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Discrete Structures II

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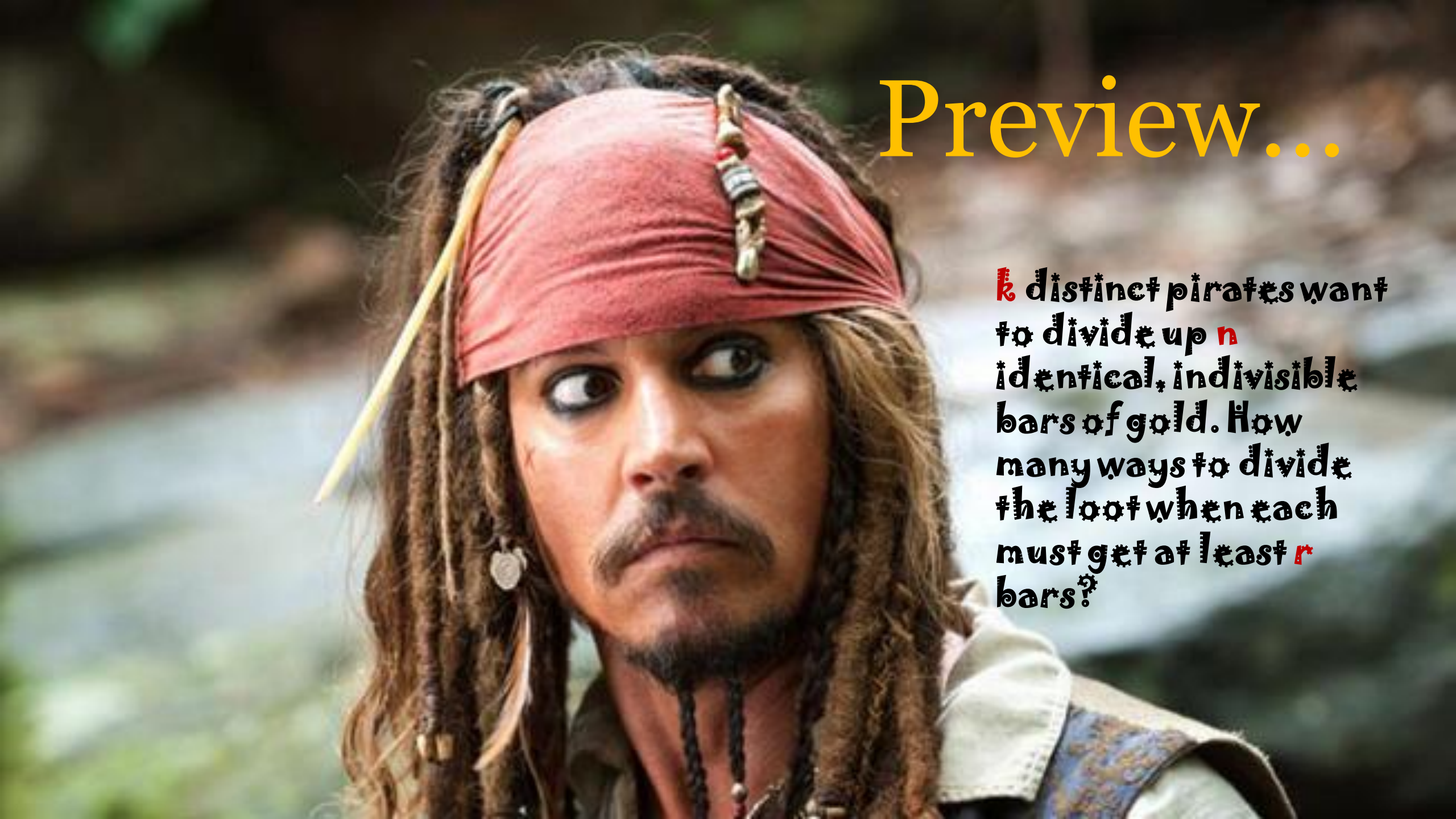


Preview…

k distinct pirates want to divide up n identical, indivisible bars of gold. How many ways to divide the loot when each must get at least r bars?

Preview…

How many integer solutions to the following equation?

$$x_1 + x_2 + \cdots + x_k = n$$

$$x_1, x_2, \ldots, x_k \geq 0$$

So Far

- ~~Proofs/Induction~~
- ~~Sum Rule~~
- ~~Partition Method~~
- ~~Difference Method~~
- ~~Bijection Method~~
- ~~Product Rule~~
- ~~Generalized product rule~~
- Permutation/Combinations
- Inclusion-Exclusion / Pigeonhole Principle
- Combinatorial Proofs and Binomial Coefficients

Last Class

- Permutations
- Combinations

Today

- Nothing

Permutations Formula

- Distinctly ordered sets are called permutations (arrangements). The number of permutations of n objects taken k at a time is given by:

$$P_k^n = \frac{n!}{(n-k)!}$$

N = number of objects
K = number of positions

Combinations Formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- If we have *n* objects and we want to choose *k* of them, we can find the total number of combinations by using the formula on the left

Permutations - Recap

- Permutations
  - Permute r out of n distinct objects – without repetitions
  - Permute r out of n distinct objects – with repetitions

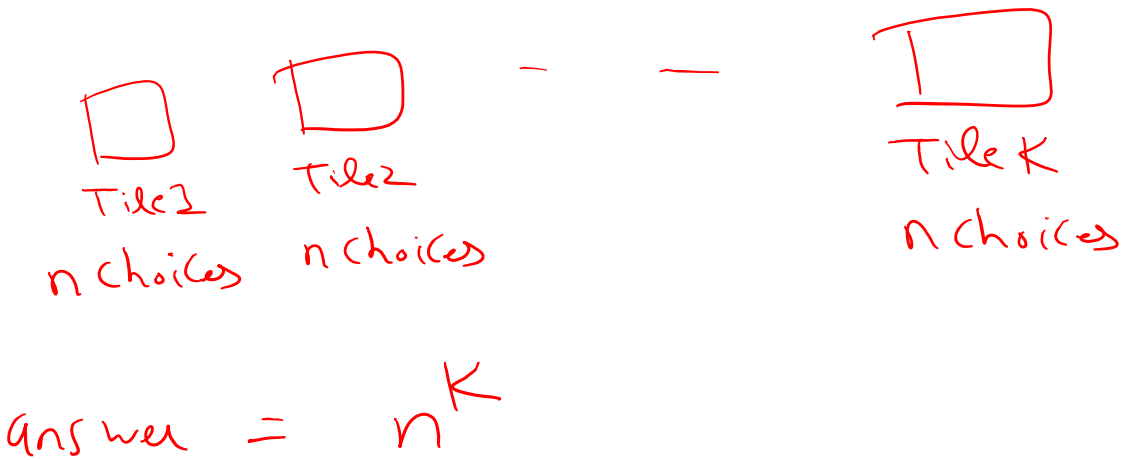
$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

$$ n^{r} $$

What is the question we are asking here?

Permutations with Repetitions

- Have n colors. Want to paint k tiles. How many ways?
  - Can reuse colors any number of times.



Permutations without Repetitions

- If 10 horses race, how many orderings of the top 3 finishers are there?

$$\frac{10!}{7!} = {}^{10}P_3$$

New Stuff

Permutations

A maths debating team consists of 4 speakers.

- In how many ways can all 4 speakers be arranged in a row for a photo?

Solution : $4x3x2x1 = 4!$ or ${}^4P_4$

- How many ways can the captain and vice-captain be chosen?

Solution : $4x3 = 12$ or ${}^4P_2$

Permutations

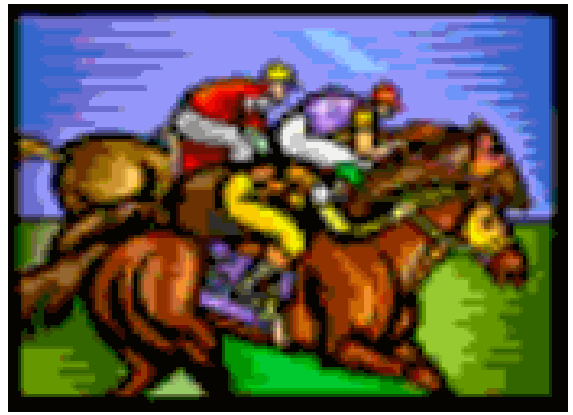
<u>A flutter on the horses</u>
There are 7 horses in a race.

- In how many different orders can the horses finish?

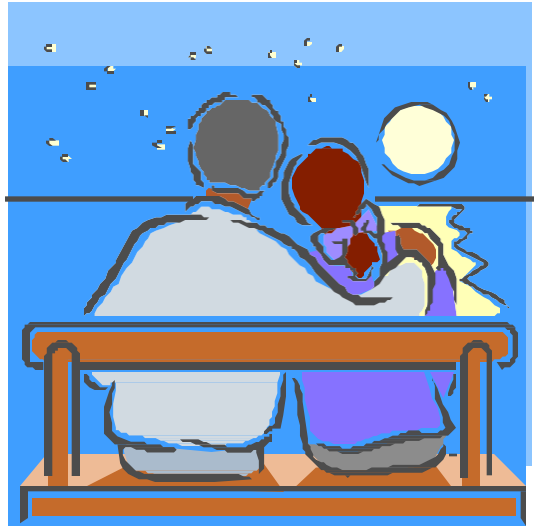
Solution : **$7x6x5x4x3x2x1 = 7!$ or ${}^7P_7$**

- How many trifectas (1^{st}, 2^{nd} and 3^{rd}) are possible?

Solution : **$7x6x5 = 210$ or ${}^7P_3$**



Permutations with Restrictions



In how many ways can 5 boys and 4 girls be arranged on a bench if

- there are no restrictions?
  Solution : 9! or $^9P_9$

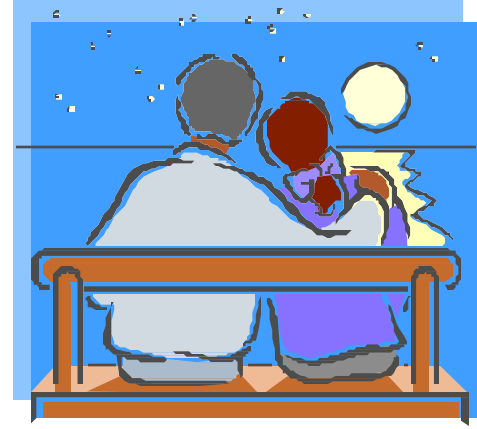
- boys and girls alternate?

  Solution : A boy will be on each end

  BGBGBGBGB = $5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

  $= 5! \times 4!$ or $^5P_5 \times {}^4P_4$

Permutations with Restrictions



In how many ways can 5 boys and 4 girls be arranged on a bench if

- boys and girls are in separate groups?

Solution : Boys & Girls or Girls & Boys

$$= 5! \times 4! + 4! \times 5! = 5! \times 4! \times 2$$

$$\text{or } {}^5P_5 \times {}^4P_4 \times 2$$

d) Anne and Jim wish to stay together?

Solution : (AJ) _ _ _ _ _ _ _

$$= 2 \times 8! \text{ or } 2 \times {}^8P_8$$

Permutations with Repetitions

If we have **n** elements of which ***x*** are alike of one kind, ***y*** are alike of another kind, ***z*** are alike of another kind, ... then the **number of ordered selections or permutations** is given by:

$$\frac{n!}{x!\,y!\,z!}$$

Permutations with Repetitions

How many permutations of the word **PARRAMATTA** are possible?

Solution :

P

A A A A

R R

M

T T

10 letters but note repetition (4 A's, 2 R's, 2 T's)

$$\text{No. of arrangements} = \frac{10!}{4!\,2!\,2!}$$

$$= 37\,800$$

Permutations with Restrictions

How many arrangements of the letters of REMAND are possible if:

- there are no restrictions?

  Solution : $^{6}P_{6} = 720$ **or** $6!$

- they begin with RE?

  Solution : R E _ _ _ _ $= {}^{4}P_{4} = 24$ **or** $4!$

- they do not begin with RE?

  Solution : Total – (b) = 6! – 4! = 696

Permutations with Restrictions

How many arrangements of the letters of REMAND are possible if:

- they have RE together in order?

Solution : **(RE)** _ _ _ _ _ **= $^5P_5$ = 120 or 5!**

- they have REM together in any order?

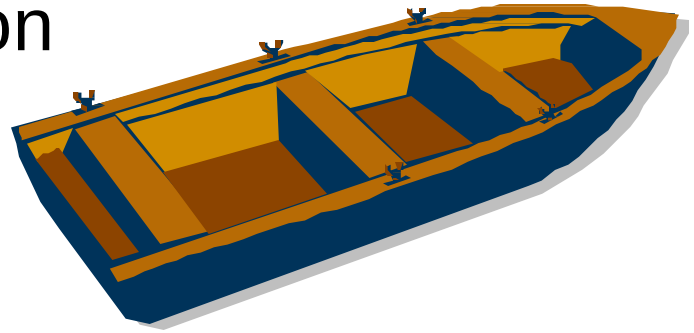
Solution : **(REM)** _ _ _ _ **= $^3P_3 \times {}^4P_4$ = 144**

- R, E and M are not to be together?

Solution : Total – (e) = 6! – 144 = 576

Permutations with Restrictions

There are 6 boys who enter a boat with 8 seats, 4 on each side. In how many ways can



- they sit anywhere? **Solution :** $^{8}P_{6}$

- two boys A and B sit on the port side and another boy W sit on the starboard side?

Solution : **A & B = $^{4}P_{2}$**

W = $^{4}P_{1}$

Others = $^{5}P_{3}$

Total = $^{4}P_{2} \times {}^{4}P_{1} \times {}^{5}P_{3}$

Permutations with Restrictions

From the digits 2, 3, 4, 5, 6

- how many numbers greater than 4,000 can be formed?

Solution : **5 digits (any) = $^5P_5$**

4 digits (must start with digit $\geq$ 4) = $^3P_1 \times {}^4P_3$

Total = $^5P_5 + {}^3P_1 \times {}^4P_3$

- how many 4 digit numbers would be even?

Even (ends with 2, 4 or 6) = _ _ _ $^3P_1$

= $^4P_3 \times {}^3P_1$