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Discrete Structures II

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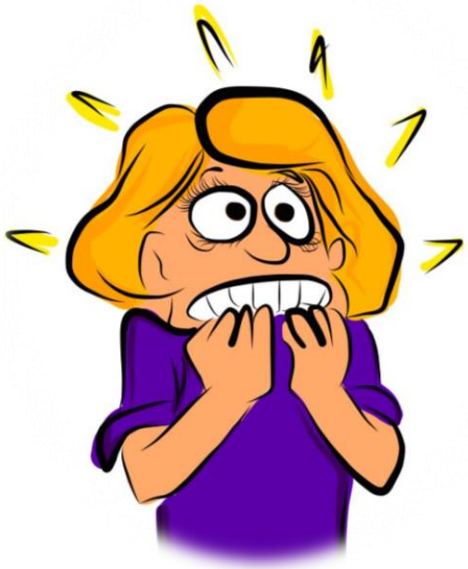
Quiz 3 – Next ~~Tuesday~~/Thursday



- More time (35 minutes)
  - + more questions ☺
- What will cover
  - Permutations with/out repetition
  - Combinations
  - Pirates Problem
  - Pirates Problem
  - Pirates Problem
    - Have you seen the extra Pirates problems?

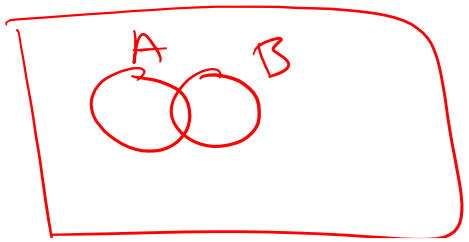
So Far

- ~~Proofs/Induction~~
- ~~Sum Rule~~
- ~~Partition Method~~
- ~~Difference Method~~
- ~~Bijection Method~~
- ~~Product Rule~~
- ~~Generalized product rule~~
- ~~Permutation/Combinations~~
- ~~Inclusion-Exclusion /~~ Pigeonhole Principle
- Combinatorial Proofs and Binomial Coefficients

Inclusion/Exclusion

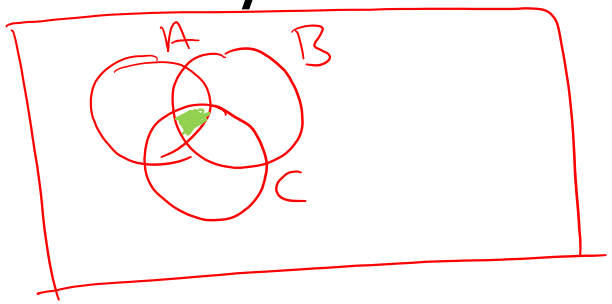
Sum Rule:
If A and B are disjoint sets, then $|A \cup B| = |A| + |B|$

- What if A and B are not disjoint? $|A \cup B| = ?$



So far...

Inclusion/Exclusion for 3 sets



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C| = (|A| + |B| + |C|) - (|A \cap B| + |B \cap C| + |A \cap C|) + (|A \cap B \cap C|)$$

Include Exclude Include

So far...

General Inclusion/Exclusion

- Solutions to $x + y + z = 15$ with $x \leq 3$ and $y \leq 4$? $x, y, z \geq 0$

$A_1 = \#$ solutions with $x \leq 3$

$A_2 = \#$ solutions with $y \leq 4$

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

$|A_1| =$

So far...

General Inclusion/Exclusion

$x, y, z \geq 0$

- Solutions to $x + y + z = 15$ with $x \leq 3$ and $y \leq 4$?

$|A_2|$ = # solutions to $x+y+z=15$ and $y \leq 4$

= (all solutions) − (solutions with $y \geq 5$)

$= \binom{17}{2} - \binom{12}{2}$

So far...

General Inclusion/Exclusion

$x, y, z \geq 0$

- Solutions to $x + y + z = 15$ with $x \leq 3$ and $y \leq 4$?

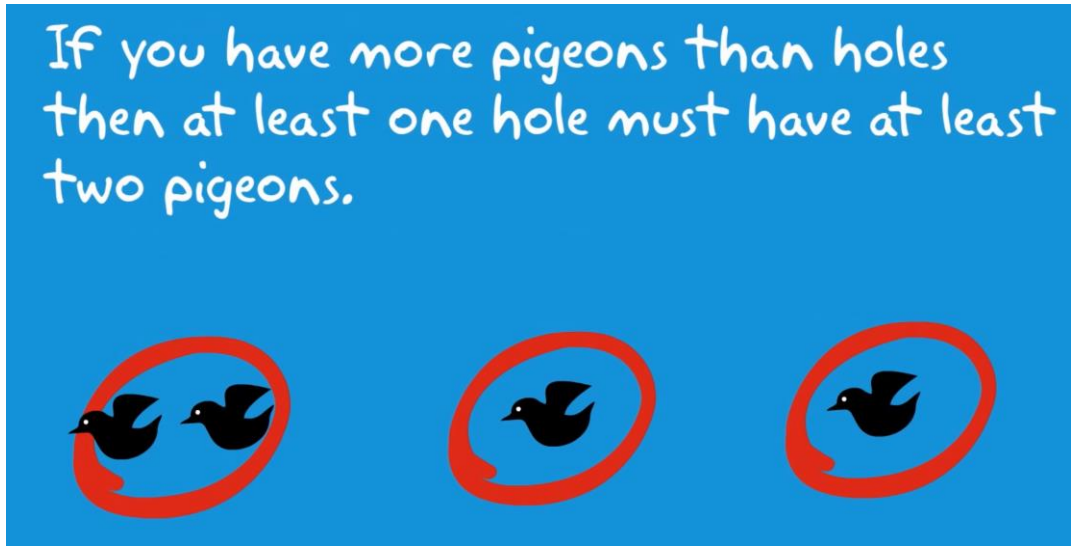
Hence, $|A_1 \cap A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

$$= \binom{17}{2} - \binom{13}{2} + \binom{17}{2} - \binom{12}{2}$$

$$- \binom{17}{2} + \binom{8}{2}$$

$$= 20$$

Pigeonhole Principle



A drawer in a room contains red, blue and green socks. How many socks must you withdraw before you see a matching pair?

Pigeonhole Principle

If there are more pigeons than holes they occupy, then at least two must be in the same hole.

Pigeonhole Principle

If m pigeons are in n holes and $m > n$, then at least $\left\lceil \frac{m}{n} \right\rceil$ pigeons are in the same hole.

$\left\lceil \frac{m}{n} \right\rceil$ = nearest integer higher than $\frac{m}{n}$

Ceiling of m over n rounds the ratio to the larger integer

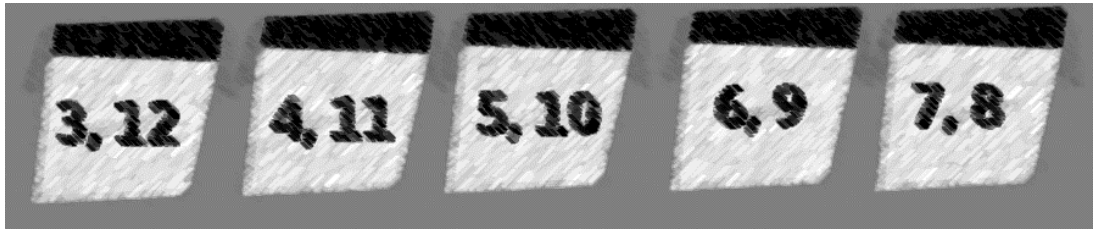
$m = 20$

$n = 9$

$\left\lceil \frac{20}{9} \right\rceil = 3$

PHP – Example 1

- Prove that if 6 integers are selected from {3,4,5,6,7,8,9,10,11,12}, there must be 2 integers whose sum is 15.
- Solution: Label 5 boxes



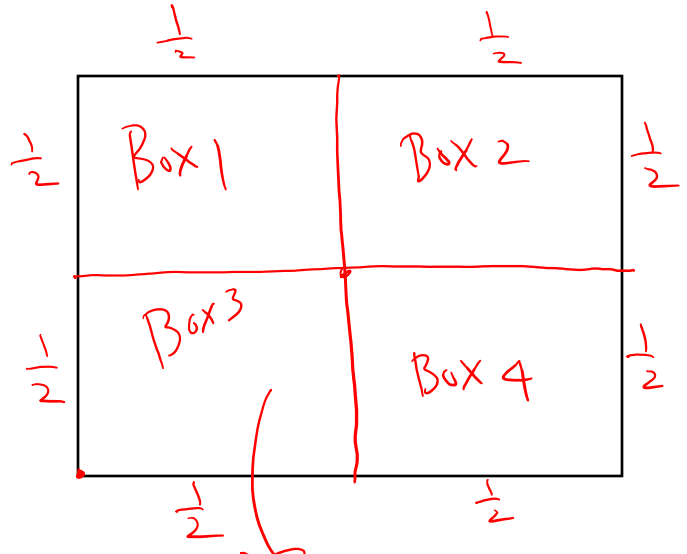
- We select 6 integers and place them in one of the boxes above, based on its label

- By PHP: One box must have at least 2 integers

Take a 5 min Break

PHP – Example 2

- Consider any 5 points in the interior of a square of unit length. Show that one can find two points that are at a distance of at most $\frac{\sqrt{2}}{2}$.



– If we pick 5 points Then by PHP some box must have ≥ 2 points

– Maximum distance between any two points in a box is when they are diagonally opposite. The distance is $\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\frac{\sqrt{2}}{2}$