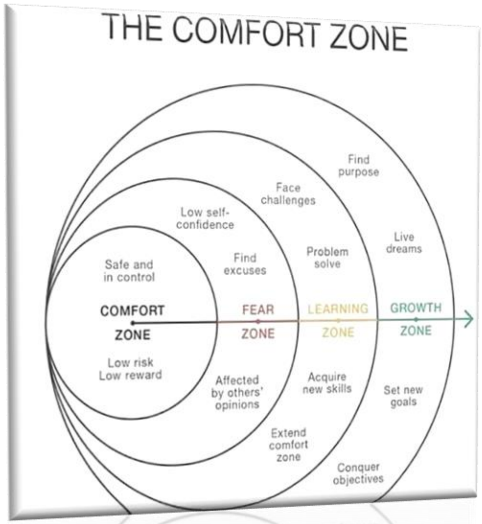

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Discrete Structures II

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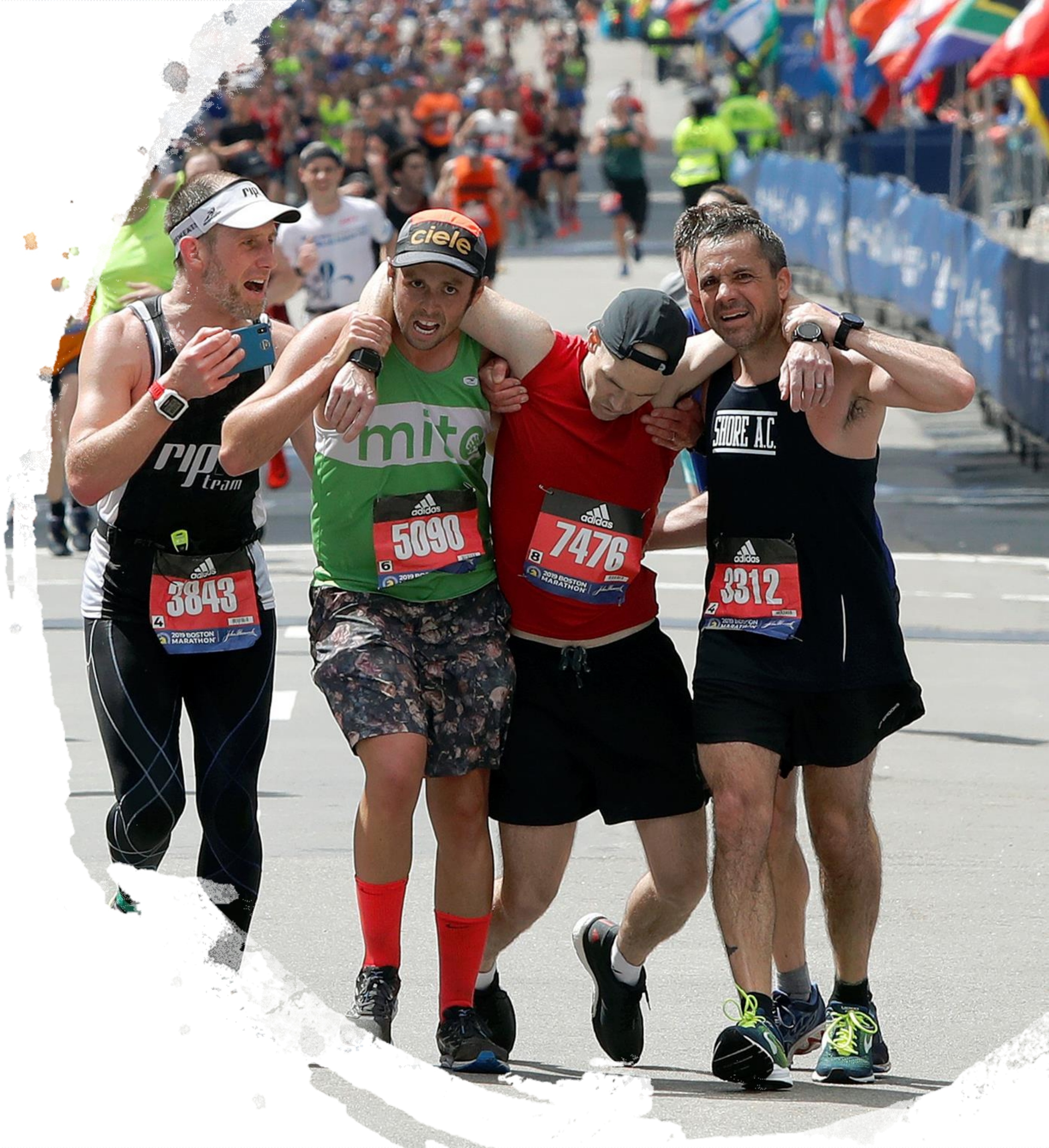
Announcements

- Assignment 2 is running
- Quiz 5 → Next week
- Extra Homework for extra credits?
    - We will find out soon…



Probabilities - Outline for this month

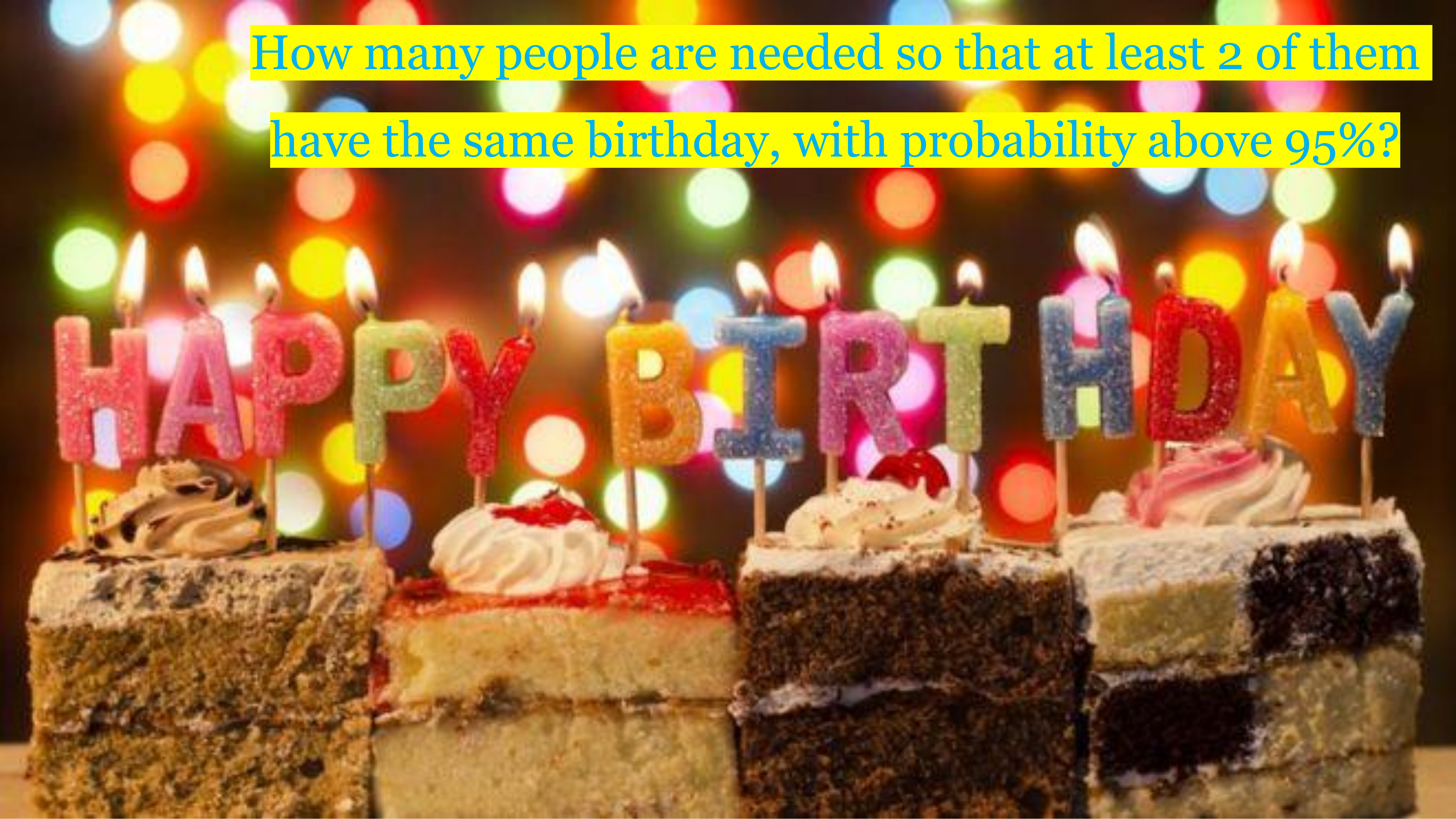
Basic building blocks

- Sample spaces and events
- Basics of probability
- Independence, conditional probability
- Random variables, expectation, variance

Intermediate

Advanced

How many people are needed so that at least 2 of them have the same birthday, with probability above 95%?



Last Class

Uniform Distribution

- 23 people are in a room. Suppose all birthdays are equally likely. What is the probability that two will have the same birthday?

Assume $\rightarrow$ 365 possible birth days

$\Omega \rightarrow$ all possible assignment of birthdays to 23 people

$|\Omega| = 365^{23}$

$A \rightarrow$ all outcomes where at least two have same birthday

Last Class

Uniform Distribution

- 23 people are in a room. Suppose all birthdays are equally likely. What is the probability that two will have the same birthday?

$P(A) = 1 - \frac{|B|}{|\Omega|}$

B → all outcomes where no two have same birthday

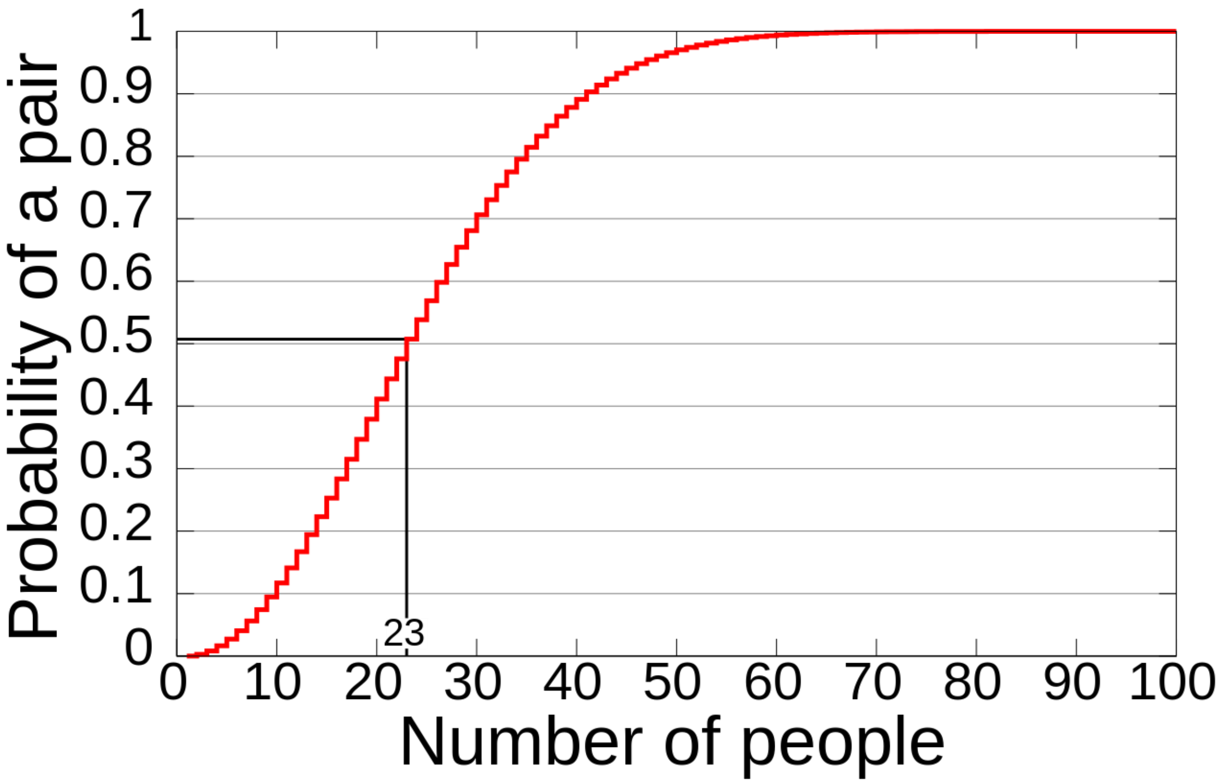
$|B| = {}_{365}P_{23} = 365 \cdot 364 \cdot 363 \cdots$

$P(A) = 1 - \frac{365P_{23}}{(365)^{23}} \approx .5027$

Uniform Distribution

- 23 people are in a room. Suppose all birthdays are equally likely. What is the probability that two will have the same birthday?

Birthday Paradox!!



Uniform Distribution

- 55% of adults consume coffee regularly, 45% consume soda and 70% regularly consume at least one of the two.
- What is probability that a randomly selected adult consumes both coffee and soda?

A → an adult consumes coffee regularly

B → an adult consumes soda regularly

$P(A) = .55,\ P(B) = .45,\ P(A \cup B) = .7$

Want: $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= .55 + .45 - .7$

Uniform Distribution

- 55% of adults consume coffee regularly, 45% consume soda and 70% regularly consume at least one of the two.
- What is the probability that a randomly selected individual doesn't consume either of the two.

$A \cup B$ = people who consume at least one of two

$(A \cup B)'$ = people who don't consume either

$P((A \cup B)') = 1 - P(A \cup B) = 1 - .7$

Last Class

Uniform Distribution

- A box contains six 40W bulbs, five 60W bulbs and four 75W bulbs. If bulbs are selected one by one in a random order, what is the probability that at least two bulbs must be selected in order to get one that is rated 75W?

identical identical identical

$A \rightarrow$ at least 2 tries for seeing 75 W

$A' \rightarrow$ see 75 W bulb on first try

$$P(A) = 1 - P(A') = 1 - \frac{|A'|}{|\Omega|}$$

$$|\Omega| = \frac{15!}{6!\,5!\,4!}, \quad |A'| = \frac{14!}{6!\,5!\,3!}$$

Next…

- Conditional Probability
  - And formula…
- Independent Events

Outline for this month

- Sample spaces and events
- Basics of probability
- Independence, conditional probability
- Random variables, expectation, variance

→ Basic building blocks

→ Intermediate

→ Advanced

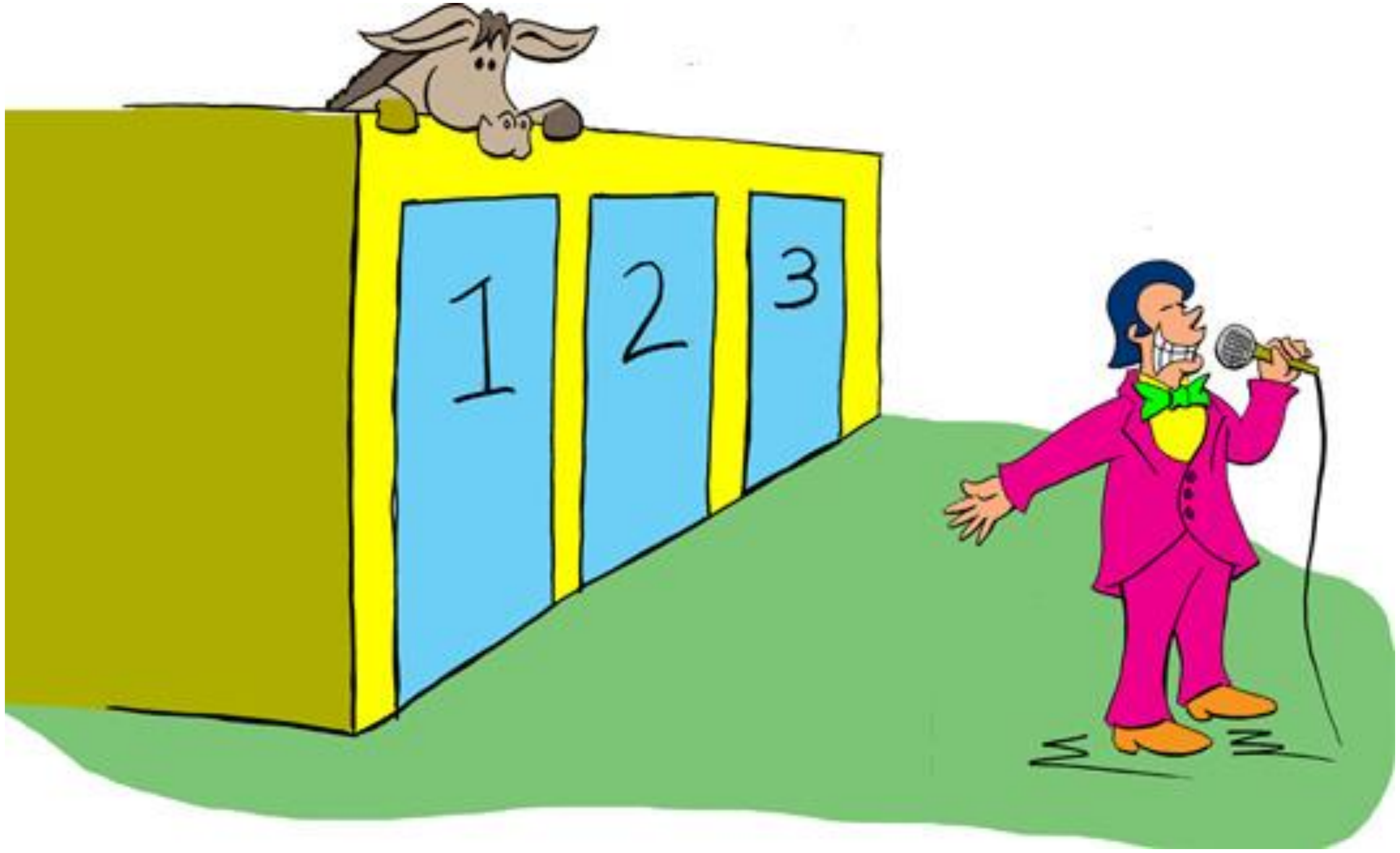
Conditional Probabilities - Example

A = man survives

- A man went on an airplane ride. → $P(A) = .1$
- Unfortunately, he fell out.
- Fortunately, he had a parachute on. → $P(A) = .7$
- Unfortunately, the parachute did not open. → $P(A) = .1$
- Fortunately, there was a haystack below him, directly in the path of his fall. → $P(A) = .5$
- Unfortunately, there was a pitchfork sticking out of the top of the haystack.
- Fortunately, he missed the pitchfork. → $P(A) = .1$
- Unfortunately, he missed the haystack.

Monty Hall Problem

Door 1 → G
Door 2 → G
Door 3 → Car



"All the News That's Fit to Print"

The New York Times

NEW YORK, SUNDAY, JULY 21, 1991

Quandary Created by Gain In Detecting Breast Cancer

New Methods Still Leave Doubts on What to Do

SAUDIS WOULD LIFT BOYCOTT IF ISRAEL HALTS SETTLEMENT

States Seeking Aid on Budgets From Pensions

After 16 Years of War, Beirut Tentatively Reclaims Peace

U.N. Says Kurds Take Most Parts Of an Iraqi City

Behind Monty Hall's Doors: Puzzle, Debate and Answer?

Communities and Their Residents Age Gracefully

Monty Hall Problem

Door 1 → Goat
Door 2 → Goat
Door 3 → Car

- Announcer hides prize behind one of 3 doors. You select some door at random. Announcer opens one of others with no prize. You can decide to keep or switch.
- What to do?

A → I win If I switch

B → door with no prize is revealed

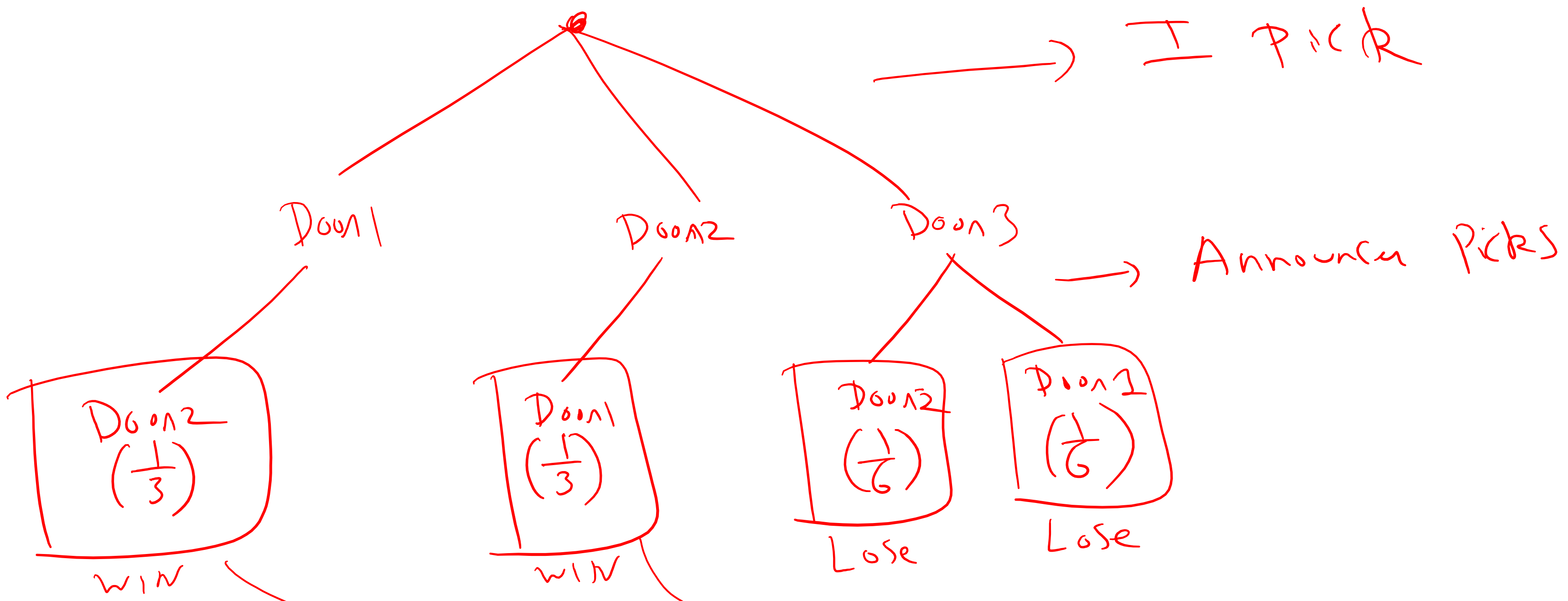
$P(A|B) > \frac{1}{2}$ or $< \frac{1}{2}$??

Will show $P(A|B) = \frac{2}{3}$

In this case $P(B) = 1$ [Because of rules of the game]

Monty Hall Problem

Door 1 → G
Door 2 → G
Door 3 → Car



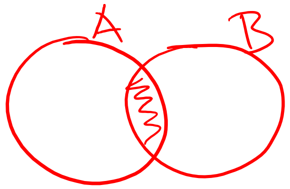
$$P(\text{win by switching}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Conditional Probabilities

- Suppose we roll a white and a black die. What is the probability that the white die is 1 given that the sum is 7?
- A = white die is 1
- B = sum if 7
- We want $P(A|B)$

→ Conditional Probability

$$P(A) = \frac{|A|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}$$



We know B has happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ → Formula for conditional probability

$$= \frac{|A \cap B|}{|B|}$$ → If equally likely outcomes