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There is a quiz today!

206
Discrete Structures II

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Quiz 1

September 2021

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6 Labor Day	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Quiz 1

- What will Quiz 1 cover?
  - Sets (Lecture 2)
  - Venn (Lecture 2)
  - Functions (Lecture 3)
  - Proofs (Lectures 3-5)
  - + What we will cover today (Sum and Product rules)

Reading for Quiz 1

Recap and Basics of Counting	Chapters 1, 2 and 5 of Rosen
Basics of Counting	Chapters 1, 2 and 5 of Rosen Chapter 15 of Lehman
Basics of Counting	Chapters 6 of Rosen Chapter 15 of Lehman

Have you seen this?

On missing quizzes due to sickness

Sep 28 at 9:52am

Konstantinos Michmizos

All Sections

Class

this announcement has important information re: asking to get a remote quiz or postponing getting it, due to a self-reported sickness.

We made anything possible to have the quizzes in-person; This is to help you, not us.

Online quizzes have a different set of challenges. They are way different in their structure, and an in-person quiz is not interchangeable to an online quiz, in terms of questions (number of them, as well as their pattern), solutions (multiple choice vs. hand-writing solutions), and other details of assessing your knowledge. Even their frequency should not be the same. We not only do not have the resources for creating both online and in-person version of quizzes but also there is no way for us to secure that the two quizzes have the same difficulty (you might be surprised to know that the remote quiz might be more difficult.)

Therefore, there is no way (reason) to have the quizzes administered both in person and remotely.

That is why, from the very first lecture, I have informed you that you will be able to drop the lowest grade quiz in the end.

In any case, the university has long-established a path that students with medical concerns can be officially excepted from taking a quiz. Although we are all doing the best we can, "self-reported" sickness does not exempt you from taking a quiz. Of course, if you do have symptoms, you should get tested asap for covid or any other disease that your physician would recommend.

Please also note that in a class of 130+ people, whatever decision I take for a particular person, I have to take the same decision for any other student, and for the entire semester, to keep everything fair in class. So, if someone sends me an email for asking a remote quiz because he/she "feels sick", I have to do exactly the same for anyone reporting the same. I believe you all understand the slippery slope that we are getting in, if I do that without a proper documentation.

Be well,

km

What we will cover today

Combinatorics

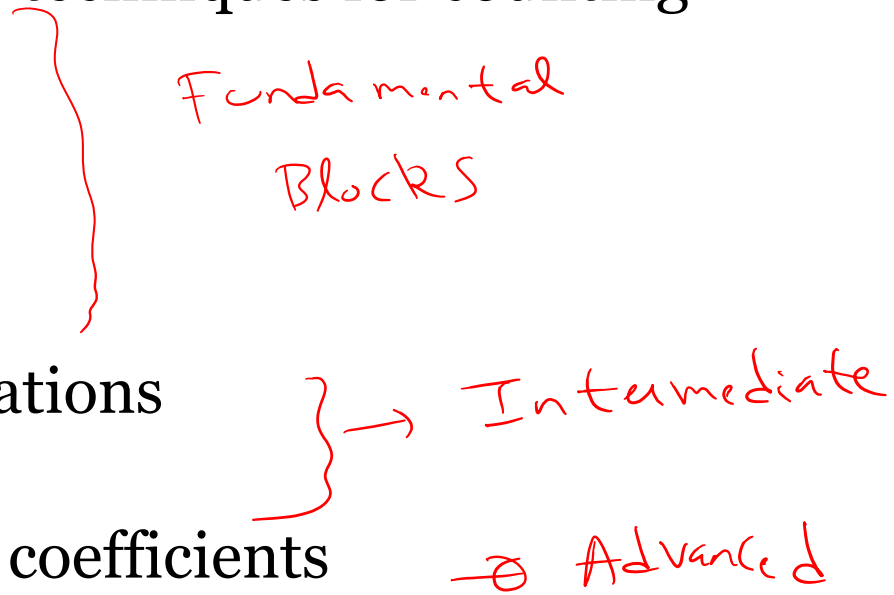
- Recap
  - Counting (Partition, Difference, Product Rule – *Combining Rules*)
- Today
  - Counting
    - Product Rule
    - *More Combining Rules!*
    - Bijection Rule
- Next
  - Permutations/Combinations
  - Pigeonhole Principle

Course Outline

- Part I
  - Recap of basics – sets, function, proofs, induction
  - Basic counting techniques
  - Pigeonhole principle
  - Generating functions
- Part II
  - Sample spaces and events
  - Basics of probability
  - Independence, conditional probability
  - Random variables, expectation, variance
  - Moment generating functions
- Part III
  - Graph Theory
  - Machine learning and statistical inference

Counting

- In the next few lectures
  - Fundamental tools and techniques for counting
  - Sum Rule
  - Product Rule
  - Difference Method
  - Bijection Method
  - Permutations/Combinations
  - Inclusion Exclusion
  - Binomial/Multinomial coefficients



So far...

Partition Method

- Possible outcomes where white and black die have different values?

S = all possible outcomes

A_1 = all outcomes with black die = 1

A_2 = black die = 2

$\vdots$

A_6 = black die = 6

$|A_1| = 5, \quad |A_2| = 5,$

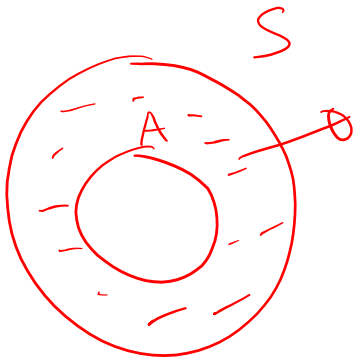
$|S| = 5 + 5 + 5 + \cdots 5 = 30$

...or we can use the Difference Method

- Possible outcomes where white and black die have different values?

A = all outcomes where white die ≠ black die

S = all outcomes, $|S| = 36$



$B = S \setminus A$
= all outcomes where white die = black die

$|B| = 6$

$\Rightarrow |A| = 36 - 6 = 30$

Lehman, 475-480

So far...

Product Rule

Product Rule:

$$|A \times B| = |A| \cdot |B|$$

- True even if A and B are not disjoint
- Useful when counting elements of a set involves dealing with tuples, sequences or a series of choices.

Insight: The Product Rule gives us how many different elements are possible

Insight #2: The multiplication finds all the possible "matches" across sets

So far...

Product Method

- If I roll a white and black die, how many possible outcomes do I see?

$A =$ all outcomes of black die

$B =$ all outcomes of white die

all outcomes $= |A \times B| = |A| \cdot |B| = 36$

Question: Can you make the above question not solvable with the product rule?

Remember: Now we are leaving behind us our ability to count elements and start developing skills that help us count sets without explicitly counting their elements

So far…

Product Rule

Product Rule:

$$|A_1 \times A_2 \times \cdots A_n| = |A_1| \cdot |A_2| \ldots |A_n|$$

Product Rule

- A restaurant has a menu with 5 Appetizers, 6 Entrees, 3 Salads, and 7 Desserts.
  - How many ways to choose a complete meal?

A = all possible complete meals

$= \{(App, Entree, Salad, Dessert), \ldots\}$

$|A| = 5 \times 6 \times 3 \times 7$

5 choices for Appetizers

6 " " Entree

3 " " Salad

7 " " Dessert

Product Rule

- A restaurant has a menu with 5 Appetizers, 6 Entrees, 3 Salads, and 7 Desserts.
  - How many ways to choose a meal if I'm allowed to skip some (or all) the courses?

$A = \{$ (App, Entree, Salad, Dessert)
(App)
(Entree)
$\vdots$ $\}$

Step 1: Make all elements the same length by including a null option. For ex: (Entree) becomes (null, Entree, null, null)

Step 2: 6 choices for Appetizer, 7 for Entree, 4 for Salad, 8 Dessert

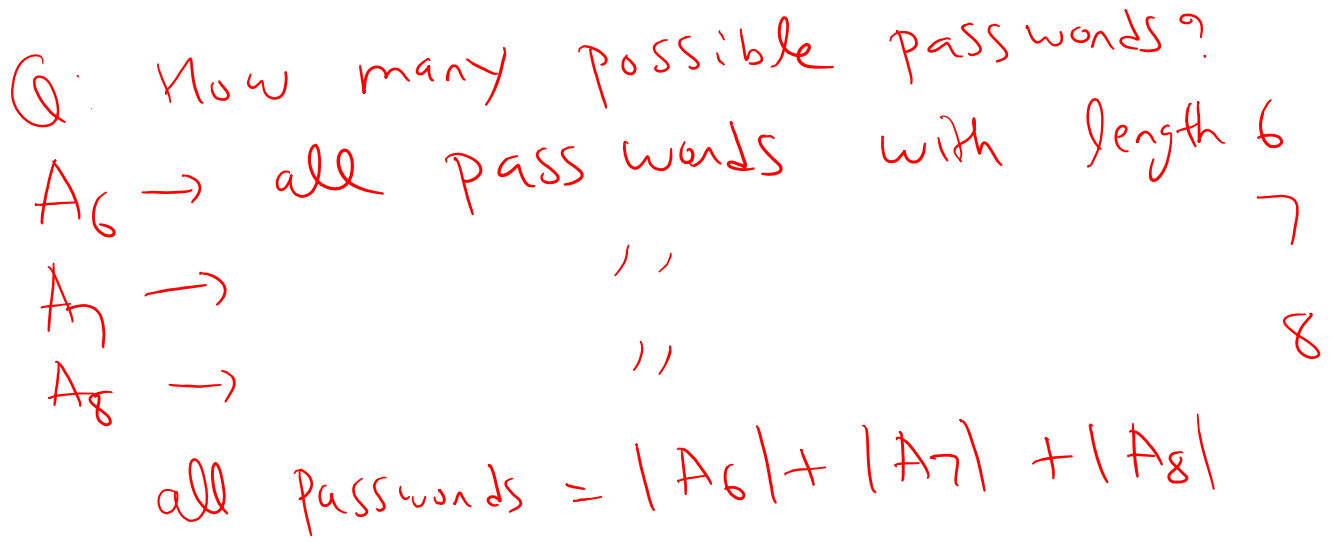
Answer = $6 \times 7 \times 4 \times 8$

So far...

Combine Methods to Count Passwords...

- You are signing up for an account on FlixBiz.com. The password has the following requirements
  - The password must be 6 to 8 characters long.
  - Each password is an uppercase letter or digit.
  - Each password must contain **at least** one digit.

Partition Method



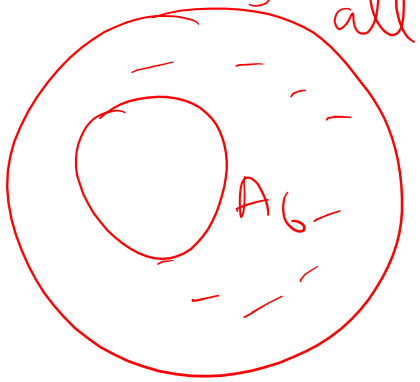
Combine Methods to Count Passwords...

A_6 = all valid passwords of length 6

S = all passwords of length 6

$B = S \setminus A_6$

B = all passwords of length 6 with no digits



Difference Method

=>

Find Contrapositive

(see Hint on next slide)

$|A_6| = |S| - |B|$

$|S| = 36^6$

$|B| = 26^6$

$\Rightarrow |A_6| = 36^6 - 26^6$

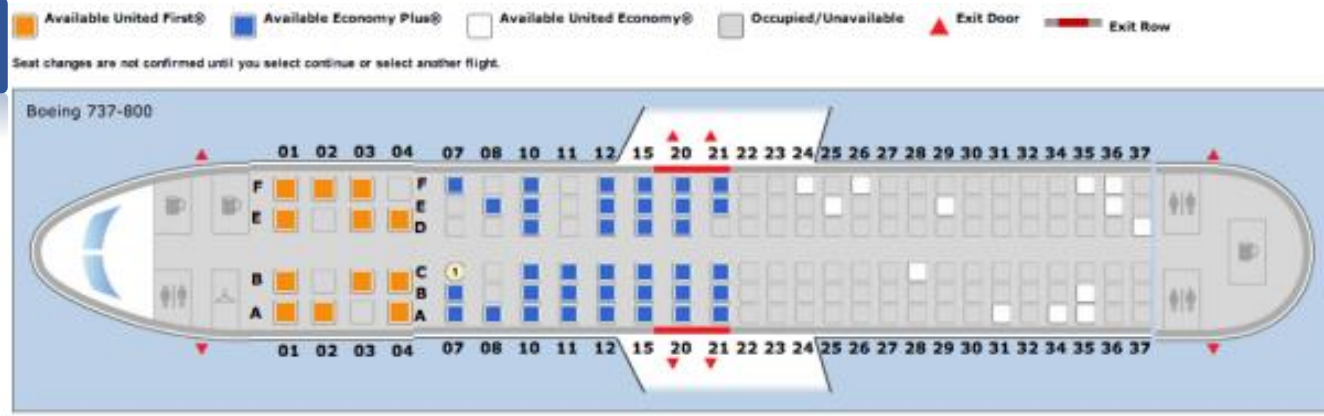
Partition Method

$|A_7| = 36^7 - 26^7$

$|A_8| = 36^8 - 26^8$

Generalized Product Rule

New Stuff



- How many ways to assign 100 passengers to 100 seats?

Let $P_1, \ldots, P_{100}$ be the passengers.

100 choices for seat of P_1

99 choices for seat of P_2

$\vdots$

1 choice for seat of P_{100}

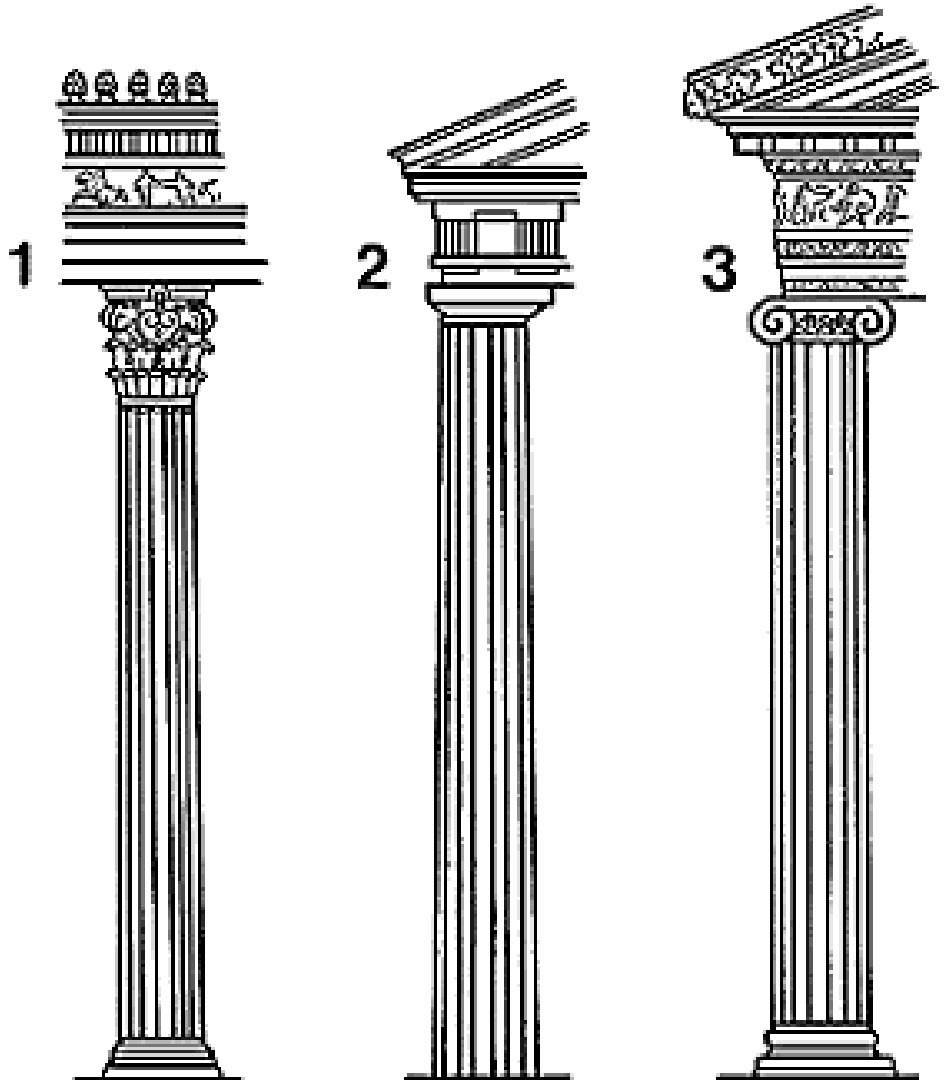
$\Rightarrow$ answer $= 100 \cdot 99 \cdot 98 \cdot 97 \cdots 3 \cdot 2 \cdot 1$

Generalized Product Rule – Order is important

- Suppose every object of a set S, can be constructed by a sequence of n choices with P_1 possibilities for the first choice, P_2 possibilities for the second choice, and so on
- **IF**
  - Each sequence of choices constructs an object in S.
  - No two different sequences **create the same** object
- THEN
  - $|S| = P_1 \times P_2 \times \cdots P_n$

Product Rule

order is important



Counting Pitfalls – and how to avoid them

- You are signing up for an account on FlixBiz.com. The password has the following requirements
  - The password must be 6 to 8 characters long.
  - Each password is an uppercase letter or digit.
  - Each password must contain at least one digit.

$A_6 \rightarrow$ all valid pass words of length 6

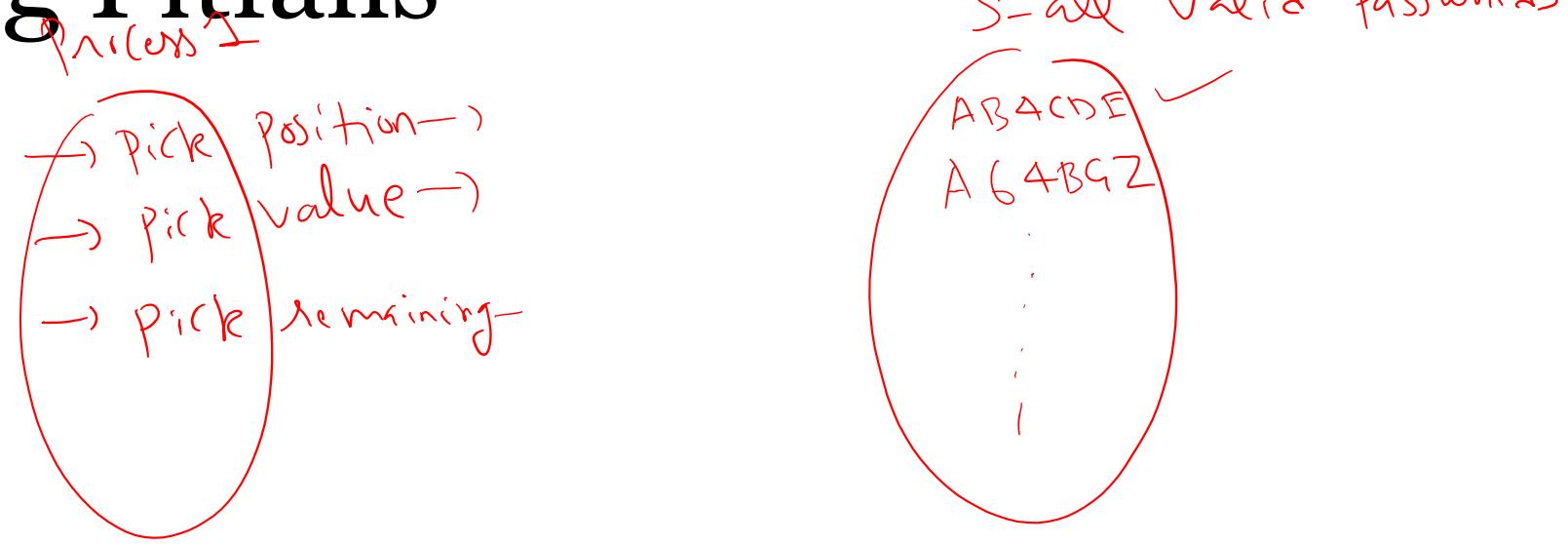
- Pick position of first digit → 6 ways
- Pick value → 10 ways → wrong
- Pick remaining values → 36^5

$|A_6| = 6 \times 10 \times 36^5 \neq 36^6 - 26^6$

Can I use the Generalized Product Rule?

We are overcounting…Why?

Counting Pitfalls



Can I use the Generalized Product Rule?

→ every choice sequence in process 1 maps to a unique element of set

→ Given element of S, must be able uniquely decode how we got to it

→ In process 1 multiple ways to reach A64BGZ

Hint! When/How to use Product Rule

- If you are counting the size of a set *S*
  - For every object in *S* you should be able to reconstruct the unique sequence of choices that led to it.

- Ask yourself
  - Am I creating objects of the right type?
  - Can I reverse engineer my choice sequence from any given object?

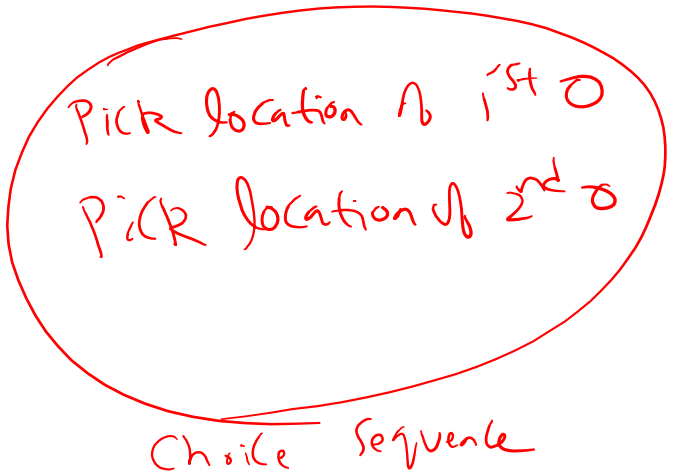
Product Rule – Counting Pitfalls cont'ed

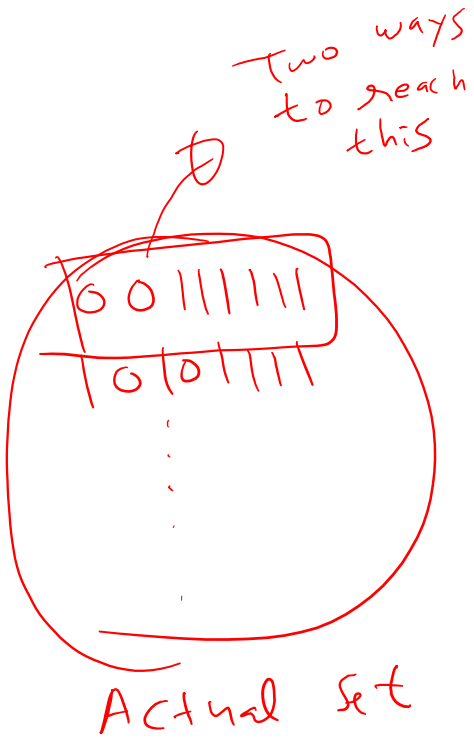
- How many binary strings of length 8 with exactly two 0's?

Pick location of 1st 0 → 8 ways

Pick location of 2nd 0 → 7 ways

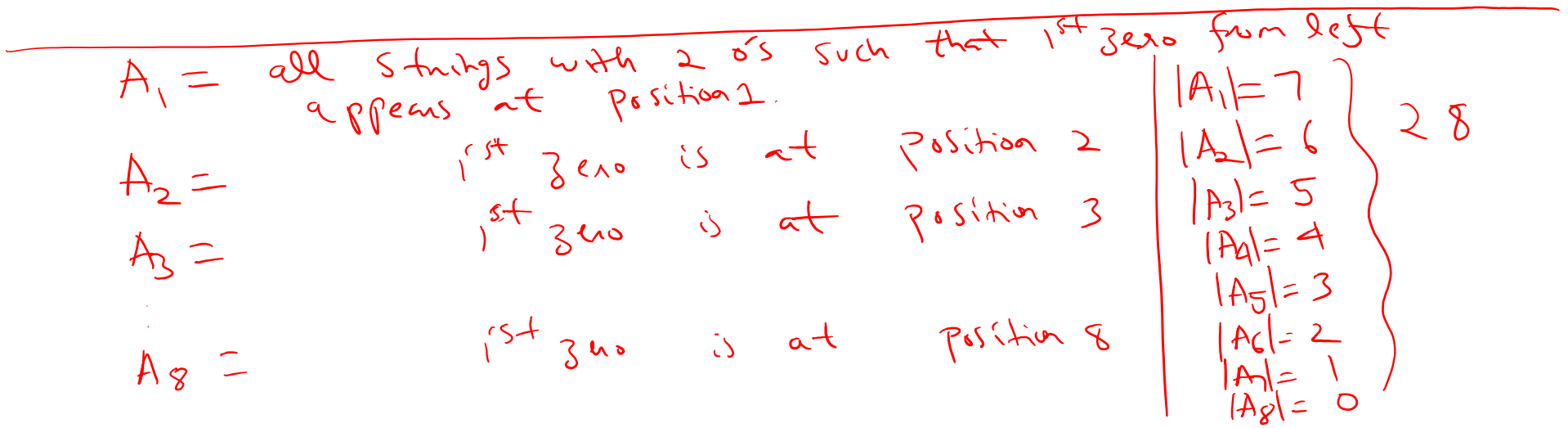
answer = 8 · 7 = 56





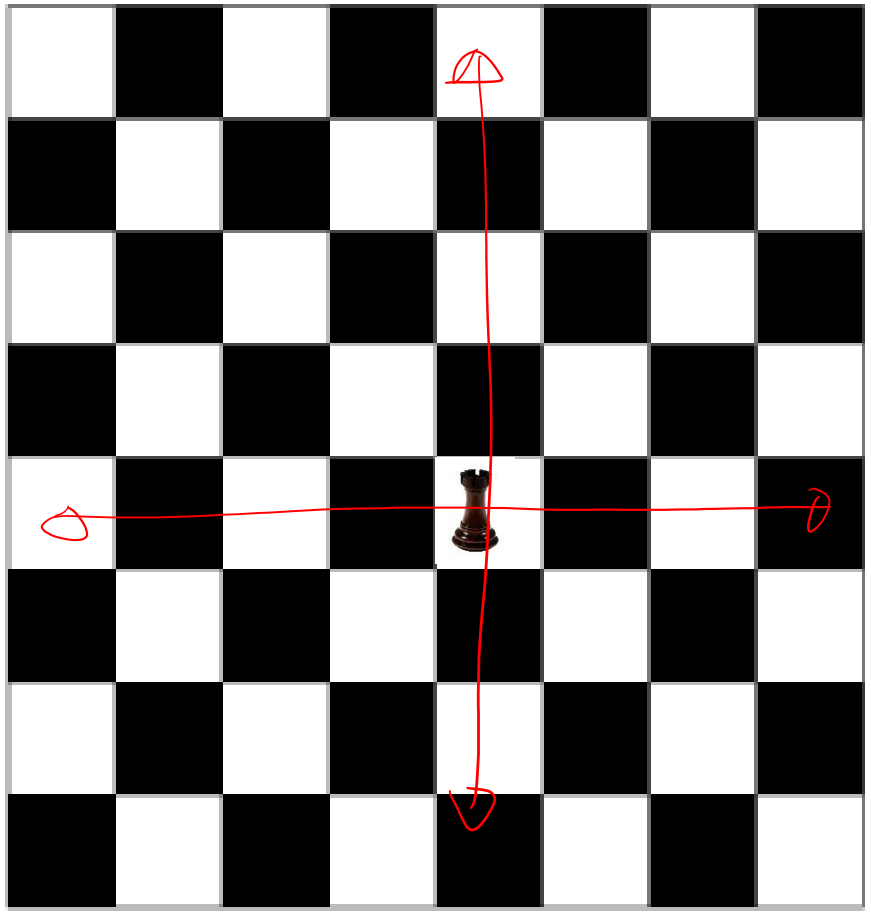
Product Rule

- How many binary strings of length 8 with exactly two 0's?



Take a 5 min Break

Generalized Product Rule



- Given two rooks labeled 1 and 2
- How many ways to place them so that they don't threaten each other?

— 64 choices for first rook
— (64-15) choices for second rook

answer (64-49)