$1000^{10}, 2n, \frac{n^2}{2}, 2^n$

1) $1000^{10} = O(2n)$

$$\lim_{n\to\infty} \frac{1000^{10}}{2n} = 0$$

0 is a constant not dependent on n so $1000^{10} = O(2n)$.

2) $2n = O(n^2/2)$

$$\lim_{n\to\infty} \frac{2n}{n^2/2} \to \lim_{n\to\infty} \frac{4\cancel{n}}{n^{\cancel{2}}} \to \lim_{n\to\infty} \frac{4}{n} = 0$$

3) $n^2/2 = O(2^n)$

$n^c = O(2^n)$

$\frac{n^2}{2} = O(n^2)$

by rule: $n^c = O(2^n)$

$n^2 = O(2^n)$

rule of transitivity:

$\frac{n^2}{2} = O(2^n)$

$f(n) \leq g(n)$

$g(n) \leq h(n)$

then

$f(n) \leq g(n)$

If: $f(n) = O(g(n))$

and $g(n) = O(h(n))$

then $f(n) = O(h(n))$

$f(n) = \Theta(g(n))$

$f(n) = O(g(n))$
$g(n) = O(f(n))$

$f(n) = O(f(n-1)) \rightarrow 4^{4^n} = O(4^{4^{n-1}})$

$f(n-1) = O(f(n)) \rightarrow 4^{4^{n-1}} = O(4^{4^n})$

False:

$f(n-1) = O(f(n))$

$\lim_{n\to\infty} (4^{4^n})^{3/4} = +\infty$

$f(n) = O(f(n-1))$

$\lim_{n\to\infty} \frac{f(n)}{f(n-1)} \rightarrow \lim_{n\to\infty} \frac{4^{4n}}{4^{4^{n-1}}} \rightarrow \lim_{n\to\infty} \frac{(4^{4^n})^1}{(4^{4^n})^{1/4}}$

$f(n) \neq O(f(n-1))$ $\quad 4^{4^{n-1}}$

$4^{n-1} = 4^n \cdot 4^{-1} = 4^n \cdot \frac{1}{4}$

$1/4$

$\therefore$

$f(n) \neq \Theta(f(n-1))$ $\quad 4^{1/4 \cdot 4^n} \rightarrow (4^{4^n})^{1/4}$

$$\lim_{n\to\infty} \frac{4^{4^n}}{4^{4^{(n-1)}}}$$

4^a

$a = 4^{(n-1)} = 4^n \cdot 4^{-1}$

$4^{-1} = \frac{1}{4^1} = \frac{1}{4}$

$$\lim_{n\to\infty} \frac{4^{4^n}}{4^{(4^n \cdot 1/4)}}$$

$$\lim_{n\to\infty} \frac{\left(4^{4^n}\right)^1}{\left(4^{4^n}\right)^{1/4}} \rightarrow \lim_{n\to\infty} \left(4^{4n}\right)^{3/4} = +\infty$$