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No quiz today!

206
Discrete Structures II

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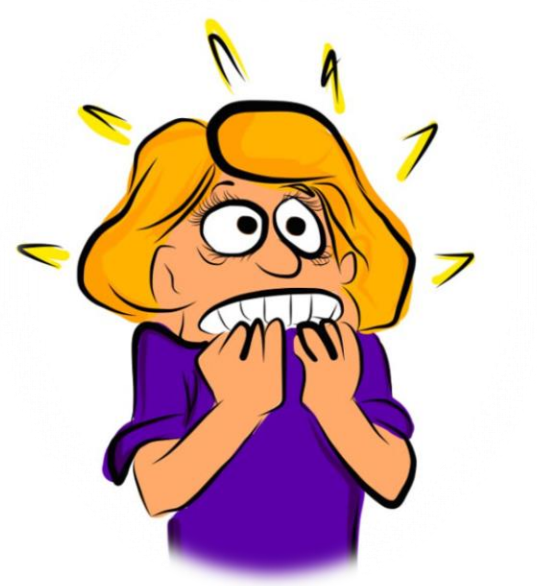
Lecture 7 | Who is Counting? | Thursday 9/23 2021

Quiz 1

September

2021

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6 Labor Day	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		



Quiz 1

- What will Quiz 1 cover?
  - Sets (Lecture 2)
  - Venn (Lecture 2)
  - Functions (Lecture 3)
  - Proofs (Lectures 3-5)
  - + What we will cover today (Sum and Product rules)

Reading for Quiz 1

Recap and Basics of Counting	Chapters 1, 2 and 5 of Rosen
Basics of Counting	Chapters 1, 2 and 5 of Rosen Chapter 15 of Lehman
Basics of Counting	Chapters 6 of Rosen Chapter 15 of Lehman

What we will cover today

Combinatorics

- Recap
  - Counting (Partition, Difference)
- Today
  - Counting
    - Product Rule
    - *Combining Rules!*
    - Bijection Rule
- Next
  - Permutations/Combinations
  - Pigeonhole Principle

Course Outline

- Part I
  - Recap of basics – sets, function, proofs, induction
  - Basic counting techniques
  - Pigeonhole principle
  - Generating functions
- Part II
  - Sample spaces and events
  - Basics of probability
  - Independence, conditional probability
  - Random variables, expectation, variance
  - Moment generating functions
- Part III
  - Graph Theory
  - Machine learning and statistical inference

New Stuff

Counting

Counting

- In the next few lectures
  - Fundamental tools and techniques for counting
  - Sum Rule
  - Product Rule
  - Difference Method
  - Bijection Method
  - Permutations/Combinations
  - Inclusion Exclusion
  - Binomial/Multinomial coefficients

Fundamental Blocks

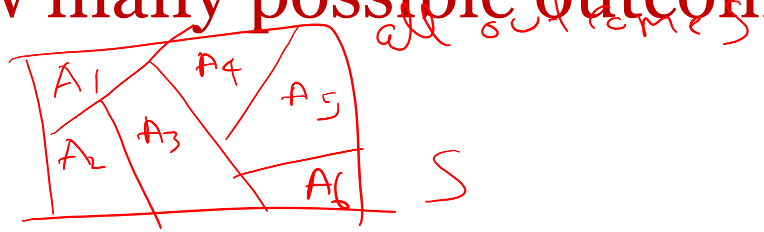
→ Intermediate

→ Advanced

So far...

Partition Method

- If I roll a white and black die, how many possible outcomes do I see?



$A_1 =$ all outcomes with black die $= 1$

$A_2 =$ all outcomes with black die $= 2$

⋮

$A_6 =$ all outcomes with black die $= 6$

$|S| = |A_1| + |A_2| + \dots + |A_6|$

$= 6 \cdot 6 = 36$

So far…

Difference Method

- To find the size of a set A,
  - Find a larger set S such that $S = A \cup B$ and
  - A and B are disjoint.
  - $|A| = |S| - |B|$

- Possible outcomes where white and black die have different values?
  - Find S with all possible outcomes $|S|$=36
  - Subtract B with the same values |B|=6
  - $|A| = |S| - |B| = 36 - 6 = 30$

So far...

Partition Method

- Possible outcomes where white and black die have different values?

$S =$ all possible outcomes

$A_1 =$ all outcomes with black die $= 1$

$A_2 =$ black die $= 2$

⋮

$A_6 =$ black die $= 6$

$|A_1| = 5, \; |A_2| = 5,$

$|S| = 5 + 5 + 5 + \cdots 5 = 30$

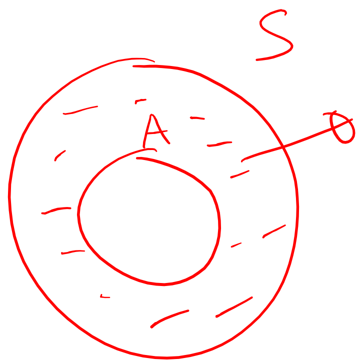
So far...

...or we can use the Difference Method

- Possible outcomes where white and black die have different values?

$A =$ all outcomes where white die $\neq$ black die

$S =$ all outcomes, $|S| = 36$



$B = S \setminus A$

$=$ all outcomes where white die $=$ black die

$|B| = 6$

$\Rightarrow |A| = 36 - 6 = 30$

Product Rule

Product Rule:
$|A \times B| = |A| \cdot |B|$

- True even if A and B are not disjoint
- Useful when counting elements of a set involves dealing with tuples, sequences or a series of choices.

Insight: The Product Rule gives us how many different elements are possible

Insight #2: The multiplication finds all the possible "matches" across sets

New Stuff

Product Method

- If I roll a white and black die, how many possible outcomes do I see?

$A =$ all outcomes of black die

$B =$ all outcomes of white die

all outcomes $= |A \times B| = |A| \cdot |B| = 36$

Question: Can you make the above question not solvable with the product rule?

Remember: Now we are leaving behind us our ability to count elements and start developing skills that help us count sets without explicitly counting their elements

Product Rule

Product Rule:
$|A_1 \times A_2 \times \cdots A_n| = |A_1| \cdot |A_2| \ldots |A_n|$

Product Rule

- A restaurant has a menu with 5 Appetizers, 6 Entrees, 3 Salads, and 7 Desserts.
  - How many ways to choose a complete meal?

A = all possible complete meals

$= \{ (\text{App}, \text{Entree}, \text{Salad}, \text{Dessert}), \ \vdots \ \}$

$|A| = 5 \times 6 \times 3 \times 7$

5 choices for Appetizers
6 " " Entree
3 " " Salad
7 " " Dessert

Product Rule

- A restaurant has a menu with 5 Appetizers, 6 Entrees, 3 Salads, and 7 Desserts.
  - How many ways to choose a meal if I'm allowed to skip some (or all) the courses?

$$A = \{ (\text{APP}, \text{Entree}, \text{Salad}, \text{Dessert}),\ (\text{APP}),\ (\text{Entree}),\ \vdots \}$$

Step 1: Make all elements the same length by including a null option. For ex: (Entree) becomes (null, Entree, null, null)

Step 2: 6 choices for Appetizer, 7 for Entree, 4 for Salad, 8 Dessert

Answer = $6 \times 7 \times 4 \times 8$

Take a Break

5 min

Combine Methods to Count Passwords…

- You are signing up for an account on FlixBiz.com. The password has the following requirements
  - The password must be 6 to 8 characters long.
  - Each password is an uppercase letter or digit.
  - Each password must contain **at least** one digit.

Partition Method

Q: How many possible passwords?

$A_6 \rightarrow$ all passwords with length 6

$A_7 \rightarrow$ " " 7

$A_8 \rightarrow$ " " 8

all passwords $= |A_6| + |A_7| + |A_8|$

Hint (or …When to think of Partition Method)

- When you are asked to count something that exists in easy-to-count ways (e.g., between 2 and 4), consider dividing the problem to the enumerable cases and then use the Partition Method
  - Note that if the different cases are too many (e.g., 100), then most probably the intention of the exercise is not to stress your patience mechanisms…

Combine Methods to Count Passwords…

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  - The password must be 6 to 8 characters long.
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  - Each password must contain **at least** one digit.

Partition Method

Q: How many possible passwords?

$A_6 \to$ all passwords with length 6

$A_7 \to$ " 7

$A_8 \to$ " 8

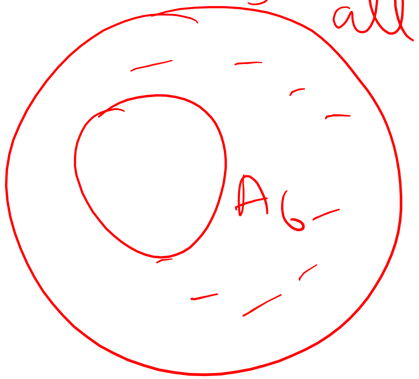
all passwords $= |A_6| + |A_7| + |A_8|$

Combine Methods to Count Passwords…

$A_6 =$ all valid passwords of length 6

$S =$ all passwords of length 6

$B = S \setminus A_6$



$B =$ all passwords of length 6 with no digits

Difference Method

=>

Find Contrapositive

(see Hint on next slide)

$|A_6| = |S| - |B|$

$|S| = 36^6$

$|B| = 26^6$

$\Rightarrow |A_6| = 36^6 - 26^6$

Partition Method

$|A_7| = 36^7 - 26^7$

$|A_8| = 36^8 - 26^8$

Hint: When to use Difference Method

When you are asked to count something that exists in

"at least" one place, consider counting the opposite

(that is "nowhere")

Which means: You need to be able to find the

"contrapositive argument".