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Bring a notepad (it will become handy)

206
Discrete Structures II

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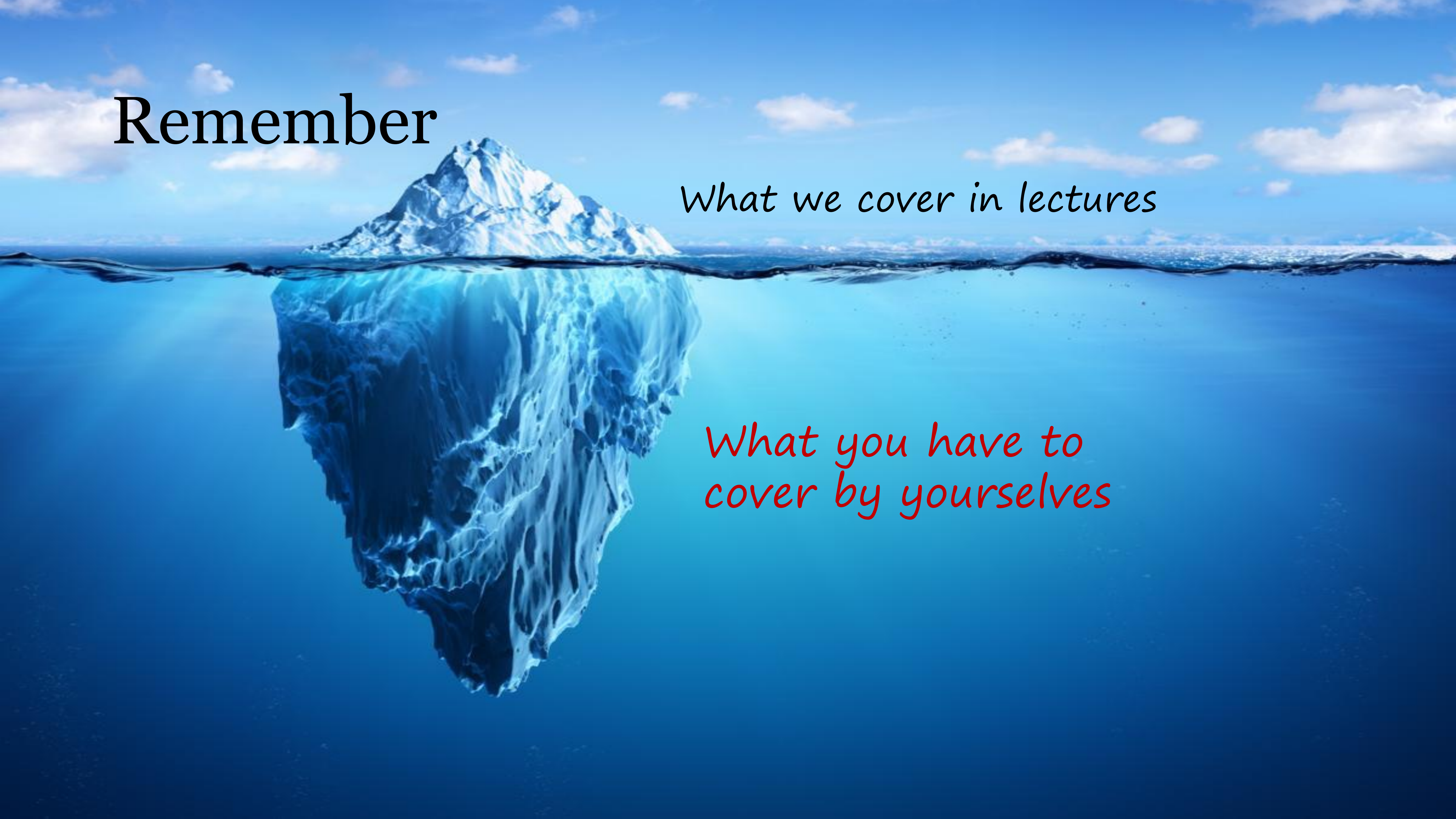
Lecture 2 | 205 Recap | Tuesday 09/07 2021

What we will cover today

- Recap
  - How to do well in the course?
- Combinatorics Intro (Recap 205)
  - Sets (Today's Lecture)
  - Break
  - Venn Diagram (Today's Lecture)
  - Functions (Today's Lecture)
  - Proofs (Thursday Lecture)
  - Induction (Thursday Lecture)

Remember


What we cover in lectures
What you have to cover by yourselves

Reading for Quiz 1 (and beyond…)

Lecture 2	Recap and Basics of Counting	Chapters 1, 2 and 5 of Rosen
Lecture 3	Basics of Counting	Chapters 1, 2 and 5 of Rosen Chapter 15 of Lehman
Lecture 4	Basics of Counting	Chapters 6 of Rosen Chapter 15 of Lehman

Recap

How to do well in the course?

- Attend lectures and ask questions
  - There are no stupid questions (use the chat wisely)
- Attend recitations

  Recitations start on Tuesday 9/14 or Thu 9/16

  Sections 5 and 6 are merged – more info to follow
  - Will introduce new material and problem solving similar to quizzes
- Form study groups
  - Find a study buddy
- Come to office hours **prepared**
- Stay up to date with the material
  - Studying the day before a quiz is a **bad** idea.

Recap

How to do well in the course?

- Stay up to date with the material
  - Studying the day before a quiz/midterm is a bad idea
  - After each lecture the slides will be posted on canvas
  - Before the end of the day spend ~20 mins going over slides and make notes of things you did not understand
  - Bring questions to next class and office hours
  - When you "attend" a lecture, have a notepad in front of you (not a PC!!!) and solve (**by hand**) the problems (till the very end)

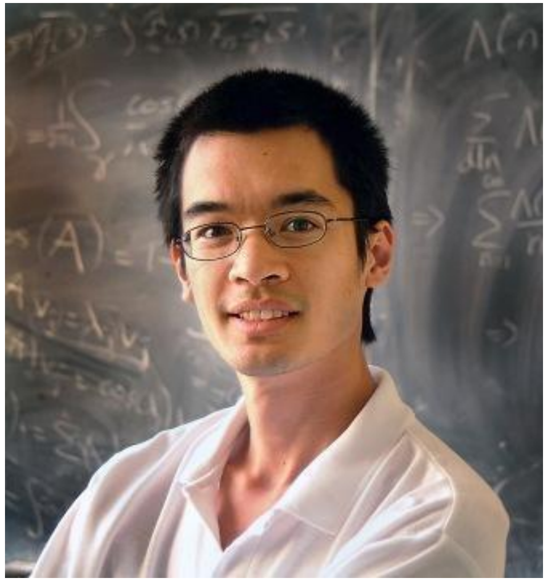
Recap

How to do well in the course?

- Lecture format
  - Each lecture will consist of introducing a concept and doing examples related to that concept
  - HWs, quizzes, recitations, study groups will give you more practice on examples related to these concepts
  - The more you practice, the better you will get

Recap

How to do well in the course?



Terence Tao

World's greatest living mathematician.
2006 Fields Medalist,
Claymath prize

"I don't have any magical ability. I look at a problem, <u>and it looks something like one I've done before</u>; I think maybe the idea that worked before will work here. When I was a kid, I had a romanticized notion of mathematics, that hard problems were solved in 'Eureka' moments of inspiration. [But] with me, it's always, 'Let's try this. That gets me part of the way, or that doesn't work. Now let's try this. Oh, there's a little shortcut here.... It's not about being smart or even fast. It's like climbing a cliff: If you're very strong and quick and have a lot of rope, it helps, but you need to devise a good route to get up there. Doing calculations quickly and knowing a lot of facts are like a rock climber with strength, quickness and good tools. You still need a plan — that's the hard part — and you have to see the bigger picture."

Combinatorics

- The study of arrangements of objects
- Studied as long ago as the 17th century, when combinatorial questions arose in the study of gambling games
- Used to solve many different types of problems
  - Examples:

  Enumeration, the **counting of objects** ***with certain properties***

  1. Counting determines the complexity of algorithms
  2. Counting determines whether there are enough resources to solve a problem
  3. …

Combinatorics

Used to solve many different types of problems

Enumeration, the **counting of objects** ***with certain properties***

Example:

1. Counting determines the complexity of algorithms
2. Counting determines whether there are enough resources to solve a problem.

- Study of discrete structures
  - Counting structures of a given kind/size

```
function TARJAN(Node* node)
    node.visited ← true
    node.index ← indexCounter
    s.push(node)
    for all successor in node.successors do
        if !node.visited then TARJAN(successor)
        end if
        node.lowlink ← MIN(node.lowlink, successor.lowlink)
    end for
    if node.lowlink == node.index then
        repeat
            successor ← stack.pop()
        until successor == node
    end if
end function
```

What questions can you ask?

Combinatorics

Used to solve many different types of problems

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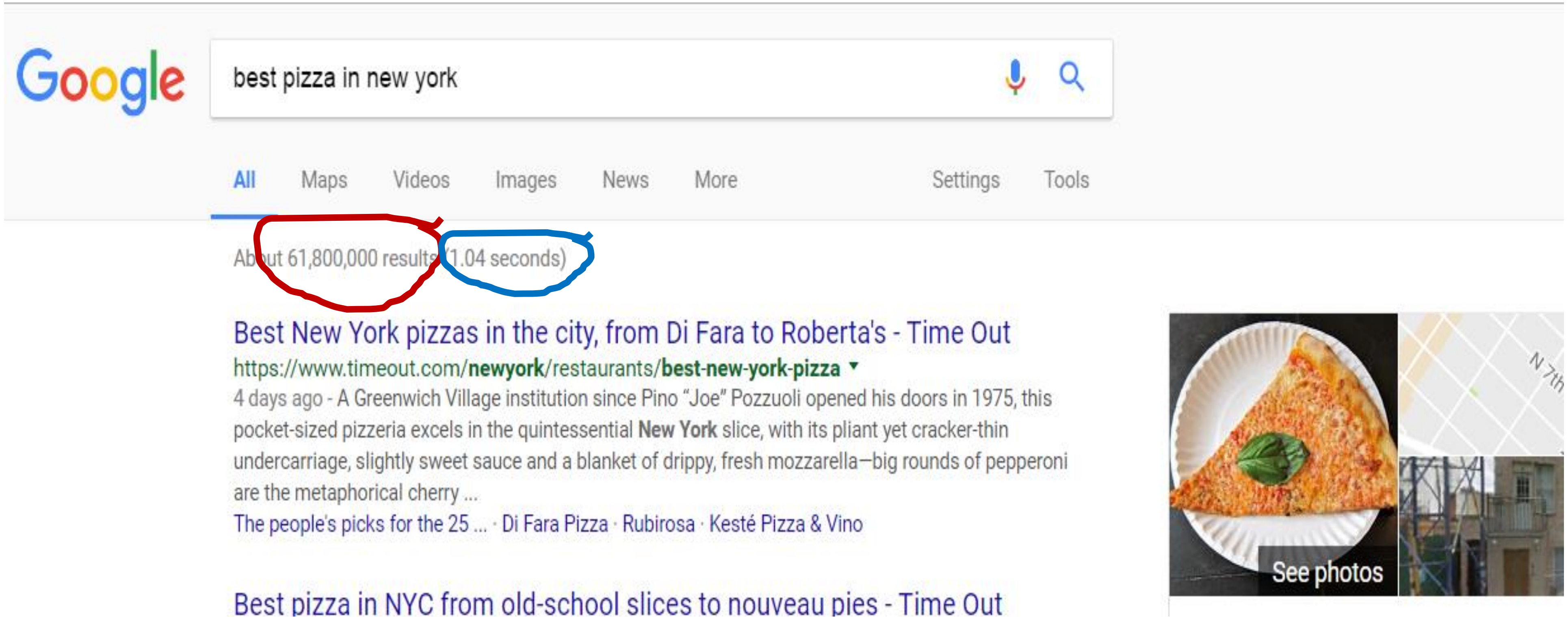
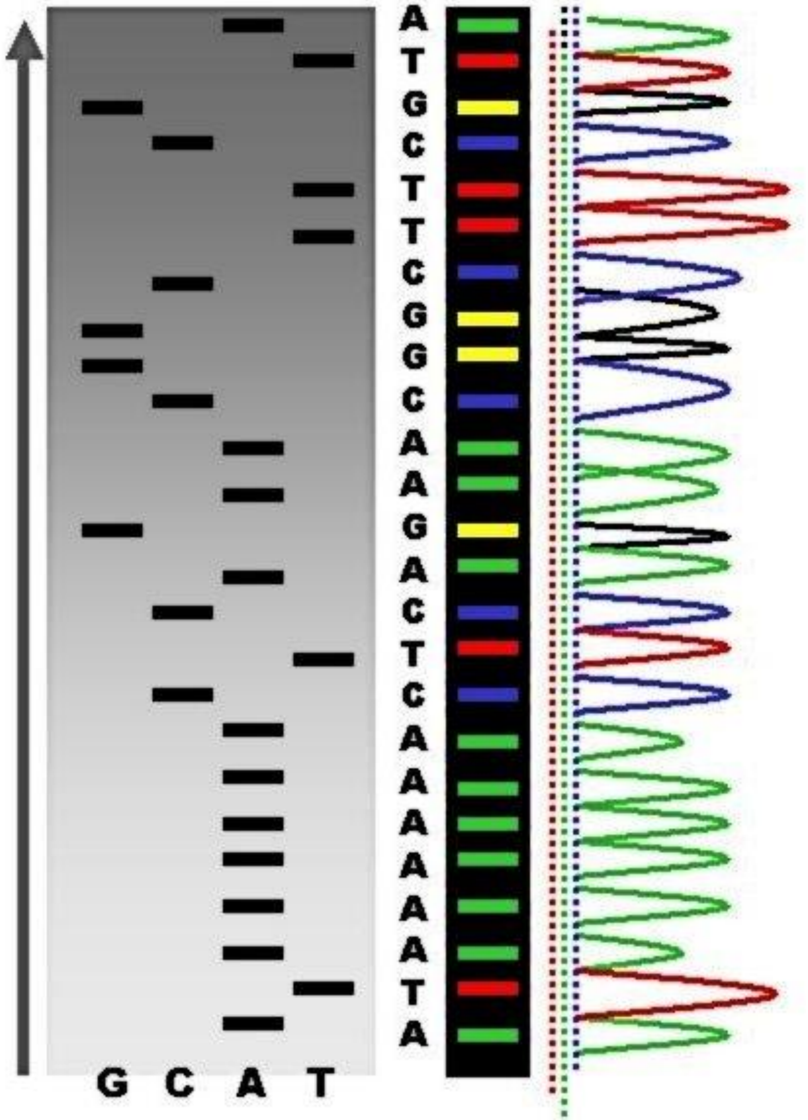
Complexity:
What is the runtime?

Resources:
What is the memory usage?

Combinatorics – Enumerating Example



Combinatorics



Recently, it has played a key role in

mathematical biology,

e.g., in sequencing DNA.

Rosen, 385

Combinatorics

- We will study the **basic rules of counting**
  - They can solve a tremendous variety of problems, such as:
    - Enumerate the **different telephone numbers** possible in the United States,
    - Enumerate the **allowable passwords** on a computer system,
    - Enumerate the different orders in which the runners in a race can finish
  - They can help us answer questions that seem hard: *What is the chance that among the 240 students in this class, we find 2 with the same birthday*?

- An important **combinatorial tool** is the *pigeonhole principle*: When objects are placed in boxes and there are more objects than boxes, then there is a box containing at least 2 objects.
  - E.g., we can use this principle to show that among a set of 15 or more students, at least 3 were born on the same day of the week

Combinatorics

Your Password:

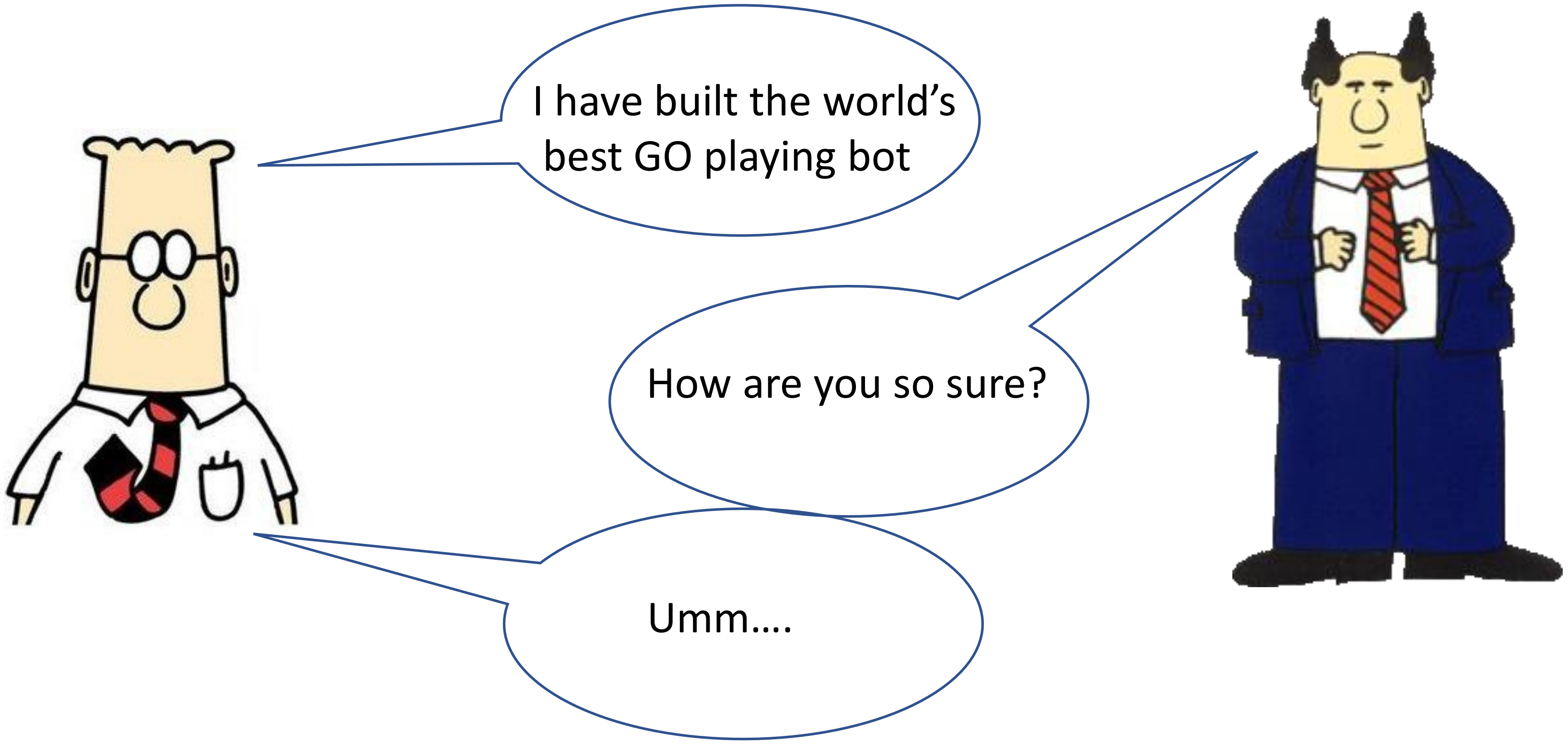
- Must be different from your User ID
- Must contain 8 to 20 characters, including one letter and number
- May include one of the following characters: %, &, _, ?, #, =, -
- Your new password cannot have any spaces and will not be case sensitive.

*REQUIRED FIELD

How many different passwords we can create?

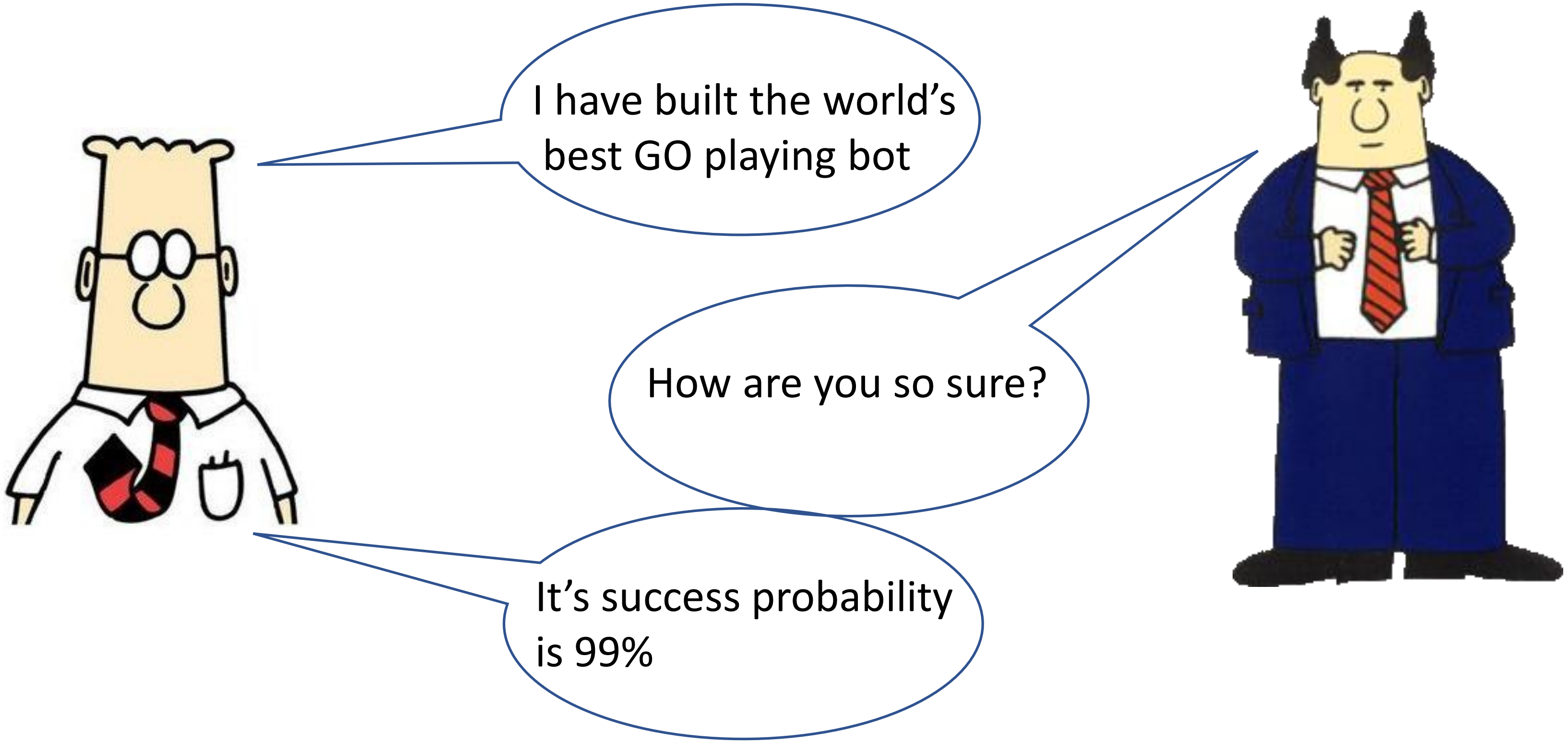
Combinatorics -> Probability Theory

- Analysis of uncertain or random phenomena



Probability Theory

- Analysis of uncertain or random phenomena



Probability Theory

- Analysis of uncertain or random phenomena

Probability Theory

- Analysis of uncertain or random phenomena

Course Outline

- Part I
  - Recap of basics – sets, function, proofs, induction
  - Basic counting techniques
  - Pigeonhole principle
  - Generating functions
- Part II
  - Sample spaces and events
  - Basics of probability
  - Independence, conditional probability
  - Random variables, expectation, variance
  - Moment generating functions
- Part III
  - Graph Theory
  - Machine learning and statistical inference

Sets

- What is a *Set?*
  - A collection of objects which are called *elements*
  - Elements are objects that share the same property
- Examples
  - Someone's followers on Twitter
  - The set of webpages for a given Google query
  - Collection of YouTube videos

Sets

- The order of elements is not significant, so $\{x, y\}$ and $\{y, x\}$ are the same set written two different ways.

- And what about $y = x$?
  - $\{x, x\} = \{x\}$

- The expression $e \in S$ asserts that e **is an element of** set S
  - E.g.,$32 \in S$ or $blue \notin S$

Sets – Common Sets

- What is a *Set?*
  - A collection of objects which are called *elements*.
- Some common sets in Math

curly braces

• $\emptyset$	Empty set	{}
• $\mathbb{N}$	Nonnegative integers	{0,1,2,3,…}
• $\mathbb{Z}$	Integers	{…,-2,-1,0,1,2,…}
• $\mathbb{Q}$	Rational numbers	{1/2, 16, -5/2}
• $\mathbb{R}$	Real numbers	{π, e, -9, $\sqrt{2}$}
• $\mathbb{C}$	Complex numbers	{i, 19/2, $\sqrt{2}$-2i}

A superscript "+" restricts a set to its positive elements; for example, $\mathbb{R}^+$ denotes the set of positive real numbers. Similarly, $\mathbb{Z}^-$ denotes the set of negative integers

Sets - Set Operations

For example
$X ::= \{1,2,3\}$
$Y ::= \{2,3,4\}$

- Union: $X \cup Y$
  - All elements present in X or Y or both.

$X \cup Y = \{1,2,3,4\}$

- Intersection: $X \cap Y$
  - All elements present in *both* X and Y.

$X \cap Y = \{2,3\}$

- Difference: $X \setminus Y$
  - All elements present in X but not in Y.
  - Not symmetric!

$X \setminus Y = \{1\}$
$Y \setminus X = \{4\}$

- Product: $X \times Y$
  - Collection of all tuples (a, b) where $a \in X$ and $b \in Y$.

$X \times Y = \{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

- Size: $|X|$
  - Number of elements in X.

$|X| = 3$
$|Y| = 3$

Sets - Set Comparisons

- Subset: $X \subset Y$
  - Every element present in X is also present in Y.
  - X is not the same as Y.

$X = \{1\}$, $Y = \{1, 2, 3\}$

$X \subset Y$

- Superset: $X \supset Y$
  - Every element present in Y is also present in X.
  - X is not the same as Y.

- Note: There is a direct analogy between [$\subset$ and $<$] and [$\subseteq$ and $\leq$]

Take a Break

5 min

Power Set

$X = \{1,2,3\}$

$Power(X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

- Let X be a set.
- $Power(X)$ = set of all subsets of X

- E.g., $Power(\{1,2\}) = \{1\}, \{2\}, and\ \{1,2\}$

- Is this correct?
  - NO!
  - $Power(\{1,2\}) = \{1\}, \{2\}, \{1,2\}, \boldsymbol{and}\ \{\}$

- Generally, if A has n elements, then there are 2^n sets in $Power(A)$

New Stuff

Set Builder Notation

- Often sets cannot be fully described by listing the elements explicitly or by taking unions, intersections, etc., of easily-described sets
- **Set builder notation** often comes to the rescue
- The idea is to define a set using a predicate; in particular, the set consists of all values that make the predicate true

Examples:

- $X = \{n \in \mathbb{N}: n \text{ is prime}\}$
- $Y = \{x \in \mathbb{R}: x^3 - 3x + 1 > 0\}$
- $Z = \{z \in YouTube\_videos: z \text{ is less than } 3 \text{ minutes long}\}$

Work on it

Exercise 1: Put everything together

$A = \{0, 1, 2\}$ $B = \{1, 4, 9\}$

- Let $A = \{n \in N: n^2 < 7\}$ and $B = \{1,4,9\}$

Find

- $A \cup B$ $A \cup B = \{0, 1, 2, 4, 9\}$
- $A \cap B$ $A \cap B = \{1\}$
- $A \times B$ $A \times B = \{(0,1), (0,4) \ldots$
- $A \setminus B$ $A \setminus B = \{0, 2\}$

Venn Diagram

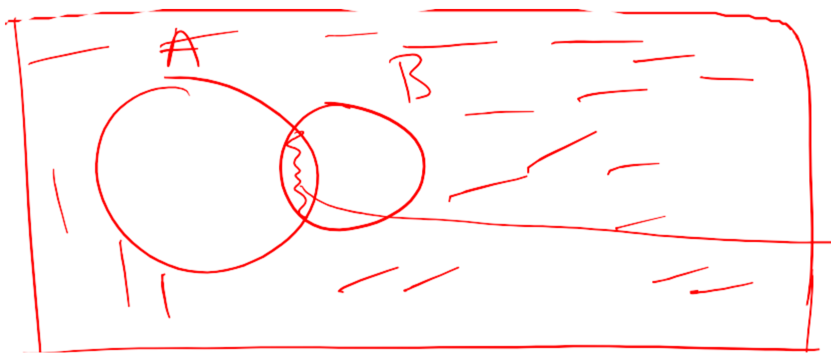
- Represent sets as circles and elements as points within it.
- Elegant way to capture relationships among sets.

Work on it

Exercise 2: Venn Diagram

- There are 131 students in CS 206.
- 100 like chocolate ice cream. 50 like vanilla ice cream.
- 20 like both chocolate and vanilla ice cream.
- Draw a Venn diagram to represent this.
- How many students do not like either flavor of ice cream.

A = chocolate
B = vanilla



$|A| = 100$
$|B| = 50$
$|A \cap B| = 20$