Examples

Temporal Models

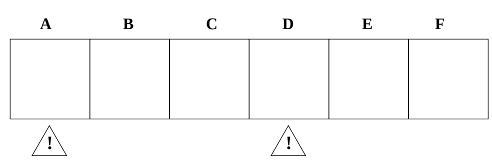
Problem 1:

You are up in your friend's apartment building, watching cars on the street below. They are far enough away that all you can see is their color. You want to catch a taxi home, so you are trying to reason about the probability that a given car is a taxi, given its color. You know that 75% of all taxis are yellow, and that only 10% of non-taxi cars are yellow. You also know that taxis are not likely to bunch up: The car following a taxi is another taxi only 25% of the time. However, non-taxi cars are followed by taxis 50% of the time. Assume 40% of all cars are taxis and 60% are non-taxis.

1. To formulate the above problem as a Hidden Markov Model (temporal model), give the transition model and the evidence model as conditional probabilities (use any correct notation you like).
2. You just saw a car but you could not tell whether it was yellow or not (no evidence at time $t = 0$), and now (at $t = 1$) you see a yellow car. What is the probability that the car you see (at $t = 1$) is a taxi?
3. What is the probability that the next car (at $t = 2$) will be a taxi?
4. You observe that the next car (at $t = 2$) is also yellow. Use this new information to update your belief that the previous car you saw (at $t = 1$) was a taxi.
5. Explain qualitatively why your new estimate in Question 4 makes sense, given your evidence and transition models (I am looking here for a short informal explanation in English).
6. Using the Viterbi algorithm, what is the most likely sequence of cars (taxi or non-taxi) at times 0, 1 and 2 given the observations at times 1 and 2?

Problem 2:

You are an interplanetary search and rescue expert who has just received an urgent message: a rover on Mercury has fallen and become trapped in Death Ravine, a deep, narrow gorge on the borders of enemy territory. You zoom over to Mercury to investigate the situation. Death Ravine is a narrow gorge 6 miles long, as shown below. There are volcanic vents at locations A and D, indicated by the triangular symbols at those locations.



The rover was heavily damaged in the fall, and as a result, most of its sensors are broken. The only ones still functioning are its thermometers, which register only two levels: *hot* and *cold*. The rover sends back evidence $E = hot$ when it is at a volcanic vent (A and D), and $E = cold$ otherwise. There is no chance of a mistaken reading. The rover fell into the gorge at position A on day 1, so $X_1 = A$. Let the rover's position on day t be $X_t \in \{A, B, C, D, E, F\}$. The rover is still executing its original programming, trying to move 1 mile east (i.e. right, towards F) every day. However, because of the damage, it only moves east with probability 0.80, and it stays in place with probability 0.20. Your job is to figure out where the rover is, so that you can dispatch your rescue-bot.

1. Filtering: Three days have passed since the rover fell into the ravine. The observations were $(E_1 = hot, E_2 = cold, E_3 = cold)$. What is $P(X_3 \mid hot_1, cold_2, cold_3)$, the probability distribution over the rover's position on day 3, given the observations? (This is a probability distribution over the six possible positions).
2. Smoothing: What is $P(X_2 \mid hot_1, cold_2, cold_3)$, the probability distribution over the rover's position on day 2, given the observations? (This is a probability distribution over the six possible positions).
3. Most Likely Explanation: What is the most likely sequence of the rover's positions in the three days given the observations $(E_1 = hot, E_2 = cold, E_3 = cold)$?

4. Prediction: What is $P(hot_4, hot_5, cold_6 \mid hot_1, cold_2, cold_3)$, the probability of observing hot_4 and hot_5 and $cold_6$ in days 4,5,6 respectively, given the previous observations in days 1,2, and 3? (This is a single value, not a distribution).

5. Prediction: You decide to attempt to rescue the rover on day 4. However, the transmission of E_4 seems to have been corrupted, and so it is not observed. What is the rover's position distribution for day 4 given the same evidence, $P(X_4 \mid hot_1, cold_2, cold_3)$?
   The same thing happens again on day 5. What is the rover's position distribution for day 5 given the same evidence, $P(X_5 \mid hot_1, cold_2, cold_3)$?