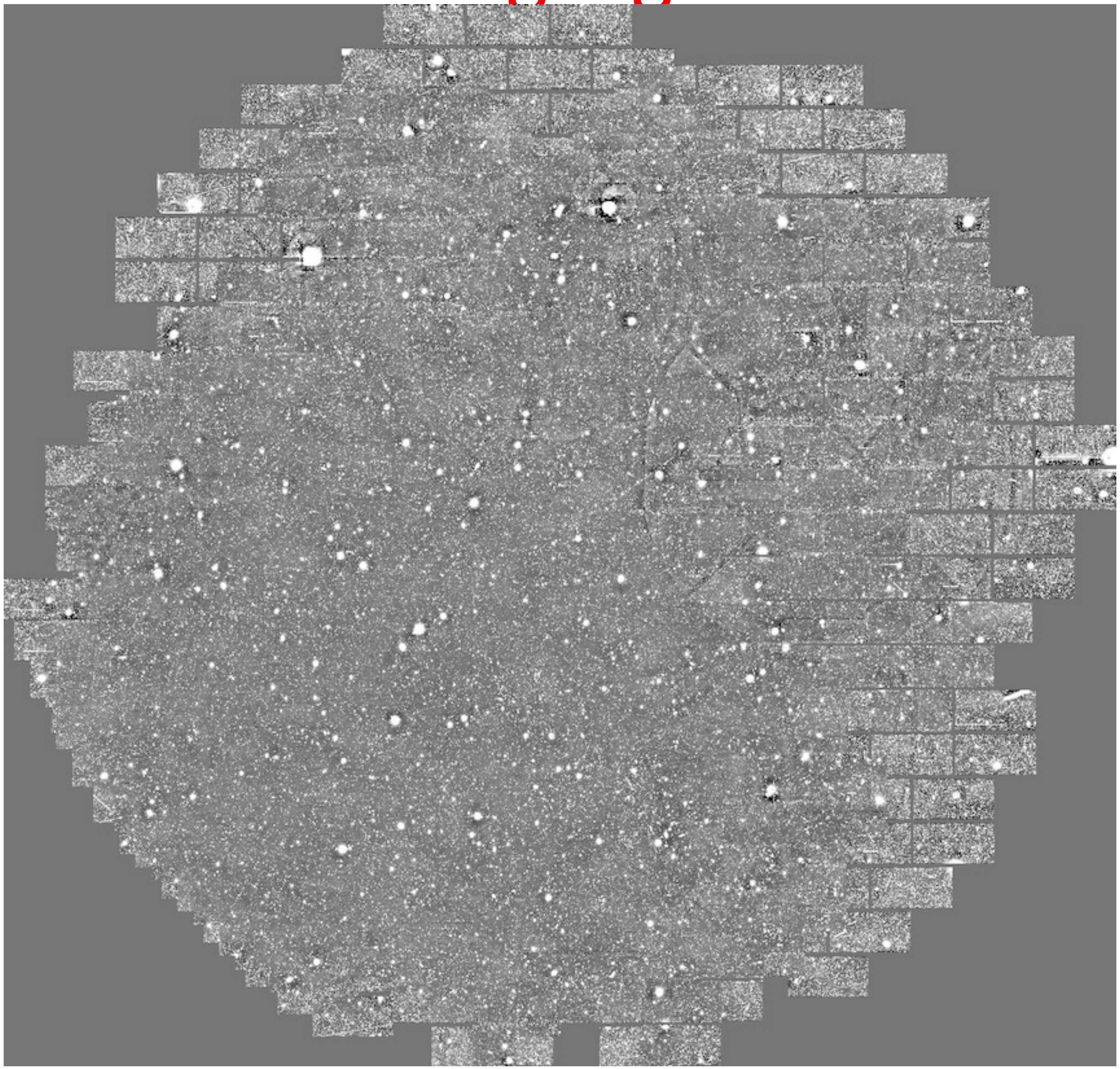
Lecture 5: Fourier transforms, convolution, cross-correlation, and calculating Signal-to-Noise

Community pipeline reduction of images from 1st night of our successful 8-night ODIN observing run on CTIO4m+DECam



Rutgers Physics 346: Observational Astrophysics   February 16, 2021

Quiz #4: Graded with solution available on Canvas.

Q: Describe what makes the Dark Energy Camera (DECam) a world-class optical imager. What type of astronomical surveys is it best suited for, and what type of surveys would it be poorly suited for? Are there other imaging cameras that offer similar capabilities to DECam? Please cite sources used to develop your answer.

A:The Dark Energy Camera offers a 3 square degree (2 degree diameter) field of view on a 4 meter telescope. This gives it one of the greatest products of field-of-view times light gathering power (called etendue) of any optical instrument, allowing it to gather deep data rapidly. It has a set of standard broad-band imaging filters and (partially thanks to the ODIN survey) several narrow-band filters.   DECam is best-suited to surveys that want to cover large areas of sky observable from the Southern hemisphere in at least one of the available filters. It is not well-suited for surveys that observe single objects, target the Northern hemisphere, or need sub-arcsecond angular resolution. Similar capabilities are offered by Subaru+HyperSuprimeCam, KPNO4m+ODI, and VST+OmegaCam, with only the first of these arguably better for surveys than DECam.   The forthcoming Vera C. Rubin Observatory will be even more powerful, offering a 10 square degree field-of-view on an effectively 6.5m diameter telescope.

Comments: It makes sense to say that DECam is poorly suited to surveys in U-band (near-UV) or near-IR past 1.1 microns; those wavelengths are focused by the telescope. But to an astronomer, it's obvious that an instrument on an optical/IR telescope is incapable of conducting surveys in X-rays, far-IR, or radio. No telescope has panchromatic capability.

Fourier transforms

FTs capture all information contained in a function of one variable as functions of a dual variable.
Example: time vs. frequency, with "time domain" the functions of time and "frequency domain" the functions of frequency.

$$\begin{aligned} H(f) &\equiv \int_{-\infty}^{\infty} h(t)\, e^{-2\pi i f t}\, dt \\ h(t) &\equiv \int_{-\infty}^{\infty} H(f)\, e^{2\pi i f t}\, df \end{aligned}$$

or equivalently in terms of *angular frequency* $\omega = 2\pi f$

$$\begin{aligned} H(\omega) &\equiv \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt \\ h(t) &\equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega)\, e^{i\omega t}\, d\omega \end{aligned}$$

H(f) is the F.T. of h(t), and h(t) is the F.T. of H(f)

Fourier transforms

Letting $h(t) = A\cos(2\pi f_0 t)$ we get the F.T.

$$H(f) = \frac{A}{2}\delta(f - f_0) + \frac{A}{2}\delta(f + f_0)$$

i.e., signal at both positive and negative frequency. That may seem strange, but it's necessary to get the input cos behavior. Take the F.T. of just a delta-function at f_0 and you get

$$\frac{A}{2}e^{2\pi i f_0 t} = \frac{A}{2}\cos(2\pi f_0 t) + i\,\frac{A}{2}\sin(2\pi f_0 t)$$

Whereas we are looking for

$$A\cos(2\pi f_0 t) = \frac{A}{2}\left(e^{2\pi i f_0 t} + e^{-2\pi i f_0 t}\right)$$

Can think of those two terms as vectors rotating in opposite directions in the complex plane, so that their sum always remains on the real axis!

Fourier transforms

F.T. of $h(t) = A\sin(2\pi f_0 t)$ **will be**

$$H(f) = -i\,\frac{A}{2}\,\delta(f - f_0) + i\,\frac{A}{2}\,\delta(f + f_0)$$

H(f) can be broken into even and odd components:

$$H(f) = \int_{-\infty}^{\infty} h(t)\,e^{-2\pi i f t}\,dt = \int_{-\infty}^{\infty} h(t)\Big(\cos(2\pi f t) - i\sin(2\pi f t)\Big)\,dt$$

Which yields the following relationships between h(t) and H(f):

- $h(t)$ is real and even $\Leftrightarrow$ $H(f)$ is real and even
- $h(t)$ is real and odd $\Leftrightarrow$ $H(f)$ is imaginary and odd
- $h(t)$ is imaginary and even $\Leftrightarrow$ $H(f)$ is imaginary and even
- $h(t)$ is imaginary and odd $\Leftrightarrow$ $H(f)$ is real and odd

Convolutions

$$f(t) \otimes g(t) = f(t) * g(t) \equiv \int_{-\infty}^{\infty} f(t')\, g(t - t')\, dt'$$

Convolution is the act of smearing one function into another. Example: optical telescope images are blurred by atmospheric turbulence so that a point source becomes a ”point spread function” (PSF).

$$\begin{aligned} f(t) * g(t) &= g(t) * f(t) \\ f(t) * \Big(g(t) * h(t)\Big) &= \Big(f(t) * g(t)\Big) * h(t) \\ f(t) * \Big(g(t) + h(t)\Big) &= f(t) * g(t) + f(t) * h(t) \end{aligned}$$

So convolutions are commutative, associative, and distributive!

The Convolution Theorem

if $y(t) \equiv f(t) * g(t)$ and $\mathcal{F}$ denotes the Fourier transform with $F(f) \equiv \mathcal{F}(f(t))$ and $G(f) \equiv \mathcal{F}(g(t))$, then

$$\mathcal{F}(y(t)) \;=\; \mathcal{F}\Big(f(t) * g(t)\Big) = \mathcal{F}\left(f(t)\right) \cdot \mathcal{F}(g(t))$$

This means that a convolution in the time domain is just a multiplication in the frequency domain (and vice versa).

Cross-correlations

$$f(t) \star g(t) \equiv \int_{-\infty}^{\infty} f(t')\, g(t+t')\, dt'$$

Just a sign difference (and different * shape) from convolution!
Let h*(t) be the complex conjugate of h(t) →

$$\begin{aligned} h(t) &= \int_{-\infty}^{\infty} H(f)\, e^{2\pi i f t}\, df \\ h^*(t) &= \int_{-\infty}^{\infty} H^*(f)\, e^{-2\pi i f t}\, df \\ &= \int_{-\infty}^{\infty} H^*(-f)\, e^{2\pi i f t}\, df \\ &= \mathcal{F}\Big(H^*(-f)\Big) \end{aligned}$$

The Correlation Theorem

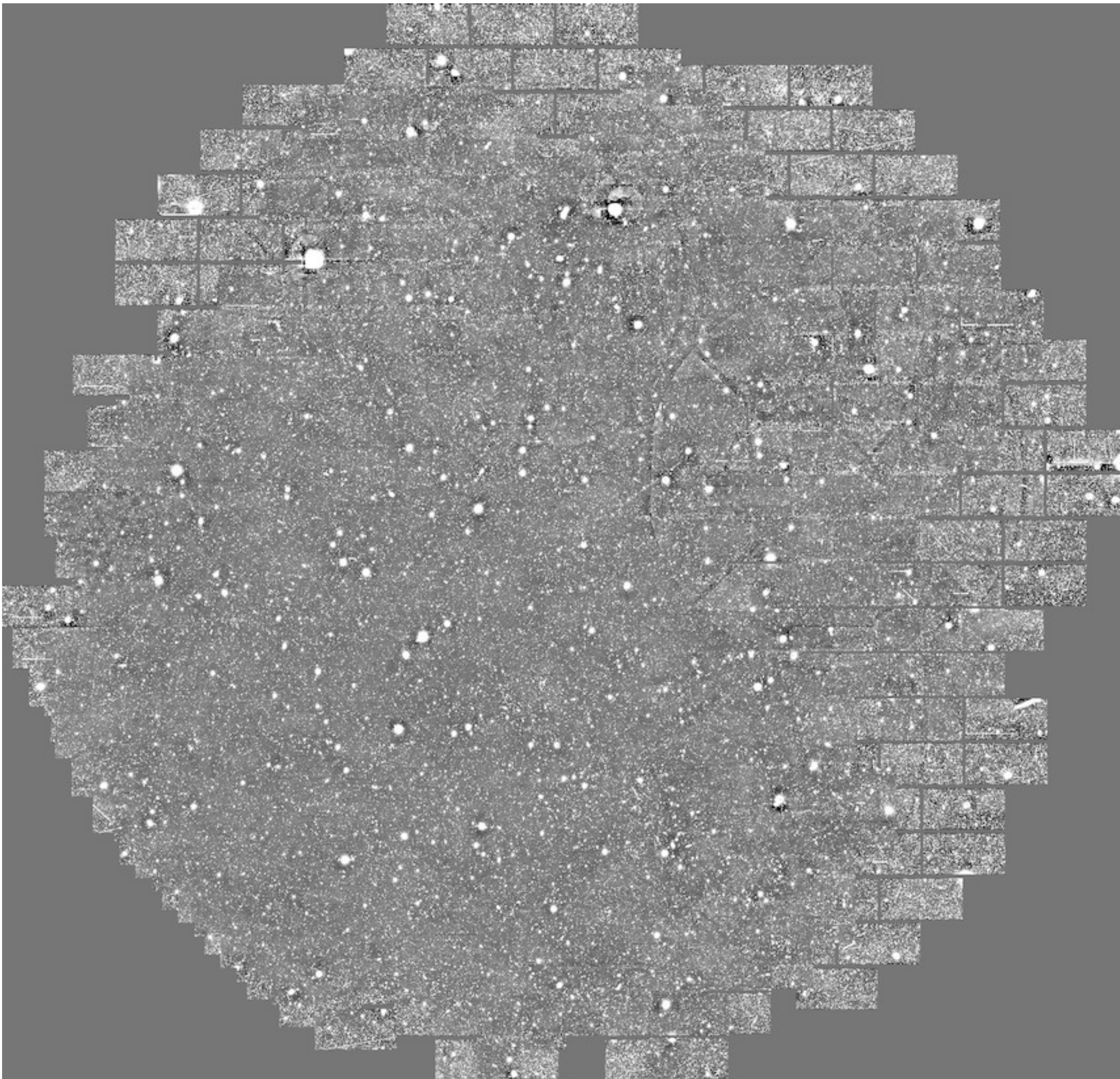
If h(t) is real, h*(t)=h(t) and the previous equation tells us that H(f)=H*(-f) i.e. H is a Hermitian function of f.
If f(t) is real, this leads to:

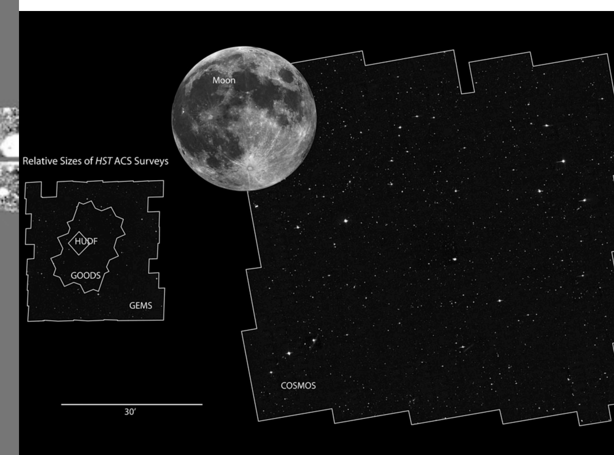
$$\begin{aligned}
\mathcal{F}\left(f(t)\star g(t)\right) &= \int_{-\infty}^{\infty} f^*(t')\,g(t+t')\,dt' \\
&= \int_{-\infty}^{\infty} f^*(-t')\,g(t-t')\,dt' \\
&= \mathcal{F}^*\left(f(t)\right)\cdot\mathcal{F}\left(g(t)\right)
\end{aligned}$$

i.e. the F.T. of the cross-correlation of f and g is the product of the complex conjugate of the F.T. of f and the F.T. of g

ODIN: Observing Feb. 7-14, 2021

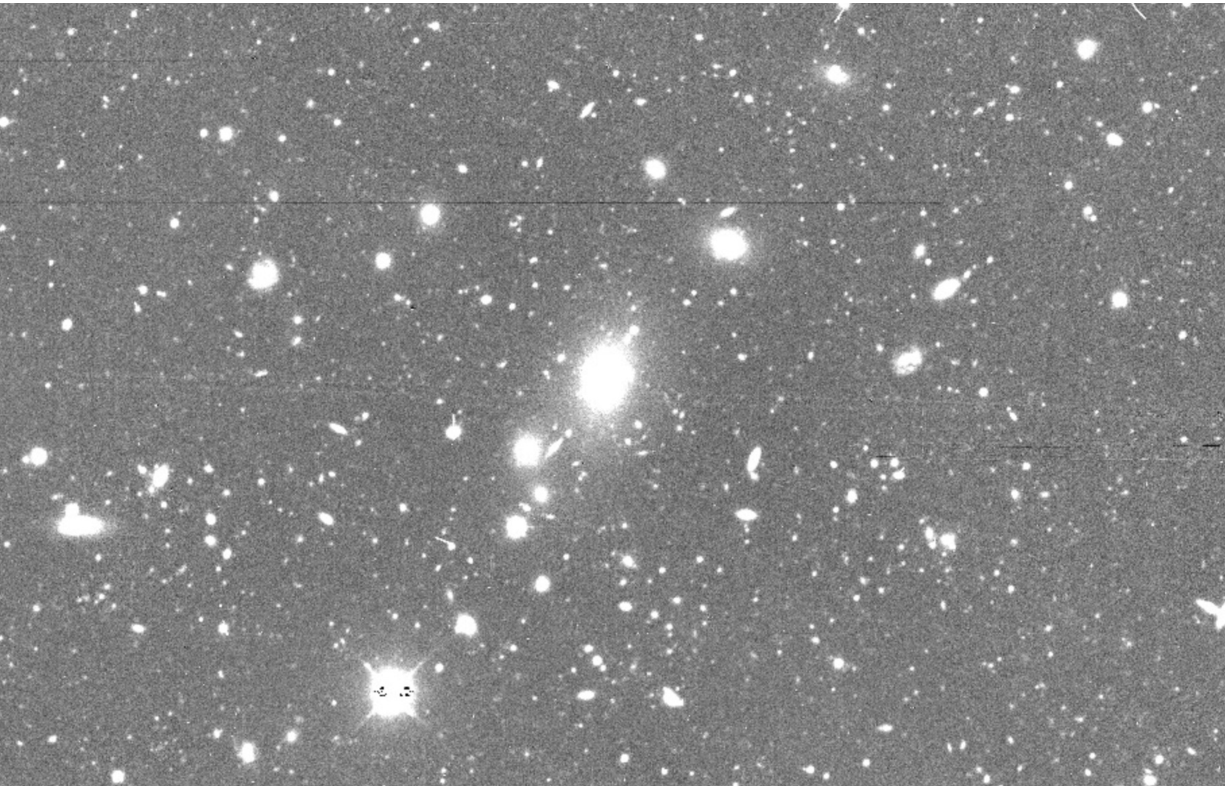
Starting to reduce the data with the DECam Community Pipeline

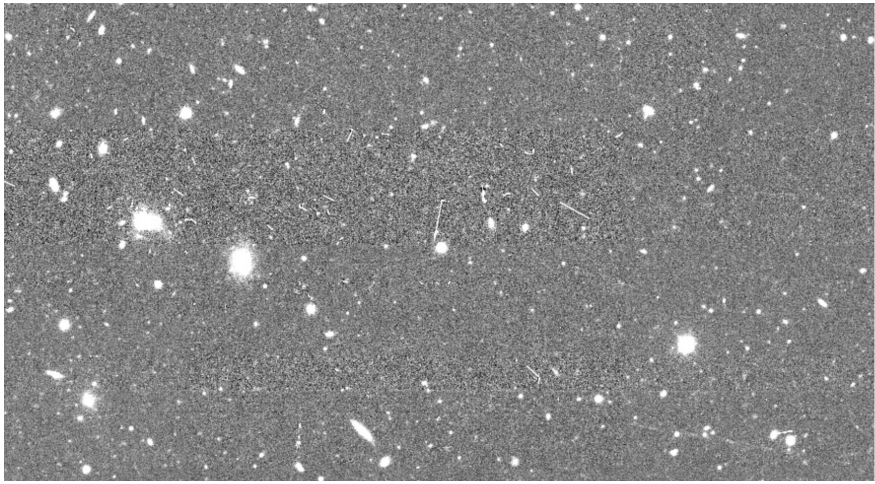


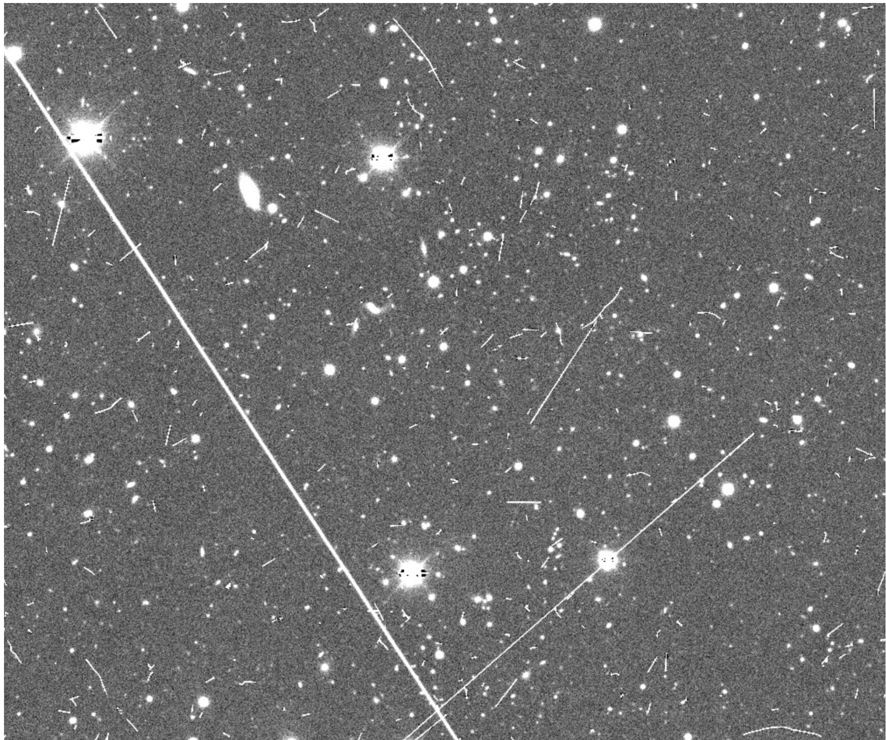


vs. the public COSMOS HST image, **to scale**

ODIN: Observing Feb. 7-14, 2021







Starting to reduce the data with the DECam Community Pipeline: zoom in enough and find a LOT of Cosmic Rays!

Calculating the S/N of an (Optical) Image

Signal = object count rate * exposure time * transparency
e.g. 1 photon/s * 1200s * 90%

Noise^2 = (object count rate* transparency + sky count rate* area)* exposure time + readout_noise^2

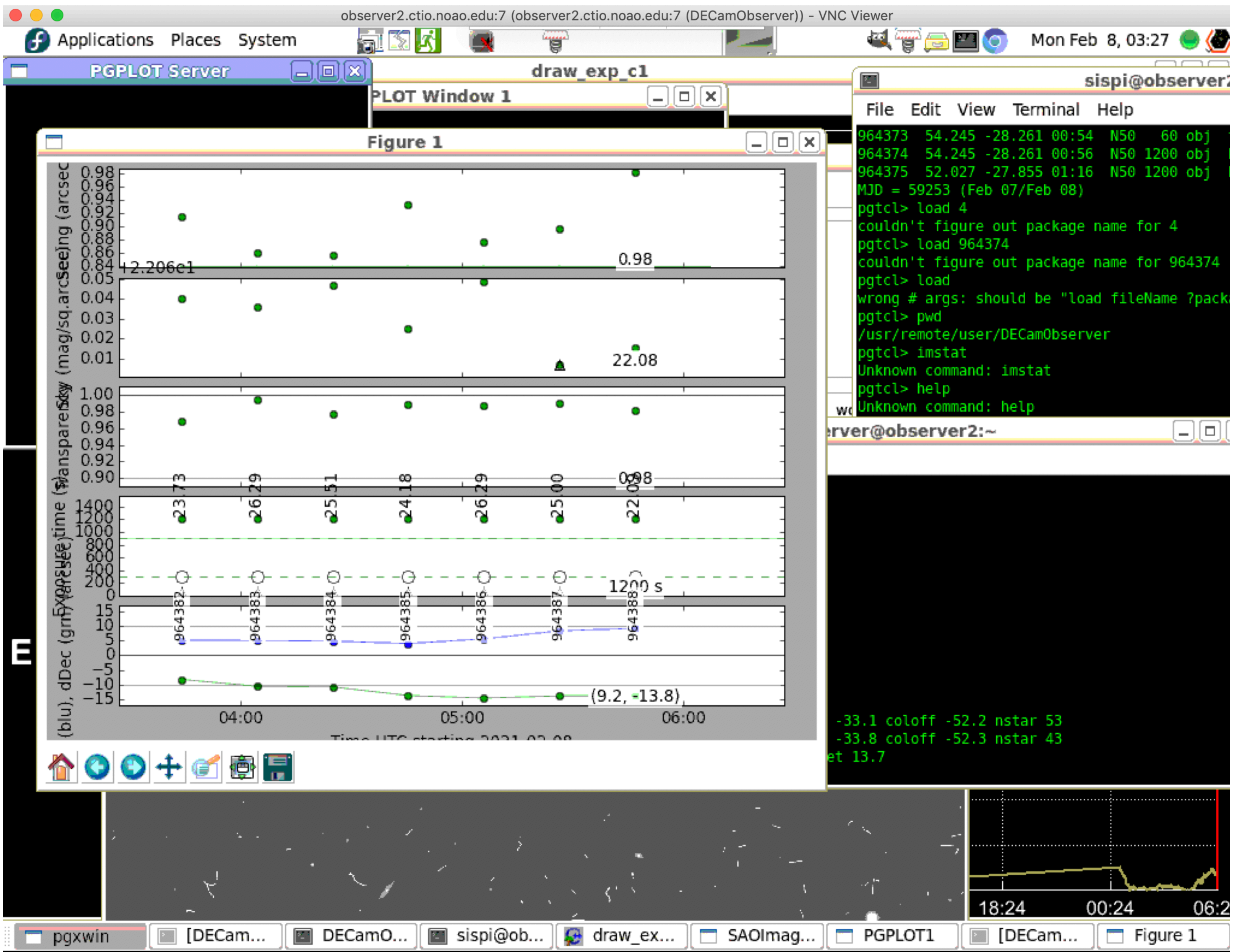
For dim, point-like objects and long enough exposures, background (sky) dominates. Photometry will be performed in an aperture whose diameter scales with the seeing →

$\text{Noise}^2 \propto$ sky count rate * seeing^2 * exposure time

S/N $\propto$ exposure $\text{time}^{1/2}$*transparency/ (sky count $\text{rate})^{1/2}$ /seeing

ODIN: Observing Feb. 7-14, 2021

Tracking seeing, sky brightness, transparency, exposure time, pointing errrors



2021A RUN – WHAT WE LEARNED

- Dustin’s copilot.py processes the data real time and estimate sky transparency
- Together with seeing and sky counts, Eric implemented an estimate of the effective depth (R_eff) in the log sheet.

ODIN_obs_progress

Local Time	Image number	Object Name	Exp time (sec)	Airmass	Seeing (arcsec)	Sky (ADU)	Transparency	R_eff (vs. 1200s)	
21:25	965288	ECDFS_N501_R1.0_24_focus	60	1.09	1.09	96	0.95	0.00	correctio
21:43	965289	ECDFS_N501_R1.0_24_focus	60	1.13	0.92	8	0.96	0.04	correctio
21:45	965290	ECDFS_N501_R1.0_24	1200	1.13	0.93	92	0.94	1.11	correctio
22:06	965291	ECDFS_N501_R1.0_28	1200	1.19	0.88	79	0.94	1.44	correctio
22:26	965292	ECDFS_N501_R1.0_19	1200	1.24	0.89	78	0.95	1.46	correctio
22:47	965293	ECDFS_N501_R1.0_2	1200	1.32	0.89	81	0.92	1.32	correctio
23:07	965294	ECDFS_N501_R1.0_33	1200	1.44	0.97	87	0.93	1.06	correctio
23:27		Telescope slew							
23:32	965295	COSMOS_N673_R1.0_23_focus	60	1.53	1.32	10	1	0.03	correctio
23:36	965296	COSMOS_N673_R1.0_23	1200	1.51	0.98	204	1	1.02	correctio
23:56	965297	COSMOS_N673_R1.0_47	1200	1.39	1.03	201	0.99	0.92	correctio
0:17	965298	COSMOS_N673_R1.0_22	1200	1.34	0.85	200	0.99	1.35	correctio
0:37	965299	COSMOS_N673_R1.0_29	1200	1.27	0.83	192	1	1.51	correctio
0:58	965300	COSMOS_N673_R1.0_6	1200	1.26	0.87	200	0.98	1.27	correctio
1:19	965301	COSMOS_N673_R1.0_48	1200	1.2	0.99	198	0.97	0.97	correctio
1:40	965302	COSMOS_N673_R1.0_0	1200	1.2	0.98	202	0.97	0.97	correctio
2:00	965303	COSMOS_N673_R1.0_39	1200	1.17	0.87	195	1	1.35	correctio
2:21	965304	COSMOS_N673_R1.0_34	1200	1.17	0.84	197	0.98	1.38	correctio
2:41	965305	COSMOS_N673_R1.0_7	1200	1.2	0.86	202	0.99	1.31	correctio
3:02	965306	COSMOS_N673_R1.0_43	1200	1.21	0.87	192	0.98	1.32	correctio
3:22	965307	COSMOS_N673_R1.0_30	1200	1.24	0.85	191	0.98	1.39	correctio
3:43	965308	COSMOS_N673_R1.0_24	1200	1.28	0.85	191	0.99	1.42	correctio
4:03	965309	COSMOS_N673_R1.0_62	1200	1.38	0.88	199	0.98	1.24	correctio
4:24	965310	COSMOS_N673_R1.0_28	1200	1.41	0.87	192	0.99	1.35	correctio
4:44	965311	COSMOS_N673_R1.0_42	1200	1.54	0.91	198	0.97	1.15	correctio
5:05	965312	COSMOS_N673_R1.0_19	1200	1.64	0.92	206	0.99	1.12	correctio
5:25	965313	COSMOS_N673_R1.0_2	1200	1.87	0.99	219	0.98	0.89	correctio
5:45		field at X>2; finished for the night							18 degre

Homework for Thursday, Feb. 18

Due: Quiz #5 will appear on Canvas Assignments at 4:40pm, due at noon tomorrow (Feb. 17).

Do: Be ready to work with your project group for most of the session.