CS 206

Recitation - Section 4

Sep. 28

Proof

1. **Direct Proof:** $p \to q$. Assume p is True, show q logistically follows.
2. **Proof by Induction:**

    Base Case: when $n = 1$ the proposition is True

    Induction Step: **Assume** $n = k$, the proposition is True. Then, to prove if the proposition holds when $n = k$, the proposition still holds when $n = k + 1$.

    Conclusion: ...

3. **Proof by Contrapositive:** $p \to q \iff \neg q \to \neg p$. Therefore, if we want to prove p implies q, we can just prove the negation of q implies the negation of p.
4. **Proof by Contradiction:** Assume $\neg p$ is True, try to find a contradiction. Therefore, because $\neg p$ is False and $\neg p \vee p = \mathrm{T}$, p must be True.
5. **Proof by Case Analysis:** Divide the problem(condition) into several smaller problems(conditions).

Counting

- Sum Rule
  - If A and B are disjoint sets, $|A \cup B| = |A| + |B|$
  - If A1, A2, …, An are disjoint sets, $|A1 \cup A2 \cup \ldots \cup An| = |A1| + |A2| + \ldots + |An|$
- Partition Method
  - To find the size of a set A
    - Partition it into a union of disjoint sets A1, A2, …, An
    - Use sum rule
- Difference Method
  - To find the size of a set A
  - Find a larger set S such that $S = A \cup B$
  - A and B are disjoint
  - $|A| = |S| - |B|$
- Product Rule
  - $|A \times B| = |A| \cdot |B|$
  - $|A1 \times A2 \times \ldots \times An| = |A1| \cdot |A2| \cdot \ldots \cdot |An|$

Problem 1

You have 6 marbles: 3 green, 2 red, 1 orange, that you want to give away to your 6 friends in sequence as you encounter them through out the day. How many different ways can you give out the 6 marbles? Use product rule.

Solution 1

Partition the sequence into subsets.
First set is number of ways you can give out the single orange marble: 6.
Assume orange marble has been given to a friend, and now you have 5 friends left for green/red marbles.
Second set is number of ways you can give out 3 green marbles to your 5 remaining friends: 10 (by enumeration). Last set has size 1 since you have 2 friends left and 2 marbles of same color.
Since these sets are independent, then by product rule total number of sequences is 6*10*1=60.

Problem 2

How many bit strings of length 8 either start with ‘1’ or end with ‘00’?

Solution 2

A = {8-bit strings start with ‘1’}
B = {8-bit strings end up with ’00’}
Since A and B are not disjoint sets,
$|A \cup B| = |A| + |B| - |A \cap B|$

$|A| = 2^7$
$|B| = 2^6$
$|A \cap B| = 2^5$
$|A \cup B| = 2^7+2^6-2^5$

Problem 3

- Amateur station call signs in the US take the format of one or two letters (the prefix), then a numeral (the call district), and finally between one and three letters (the suffix). Assume all letters are uppercase and the numeral is a digit 0-9.
    - How many amateur station call signs are possible that contain at least four letters without a repeat letter?

Solution 3

- Case 1 - 1+3 - 10x26x25x24x23
- Case 2 - 2+2 - 10x26x25x24x23
- Case 3 - 2+3 - 10x26x25x24x23x22 … and sum