Filtering

$$P(X_t \mid e_{1:t}) = \alpha \underbrace{P(e_t \mid X_t)}_{observation\ \ model} \sum_{x_{t-1}} \underbrace{P(X_t \mid x_{t-1})}_{transition\ \ model} \underbrace{P(x_{t-1} \mid e_{1:t-1})}_{previous\ \ filtering} .$$

$P(X_t \mid e_{1:t})$ is computed recursively, starting from the prior $P(X_1)$, and $P(X_1|e_1) = \alpha P(e_1|X_1)P(X_1)$. The answer is a vector of probabilities that add to one.

Prediction I

$$P(e_{t+1} \mid X_t) = \sum_{x_{t+1}} \underbrace{P(e_{t+1} \mid x_{t+1})}_{Observation\ \ model} \underbrace{P(x_{t+1} \mid X_t)}_{Transition\ \ model}$$

If the value of X_t is not specified, then the answer is a vector of probabilities, each one is computed for a specific value of X_t. These probabilities do not have to add to one.

Prediction II

$$P(e_{t+1} \mid e_{1:t}) = \sum_{x_t} \underbrace{P(x_t \mid e_{1:t})}_{Filtering} \underbrace{P(e_{t+1} \mid x_t)}_{Prediction\ \ I}$$

Prediction I is explained in the previous slide.
The answer here is a single number, like 0.42.

Prediction III

$$P(X_{t+1} \mid e_{1:t}) = \sum_{x_t} \underbrace{P(x_t \mid e_{1:t})}_{Filtering} \ \underbrace{P(X_{t+1} \mid x_t)}_{Transition\ \ model}$$

If the value of X_{t+1} is not specified, then the answer is a vector of probabilities, each one is computed for a specific value of X_{t+1}. These probabilities add to one.

Smoothing

$$P(X_t \mid e_{1:t+1}) = \alpha \underbrace{P(X_t \mid e_{1:t})}_{Filtering} \underbrace{P(e_{t+1} \mid X_t)}_{Prediction\ \ I}$$

The answer is a vector of probabilities that add to one, each probability is computed for a specific value of X_t.