CS206 Quiz 6 **Dec 7, 2021** **A**

Name: ______________________________

NetID: _____________(Please **PRINT**)

Section No.: _____________

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1. (20%) Which of the following are true statements? Briefly explain.

   (a) $P(-A|B) = 1 - P(A|B)$

   Solution: True

   (b) $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

   Solution: True

   (c) $P(A|B) = \frac{P(B \bigcap A)}{P(B)}$

   Solution: True

   (d) The probability of flipping an unbiased coin and getting 2 heads in a row is $P(\{H, H\}) = 0.5 + 0.5 = 1.0$

   Solution: False

2. A toy car company produces 2 types of model cars; 40% are sedans and 60% are SUVs. Additionally, there each type of car can be either red or blue. Sedans are 50% red and SUVs are 75% red. After the production process, cars randomly partitioned into boxes, with 100 cars per box. You bought one of these boxes.

   (a) (5%) Draw the probability tree for the model car manufacturing process. Label only the branch probabilities involved.(Don't calculate leave probabilities)

   Solution: From root 2 branches with nodes: 0.4 sedan node and 0.6 SUV node. Sedan node has 2 branches with nodes: 0.5 blue color node and 0.5 red color node. SUV has 2 branches with nodes: 0.75 red color and 0.25 blue color.

   (b) (10%) What is the probability that you choose a blue car from the box?

   Solution: Let B be the event that the car you choose is blue and C be the event that the car type you choose is a sedan. Then, $P(B) = p(B|C)P(C) + p(B|C')P(C') = (0.5 \times 0.4) + (0.25 \times 0.6) = 0.35$

(c) (10%) What is the probability that the blue car that you choose in part b is an SUV?

Solution: By bayes theorem and part b, we have $P(C'|B) = \frac{P(B|C')P(C')}{P(B)} = \frac{0.25\times 0.6}{0.35} = 0.429$

3. A test for a rare medical disease has a probability of 0.95 to positively classify a person suffering with the disease. Also, it has a probability of 0.1 to positively classify a non-diseased person. There is a probability of 0.005 for any given person to have the disease. Given an arbitrary person, what is the probability that:

Solution: Let C =person is classified positively, D =person is diseased,W =person is classified incorrectly

(a) (5%) The test classification will be positive?

Solution: $P(C) = P(C|S)P(D)+P(C|-D)P(-D) = (0.95\times 0.005)+(0.1\times 0.995) = 0.10425$

(b) (10%) The person is diseased, given a positive classification?

Solution: $P(D|C) = \frac{P(C|D)P(D)}{P(C|S)P(D)+P(C|-D)P(-D)} = \frac{0.95\times 0.005}{(0.95\times 0.005)+(0.1\times 0.995)} = 0.0455$

(c) (10%) The person is not diseased, given a negative classification?

Solution: $P(-D|-C) = \frac{P(-C|-D)P(-D)}{P(-C)} = \frac{0.9\times 0.995}{1-0.10425} = 0.9997$

(d) (10%) the person is classified incorrectly?

Solution: $P(W) = P(C\bigcap -D)+P(-C\bigcap D) = P(C|-D)P(-D)+P(-C|D)P(D) = (0.1\times 0.995)+(0.05\times 0.005) = 0.09975$

4. (20%) A box contains three coins: two regular coins and one fake two-headed coin ($P(H) = 1$). You pick a coin at random and toss it. What is the probability that it lands heads up?

Solution: 2/3

5. (extra credits – 20%) Suppose that there are two slot machines, one of which pays out 10% of the time and the other pays out 20% of the time. Unfortunately, you have no idea which is which. Suppose you randomly choose a machine and put in a quarter.

(a) If you don't get a jackpot, what is the chance that you chose the machine that pays out 20% of the time?

Solution: $P(S^C|J^C) = \frac{0.8*0.5}{0.8*0.5+0.9*0.5} = 8/17 = 0.471$

(b) If you had instead gotten a jackpot, what would be the chance that you chose the one that pays out 20% of the time?

Solution: $P(S^C|J) = \frac{0.2*0.5}{0.1*0.5+0.2*0.5} = 2/3 = 0.667$

CS206 Quiz 6 **Dec 9, 2021** **B**

Name: ____________________________

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Section No.: ____________

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1. (20%) Which of the following are true statements? Explain briefly.

   (a) $P(-A|B) = P(A|B) - 1$

   Solution: False

   (b) $P(B|A)P(A) = P(A|B)P(B)$

   Solution: True

   (c) $P(A|B) = \frac{P(B)}{P(B \bigcap A)}$

   Solution: False

   (d) The probability of flipping an unbiased coin and getting 2 heads in a row is $P(\{H, H\}) = 0.5 \times 0.5 = 0.25$

   Solution: True

2. A toy car company produces 2 types of model cars; 40% are sedans and 60% are SUVs. Additionally, there each type of car can be either red or blue. Sedans are 50% red and SUVs are 75% red. After the production process, cars randomly partitioned into boxes, with 100 cars per box. You bought one of these boxes.

   (a) (5%) Draw the probability tree for the model car manufacturing process. Label only the branch probabilities involved.(Don't calculate leaf probabilities)

   Solution: From root 2 branches with nodes: 0.4 sedan node and 0.6 SUV node. Sedan node has 2 branches with nodes: 0.5 blue color node and 0.5 red color node. SUV has 2 branches with nodes: 0.75 red color and 0.25 blue color.

   (b) (10%) What is the probability that you choose a blue car from the box?

   Solution: Let B be the event that the car you choose is blue and C be the event that the car type you choose is a sedan. Then, $P(B) = p(B|C)P(C) + p(B|C')P(C') = (0.5 \times 0.4) + (0.25 \times 0.6) = 0.35$

(c) (10%) What is the probability that the blue car that you choose in part b is an SUV?

Solution: By bayes theorem and part b, we have $P(C'|B) = \frac{P(B|C')P(C')}{P(B)} = \frac{0.25\times0.6}{0.35} = 0.429$

3. A test for a rare medical disease has a probability of 0.9 to positively classify a person suffering with the disease. Also, it has a probability of 0.15 to positively classify a non-diseased person. There is a probability of 0.01 for any given person to have the disease. Given an arbitrary person, what is the probability that:

   Solution: Let C =person is classified positively, D =person is diseased,W =person is classified incorrectly

   (a) (5%) The test classification will be positive?

   Solution: $P(C) = P(C|S)P(D) + P(C|-D)P(-D) = (0.9 \times 0.01) + (0.15 \times 0.99) = 0.1575$

   (b) (10%) The person is diseased, given a positive classification?

   Solution: $P(D|C) = \frac{P(C|D)P(D)}{P(C|S)P(D)+P(C|-D)P(-D)} = \frac{0.9\times0.01}{(0.9\times0.01)+(0.15\times0.99)} = 0.0571$

   (c) (10%) The person is not diseased, given a negative classification?

   Solution: $P(-D|-C) = \frac{P(-C|-D)P(-D)}{P(-C)} = \frac{0.85\times0.99}{1-0.1575} = 0.9988$

   (d) (10%) the person is classified incorrectly?

   Solution: $P(W) = P(C\bigcap -D) + P(-C\bigcap D) = P(C|-D)P(-D) + P(-C|D)P(D) = (0.15 \times 0.99) + (0.1 \times 0.01) = 0.1495$

4. (20%) A box contains three coins: two regular coins and one fake two-headed coin ($P(H) = 1$). You pick a coin at random and toss it. What is the probability that it lands heads up?

   Solution: Let C1 be the event that you choose a regular coin, and let C2 be the event that you choose the two-headed coin. $P(H) = P(H|C1)P(C1) + P(H|C2)P(C2) = 1/2 * 2/3 + 1 * 1/3 = 2/3$

5. (extra credits – 20%) Suppose that there are two slot machines, one of which pays out 10% of the time and the other pays out 20% of the time. Unfortunately, you have no idea which is which. Suppose you randomly choose a machine and put in a quarter.

(a) If you don't get a jackpot, what is the chance that you chose the machine that pays out 20% of the time?

Solution: $P(S^C|J^C) = \frac{0.8*0.5}{0.8*0.5+0.9*0.5} = 8/17 = 0.471$

(b) If you had instead gotten a jackpot, what would be the chance that you chose the one that pays out 20% of the time?

Solution: $P(S^C|J) = \frac{0.2*0.5}{0.1*0.5+0.2*0.5} = 2/3 = 0.667$