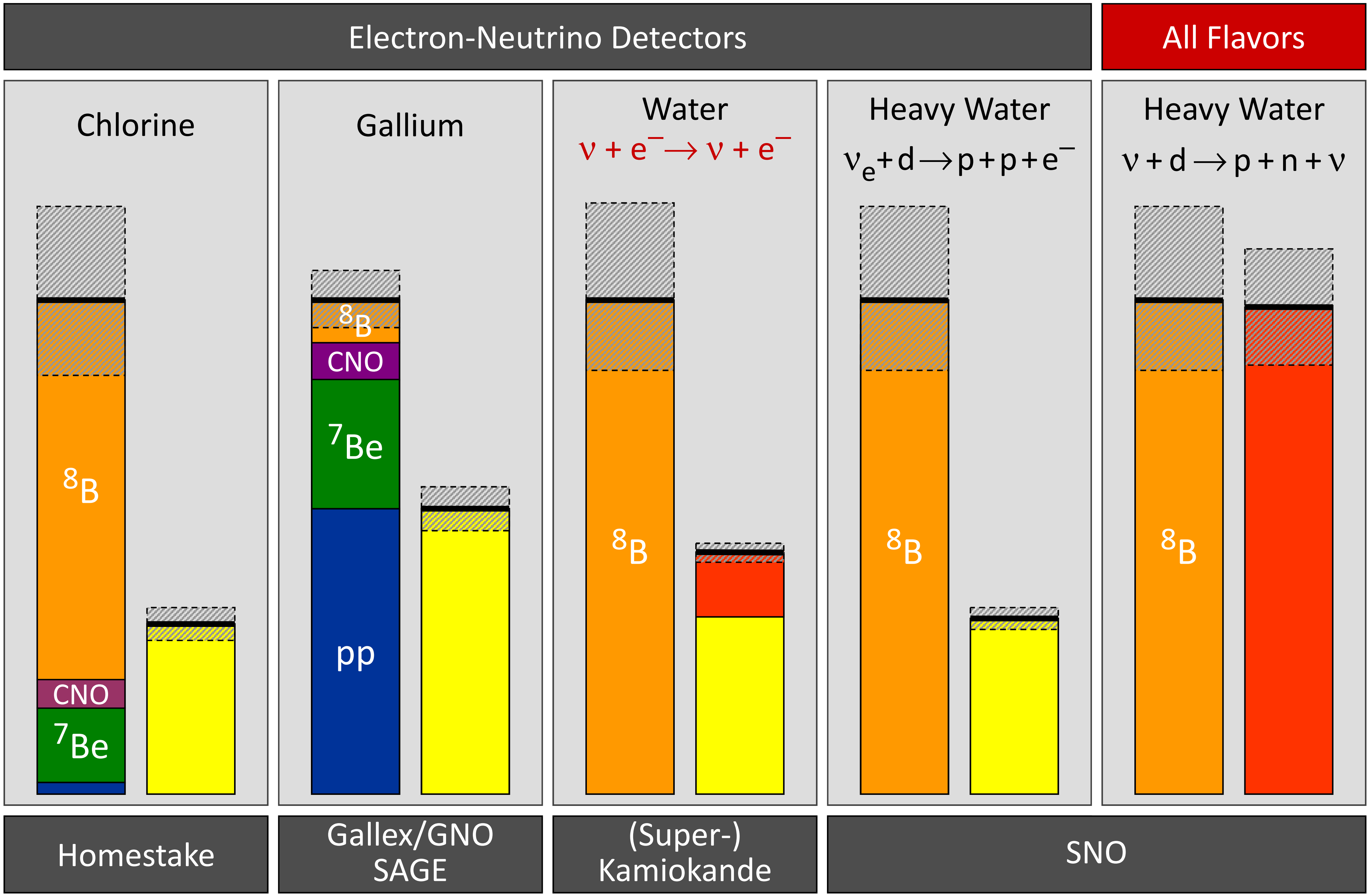
Lecture 10: Nuclear Reactions in Stars

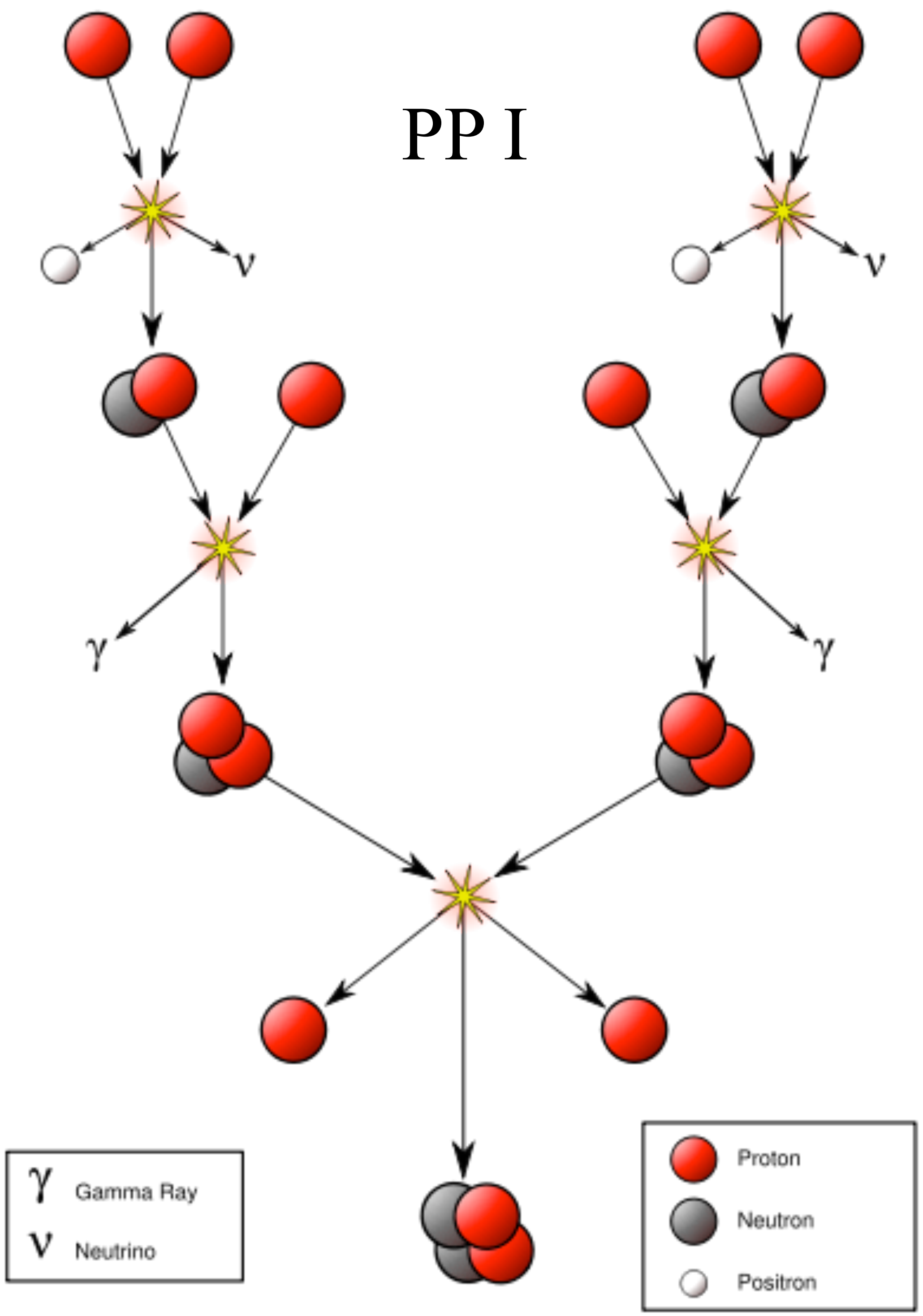
Phillips Ch. 4
Lamers & Levesque Ch. 8



from Raffelt (2012), https://arxiv.org/abs/1201.1637

Rutgers Physics 441/541: Stars & Star Formation   February 18, 2022

p-p chain



REACTIONS OF THE PROTON-PROTON CHAIN

$p + p \rightarrow d + e^+ + \nu_e$ $\quad S_{pp}(0) = 3.8 \times 10^{-22}$ keV barns.

$p + d \rightarrow {}^3\mathrm{He} + \gamma$ $\quad S_{pd}(0) = 2.5 \times 10^{-4}$ keV barns,

${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p$

${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$

$e^- + {}^7\mathrm{Be} \rightarrow {}^7\mathrm{Li} + \nu_e$

$p + {}^7\mathrm{Li} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$

$p + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + \gamma$

${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu_e$

${}^8\mathrm{Be}^* \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$

BRANCH I	BRANCH II	BRANCH III
$Q_{eff} = 26.2$ MeV	$Q_{eff} = 25.7$ MeV	$Q_{eff} = 19.1$ MeV
[85%]	[15%]	[0.02%]

Fig. 4.4 The three competing branches of the proton–proton chain with the net result $4p \rightarrow {}^4\mathrm{He} + Q_{eff}$. Here Q_{eff} is the effective energy released by the branch; it includes the energy from the annihilation of positrons, but it does not include any of the energy carried away by neutrinos. Note, a pre-existing ${}^4\mathrm{He}$ nucleus acts as a catalyst in branches II and III, its destruction leading to two new ${}^4\mathrm{He}$ nuclei. According to the Standard Solar Model, Bahcall (1989), the proton–proton chain in the sun is terminated by branch I 85% of the time, by branch II 15% of the time and by branch III 0.02% of the time.

from http://en.wikipedia.org/wiki/Proton-proton_chain_reaction and Phillips Figure 4.4

CNO cycle

REACTIONS OF THE CARBON–NITROGEN CYCLE

$p + {}^{12}C \rightarrow {}^{13}N + \gamma$ $\qquad [S(0) = 1.5 \text{ keV barns}]$

$\quad \hookrightarrow {}^{13}C + e^{+} + \nu_e$

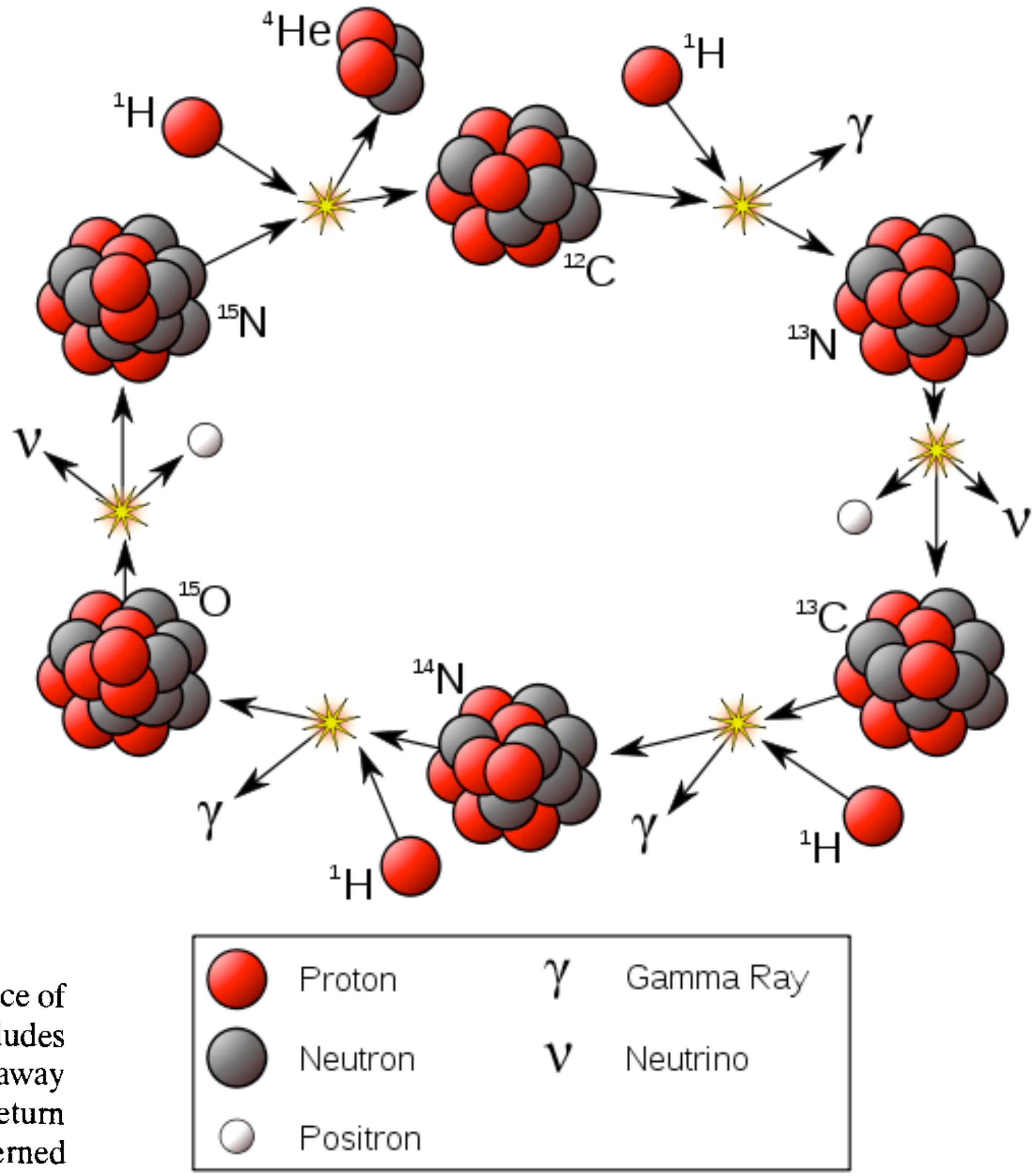
$p + {}^{13}C \rightarrow {}^{14}N + \gamma$ $\qquad [S(0) = 5.5 \text{ keV barns}]$

$p + {}^{14}N \rightarrow {}^{15}O + \gamma$ $\qquad [S(0) = 3.3 \text{ keV barns}]$

$\quad \hookrightarrow {}^{15}N + e^{+} + \nu_e$

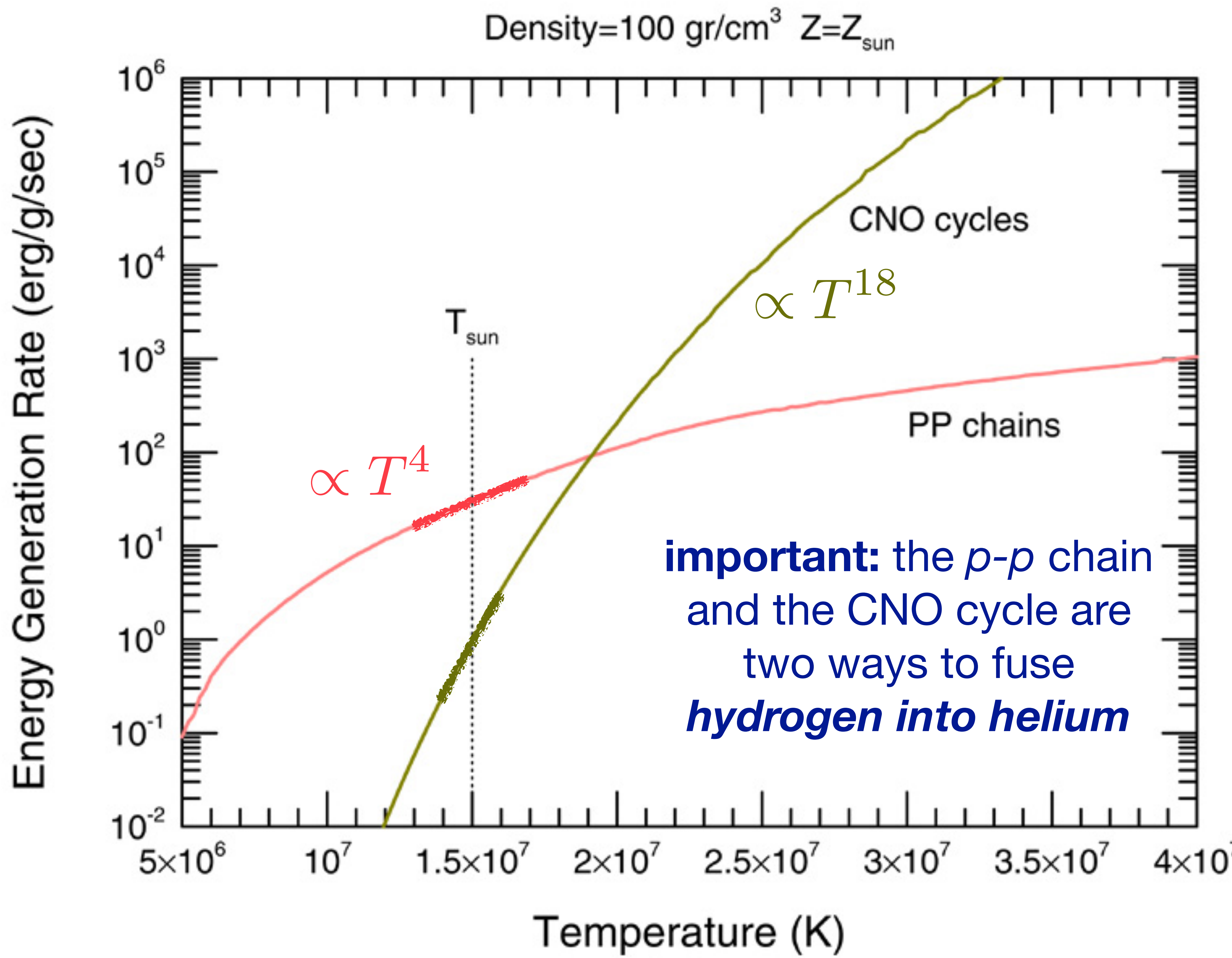
$p + {}^{15}N \rightarrow {}^{12}C + {}^{4}He$ $\qquad [S(0) = 78 \text{ keV barns}]$

Fig. 4.5 Hydrogen burning by the carbon–nitrogen cycle. The net result of this sequence of reactions is $4p \rightarrow {}^{4}He + Q_{eff}$. The effective energy released Q_{eff} is 23.8 MeV; this includes the energy from the annihilation of positrons, but it does not include the energy carried away by neutrinos. Note that nuclei of carbon and nitrogen are temporarily transformed but return to take part in subsequent operations of the cycle. The rates for these reactions are governed by the relevant Coulomb barriers and the approximate S factors indicated.



Phillips Figure 4.5 and http://en.wikipedia.org/wiki/Cno_cycle

p-p chain or CNO cycle



from http://cococubed.asu.edu/images/net_ppcno/ppcno_sdot_web.jpg

Solar neutrinos

$$4\ {}^1_1\mathrm{H} \ \longrightarrow\ {}^4_2\mathrm{He}\ +\ 2\ e^+\ +\ 2\ \nu_e\ +\ \mathrm{photons}$$

The cross section for a typical solar neutrino to interact with an atomic nucleus is $\sigma \sim 10^{-43}\ \mathrm{cm}^2$. The cross section also depends on the neutrino energy, with $\sigma \propto E_\nu^2$. This cross-section is many orders of magnitude less than typical nuclear or photon cross sections, and explains why neutrinos can easily escape from the Sun. Recall that the mean free path $\ell = 1/n\sigma$, where n is the number density of targets, in this case atomic nuclei. If we assume the Sun was made entirely of hydrogen, we could estimate

$$n_{\mathrm{nuclei}} \sim \frac{\rho}{m_p} \sim \frac{M_\odot}{V_\odot m_p} = \frac{3\ M_\odot}{4\pi R_\odot^3 m_p} = 8.4 \times 10^{23}\ \mathrm{cm}^{-3}$$

Thus the mean free path for neutrinos in the Sun is approximately

$$\ell \sim \frac{1}{n\sigma} \sim \frac{1}{(8.4 \times 10^{23}\ \mathrm{cm}^{-3})(10^{-43}\ \mathrm{cm}^2)} \sim 10^{19}\ \mathrm{cm} \sim 10^8\ R_\odot$$

This is many orders of magnitude larger than the radius of the Sun! In other words, the probability of a neutrino interacting with nucleus on its way out of the Sun is just $p \sim L/\ell \sim 10^{-8}$. The vast majority of the neutrinos stream right out into space.

Solar neutrinos

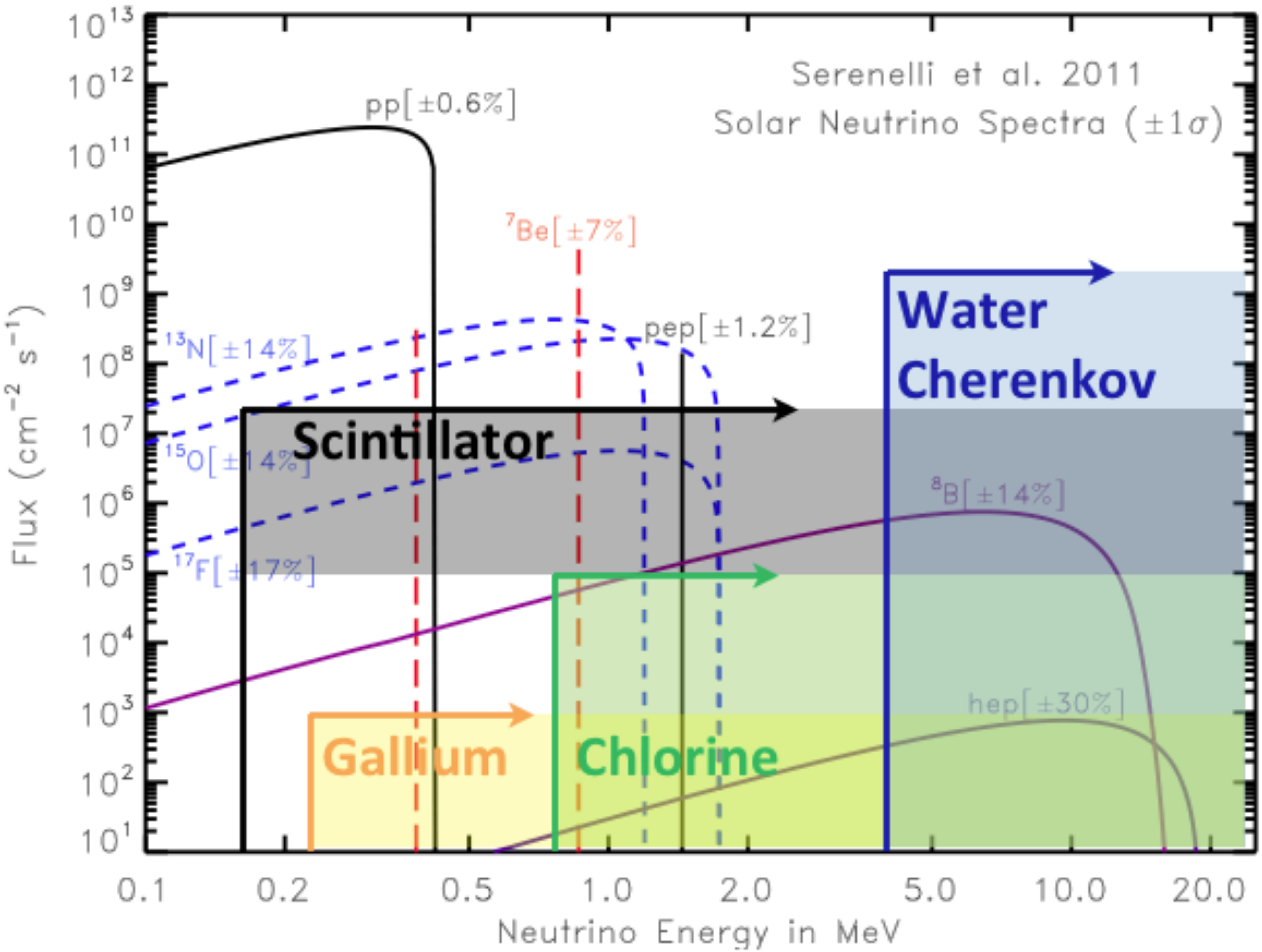
Here are the specific reactions in which the neutrinos are produced:

$$
\begin{array}{llcll}
\text{PP I:} & {}^1_1\mathrm{H} + {}^1_1\mathrm{H} & \longrightarrow & {}^2_1\mathrm{H} + e^+ + \nu_e & \\
\text{PP II:} & {}^7_4\mathrm{Be} + e^- & \longrightarrow & {}^7_3\mathrm{Li} + \nu_e & \\
\text{PP III:} & {}^8_5\mathrm{B} & \longrightarrow & {}^8_4\mathrm{Be} + e^+ + \nu_e & **** \\
\text{CNO I:} & {}^{13}_7\mathrm{N} & \longrightarrow & {}^{13}_6\mathrm{C} + e^+ + \nu_e & \\
 & {}^{15}_8\mathrm{O} & \longrightarrow & {}^{15}_7\mathrm{N} + e^+ + \nu_e & \\
\text{CNO II:} & {}^{17}_9\mathrm{F} & \longrightarrow & {}^{17}_8\mathrm{O} + e^+ + \nu_e &
\end{array}
$$

The boron decay reaction is rare (only 1 out of every 5000 PP chains follows the PP III branch), but it is responsible for most of the solar neutrinos seen on Earth, because in this reaction the mass differences in the nuclei are such that the neutrinos produced can have a high energy (up to 14 MeV, compared to up to 2.8 MeV for the CNO cycle neutrinos, or less than 1 MeV for the other PP neutrinos). The high energy neutrinos are easier to detect.

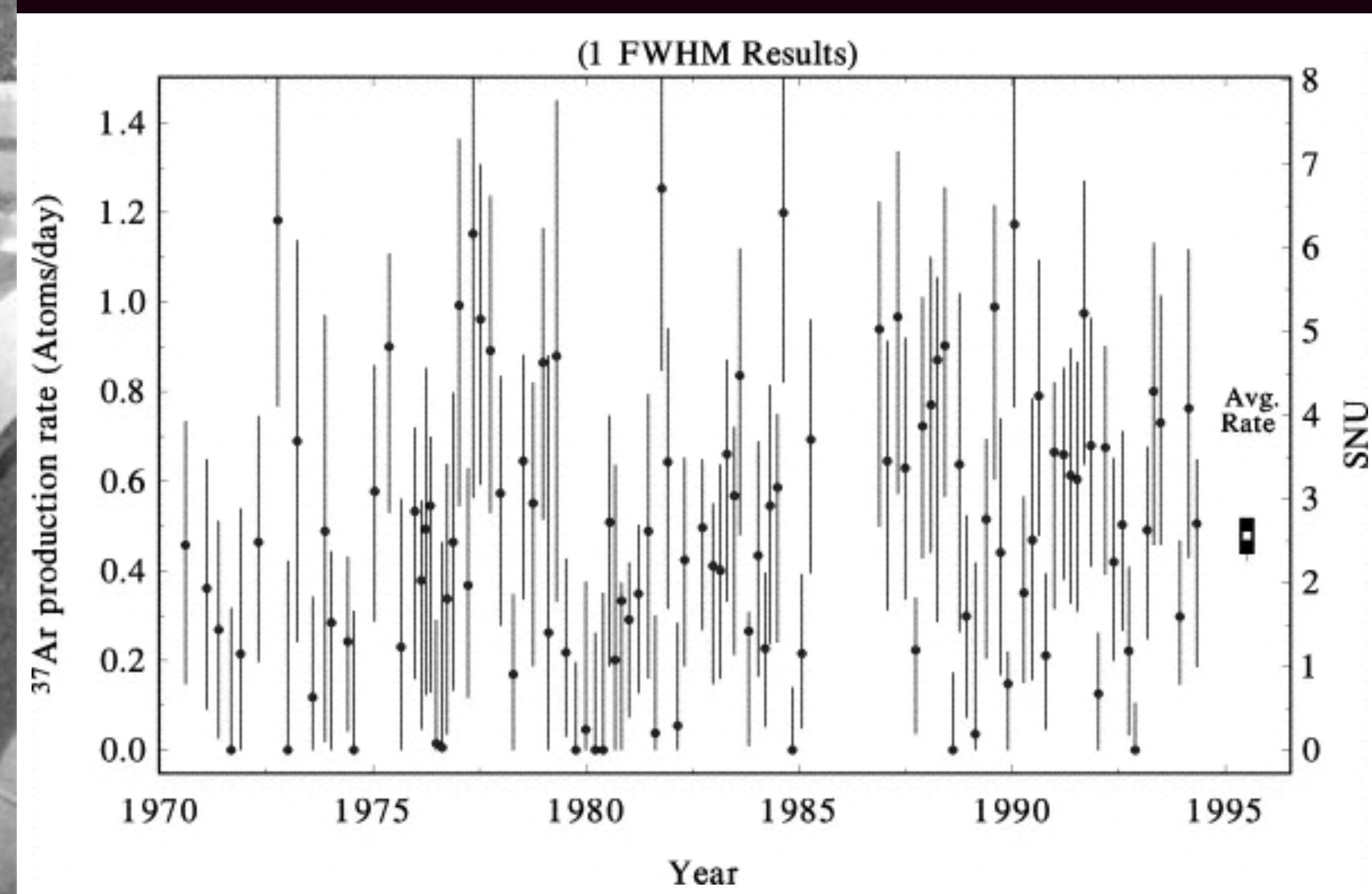
The reaction rate for the sequence that produces boron neutrinos has a steep temperature dependence of $r \propto T^\alpha$ with $\alpha \approx 25$. So if we change the model temperature a little bit, the model reaction rate will change a lot. Turning this around, the fact that the neutrino data and model agree so well indicates that we know the temperature at the center of the Sun quite well!

Solar neutrino spectrum



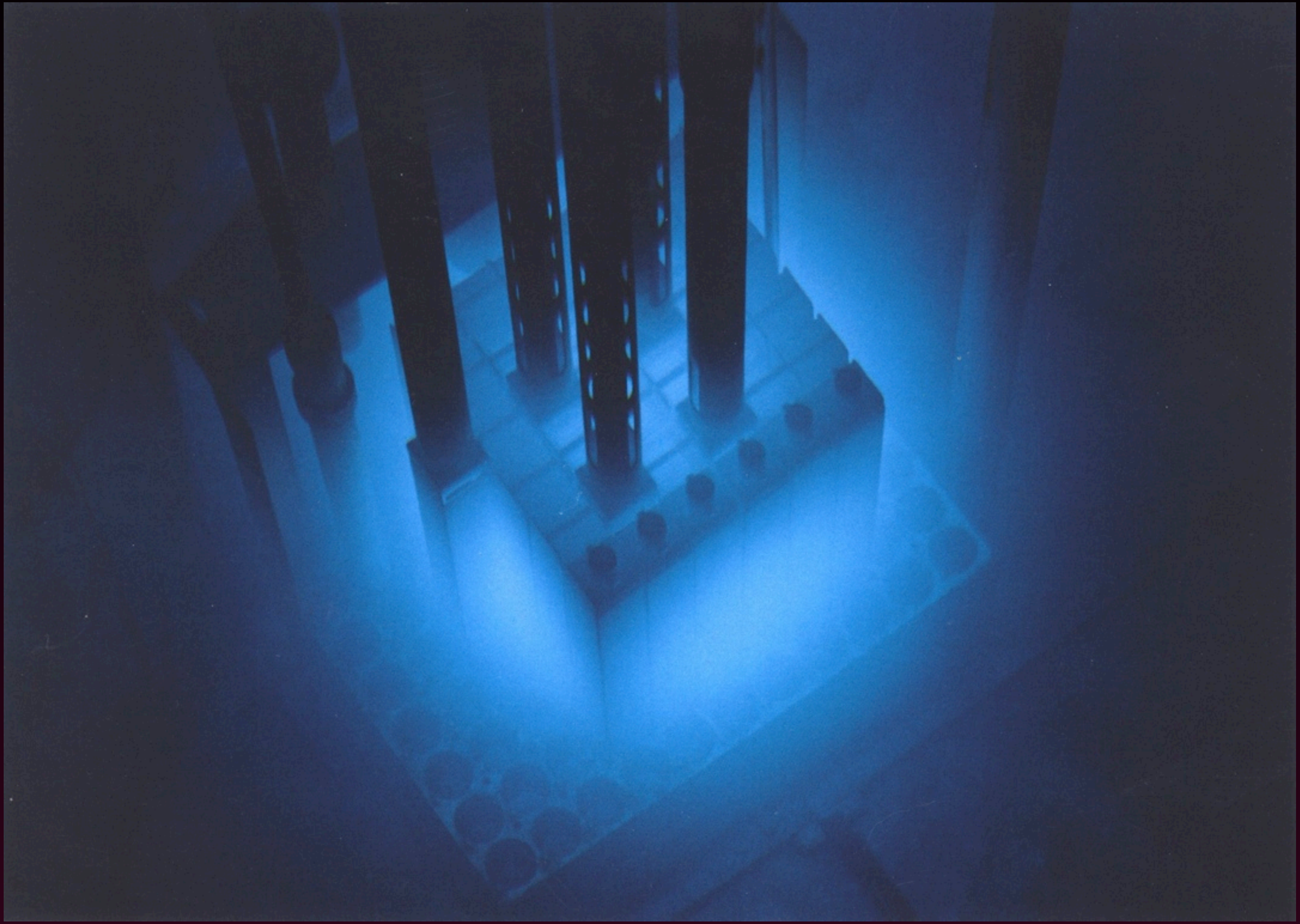
from Orebi Gann 2015, https://arxiv.org/abs/1504.02154v2

catching neutrinos with cleaning fluid



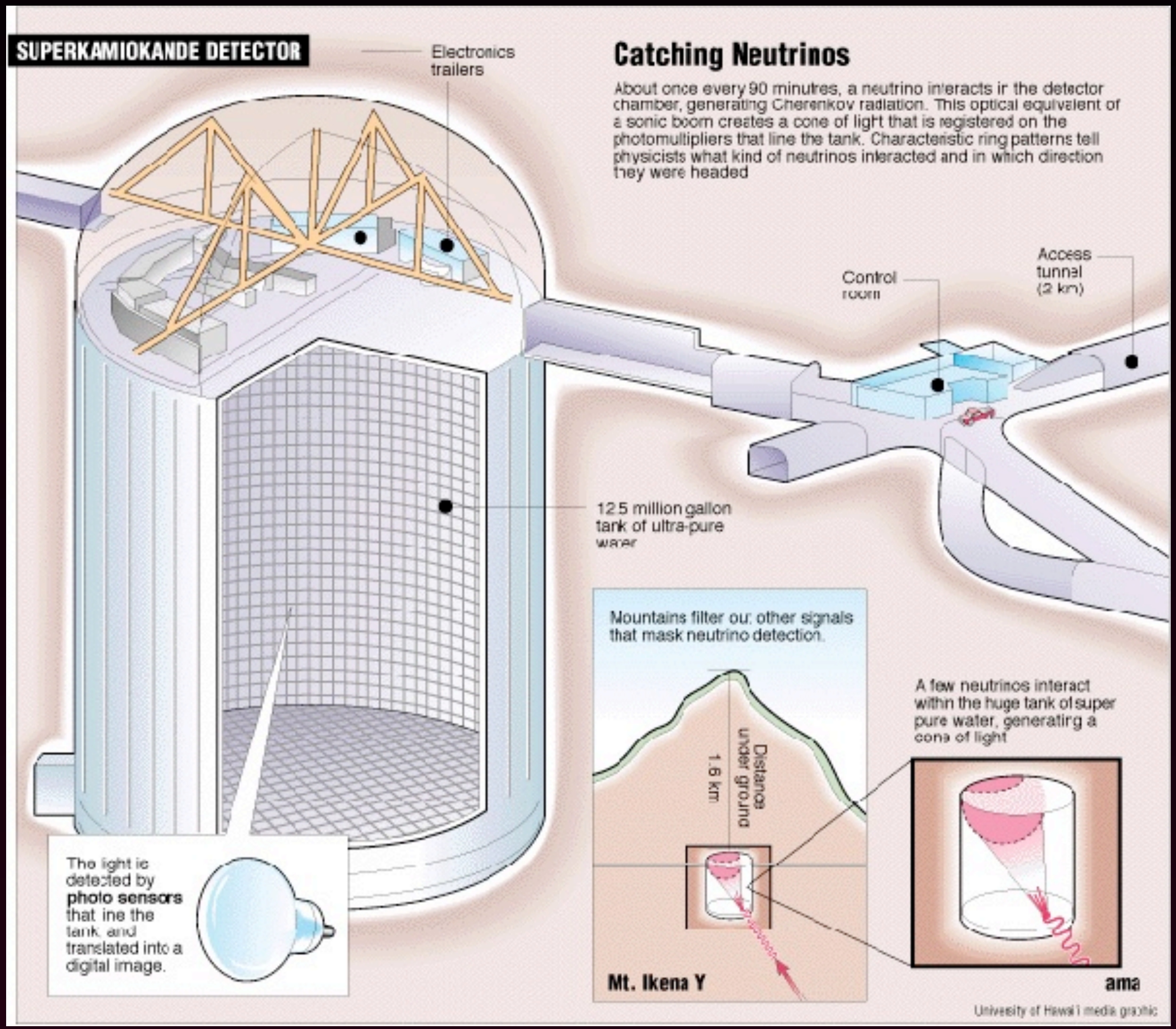
Homestake gold mine, Lead, South Dakota from http://www.sns.ias.edu/~jnb/Papers/Popular/Scientificamerican69/scientificamerican69.html, Davis and Bahcall photo, and results from Cleveland et al. (1998, ApJ, 496, 505)

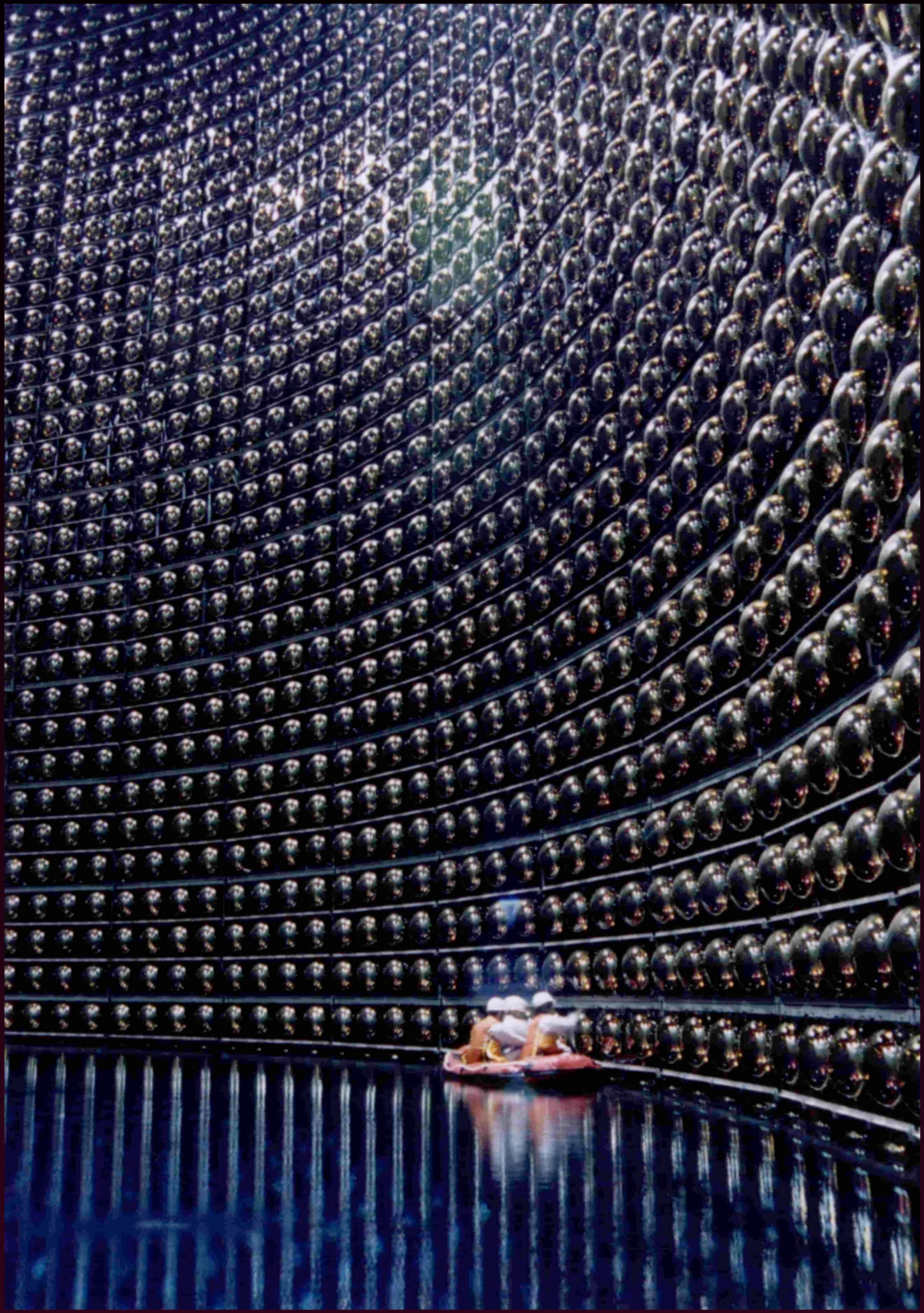
Cerenkov radiation



from https://engineering.purdue.edu/NE/Research/Research/Facilities/reactor.html

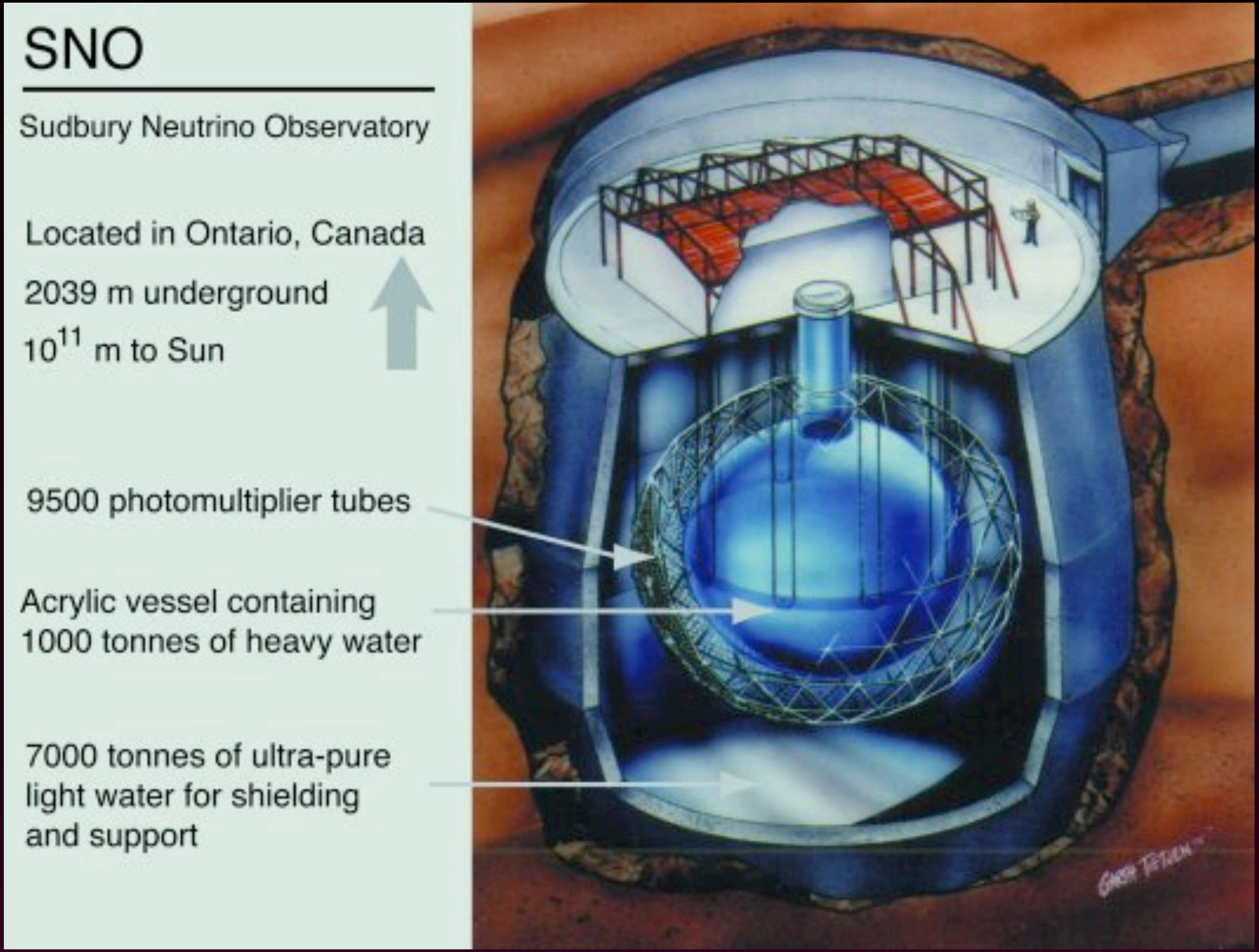
lights in the water: Super-Kamiokande





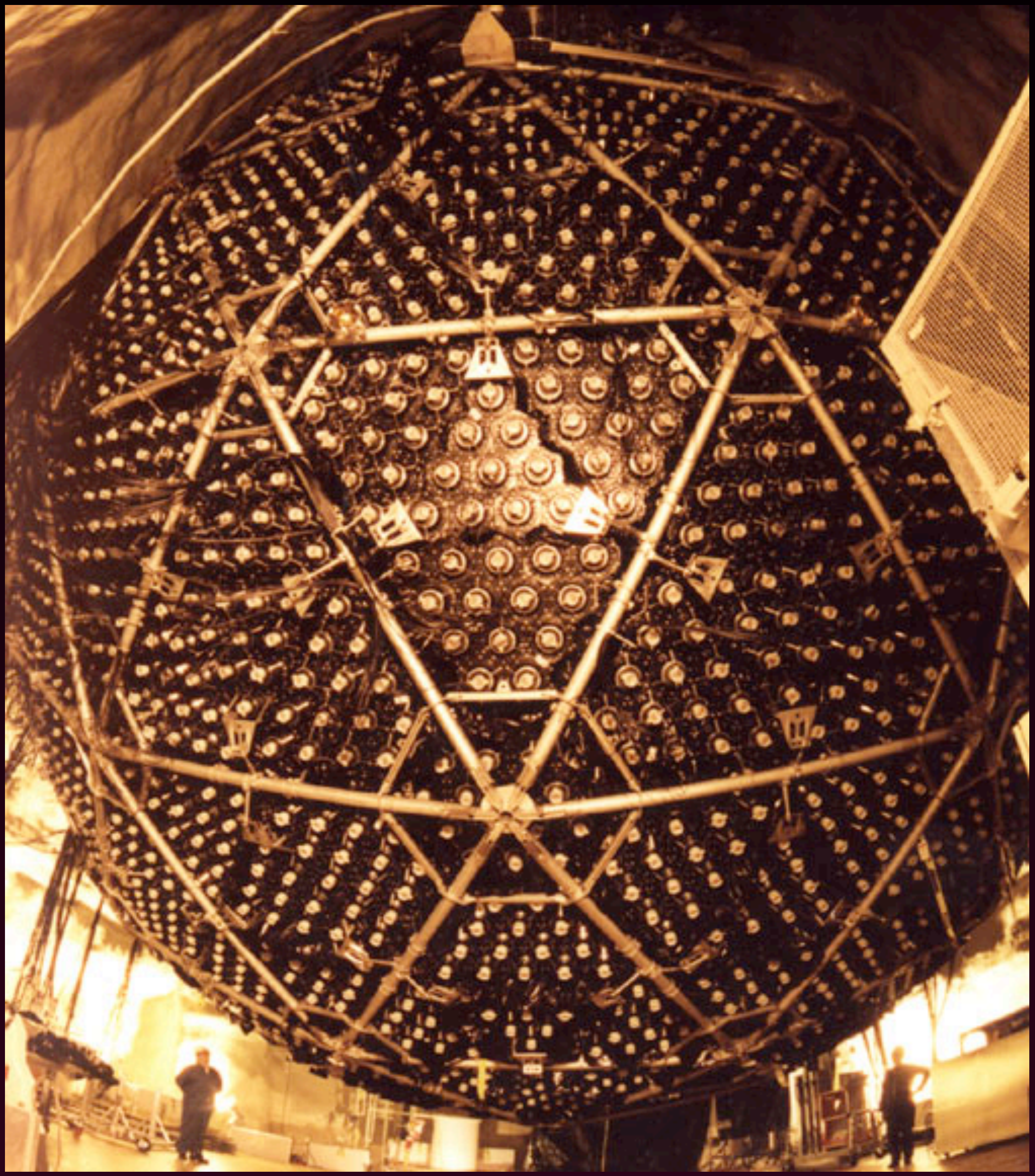
from http://www-sk.icrr.u-tokyo.ac.jp/sk/photo/high.html

Sudbury Neutrino Observatory



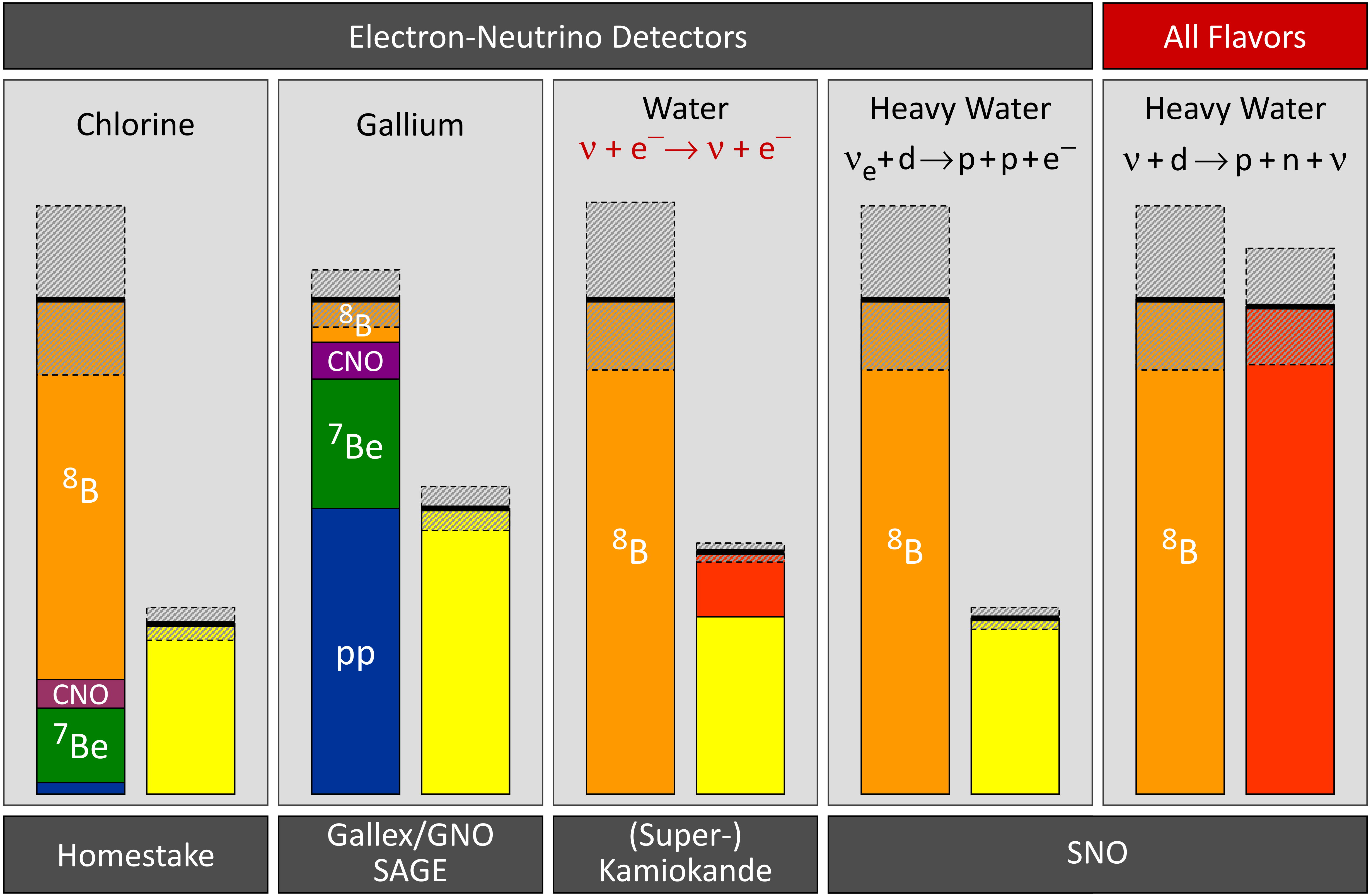
from http://www.europhysicsnews.com/full/11/article6/article6.html

Sudbury Neutrino Observatory



detectors around SNO from http://ww.pbs.org/wgbh/nova/neutrino/danc-10.html

Solar neutrino measurements



from Raffelt (2012), https://arxiv.org/abs/1201.1637

Color-Magnitude (or H-R) Diagram

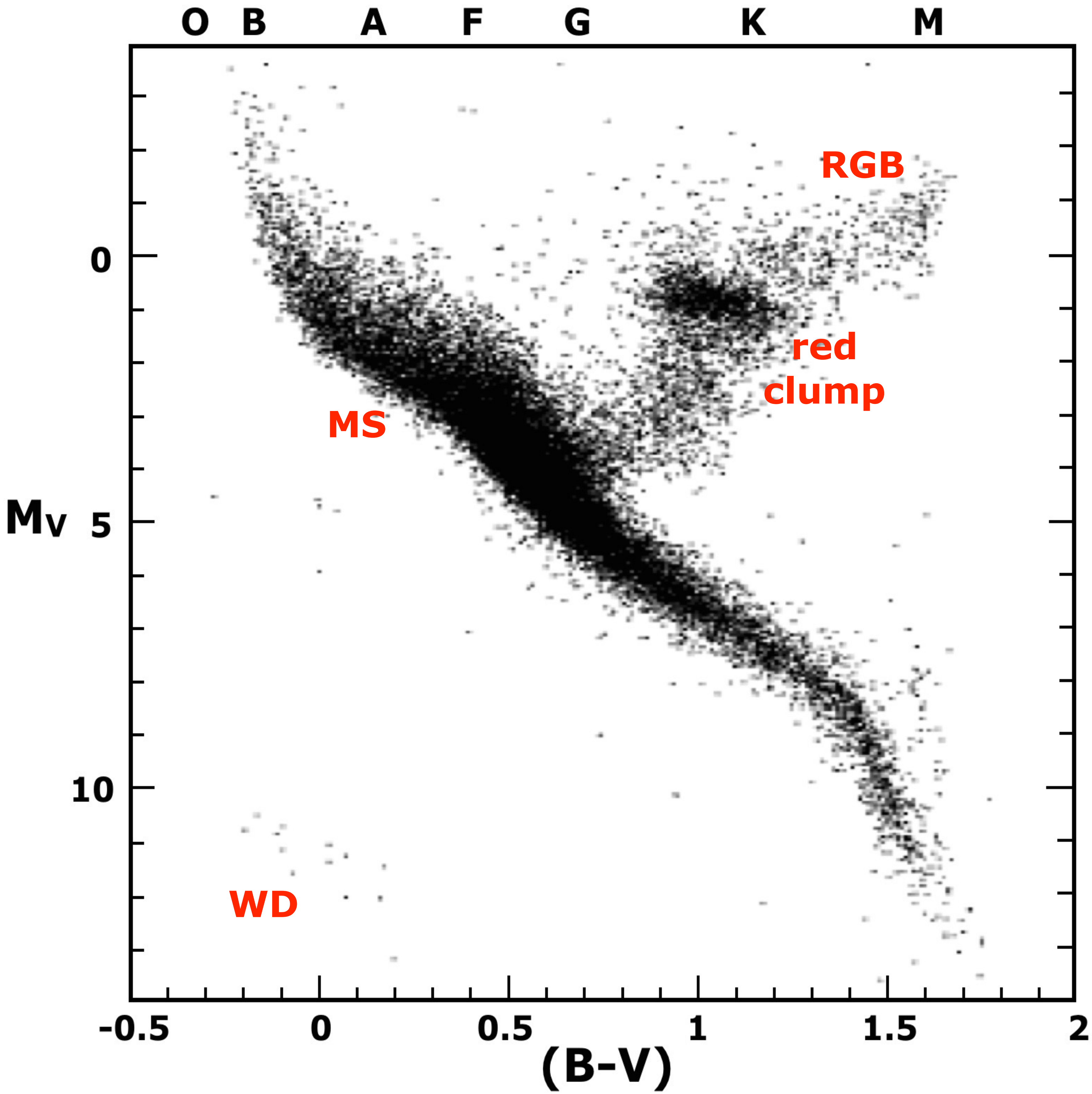
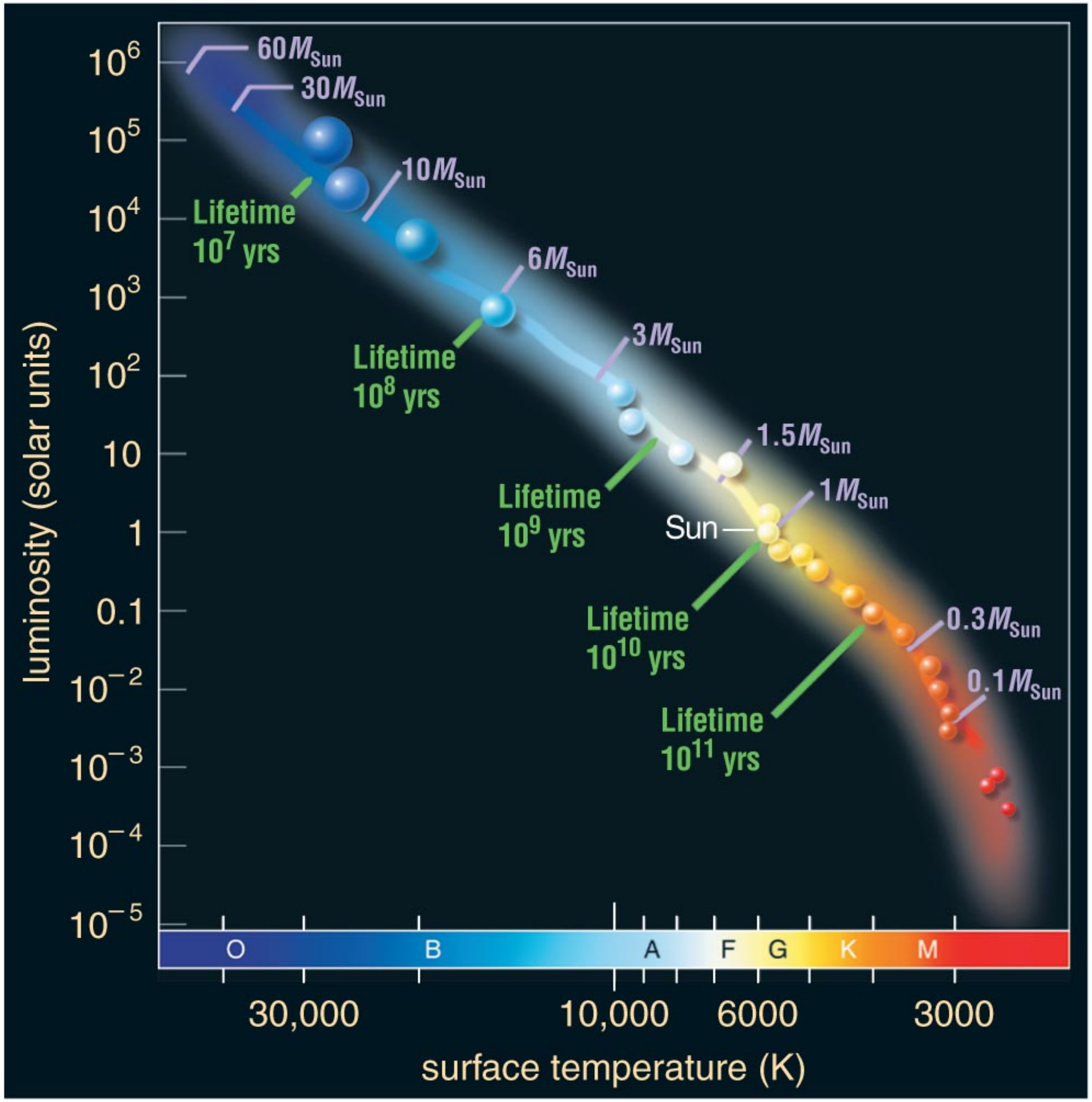


Figure courtesy of ESA.

Main-sequence stars



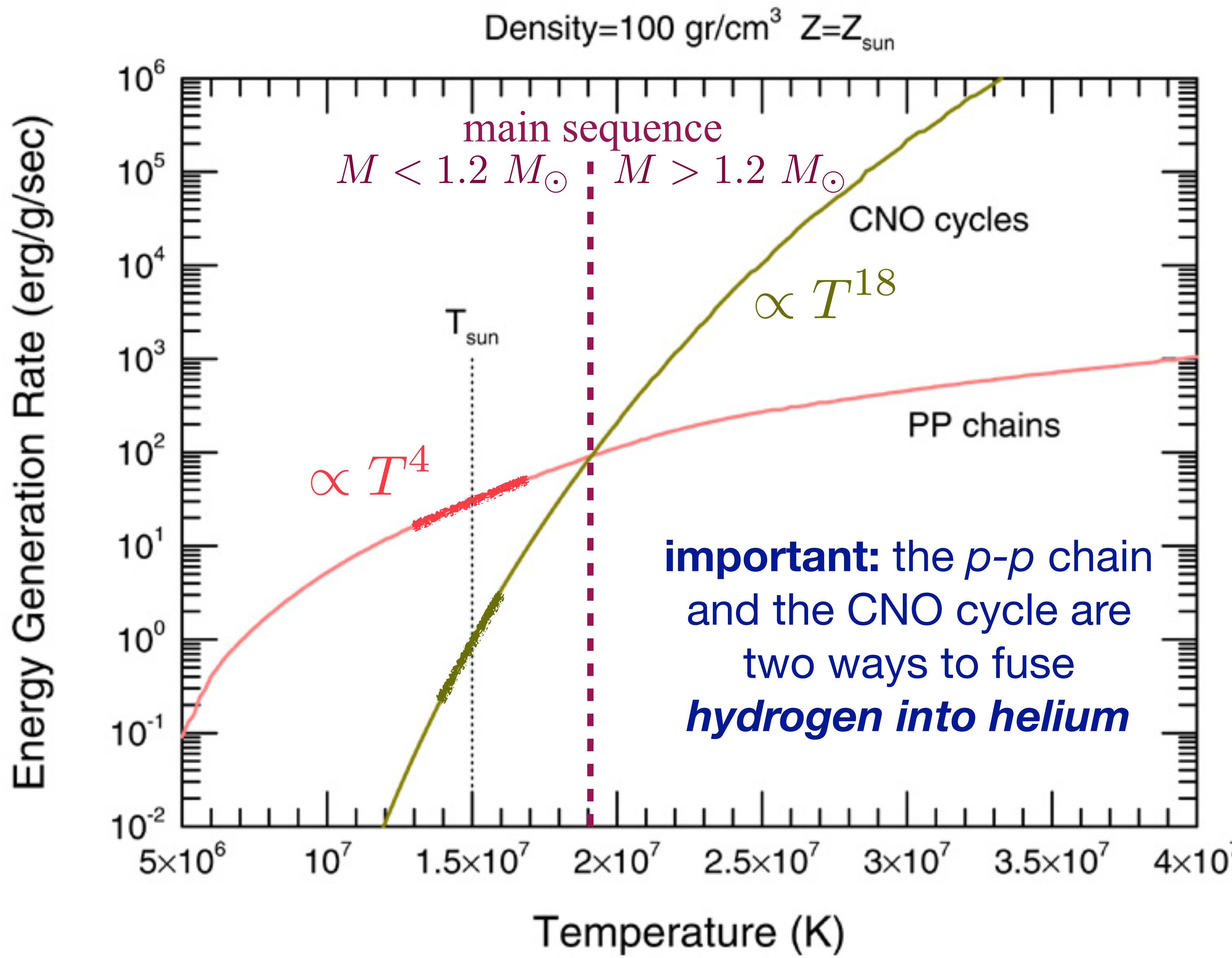
Main-sequence stars are fusing hydrogen into helium in their cores.

The Sun is a main-sequence star.

Luminous main-sequence stars are hot (blue).

Less luminous ones are cooler (yellow or red).

p-p chain or CNO cycle



from http://cococubed.asu.edu/images/net_ppcno/ppcno_sdot_web.jpg

Helium burning: triple α process

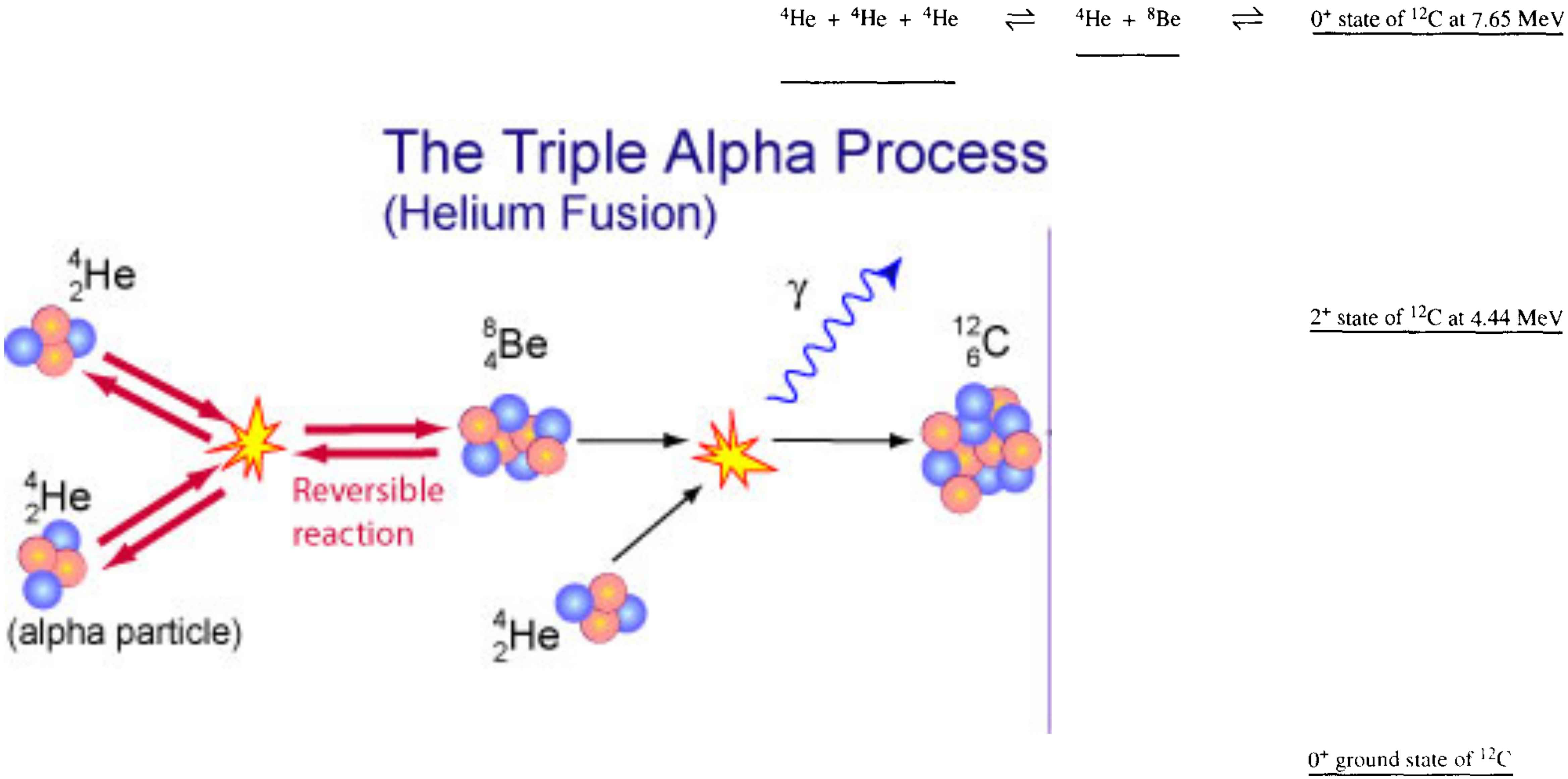


Fig. 4.6 Threshold energies and energy levels of carbon-12 relevant to helium burning. The 0^+ state of carbon-12 at 7.65 MeV, denoted by $^{12}C^*$ in the text, is only 0.3795 MeV above the threshold energy for three ^4He nuclei. Carbon is produced by establishing transient populations of unstable ^8Be and $^{12}C^*$ nuclei which coexist with ^4He nuclei at high temperature and density. A small proportion of the $^{12}C^*$ nuclei opt out of this dynamic coexistence by decaying to the ground state of carbon-12. The activation energy for carbon production is the energy needed to produce a $^{12}C^*$ nucleus, 0.3795 MeV. The energy released by carbon production is the difference in energy between the threshold for three ^4He nuclei and the ground state of carbon-12, 7.275 MeV.

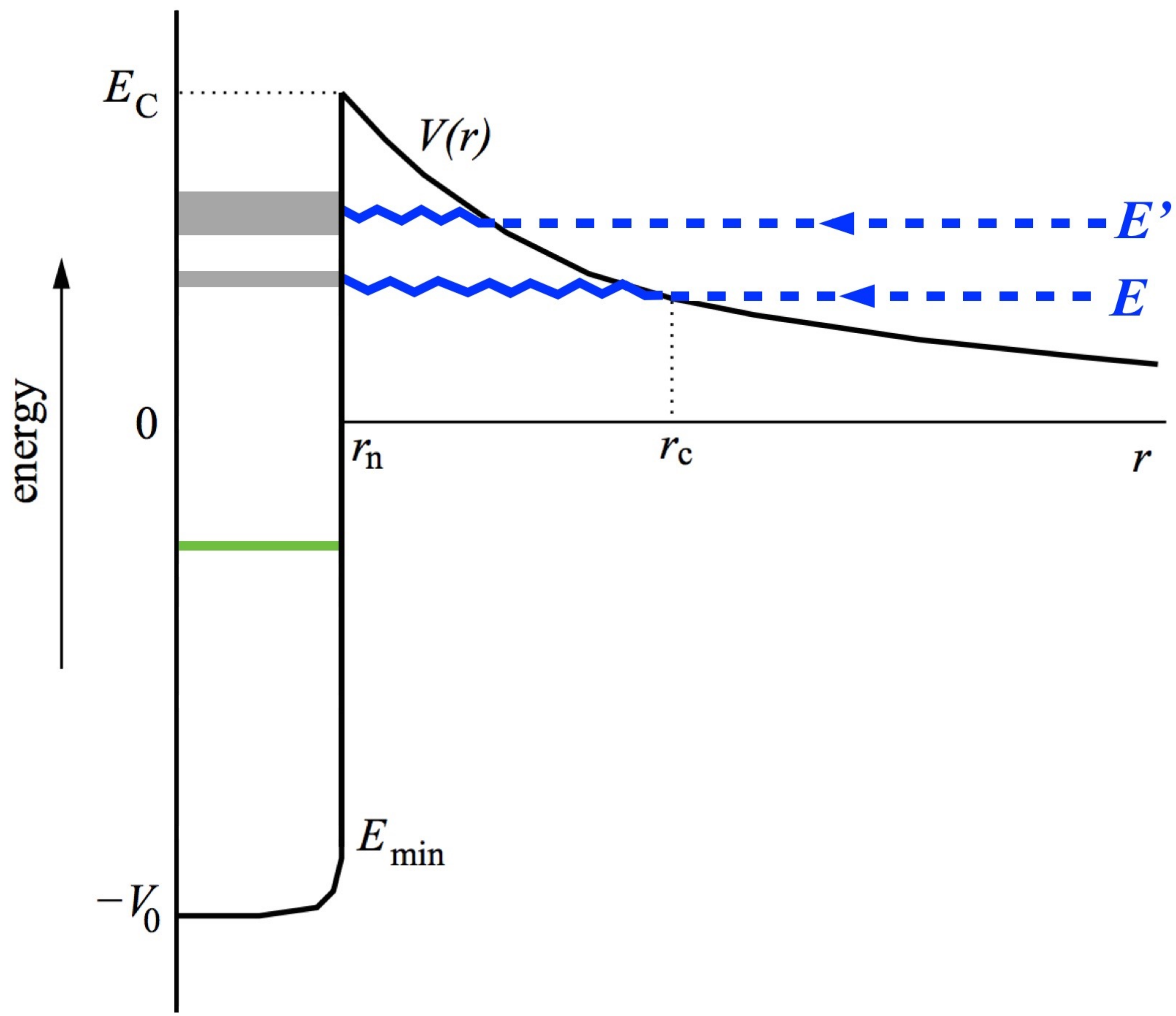
from http://www.atnf.csiro.au/outreach//education/senior/astrophysics/stellarevolution_postmain.html and Phillips Figure 4.6

Helium burning: triple α process

He fusion - the triple-alpha process ($T>10^8$ K)

$${}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \leftrightarrows {}^{8}_{4}\mathrm{Be}$$ ⟵ equilibrium reaction; ^{8}Be lifetime ~3×10^{-16} s!

$${}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C}$$ ⟵ reaction only possible at $T>10^8$ K



Normally the probability of a reaction just increases with the probability of quantum tunneling…

…but at E' there is a **resonance** with a discrete energy level and the probability is several orders of magnitude higher.

Adapted from Onno Pols, 2011.

Helium burning: triple α process

He fusion - the triple-alpha process ($T>10^8$ K)

$${}^4_2\mathrm{He} + {}^4_2\mathrm{He} \leftrightarrows {}^8_4\mathrm{Be} \longleftarrow$$ equilibrium reaction; ^{8}Be lifetime ~3×10^{-16} s!

$${}^8_4\mathrm{Be} + {}^4_2\mathrm{He} \rightarrow {}^{12}_6\mathrm{C} \longleftarrow$$ reaction only possible at $T>10^8$ K

${}^{12}_6$C has a resonant energy level precisely at the Gamow peak of the ${}^8_4$Be + ${}^4_2$He reaction, ~7.7 MeV.

$$3\ {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + 7.3\ \mathrm{MeV}\,,\quad \epsilon_{3\alpha} \sim Y^3 \rho^2\, T^{40}$$

Helium burning: triple α process

He fusion - the triple-alpha process ($T>10^8$ K)

$^{4}_{2}He + ^{4}_{2}He \leftrightarrows ^{8}_{4}Be$ ⟵ equilibrium reaction; ^{8}Be lifetime ~$3x10^{-16}$ s!

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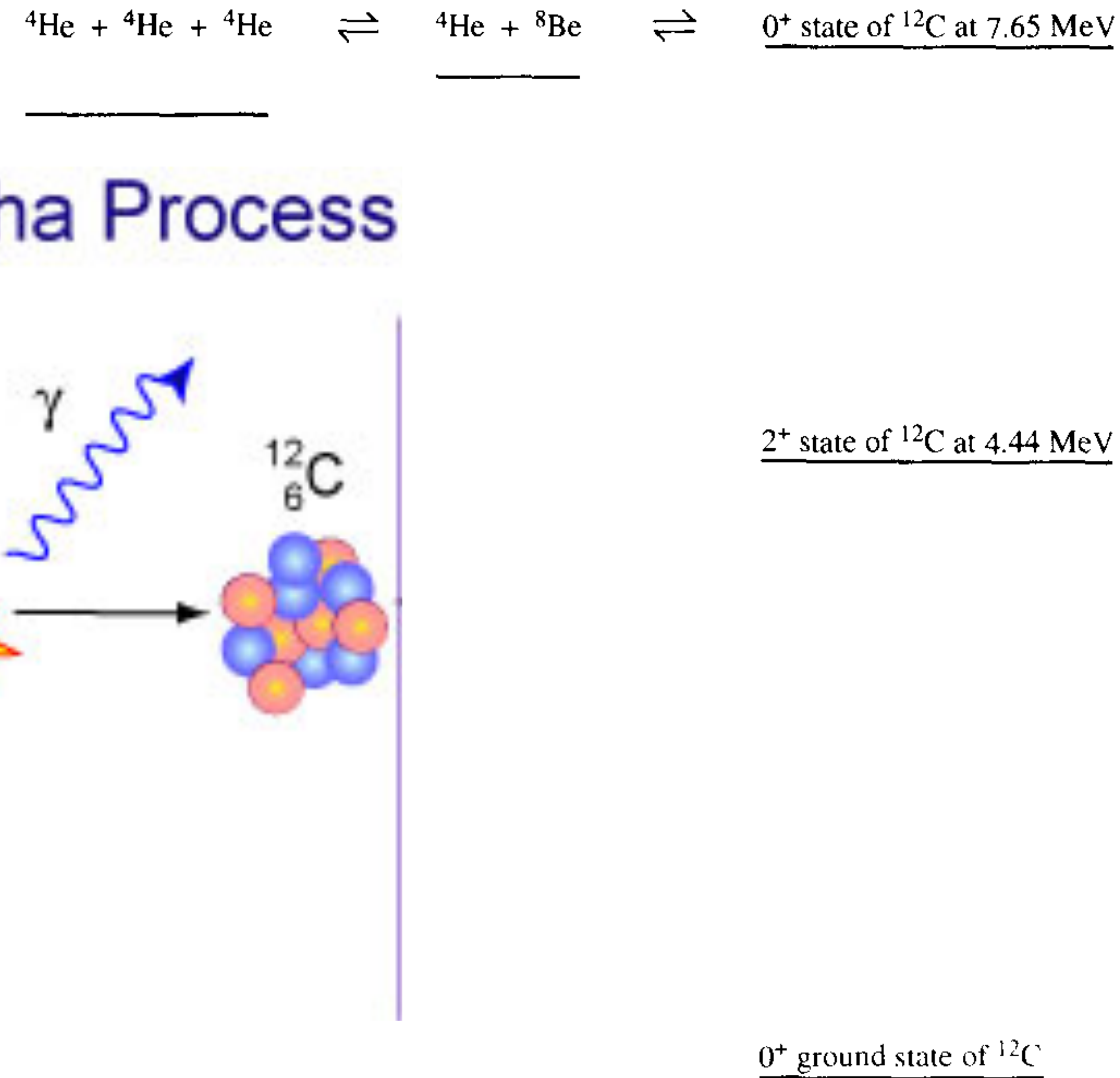
$^{12}_{6}C$ has a resonant energy level precisely at the Gamow peak of the $^{8}_{4}Be + ^{4}_{2}He$ reaction, ~7.7 MeV.

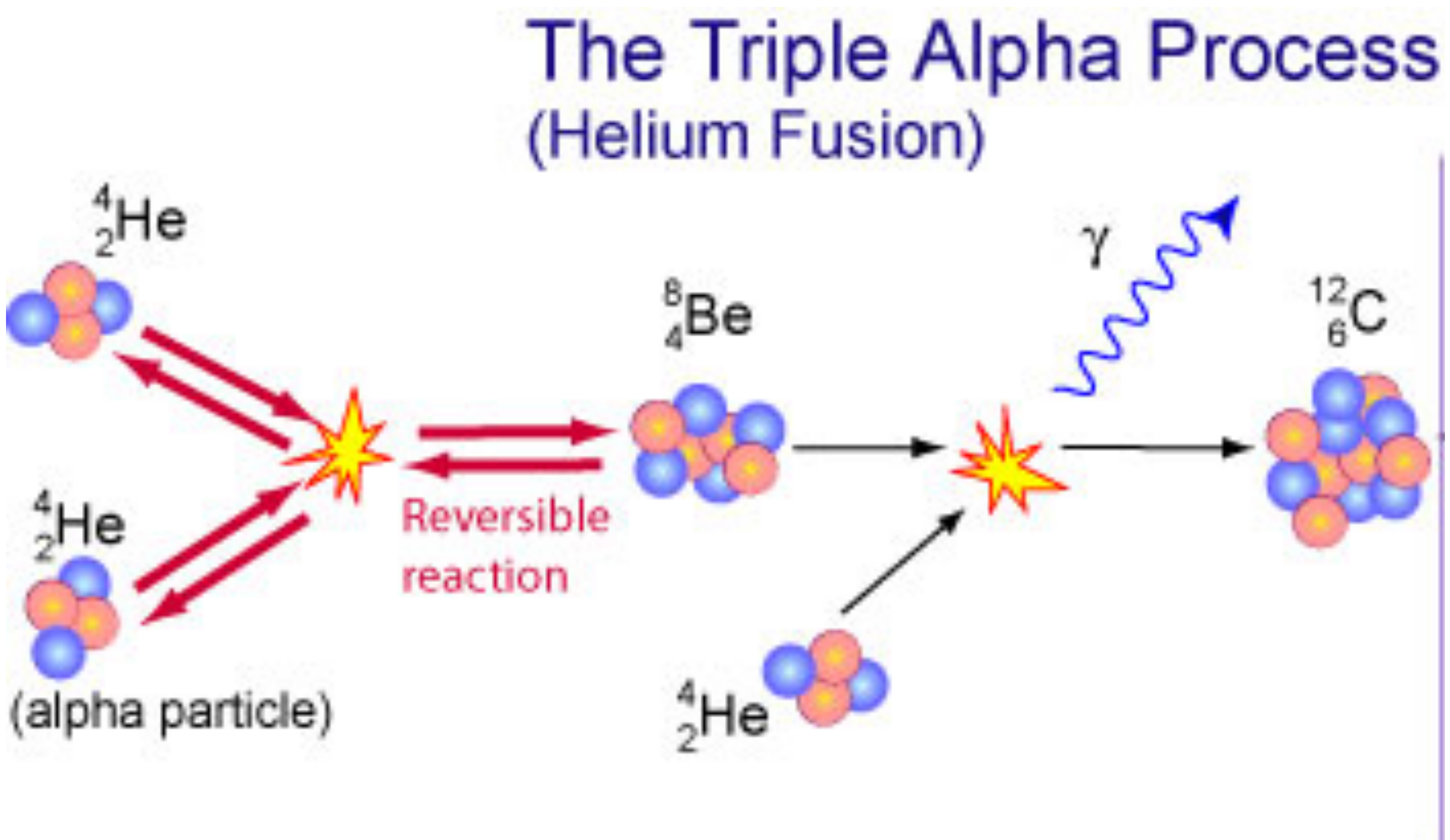
$$3\ ^{4}\mathrm{He} \rightarrow ^{12}\mathrm{C} + 7.3\ \mathrm{MeV}\ ,\ \epsilon_{3\alpha} \sim Y^3 \rho^2 T^{40}$$

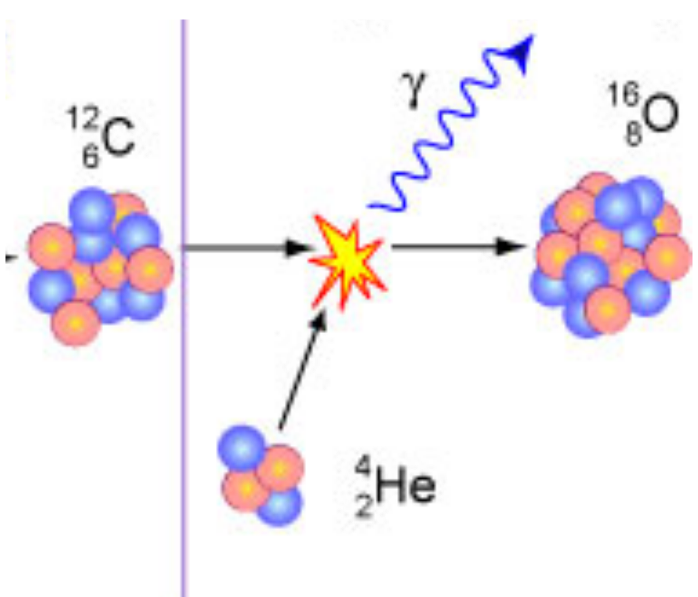
Finally, once we've built up some ^{12}C (near the end of the He-fusion phase) we get:

$^{12}_{6}C + ^{4}_{2}He \rightarrow ^{16}_{8}O + 7.16$ MeV (another reaction made possible by resonance…)

Helium burning: triple α process







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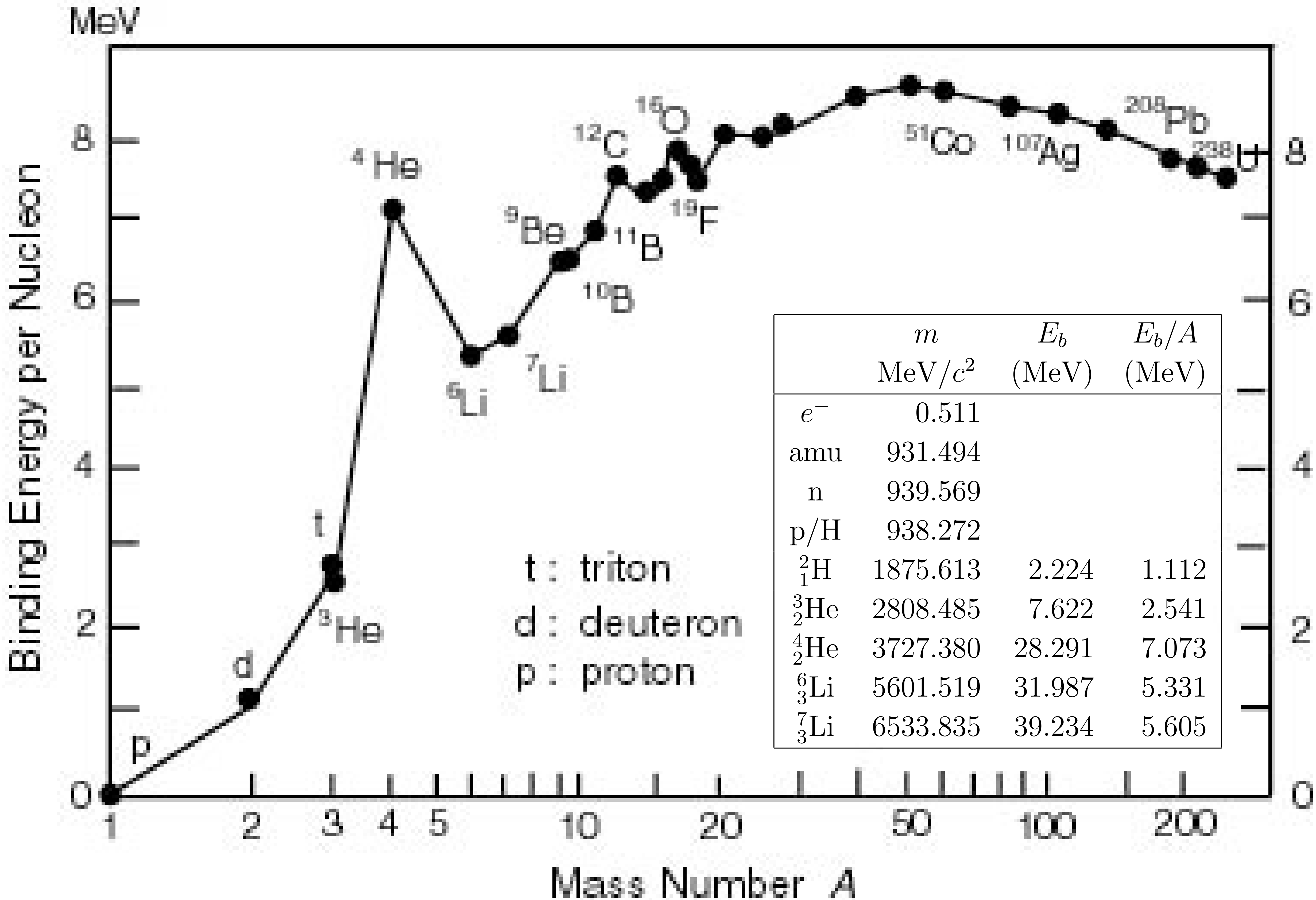
from http://www.atnf.csiro.au/outreach//education/senior/astrophysics/stellarevolution_postmain.html and Phillips Figure 4.6

Nuclear binding energy per nucleon



	m MeV/c^2	E_b (MeV)	E_b/A (MeV)
e^-	0.511		
amu	931.494		
n	939.569		
p/H	938.272		
$^2_1$H	1875.613	2.224	1.112
$^3_2$He	2808.485	7.622	2.541
$^4_2$He	3727.380	28.291	7.073
$^6_3$Li	5601.519	31.987	5.331
$^7_3$Li	6533.835	39.234	5.605

from http://www.kutl.kyushu-u.ac.jp/seminar/MicroWorld3_E/3Part2_E/3P23_E/nuclear_mass_E.htm

Advanced burning: massive stars

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{16}_{8}\mathrm{O} + \gamma \\
{}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma
\end{aligned}
$$

T > 10^8 K

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} &\rightarrow {}^{16}_{8}\mathrm{O} + 2\,{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{23}_{11}\mathrm{Na} + p \\
&\rightarrow {}^{23}_{12}\mathrm{Mg} + n \\
&\rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma
\end{aligned}
$$

carbon burning,
T > 6 × 10^8 K

oxygen burning,
T > 10^9 K

$$
\begin{aligned}
{}^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} &\rightarrow {}^{24}_{12}\mathrm{Mg} + 2\,{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{31}_{15}\mathrm{P} + p \\
&\rightarrow {}^{31}_{16}\mathrm{S} + n \\
&\rightarrow {}^{32}_{16}\mathrm{S} + \gamma
\end{aligned}
$$

silicon burning,
T > 3 × 10^9 K

$$
\begin{aligned}
{}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{32}_{16}\mathrm{S} + \gamma \\
{}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{36}_{18}\mathrm{Ar} + \gamma \\
&\vdots \\
{}^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{56}_{28}\mathrm{Ni} + \gamma
\end{aligned}
$$

photodisintegration!

Fusion temperatures/timescales

Phillips Table 4.2

TABLE 4.2 The time scale for the nuclear burning stages for a star of mass 25 $M_\odot$, and the central temperature and density at which they take place. This data is based on the calculations of Weaver, cited by Rolfs and Rodney (1988).

Stage	Time scale	Temperature (10^9 K)	Density (kg m^{-3})
Hydrogen burning	7×10^6 years	0.06	5×10^4
Helium burning	5×10^5 years	0.23	7×10^5
Carbon burning	600 years	0.93	2×10^8
Neon burning	1 year	1.7	4×10^9
Oxygen burning	6 months	2.3	1×10^{10}
Silicon burning	1 day	4.1	3×10^{10}

25 $M_\odot$ star

Lamers & Levesque Table 8.4

Table 8.4. Summary of the Most Important Reaction Rates in Stars

Fuel (1)	Process (2)	$T_{\rm thresh}$ 10^6 K (3)	Product (4)	$E_{\rm net}$ MeV/nucl (5)	T_c 10^6 K (6)	$L_{\rm net}/L$ (7)	Duration yr (8)
H	p-p chain	4	He	6.55	–	–	–
H	CNO cycle	15	He	6.25	35	0.94	1.1×10^7
He	3-α fusion	100	C,O	0.61	180	0.96	2.0×10^6
C	C-fusion	600	Ne,Mg,Na,O	0.54	810	0.16	2.0×10^3
Ne	Ne photdis	900	O,Mg,Si		1600	5.3×10^{-4}	0.7
O	O-fusion	1000	S,Si,P,Mg	0.30	1900	8.2×10^{-5}	2.6
Si	Si nucl equil.	3000	Fe,Ni,Cr,Ti	<0.18	3300	5.8×10^{-7}	0.05

Notes. photdis = photodisintegration. **nucl equil** = nuclear equilibrium = photodisintegration + capture of p, n, and He. Column (5) = energy generated per nucleon (He has 4 nucleons, C has 12 nucleons etc.). Columns (6), (7), and (8) refer to the evolution of a star of 15$M_\odot$ (Based on Woosley & Janka 2005, and Maeder 2009).

15 $M_\odot$ star

Fusion and Stellar Evolution

Minimum core mass for ignition

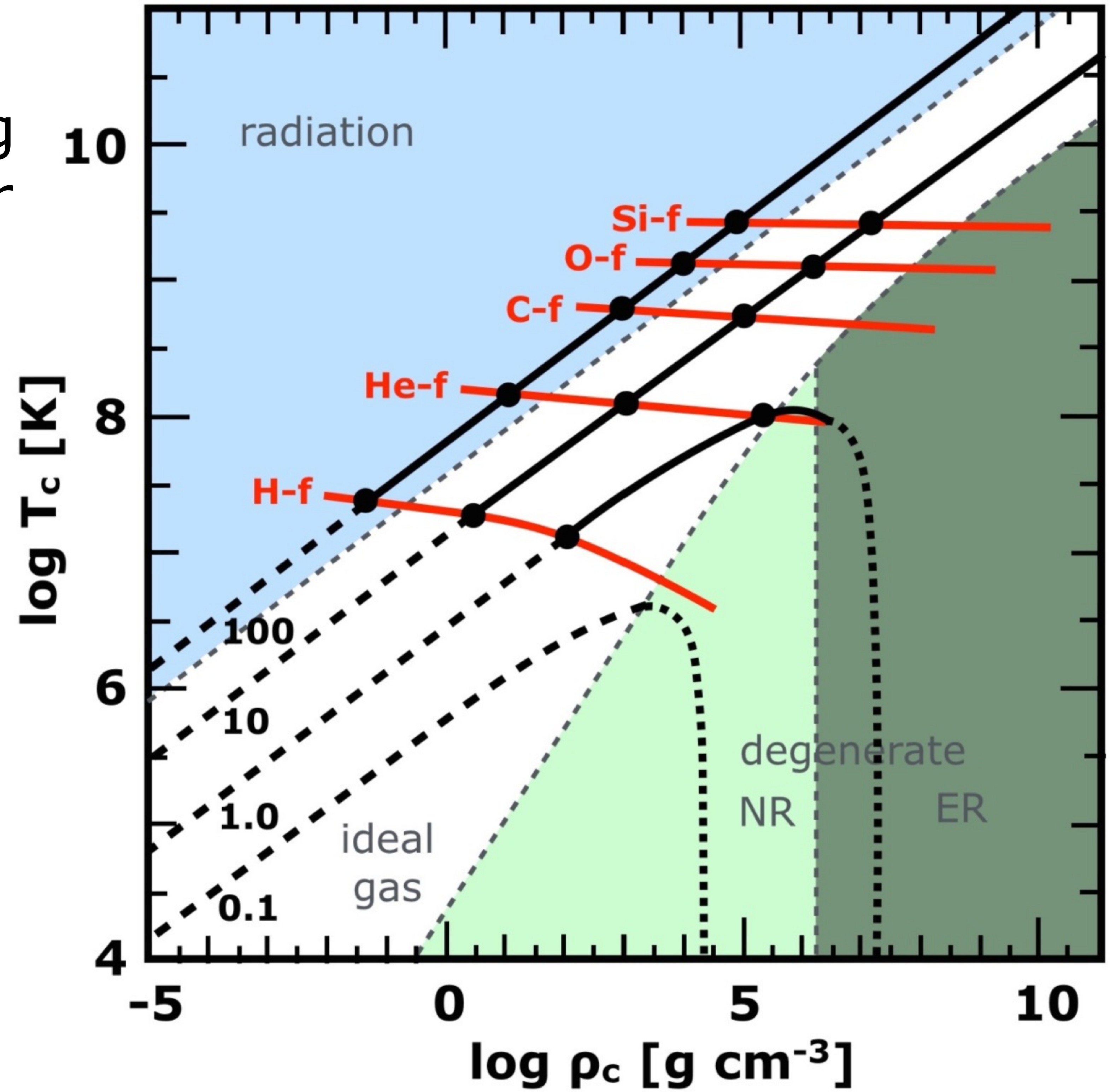
This is why:

—the threshold for calling something a massive star

is ~8M⊙

—most stellar models use a minimum stellar

mass ~of 0.1M⊙



Adapted from Onno Pols, 2011.