Helioseismology

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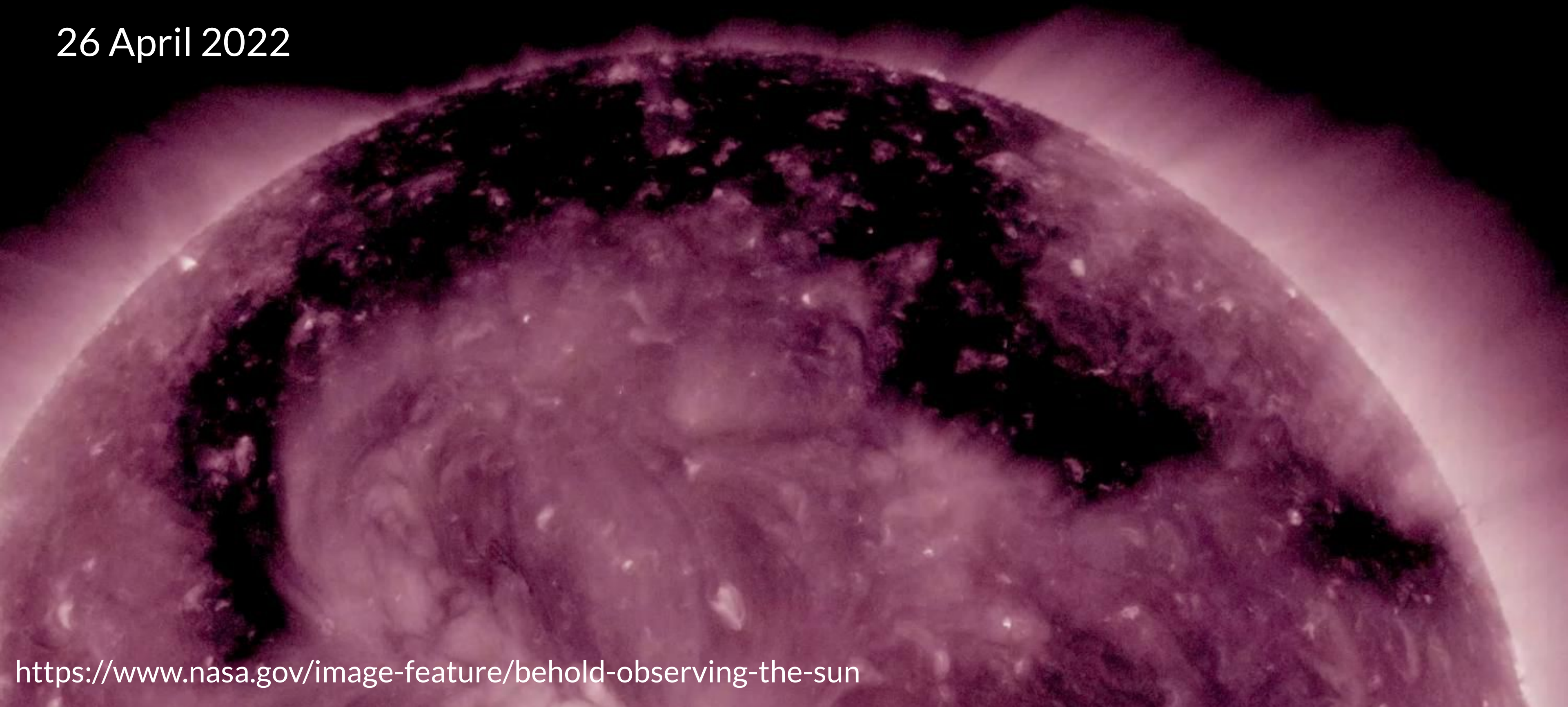
Seung Hee Sung

26 April 2022



https://www.nasa.gov/image-feature/behold-observing-the-sun

Overview

- Motivation
- Definition
- Basic Physics
- Relevant Equations
- Relevant Derivations
- Examples

Why study helioseismology?

Pulsating stars have two tones at most while the Sun has millions of resonant tones. (Chaplin

The oscillations/vibrations of the sun creates magnetic fields, which reach the surface to create sunspots, released as

Definition

The study of the solar interior

- The study of the natural, resonant oscillations of the sun (Chaplin, 2006), also known as its seismic activity.
- Helioseismology is divided by local and global seismology. (L. Gizon et al., page 2, 2006)
- Local applies to sunspots.
- Global applies to the Sun as a whole
- Asteroseismology is the study of seismic activity in other Stars.

Basic Physics

Oscillations

- Resonates on a period of about 5 minutes .
- 180 decibels of sound.
    - A normal conversation is 60 decibels

Radiation

- Sound wave effects are coded in the EM radiation received from the Sun and are decoded from them.

Methods for Observing Oscillations

Doppler Shift

- Velocities are measured in km*s^-1
- Measures the motions of the shift in light waves on the surface of the Sun.

Gas Compression

- Its compression increases temperature which increase light intensity emitted.
- Gas turbulence produces sound (p9) causing resonance

Basic Physics

Mechanism

- Turbulence at top layer of convection zone drives and damps oscillations
- Resulting noise interferes to form standing waves based on properties of the Sun's interior
- Resonates like a conical pipe – with Sun center as apex of cone

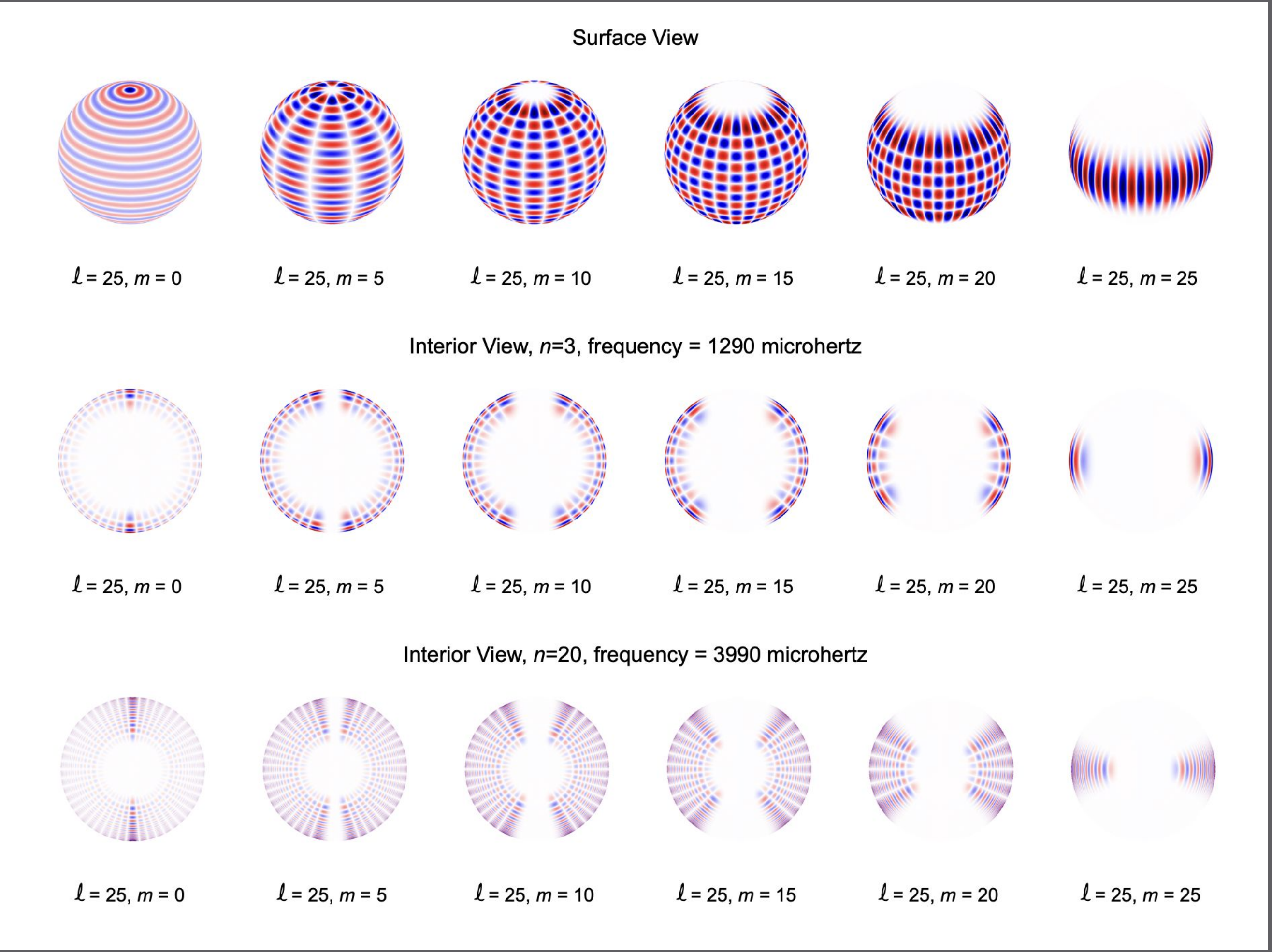
Modes

- Radial (up-and-down) and transverse (sideways) modes
- Described by spherical harmonics
- Appears as a checkerboard pattern
  - Patches move with individual displacement and velocity
- Over a million of these modes!

Helioseismology vs. Asteroseismology

- Other stars may have other mechanisms of driving oscillations
  - “Solar-like oscillator”
- Low resolution of modes in distant stars
  - Restricts to low-degree modes ($l<4$)

http://solar-center.stanford.edu/sosh/



The figure shows the harmonic oscillations of a spherical model of the Sun.

Use the link above to see and here the models by SOSH.

Relevant Equations

Energy Flux, the energy invested into the seismic transient.

$$\varepsilon \sim \left(\frac{\Delta I_c}{I_c}\right)^2,$$

- It is proportional to a fraction of order

$$\frac{\Delta I_c}{I_c}$$

times the radiative energy emitted by the flare.

- There is more that is needed to describe acoustics of solar flares.

Relevant Equations

2nd-order equation for radial displacement eigenfunction

$$\frac{d^2 \xi_r}{dr^2} = \frac{\omega^2}{c_s^2} \left(1 - \frac{N^2}{\omega^2}\right) \left(\frac{S_\ell^2}{\omega^2} - 1\right) \xi_r$$

- Buoyancy frequency *N* and Lamb frequency S_l given by

$$N^2 = g \left(\frac{1}{\Gamma_1 P} \frac{dp}{dr} - \frac{1}{\rho} \frac{d\rho}{dr} \right)$$

$$S_\ell^2 = \frac{\ell(\ell+1)c_s^2}{r^2}$$

- *r* is radial coordinate, ω is frequency of mode, c_s is sound speed inside the star

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- Reduced from system of 4 differential equations
- Assumes perturbations are adiabatic, perturbation to gravitational potential is negligible, & star's structure varies more slowly with radius than oscillation mode
- Displacement proportional to its second derivative – harmonic!

Derivation of Seismic Heating by Photospheric Emission

$$\delta p = \rho g \delta z,$$

$$\Delta W = \frac{1}{2}\delta p \delta z = \frac{(\delta p)^2}{2\rho g}.$$

$$\frac{\delta p}{p_0} = \frac{\delta T}{T_0},$$

$$\tau_r \sim \frac{2H}{c},$$

1. The pressure increment approximation, density for the solar medium, g is gravity, z is the location
2. The area (delW), results in a triangular hysteresis loop in the phase diagram escaping the transient.
3. Estimation of the pressure perturbation from heating of the low photosphere, based on Boyl’es law. T_o=reference temperature.
4. Recoil time of pressure excess in the photosphere. H = density of e-folding height of the medium.

C. Lindsey, A.-C. Donea, p 628

Derivation of Seismic Heating by Photospheric Emission

$$\frac{\delta T}{T_0} \sim \frac{1}{4} \frac{\delta F_{\rm rad}}{F_{\rm rad0}}.$$

$$\Delta W \sim \frac{p_0^2}{32 \rho_0 g} \frac{(\delta F_{\rm rad})^2}{F_{\rm rad0}^2}.$$

5. Approximating the heat of excess delF_rad in continuum flux using the Stefan-Boltzmann law.

6. Finally the energy deposited into transient emission is as such on the left.

C. Lindsey, A.-C. Donea, p 628

Seismic Emissions From Solar Flares

- Measuring the acoustic emissions produced by the flare
  - By the *time-distance technique* by Kosovichev and Zharkova (1998).
  - Computational seismic holography to image the acoustic source. (J.C. Martínez-Oliveros *et al.*)
    - Forward in time
    - or backward in time (acoustic egression)

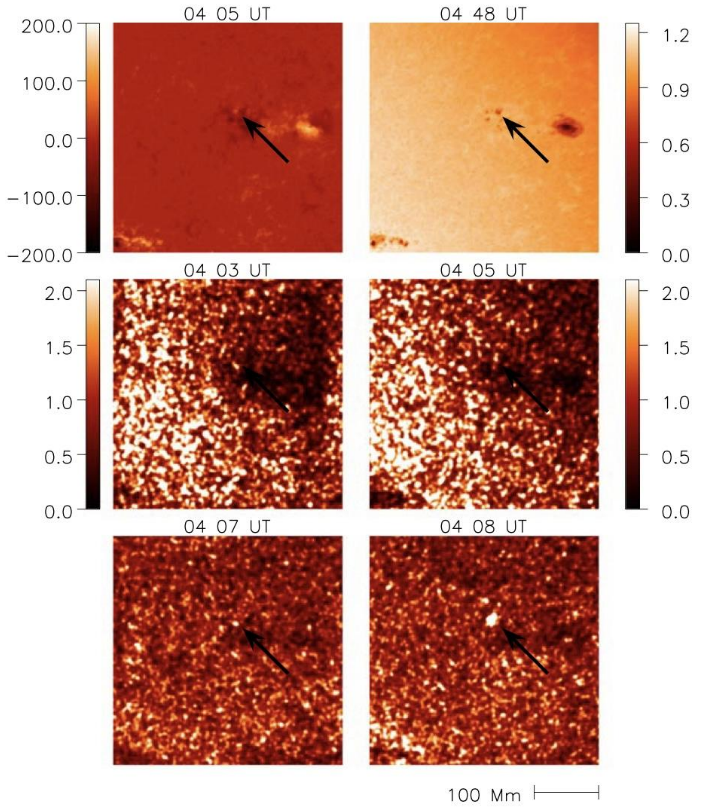


Figure 2 Egression and acoustic power snapshots of AR 9348 on 10 March 2001 integrated over 2.0 – 4.0 mHz and 5.0 – 7.0 mHz frequency bands and taken at the maximum of the correspondence frequency. Top frames show MDI magnetogram of the active region (right) at 04:05 UT and a visible continuum image at 04:08 UT (left). Second row shows egression power at 3 mHz (left) and 6 mHz (right) at the respective maxima. The bottom row shows acoustic power. Times are indicated above the respective panels, with arrows inserted to indicate the location of the seismic source.

Seismic Emissions From Solar Flares

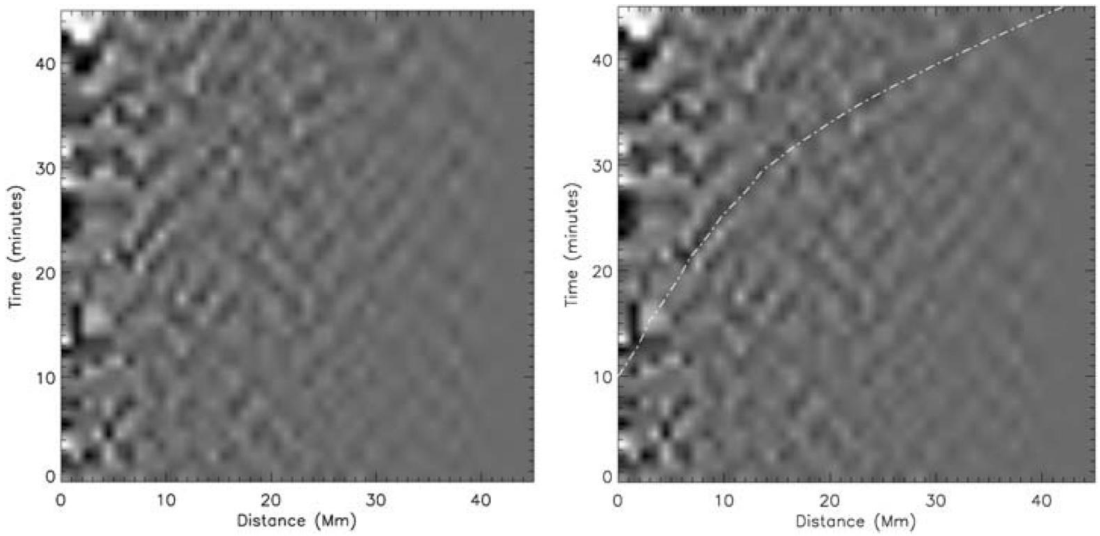
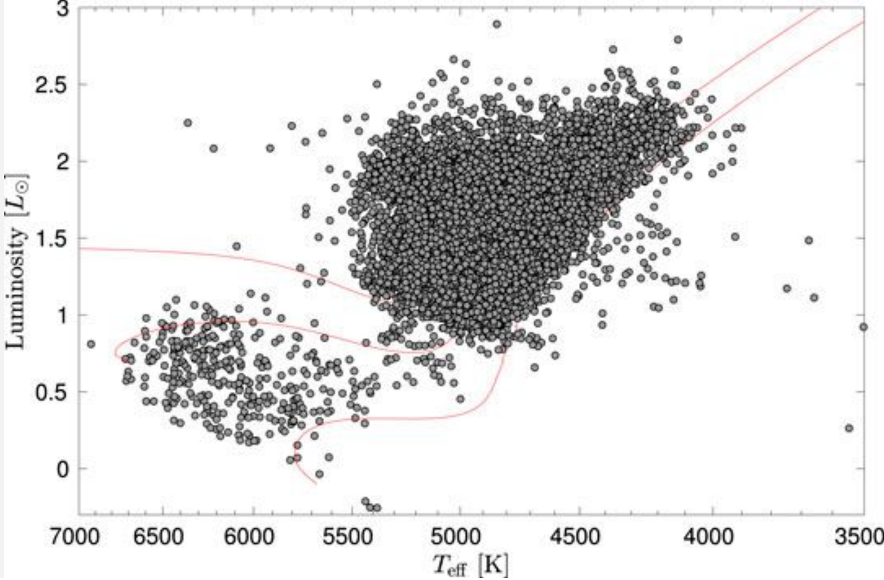
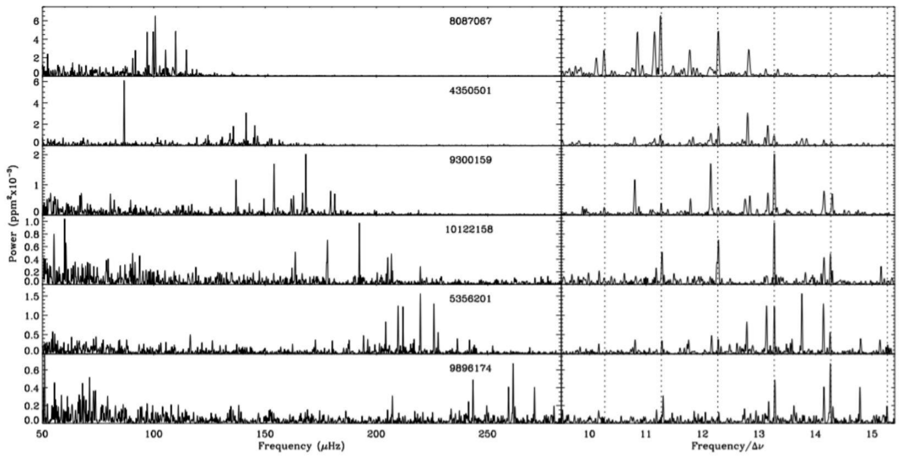


Figure 1 Time–distance plot of the amplitude of the surface ridge averaged over curves of constant radius in the azimuth range $+135°$ to $+225°$ (rendered in gray in both frames). The white curve superimposed on the right frame represents the wave time travel for a standard model of the solar interior. The time represented as 0 along the vertical axis of the plot is 04:07 UT.

The *Kepler* Era

- *Kepler* has measured fluctuations of tens of thousands of stars
- Enables observations of low-luminosity stars
- Solar-like oscillations in red giants





Bedding et al. (2010)

Conclusion

- The sun provides intricate and detailed information that even the closest star cannot provide, so that we can determine the minutiae of the structure of stars.
- It is the study of oscillations of the interior to the atmosphere of the Sun
- The study of thermal physics, quantum physics and acoustics is the foundation of helioseismology.
- Geometry, Fourier Analysis and Taylor Series expansion are the most common mathematics found in helioseismology.

Conclusion

- A Relevant Equation is the energy flux but many more mathematics are needed to describe acoustics of solar flares.
- A Relevant Derivation is the energy deposited into transient emission.
- An example includes modeling magnetic field strengths and seismic emissions from solar flares.

References

Aerts, C., et al. “The Current Status of Asteroseismology.” *Solar Physics*, vol. 251, no. 1-2, 25 Apr. 2008, pp. 3–20., https://doi.org/DOI: 10.1007/s11207-008-9182-z.

Chaplin, William J., and Sarbani Basu. “Perspectives in Global Helioseismology and the Road Ahead.” Solar Physics, vol. 251, no. 1-2, 3 Mar. 2008, pp. 53–75., https://doi.org/10.1007/s11207-008-9136-5.

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García, R. A., et al. “Influence of Low-Degree High-Order P-Mode Splittings on the Solar Rotation Profile.” *Solar Physics*, vol. 251, no. 1-2, 4 Sept. 2008, pp. 119–133., https://doi.org/10.1007/s11207-008-9144-5.

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To be able to see the machinery of a star throbbing with activity is the most instructive for the development of our knowledge.

- Sir Arthur Eddington

Beyond Stars

The Sun is a powerful testing ground for basic astrophysical principles.

Stellar interiors exhibit extreme exotic conditions that are not easily recreated.

Helioseismology allows us to gather and interpret meaningful solar data.

Implications range from particle physics to cosmology and theories of relativity.

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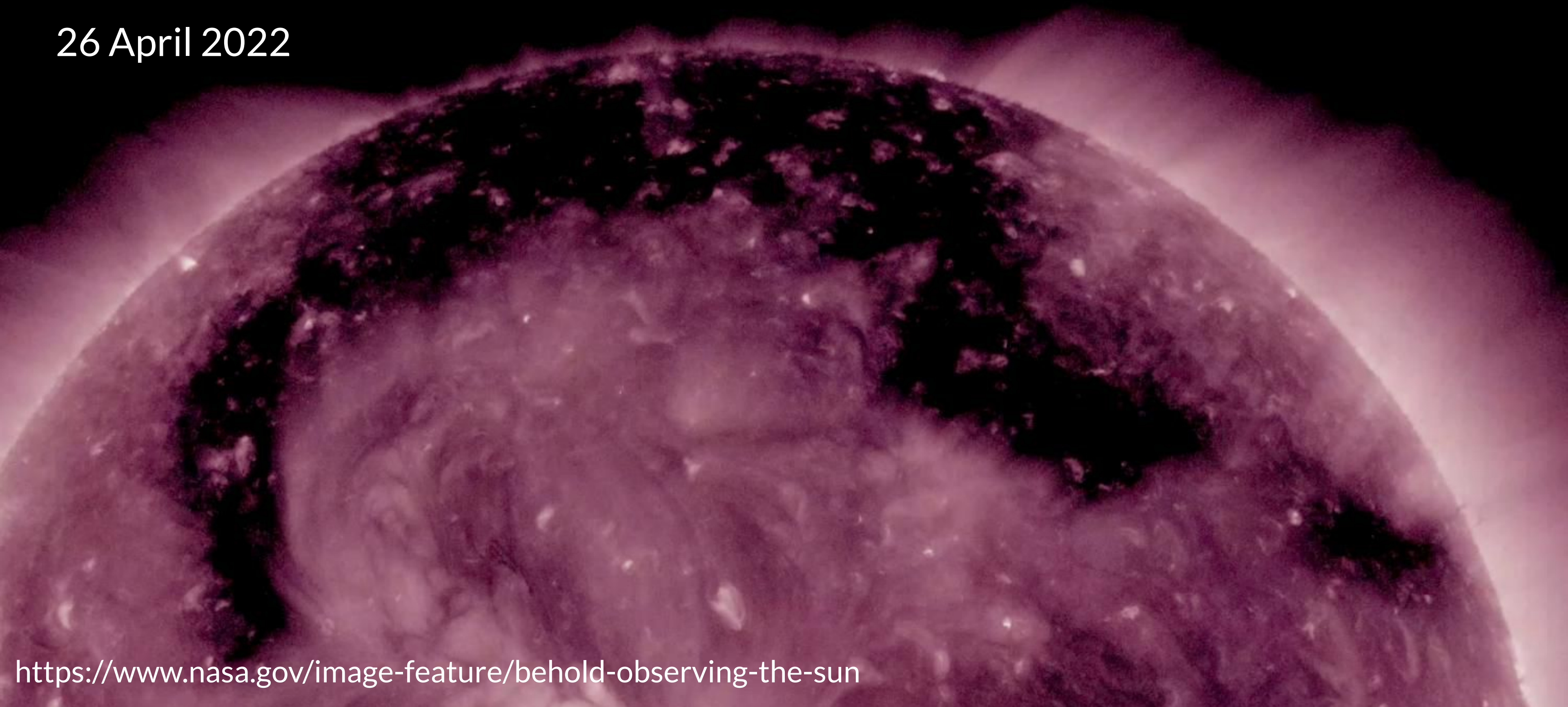
THANK YOU

26 April 2022



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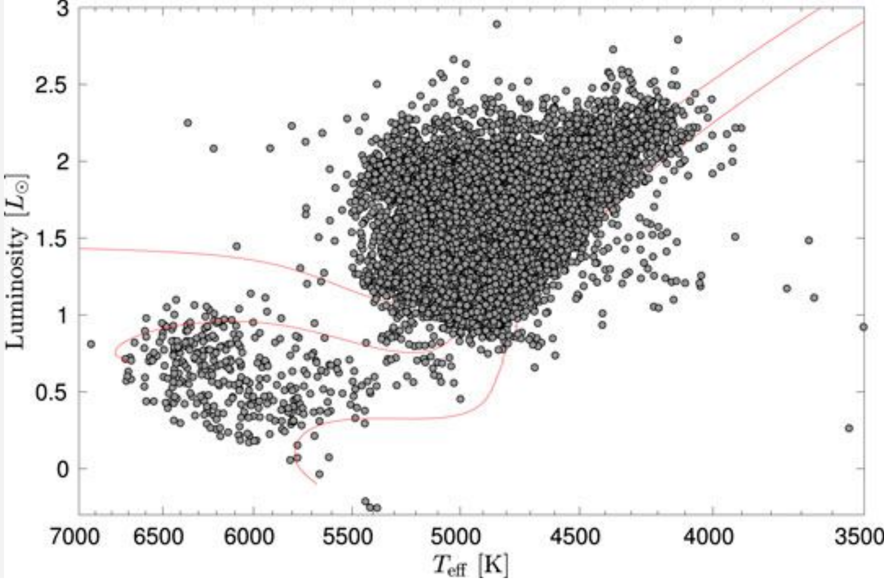
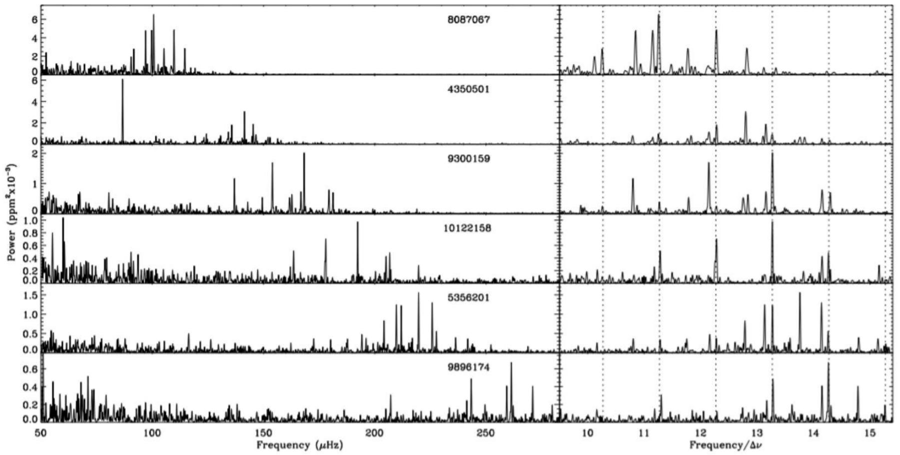




Bedding et al. (2010)

Helioseismology vs. Asteroseismology

- Other stars may have other mechanisms of driving oscillations
    - “Solar-like oscillator”
- Low resolution of modes in distant stars
    - Restricts to low-degree modes (l<4)

Second Example

Magnetic Field Strengths

-

Spherical Harmonics

$$\nu_{n\ell} \sim \Delta\nu\left(n + \frac{\ell}{2} + \epsilon\right)$$

- v_{nl} is frequency,
- Δv is a measure of inverse sound travel time over a star’s diameter, and
- ϵ is a function of frequency mainly determined by conditions near the surface
- Good for high *n*

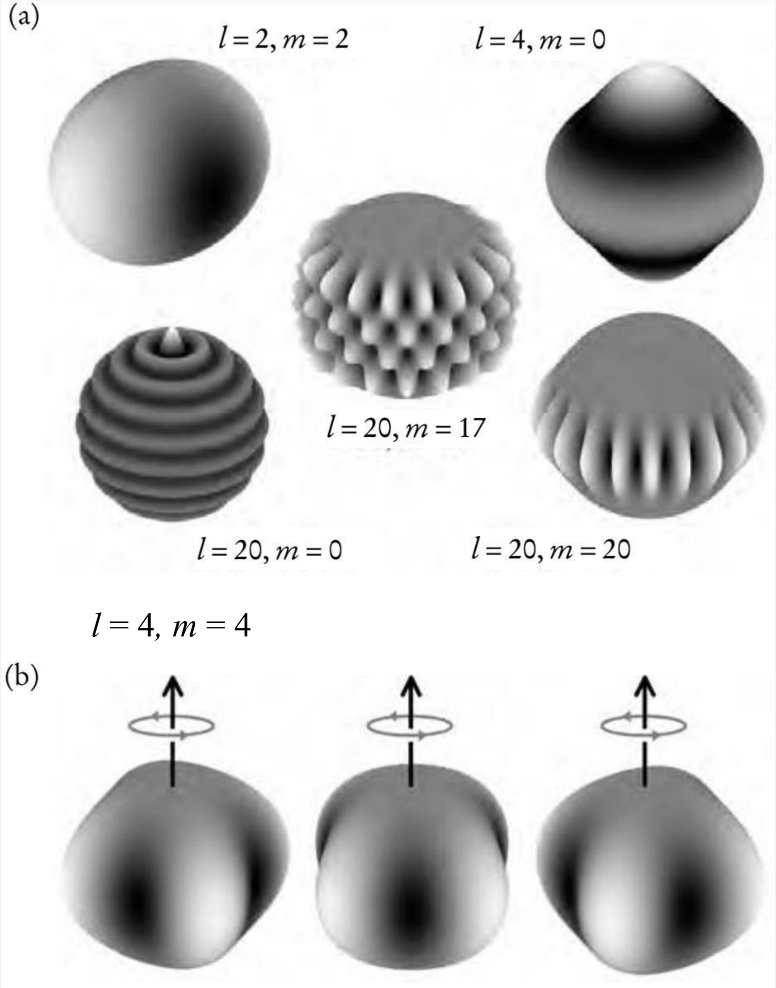


Figure 3.3 Chaplin (50)

Spherical Harmonics

$$\nu_m = \nu_0 + m \int_0^R K(r) \frac{\Omega(r)}{2\pi} \frac{\mathrm{d}r}{R}$$

where v_m is frequency, Ω is angular velocity, and

$$K(r) = \frac{\{\xi_r^2 - 2\xi_r\xi_h + [\ell(\ell+1) - 1]\xi_h^2\}r^2\rho}{\int_0^R \frac{\mathrm{d}r}{R}[\xi_r^2 + \ell(\ell+1)\xi_h^2]r^2\rho}$$

- Ignores higher-order and magnetic effects
- Under rotation, transverse modes undergo splitting of frequency into multiplets.

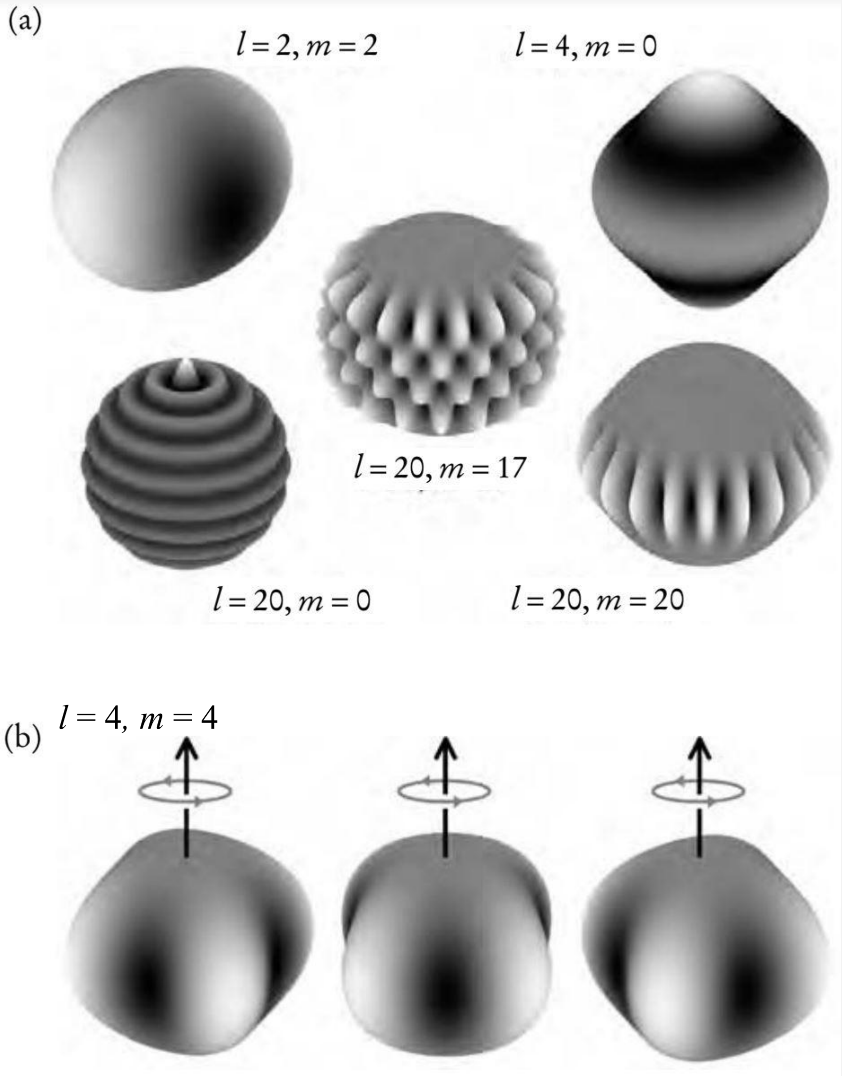


Figure 3.3
Chaplin (50)

Relationships

- Frequencies of oscillations are related to temperature, mean molecular weight, pressure, and/or density
    - Dependent on speed at which waves travel
- Can reveal mass, radius, structure, rotation, and interior profile
- More modes means more information.

$$\nu_{nl} \propto c \propto \sqrt{\frac{T}{\mu}} \propto \sqrt{\frac{P}{\rho}}$$

p Mode Peak Height Calculation

$$H = \frac{2V^2}{\pi\Delta}$$

- where H is peak height (maximum power spectral density),
- V is mode amplitude (Doppler velocity),
- and peak width Δ is related to the linear damping constant η by

$$\Delta = \eta/\pi$$

- *p* modes – established by sound waves (instead of gravity)
- Power spectral density proportional to square of amplitude divided by peak width
- Greater damping produces wider, shorter peaks

Other Relevant Equations

See Thompson and Zharkov

- Pp. 2–5 for equations relating to time-distance helioseismology
- Pp. 5-6 for eqns pertinent to forward modelling

See Garcia et al.

- For a quantitative analysis of Sun rotation and *p* mode splitting

Both can be found in Gizon et al. editors "Helioseismology, Asteroseismology, and MHD Connections" (previously published in **Solar Physics*****) along with extensive discussion and research on helioseismology.***

https://www.nasa.gov/sites/default/files/styles/full_width_feature/public/images/655928main_solar-anatomy-MOS-orig_full.jpg

Solar Interior

Missing:

Prominence

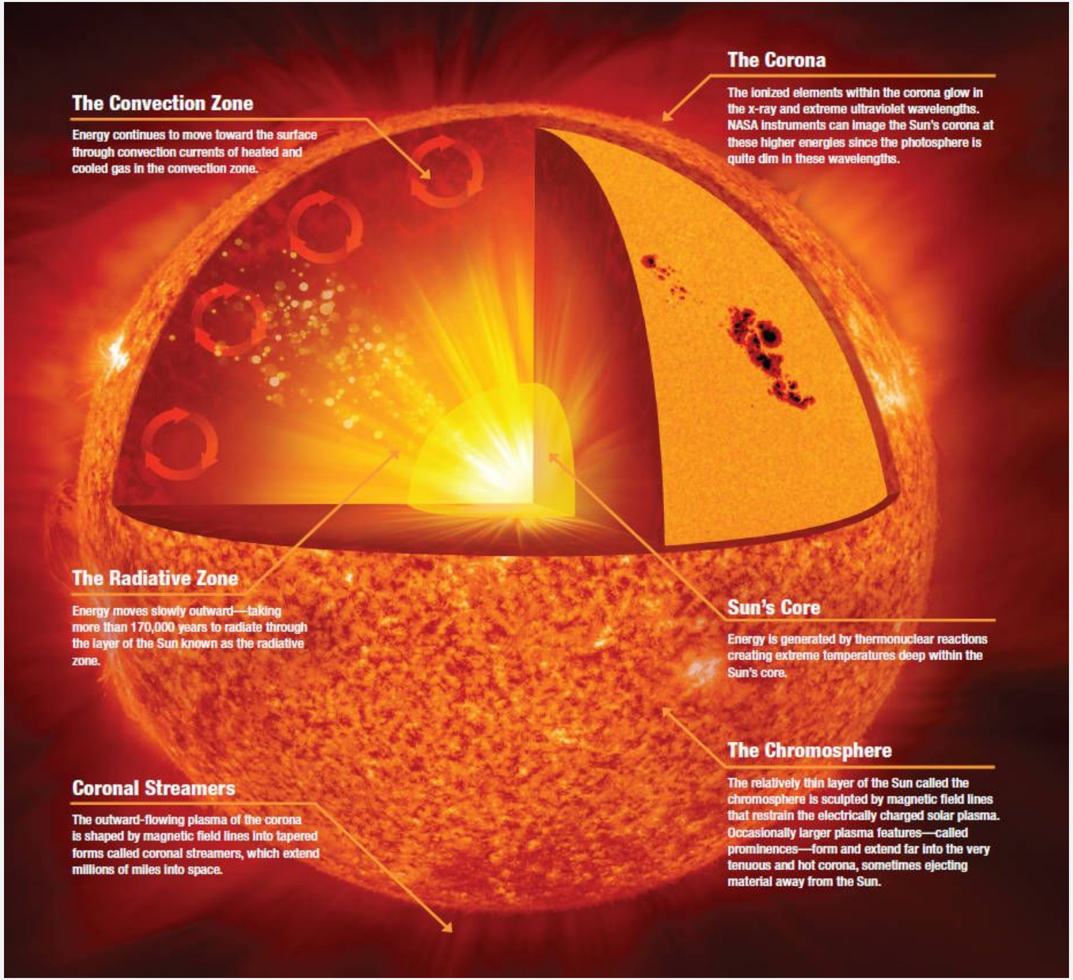
Sunspots

Photosphere

Coronal hole

Flare

Tacholine* p248



Methods for Observing Oscillations

Doppler Shift p8

- Velocities are measured in km*s^-1
- Measures the motions of the shift in light waves on the surface of the Sun.

Gas Compression p8

- Its compression increases temperature which increase light intensity emitted.
- Gas turbulence produces sound (p9) causing resonance

Relevant Equations

Fraunhofer Lines p12

- Show the sunlight spectrum of light.

From textbook

- P11, 18, 39-45 (all ebk pgs).
- P41-42, 47-48, 51, 54,68-69, 79-83, 95, 96, 103-115, 120, 125-127, 151, 158-159, 161, 165-166, 175, 181-183, 194, 202, 206, 218, 226-230