Introduction to Algorithms and Asymptotic Analysis

Recitation 2

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Announcements

- Office hours (mine are Wed 2:30-3:30)
- Sign up for Piazza!
- Set up latex (if you haven’t already)
- HackHers

Short Lecture Recap

- Problem: A mapping from the set of all potential inputs to the correct answer(s) for each input.
- Algorithms: A sequence of simple and well-defined steps for outputting the correct answer to any input of a given problem.
- When designing an algorithm, we must provide the algorithm, proof of correctness, and analyze the runtime.

Short Lecture Recap (cont.)

- $f(n) = O(g(n))$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \le C.$$

- $f(n) = \Omega(g(n)$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \ge C.$$

- $f(n) = o(g(n))$ if and only if $f(n) = O(g(n))$ but $f(n) \,!= \Theta(g(n))$

- $f(n) = \omega(g(n))$ if and only if $f(n) = \Omega(g(n))$ but $f(n) \,!= \Theta(g(n))$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 1

Rank the following functions based on their asymptotic value in the increasing order, i.e., list them as functions f1, f2, f3 such that f1 = O(f2), f2 = O(f3).

1000^{10} $\quad$ 2^n $\quad$ $n^2/2$ $\quad$ $2n$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 1 Solution

Rank the following functions based on their asymptotic value in the increasing order, i.e., list them as functions f1, f2, f3 such that f1 = O(f2), f2 = O(f3).

1000^{10} $\quad 2^n$ $\quad n^2/2$ $\quad 2n$

Order:

1000^{10} $\quad 2n$ $\quad n^2/2$ $\quad 2^n$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 2

For the function

$$f(n) = 4^{4^n}$$

determine whether the following statement is true or false. Prove it.

$$f(n) = \Theta(f(n-1))$$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 2 Solution

False.

f(n) != O(f(n-1)) since $\lim_{n \to \infty} \frac{4^{4^n}}{4^{4^{n-1}}} = +\infty.$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 3

Determine whether the following statements are true. If false, show a counterexample.

1. $f(n)+g(n)=\Theta(min(f(n),g(n)))$
2. $f(n) = O(g(n))$ implies $2^{f(n)} = O\left(2^{g(n)}\right)$
3. $f(n)+o(f(n))=\Theta(f(n))$
4. $f(n)=\Theta(f(n/2))$
5. $f(n) = O((f(n))^2)$

$$n^c = O(n^{c+1}) \qquad (\log n)^c = O(n) \qquad n^c = O(2^n) \qquad c^n = O((c+1)^n).$$

Practice Problem 3 Solution

1. False. Example: $n^2 + n \neq \Theta(min(n^2, n)) = \Theta(min(n))$
2. False. Let f(n) = 2n and g(n) = n.
3. True.
4. False. Let $f(n) = 4^n$.
5. False. It doesn't hold if f(n) < 1.