252 ELEMENTARY DIFFERENTIAL EQUATIONS: HW4

(1) Let $Y_1(t), Y_2(t)$ be two solutions to the linear system

$$Y' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} Y.$$

Let $D(t) = \det \begin{pmatrix} Y_1(t) & Y_2(t) \end{pmatrix}$.

(a) Show that D satisfies the differential equation

$$D' = \operatorname{tr} A \cdot D, \quad \text{where } \operatorname{tr} A = a + d.$$

(b) Conclude that if $Y_1(0), Y_2(0)$ is linearly independent, then $Y_1(t), Y_2(t)$ is linearly independent for all t.

(2) Solve the IVP:

$$\frac{dY}{dt} = \begin{pmatrix} -2 & -2 \\ -2 & 1 \end{pmatrix} Y, \quad Y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(3) Find the general solution to the following system:

$$\frac{dY}{dt} = \begin{pmatrix} -3 & -5 \\ 3 & 1 \end{pmatrix} Y, \quad .$$

(4) Solve the IVP:

$$\frac{dY}{dt} = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix} Y, \quad Y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(5) Sketch the phase portraits of the system given in Q2, 3, 4.

(6) Let B be a matrix with a repeated zero eigenvalues. Then show that $B^2 = 0$ (the 2×2 zero matrix). Use this to show: if A has a repeated eigenvalue λ_0, then $(A - \lambda_0 I)^2 = 0$.

(7) Let A be a 2×2 matrix. Assume that

$$Y_1(t) = \begin{pmatrix} e^t \\ -2e^t \end{pmatrix}, \quad Y_2(t) = \begin{pmatrix} 3e^{-2t} \\ e^{-2t} \end{pmatrix}$$

and both solutions to the system $Y' = AY$. Then solve the IVP

$$Y' = AY, \quad Y(0) = \begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$