September 27, 2019

1. You have 6 marbles: 3 green, 2 red, 1 orange, that you want to give away to your 6 friends in sequence as you encounter them through out the day. For each color, the marbles not distinguishable. How many different ways can you give out the 6 marbles? Use product rule.

   Solution: Break up the sequence into subsets by marble color. The first subset is the number of ways you can give out the single orange marble to any of your 6 friends: 6. Assume orange the marble has been given to a friend, and now you have 5 friends left for green/red marbles. The second subset is the number of ways you can give out 3 green marbles to your 5 remaining friends: 10 (by enumeration). Last set has size 1 since you have 2 friends left and 2 marbles of the same color. Since these sets are independent, then by product rule the total number of sequences is 6*10*1=60.

2. You need to make a user name consisting of 4 or 5 characters. You can use lower case English letters and digits. Because of heavy use, usernames can't start with letters "u,v,a" and can't end with digits "4,5". There are also $30,000,000$ usernames created already in the system that meet the above constraints. How many different possible user names are left available?

   Solution: Partition the problem into 2 sets: 1) 4 char usernames, 2) 5 char user names. Set 1 is of size $((26-3)+10)*((26)+10)*((26)+10)*((26)+8) = 1,454,112$. Set 2 is of size $((26-3)+10)*((26)+10)*((26)+10)*((26)+10)*((26)+8) = 52,348,032$. By partition method, total possible usernames meeting the constraints are $1,454,112+52,348,032 = 53,802,144$. By difference method, there are $53,802,144 - 30,000,000 = 23,802,144$ available usernames.