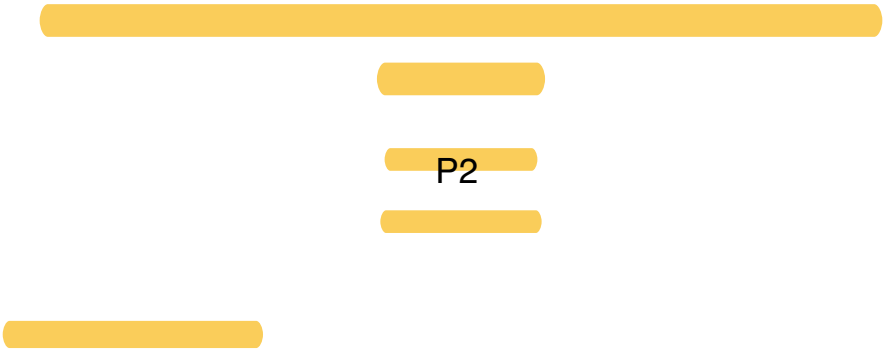



**(a)**

$$\begin{aligned} Pr(A, B, C, D, E) &= Pr(A) \times Pr(B) \times Pr(C) \times Pr(D|A, B) \times Pr(E|B, C) \\ &= 0.2 \times 0.5 \times 0.8 \times 0.1 \times 0.3 \\ &= 0.0024. \end{aligned}$$

**(b)**

$$\begin{aligned} Pr(\neg A, \neg B, \neg C, \neg D, \neg E) &= Pr(\neg A) \times Pr(\neg B) \times Pr(\neg C) \times Pr(\neg D|\neg A, \neg B) \times Pr(\neg E|\neg B, \neg C) \\ &= 0.8 \times 0.5 \times 0.2 \times 0.1 \times 0.8 \\ &= 0.0064. \end{aligned}$$

**(c)**

$$\begin{aligned} Pr(\neg A|B, C, D, E) &= \frac{Pr(\neg A, B, C, D, E)}{Pr(A, B, C, D, E) + Pr(\neg A, B, C, D, E)} \\ &= \frac{Pr(\neg A) \times Pr(B) \times Pr(C) \times Pr(D|\neg A, B) \times Pr(E|B, C)}{0.0024 + Pr(\neg A) \times Pr(B) \times Pr(C) \times Pr(D|\neg A, B) \times Pr(E|B, C)} \\ &= \frac{0.8 \times 0.5 \times 0.8 \times 0.6 \times 0.3}{0.0024 + 0.8 \times 0.5 \times 0.8 \times 0.6 \times 0.3} \\ &= \frac{0.0576}{0.0024 + 0.0576} \\ &= 0.96. \end{aligned}$$

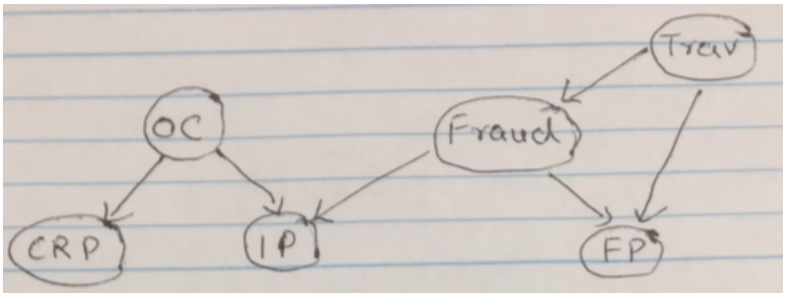
# P3

$$
\begin{aligned}
Pr(B|JC=t,MC=t) &= \alpha Pr(B,JC=t,MC=t) \\
&= \alpha \sum_E \sum_A Pr(B,E,A,JC=t,MC=t) \\
&= \alpha \sum_E \sum_A Pr(B)Pr(E)Pr(A|B,E)Pr(JC=t|A)Pr(MC=t|A) \\
&= \alpha Pr(B) \sum_E Pr(E) \sum_A Pr(A|B,E)Pr(JC=t|A)Pr(MC=t|A) \\
&= \alpha \times 0.001 \times [0.002 \times \{0.95 \times 0.9 \times 0.7 + 0.05 \times 0.05 \times 0.01\} + \\
&0.998 \times \{0.94 \times 0.9 \times 0.7 + 0.06 \times 0.05 \times 0.01\}] \\
&= \alpha \times 0.001 \times [0.002 \times 0.5985 + 0.998 \times 0.5922] \\
&= \alpha \times 0.001 \times 0.5922 \\
&= 0.0005922\alpha
\end{aligned}
$$

$$
\begin{aligned}
Pr(\neg B|JC=t,MC=t) &= \alpha Pr(\neg B,JC=t,MC=t) \\
&= \alpha \sum_E \sum_A Pr(\neg B,E,A,JC=t,MC=t) \\
&= \alpha \sum_E \sum_A Pr(\neg B)Pr(E)Pr(A|B,E)Pr(JC=t|A)Pr(MC=t|A) \\
&= \alpha Pr(\neg B) \sum_E Pr(E) \sum_A Pr(A|\neg B,E)Pr(JC=t|A)Pr(MC=t|A) \\
&= \alpha \times 0.999 \times [0.002 \times \{0.29 \times 0.9 \times 0.7 + 0.71 \times 0.05 \times 0.01\} + \\
&0.998 \times \{0.001 \times 0.9 \times 0.7 + 0.999 \times 0.05 \times 0.01\}] \\
&= \alpha \times 0.999 \times [0.002 \times 0.1831 + 0.998 \times 0.0011] \\
&= \alpha \times 0.999 \times 0.0015 \\
&= 0.0014985\alpha
\end{aligned}
$$

Thus, $P(B|JC=t,MC=t) = 478.3087 \times 0.0005922 = 0.2833$.

$$\alpha = \frac{1}{0.0005922 + 0.0014985} = 478.3087$$




P(Trav) = 0.05
P(OC) = 0.75

| Trav | P(Fraud) |
|---|---|
| t | 0.01 |
| f | 0.004 |

| Fraud | Travel | P(FP) |
|---|---|---|
| t | t | 0.9 |
| t | f | 0.9 |
| f | t | 0.1 |
| f | f | 0.01 |

| Fraud | OC | P(IP) |
|---|---|---|
| t | t | 0.02 |
| t | f | 0.011 |
| f | t | 0.01 |
| f | f | 0.001 |

| OC | P(CRP) |
|---|---|
| t | 0.1 |
| f | 0.001 |

# 3 Problem 4

$$\begin{aligned}
P(Fraud) &= P(Fraud|Trav)P(Trav) + P(Fraud|\neg Trav)P(\neg Trav) \\
&= 0.01 \times 0.05 + 0.004 \times 0.95 \\
&= 0.0005 + 0.0038 \\
&= 0.0043.
\end{aligned}$$

$$\begin{aligned}
P(Fraud|FP, \neg IP, CRP) &= \alpha P(Fraud, FP, \neg IP, CRP) \\
&= \alpha \sum_{Trav} \sum_{OC} P(Fraud, FP, \neg IP, CRP, Trav, OC) \\
&= \alpha \sum_{Trav} P(Trav)P(Fraud|Trav)P(FP|Trav, Fraud)\times \\
&(\sum_{OC} P(OC)P(CRP|OC)P(\neg IP|Fraud, OC)) \\
&= \alpha \times [0.05 \times 0.01 \times 0.9 \times \{0.75 \times 0.1 \times 0.98 + 0.25 \times 0.001 \times 0.989\}+ \\
&0.95 \times 0.004 \times 0.1\{0.75 \times 0.1 \times 0.98 + 0.25 \times 0.001 \times 0.989\}] \\
&= \alpha \times [0.00045 \times (0.0735 + 0.00024725) + 0.00038 \times (0.0735 + 0.00024725)] \\
&= \alpha \times 0.0737 \times 0.00387 \\
&= 0.00028\alpha
\end{aligned}$$

$$\begin{aligned}
P(\neg Fraud|FP, \neg IP, CRP) &= \alpha P(\neg Fraud, FP, \neg IP, CRP) \\
&= \alpha \sum_{Trav} \sum_{OC} P(\neg Fraud, FP, \neg IP, CRP, Trav, OC) \\
&= \alpha \sum_{Trav} P(Trav)P(\neg Fraud|Trav)P(FP|Trav, \neg Fraud)\times \\
&(\sum_{OC} P(OC)P(CRP|OC)P(\neg IP|\neg Fraud, OC)) \\
&= \alpha \times [0.05 \times 0.99 \times 0.1 \times \{0.75 \times 0.1 \times 0.99 + 0.25 \times 0.001 \times 0.999\}+ \\
&0.95 \times 0.996 \times 0.01\{0.75 \times 0.1 \times 0.99 + 0.25 \times 0.001 \times 0.999\}] \\
&= \alpha \times [0.00495 \times (0.07425 + 0.00024975) + 0.009462 \times (0.07425 + 0.00024975)] \\
&= \alpha \times 0.0745 \times 0.01441 \\
&= 0.00107\alpha
\end{aligned}$$

$$\begin{aligned}
\alpha &= \frac{1}{0.00028 + 0.00107} \\
&= 740.7.
\end{aligned}$$

Thus, $P(Fraud|FP, \neg IP, CRP) = 740.7 \times 0.00028 \approx 0.207$.