CS206 Quiz 2 **Oct 14, 2021** **A**

Name: ____________________________

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1. (20%) True/False

   (a) For finite sets X and Y, the number of possible binary relations (x, y) where $x \in X$ and $y \in Y$, is $|X| \cdot |Y|$.

   Solution: True, since every element of X can map onto any other element of Y, total number of mappings is product of elements in X and Y.

   (b) Given a set X, its permutation set has less elements than its combination set.

   Solution: False. Permutations take into account ordering of the elements, while combinations do not. Hence, multiple permutations count as a single combination, so the set of permutations must be larger than set of combinations.

2. (20%) For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9.

   (a) How many area codes were possible?

   Solution: $8 * 2 * 9 = 144$. The size of set from which first digit is drawn is $|D_1| = 8$, second digit is drawn is $|D_2| = 2$,third digit is drawn is $|D_3| = 9$. Since we are trying to find all the chains of maps $D_1 \to D_2 \to D_3$, we apply product rule.

   (b) How many area codes starting with a 4 were possible?

   Solution: $2 * 9 = 18$. The logic is same as above, except now we fix the first digit to 4. This means that the $|D_1| = 1$

3. (20%) There are 100 airline passengers waiting to board a plane.
   (a) In how many ways can I arrange 30 of them in Business class?

   Solution: $P(100, 30) = \frac{100!}{(100-30)!}$. In this case order of seating arrangement matters. Hence, we want to obtain the number of ways we can permute 100 people into 30 slots.

   (b) In how many ways can I create randomly the first boarding group of 30 people?

   Solution: $\binom{100}{30} = \frac{100!}{30!(100-30)!}$. In this case the order of the group doesn't matter, hence we get the number of combinations of 30 people from group of 100 people.

4. (20%) How many different words(existing and non-existing) can be formed from the letters of the word "TROOPER".

   Solution: $\frac{7!}{2!2!} = 1260$. Order of letters results in different words, hence order matters. There are 7 letters to arrange total, with 2 letters both repeating twice. So we divide by 2! for each repeating letter, since the swapping of repeating letters does not produce a different word.

5. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "CAMPER", such that the two vowels "A","E" are always next to each other.

   Solution: $5!2! = 240$. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 4! arrangements of 4 letters. We must also account for the 2 arrangements of the 2 vowel letter through 2!, since they are different.

6. (Extra Credits - 20%) How many different possible words(existing and not-existing) can be made from the word "WALLET" such that the vowels are *never* together?

   Solution: Total number of permutations of WALLET is 6!/2!=360, because we have 6 letters with one letter repeating twice.
   Permutations with vowels together is (5!/2!)*2!=120, because vowels are assumed to be one letter, resulting in arrangement of only 5 letters with one letter repeating twice. We also taking into account vowel arrangements by multiplying by 2!
   Thus, permutations with vowels never together is 360-120=240, by difference method

CS206 Quiz 2 **Oct 19, 2021** **B**

Name: ______________________________

NetID: ____________(Please **PRINT**)

Section No.: ____________

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1. (20%) True/False

   (a) For finite set X, the number of possible binary relations (x_1, x_2) where $x_1 \in X$ and $x_2 \in X$, is $|X|^2$.

   Solution: TRUE, since every element of X can map onto any other element of X including itself.

   (b) Given a set X, its permutation set has less elements than its combination set.

   Solution: False. Permutations take into account ordering of the elements, while combinations do not. Hence, multiple permutations count as a single combination, so the set of permutations must be larger than set of combinations.

2. (20%) For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9.

   (a) How many area codes were possible?

   Solution: $8 * 2 * 9 = 144$. The size of set from which first digit is drawn is $|D_1| = 8$, second digit is drawn is $|D_2| = 2$,third digit is drawn is $|D_3| = 9$. Since we are trying to find all the chains of maps $D_1 \to D_2 \to D_3$, we apply product rule.

   (b) How many area codes starting with a 8 were possible?

   Solution: $2 * 9 = 18$. The logic is same as above, except now we fix the first digit to 8. This means that the $|D_1| = 1$

3. (20%) There are 100 airline passengers waiting to board a plane.

(a) In how many ways can I arrange 40 of them in Business class?

Solution: $P(100,40) = \frac{100!}{(100-40)!}$. In this case order of seating arrangement matters. Hence, we want to obtain the number of ways we can permute 100 people into 40 slots.

(b) In how many ways can I create randomly the first boarding group of 40 people?

Solution: $C(100,40) = \frac{100!}{40!(100-40)!}$. In this case the order of the group doesn't matter, hence we get the number of combinations of 40 people from group of 100 people.

4. (20%) How many different words(existing and non-existing) can be formed from the letters of the word "ROOFER".

Solution: $\frac{6!}{2!2!} = 180$. Order of letters results in different words, hence order matters. There are 6 letters to arrange total, with 2 letters both repeating twice. So we divide by 2! for each repeating letter, since the swapping of repeating letters does not produce a different word.

5. (20%) How many different words (existing and non-existing) can be formed from the letters of the word "SKIER", such that the two vowels "I","E" are always next to each other.

Solution: $4!2! = 48$. Since we need to count all arrangements with vowels neighboring each other, we can treat the vowels as 1 letter. This results in 4! arrangements of 4 letters. We must also account for the 2 arrangements of the 2 vowel letter through 2!, since they are different.

6. (Extra Credits - 20%) How many different possible words(existing and not-existing) can be made from the word "TRAPPER" such that the vowels are *never* together?

Solution: Total number of permutations of TRAPPER is 7!/(2!*2!)=1260, because we have 7 letters with two letters repeating twice.
Permutations with vowels together is (6!/(2!*2!))*2!=360, because vowels are assumed to be one letter, resulting in arrangement of only 6 letters with two letters repeating twice. We also taking into account vowel arrangements by multiplying by 2!
Thus, permutations with vowels never together is 1260-360=900, by difference method