CS 206

Recitation - Section 4

Oct. 5

Problem 1

How many permutations of word “REMAINS” are there such that vowels are always in odd places of the word?

vowels: A, E, I, O, and U.

Solution 1

There are 4 odd places in the 7-letter word, so p(4,3)=24 ways to assign 3 vowels to 4 places.
There rest of the places are assigned a consonant p(4,4)=4!=24.
So total is 24*24=576 ways

Problem 2

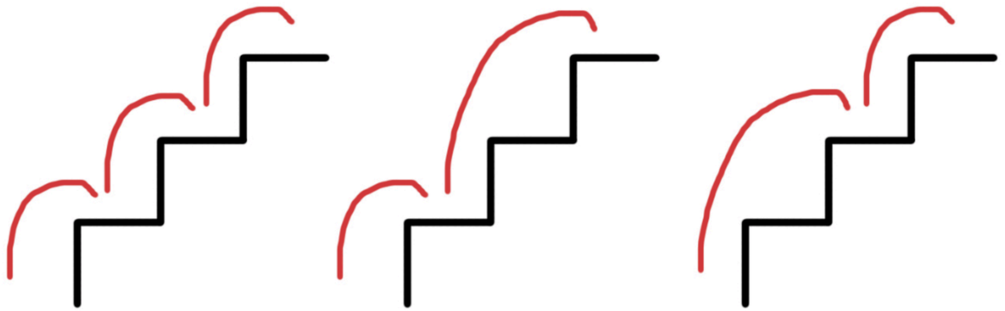
Given word “SUPER”, how many words can be formed such that vowels are grouped together?

Solution 2

Assume the two vowels are one letter, then number of ways to arrange the 4 letters are 4!=24.
And the number of ways to arrange 2 vowels is 2!=2.
So total number of words you can form given constraints is 24*2=48

Problem 3

How many distinct ways can you climb to the top of a stairway with 8 steps.
Each time you can climb either 1 or 2 steps.



Solution 3

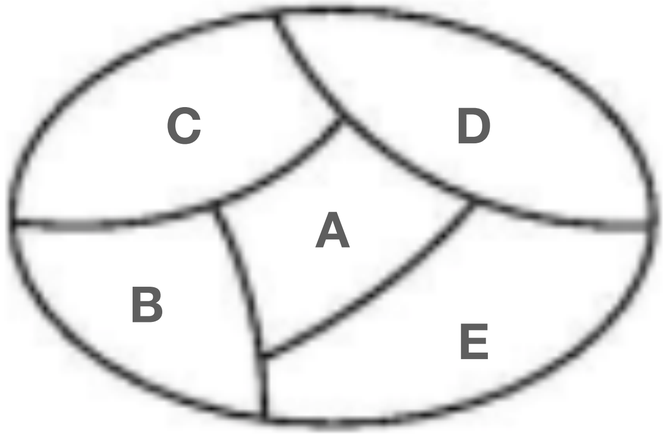
4 movements - 4*2: **1**
5 movements - 3*2+2*1: C(5,2) = **10**
6 movements - 2*2+4*1: C(6,2) = **15**
7 movements - 1*2+6*1: C(7,1)=**7**
8 movements - 8*1: **1**

1+10+15+7+1 = 34

Problem 4

Label each area with a color
- Adjacent areas must be labeled different colors
- 4 available colors - DO NOT have to use all 4 colors

How many different ways can you label the areas?



Solution 4

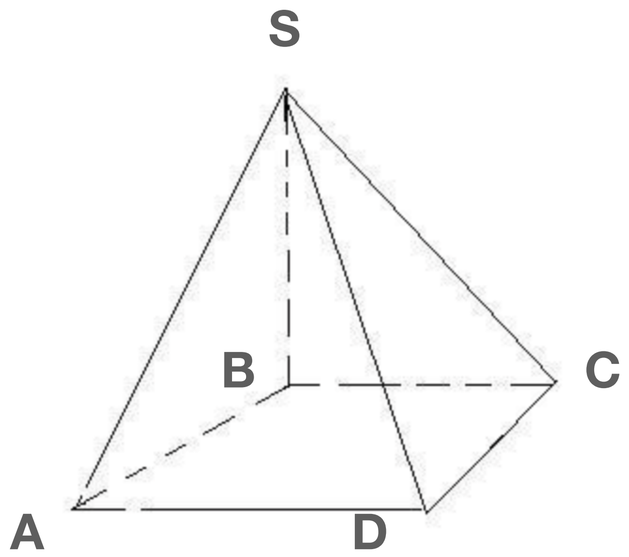
Assume we label them in the order of A->B->C->D->E

A: 4
B: 3
C: 2
D: 2 = 1(same as B’s)+1(different from B’s)
E: depends on B’s and D’s colors
    If B and D are same color: 2
    If B and D are different color: 1

4*3*2*1*2+4*3*2*1*1=72

Problem 5

Label each vertex of a quadrilateral pyramid with a color
Adjacent vertices must be labeled different colors
5 available colors - DO NOT have to use all 5 colors
How many different ways can you label the vertices?



Solution 5.1

S -> A -> B -> C -> D

S: 5
A: 4 (adjacent to S)
B: 3 (adjacent to S, A)
C: 3 (adjacent to S, B)
    If C's color == A's color:
        C:1 D: 3
    If C's color != A's color:
        C:2 D: 2

5*4*3*1*3 + 5*4*3*2*2 = 420

Solution 5.2

5 colors / 4 colors / 3 colors

5 colors: P(5,5)

4 colors: A,C are same color or B,D are same color

P(5,4) P(5,4)

3 colors: A,C are same color and B,D are same color

5*4*3

P(5,5) + 2 * P(5,4) + P(5,3)= 420

Problem 6

Tom has 10 books that he is going to put on his bookshelf. Of these, 4 are Python books, 3 are Java books, 2 are C# books, and 1 is a Swift book. Tom wants to arrange his books so that all the books dealing with the same programming language are together on the shelf.

How many different arrangements are possible?

Solution 6

There are 4!*3!*2!*1! arrangements such that the Python books are first in line, then the Java books, then the C# books, and then the Swift book.
Similarly, for each possible ordering of the subjects, there are 4!*3!*2!*1! possible arrangements.
Hence, as there are 4! possible orderings of the subjects, the desired answer is

4! * 4! * 3! * 2! * 1! = 6912