Lecture 18: Binary stellar evolution

Lamers & Levesque Ch. 28, 29

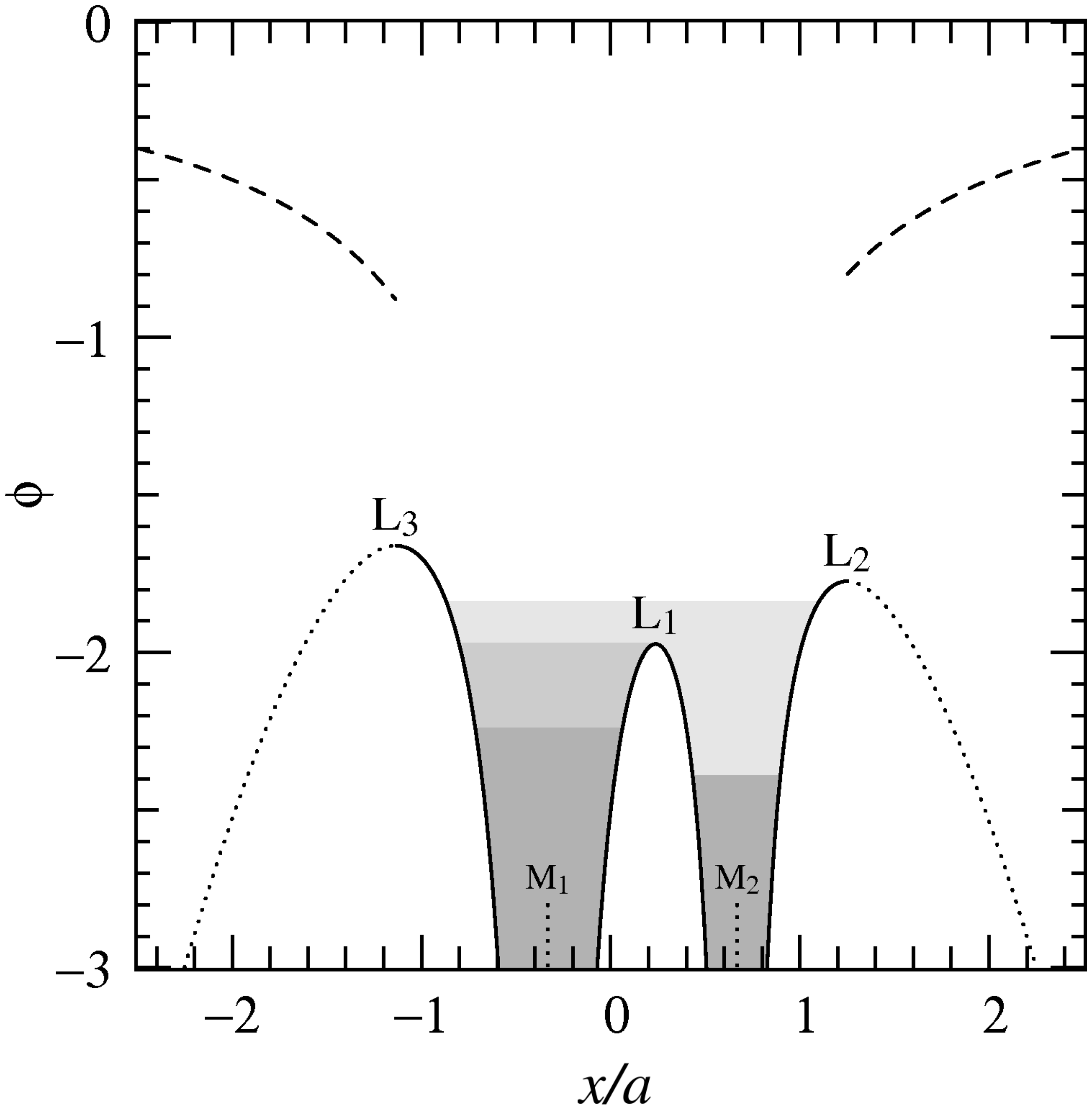
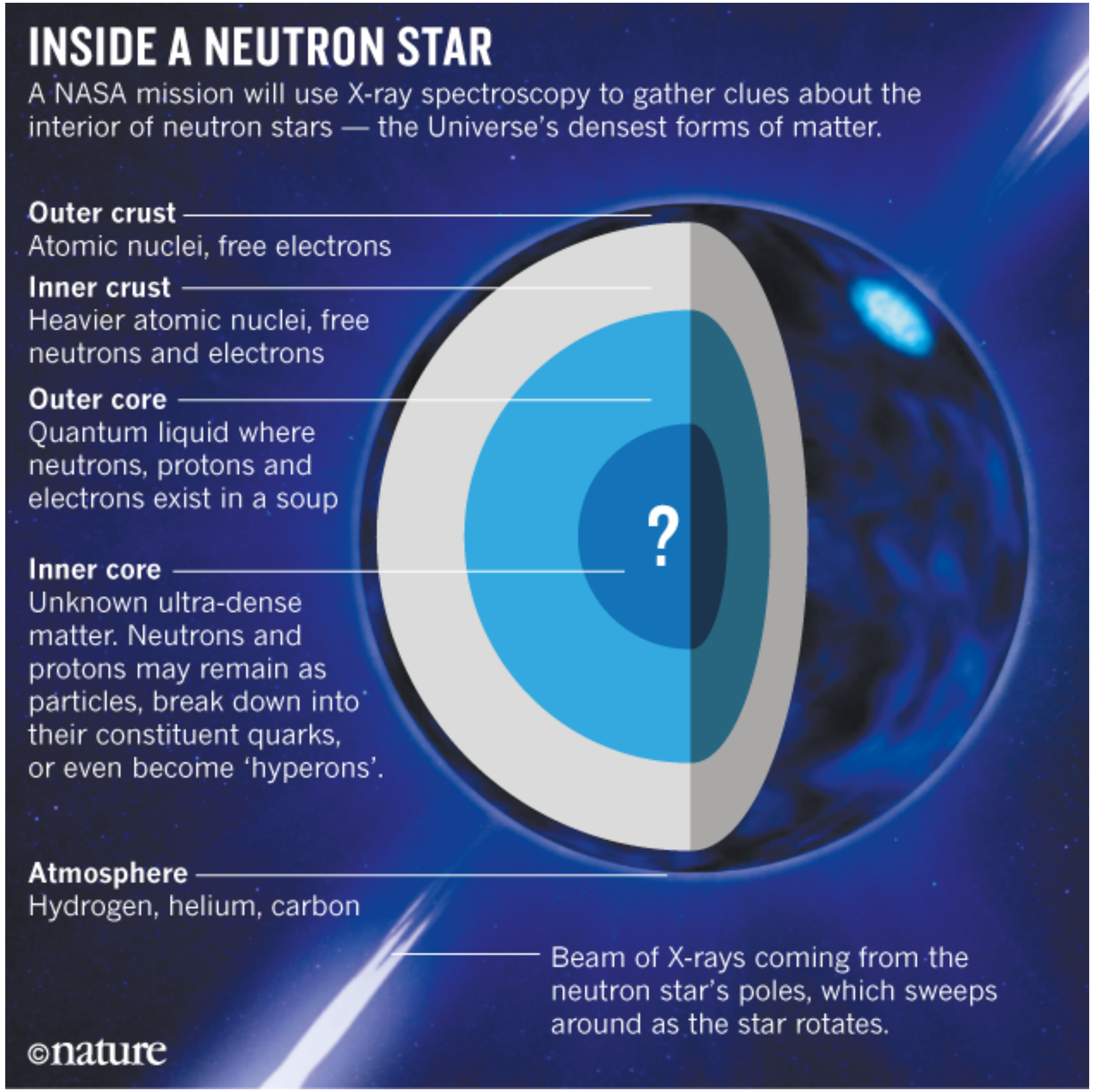


Figure 6.1. Shape of the Roche potential (solid line) along the line connecting the two stars, for a binary with mass ratio $q = M_1/M_2 = 2$. The horizontal scale is in units of the semi-major axis a, and the potential ϕ is in units of $G(M_1 + M_2)/a$. The locations of the centres of mass of the two stars are indicated by M_1 and M_2, and the Lagrangian points by L_1, L_2 and L_3. Gray scales indicate the three possible stable binary configurations: detached (dark), semi-detached (middle) and contact (light gray).

Matter located outside L_2 and L_3 cannot maintain corotation with the orbit, and the Roche potential (shown as dotted lines) loses its physical meaning. Such matter is still bound to the system, as indicated by the dashed lines that represent the gravitational potential of the binary at large distance.

from Onno Pols, http://www.astro.ru.nl/~onnop/education/binaries_utrecht_notes/

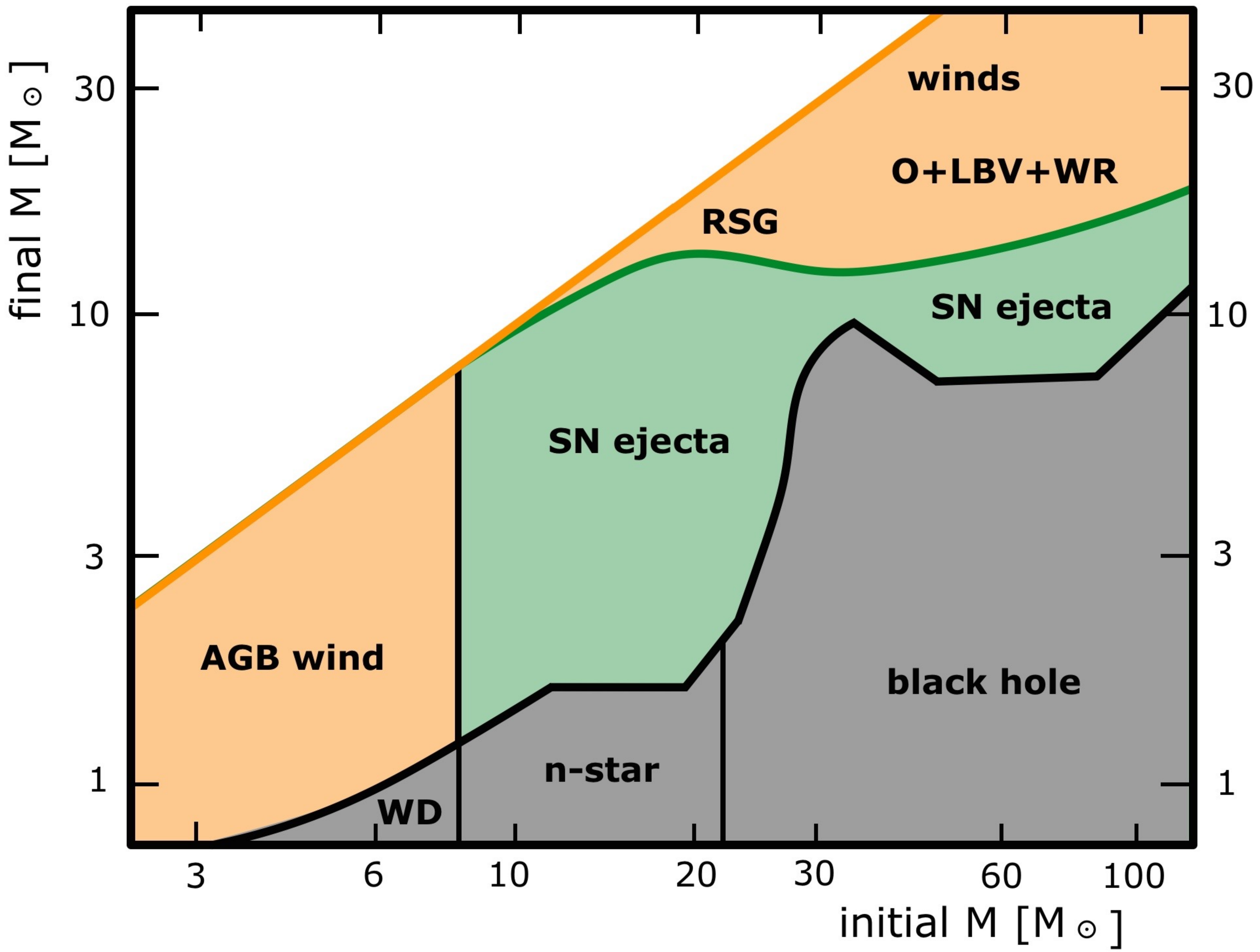
Neutron stars: interior structure



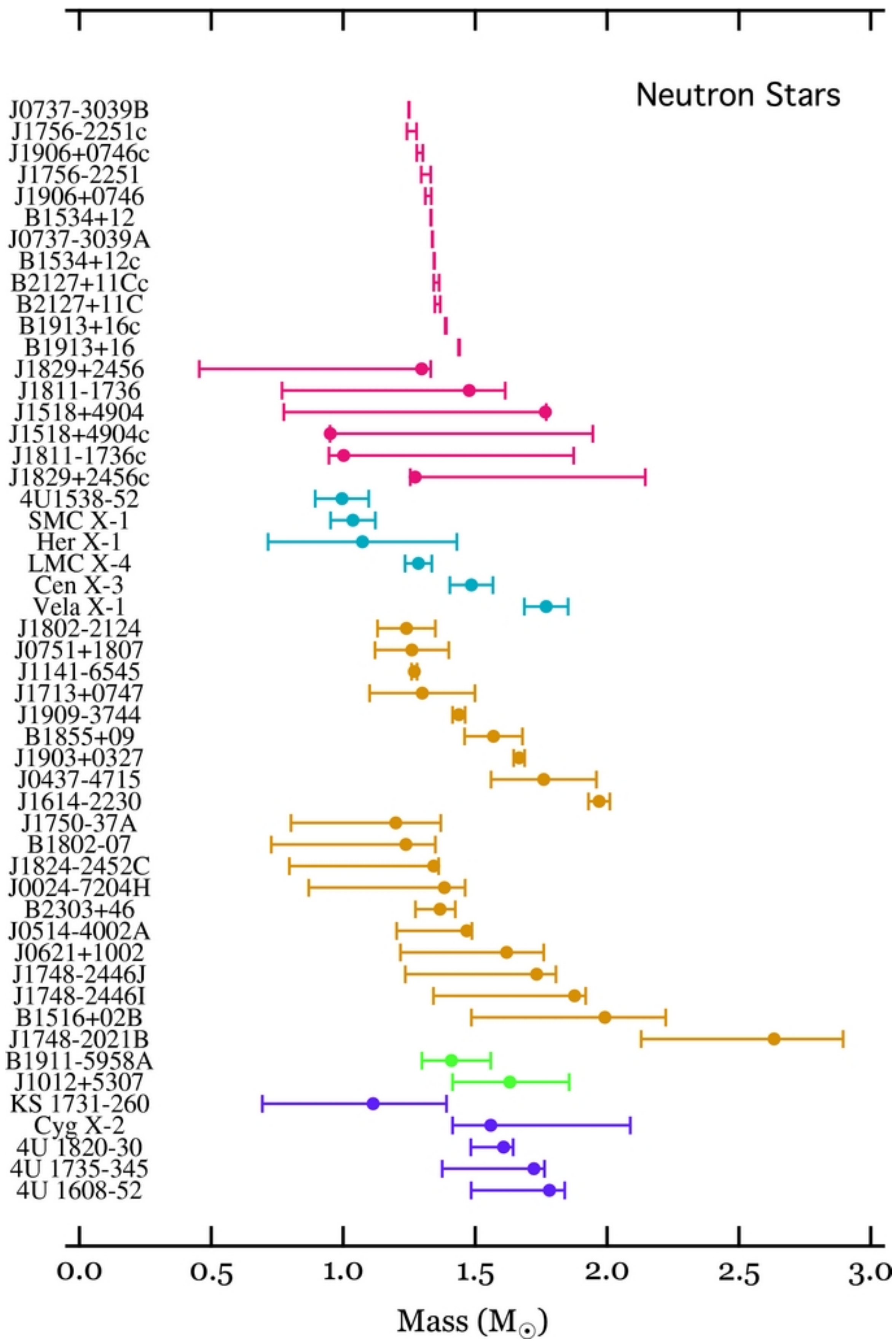
from https://www.nature.com/news/neutron-stars-set-to-open-their-heavy-hearts-1.22070

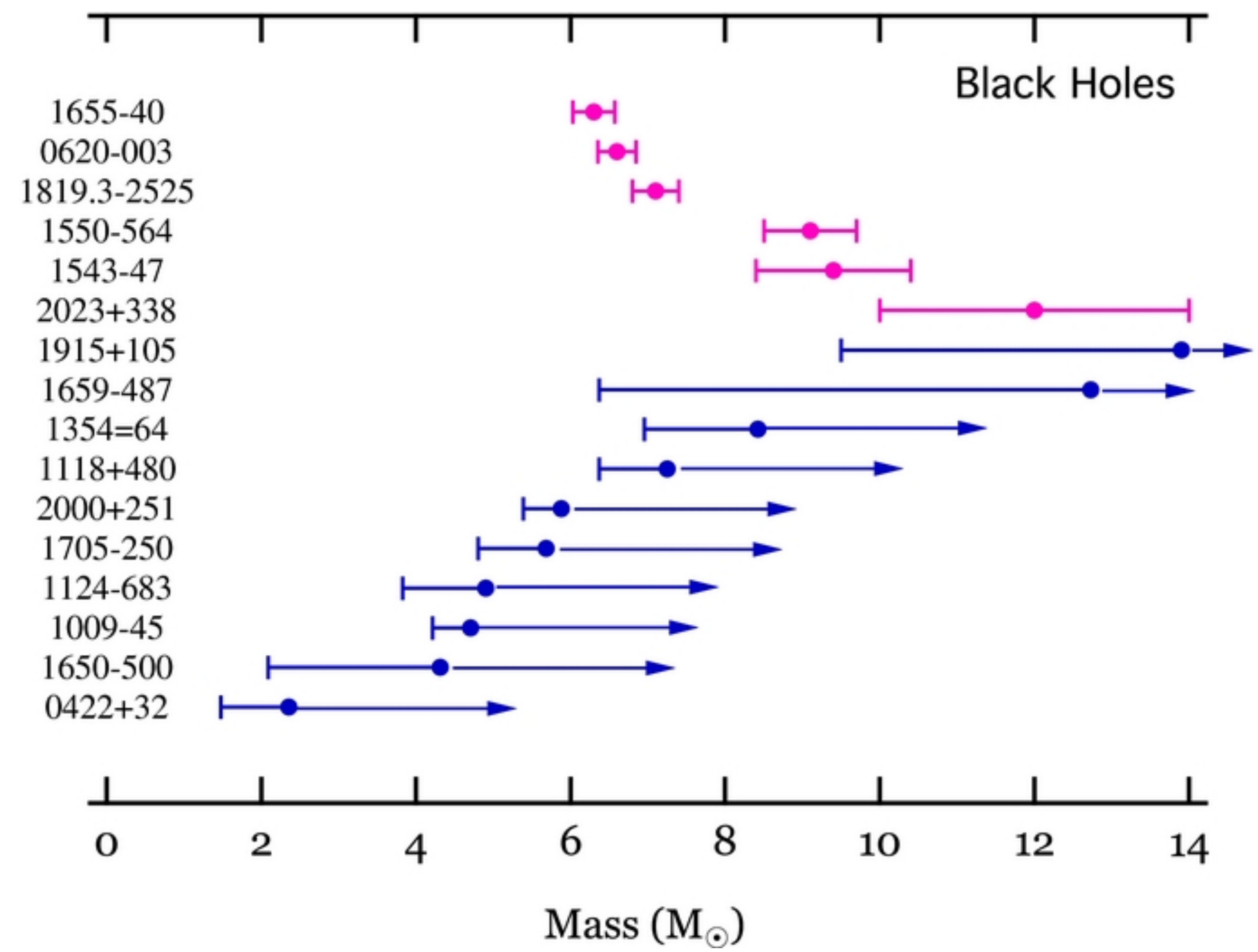
Supernovae - remnants

Our simplified model illustrates that stars with different initial masses end their lives differently…



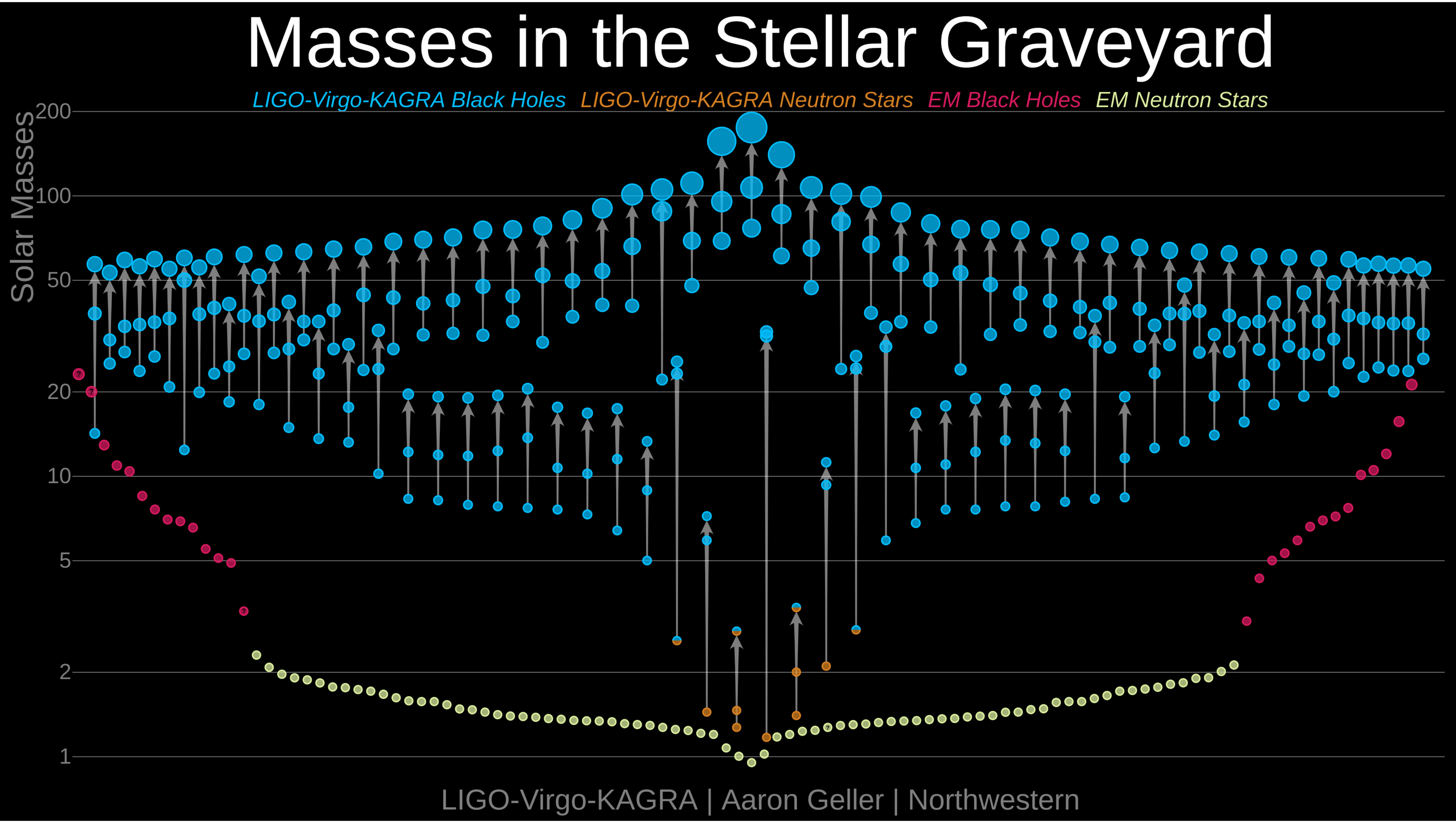
Neutron stars & black holes





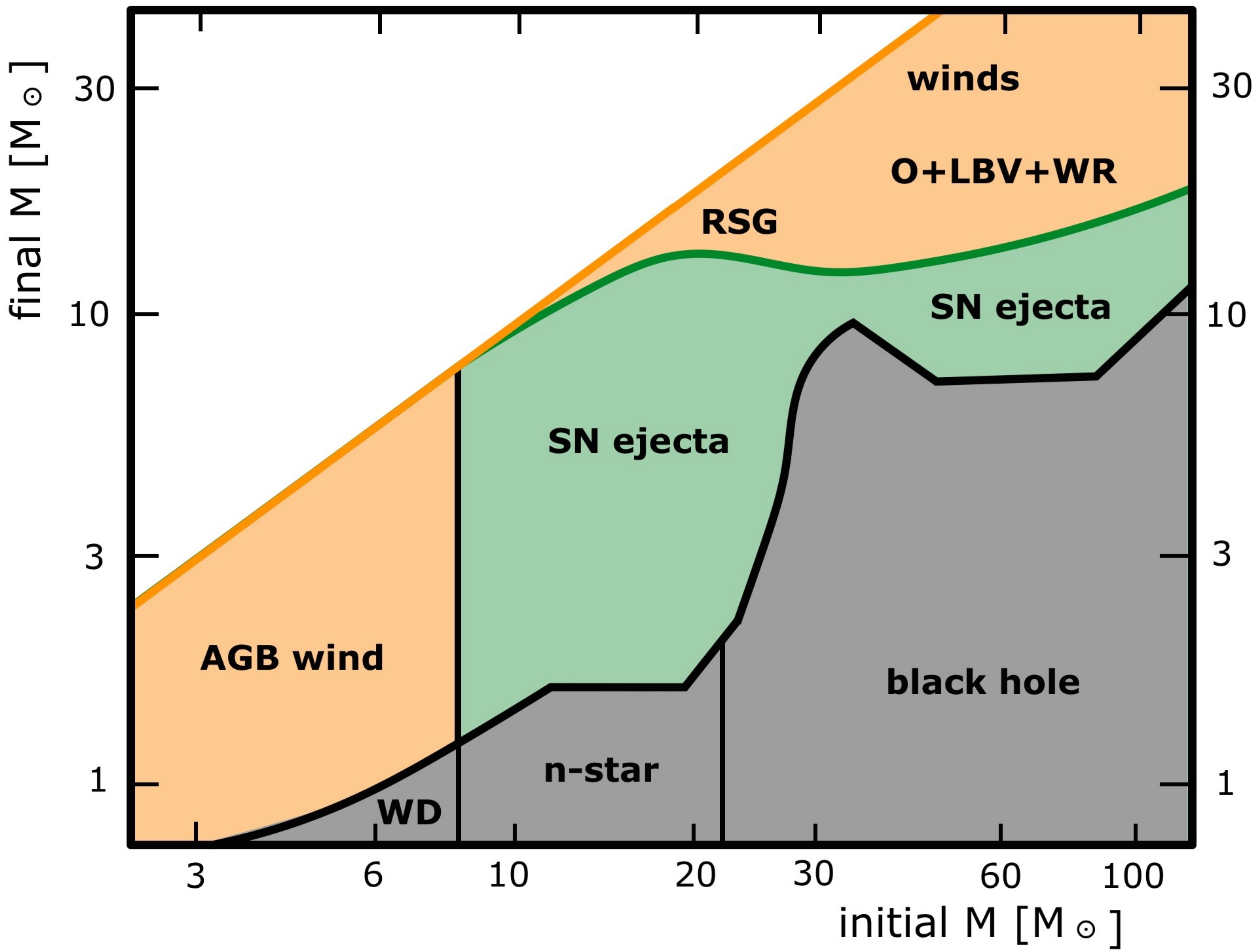
from Özel et al. (2012)

Neutron stars & black holes



Supernovae - remnants

Our simplified model illustrates that stars with different initial masses end their lives differently…

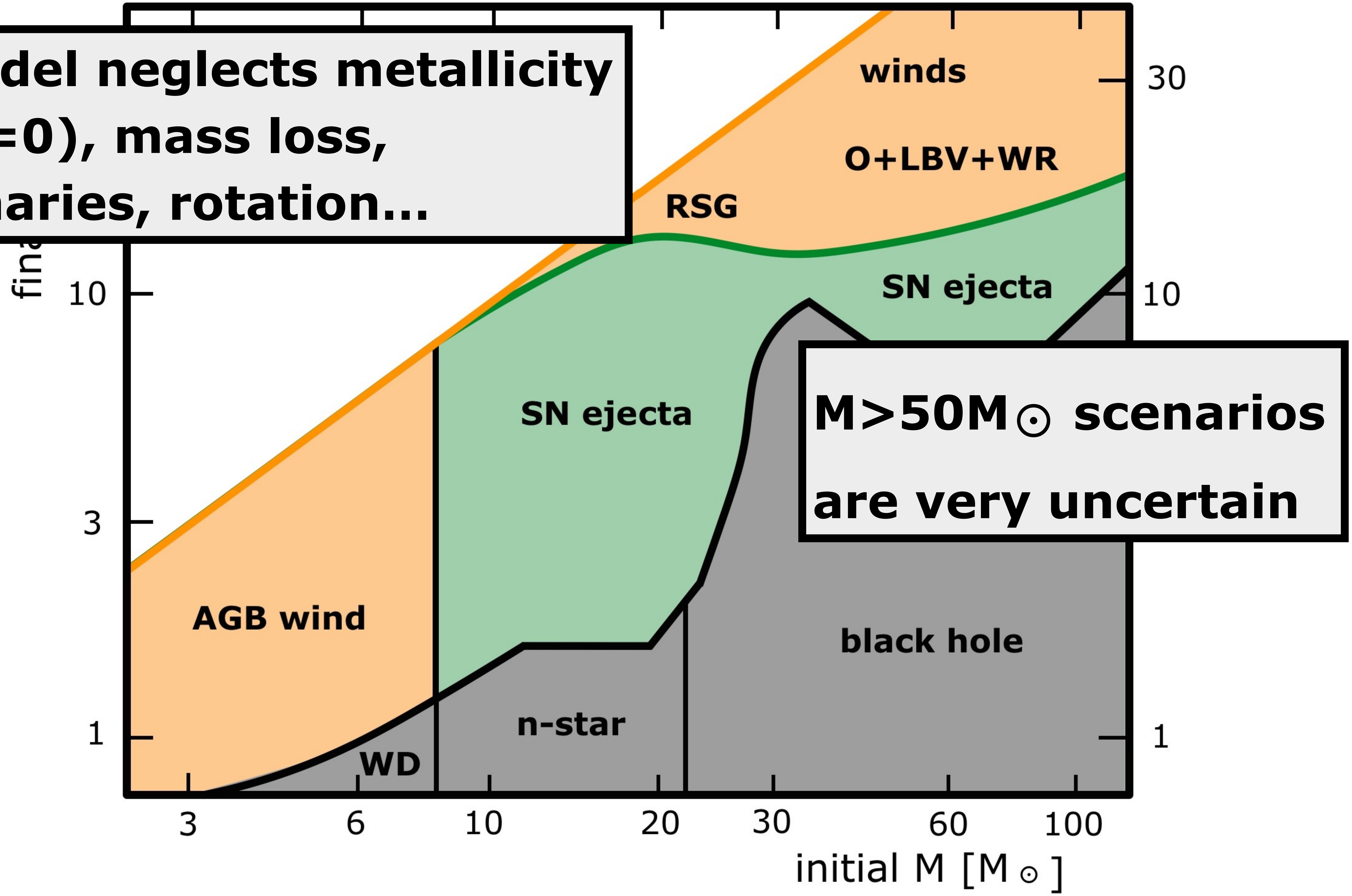


Supernovae - remnants

Our simplified model illustrates that stars with different initial masses end their lives differently…however…

Model neglects metallicity (Z=0), mass loss, binaries, rotation…

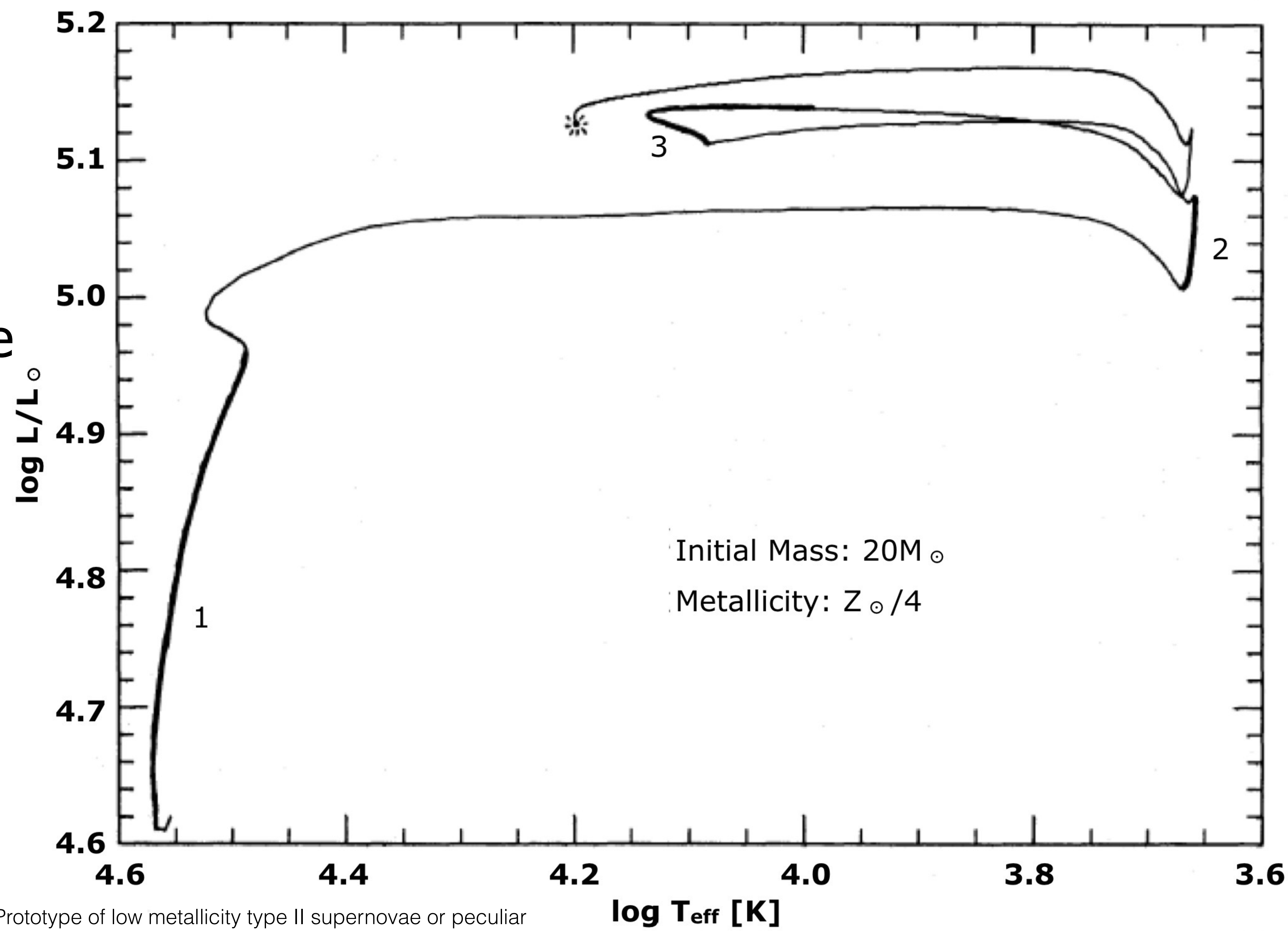
M>50M$_\odot$ scenarios are very uncertain



Supernova 1987A

SN 1987A in the LMC is the closest and best-studied example of a core-collapse supernova:

A single-star model can match the progenitor star but is dependent on the blue loop. More recent models adopt a pre-SN merger model with a binary companion…



SN 1987A in the LMC

from https://www.nasa.gov/feature/goddard/2017/the-dawn-of-a-new-era-for-supernova-1987a

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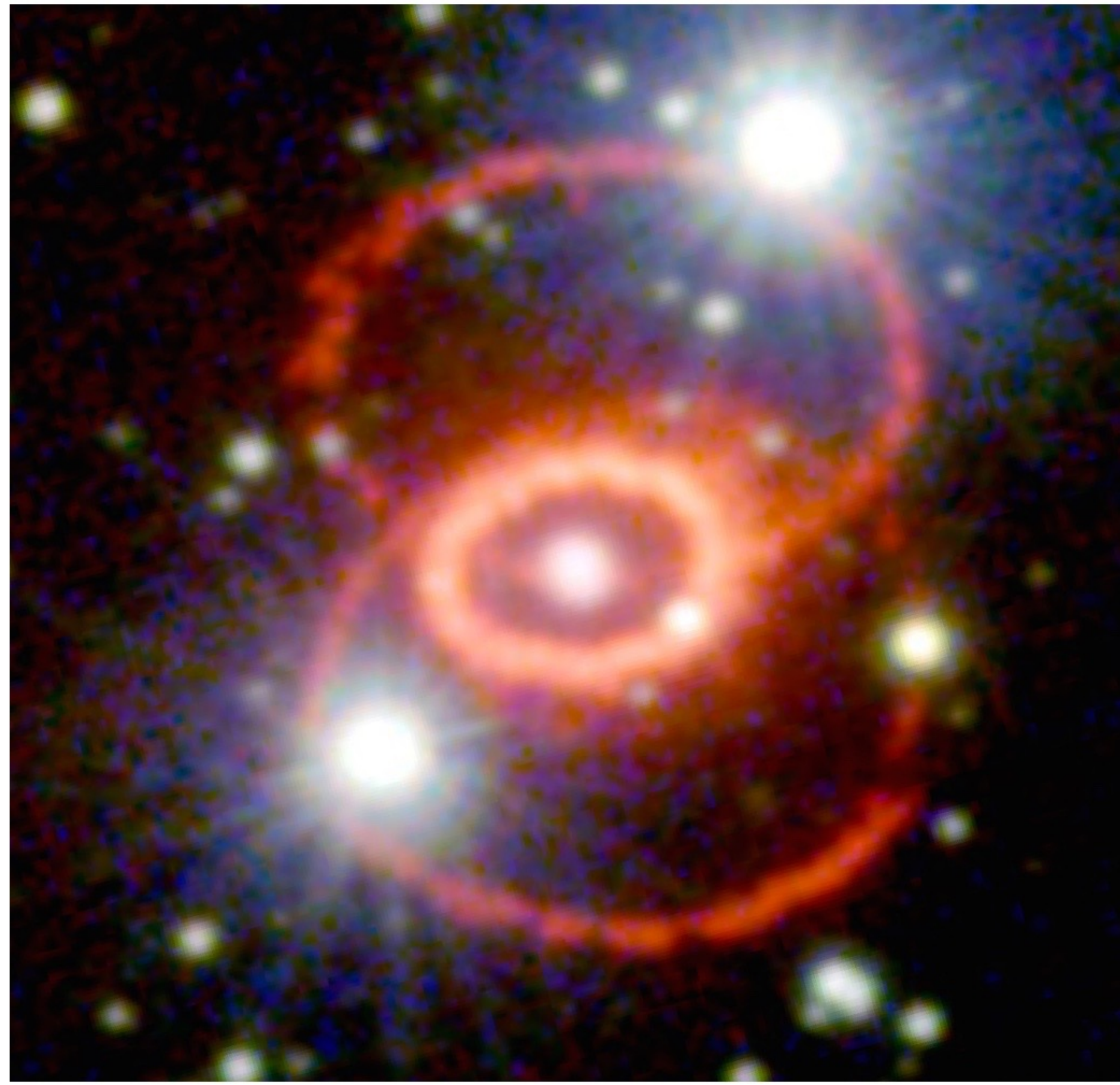
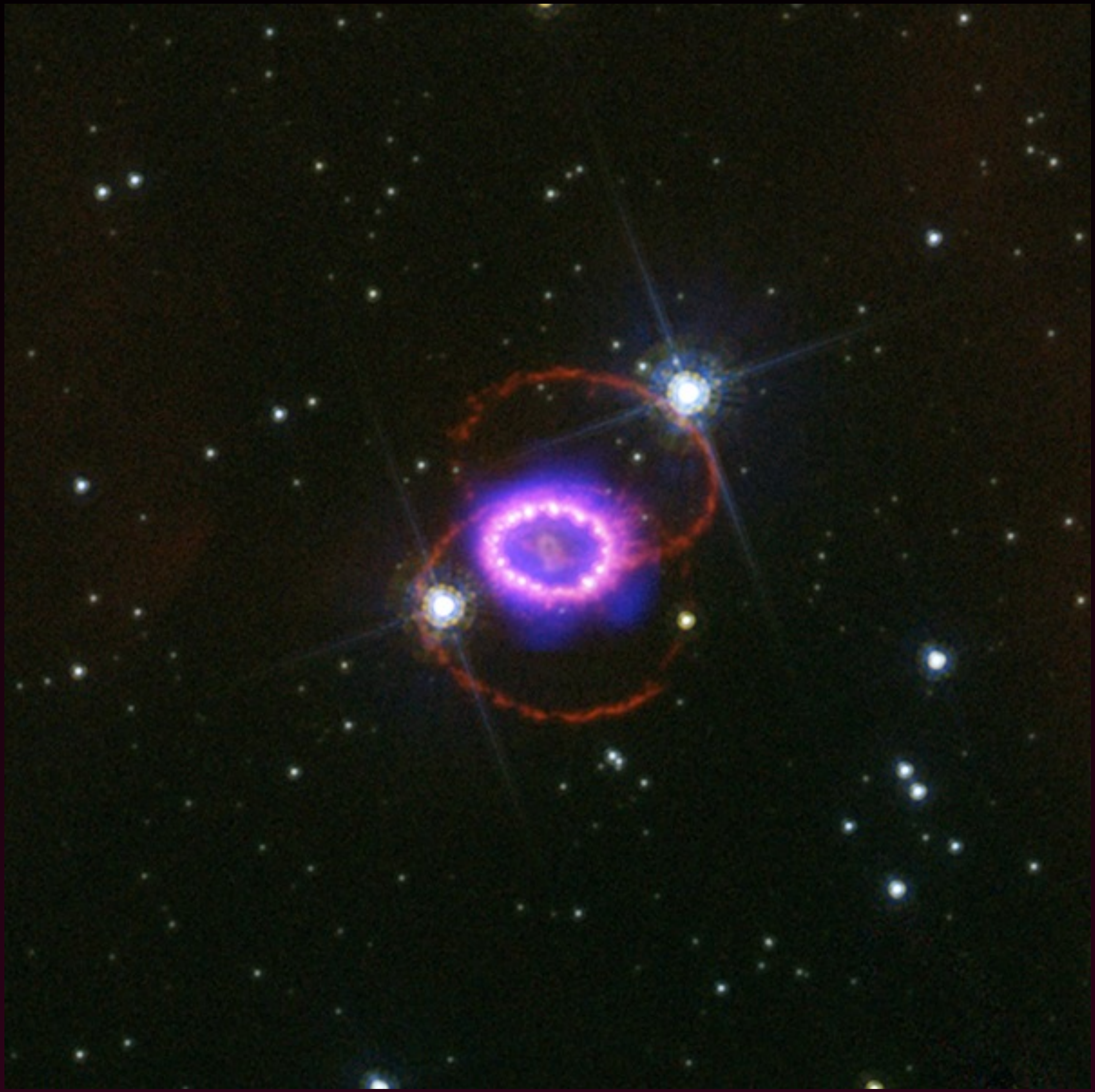


The triple-ring nebula remnant around SN 1987A originate from slow-moving (inner) and fast-moving (outer) material ejected from the star ~20,000y prior to explosion and illuminated by the UV flash of the SN explosion.

Figure courtesy of ESA.

SN 1987A in the LMC



optical and x-ray image of SN 1987A taken in 2007
from http://chandra.harvard.edu/photo/2007/sn87a/

SN 1987A in the LMC

1994

from https://www.nasa.gov/feature/goddard/2017/the-dawn-of-a-new-era-for-supernova-1987a

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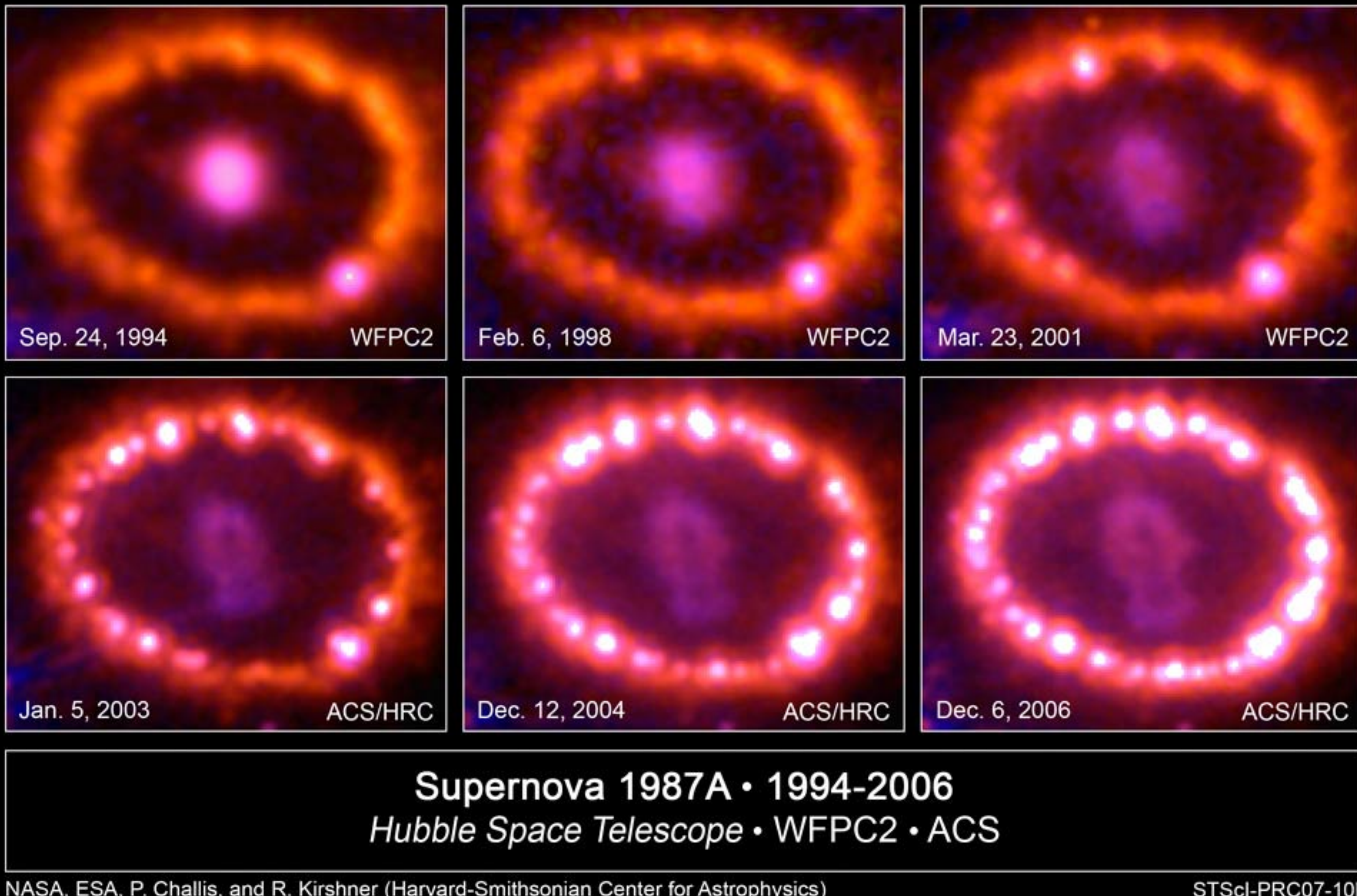
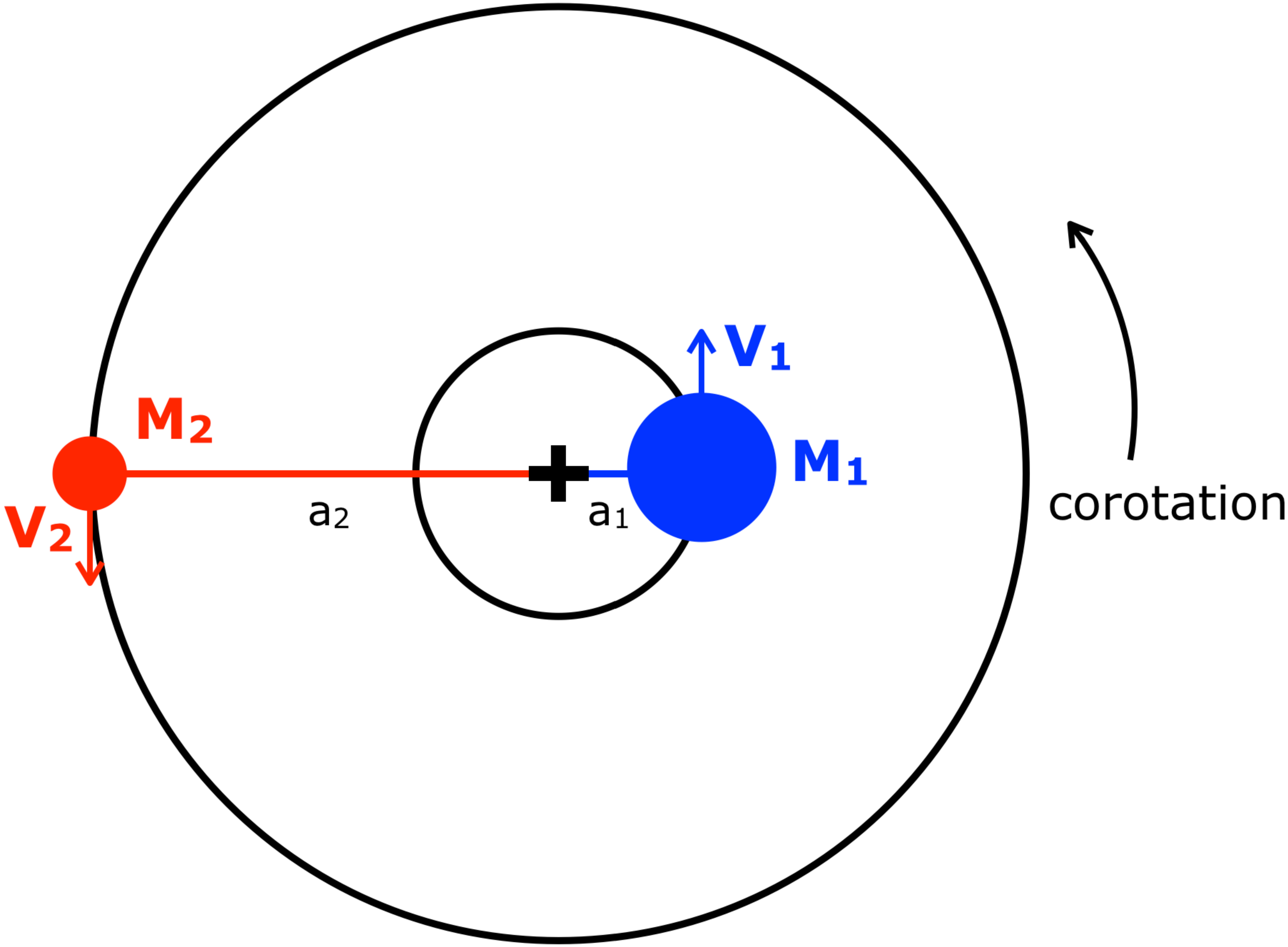


from http://hubblesite.org/gallery/album/star/supernova/pr2007010b/npp/all/

Stars in Binaries: Principles

Periods and angular momentum



Stars in Binaries: Principles

Periods and angular momentum

The period of a binary in a circular orbit is described by:

$$\left(\frac{2\pi}{P}\right)^2 = \omega^2 = \frac{G(M_1 + M_2)}{a^3}$$ (Kepler's 3rd Law)

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The angular momentum of the system around the center of gravity is:

$$J = M_1\, a_1\, \mathrm{v}_1 + M_2\, a_2\, \mathrm{v}_2$$

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$$M_1\, a_1 = M_2\, a_2 = \frac{M_1\, M_2}{(M_1 + M_2)}\, a$$

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The velocities of the system are:

$$\mathrm{v} = \mathrm{v}_1 + \mathrm{v}_2 = \omega a_1 + \omega a_2 = \omega a$$

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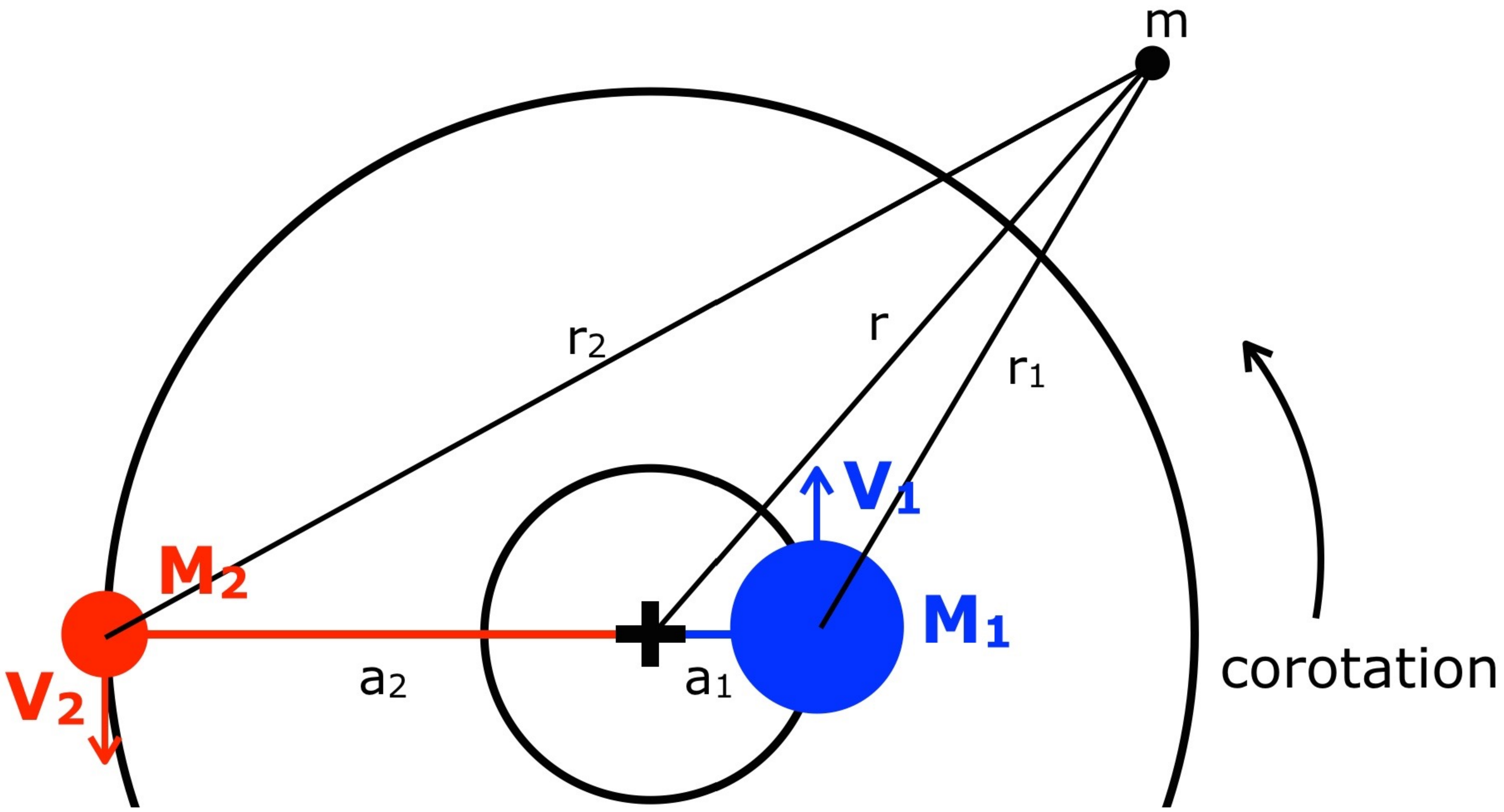
The velocities of the system are:

$$\mathrm{v} = \mathrm{v}_1 + \mathrm{v}_2 = \omega\, a_1 + \omega\, a_2 = \omega\, a$$

Substituting into the angular momentum equation gives:

$$\boxed{J^2 = G\, a\, \frac{M_1^2 M_2^2}{(M_1 + M_2)}}$$

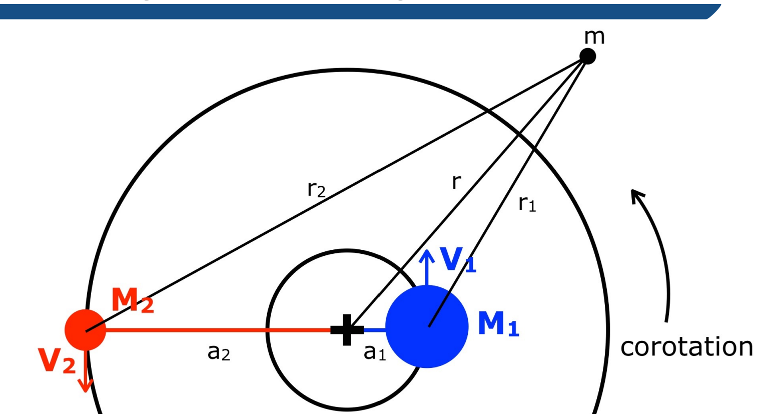
Stars in Binaries: Principles



Stars in Binaries: Principles

see derivations at
https://en.wikipedia.org/wiki/Rotating_reference_frame

position in the rotating frame $\vec{r}$



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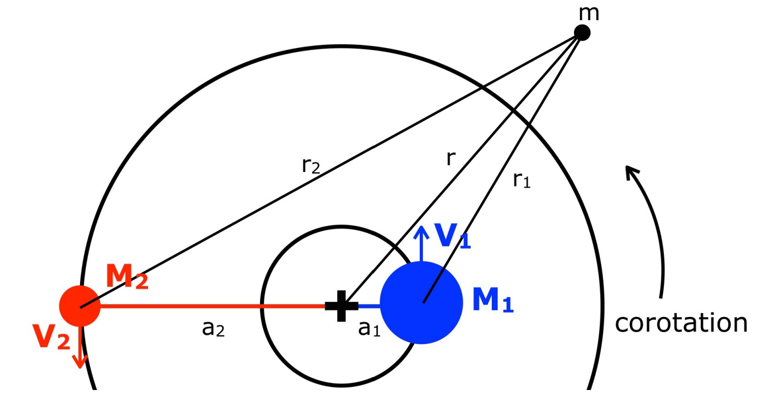

position in the rotating frame $\vec{r}$

velocity in the rotating frame $\vec{v} = \vec{v}_i - \vec{\omega} \times \vec{r}$

velocity in inertial frame

angular rotation vector

Stars in Binaries: Principles

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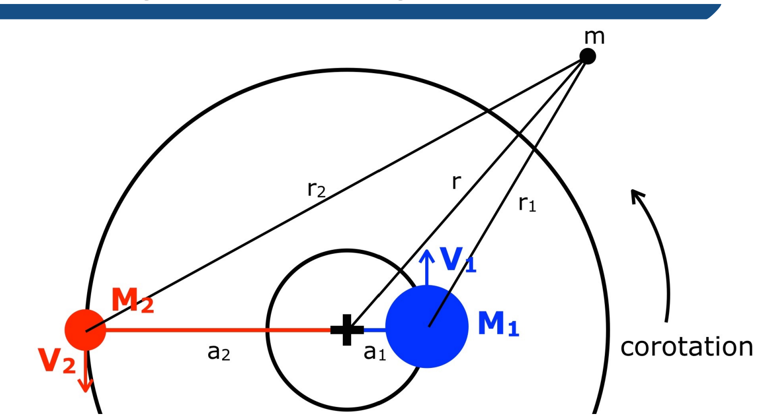

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acceleration in inertial frame

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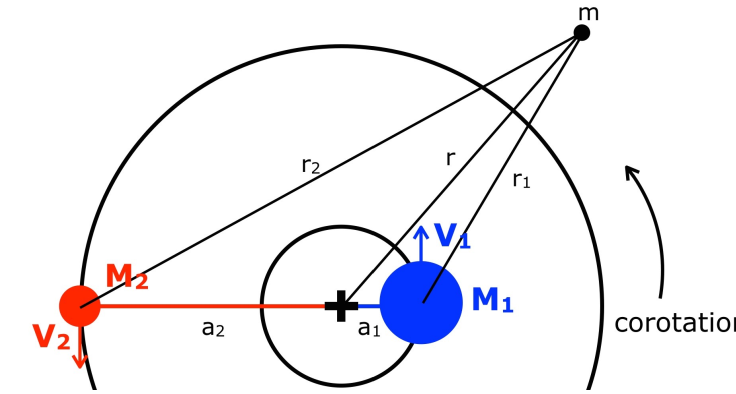

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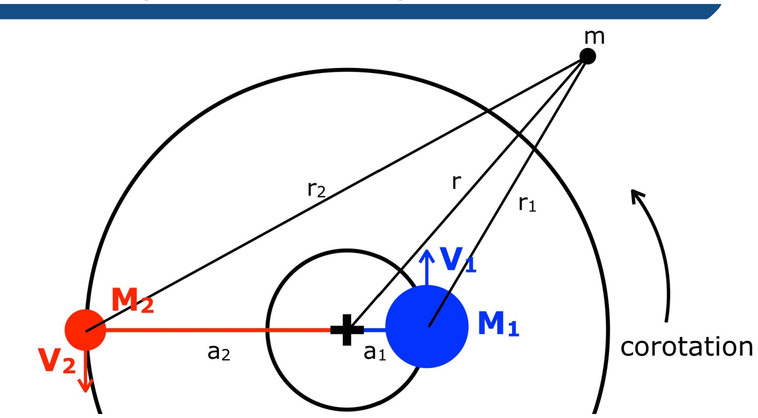

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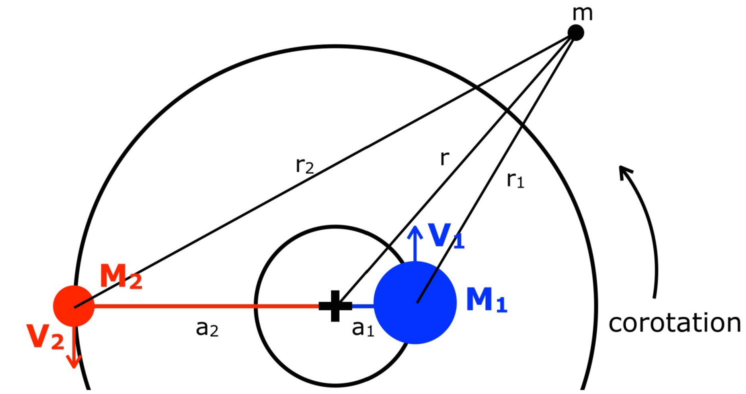

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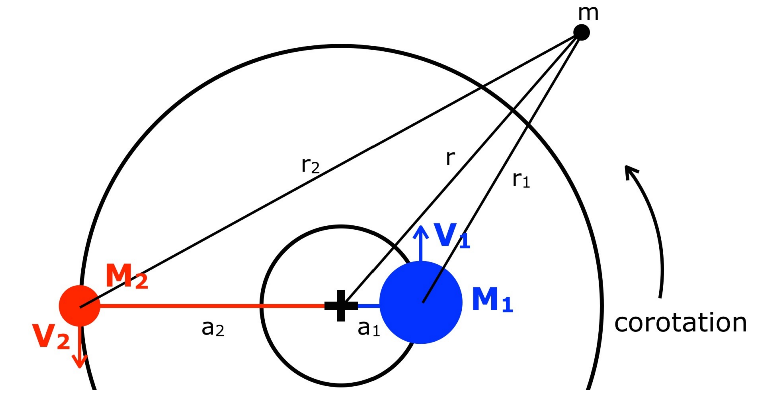

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velocity in inertial frame — angular rotation vector

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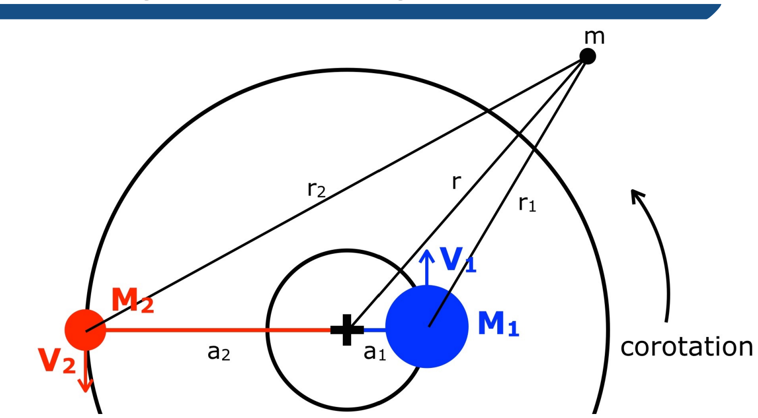

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neglect for now
(subdominant if v is low)

doesn't apply (rotation
rate is not changing)

Stars in Binaries: Principles

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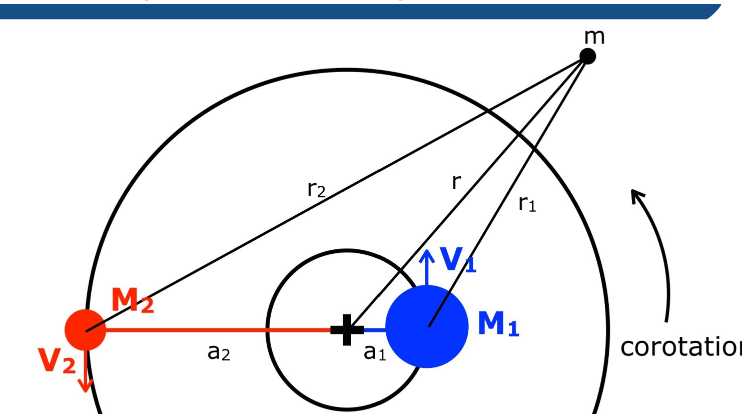

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velocity in inertial frame ($\vec{v}_i$); angular rotation vector ($\vec{\omega}$)

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Coriolis force: neglect for now (subdominant if v is low)

Euler force: doesn't apply (rotation rate is not changing)

If we consider motion in the plane of the two stars, so that $\vec{r}$ is orthogonal to $\vec{\omega}$, then $\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2\,\vec{r}$ and the centrifugal force term simplifies so that the effective force is

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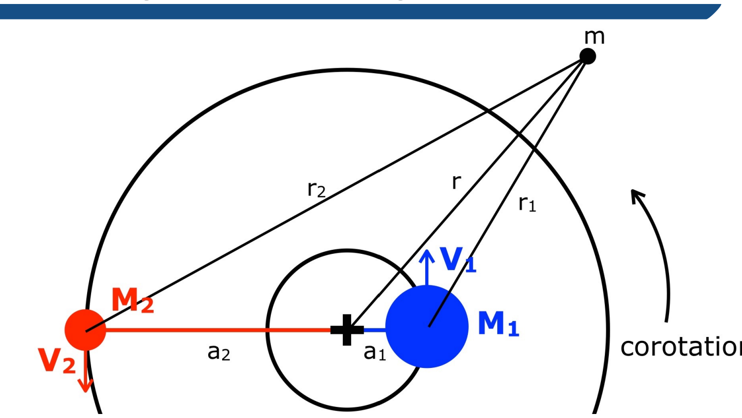

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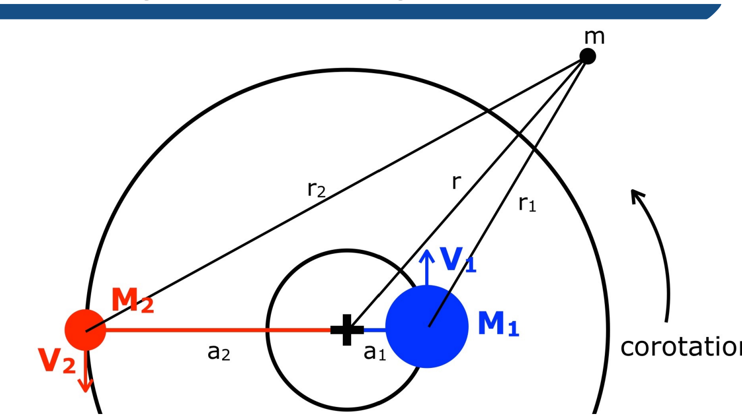

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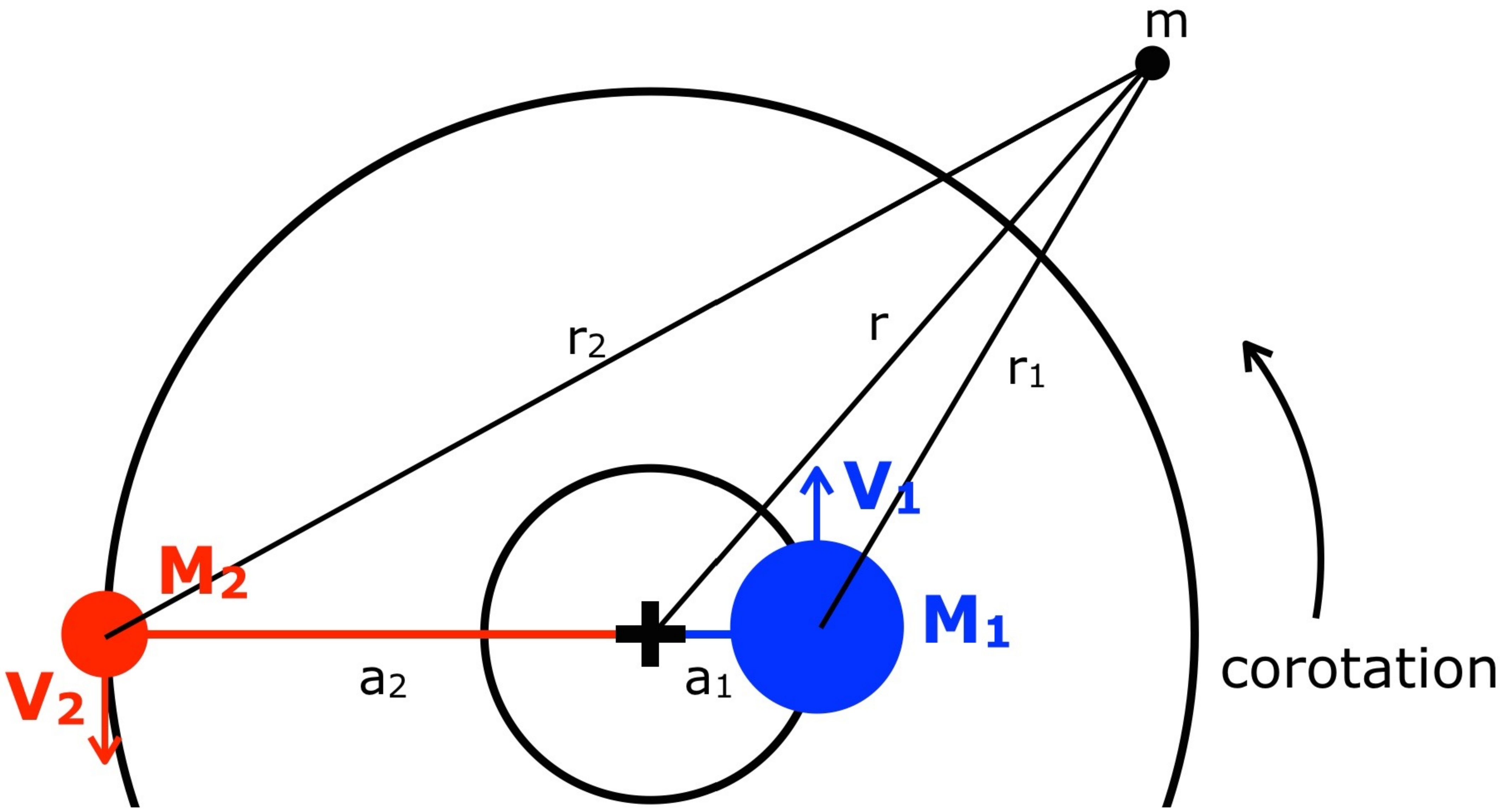
The effective potential energy is then
$$\begin{aligned} U_{\rm eff} &= -\int \vec{F}_{\rm eff}\cdot d\vec{r} \\ &= -\int \vec{F}_{\rm true}\cdot d\vec{r} - \int m\,\omega^2\,\vec{r}\cdot d\vec{r} \\ &= U_{\rm true} - \frac{1}{2}\,m\,\omega^2\,r^2 \end{aligned}$$

We know the true potential energy for the two stars, so the effective potential is

$$\Phi_{\rm eff} \equiv \frac{U_{\rm eff}}{m} = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\,\omega^2\,r^2$$

Stars in Binaries: Principles



Stars in Binaries: Principles

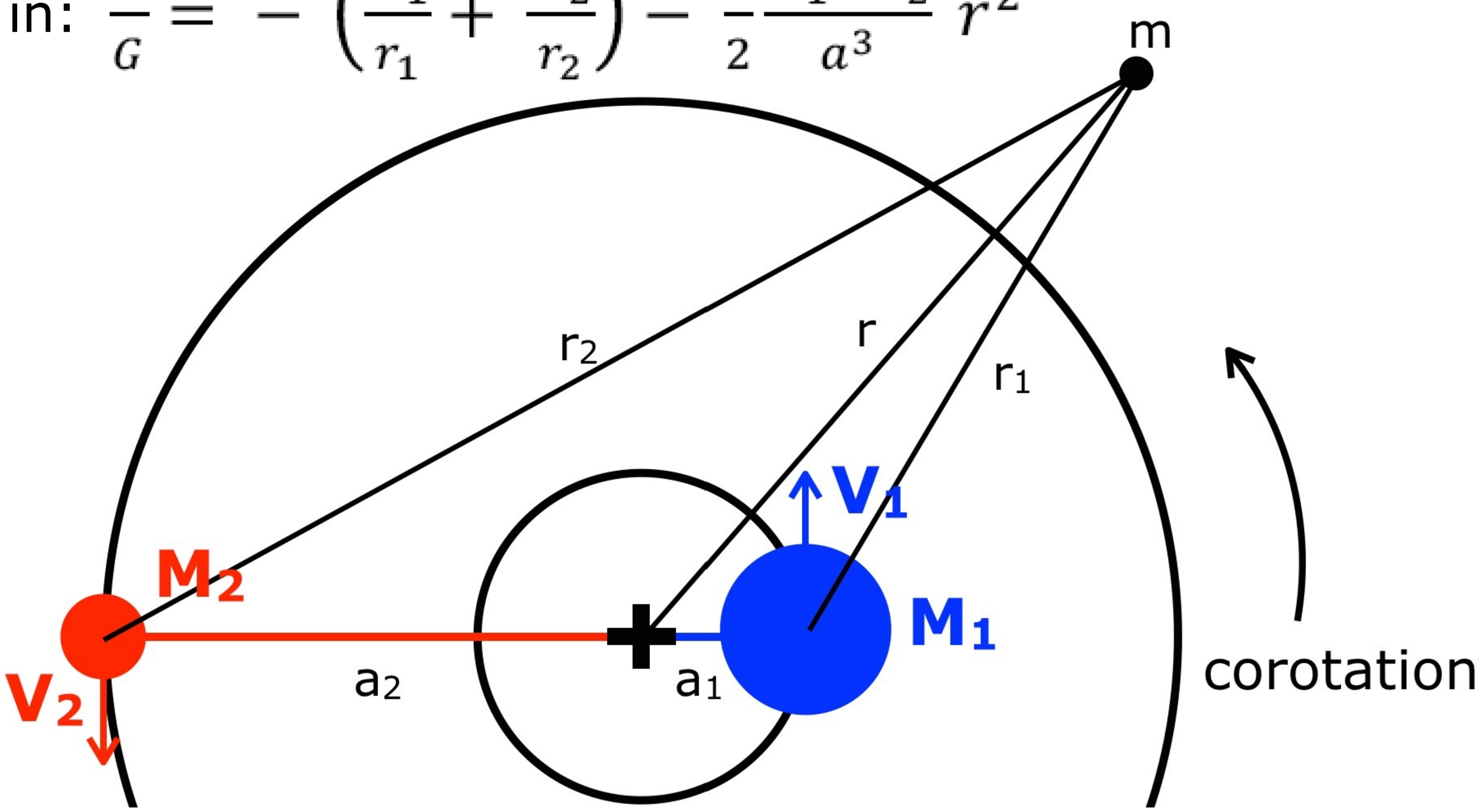
Equipotential surfaces of binaries

The particle located at *r* in the system has an effective potential energy of:

$$\Phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \Phi_c$$

where $-\Phi_c = \frac{1}{2}\,\omega^2\, r^2$ = potential of the centrifugal acceleration at *r*.

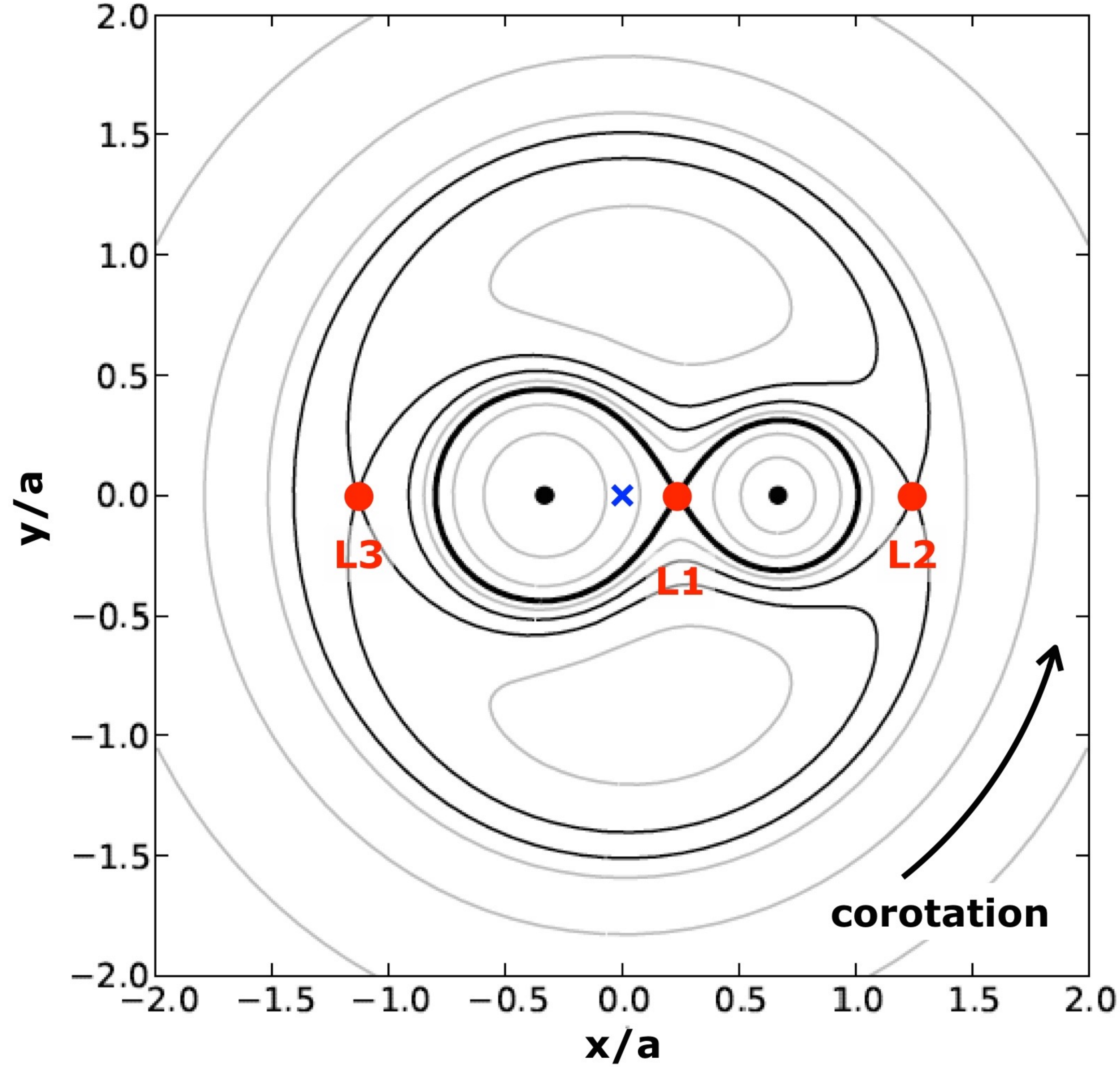
This results in: $\frac{\Phi}{G} = -\left(\frac{M_1}{r_1} + \frac{M_2}{r_2}\right) - \frac{1}{2}\frac{M_1+M_2}{a^3}\, r^2$



Stars in Binaries: Principles

Equipotential surfaces of binaries

Equipotential surface: locus of points with same value of Ω.



Stars in Binaries: Principles

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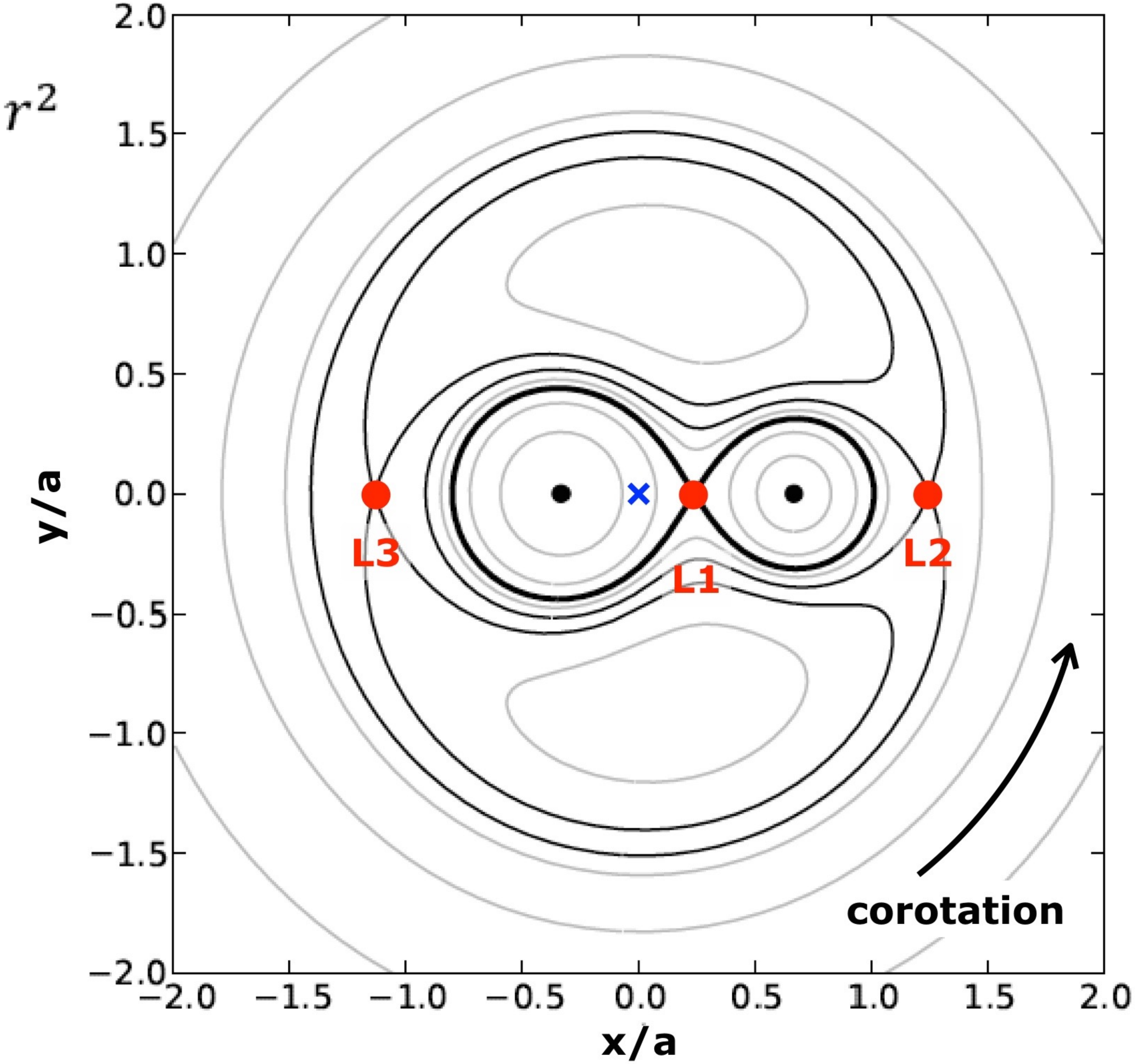
In 3D:

$$\frac{\Phi}{G} = -\left(\frac{M_1}{r_1^z} + \frac{M_2}{r_2^z}\right) - \frac{1}{2}\frac{M_1+M_2}{a^3}\, r^2$$

where $(r_1^z)^2 = r_1^2 + z^2$

$$(r_2^z)^2 = r_2^2 + z^2$$



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Equipotential surface: locus of points with same value of Ω.

In 3D:

$$\frac{\Phi}{G} = -\left(\frac{M_1}{r_1^z} + \frac{M_2}{r_2^z}\right) - \frac{1}{2}\frac{M_1+M_2}{a^3}\, r^2$$

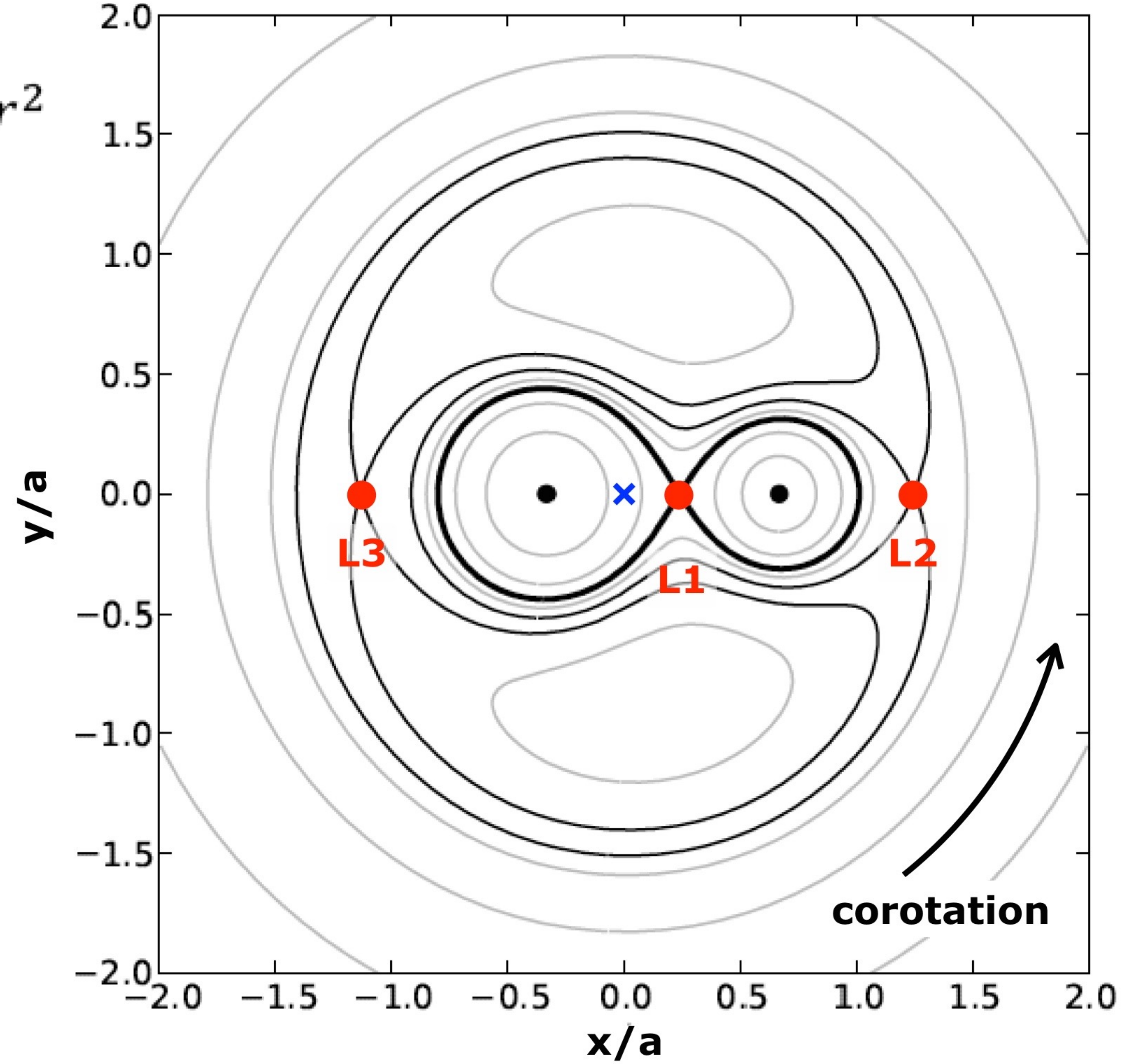
where $(r_1^z)^2 = r_1^2 + z^2$

$$(r_2^z)^2 = r_2^2 + z^2$$

The thick line is the **Roche lobe**, the tightest equipotential surface that includes both stars.

The effects of binary evolution depend on the volume of the Roche lobe of each star:

$$\mathrm{Volume(Roche\ lobe)} = \frac{4\pi}{3}\, R_L^3$$



Stars in Binaries: Principles

Equipotential surfaces of binaries

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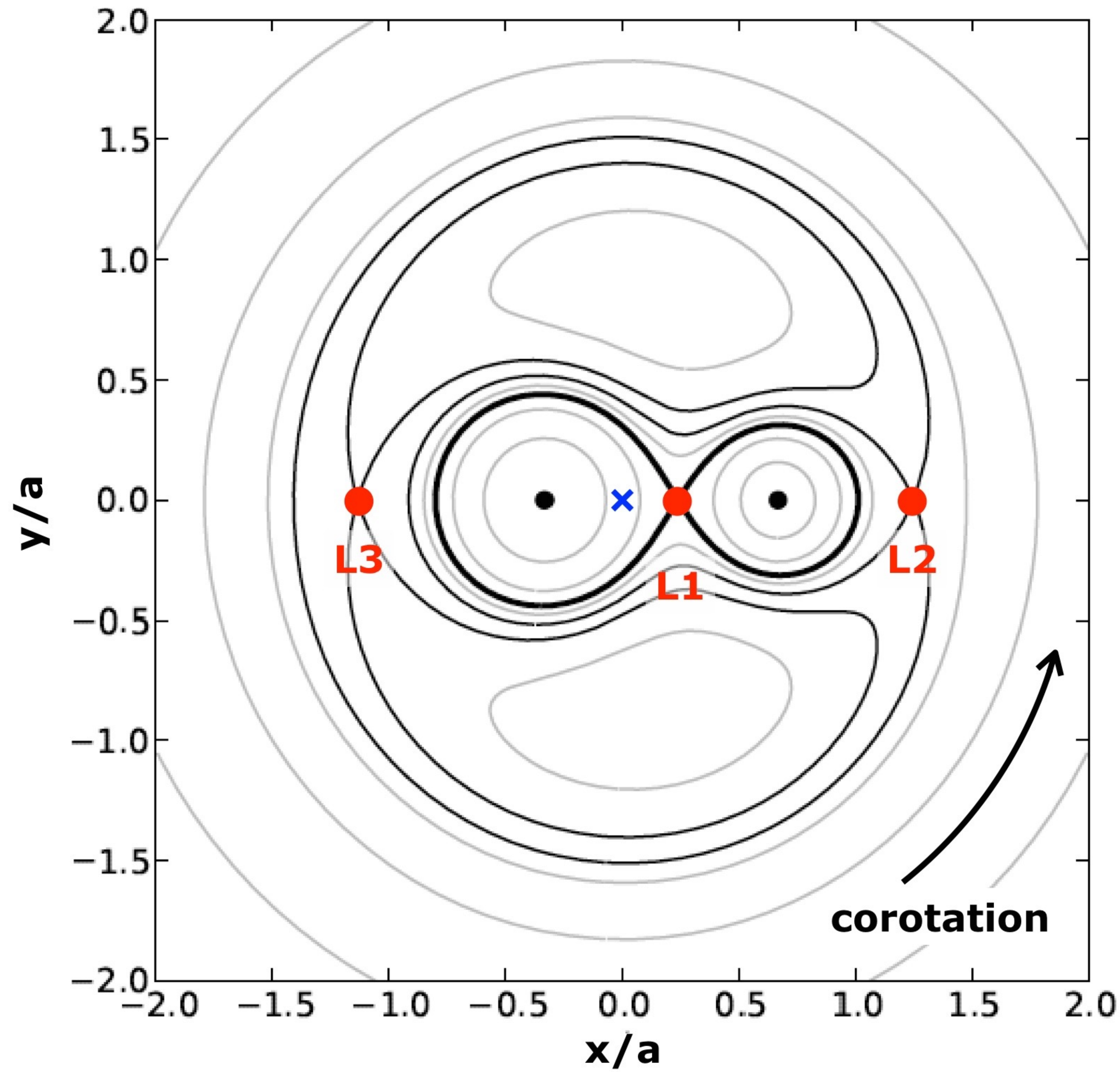

Stars in Binaries: Principles

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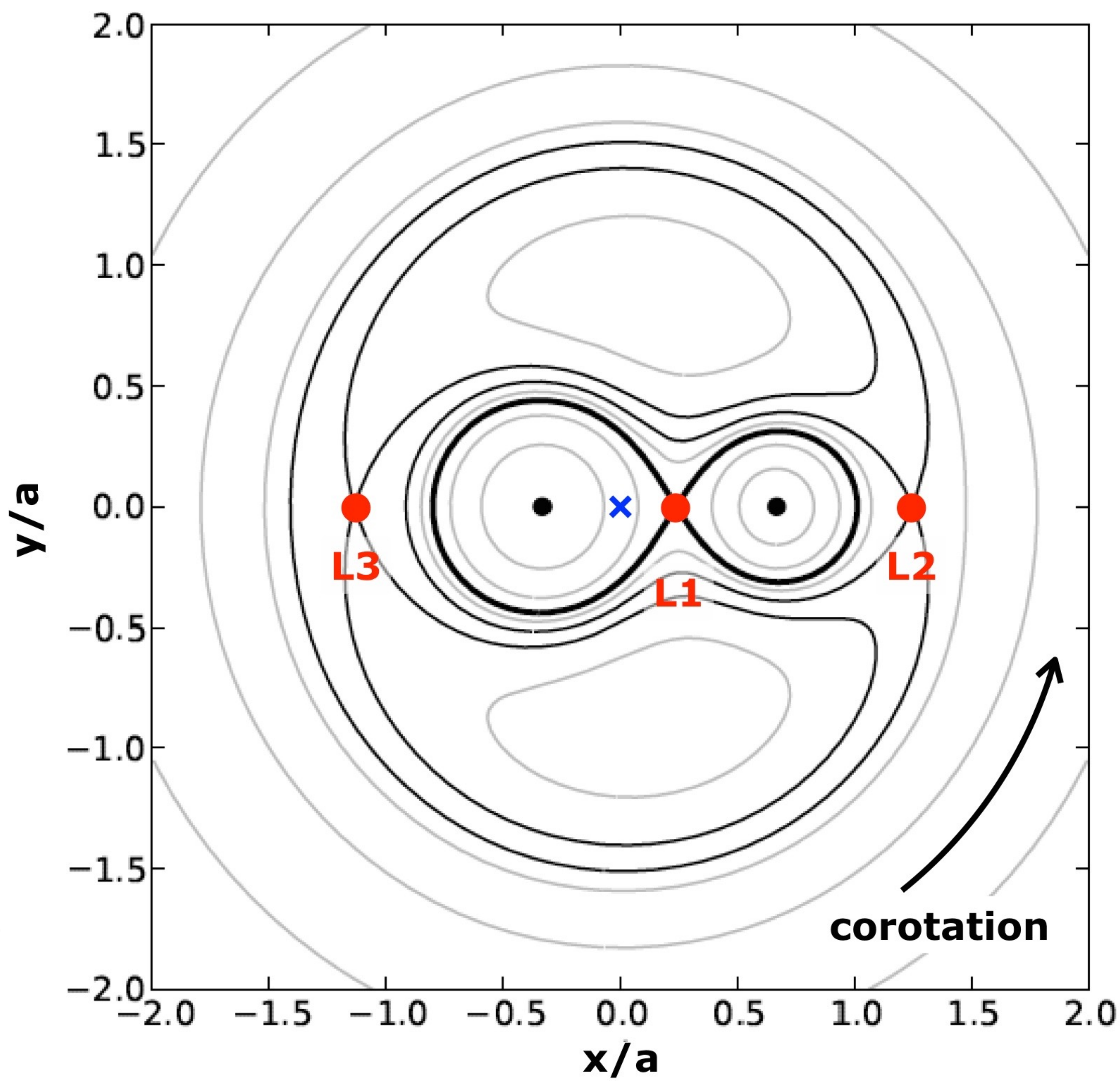
$$\text{Volume(Roche lobe)} = \frac{4\pi}{3} R_L^3$$

$$\frac{R_{L,1}}{a} \cong \frac{0.49x^{2/3}}{0.6\,x^{2/3}+ln\left(1+x^{1/3}\right)}$$

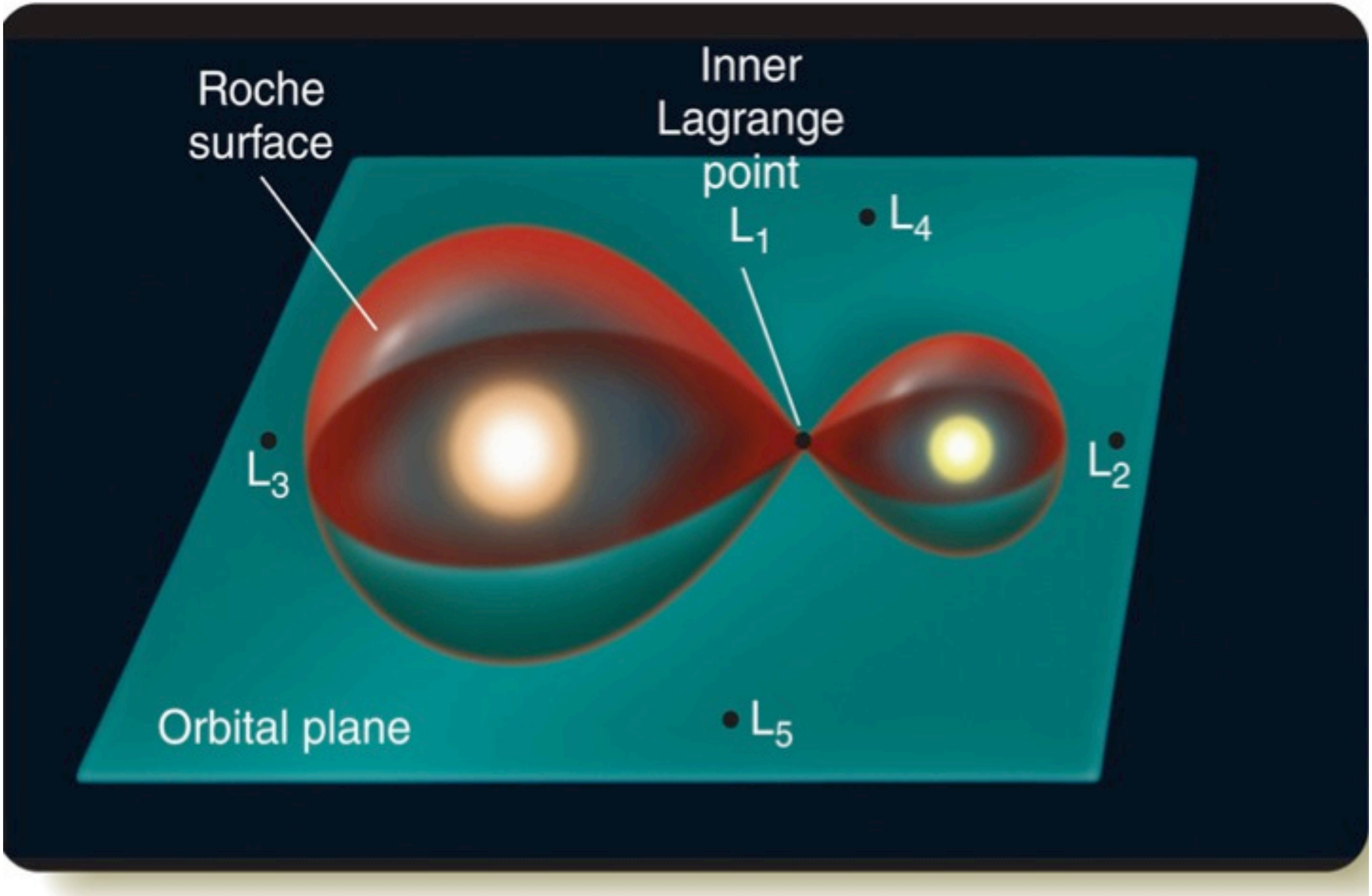
with $x = M_1 / M_2$

$$\frac{R_{L,2}}{a} \cong \frac{0.49y^{2/3}}{0.6\,y^{2/3}+ln\left(1+y^{1/3}\right)}$$

with $y = M_2 / M_1$



Stars in Binaries: Roche lobe

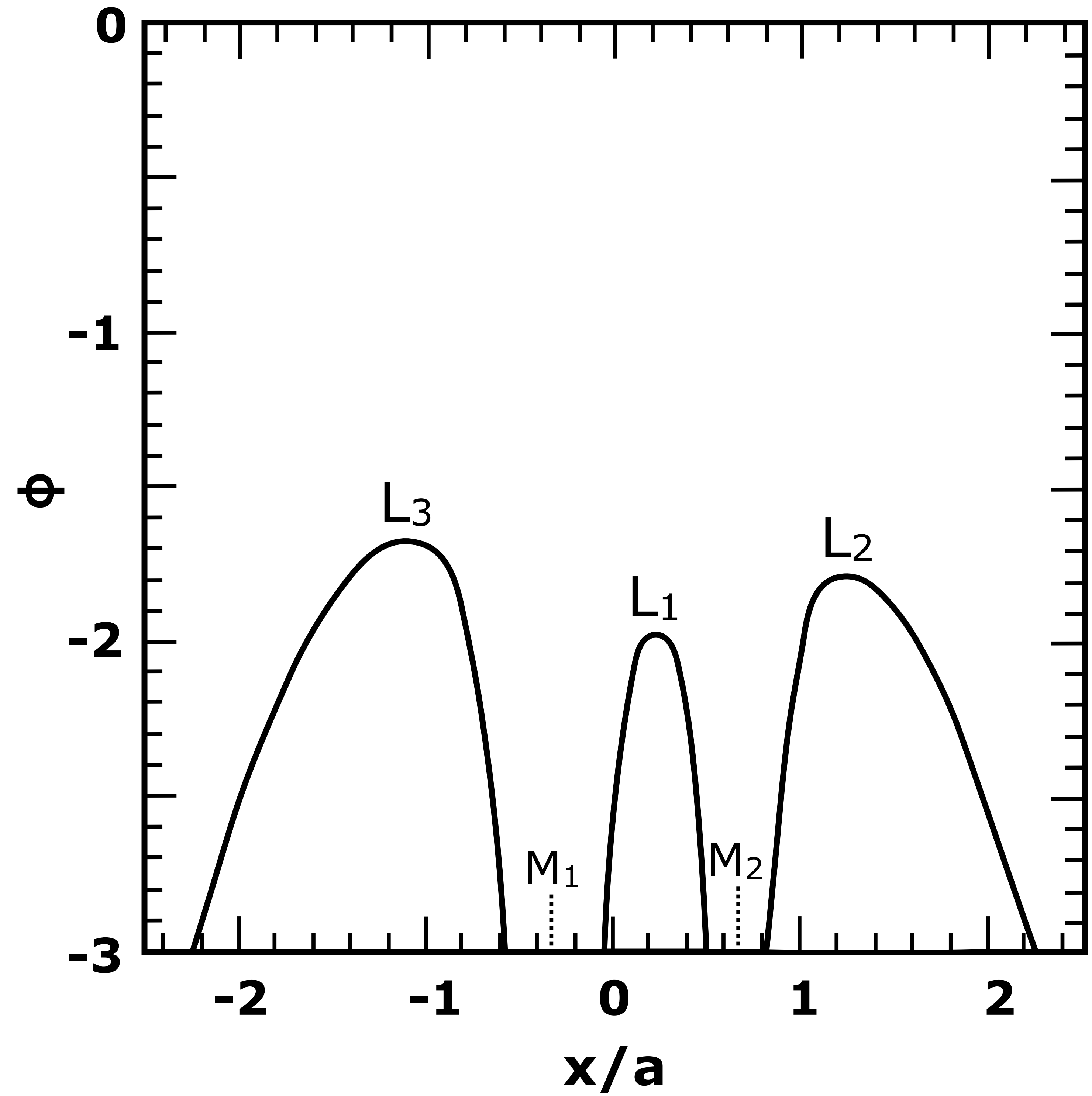


from http://crab0.astr.nthu.edu.tw/~hchang/ga1/ch10-05.htm (see also L&L eqn. 28.8)

Stars in Binaries: Principles

Contact phases

Binaries interact when the size of one of the two components reaches or overflows its Roche lobe.



Adapted from Onno Pols, 2011.

Stars in Binaries: Principles

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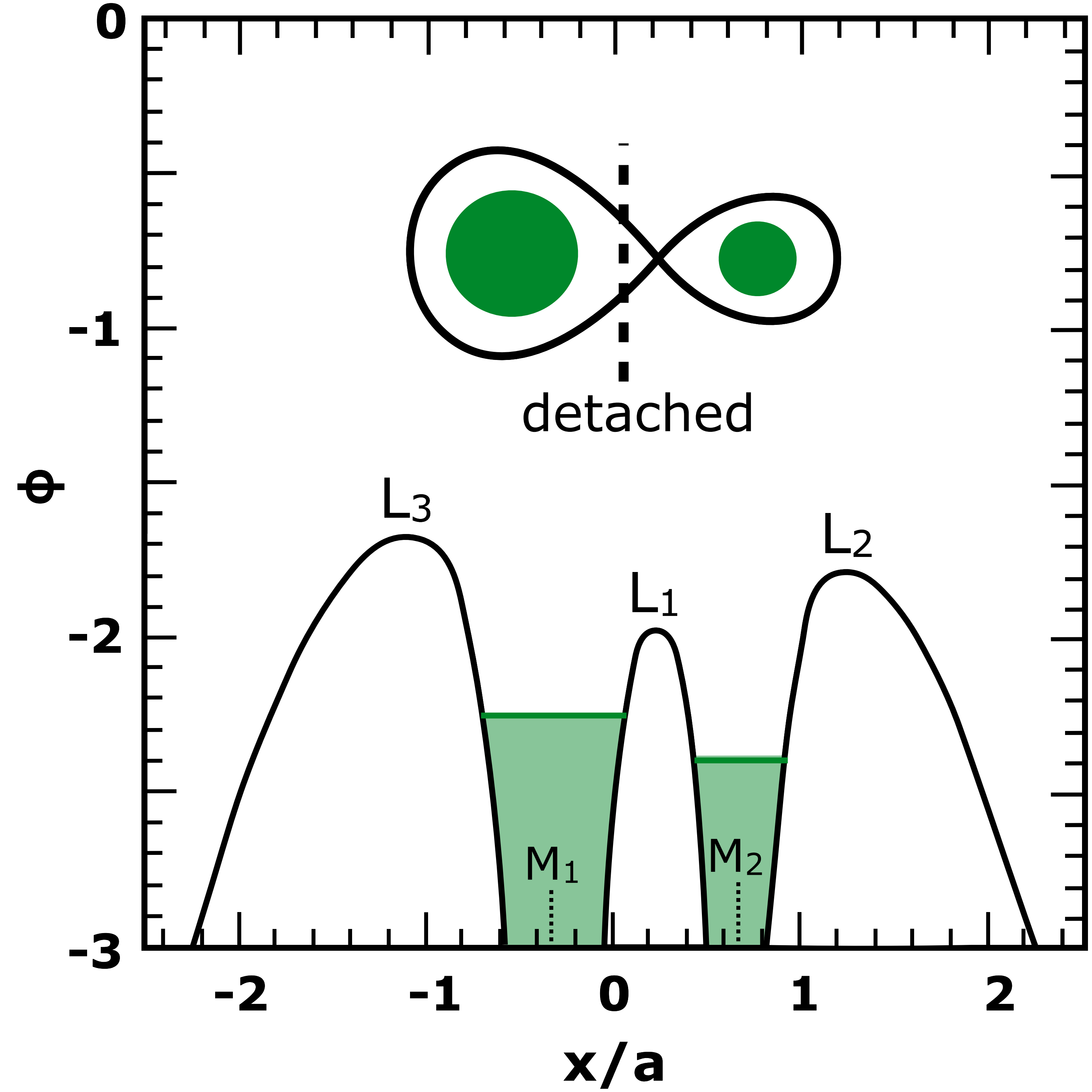
Detached system: both stars safely fit inside their Roche lobes; matter cannot flow between them



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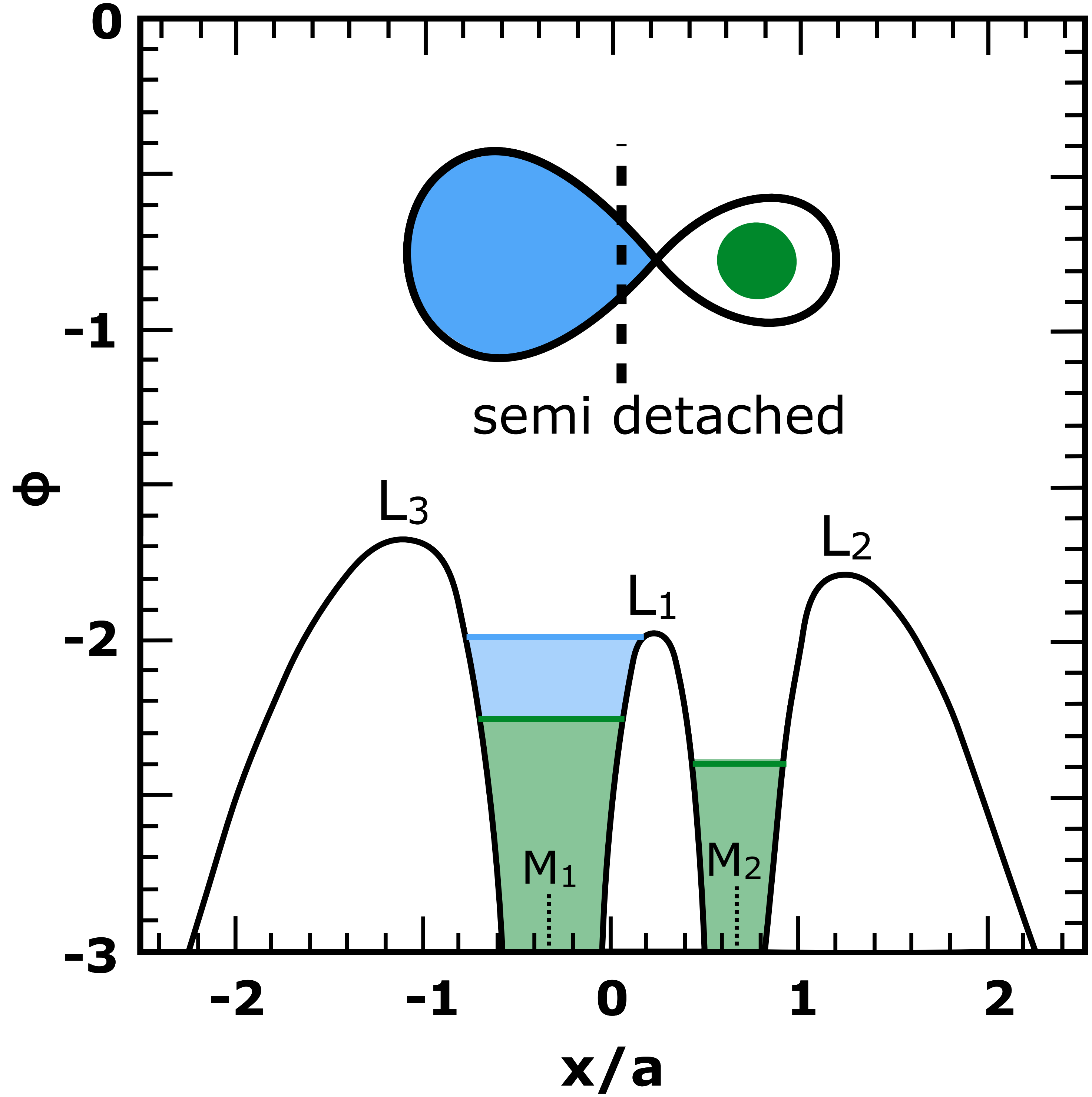
Semi-detached system: M_1 (the more massive component) fills its Roche lobe; gas can flow freely from M_1 to M_2.



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Stars in Binaries: Principles

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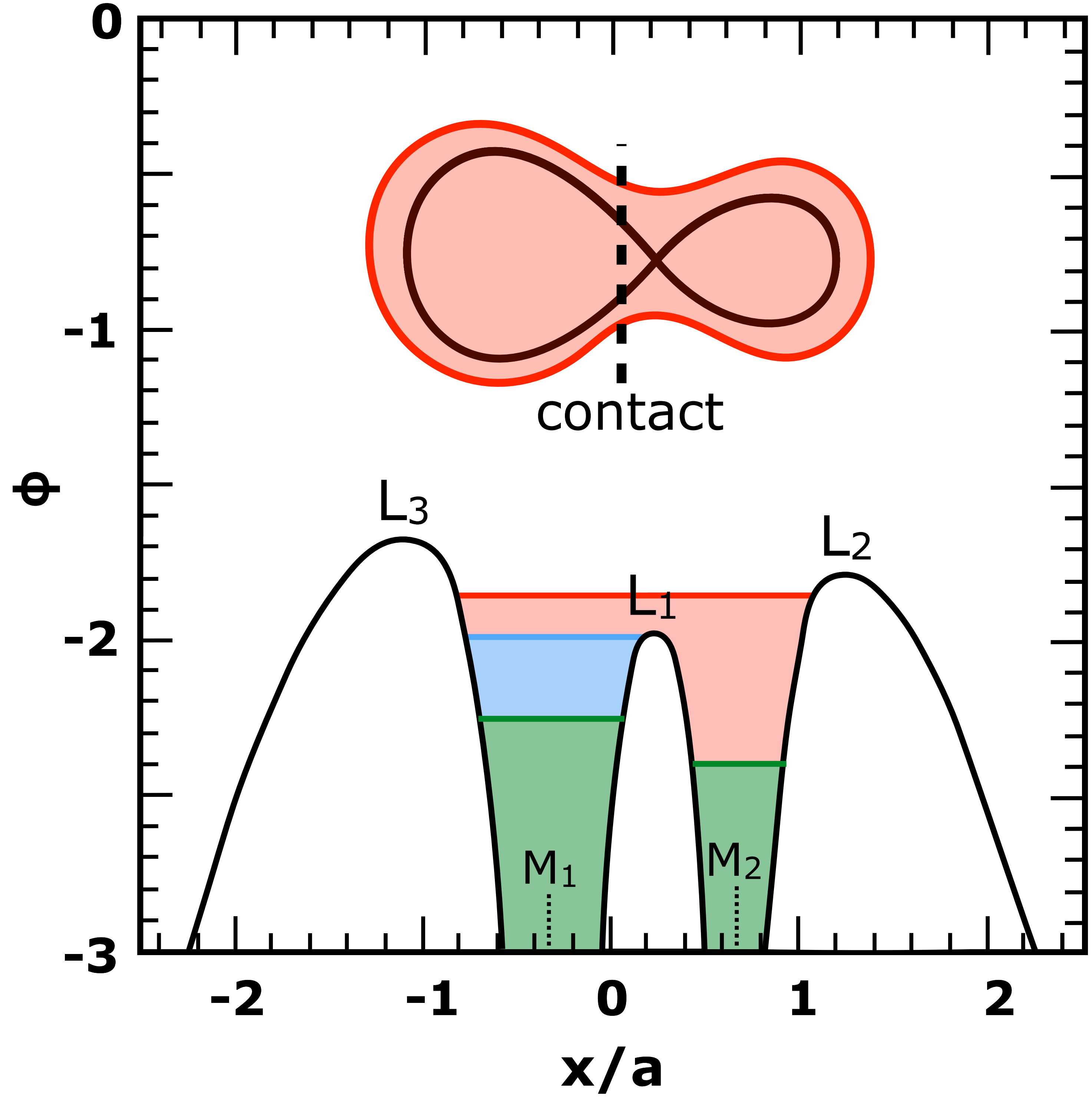
Detached system: both stars safely fit inside their Roche lobes; matter cannot flow between them

Semi-detached system: M_1 (the more massive component) fills its Roche lobe; gas can flow freely from M_1 to M_2.

Contact system: both stars have filled their Roche lobes and are in contact with one another.



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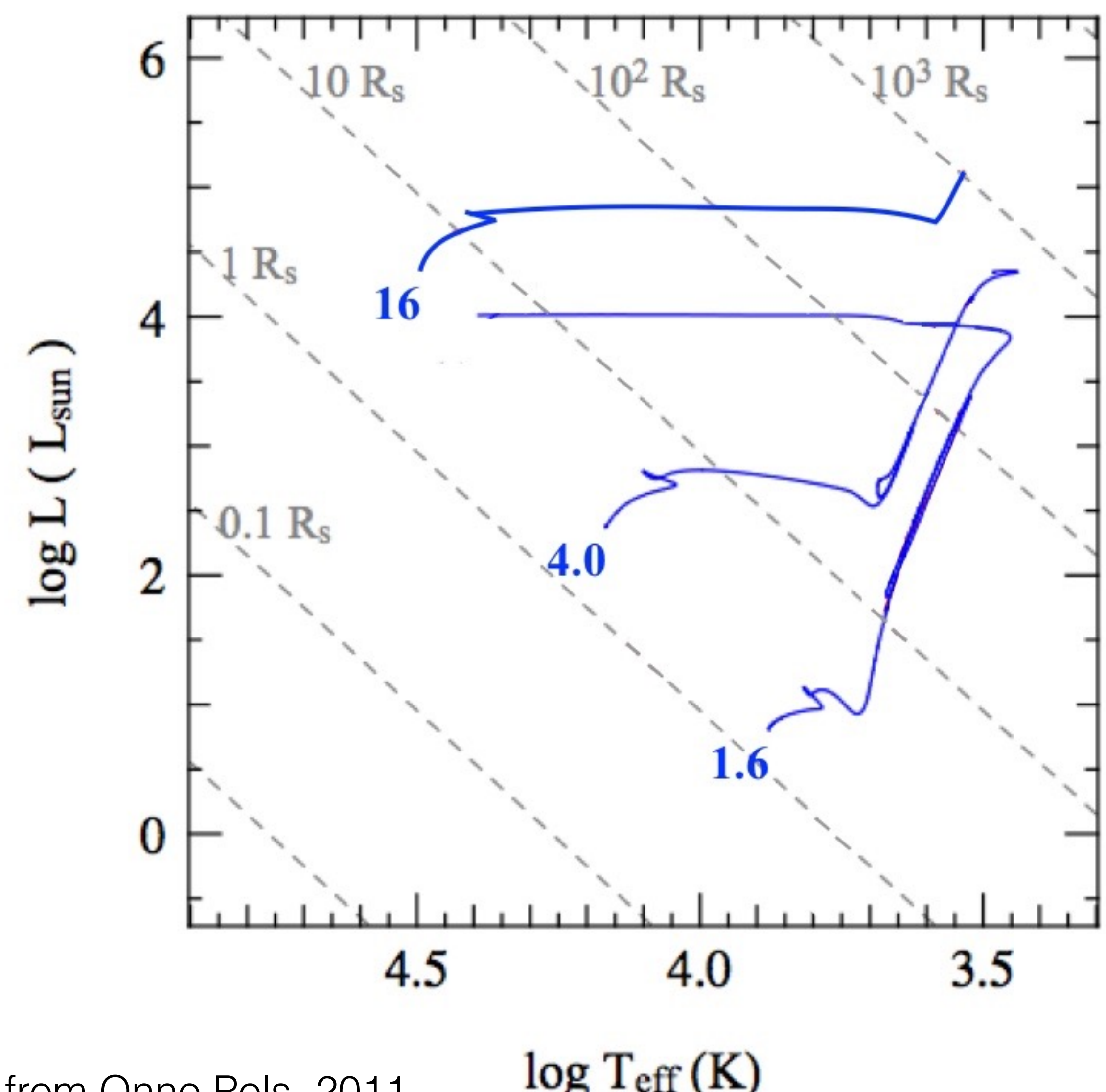
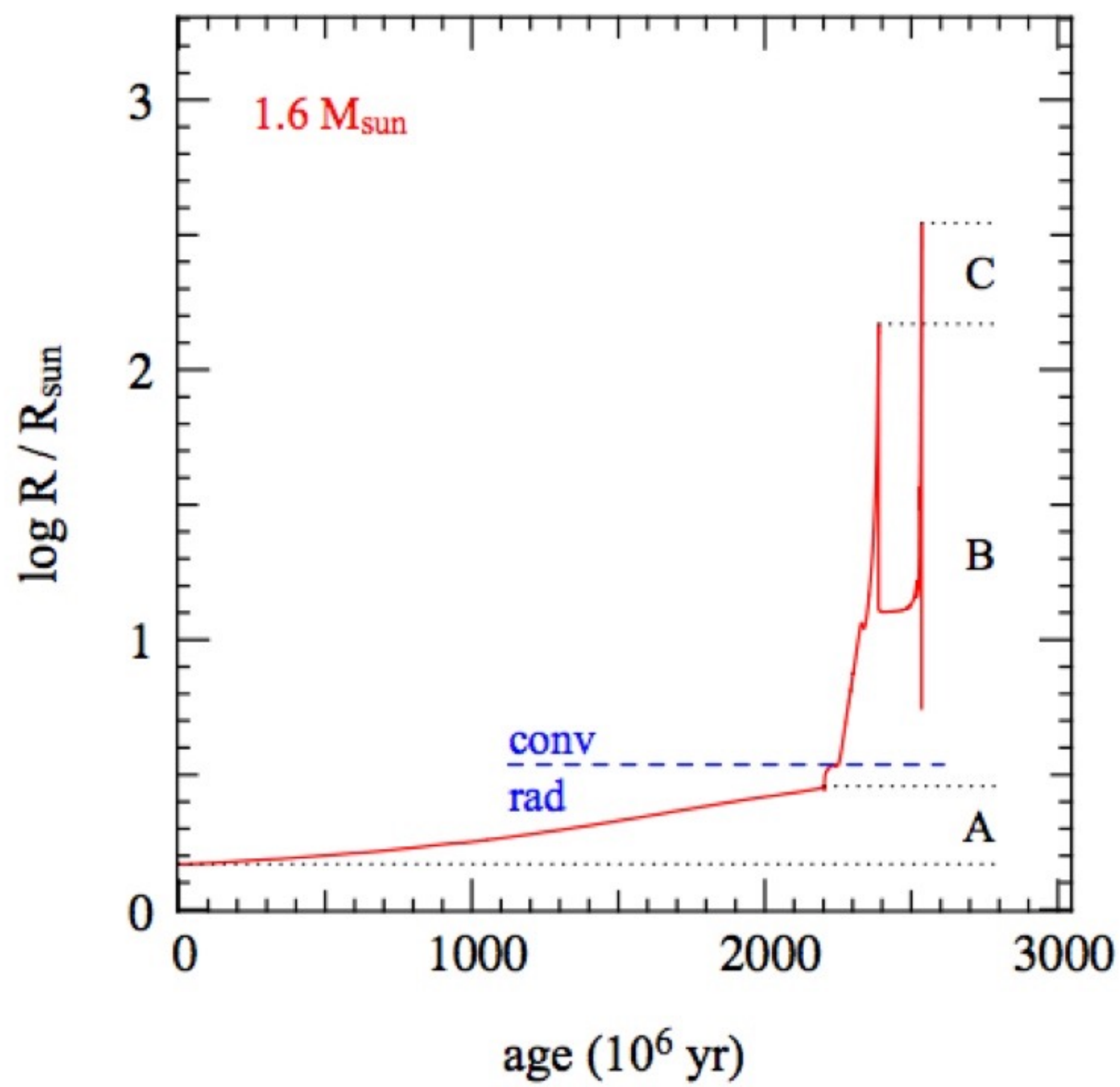
Case A: first contact during main sequence phase
Case B: first contact during H shell fusion
Case C: first contact while the star is on the Hayashi track





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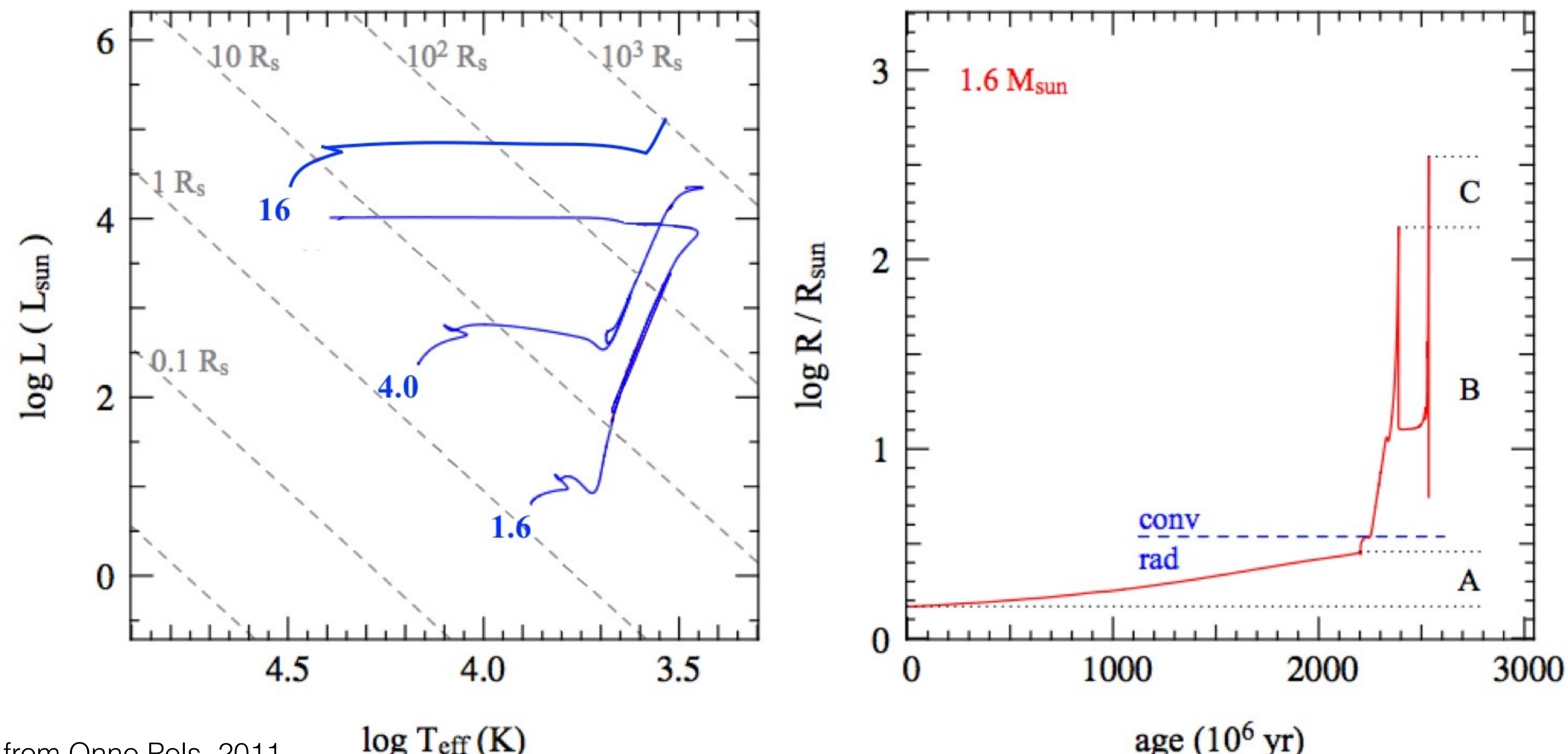

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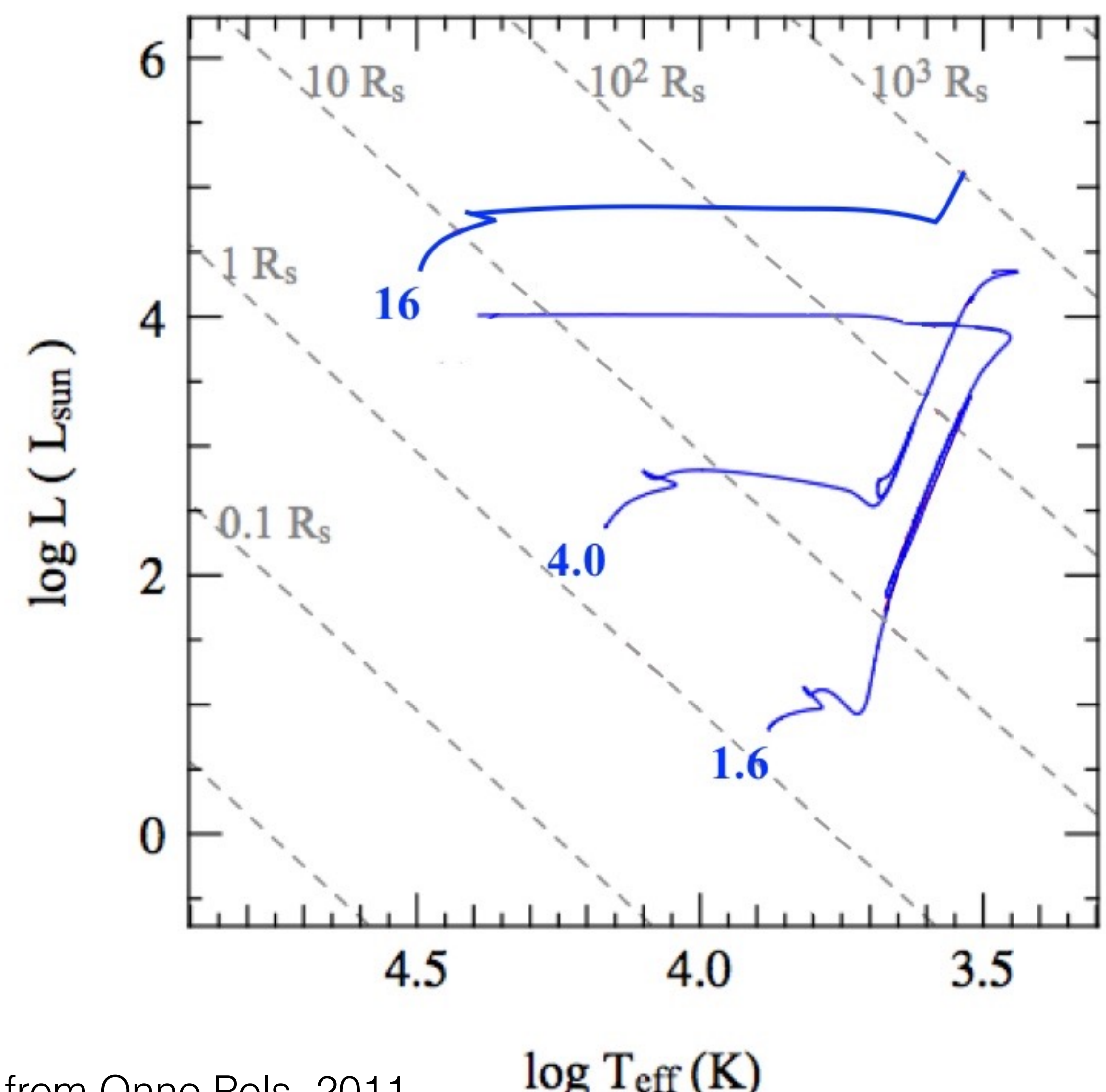

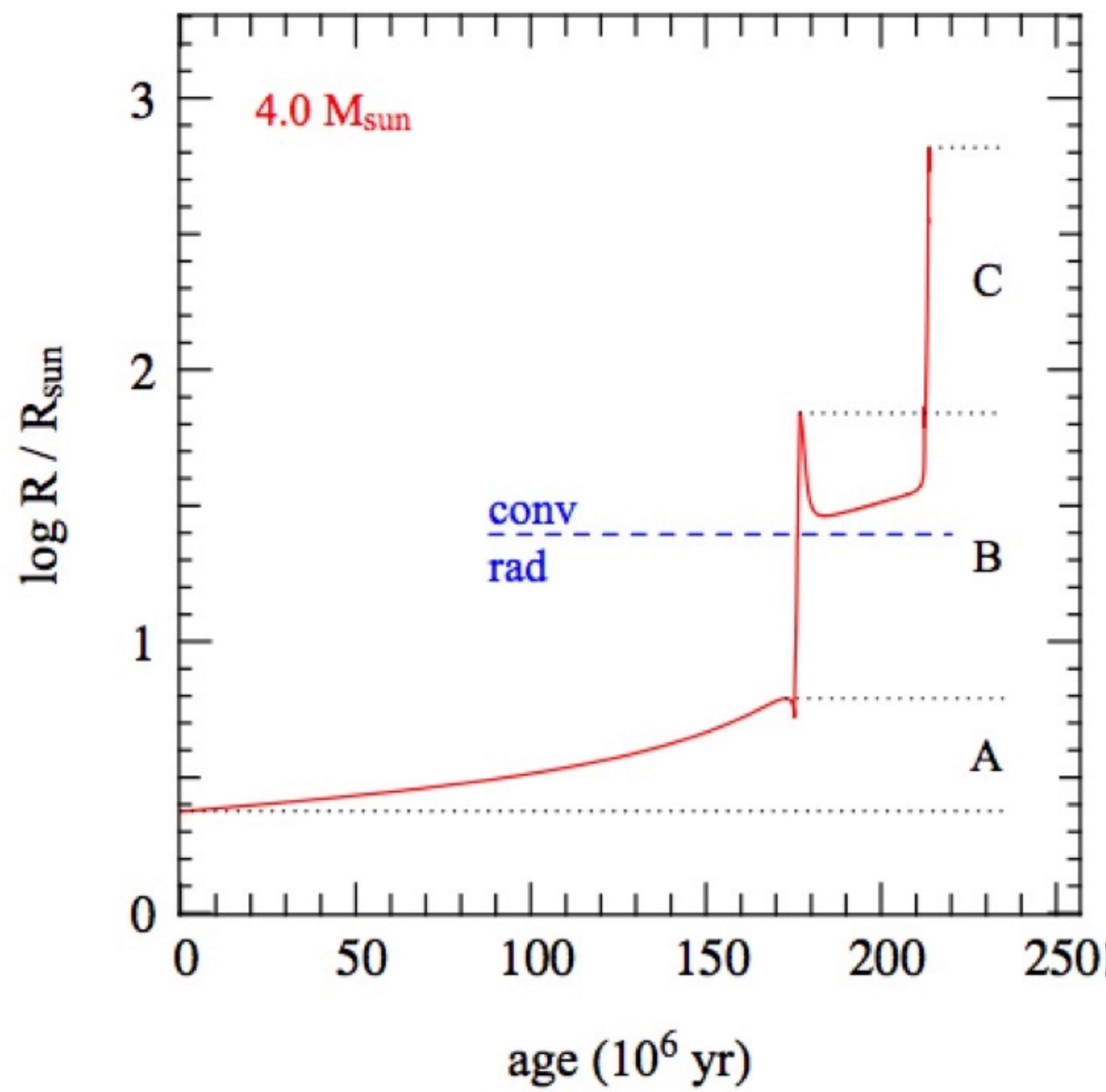

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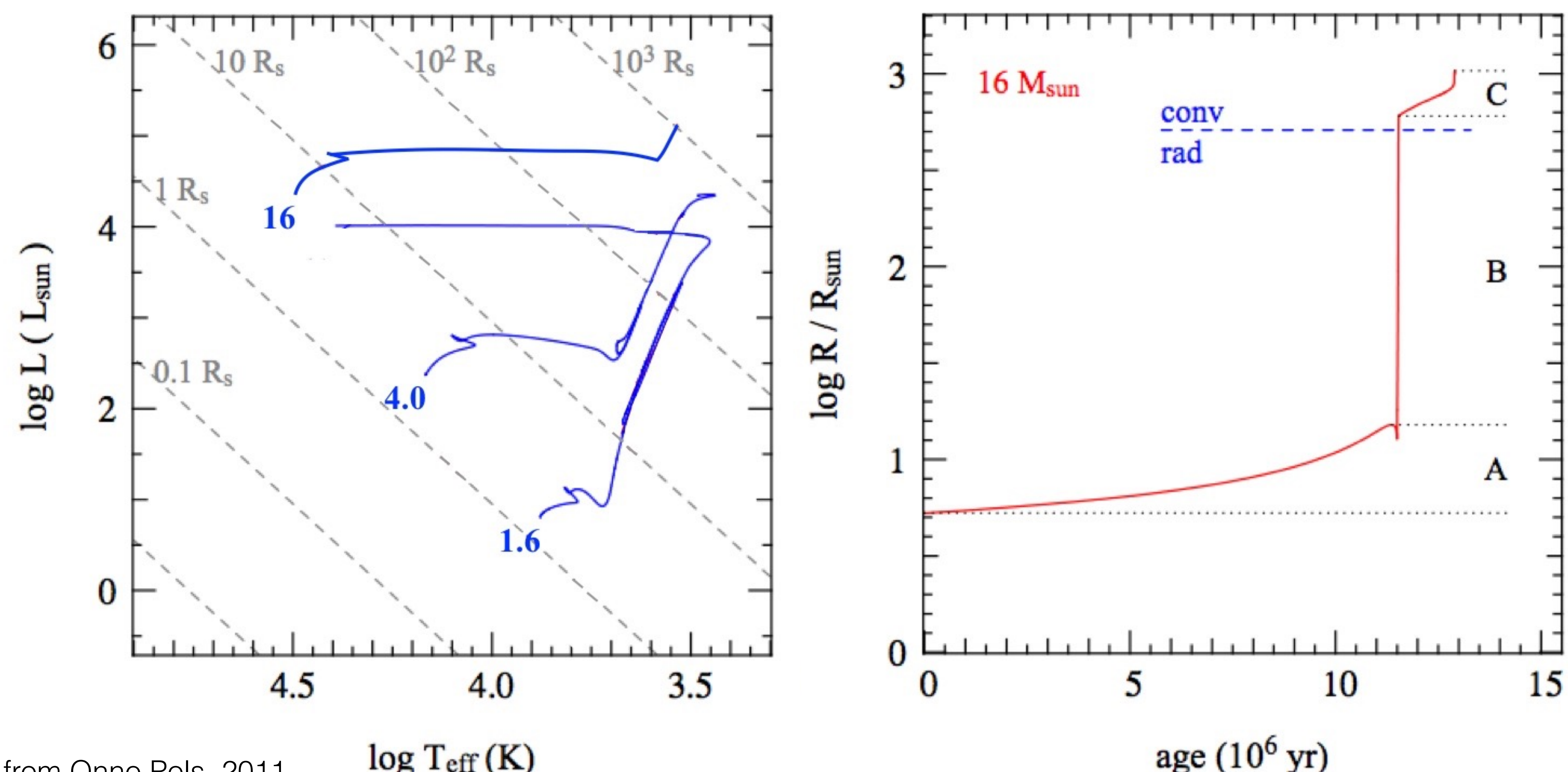

Adapted from Onno Pols, 2011.

Stars in Binaries: Principles

Changes during mass transfer

Mass transfer changes the period and separation of a binary. If we consider the time derivatives of the stellar masses $\dot{M}_1$ and $\dot{M}_2$, in the case of conservative mass transfer:

$$-\dot{M}_1 = \dot{M}_2 \quad \text{and} \quad \dot{J} = 0$$

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$$2\frac{\dot{J}}{J} = \frac{\dot{a}}{a} + 2\,\frac{\dot{M}_1}{M_1} + 2\,\frac{\dot{M}_2}{M_2} - \frac{\dot{M}_1 + \dot{M}_2}{M_1 + M_2}$$

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If we then take the time derivative and use $\mathrm{d}\ln(x)/\mathrm{d}t = \dot{x}/x$:

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The change in a can be derived directly from the conditions that J and $M_1 + M_2$ are both constant. From this:

$$M_1^2 \; M_2^2 \; a \; = \; J^2 \, (M_1 + M_2)/G = \text{constant}$$

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This implies that after mass transfer we can express the ratio between the final separation and period (a and P) and their initial values (a_i and P_i) as:

$$\frac{a}{a_{\mathrm{i}}} = \left(\frac{M_{1i}}{M_1} \cdot \frac{M_{2i}}{M_2}\right)^2 \quad \text{and} \quad \frac{P}{P_{\mathrm{i}}} = \left(\frac{M_{1i}}{M_1} \cdot \frac{M_{2i}}{M_2}\right)^3$$

(the last equation follows from Kepler’s 3rd law…)

Stars in Binaries: Principles

Three time scales associated with single stars are important for the study of binary evolution. In order of increasing length these are:

the dynamical time scale This is the time scale on which a star counteracts a perturbation of its hydrostatic equilibrium. It is given by the ratio of the radius of the star R and the average sound velocity of the stellar matter c_s:

$$\tau_{\rm dyn} = \frac{R}{c_s} \approx 0.04 \left(\frac{M_\odot}{M}\right)^{1/2} \left(\frac{R}{R_\odot}\right)^{3/2} \text{ day} \tag{6.3}$$

the thermal or Kelvin-Helmholtz time scale This is the time scale on which a star reacts when energy loss and energy production are no longer in equilibrium. It is given by the ratio of the thermal energy content of the star $E_{\rm th}$ and the luminosity L:

$$\tau_{\rm KH} = \frac{E_{\rm th}}{L} \approx \frac{GM^2}{2RL} \approx 1.5 \times 10^7 \left(\frac{M}{M_\odot}\right)^2 \frac{R_\odot}{R} \frac{L_\odot}{L} \text{ yr} \tag{6.4}$$

the nuclear time scale This is the time scale on which a star uses its nuclear fuel. It is given by the product of the available fusable matter $M_{\rm core}$ and the fusion energy per unit mass Q, divided by the stellar luminosity. For hydrogen fusion with $Q = 0.007c^2$, this is:

$$\tau_{\rm nuc} = 0.007 \frac{M_{\rm core} c^2}{L} \approx 10^{10} \frac{M}{M_\odot} \frac{L_\odot}{L} \text{ yr} \tag{6.5}$$

from Onno Pols, http://www.astro.ru.nl/~onnop/education/binaries_utrecht_notes/

Stars in Binaries: Principles

Stable and runaway mass transfer

1) <u>Stable mass transfer on the evolution timescale of the donor</u>
   Occurs when the donor star radius decreases, due to mass transfer, faster than the size of the Roche lobe. Mass transfer will shrink the donor star; it then expands again due to its evolution and mass transfer begins again. Happens in Case A mass transfer when the donor is still on the MS with a radiative envelope

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2) <u>Runaway mass transfer: dynamically unstable mass transfer</u>
   Occurs when mass transfer shrinks the Roche lobe, while the donor radius does not shrink (or even keeps expanding). In this case the donor is out of hydrostatic equilibrium. Happens in Case C mass transfer in stars with deep convection zones (i.e., on the Hayashi track)

   This happens because these stars' L is set by M_c and their T_{eff} is almost constant, making the radius independent of the mass of the envelope and causing a runaway mass transfer process from the star's envelope. Ends when the donor contracts to become a WD (low mass) or WR star (massive).

Stars in Binaries: Principles

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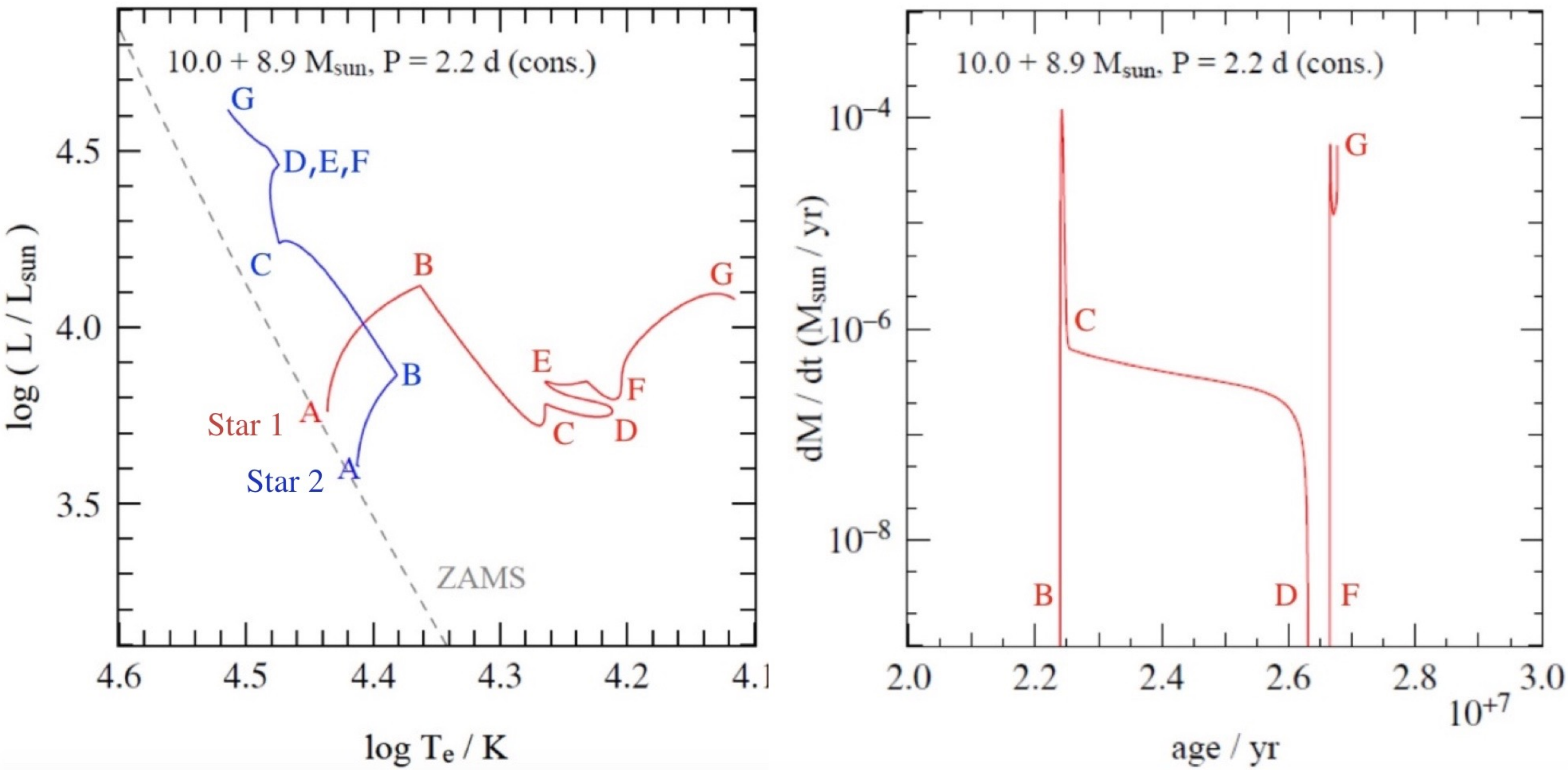
3) Unstable mass transfer on thermal timescale of donor

A middle ground between the two extremes above. The donor star is out of thermal equilibrium but mass transfer is slow enough to maintain hydrostatic equilibrium; the timescale for mass transfer is shorter than the dynamical timescale. Readjusting to thermal equilibrium occurs on a K-H timescale. This happens in Case B mass transfer, when stars have radiative envelopes that are expanding post-MS but have not reached the Hayashi track yet.

Stars in Binaries: Evolution

Example: Case A

Algol system: (sub)giant filling its Roche lobe + more massive MS star



B: primary (star 1) fills its Roche lobe

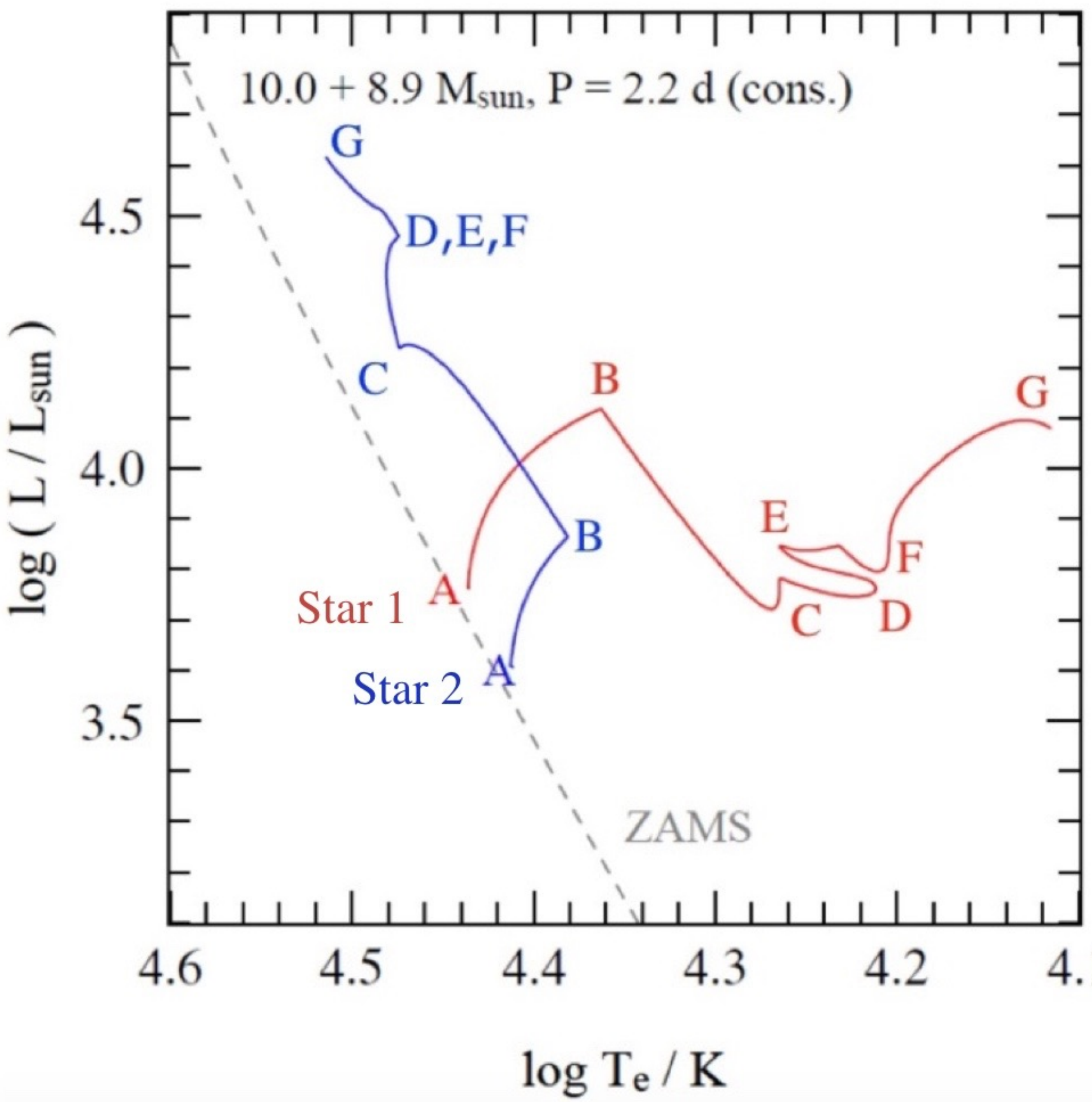
B-C: mass transfers from primary to secondary

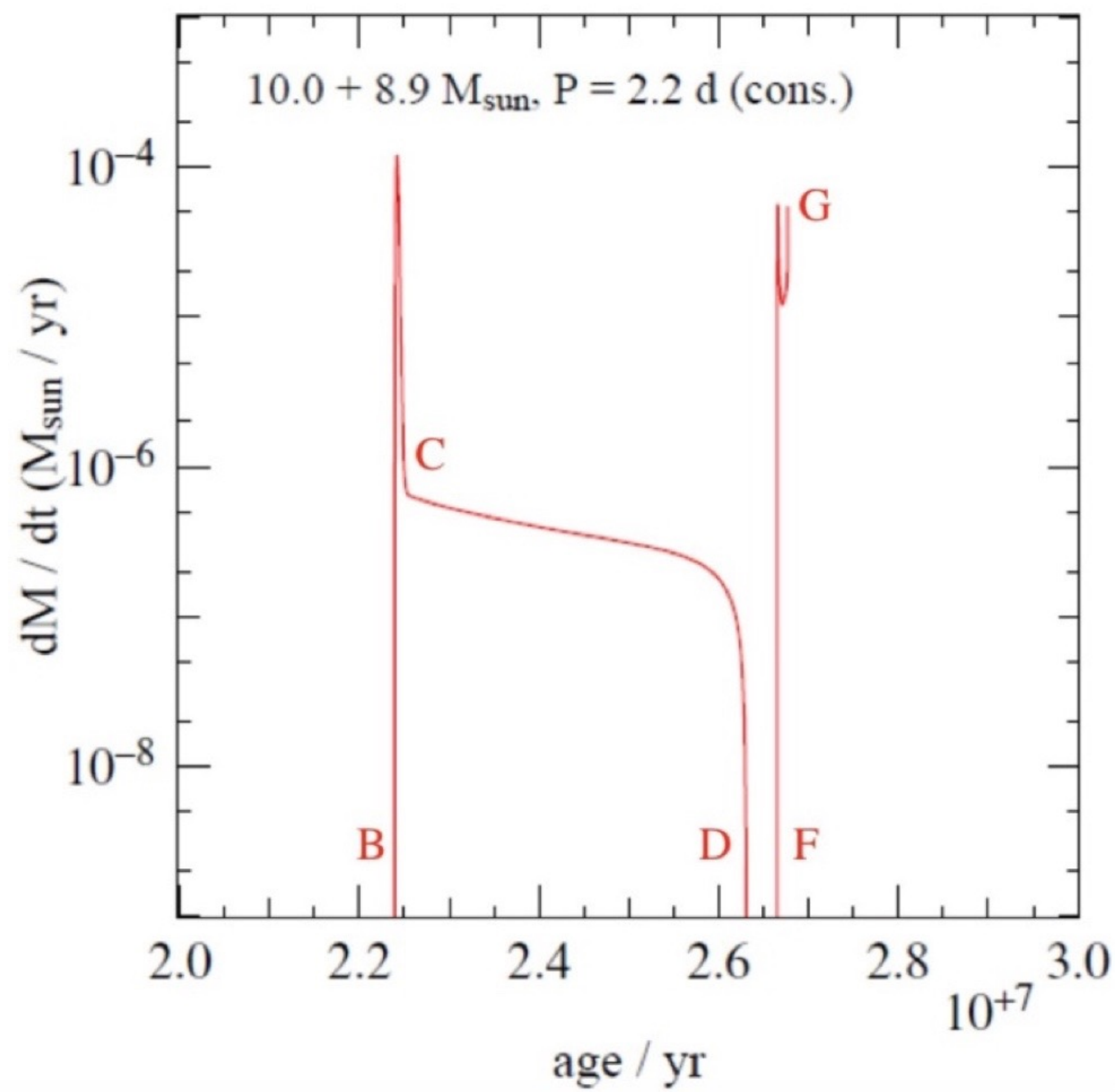
C-D: mass transfer has settled to the nuclear timescale of star 1

Stars in Binaries: Evolution

Example: Case A

Algol system: (sub)giant filling its Roche lobe + more massive MS star





D: end of the MS phase for primary; star 1 shrinks briefly

D-E-F: mass transfer stops

F-G: star expands; high mass transfer rate occurs on K-H timescale

Stars in Binaries: Evolution

Example: Case A

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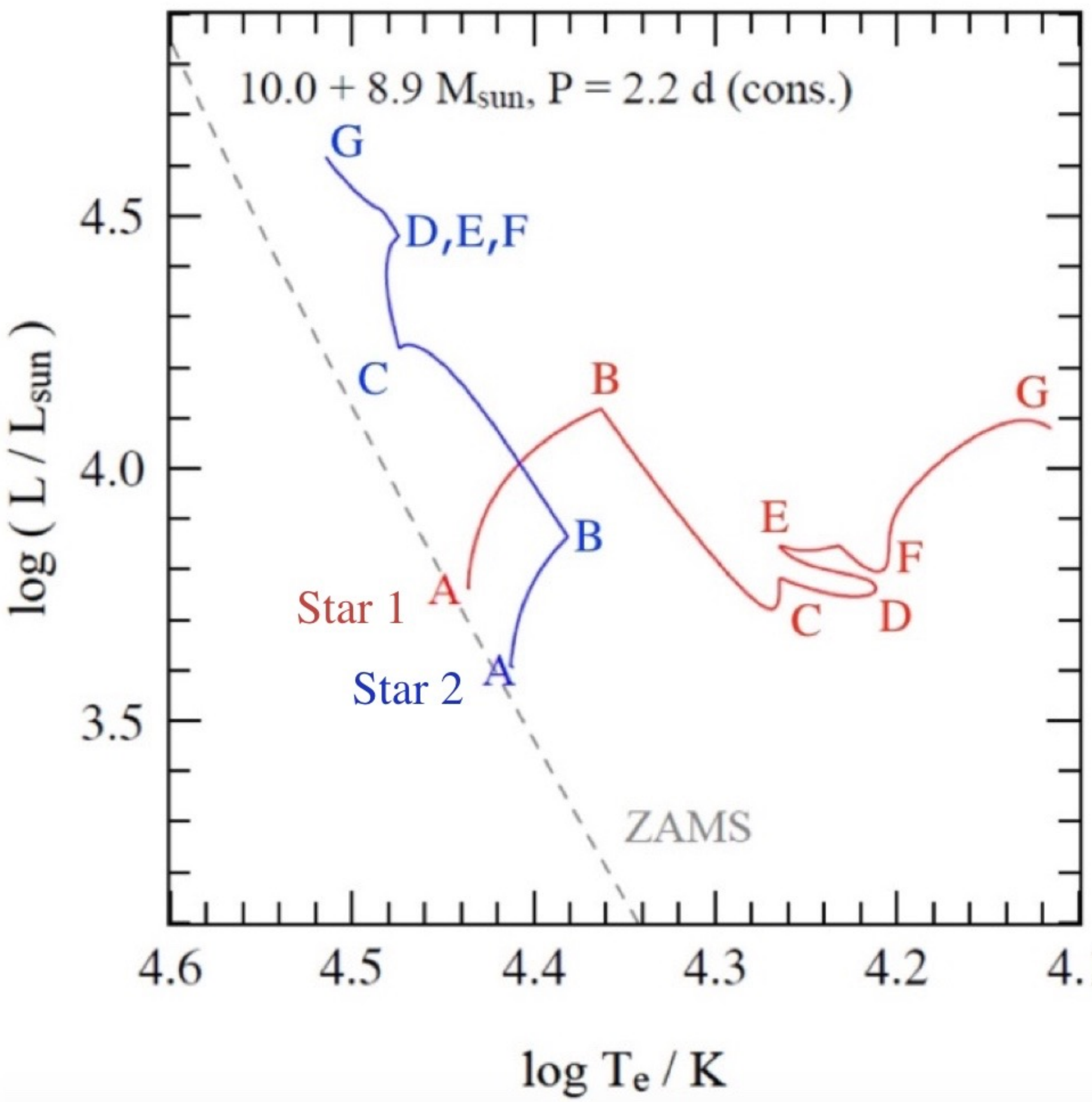
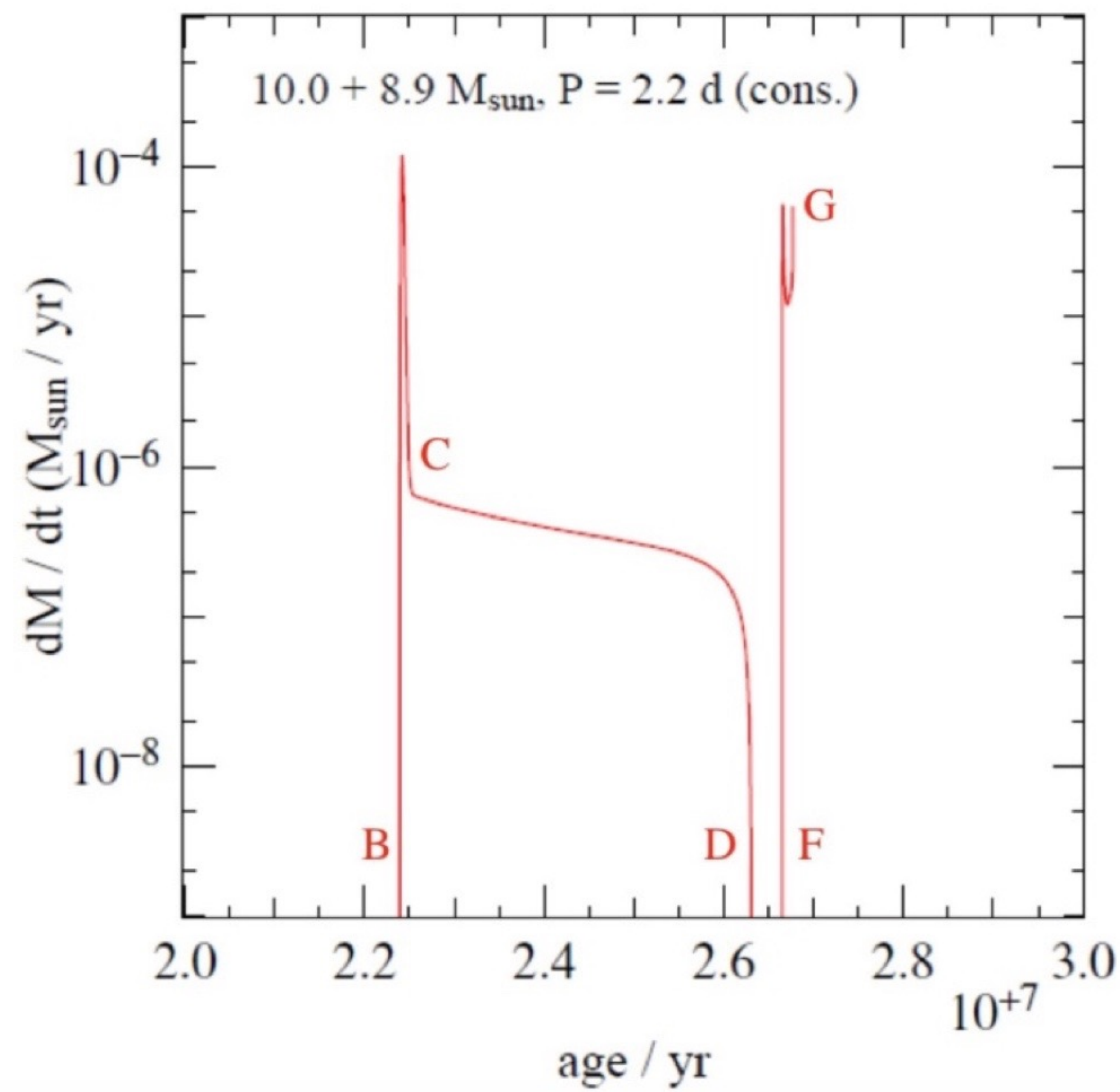


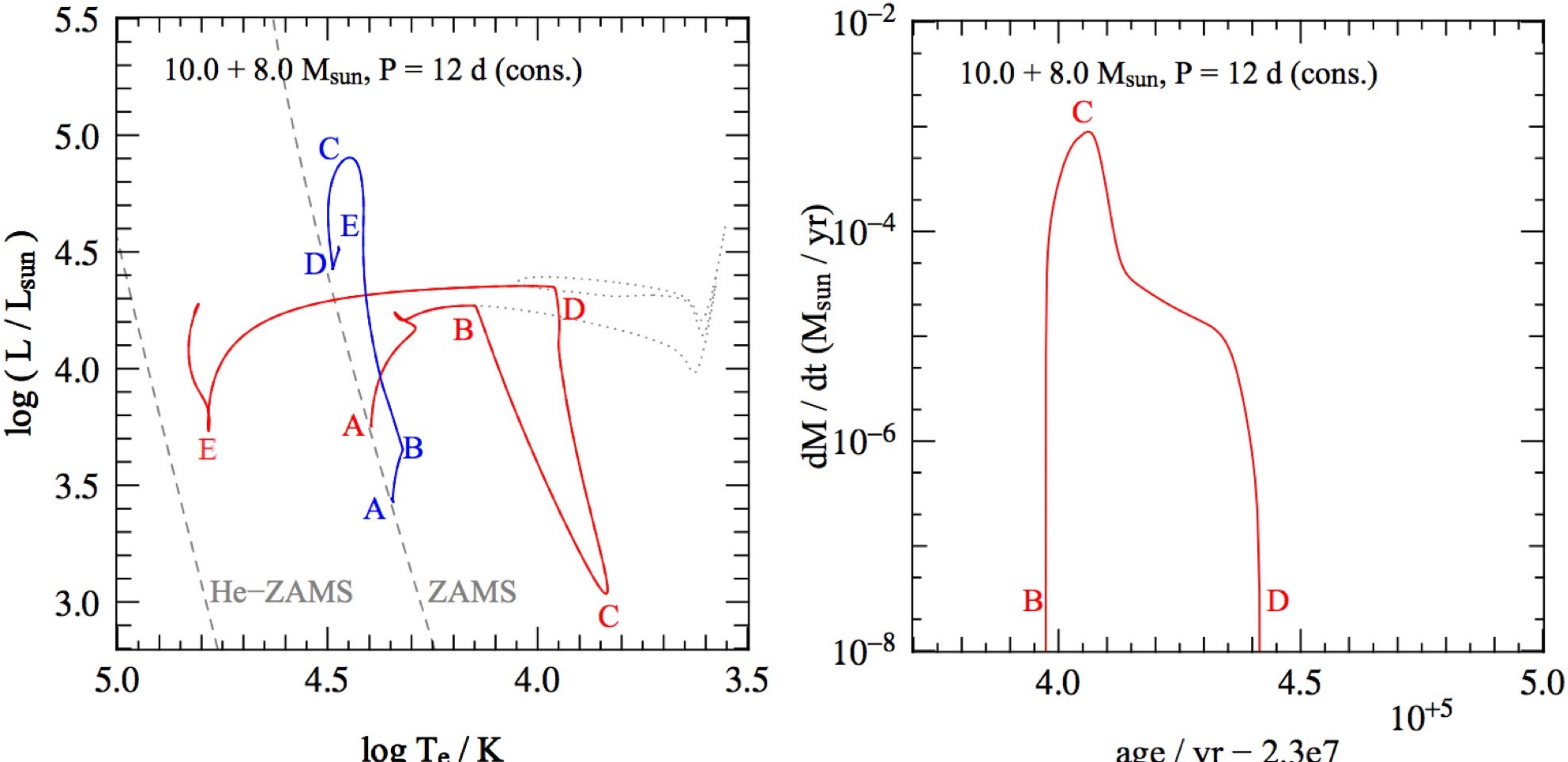


In clusters star 2 can become a **blue straggler**, appearing as a MS star with a mass above the turnoff point.

Stars in Binaries: Evolution

Example: Case B

Two massive stars with a close initial orbital separation ($a=60R_\odot$)…



B: primary (star 1) fills Roche lobe during H shell fusion (crossing gap)

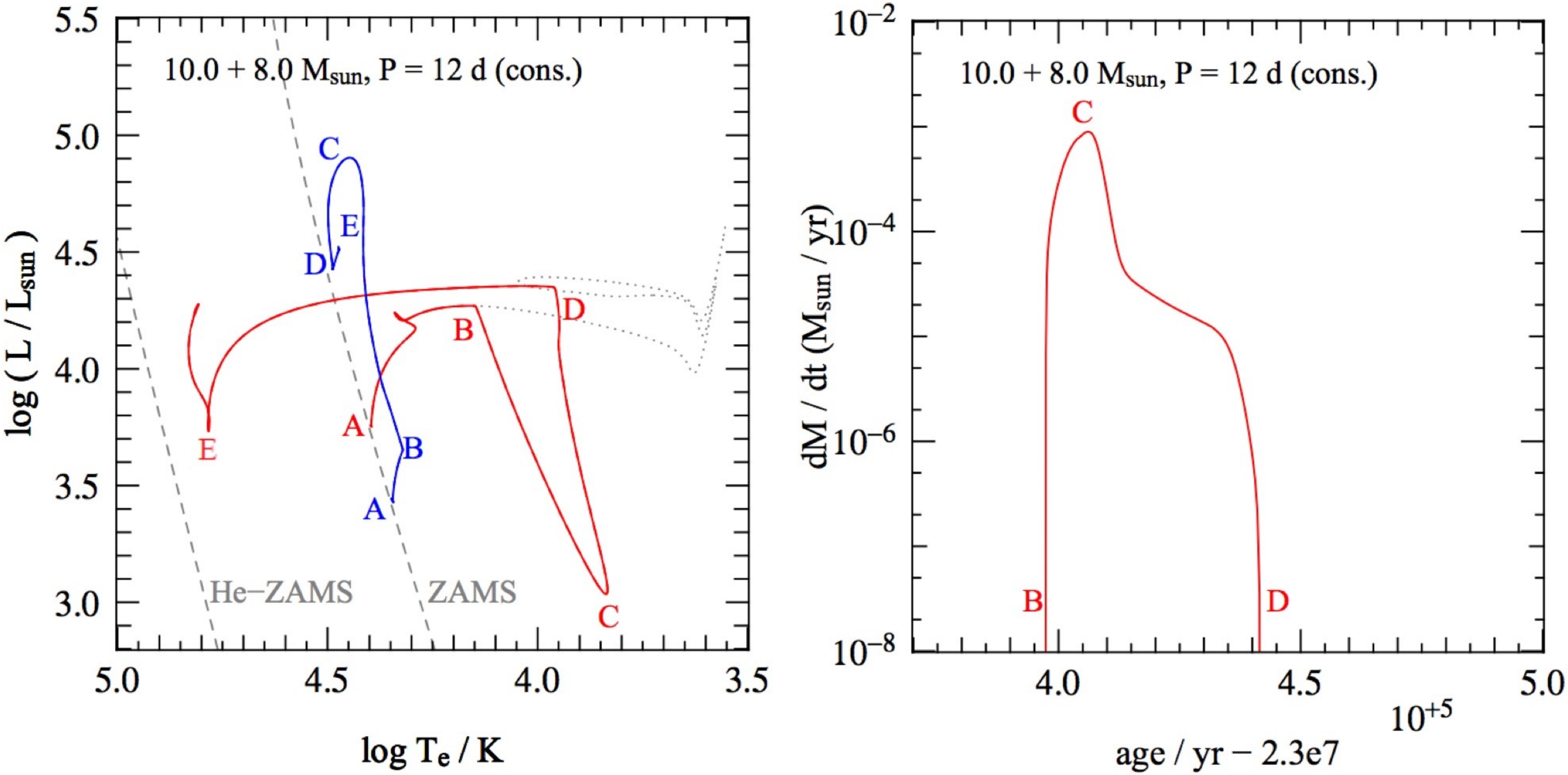
B-C: L of star 1 drops dramatically; star 2 gains mass and increases L

C: stars reach ~equal mass, system reaches minimum a

Stars in Binaries: Evolution

Example: Case B

Two massive stars with a close initial orbital separation ($a=60R_\odot$)…



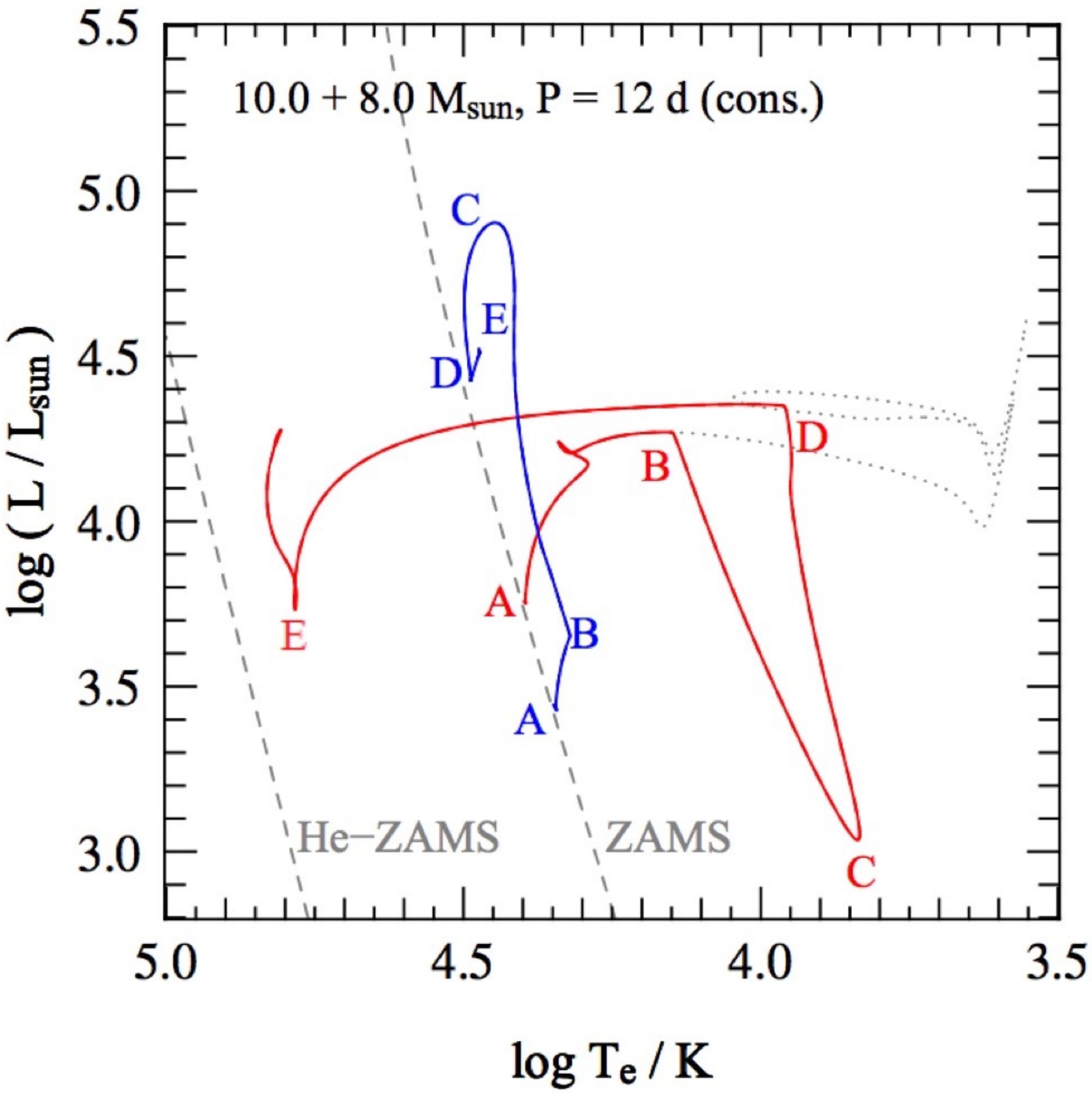
C-D: a increases as mass is transferred to star 2; mass transfer rate drops; star 1 regains TE and its L increases

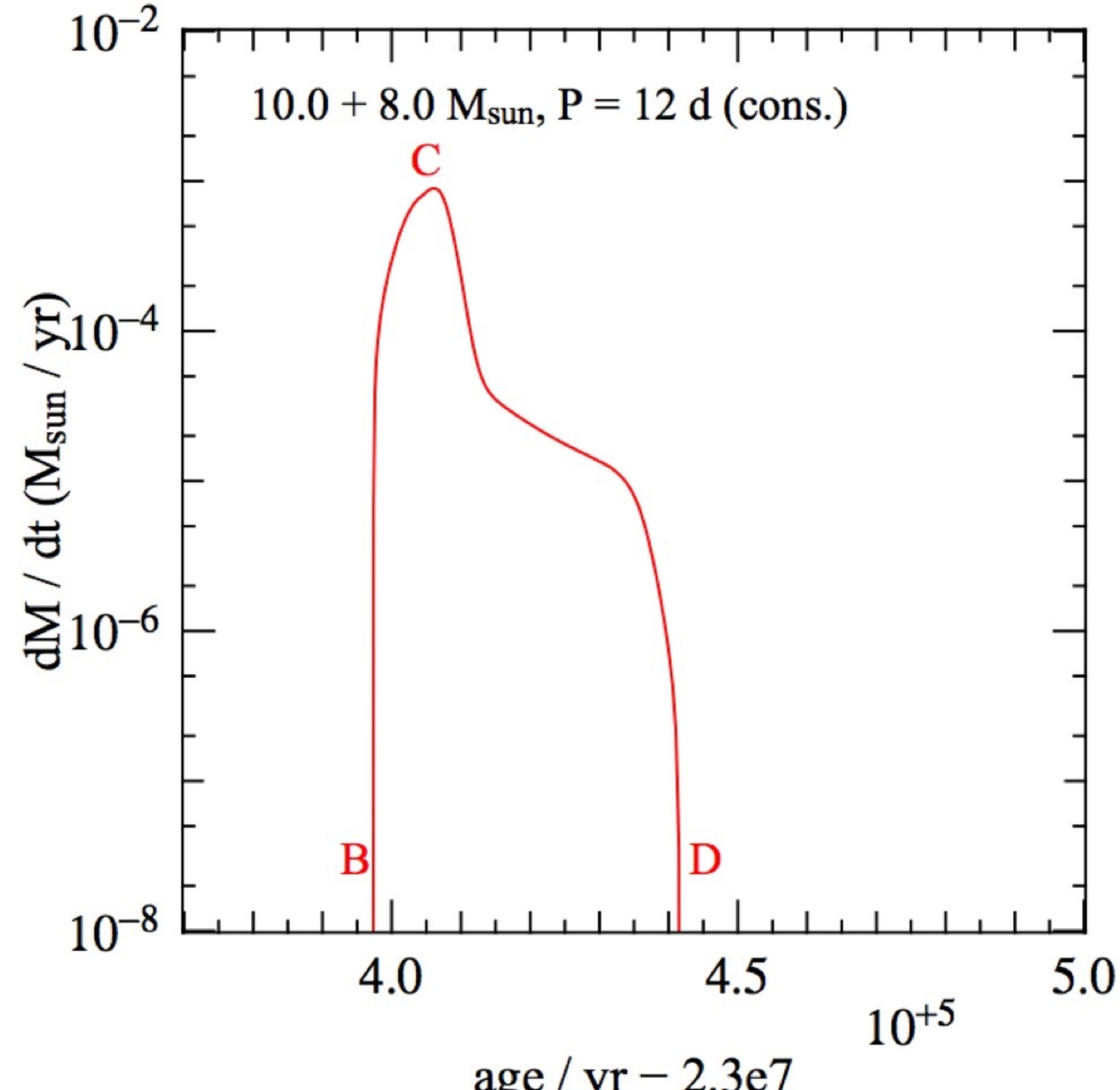
D: He fusion ignites in star 1; it shrinks and mass transfer stops

Stars in Binaries: Evolution

Example: Case B

Two massive stars with a close initial orbital separation ($a=60R_\odot$)...





E: star 1 is a WR star; star 2 has reaches its final mass and continues its evolution. Star 1 is now a low-mass WR star; star 2 is a more massive O star

Stars in Binaries: Evolution

Table 6.1. End products of stellar evolution as a function of stellar mass, for single stars and for components of close binary stars (roughly, those undergoing case B mass transfer). The columns 'He-core mass' give the maximum mass of the helium core reached by a star of the given initial mass range, for single and close binary stars. The values given are only indicative, and depend on the metallicity (assumed to be solar) and uncertainties in mass loss rates and convective overshooting (mild overshooting was assumed). For binary stars, they also depend on the orbital period and mass ratio.

	single star		close binary star	
initial mass	He-core mass	final remnant	He-core mass	final remnant
$\lesssim 2.0\ M_\odot$	$\approx 0.6\ M_\odot$	CO white dwarf	$< 0.47\ M_\odot$	He white dwarf
$2.0 - 6\ M_\odot$	$0.6 - 1.7\ M_\odot$	CO white dwarf	$0.4 - 1.3\ M_\odot$	CO white dwarf
$6 - 8\ M_\odot$	$1.7 - 2.2\ M_\odot$	ONe white dwarf	$1.3 - 1.7\ M_\odot$	CO white dwarf
$8 - 10\ M_\odot$	$2.2 - 3.0\ M_\odot$	neutron star	$1.7 - 2.2\ M_\odot$	ONe white dwarf
$10 - 25\ M_\odot$	$3.0 - 10\ M_\odot$	neutron star	$2.2 - 8\ M_\odot$	neutron star
$\gtrsim 25\ M_\odot$	$> 10\ M_\odot$	black hole	$> 8\ M_\odot$	neutron star/black hole

from Onno Pols, http://www.astro.ru.nl/~onnop/education/binaries_utrecht_notes/