the area of an ellipse when an error of **1.5 %** is made in measuring its major and minor axes. [4 $Marks$]

(c) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [4 $Marks$]

Q.5 Solve any TWO:

(a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\, dy\, dx$. [6 $Marks$]

(b) Change the order of integration and evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y}\, dx\, dy.$ [6 $Marks$]

(c) Evaluate the integral $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dx\, dy$. [6 $Marks$]

Q.6

(a) State D' Alembert's ratio test, and hence check the convergence of the series:
$\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$. [6 $Marks$]

(b) State Cauchy's root test, and hence check the convergence of the series:
$\sum \frac{[(2n+1)\, x]^n}{n^{n+1}}$ $(x > 0)$. [6 $Marks$]

****************************Paper End*****************************

SLR-TC – 1

Seat No.	

Set **P**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
2) Solve Q. No. ***1*** *in* ***first 30 minutes****.* ***Each*** *question carries* ***one*** *mark.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*
4) ***Use*** *of calculator is* ***allowed****.*
5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n\cdot n!}{(x-2)^{n+1}}$

2) If $y = xe^{3x}$ then $y_n =$

a) $3n \,!\, xe^{3x}$ b) $3^n x\, e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$ d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

3) Expansion of sinhx in powers of x is

a) $x-\frac{x^3}{3!}+\frac{x^5}{5!}$ ...... b) $1+x+\frac{x^3}{3!}+$...... c) $1+\frac{x^2}{2}+\frac{x^4}{4}+$...... d) None of these

4) Taylor's series expansion of $y = \frac{1}{x}$ about x = 1 is

a) $1+(x-1)+\frac{(x-1)^2}{2!}+$ ...... b) $1-(x-1)+(x-1)^2-$......

c) $1-(x-1)+\frac{(x-1)^2}{2!}-$...... d) None of these

P.T.O.

5) Which of the following is true ?

a) cotix = icothx
b) sechix = isecx
c) tanix = –itanhx
d) sinhx = –isinix

6) Cosh(x + iy) =

a) coshx cosy + isinhx siny
b) coshx cosy – isinhx siny
c) coshx coshy + isinhx sinhy
d) sinhx siny + icoshy cosx

7) The modulus and amplitude of $z - 2\sqrt{3}i$ are

a) $4\sqrt{3}, \frac{-\pi}{3}$
b) $4, \frac{-\pi}{3}$
c) $4, \frac{-\pi}{6}$
d) $4, \frac{-2\pi}{3}$

8) If the determinant of square matrix A of order m is equal to zero, then the rank of A is

a) Less than m
b) Greater than m
c) Equal to m
d) None of these

9) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is

a) n
b) r
c) n – r
d) n + r

10) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to

a) 9
b) 24
c) $\frac{1}{24}$
d) $\frac{1}{9}$

11) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$

a) $\frac{1}{\sqrt{y^2 - x^2}}$
b) $\frac{x}{\sqrt{y^2 \quad x^2}}$
c) $\frac{y}{\sqrt{y^2 \quad x^2}}$
d) $\frac{1}{\sqrt{x^2 \quad y^2}}$

12) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) 4u
b) 20u
c) $\frac{1}{20}u$
d) 5u

13) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1
b) –1
c) u
d) –u

14) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error
b) Percentage error
c) Relative error
d) None of these

Set P

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

***Instructions** : 1) **All** questions are **compulsory**.*
*2) Figures to the **right** indicate **full** marks.*
*3) **Use** of calculator is **allowed**.*

SECTION – I

2. Solve **any three** : **9**

a) Find n^{th} derivative of $\dfrac{x^2}{(x\quad 1)(2x\quad 3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)\quad i\cos\left(\frac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3}\quad 0$.

c) Prove that $\text{sech}^{-1}(\sin\theta)\quad \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find n^{th} derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

Set P

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x \cdot \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \ldots\ldots\ldots\ldots\ldots$$

c) If $i^{i^{i\ldots\infty}} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

ii) Many solution

iii) No solution.

c) If $u = \frac{x^2 + y^2}{x + y}$, Show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} \quad 2 \quad x^2 \quad y^2 \;{}^{1/2} \frac{\partial z}{\partial x}$.

e) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

6. Solve **any three** of the following : **9**

a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ 6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}.$$

b) Find the eigen value of the matrix A and also find eigen values of A^2

$$A \quad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

c) If u = f(r, s), where $r \quad \frac{x \quad y}{xy}$, $s \quad \frac{z \quad x}{zx}$, prove that $x^2 \frac{u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0.$

d) If $u \quad \frac{xy}{z}$, $v \quad \frac{yz}{x}$, $w = \frac{zx}{y}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

$$A \quad \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial z} \quad \sin 2u.$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} \quad \sin 4u \quad \sin 2u.$

c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Set P

SLR-TC – 1

Seat No.	

Set **Q**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
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5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) If the determinant of square matrix A of order m is equal to zero, then the rank of A is
   a) Less than m b) Greater than m c) Equal to m d) None of these

2) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is
   a) n b) r c) n – r d) n + r

3) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to
   a) 9 b) 24 c) $\frac{1}{24}$ d) $\frac{1}{9}$

4) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$
   a) $\frac{1}{\sqrt{y^2 - x^2}}$ b) $\frac{x}{\sqrt{y^2 \; x^2}}$ c) $\frac{y}{\sqrt{y^2 \; x^2}}$ d) $\frac{1}{\sqrt{x^2 \; y^2}}$

5) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$
   a) 4u b) 20u c) $\frac{1}{20}u$ d) 5u

P.T.O.

6) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) −1 c) u d) −u

7) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error
b) Percentage error
c) Relative error
d) None of these

8) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n \cdot n!}{(x-2)^{n+1}}$

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10) Expansion of sinhx in powers of x is

a) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ...... b) $1 + x + \frac{x^3}{3!} + ...$ c) $1 + \frac{x^2}{2} + \frac{x^4}{4} + ...$ d) None of these

11) Taylor's series expansion of y = $\frac{1}{x}$ about x = 1 is

a) $1 + (x-1) + \frac{(x-1)^2}{2!} + ...$
b) $1 - (x-1) + (x-1)^2 - ......$
c) $1 - (x-1) + \frac{(x-1)^2}{2!} - ......$
d) None of these

12) Which of the following is true ?
a) cotix = icothx
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b) coshx cosy − isinhx siny
c) coshx coshy + isinhx sinhy
d) sinhx siny + icoshy cosx

14) The modulus and amplitude of $z - 2\sqrt{3}i$ are

a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

____

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

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SECTION – I

2. Solve **any three** : **9**

a) Find n[th] derivative of $\dfrac{x^2}{(x\quad 1)(2x\quad 3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)\quad i\cos\left(\frac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

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b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3}\quad 0$.

c) Prove that $\operatorname{sech}^{-1}(\sin\theta)\quad \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find n[th] derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

Set Q

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, prove that $y_{n+2}(0)=-n^2y_n(0)$.

b) By using standard expansion prove that

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c) If $i^{i^{\cdots\infty}}=\alpha+i\beta$, prove that $\alpha^2+\beta^2=e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix}1 & 1 & -1 & 1\\ 1 & 3 & 2 & 1\\ 2 & 0 & 3 & 2\\ 3 & 3 & 3 & 3\end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

ii) Many solution

iii) No solution.

c) If $u=\frac{x^2+y^2}{x+y}$, Show that $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u}-v\frac{\partial z}{\partial v}\quad 2\quad x^2\quad y^2\ {}^{1/2}\frac{\partial z}{\partial x}$.

e) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

6. Solve **any three** of the following : **9**

a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ 6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}.$$

b) Find the eigen value of the matrix A and also find eigen values of A^2

$$A \quad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

c) If u = f(r, s), where r $\frac{x \quad y}{xy}$, s $\frac{z \quad x}{zx}$, prove that $x^2 \frac{u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.

d) If u $\frac{xy}{z}$, v $\frac{yz}{x}$, $w = \frac{zx}{y}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

$$A \quad \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial z} \quad \sin 2u.$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \quad \sin 4u \quad \sin 2u.$

c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Set Q

Seat No.	

Set **R**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
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5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) Which of the following is true ?
a) cotix = icothx b) sechix = isecx
c) tanix = –itanhx d) sinhx = –isinix

2) Cosh(x + iy) =
a) coshx cosy + isinhx siny b) coshx cosy – isinhx siny
c) coshx coshy + isinhx sinhy d) sinhx siny + icoshy cosx

3) The modulus and amplitude of $z - 2\sqrt{3}i$ are
a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

4) If the determinant of square matrix A of order m is equal to zero, then the rank of A is
a) Less than m b) Greater than m c) Equal to m d) None of these

5) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is
a) n b) r c) n – r d) n + r

6) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to
a) 9 b) 24 c) $\frac{1}{24}$ d) $\frac{1}{9}$

P.T.O.

7) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$

a) $\frac{1}{\sqrt{y^2 - x^2}}$ b) $\frac{x}{\sqrt{y^2 \quad x^2}}$ c) $\frac{y}{\sqrt{y^2 \quad x^2}}$ d) $\frac{1}{\sqrt{x^2 \quad y^2}}$

8) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) 4u b) 20u c) $\frac{1}{20}u$ d) 5u

9) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) –1 c) u d) –u

10) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error b) Percentage error
c) Relative error d) None of these

11) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n (n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n . n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n (n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n \cdot n!}{(x-2)^{n+1}}$

12) If $y = xe^{3x}$ then $y_n =$
a) $3n ! \, xe^{3x}$ b) $3^n x \, e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$ d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

13) Expansion of sinhx in powers of x is
a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots$ b) $1 + x + \frac{x^3}{3!} + \ldots\ldots$ c) $1 + \frac{x^2}{2} + \frac{x^4}{4} + \ldots\ldots$ d) None of these

14) Taylor's series expansion of $y = \frac{1}{x}$ about x = 1 is

a) $1 + (x-1) + \frac{(x-1)^2}{2!} + \ldots\ldots$ b) $1 - (x-1) + (x-1)^2 - \ldots\ldots$

c) $1 - (x-1) + \frac{(x-1)^2}{2!} - \ldots\ldots$ d) None of these

SLR-TC – 1

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1) **All** questions are **compulsory**.*
*2) Figures to the **right** indicate **full** marks.*
*3) **Use** of calculator is **allowed**.*

SECTION – I

2. Solve **any three** : **9**

a) Find nth derivative of $\dfrac{x^2}{(x\quad 1)(2x\quad 3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)\quad i\cos\left(\frac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3}\quad 0$.

c) Prove that $\operatorname{sech}^{-1}(\sin\theta)\quad \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find nth derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

Set R

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x\cdot\sin x}=1+x^2+\frac{x^4}{3}+\frac{x^6}{120}+\ \ldots\ldots\ldots\ldots\ldots$$

c) If $i^{i^{\ldots\infty}}=\alpha+i\beta$, prove that $\alpha^2+\beta^2=e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

ii) Many solution

iii) No solution.

c) If $u=\frac{x^2+y^2}{x+y}$, Show that $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u}-v\frac{\partial z}{\partial v}\quad 2\quad x^2\quad y^2\ ^{1/2}\frac{\partial z}{\partial x}$.

e) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

6. Solve **any three** of the following : **9**

   a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ 6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}.$$

   b) Find the eigen value of the matrix A and also find eigen values of A^2

$$A \quad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

   c) If u = f(r, s), where $r \quad \frac{x \quad y}{xy}$, $s \quad \frac{z \quad x}{zx}$, prove that $x^2 \frac{u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.

   d) If $u \quad \frac{xy}{z}$, $v \quad \frac{yz}{x}$, $w = \frac{zx}{y}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

   e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

   a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

$$A \quad \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

   b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

      i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial z} \quad \sin 2u.$

      ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} \quad \sin 4u \quad \sin 2u.$

   c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

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Set R

SLR-TC – 1

Seat No.	

Set **S**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70

Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*

2) Solve Q. No. ***1*** *in* ***first 30 minutes****.* ***Each*** *question carries* ***one*** *mark.*

3) Figures to the ***right*** *indicate* ***full*** *marks.*

4) ***Use*** *of calculator is* ***allowed****.*

5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to

a) 9 b) 24 c) $\frac{1}{24}$ d) $\frac{1}{9}$

2) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$

a) $\frac{1}{\sqrt{y^2 - x^2}}$ b) $\frac{x}{\sqrt{y^2 \quad x^2}}$ c) $\frac{y}{\sqrt{y^2 \quad x^2}}$ d) $\frac{1}{\sqrt{x^2 \quad y^2}}$

3) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) 4u b) 20u c) $\frac{1}{20}u$ d) 5u

4) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) –1 c) u d) –u

P.T.O.

5) If δx is an error in x, then $\dfrac{\delta x}{x}$ is called

a) Absolute error b) Percentage error
c) Relative error d) None of these

6) The n^{th} derivative of $\dfrac{1}{(x+2)^2}$ is

a) $\dfrac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\dfrac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\dfrac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\dfrac{(-1)^n \cdot n!}{(x-2)^{n+1}}$

7) If $y = xe^{3x}$ then y_n =
a) $3n!\, xe^{3x}$ b) $3^n x\, e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$ d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

8) Expansion of sinhx in powers of x is
a) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$ ...... b) $1 + x + \dfrac{x^3}{3!} + ...$ c) $1 + \dfrac{x^2}{2} + \dfrac{x^4}{4} + ...$ d) None of these

9) Taylor's series expansion of $y = \dfrac{1}{x}$ about x = 1 is

a) $1+(x-1) + \dfrac{(x-1)^2}{2!} + ...$ b) $1 - (x-1) + (x-1)^2 - ......$

c) $1 - (x-1) + \dfrac{(x-1)^2}{2!} - ......$ d) None of these

10) Which of the following is true ?
a) cotix = icothx b) sechix = isecx c) tanix = –itanhx d) sinhx = –isinix

11) Cosh(x + iy) =
a) coshx cosy + isinhx siny b) coshx cosy – isinhx siny
c) coshx coshy + isinhx sinhy d) sinhx siny + icoshy cosx

12) The modulus and amplitude of $z - 2\sqrt{3}i$ are

a) $4\sqrt{3}, \dfrac{-\pi}{3}$ b) $4, \dfrac{-\pi}{3}$ c) $4, \dfrac{-\pi}{6}$ d) $4, \dfrac{-2\pi}{3}$

13) If the determinant of square matrix A of order m is equal to zero, then the rank of A is
a) Less than m b) Greater than m c) Equal to m d) None of these

14) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is
a) n b) r c) n – r d) n + r

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Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
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SECTION – I

2. Solve **any three** : **9**

a) Find n[th] derivative of $\frac{x^2}{(x\quad 1)(2x\quad 3)}$.

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c) Simplify $\left[\frac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)\quad i\cos\left(\frac{\pi}{8}\right)}\right]^8$.

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3. Solve **any three** : **9**

a) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim_{x\to 0}\frac{\sin x + ax + bx^3}{x^3}\quad 0$.

c) Prove that $\operatorname{sech}^{-1}(\sin\theta)\quad \log\cot\left(\frac{\theta}{2}\right)$.

d) Find n[th] derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\frac{3i}{4}\right)$.

Set S

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x \cdot \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \ldots\ldots\ldots\ldots\ldots$$

c) If $i^{i^{\cdots\infty}} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

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c) If $u = \frac{x^2+y^2}{x+y}$, Show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} \quad 2 \quad x^2 \quad y^2 \ {}^{1/2}\frac{\partial z}{\partial x}$.

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b) Find the eigen value of the matrix A and also find eigen values of A^2

$$A \quad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

c) If u = f(r, s), where r $\frac{x \quad y}{xy}$, s $\frac{z \quad x}{zx}$, prove that $x^2\frac{u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0.$

d) If u $\frac{xy}{z}$, v $\frac{yz}{x}$, w = $\frac{zx}{y}$, find $\frac{\partial(u,\, v,\, w)}{\partial(x,\, y,\, z)}$.

e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

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a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

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b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that

i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial z} \quad \sin 2u.$

ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2} \quad \sin 4u \quad \sin 2u.$

c) Find the extreme values of the function f(x, y) = $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

____

Set S

SLR-TC – 1

Seat No.	

Set **P**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70
Time : 10.00 a.m. to 1.00 p.m.

Instructions : 1) ***All*** *questions are* ***compulsory****.*
2) *Solve Q. No.* ***1*** *in* ***first 30 minutes****.* ***Each*** *question carries* ***one*** *mark.*
3) *Figures to the* ***right*** *indicate* ***full*** *marks.*
4) ***Use*** *of calculator is* ***allowed****.*
5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n\cdot n!}{(x-2)^{n+1}}$

2) If $y = xe^{3x}$ then y_n =

a) $3n\,!\,xe^{3x}$
b) $3^n\,x\,e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$
d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

3) Expansion of sinhx in powers of x is

a) $x-\frac{x^3}{3!}+\frac{x^5}{5!}$ ...... b) $1+x+\frac{x^3}{3!}+$...... c) $1+\frac{x^2}{2}+\frac{x^4}{4}+$...... d) None of these

4) Taylor's series expansion of $y = \frac{1}{x}$ about x = 1 is

a) $1+(x-1)+\frac{(x-1)^2}{2!}+$ ......
b) $1-(x-1)+(x-1)^2-$......
c) $1-(x-1)+\frac{(x-1)^2}{2!}-$......
d) None of these

P.T.O.

5) Which of the following is true ?

a) cotix = icothx b) sechix = isecx

c) tanix = –itanhx d) sinhx = –isinix

6) Cosh(x + iy) =

a) coshx cosy + isinhx siny b) coshx cosy – isinhx siny

c) coshx coshy + isinhx sinhy d) sinhx siny + icoshy cosx

7) The modulus and amplitude of $z - 2\sqrt{3}i$ are

a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

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12) If u = $\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

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13) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) –1 c) u d) –u

14) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error b) Percentage error

c) Relative error d) None of these

Set P

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
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SECTION – I

2. Solve **any three** : **9**

a) Find n^{th} derivative of $\dfrac{x^2}{(x-1)(2x+3)}$.

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c) Simplify $\left[\dfrac{1+\sin\left(\dfrac{\pi}{8}\right)+i\cos\left(\dfrac{\pi}{8}\right)}{1+\sin\left(\dfrac{\pi}{8}\right)-i\cos\left(\dfrac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

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Set P

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SECTION – II

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d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = 2\left(x^2 - y^2\right)^{1/2}\frac{\partial z}{\partial x}$.

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c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

____

Set P

SLR-TC – 1

Seat No.	

Set **Q**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70

Time : 10.00 a.m. to 1.00 p.m.

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5) Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

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1) If the determinant of square matrix A of order m is equal to zero, then the rank of A is

a) Less than m b) Greater than m c) Equal to m d) None of these

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a) $3n ! xe^{3x}$
b) $3^n x e^{3x}$
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d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

10) Expansion of sinhx in powers of x is

a) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ...... b) $1 + x + \frac{x^3}{3!} + ...$ c) $1 + \frac{x^2}{2} + \frac{x^4}{4} + ...$ d) None of these

11) Taylor's series expansion of $y = \frac{1}{x}$ about x = 1 is

a) $1+(x-1) + \frac{(x-1)^2}{2!} +$ ......
b) $1 - (x - 1) + (x - 1)^2 -$ ......
c) $1 - (x - 1) + \frac{(x-1)^2}{2!} -$ ......
d) None of these

12) Which of the following is true ?

a) cotix = icothx
b) sechix = isecx
c) tanix = –itanhx
d) sinhx = –isinix

13) Cosh(x + iy) =

a) coshx cosy + isinhx siny
b) coshx cosy – isinhx siny
c) coshx coshy + isinhx sinhy
d) sinhx siny + icoshy cosx

14) The modulus and amplitude of $z - 2\sqrt{3}i$ are

a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*
3) ***Use*** *of calculator is* ***allowed****.*

SECTION – I

2. Solve **any three** : **9**

a) Find n^{th} derivative of $\dfrac{x^2}{(x-1)(2x+3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\dfrac{\pi}{8}\right)+i\cos\left(\dfrac{\pi}{8}\right)}{1+\sin\left(\dfrac{\pi}{8}\right)-i\cos\left(\dfrac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3} = 0$.

c) Prove that $\operatorname{sech}^{-1}(\sin\theta) = \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find n^{th} derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x \cdot \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \text{..............}$$

c) If $i^{i^{i \ldots \infty}} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

ii) Many solution

iii) No solution.

c) If $u = \frac{x^2 + y^2}{x + y}$, Show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = 2\left(x^2 - y^2\right)^{1/2}\frac{\partial z}{\partial x}$.

e) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

6. Solve **any three** of the following : **9**

a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

b) Find the eigen value of the matrix A and also find eigen values of A^2

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

c) If u = f(r, s), where $r = \frac{x-y}{xy}$, $s = \frac{z-x}{zx}$, prove that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.

d) If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$, $w = \frac{zx}{y}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial z} = \sin 2u.$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u.$

c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Set Q

Seat No.	

Set **R**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70

Time : 10.00 a.m. to 1.00 p.m.

Instructions : *1)* ***All*** *questions are* ***compulsory****.*

2) Solve Q. No. ***1*** *in* ***first 30 minutes****.* ***Each*** *question carries* ***one*** *mark.*

3) Figures to the ***right*** *indicate* ***full*** *marks.*

4) ***Use*** *of calculator is* ***allowed****.*

5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) Which of the following is true ?
   a) cotix = icothx b) sechix = isecx
   c) tanix = –itanhx d) sinhx = –isinix

2) Cosh(x + iy) =
   a) coshx cosy + isinhx siny b) coshx cosy – isinhx siny
   c) coshx coshy + isinhx sinhy d) sinhx siny + icoshy cosx

3) The modulus and amplitude of $z-2\sqrt{3}i$ are
   a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

4) If the determinant of square matrix A of order m is equal to zero, then the rank of A is
   a) Less than m b) Greater than m c) Equal to m d) None of these

5) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is
   a) n b) r c) n – r d) n + r

6) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to
   a) 9 b) 24 c) $\frac{1}{24}$ d) $\frac{1}{9}$

P.T.O.

7) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$

a) $\frac{1}{\sqrt{y^2 - x^2}}$ b) $\frac{x}{\sqrt{y^2 - x^2}}$ c) $\frac{y}{\sqrt{y^2 - x^2}}$ d) $\frac{1}{\sqrt{x^2 - y^2}}$

8) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) 4u b) 20u c) $\frac{1}{20}u$ d) 5u

9) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) –1 c) u d) –u

10) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error b) Percentage error
c) Relative error d) None of these

11) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n \cdot n!}{(x-2)^{n+1}}$

12) If $y = xe^{3x}$ then $y_n =$
a) $3n\,!\,xe^{3x}$ b) $3^n x e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$ d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

13) Expansion of sinhx in powers of x is
a) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ...... b) $1 + x + \frac{x^3}{3!} + ...$ c) $1 + \frac{x^2}{2} + \frac{x^4}{4} + ...$ d) None of these

14) Taylor's series expansion of $y = \frac{1}{x}$ about x = 1 is

a) $1+(x-1) + \frac{(x-1)^2}{2!} + ...$ b) $1 - (x - 1) + (x - 1)^2 - ......$

c) $1 - (x - 1) + \frac{(x-1)^2}{2!} - ......$ d) None of these

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : 1) ***All*** *questions are* ***compulsory****.*
2) *Figures to the* ***right*** *indicate* ***full*** *marks.*
3) ***Use*** *of calculator is* ***allowed****.*

SECTION – I

2. Solve **any three** : **9**

a) Find n^{th} derivative of $\dfrac{x^2}{(x-1)(2x+3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\dfrac{\pi}{8}\right)+i\cos\left(\dfrac{\pi}{8}\right)}{1+\sin\left(\dfrac{\pi}{8}\right)-i\cos\left(\dfrac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3} = 0$.

c) Prove that $\text{sech}^{-1}(\sin\theta) = \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find n^{th} derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x \cdot \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \ldots\ldots\ldots\ldots\ldots$$

c) If $i^{i^{i \ldots \infty}} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

ii) Many solution

iii) No solution.

c) If $u = \dfrac{x^2 + y^2}{x + y}$, Show that $\left(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\dfrac{\partial z}{\partial u} - v\dfrac{\partial z}{\partial v} = 2\left(x^2 - y^2\right)^{\frac{1}{2}}\dfrac{\partial z}{\partial x}$.

e) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

6. Solve **any three** of the following : **9**

   a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

   $$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

   b) Find the eigen value of the matrix A and also find eigen values of A^2

   $$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

   c) If u = f(r, s), where $r = \frac{x-y}{xy}$, $s = \frac{z-x}{zx}$, prove that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0.$

   d) If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$, $w = \frac{zx}{y}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

   e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

   a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

   $$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

   b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

      i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial z} = \sin 2u.$

      ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u.$

   c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

SLR-TC – 1

Seat No. | |

Set **S**

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Max. Marks : 70
Time : 10.00 a.m. to 1.00 p.m.

Instructions : 1) ***All*** *questions are* ***compulsory****.*
2) *Solve Q. No.* ***1*** *in* ***first 30 minutes****.* ***Each*** *question carries* ***one*** *mark.*
3) *Figures to the* ***right*** *indicate* ***full*** *marks.*
4) ***Use*** *of calculator is* ***allowed****.*
5) ***Answer MCQ/Objective type questions on Page No. 3 only. Don't forget to mention, Q.P. Set (P/Q/R/S) on Top of Page.***

MCQ/Objective Type Questions

Duration : 30 Minutes Marks : 14

1. Choose the correct alternative : **(14×1=14)**

1) If 2, 3, 4 are the eigen values of matrix A, then |A| is equal to

a) 9 b) 24 c) $\frac{1}{24}$ d) $\frac{1}{9}$

2) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial z}{\partial x} =$

a) $\frac{1}{\sqrt{y^2 - x^2}}$ b) $\frac{x}{\sqrt{y^2 - x^2}}$ c) $\frac{y}{\sqrt{y^2 - x^2}}$ d) $\frac{1}{\sqrt{x^2 - y^2}}$

3) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) 4u b) 20u c) $\frac{1}{20}u$ d) 5u

4) If x = ucosv, y = usinv, then $\frac{\partial(x, y)}{\partial(u, v)} =$

a) 1 b) –1 c) u d) –u

5) If δx is an error in x, then $\frac{\delta x}{x}$ is called

a) Absolute error b) Percentage error
c) Relative error d) None of these

6) The n^{th} derivative of $\frac{1}{(x+2)^2}$ is

a) $\frac{(-1)^n(n+1)!}{(x+2)^{n+2}}$ b) $\frac{(-1)^n.n!}{(x+2)^{n+2}}$ c) $\frac{(-1)^n(n+1)!}{(x-2)^{n+1}}$ d) $\frac{(-1)^n \cdot n!}{(x-2)^{n+1}}$

7) If $y = xe^{3x}$ then y_n =

a) $3n!\, xe^{3x}$ b) $3^n x\, e^{3x}$
c) $3^n e^{3x} x + n3^{n-1} e^{3x}$ d) $3^n e^{3x} x^2 + n3^{n-2} e^{3x}$

8) Expansion of sinhx in powers of x is

a) $x-\frac{x^3}{3!}+\frac{x^5}{5!}$ ...... b) $1+x+\frac{x^3}{3!}+$...... c) $1+\frac{x^2}{2}+\frac{x^4}{4}+$...... d) None of these

9) Taylor's series expansion of y = $\frac{1}{x}$ about x = 1 is

a) $1+(x-1)+\frac{(x-1)^2}{2!}+$ ...... b) $1-(x-1)+(x-1)^2-$......
c) $1-(x-1)+\frac{(x-1)^2}{2!}-$...... d) None of these

10) Which of the following is true ?

a) cotix = icothx b) sechix = isecx c) tanix = –itanhx d) sinhx = –isinix

11) Cosh(x + iy) =

a) coshx cosy + isinhx siny b) coshx cosy – isinhx siny
c) coshx coshy + isinhx sinhy d) sinhx siny + icoshy cosx

12) The modulus and amplitude of $z-2\sqrt{3}i$ are

a) $4\sqrt{3}, \frac{-\pi}{3}$ b) $4, \frac{-\pi}{3}$ c) $4, \frac{-\pi}{6}$ d) $4, \frac{-2\pi}{3}$

13) If the determinant of square matrix A of order m is equal to zero, then the rank of A is

a) Less than m b) Greater than m c) Equal to m d) None of these

14) If the rank of A is r and number of variables is n then the number of linearly independent solutions of the system AX = 0 is

a) n b) r c) n – r d) n + r

SLR-TC – 1

Seat No.	

F.E. (Part – I) (CBCS) Examination, 2018
ENGINEERING MATHEMATICS – I

Day and Date : Thursday, 3-5-2018 Marks : 56
Time : 10.00 a.m. to 1.00 p.m.

Instructions : 1) ***All*** *questions are* ***compulsory****.*
2) *Figures to the* ***right*** *indicate* ***full*** *marks.*
3) ***Use*** *of calculator is* ***allowed****.*

SECTION – I

2. Solve **any three** : **9**

a) Find n^{th} derivative of $\dfrac{x^2}{(x-1)(2x+3)}$.

b) Find all the values of $(-i)^{1/3}$.

c) Simplify $\left[\dfrac{1+\sin\left(\dfrac{\pi}{8}\right)+i\cos\left(\dfrac{\pi}{8}\right)}{1+\sin\left(\dfrac{\pi}{8}\right)-i\cos\left(\dfrac{\pi}{8}\right)}\right]^8$.

d) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x + 2)$.

e) By using Maclaurins series expand $e^x \cdot \sin x$.

3. Solve **any three** : **9**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$.

b) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{\sin x + ax + bx^3}{x^3} = 0$.

c) Prove that $\operatorname{sech}^{-1}(\sin\theta) = \log\cot\left(\dfrac{\theta}{2}\right)$.

d) Find n^{th} derivative of sinx sin2x sin3x.

e) Separate into real and imaginary parts of $\sin^{-1}\left(\dfrac{3i}{4}\right)$.

Set S

4. Solve **any two** : **10**

a) State Leibnitz theorem.

If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$, prove that $y_{n+2}(0) = -n^2 y_n(0)$.

b) By using standard expansion prove that

$$e^{x \cdot \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \ldots\ldots\ldots\ldots\ldots$$

c) If $i^{i^{i\ldots\infty}} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

SECTION – II

5. Solve **any three** of the following : **9**

a) Find the rank of the following matrix by reducing it into normal form.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

b) Find the value of λ and μ for which the system of equations : $x + 2y + 3z = 5$; $x + 3y - z = 4$; $x + 4y + \lambda z = \mu$ has a

i) Unique solution

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c) If $u = \frac{x^2 + y^2}{x + y}$, Show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

d) If $z = f(x, y)$ and $x = u^2 + v^2$, $y = 2uv$. Show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = 2\left(x^2 - y^2\right)^{1/2}\frac{\partial z}{\partial x}$.

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   a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix

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   b) Find the eigen value of the matrix A and also find eigen values of A^2

   $$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

   c) If u = f(r, s), where $r = \dfrac{x-y}{xy}$, $s = \dfrac{z-x}{zx}$, prove that $x^2 \dfrac{\partial u}{\partial x} + y^2 \dfrac{\partial u}{\partial y} + z^2 \dfrac{\partial u}{\partial z} = 0.$

   d) If $u = \dfrac{xy}{z}$, $v = \dfrac{yz}{x}$, $w = \dfrac{zx}{y}$, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

   e) Find the percentage error in the area of the ellipse. When an error of +1% is made by measuring major and minor axis.

7. Solve **any two** of the following : **10**

   a) Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}

   $$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

   b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$, prove that

      i) $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial z} = \sin 2u.$

      ii) $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u.$

   c) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Set S