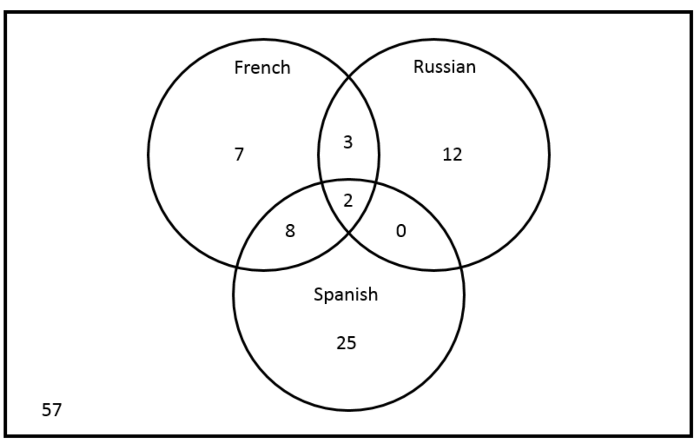
CS206 Recitation

Week of September 13, 2021

1. **Venn Diagram:**
   Students in CS 206 class are known multiple languages:

   - 20 students know French
   - 18 students know Russian
   - 35 students know Spanish
   - 5 students know both Russian and French
   - 2 students know French, Russian Spanish
   - 10 students know Spanish and French

   Draw a Venn diagram for the above. How many students are there in the class?



   Solution: Solution: 7+12+25+3+8+2=57 students in class

2. **Working with Sets:**
   Let $A = \{n \in \mathcal{N} : n > 0 \ \& \ \text{n is even} \ \& \ n < 12\}$ and $B = \{n \in \mathcal{Z} : |n| > 6, |n| < 13\}$

   Questions:

   (a) List elements of set A.
   Solution: $A = \{2, 4, 6, 8, 10\}$

   (b) List elements of set B.
   Solution: $B = \{-12, -11, -10, -9, -8, -7, 7, 8, 9, 10, 11, 12\}$

(c) What is set $A \bigcup B$?

Solution: $A \bigcup B = \{-12, -11, -10, -9, -8, -7, 2, 4, 6, 7, 8, 9, 10, 11, 12\}$

(d) What is set $A \bigcap B$?

Solution: $A \bigcap B = \{8, 10\}$

(e) What is set $A \times (A \bigcap B)$?

Solution: $A \times (A \bigcap B) = \{(2, 8), (4, 8), (6, 8), (8, 8), (10, 8), (2, 10), (4, 10), (6, 10)$
$, (8, 10), (10, 10)\}$

(f) What is set $A \setminus B$?

Solution: $A \setminus B = \{2, 4, 6\}$

(g) What is set $|A|$?

Solution: $|A| = 5$

3. **Power Set:**
For any set A, let $\mathcal{P}(A)$ be its power set. Let $\emptyset$ denote the empty set.

(a) Write down all the elements of $\mathcal{P}(\{1, 2, 3\})$.

(b) Write down all the elements of $\mathcal{P}(\{\emptyset, \{\emptyset\}\})$.

(c) How many elements are there in $\mathcal{P}(\{1, 2, 3, 4, 5, 6, 7, 8\})$?

Solution: Solutions:

$$\mathcal{P}(\{1, 2, 3\}) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \emptyset, \{1, 2, 3\}\}$$

$$\mathcal{P}(\{\emptyset, \{\emptyset\}\}) = \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}, \emptyset\}$$

$$|\mathcal{P}(\{1, 2, 3, 4, 5, 6, 7, 8\})| = 2^8$$

4. **Set and Propositional Formulas:**

(a) Show

$$P \iff (P \land -Q) \lor (P \land Q) \tag{1}$$

Solution: By distributive,inverse, and identity laws, respectively:

$$\begin{align}(P \wedge -Q) \vee (P \wedge Q) &\iff P \wedge (-Q \vee Q) \tag{2}\\ &\iff P \wedge T \tag{3}\\ &\iff P \tag{4}\\ \tag{5}\end{align}$$

QED

(b) Show $\forall A, B$ sets:

$$A = (A - B) \cup (A \cap B) \tag{6}$$

Solution: Proof:
Suppose there is an arbitrary element x such that,

$$\begin{gathered} x \in (A - B) \cup (A) \\ x \in (A - B) \vee x \in (A \cap B) \\ (x \in A \wedge x \notin B) \vee (x \in A \vee x \in B) \\ x \in A \wedge (x \notin B \vee x \in B) \\ x \in A \wedge (T) \\ x \in A \end{gathered}$$

By distributive,inverse,identity laws, the above equality holds. QED

5. **Direct Proof:**
Prove the following:
$\forall n \in Z$, if n is odd, then $n^2 + 2n + 1$ is even.

Solution: Proof:
Assume n is odd.
By defintion of an odd number,

$$n = 2j + 1$$

where $j \in Z$.
Then,

$$\begin{aligned}
n^2 + 2n + 1 &= (2j+1)(2j+1) + 2(2j+1) + 1 \\
&= 4j^2 + 4j + 1 + 4j + 2 + 1 \\
&= 4j^2 + 8j + 4 \\
&= 2(2j^2 + 4j + 2)
\end{aligned}$$

since $(2j^2+4j+2) \in Z$ it must be that $2(2j^2+4j+2)$ is an even number by definition, hence the above statement must be true. QED