CS206 Recitation Problem Sets Section 06

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1. For finite sets X and Y, the number of possible binary relations (x, y) where $x \in X$ and $y \in Y$, is $|X| \cdot |Y|$.

   Solution: True, since every element of X can map onto any other element of Y, total number of mappings is product of elements in X and Y.

2. How many different possible words(existing and not-existing) can be made from the word "WALLET" such that the vowels are *never* together?

   Solution: By separating "A" and "E", we have 3 spaces to insert 4 letters "WLLT". To simplify the question, treat 4 letters as the same letter first, and later we multiply it by $4!/2! = 12$
   To keep A and E always stay apart from each other, we have to insert one letter in the middle space. We then have to calculate the number of possible ways to put $4 - 1 = 3$ same letters into 3 different spaces.
   Instead of putting 3 same letters into 3 spaces, we can think it as using 2 separator to separate 3 same letters, and that will be $\frac{5!}{2!3!} = 10$ ways. Or we could use the combinations with repetition formula $\binom{n+r-1}{n}$ where n = 3 and r = 3, and get the same result $\binom{3+3-1}{3} = \binom{5}{3} = \frac{5!}{3!(5-3)!} = 10$.
   Also we have to take vowel arrangements into account by multiplying by 2!.
   Thus, permutations with vowels never together is $12 \times 10 \times 2! = 240$.

   Solution: Total number of permutations of WALLET is 6!/2!=360, because we have 6 letters with one letter repeating twice.
   Permutations with vowels together is (5!/2!)*2!=120, because vowels are assumed to be one letter, resulting in arrangement of only 5 letters with one letter repeating twice. We also taking into account vowel arrangements by multiplying by 2!
   Thus, permutations with vowels never together is 360-120=240, by difference method