October 4, 2021

1. PERMUTATION PROBLEMS:

   (a) Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

   Solution: There are 4! 3! 2! 1! arrangements such that the mathematics books are first in line, then the chemistry books, then the history books, and then the language book. Similarly, for each possible ordering of the subjects, there are 4! 3! 2! 1! possible arrangements. Hence, as there are 4! possible orderings of the subjects, the desired answer is 4! 4! 3! 2! 1! = 6912.

   (b) How many permutations of work "REMAINS" are there such that vowels are always in odd places of the word?

   Solution: First, vowel letters are A, E, I, O, and U. Then there are 4 odd places in the 7 letter word, so p(4,3)=24 ways to assign 3 vowels to 4 places. There rest of the places are assigned a consonant p(4,4)=4!=24. So total is 24*24=576 ways

   (c) Given word "SUPER", how many words can be formed such that vowels are grouped together?

   Solution: Assume the two vowels are one letter, then number of ways to arrange the 4 letters are 4!=24. And the number of ways to arrange 2 vowels is 2!=2. So total number of words you can form given constraints is 24*2=48

   (d) What about word "BUTTER"?

   Solution: Again only two vowels which we assume are one letter. So now we have 5 letters of which two are the same, so number of words to form is 5!/2!=60. Again, the number of ways to arrange 2 vowels is 2!=2. So in total its 60*2=120

2. COMBINATION PROBLEMS:

   (a) How many different letter arrangements can be formed from the letters **PEPPER**??

   Solution: There are 6!/(3! 2!) = 60 possible letter arrangements of the letters PEPPER.

   (b) From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed? What if 2 of the men are feuding and refuse to serve on the committee together?

Solution: (a) C(5,2)*C(7,3) = 350
(b) Now suppose that 2 of the men refuse to serve together. Because a total of C(2,2)*C(5,1)=5 out of the C(7,3) = 35 possible groups of 3 men contain both of the feuding men, it follows that there are 35 - 5 = 30 groups that do not contain both of the feuding men. Because there are still C(5,2) = 10 ways to choose the 2 women, there are 30 * 10 = 300 possible committees in this case.

3. Use Bijection Method: Out of 10 different animals, how many ways can you choose 4 in any order?

Solution: Let $|P|$ be number of permutations of 4 animals chose from 10. Let $|C|$ be the combinations of 4 animals. Then one element c maps to many elements in p. By bijection method

$$|C| = \frac{|P|}{4!} = \frac{nPr}{4!} \tag{15}$$
$$= \frac{10!}{6!4!} \tag{16}$$
$$\tag{17}$$

4. How many strings of length 4 over the alphabet $0, 1, ..., 9$ do not begin with 0?

Solution: By product rule: $9 * 10^3 = 9000$ of length 4 that do not begin with a 0.

5. You need to come up with a password that at minimum need to have 5 characters and at most 7. Only uppercase letters can be used and digits $0, 1, ..., 9$. There must be at least 1 digit in a password. How many passwords are possible?

Solution: By partition method, we separately count number of possible passwords of size 5, P_5, of size 6, P_6, and of size 7, P_7. Hence, total number of passwords is $P = P_5 + P_6 + P_7$. To account for the single digit constraint, we do indirect counting using difference method. So we first count how many illegal password there are: 26^5 for passwords of length 5. Then number of legal passwords by difference method is $P_5 = 36^5 - 26^5$. Similarly, $P_6 = 36^6 - 26^6$ and $P_7 = 36^7 - 26^7$. Then $P = P_5 + P_6 + P_7 = (36^5 - 26^5) + (36^6 - 26^6) + (36^7 - 26^7)$

6. California license plate begins with nonzero digit, followed by 3 uppercase English letters, followed by 3 digits from $0, 1, ..., 9$. How many numbers have

- no repeated digit? **Solution:** $9 * 26^3 * 9 * 8 * 7$
- no repeated letter? **Solution:** $9 * 26 * 25 * 24 * 10^3$
- no repeated symbol? **Solution:** $9 * 26 * 25 * 24 * 9 * 8 * 7$