Differential Equations (D.E.)

- An equation involving
  - the variable t,
  - a function $y(t)$, and
  - it's derivative $y'(t)$, $y''(t)$, ...

e.g. $y' = y + t$

$(y'')^2 + (y')^2 + \cos(ty) = 0$. ...

~~An~~ A differential equation is called first order if it does not involve $y''(t), y'''(t), y^{(4)}(t), \ldots$

e.g. $(y')^2 + e^y = t$.

$y' = 2y + 3$,

it's called explicit if y' can be ~~made~~ isolated: i.e.

$$\boxed{y' = f(t, y).}$$

e.g. $y' = \cos y + t^2$. $\quad (f(t,y) = \cos y + t^2)$

$y' = (\sin t) y$.

Remark: We used the shorthand notation y: e.g.

$$y' = f(t, y)$$

means $y'(t) = f(t, y(t))$ for all t where $y(t)$ is defined.

Goal:

① Given a D.E., find $\boxed{\text{all}}$ the possible solutions $\boxed{\text{explicitly}}$

A function $y_0(t)$ is called a solution to $y' = f(t, y)$ if $y_0'(t) = f(t, y_0(t))$ for all t where $y_0(t)$ is defined.

e.g. Given $y' = \frac{y^2 - 1}{t^2 + 2t}$. Then $y_0(t) = 1 + t$ is a solution:

$$y_0'(t) = (1+t)' = 1, \quad \text{and}$$

$$\frac{(y_0(t))^2 - 1}{t^2 + 2t} = \frac{(1+t)^2 - 1}{t^2 + 2t}$$

$$= \frac{\cancel{1} + 2t + t^2 \cancel{- 1}}{t^2 + 2t}$$

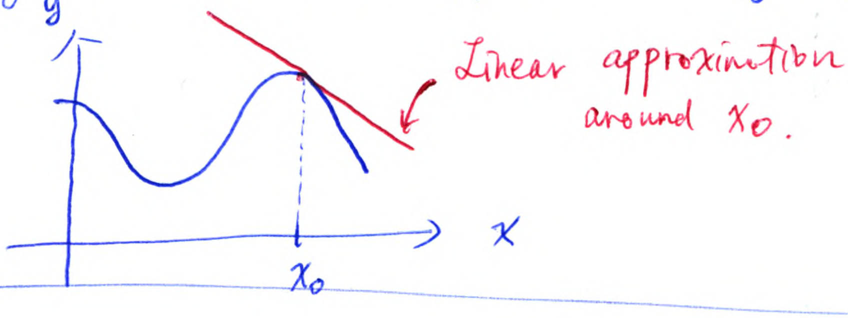
$$= 1 = y_0'(t).$$

② If ① is not possible ~~(i.e.~~,

- Qualitatively Analysis (solution curve sketching, behavior when $t \to +\infty$, ...)
- Numerical Method.

III Stability: Study nonlinear system of D.E.s by it's Linearization

(Analogous to the linear approximation of a function)



Background Needed:

- Techniques in integration.
- Fundamental Theorem of Calculus (FTC)
- Linear algebra (matrix, eigenvalues, eigenvectors)