252 ELEMENTARY DIFFERENTIAL EQUATIONS: HW3

No need to hand in

(1) Solve the following intial value problem

$$\begin{cases} \frac{dx}{dt} = 3x, \\ \frac{dy}{dt} = 4y - x^2 \end{cases}$$

and $(x(0), y(0)) = (1, 2)$.

(2) Consider the system

$$\begin{cases} x' = x^2 + y \\ y' = x^2 y^2 \end{cases}$$

Show that, for the solution $(x(t), y(t))$ with intial condition $(x(0), y(0)) = (0, 1)$, there is a time t_* such that $x(t) \to +\infty$ as $t \to t_*$. In other words the solution blows up in finite time (Hint: show that $y' \geq 0$ for all x, y).

(3) Rewrite the following system of differetial equations in matrix form:

$$\frac{dp}{dt} = 2p - q + 6r,$$
$$\frac{dq}{dt} = -p + 3r,$$
$$\frac{dr}{dt} = 7q + 2r.$$

(4) Find the equilibria of the following systems of differential equations:

$$\begin{cases} x' = -3y(1 - x - y) \\ y' = x(3 - 2x - y) \end{cases}$$

(5) Consider the following system of differential equations:

$$\begin{cases} \frac{dx}{dt} = -3y(1 + x^2 + y^2) \\ \frac{dy}{dt} = 2x(1 + 2x^2 + 2y^2) \end{cases}$$

(a) Show that $(\cos 6t, \sin 6t)$ is one of the solution.
(b) Show that if $(x(t), y(t))$ is another solution with $(x(1), y(1)) = (0.5, 0.5)$, then $x(t)^2 + y(t)^2 < 1$ for all t.

(6) In each of the following, factor the matrix A into a product $S\Lambda S^{-1}$, where Λ a diagonal matrix.

(a) $A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$.

(b) $A = \begin{pmatrix} 5 & 6 \\ -1 & -2 \end{pmatrix}$.

(7) For each of the matrix A in question 6, calculate A^4.

(8) For each of the matrix A in question 6, calculate e^{At}.