01:640:252 ELEMENTARY DIFFERENTIAL EQUATIONS: HW5

(1) Find the general solution to the following differential equations:

$$y'' - y' - 6y = e^{4t}.$$

(2) Find the solution to the following initial value problem:

$$y'' + 4y' + 20y = -3\sin 2t, \text{ with } y(0) = y'(0) = 0.$$

(3) Use the angle sum formula

$$\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$$

to write

$$A\sin\theta + B\cos\theta = C\sin(\theta + \phi).$$

Write C, ϕ in terms of A, B.

(4) Use the angle sum formulae

$$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2,$$
$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

to show

$$\cos\phi_1 - \cos\phi_2 = -2\sin\left(\frac{\phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_1 - \phi_2}{2}\right).$$

(5) Solve the following initial value problem and sketch the solution.

$$y'' + 4y = \cos 2t, \quad y(0) = y'(0) = 0.$$

(6) Find the general solution to the following first order (non-homogeneous) linear system

$$Y' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} Y + \begin{pmatrix} e^{2t} \\ e^{t} \end{pmatrix}.$$

(7) Find the general solution to the following second order differential equation:

$$y'' - 2y' - 3y = te^{-t}.$$

(8) Show directly that

$$y_p(t) = \int_0^t (t-u)e^{t-u} f(u)du$$

is a particular solution to

$$y'' - 2y' + y = f(t).$$