Lecture 8: Aperture Synthesis in Radio Astronomy

Image of Very Large Array from https://public.nrao.edu/gallery

Rutgers Physics 346: Observational Astrophysics March 9, 2021

Homework for Thursday, Mar. 11

Due: Group project reports and **individual contribution statements** are due at 11:59pm tonight via Canvas. Only one report needs to be uploaded, but everyone needs to fill out a brief contribution statement.

Due: Quiz #7 will appear on Canvas Assignments at 4:40pm, due at noon **Thursday**.

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Quiz #6: Which of the following observations would have higher angular resolution: the 100m diameter Green Bank Telescope observing at 100 GHz, or the 30m diameter IRAM telescope observing at 250 GHz?

A:

As noted in lecture, angular resolution is proportional to the wavelength divided by effective telescope diameter. We can rewrite this as

$\theta \propto c / (D_{eff} * \nu)$ where c is the speed of light and is just a constant here so can be dropped for the comparison.

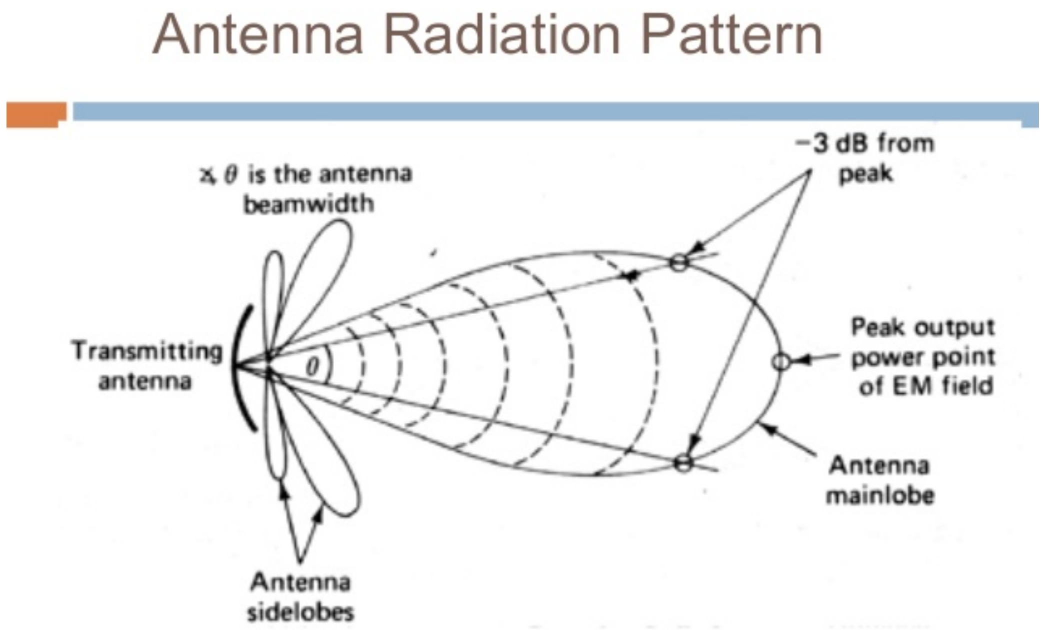
GBT wins, since 1/(100m*100GHz) is smaller than 1/(30m*250GHz), and **higher angular resolution means a smaller value for θ.**

Using the full formula of θ [radians] = $1.22 * \lambda / D_{eff}$, and turning radians into arcsec by multiplying by $(180*3600/\pi) = 206265$, we get resolutions of 7 arcsec for GBT and 10 arcsec for IRAM.

Terminology is funny; I did not take points off for assuming (incorrectly) that higher resolution means larger θ.

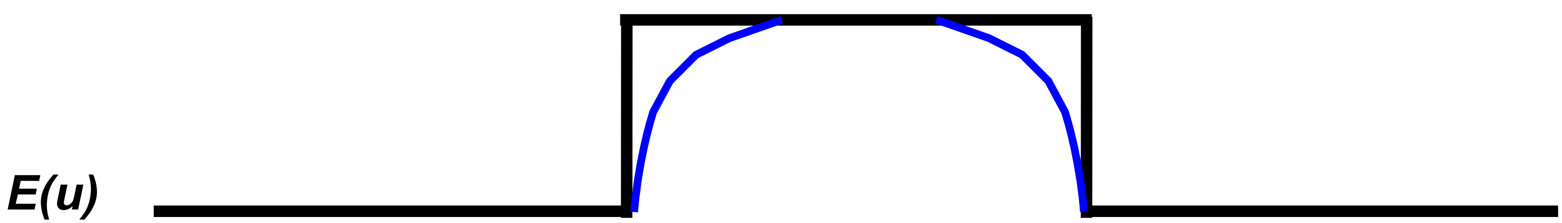
Be careful: you can convert wavelength to frequency at 1 GHz, but that does not tell you the conversion factor at 100 GHz!

Tapering



Aperture efficiency η_A is relevant for observing point sources;
Beam efficiency η_B is relevant for mapping extended sources.

We can intentionally modify the telescope illumination pattern in order to reduce sidelobes and improve beam efficiency.



Taper applied to aperture illumination pattern:

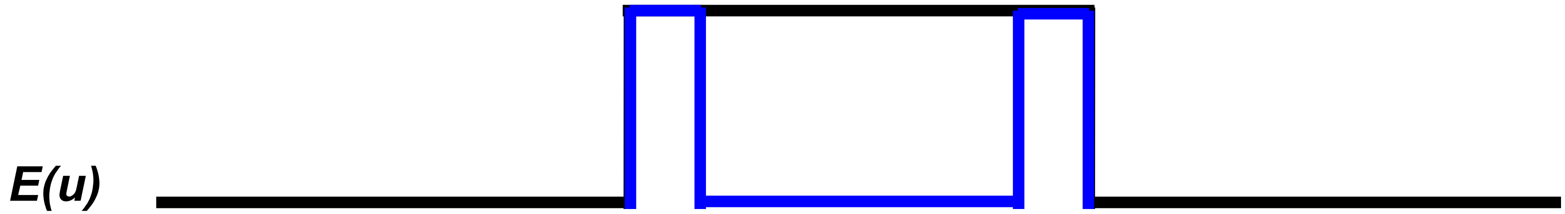
- **reduces strength of sidelobes, so improves beam efficiency**
- **reduces effective collecting area, so worsens aperture efficiency**
- **reduces effective diameter, so worsens angular resolution**

Surface irregularities

The aperture efficiency η_A is also affected by the smoothness of the antenna surface.
Surface irregularities scatter incident radiation out of the main beam into a bigger, fatter "error beam".
Example: if antenna's surface comprised of panels of size L, the error beam will have FWHM $\propto \frac{\lambda}{L}$
Three cases:
1. L is small: error is spread all over, so still a well-defined beam and telescope is ok for mapping
2. L is close to antenna diameter: errors just broaden the existing beam slightly, still ok for mapping
3. L is intermediate – watch out! Error beam is a plateau extending from the main beam. Need to work very hard to maintain surface accuracy, including gravitational deformation of the surface on these scales.

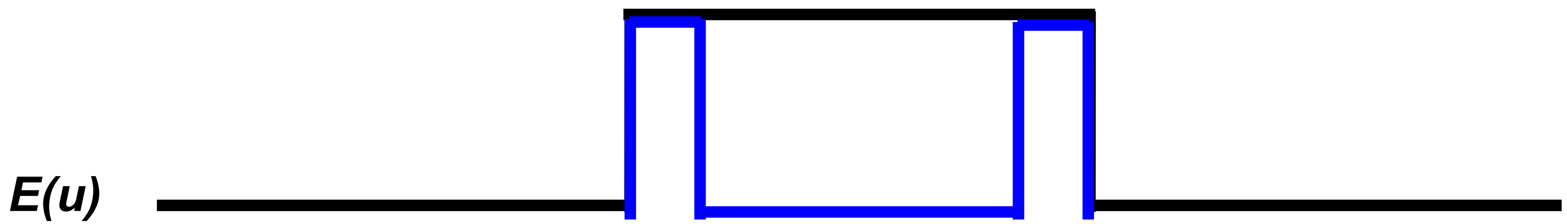
Aperture synthesis: motivation

To build a radio telescope with 1” resolution at $\lambda = 3$ mm, need diameter of 755 m. Good luck with cost and surface!



Thinking outside the box... what happens if we do the opposite of tapering, keeping the outer parts of an antenna and getting rid of the middle?

Aperture synthesis: motivation



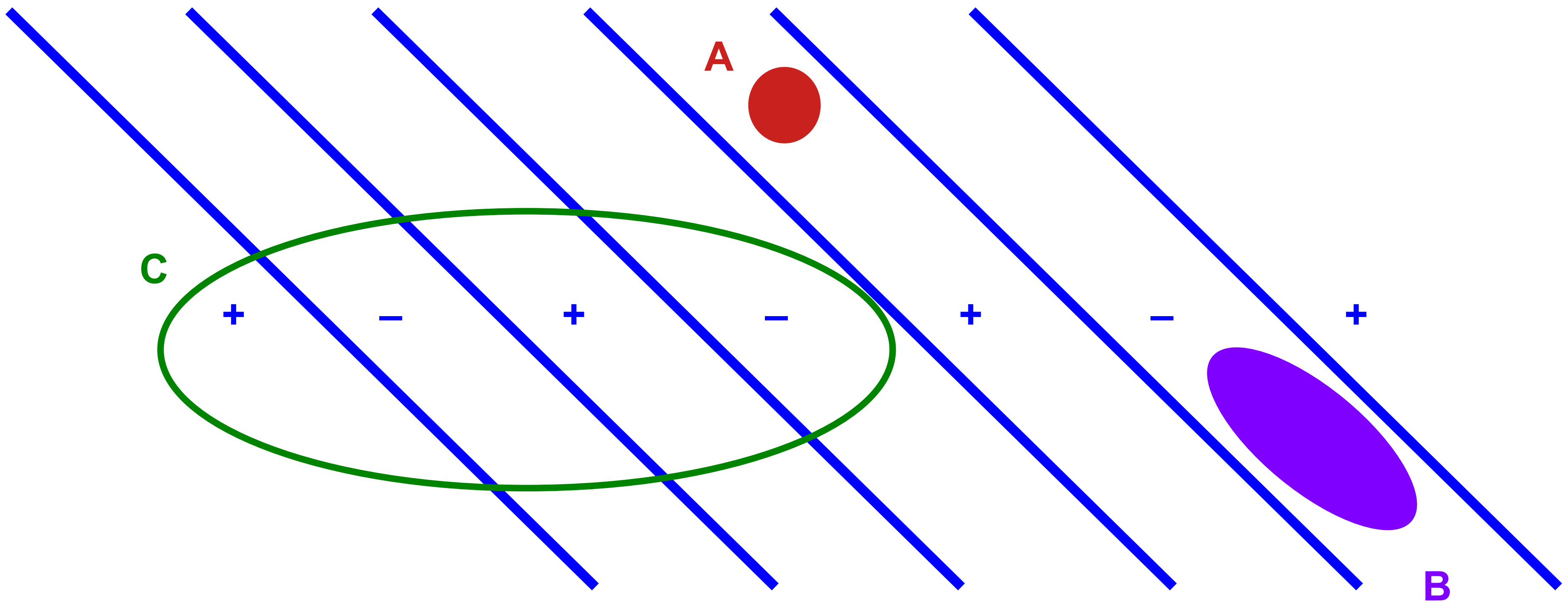
Resulting far-field illumination function looks like a cosine:

$$
\begin{aligned}
E(u) &\propto \delta\Big(u - \frac{D}{2\lambda}\Big) + \delta\Big(u + \frac{D}{2\lambda}\Big) \\
F(\ell) &\propto \cos\left(2\pi \frac{D}{2\lambda}\,\ell\right) = \cos\left(\pi \frac{D}{\lambda}\,\ell\right)
\end{aligned}
$$

$$F(\ell) \propto \cos\left(\pi \frac{B}{\lambda}\,\ell\right)$$

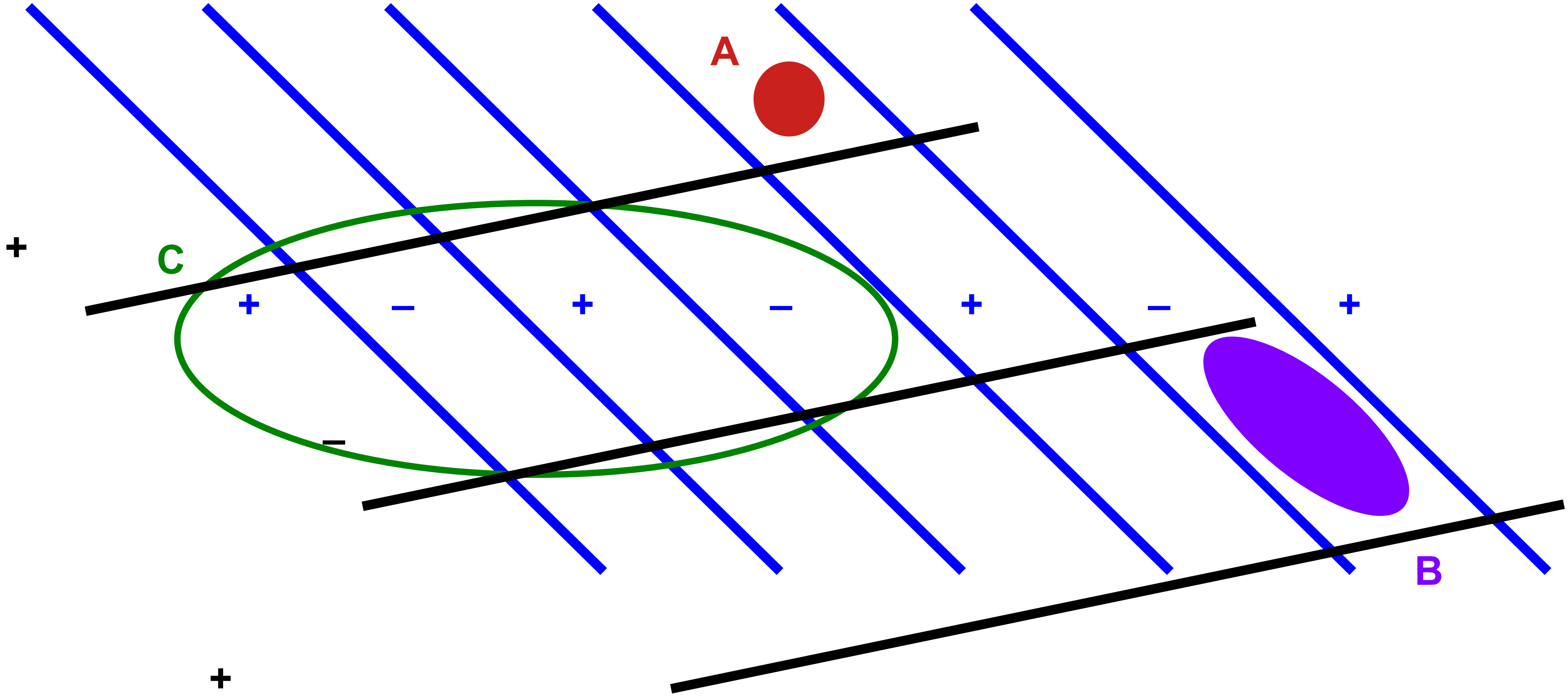
B no longer tied to a physical antenna diameter – if two small antennae, we can make their separation arbitrarily large for better resolution with fixed (small) collecting area.
Basically measures a single Fourier component of the sky brightness distribution for a given source. Not sensitive to extended features larger than cosine oscillation length.

Field pattern for a single baseline



Source A = strong positive signal
Source B = strong negative signal
Source C = nearly zero signal

Different baselines make complementary measurements



Aperture synthesis: summary

- **any pair of antennas in an array corresponds to a baseline *B*, defined by a length and an orientation**

- **when viewed "from the point of the source" at a particular time, this becomes a projected baseline B_p**

- **the *x* and *y* components of a projected baseline and the observing wavelength λ determine the dimensionless coordinates $u = B_{p,x}/\lambda$ and $v = B_{p,y}/\lambda$**

- **a given (*u*,*v*) pair defines a two-dimensional spatial frequency; the power measured at that (*u*,*v*) corresponds to how "ripply" the sky brightness distribution is at a particular direction and periodicity**

Aperture synthesis: summary

- when we have many pairs of antennas (i.e., baselines) and make measurements at many different times, we can measure power at many spatial frequencies

- angular resolution ~ $\lambda/B_{p,max}$ – so longer baselines (and shorter wavelengths) give higher resolution!

- largest angular scale ~ $\lambda/B_{p,min} \geq \lambda/D$ (for D = diameter of an antenna) – so smaller antennas do a better job of recovering smoothly extended emission!

- if the sky brightness distribution includes emission that is larger than λ/D, the interferometer can't see it

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Any questions?