October 18, 2021

1. A group of twenty people, consisting of ten married couples, is randomly seated in a row of twenty seats.
   (a) Suppose that one of the couples is Alice and Bob. The number of possible arrangements that Alice and Bob are seated next to each other.

   Solution: $2! * 19!$

   19! represents arrangements of all individuals plus AB as one individual. 2! is the interchanging of A and B.

   (b) The number of possible arrangements that everyone is seated next to their spouse (i.e. that every couple is seated
   together)

   Solution: $2^{10} * 10!$

   10! is the number of arrangements of the couples, assume couples are unitary. $2^{10} = (2!)^{10}$ accounts for additional arrangements if you are counting the different ordering of each couple.

2. A police department in a small city consists of 10 officers. If the department policy is to have 5 of the officers patrolling the streets, 2 of the officers working full time at the station, and 3 of the officers on reserve at the station, how many different divisions of the 10 officers into the 3 groups are possible?

   Solution: $\binom{10}{5}\binom{5}{3}\binom{2}{2}$

3. We roll 6 standard 6-sided dice. Find the number of possible outcomes possible if order of dice does not matter.

   Solution:
   $$\frac{(5+6)!}{(5!6!)} \tag{18}$$

4. We roll 6 standard 6-sided dice. Find the number of outcomes with at least two dice showing 6 if order does not matter.

   Solution:
   $$\frac{(5+4)!}{(5!4!)} \tag{19}$$

5. We roll 6 standard 6-sided dice. Find the number of possible outcomes possible if order of dice does matters.

Solution: 6^6

6. We roll 6 standard 6-sided dice. Find the number of outcomes with at least two dice showing 6 if order matters.

Solution: $6^6 - (5^6 + \binom{6}{1}5^5)$

Using difference method, we subtract from the total number of arrangements 6^6 the amount of arrangements that do not include 2 dice with a 6. In other words, 5^6 gives us the number of arrangements where the 6 dice do not have a value of 6. Finally, the second term, $\binom{6}{1}5^5$ accounts for all the arrangements without a value of six in 5 remaining dice(since we accounted for the first die with term, 5^6.