CS 206

Recitation - Section 4

Oct. 12

Problem 1

Place 5 different marbles {M1, M2, M3, M4, M5} in a row.
How many different arrangements if there is one and only one marble between M1 and M2?

Solution 1

Regard M1, M2 and another marble as a whole:

1. Place M1 and M2: P(2,2)
2. Choose 1 marble Mx from marbles except for M1, M2: C(3,1)
3. Place {M1,M2,Mx} and the other 2 marbles: P(3,3)

P(2,2) * C(3,1) * P(3,3) = 36

Problem 2

Place 7 different marbles {M1, M2, M3, M4, M5, M6, M7} in a row.
How many different arrangements if neither M1 nor M2 is next to M3?

Solution 2

M3 is the first/last：

C(4,1) * P(5,5) = 480

M3 is in the middle:

C(5,1) * P(4,2) * P(4,4) = 1440

480 + 480 + 1440 = 2400

Problem 3

Place 7 different marbles (5 green and 2 red) {G1,G2,G3,G4,G5,R1,R2} in a row. How many different arrangements if there are 2 and only 2 green marbles between 2 red marbles?

Solution 3

Regard R1, R2 and another two green marbles between them as a whole:

1. Place R1 and R2: P(2,2)
2. Choose 2 green marbles {Gx1,Gx2} and place them : P(5,2)
3. Place {R1,R2,Gx1,Gx2} and other 3 green marbles: P(4,4)

P(2,2) * P(5,2) * P(4,4) = 960

Problem 4

From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed?
What if 2 of the men are feuding and refuse to serve on the committee together?

Solution 4

(a) C(5,2) * C(7,3) = 350
(b) C(5,2) * (C(7,3) - C(5,1)) = 300

Problem 5

There are 6 books on the shelf.
Now put in 3 more books.
- DO NOT change in the relative order of the original six books.

How many different arrangements?

Solution 5

Place these 9 books:

P(9,9)

However, some of them change the relative order of the original six books:

Every 1 of P(6,6) arrangements doesn’t change the order.

P(9,9) / P(6,6) = 504

Problem 6

Given an m*n grid in Cartisian coordinates starting from (0,0) to (m,n), how many different paths are there starting from the bottom-left corner to the top-right corner? You can only go rightward or upward, i.e., no returning routes.

Solution 6

Totally, you need n rightward movements and m upward movements.
Place n rightward movements into m+n spaces.

C(m+n, n)

Problem 7

A police department in a small city consists of 10 officers. If the department policy is to have 5 of the officers patrolling the streets, 2 of the offices working full time at the station, and 3 of the officers on reserve at the station, how many different divisions of the 10 officers into 3 groups are possible?

Solution 7

1. Choose 5 of 10
2. Choose 2 of 10-5
3. Choose 3 of 10-5-3

$C(10,5) * C(5,2) * C(3,3)$