$f, g : \mathbb{N} \to \mathbb{R}$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \leq c \triangleq f = O(g).$$

$$= c - \delta \quad (\delta \geq 0)$$

$$\begin{cases} \forall \varepsilon > 0, \exists N \text{ s.t.} \\ \quad \forall n \geq N \quad \left| \frac{f(n)}{g(n)} - (c-\delta) \right| < \varepsilon \end{cases}$$

$$\Downarrow$$

$$\forall \varepsilon > 0, \exists N \text{ s.t.}$$

$$\forall n \geq N, \quad \frac{f(n)}{g(n)} \leq c + \varepsilon$$

$$\boxed{\to \; f(n)/g(n) \leq c}$$

$$\to \iff f(n) \leq c \cdot g(n)$$

$$f(n) = \overline{100n}, \; g(n) = \overline{n^2}.$$

$$\to n = 1, \; f(n) = 100, \; g(n) = 1.$$

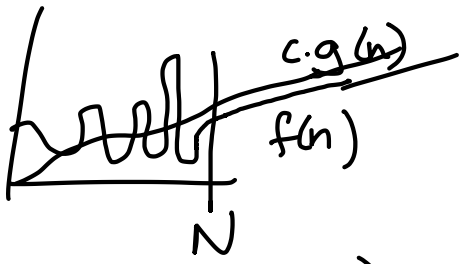
$$\to n = 1000, \; \hookrightarrow 10^5 \quad \hookrightarrow 10^6$$

$$n \geq 1000.$$

$\log(n) \stackrel{?}{=} O(\log\log n)$ ✗

$10^5 \sqrt{\log(n)} \stackrel{?}{=} O(\log n)$

$f = O(g)$ if



$\exists c, N$ s.t.

$\forall n \geq N,\ f(n) \leq c \cdot g(n)$

$\log(n) \geq \log\log(n)$ $\sqrt{n}$ 2^n

$\log_2$

$2^{\log n} \geq 2^{\log\log n}$

$n \geq \log n$

fix n: $\log n \geq \log\log n$

$2^{\log n} \geq 2^{\log\log n}$

$\log\log n$ vs. $\log n$

$m = \log\log n$

$\rightarrow m$ vs. 2^m