Lecture 6: Radio telescopes

Image of Very Large Array from https://public.nrao.edu/gallery

Rutgers Physics 346: Observational Astrophysics February 23, 2021

Quiz #5: Describe, using pictures if you want, an example of how convolution can be relevant to an astronomical observation.

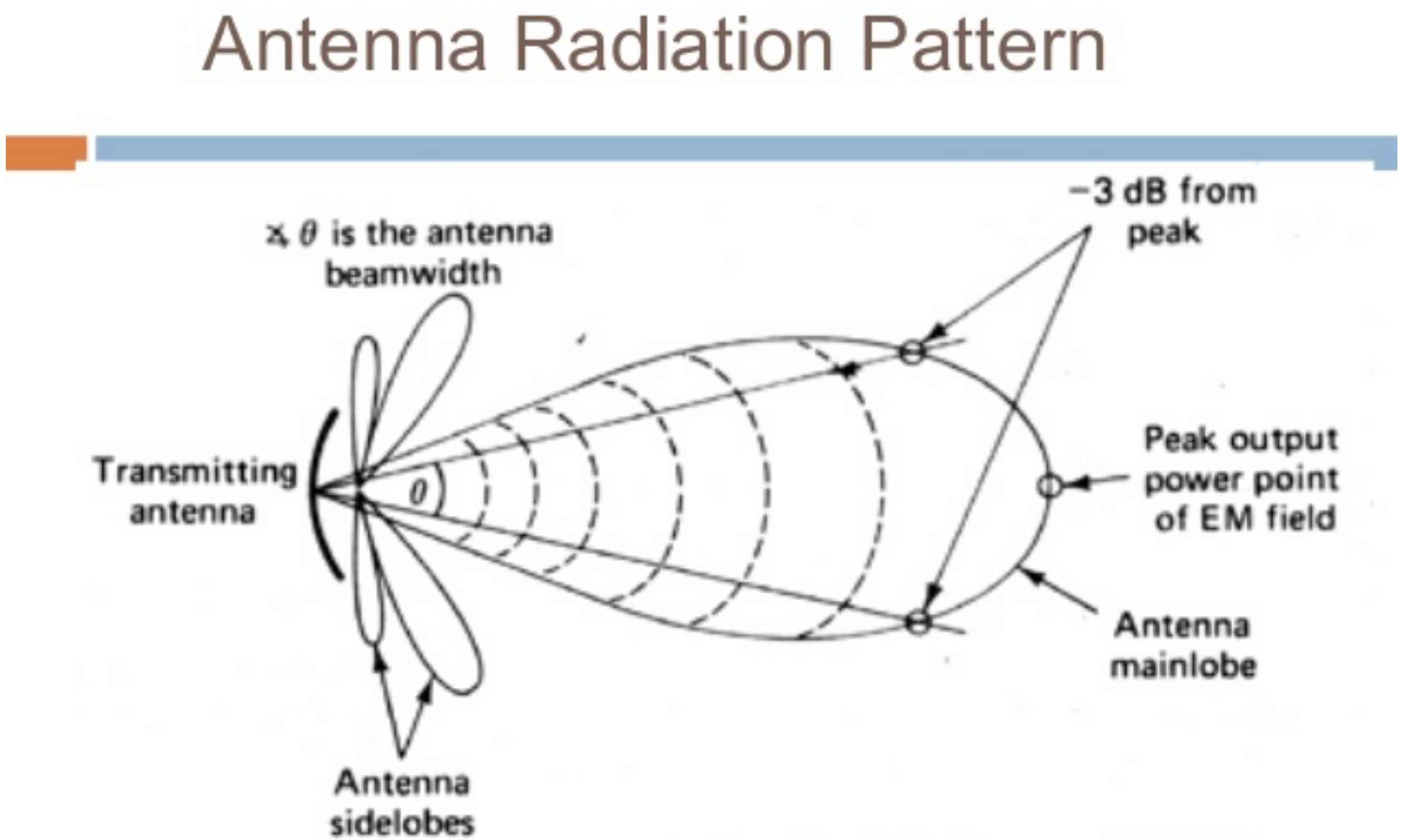
A: Convolution is often relevant to astronomical observations. A source that occupies a smaller angular extent on the sky than the angular resolution of the telescope is referred to as a point source. Such a source is seldom observed in a single pixel; rather, due to atmospheric turbulence (a.k.a. seeing), it appears as a roughly gaussian distribution of light. This is because the entire input image is being convolved with a nearly-two-dimensional-gaussian point spread function (PSF). Even extended galaxies on the sky look somewhat larger in the image, as they too are convolved with this PSF.

It might seem like this does not occur for space-based observations, which lack atmospheric turbulence. But every telescope has an intrinsic point spread function caused by its diffraction limit that convolves the input image. And even if the pixels are large enough that all light from an object falls in a single square pixel, that means that a point has been turned into a square, reminding us that sources are convolved with the shape of pixels as well as the PSF. In typical optical images, the seeing is much larger than individual pixels or the diffraction limit of the telescope, and convolution with the atmospheric PSF dominates the observed image.
Another example of convolution is when one intentionally smoothes a noisy image or spectrum to see an underlying signal that is weaker than the initial pixel-to-pixel noise.

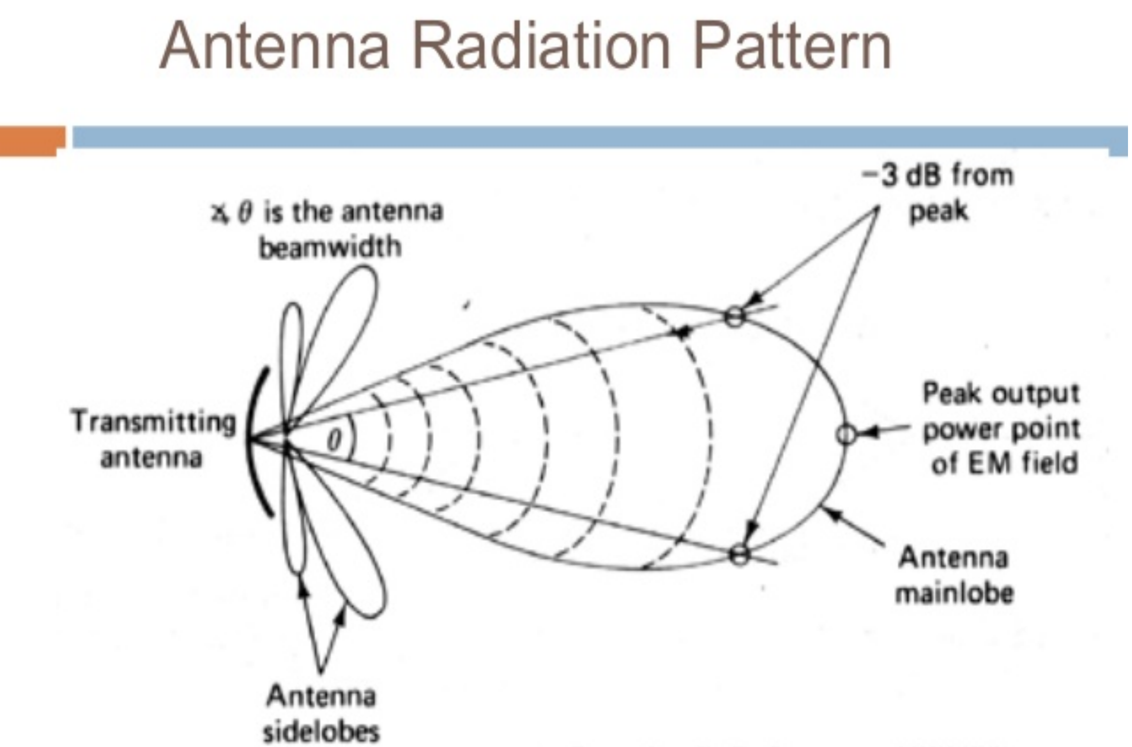
Comments: Generally good answers! Be careful: if you take an image from a website, you must identify the image as having come from there; simply listing that URL as an overall source is not sufficient.

Normalized power pattern

Describes directional dependence of reponsiveness of an antenna to incoming EM radiation in terms of θ (measured from the symmetry axis) and ϕ (azimuthal around that symmetry axis). This is equivalent to the antenna's ability to transmit power!



Normalized power pattern



Normalization condition:

$$P_{\mathrm{n}}(\theta,\phi) = 1 \text{ at}$$

$$\theta = 0 \text{ (on-axis)}$$

$P_{\mathrm{n}}(\theta_{1/2},\phi) = 0.5$ identifies the *beam width* $2 \times \theta_{1/2}$

Also referred to as full width at half-maximum (FWHM) or half-power beamwidth (HPBW)

Power collected

Over a bandwidth $\Delta\upsilon$ and antenna geometric area A, with aperture efficiency η_A and polarization response *p* (usually ½), when observing a source whose specific intensity as a function of position is $I_\nu(\theta,\phi)$ we receive power P given by

$$P = p\,A\,\eta_{\mathrm{A}}\,\Delta\nu \int_{4\pi} P_{\mathrm{n}}(\theta,\phi)\,I_\nu(\theta,\phi)\,d\Omega$$

Now define the *effective solid angle* of the antenna:

$$\Omega_{\mathrm{A}} \equiv \int_{4\pi} P_{\mathrm{n}}(\theta,\phi)\,d\Omega$$

Effective solid angle vs. effective antenna area

Imagine that we place our antenna in a blackbody enclosure at temperature T and measure its response. Power received will be

$$P_{\rm in} = p\, B_\nu(T)\, \Delta\nu\, \Omega_A\, A\, \eta_{\rm A}$$

Modeling antenna as a resistor at temperature T, power generated is

$$P_{\rm out} = h\nu\, \frac{1}{e^{h\nu/kT} - 1}\, \Delta\nu$$

Assuming thermal equilibrium i.e., $P_{\rm in} = P_{\rm out}$ we find a relationship that implies that larger effective antenna area $(\eta_{\rm A}\, A)$ correspond to smaller effective solid angle, and vice versa:

$$\frac{\nu^2}{c^2}\, \Omega_A\, A\, \eta_{\rm A} \quad = \quad 1$$

$$\Omega_{\rm A} \quad = \quad \frac{\lambda^2}{A\, \eta_{\rm A}}$$

Antenna temperature

If we observe a point source with flux density f_{υ} we'll receive power

$$P_{\text{in}} = p\, A\, \eta_A\, f_\nu\, \Delta\nu$$

Now define the *antenna temperature* in terms of power per unit freq.:

$$T_{\text{A}} \equiv \frac{P_{\text{in}}}{k\Delta\nu}$$

And let the *system temperature* $(T_{\text{sys}} > 0)$ be what you measure when not observing a source.

Beam efficiency

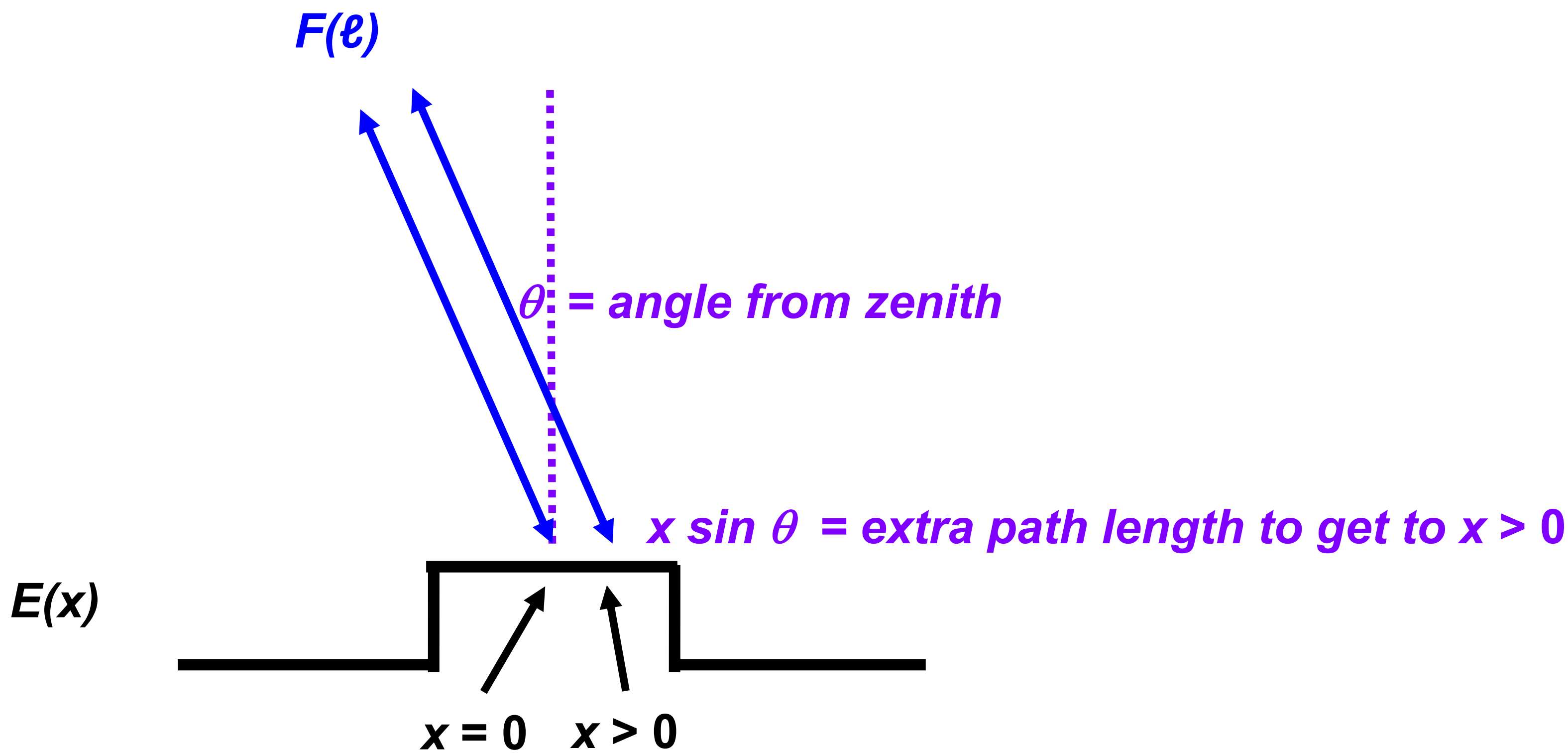
$$\Omega_{\mathrm{mb}} \equiv \int_{\mathrm{mainbeam}} P_n(\theta, \phi)\, d\Omega$$

Gives the effective solid angle of the main beam (no sidelobes)
Now define *beam efficiency* as

$$\eta_B \equiv \frac{\Omega_{\mathrm{mb}}}{\Omega_A} \leq 1$$

Aperture efficiency η_A is relevant for observing point sources;
Beam efficiency η_B is relevant for mapping extended sources.

Telescope Resolution

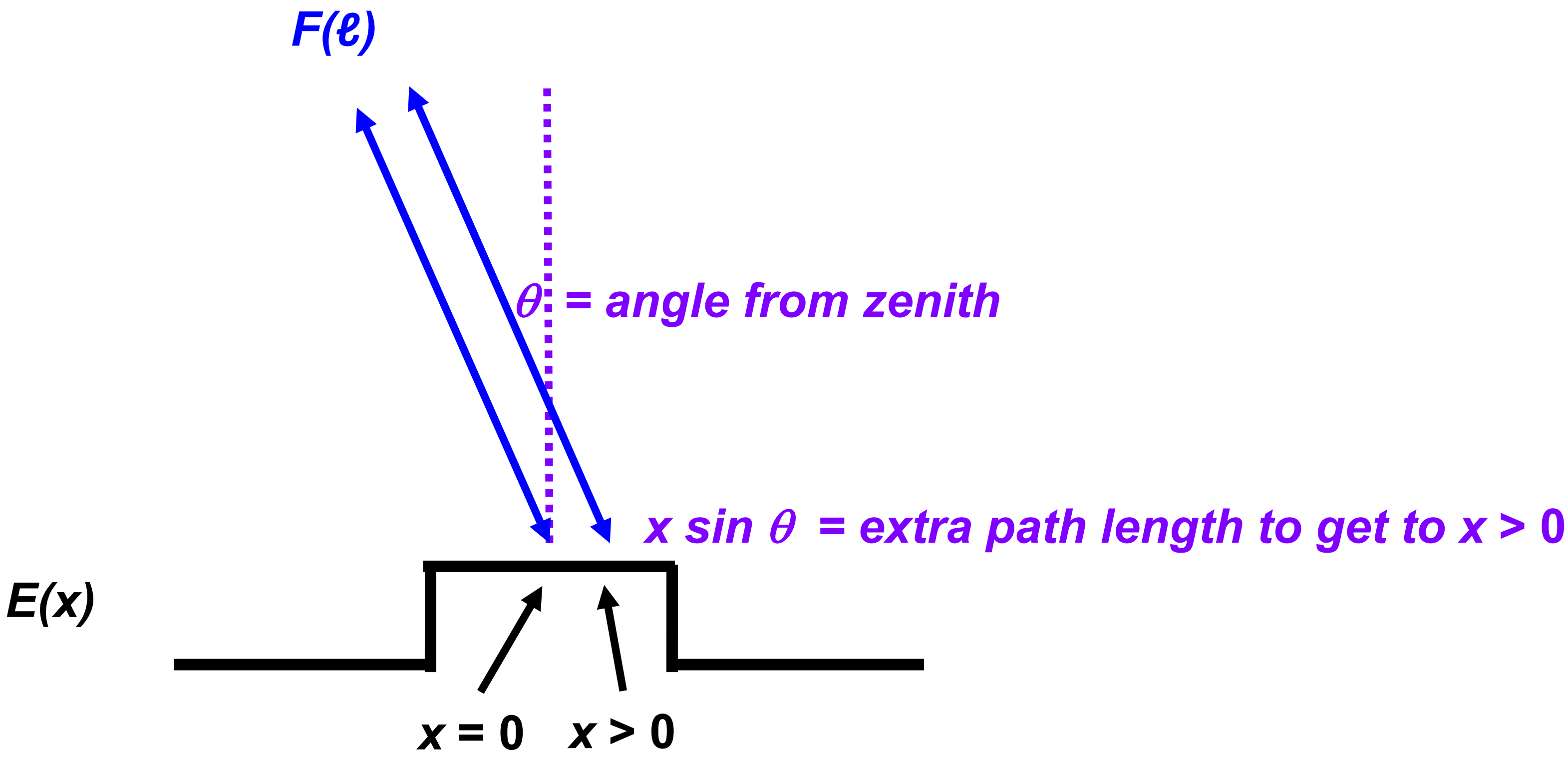


Consider a one-dimensional aperture with electric field distribution E(x) transmitting towards some distant source at zenith angle θ.

This creates an extra path length to the source from detector position x and an infinitesimal part of the detector from x to x+dx contributes E-field

$$E(x)\, e^{-2\pi i x \sin\theta/\lambda}\, dx$$

Telescope Resolution



Substitute $u = \frac{x}{\lambda}$ and $\ell \equiv \sin\theta$ and integrate over the full detector to get

$$F(\ell) = \int E(u)\, e^{-2\pi i u \ell}\, du$$

$$E(u) = \int F(\ell)\, e^{2\pi i u \ell}\, d\ell$$

A Fourier pair! F is the *field pattern* and E is the *aperture Illumination pattern*.

Telescope Resolution

Try the simplest possible illumination pattern, uniform, and generalize to 2-D:

$$F(\ell, m) \propto \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(u, v) e^{-2\pi i(\ell u + mv)} \, du \, dv$$

If we have a rectangular aperture for which E(u,v) is a boxcar, its F.T. will be a sinc function (sinc(x) = sin x /x) in both directions:

$$F(\ell, m) \propto \operatorname{sinc}\left(\frac{\ell D_x}{\lambda}\right) \operatorname{sinc}\left(\frac{m D_y}{\lambda}\right)$$

$$P_n(\ell, m) = \operatorname{sinc}^2\left(\frac{\ell D_x}{\lambda}\right) \operatorname{sinc}^2\left(\frac{m D_y}{\lambda}\right)$$

Where the last equation gives the normalized power pattern

Telescope Resolution: the formula

θ [radians] = 1.22 $\frac{\lambda}{D_{eff}}$

(wavelength and effective diameter must be in same units)

Telescope Resolution

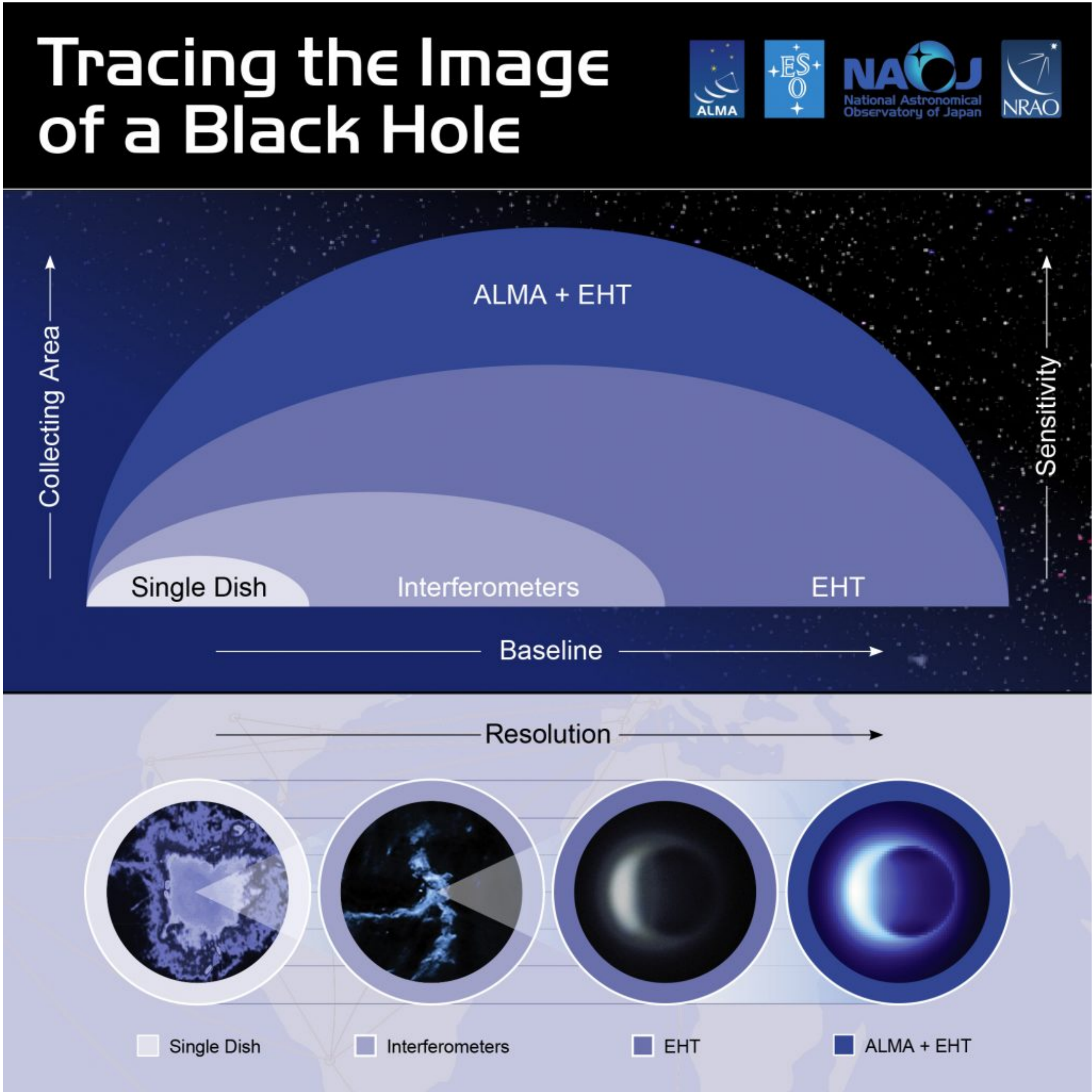


Figure shows how sensitivity varies with collecting area but resolution tracks baseline

Figure is from https://public.nrao.edu/gallery

Homework for Thursday, Feb. 25

Due: Quiz #6 will appear on Canvas Assignments at 4:40pm, due at noon tomorrow (Feb. 24).

Do: Be ready to work with your project group for most of the session. You should aim to finish your analysis by the end of it.

Due: Group presentations will take place in lecture next Tuesday, Mar. 2. Rough drafts are due by noon on March 1 – please email them to both Jack & Eric as links or attachments.