CS206 Recitation Problem Sets Section 06

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1. A group of twenty people, consisting of ten married couples, is randomly seated in a row of twenty seats.

   (a) Suppose that one of the couples is A and B. The number of possible arrangements that Alice and Bob are seated next to each other.

   Solution: $19! \times 2!$
   19! represents arrangements of all individuals plus AB as on individual.
   2! is the interchanging of A and B.

   (b) The number of possible arrangements that everyone is seated next to their spouse (i.e. that every couple is seated together)

   Solution: $10! \times 2^{10}$
   10! is the number of arrangements of the couples, assume couples are unitary.
   $(2!)^{10} = 2^{10}$ accounts for additional arrangements if you are counting the different ordering of each couple.

2. How many ways are there to choose a collection of 4 characters from **CHANNEL**?

   Solution:
   Using partition method:

   Case 1: Choose both **N**.
   $\binom{5}{2}$ ways to choose $4 - 2 = 2$ characters from **CHAEL**.

   Case 2: Choose exactly one **N**
   $\binom{5}{3}$ ways to choose $4 - 1 = 3$ characters from **CHAEL**.

   Case 3: None of the **N** is chosen
   $\binom{5}{4}$ ways to choose 4 characters from **CHAEL**.

   Ans: $\binom{5}{2} + \binom{5}{3} + \binom{5}{4}$

3. We roll 6 standard 6-sided dice. Find the number of possible outcomes possible if order of dice does not matter.

   Solution:
   $$\frac{(5+6)!}{5!6!} \tag{1}$$

4. How many ways are there to put 5 rings onto 4 fingers?

   (a) What if the 5 rings are different?

   Solution: Let the ring be a, b, c, d, e. By putting the rings onto 4 fingers, we are actually separating the rings into 4 groups. We can use 3 separator | to indicate the separate points. Now the question becomes how many permutations are there for $abcde|||$, and the answer is $\frac{(5+4-1)!}{(4-1)!} = \frac{8!}{3!}$.

   (b) What if the 5 rings are identical?

   Solution: The order of the rings doesn't matter since they are identical. Permutations of 5 rings are 5!. We then get our answer $\frac{8!}{3!5!}$, by dividing the answer we got from the last question by 5!.

   (c) What if 3 out of 5 are identical?

   Solution: Similar as the last question. We can get our answer $\frac{8!}{3!3!}$, by dividing the answer we got from the first question by 3!.

5. We have tea bags with 4 different kinds of flavor. How many ways we can pick 10 tea bags (order does not matter)?

   (a) What if we have 100 (or infinite) tea bags for each flavor?

   Solution: We can think of this question as separating 10 tea bags into 4 groups (flavors), and we can graph the problem as $+++++++++++|||$, where $+$ indicates the tea bag and | indicates the separator. The question now becomes finding the permutations of the graph, and the answer is $\frac{(10+4-1)!}{(4-1)!10!} = \frac{13!}{3!10!}$.

   (b) What if we have only 9 tea bags for each flavor?

   Solution: We can not choose 10 tea bags with the same flavor and there are 4 ways to choose. Hence, we can obtain the answer $\frac{13!}{3!10!} - 4$ by using difference rule and the answer we got from the last question.