CS206 Quiz 5 **Nov 30, 2021** **A**

Name: ____________________________

NetID: ____________(Please **PRINT**)

Section No.: ____________

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1. (20%) Let A and B be two events with $A \subseteq B$ and $0 < P(A) < 1$ and $0 < P(B) < 1$. Which of the following are true statements.

   (a) $P(A|B) = 1$

   Solution: FALSE

   (b) $P(B|A) = 1$

   Solution: TRUE

   (c) A and B are not independent

   Solution: TRUE

   (d) $P(A \cap B) > P(A)$

   Solution: FALSE

2. (15%) Using the axioms of probability, prove that for any experiment and associated sample space Ω, $P(\emptyset) = 0$. In other words, prove that the probability of the event corresponding to the empty set is 0.

   Solution: We know that $\emptyset = \Omega'$. We also know that $P(A) + P(A') = 1$ for any event A. Furthermore, $P(\Omega) = 1$. Hence, $P(\emptyset) = 0$.

3. (15%) Three fair dice colored red, blue and green are rolled. What is the probability that **at least** two of them roll the same number?

   Solution: The size of $\Omega = 6^3$. We are looking at all outcomes where all three have the same number or exactly two of them have the same number. There are 6 outcomes where they all have the same number and $\binom{3}{2} * 6 * 5$ outcomes where exactly two of them have the same number. Hence the probability that at least two have the same number $= \frac{16}{36} = \frac{4}{9}$.

4. (15%) Three fair dice colored red, blue and green are rolled. What is the probability that **exactly** two of them roll the same number?

   Solution: The size of $\Omega = 6^3$. There are $\binom{3}{2} * 6 * 5$ outcomes where exactly two of them have the same number. Hence, the probability that at least two have the same number $= \frac{15}{36} = \frac{5}{12}$

5. (15%) Three fair dice colored red, blue and green are rolled. What is the probability that at exactly two of them roll the same number **given** that the sum of the three numbers is 10?

Solution: Let B be the outcomes where the sum of the 3 numbers is 10. Let A be the outcomes where exactly two roll the same number. We want $P(A|B)$ which is equal to $\frac{P(A\cap B)}{P(B)}$. Since the outcomes are equally likely, this equals $\frac{|A\cap B|}{|B|}$.
$A \cap B$ contains the following elements: All permutations of (2,2,6), all permutations of (3,3,4), all permutations of (4,4,2). Hence, the sice of $A \cap B$ is 9. The size of B is all solutions to $x_1 + x_2 + x_3 = 10$, where $1 \leq x_i \leq 6$, for all $i = 1, 2, 3$. This number is 27. So, the probability of A conditioned on B equals $\frac{9}{27} = \frac{1}{3}$.

6. (10%) 8 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid 1 gets at least 3 chocolates.

Solution: Total outcomes $\binom{10}{2}$. Outcomes where kid 1 gets at least 3 = $\binom{7}{2}$. Hence, probability is $\frac{21}{45} = \frac{7}{15}$.

7. (10%) 8 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid 1 gets at least 3 chocolates given that kid 2 received 3 chocolates.

Solution: Let B = outcomes where kid 2 gets 3 chocolates, and A = outcomes where kid 1 get at least 3 chocolates. Then —B— = 6 and $|A \cap B| = 3$. Hence, probability is $\frac{3}{6} = \frac{1}{2}$.

8. (extra credits: 20%) In the hope of having a dry outdoor wedding, John and Mary decide to get married in the desert, where the average number of rainy days per year is 10. Unfortunately, the weather forecaster is predicting rain for tomorrow, the day of John and Mary's wedding. Suppose that the weather forecaster is not perfectly accurate: If it rains the next day, 90% of the time the forecaster predicts rain. If it is dry the next day, 10% of the time the forecaster still (incorrectly) predicts rain. Given this information, what is the probability that it will rain during John and Mary's wedding?

Solution: Let A be the event that it rains tomorrow and B the event that rain is predicted for tomorrow. Then we want to compute $P(A|B) = \frac{P(A \cap B}{P(B)}$. By Bayes theorem, this is equal to $\frac{P(A)(B|A)}{P(A)P(B|A)+P(A')P(B|A')}$. We are given that $P(A) = \frac{10}{365} = 0.027$, $P(B|A) = 0.9$, and $P(B|A') = 0.1$. Substituting we get that $P(A|B) = \frac{(0.027)(0.90)}{(0.027)(0.90)+(0.972)(0.1)}$

CS206 Quiz 5 **Dec 2, 2021** **B**

Name: ______________________________

NetID: _____________(Please **PRINT**)

Section No.: _____________

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1. (20%) Let A and B be two events with $A \subseteq B$ and $0 < P(A) < 1$ and $0 < P(B) < 1$. Which of the following are true statements.

   (a) $P(B|A) = 1$

   Solution: TRUE

   (b) $P(A|B) = 1$

   Solution: False

   (c) A and B are independent

   Solution: False

   (d) $P(A \cap B) < P(A)$

   Solution: FALSE

2. (15%) Using the axioms of probability, prove that for any experiment and associated sample space Ω, $P(\emptyset) = 0$. In other words, prove that the probability of the event corresponding to the empty set is 0.

   Solution: We know that $\emptyset = \Omega'$. We also know that $P(A) + P(A') = 1$ for any event A. Furthermore, $P(\Omega) = 1$. Hence, $P(\emptyset) = 0$.

3. (15%) Four fair dice colored red, blue, yellow and green are rolled. What is the probability that **at least** three of them roll the same number?

   Solution: The size of $\Omega = 6^4$. We are looking at all outcomes where all Four have the same number or none of them have the same number. There are 6 outcomes where they all have the same number and $\binom{4}{3} * 6 * 5$ outcomes where exactly Three of them have the same number. Hence the probability that at least three have the same number $= \frac{6+4*6*5}{6^4} = \frac{21}{216}$.

4. (15%) Four fair dice colored red, blue, yellow and green are rolled. What is the probability that **exactly** three of them roll the same number?

   Solution: The size of $\Omega = 6^4$. There are $\binom{4}{3} * 6 * 5$ outcomes where exactly three of them have the same number. Hence, the probability that exactly three have the same number $= \frac{4*5}{216} = \frac{5}{54}$

5. (15%) Three fair dice colored red, blue and green are rolled. What is the probability that at exactly two of them roll the same number **given** that the sum of the three numbers is 10?

> **Solution:** Let B be the outcomes where the sum of the 3 numbers is 10. Let A be the outcomes where exactly two roll the same number. We want $P(A|B)$ which is equal to $\frac{P(A \cap B)}{P(B)}$. Since the outcomes are equally likely, this equals $\frac{|A \cap B|}{|B|}$.
> $A \cap B$ contains the following elements: All permutations of (2,2,6), all permutations of (3,3,4), all permutations of (4,4,2). Hence, the sice of $A \cap B$ is 9. The size of B is all solutions to $x_1 + x_2 + x_3 = 10$, where $1 \leq x_i \leq 6$, for all $i = 1, 2, 3$. This number is 27. So, the probability of A conditioned on B equals $\frac{9}{27} = \frac{1}{3}$.

6. (10%) 9 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid 1 gets at least 3 chocolates.

> **Solution:** Total outcomes $\binom{11}{2}$. Outcomes where kid 1 gets at least 3 = $\binom{8}{2}$. Hence, probability is $\frac{28}{55}$.

7. (10%) 9 identical chocolates are randomly divided among 3 kids. Assume that each possible way to divide is equally likely. What is the probability that kid 1 gets at least 3 chocolates given that kid 2 received 3 chocolates.

> **Solution:** Let B = outcomes where kid 2 gets 3 chocolates, and A = outcomes where kid 1 get at least 3 chocolates. Then $|B| = 7$ and $|A \cap B| = 4$. Hence, probability is $\frac{4}{7}$.

8. (extra credits: 20%) In the hope of having a dry outdoor wedding, John and Mary decide to get married in the desert, where the average number of rainy days per year is 10. Unfortunately, the weather forecaster is predicting rain for tomorrow, the day of John and Mary's wedding. Suppose that the weather forecaster is not perfectly accurate: If it rains the next day, 90% of the time the forecaster predicts rain. If it is dry the next day, 10% of the time the forecaster still (incorrectly) predicts rain. Given this information, what is the probability that it will rain during John and Mary's wedding?

Solution: Let A be the event that it rains tomorrow and B the event that rain is predicted for tomorrow. Then we want to compute $P(A|B) = \frac{P(A \cap B}{P(B)}$. By Bayes theorem, this is equal to $\frac{P(A)(B|A)}{P(A)P(B|A)+P(A')P(B|A')}$. We are given that $P(A) = \frac{10}{365} = 0.027$, $P(B|A) = 0.9$, and $P(B|A') = 0.1$. Substituting we get that $P(A|B) = \frac{(0.027)(0.90)}{(0.027)(0.90)+(0.972)(0.1)}$.