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206
Discrete Structures II

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Computational Brain Lab

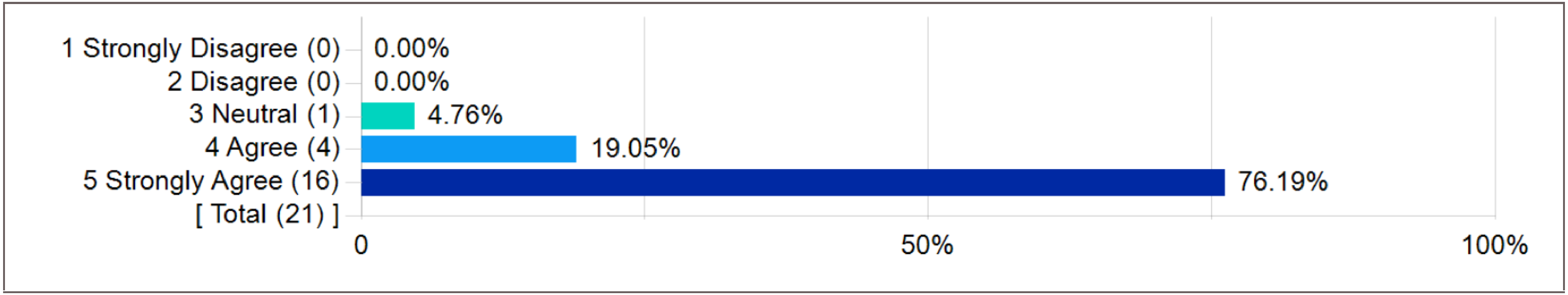
Computer Science | Rutgers University | NJ, USA

Midterm Survey Results

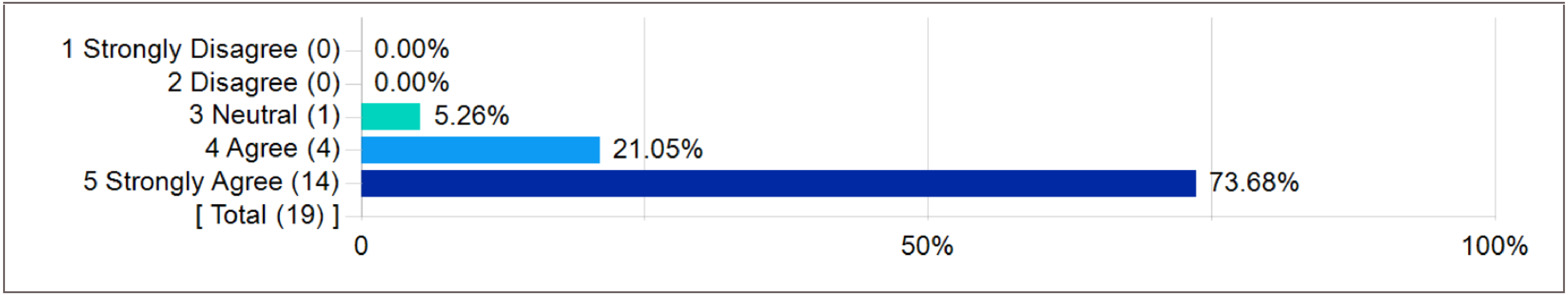
✓Slides + explanations
✓Focus on method not solution
✓Large number of problems
✓Recaps(!)
✓Small Break (!!)
✓Quiz aligns with lectures

֍None
֍Online vs in-person
֍Time is limited in Quizzes
֍More practice problems
֍Post lecture notes before lecture
֍TA issues

The instructor Konstantinos Michmizos was prepared for class and presents material in an organized manner.



The instructor Konstantinos Michmizos responds effectively to student comments and questions.



Two “negative” answers - 1

“To me, it feels like the professor explains the topics in a way where he already knows the answer and then just explains it to us rather than figuring it out together with the rest of the class. I think it would be beneficial to walk us through the problem by hand, that is, if possible, write everything down on a whiteboard that the whole class can see (like share the screen on Microsoft Whiteboard), rather than giving us the whole answer at once on the slides. I also personally do not like when the professor gets frustrated when answering a question that has already been asked. YES! I know it's annoying but, a lot of times, it helps not only the one person asking the question, but many other students in the class. And when it seems like a burden to ask questions, it discourages students from participating altogether (because who would PURPOSELY ask a question that has already been asked?)”

Two “negative” answers - 2

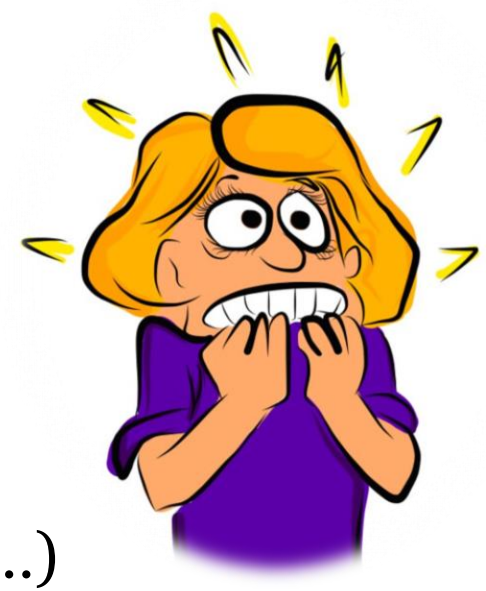
“I'm a little confused as to why this course exists in the first place. Don't get me wrong, I'm not confused as to why computer science students are learning discrete structures. I recognize the importance of the material. This is probably more of a problem with the fact that I didn't take CS205 at Rutgers and instead transferred from my local community college. It's about half way through the semester and I can't say that I've learned much new material that I didn't learn from my community college's Discrete I, and the syllabus doesn't seem to indicate much of a difference either. So again, maybe this is entirely my fault in that I didn't take CS205 here at Rutgers, and it certainly has nothing to do with the professor, they've been great. But for me, the content of this course doesn't justify it being a separate course. Maybe that will change as the semester progresses, but I'm not counting on it.”

The answer

"It's not important what we cover in the class, it's important what you discover

To be truly educated from this point of view means to be in a position to inquire and to create on the basis of the resources available to you which you've come to appreciate and comprehend. To know where to look, to know how to formulate serious questions, to question a standard doctrine if that's appropriate, to find your own way, to shape the questions that are worth pursuing, and to develop the path to pursue them...."

Quiz 2 – When and What?



- When
  - Sections 5 and 6: Thursday 10/14, during recitation (today…)
  - Section 4: Tuesday 10/19, during recitation
- What will cover
  - Sum rule (Week 4 Lectures)
  - Product rule (Week 4-5 Lectures)
  - Permutations with and without repetitions (Week 6 & Tuesday's Lectures)

So Far

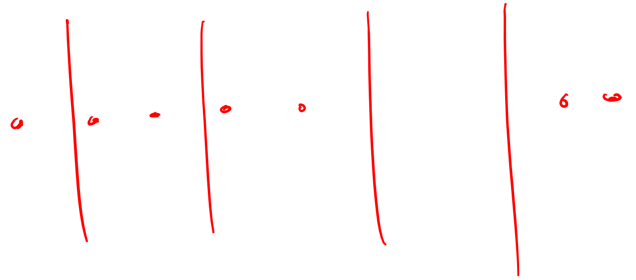
- ~~Proofs/Induction~~
- ~~Sum Rule~~
- ~~Partition Method~~
- ~~Difference Method~~
- ~~Bijection Method~~
- ~~Product Rule~~
- ~~Generalized product rule~~
- ~~Permutation/Combinations~~
- Inclusion-Exclusion / Pigeonhole Principle
- Combinatorial Proofs and Binomial Coefficients

Combinations with Repetitions

- 5 distinct pirates want to divide up 20 identical, indivisible bars of gold. How many ways to divide the loot?

Count all sequences of (a,b,c,d,e) such that $a+b+c+d+e=20$

$a =$ # pirate 1 gets
$b =$ # 2 gets
$c =$ # 3 gets
$d =$ 4
$e =$ 5 gets



answer = all ways to arrange 20 dots and 4 lines

$$= \frac{(24)!}{(20!)(4!)} = \binom{24}{4}$$

Combinations with Repetitions

- How many integer solutions to the following equation?
  - $x_1 + x_2 + \cdots + x_5 = 20$
  - $x_1, x_2, \ldots, x_5 \geq 0$

$(x_1, x_2, x_3, x_4, x_5)$ such that $\sum x_i = 20$

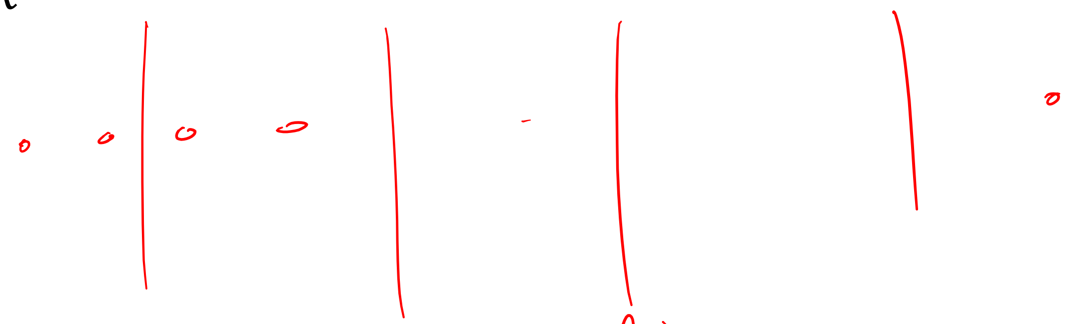
$\Rightarrow$ all arrangements of 20 dots and 4 lines

$= \dfrac{(24)!}{(20!)(4!)}$

So far...

Combinations with Repetitions

- How many integer solutions to the following equation?
  - $x_1 + x_2 + \cdots + x_k = n$
  - $x_1, x_2, \ldots, x_k \geq 0$

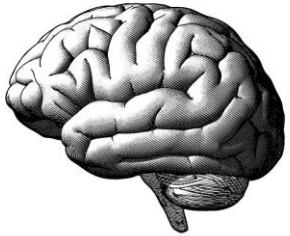


K-1 lines

→ all arrangements of n dots and K-1 lines

$$\to \frac{(n+k-1)!}{n!\,(k-1)!} = \binom{n+k-1}{k-1}$$

New Stuff

Get your  in gear

- How many ways to seat 6 boys and 8 girls in a row such that no two boys are seated next to each other.

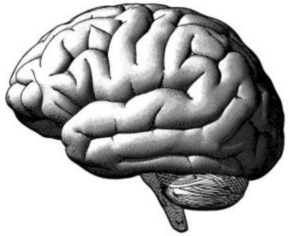
— First seat 8 girls

— There are 9 empty places next to them

— Pick 6 out of 9 and seat the boys

answer = $(8!)\binom{9}{6}(6!)$

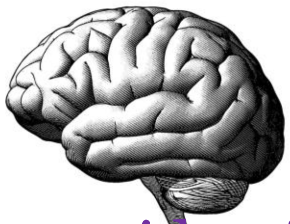
New Stuff

Get your  in gear

- How many bit strings of length 8 either start with a 1 or end with a 00?

Solution: Use the **partition method**.
Let A1 = number of bit strings that start with 1 and end with 00.
Let A2 = number of strings that start with 1 and do not end with 00.
Let A3 = number of strings that start with 0 and end in 00.
We have $|A1| = 2^5$, $|A2| = 2^5 \cdot 3$, $|A3| = 2^5$.
Hence the total number of strings = $2^5 + 2^5 \cdot 3 + 2^5$.

Explanation for |A2|. In A2 we are counting all string that start with 1 and do not end with 00. Again using the **partition method**, we can divide the outcomes into 3 possible subsets: start with 1 and end with 01, start with 1 and end with 10, start with 1 and end with 11. In each cases, there are 2^5 choices for the remaining 5 elements.

Get your  in gear

- 5 pirates want to divide 20 identical bars of gold among them. How many ways to divide if each pirate wants at least 2 bars and no pirate can get more than 8 bars.

First give 2 bars to each pirate. We are left with 10 bars. No pirate can get more than 6 of them.

Difference method. Count all possible ways to divide 10 bars among 5 pirates and subtract the number of ways in which some pirate gets more than 6 bars.

There are $\binom{14}{4}$ ways to divide 10 bars among 5 pirates.

Now, let's count the ways in which some pirate gets more than 6 bars.

<u>*Notice that only one pirate can get more than 6 bars*</u>. There are five cases:

- – All ways to distribute gold such that pirate 1 gets more than 6 bars (7 bars for sure…)
- – All ways to distribute gold such that pirate 2 gets more than 6 bars (7 bars for sure…)
- – All ways to distribute gold such that pirate 3 gets more than 6 bars (7 bars for sure…)
- – All ways to distribute gold such that pirate 4 gets more than 6 bars (7 bars for sure…)
- – All ways to distribute gold such that pirate 5 gets more than 6 bars (7 bars for sure…)

For each case, the answer is $\binom{7}{4}$. In total there are $5 \cdot \binom{7}{4}$ ways to distribute such that some pirate gets more than 6. By the **difference method**, the final answer is $\binom{14}{4} - 5 \cdot \binom{7}{4}$.

1. (15%) How many 8 letter words contain only vowels?
Hint: There are 5 vowels

Solution: 5^8. There are 5 vowels. Hence, for each letter there are 5 choices and these are repeatable, $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$.

How many 8 letter words contain only consonants?

Solution: 21^8. Same as before. There are 21 consonants. Hence, for each letter there are 21 choices.

How many 8 letter words contain both vowels and consonants?

Solution: **DIFFERENCE METHOD**: $26^8 - 5^8 - 21^8$. Let S be the set of all 8-letter words, S_1 be the set of all 8-letter words containing only vowels, and S_2 be the set of all 8-letters words containing only consonants. Then by the difference method, the number of 8-letter words that contain both vowels and consonants is $|S| - |S_1| - |S_2| = 26^8 - 5^8 - 21^8$.

2. (10%) How many ways to divide 10 identical Hershey bars among 3 kids?

Solution: This is the same as number of non-negative solutions to $x_1 + x_2 + x_3 = 10$. And the same as counting the ways for arranging 3-1 = 2 dividers within the 10 dots, as in lecture. Hence, $\binom{12}{2}$.

How many ways to divide a subset of 10 identical Hershey bars among 3 kids when you can decide to keep some of the bars for yourself.

Solution: This is the same as number of non-negative solutions to $x_1 + x_2 + x_3 + x_4 = 10$. Hence, $\binom{10+3}{3}$.