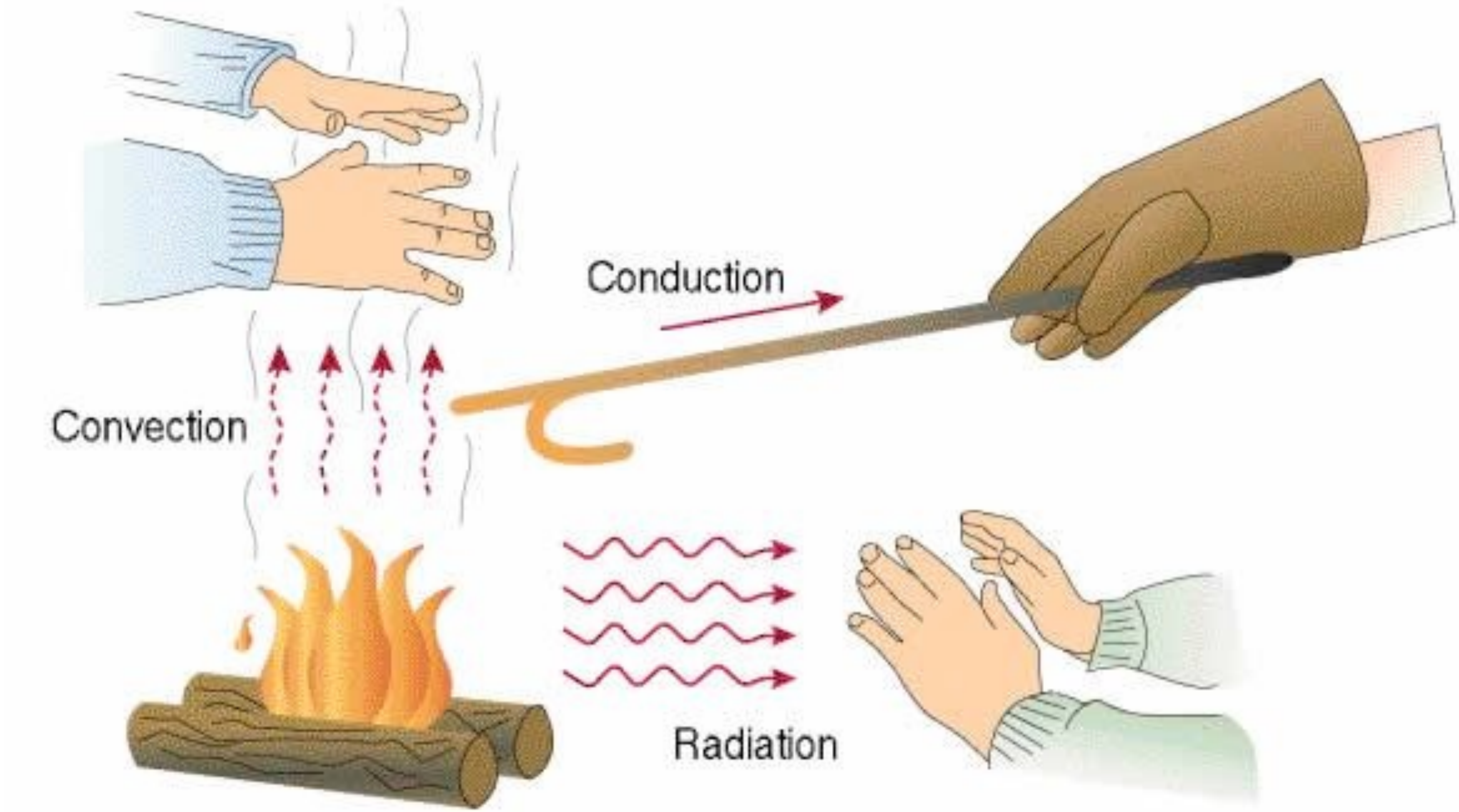
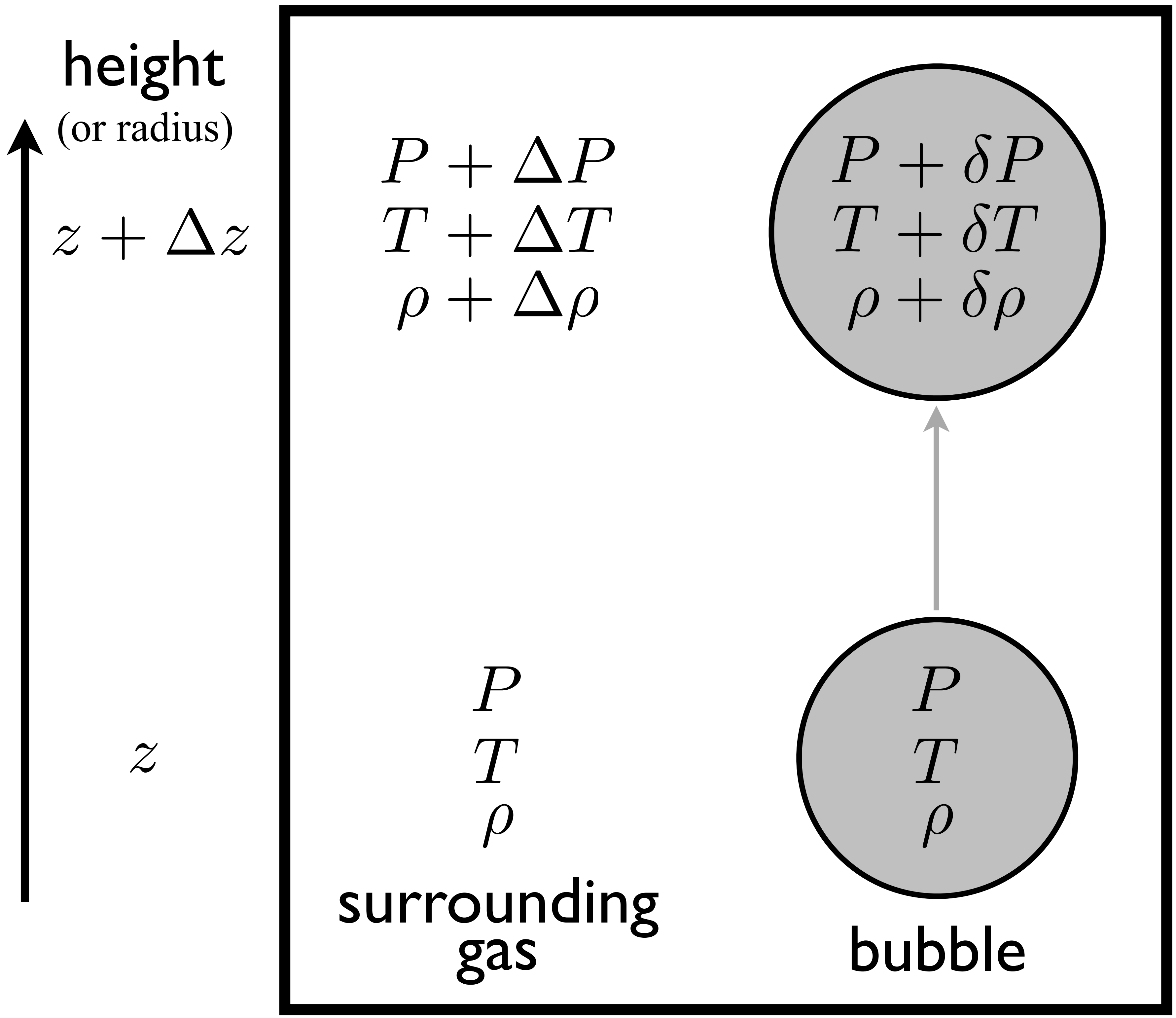
Lecture 6: Energy Transport — Convection

Lamers & Levesque Ch. 7
Phillips Ch. 3.2



Convective instability



adapted from Carroll & Ostlie Figure 10.10, see also Phillips Fig 3.2 and L&L Fig 7.1

adiabatic process

$$dU = dQ - dW$$
change in energy

adiabatic: $dQ = 0$
no heat in or out

work done by gas: $dW = PdV$
$$\Rightarrow \; dU = -PdV$$

let's assume we can use the ideal gas equation of state

pressure: $$P = nkT = \frac{N}{V}kT \;\Rightarrow\; PV = NkT \qquad d(PV) = NkdT$$

energy:
s degrees of freedom

$$U = \frac{s}{2}NkT \qquad dU = \frac{s}{2}NkdT$$

$$dU = \frac{s}{2}d(PV) = \frac{s}{2}(PdV + VdP) = -PdV$$

$$\left(\frac{s}{2}\right)VdP = -\left(\frac{s}{2}+1\right)PdV$$

$$\frac{dP}{P} = -\left(\frac{s/2+1}{s/2}\right)\frac{dV}{V}$$

$$\ln P = -\gamma \ln V + const$$

$$P = const \times V^{-\gamma}$$

$$PV^{\gamma} = const \qquad \text{or} \qquad P \propto \rho^{\gamma}$$

e.g., $s = 3 \;\Rightarrow\; \gamma = 5/3$

$$\gamma = \frac{s/2+1}{s/2}$$ adiabatic index; polytropes!

$$\gamma = \frac{C_P}{C_V} \qquad C_P = \left.\frac{dQ}{dT}\right|_P \qquad C_V = \left.\frac{dQ}{dT}\right|_V,$$

adiabatic index also related to heat capacities
(at const pressure or const volume)

convective instability

surrounding gas: $$P = nkT = \frac{\rho k T}{\mu m_p} \quad \Rightarrow \quad \rho = \frac{m_p}{k}\frac{\mu P}{T}$$

$$\ln \rho = \ln m_p - \ln k + \ln \mu + \ln P - \ln T$$

$$\frac{d\rho}{\rho} = \frac{d\mu}{\mu} + \frac{dP}{P} - \frac{dT}{T}$$

assume homogeneous composition $$\frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} - \frac{\Delta T}{T}$$

rising bubble: $$P \propto \rho^{\gamma} \ \Rightarrow \ \ln P = \gamma \ln \rho + const$$

$$\frac{\delta P}{P} = \gamma \frac{\delta \rho}{\rho} \iff \frac{\delta\rho}{\rho} = \frac{1}{\gamma}\frac{\delta P}{P}$$

The bubble will be buoyant, and want to continue to rise, if

$$\begin{aligned} \rho + \delta\rho \ &< \ \rho + \Delta\rho \\ \delta\rho \ &< \ \Delta\rho \end{aligned}$$

convective instability

The bubble will be buoyant, and want to continue to rise, if

$$\begin{aligned} \rho + \delta\rho &< \rho + \Delta\rho \\ \delta\rho &< \Delta\rho \\ \frac{\delta\rho}{\rho} &< \frac{\Delta\rho}{\rho} \\ \frac{1}{\gamma}\frac{\Delta P}{P} &< \frac{\Delta P}{P} - \frac{\Delta T}{T} \end{aligned}$$

as parcel moves, volume will adjust to any pressure difference, so $\delta P = \Delta P$

Rearranging, we can say that the bubble will rise if

$$\Delta T < \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\,\Delta P \quad \Rightarrow \quad \frac{dT}{dz} < \frac{\gamma - 1}{\gamma}\,\frac{T}{P}\,\frac{dP}{dz}$$

In general, T and P both decrease with height, so the derivatives are both negative. Switching to absolute values so we work with positive quantities, we have the condition for the bubble to rise being

$$\left|\frac{dT}{dz}\right| > \frac{\gamma - 1}{\gamma}\,\frac{T}{P}\,\left|\frac{dP}{dz}\right|$$

Schwarzschild criterion for convection

In other words, if the temperature gradient exceeds this threshold, the system is unstable: if you take a little bubble and nudge it upwards it will continue to rise (as opposed to falling back to where it started). This is called the **convective instability**. Any little perturbation will cause bubbles to start rising—convection.

$$\left|\frac{dT}{dz}\right| > \frac{\gamma-1}{\gamma}\,\frac{T}{P}\,\left|\frac{dP}{dz}\right|$$

$$\left|\frac{1}{T}\frac{dT}{dz}\right| > \frac{\gamma-1}{\gamma}\left|\frac{1}{P}\frac{dP}{dz}\right| \quad \Rightarrow \quad \left|\frac{d\ln T}{dz}\right| > \frac{\gamma-1}{\gamma}\left|\frac{d\ln P}{dz}\right|$$

and we can also write this in terms of just pressure and temperature:

$$\boxed{\frac{d\ln T}{d\ln P} > \frac{\gamma-1}{\gamma}.}$$

if convection *can* happen, it *does* happen,
and then convection dominates the energy transport

convection and hydrostatic equilibrium

if the gas is in hydrostatic equilibrium then

$$\frac{dP}{dz} = -\rho g = -\frac{\mu m_p P}{kT} g \qquad \Rightarrow \quad \frac{T}{P}\frac{dP}{dz} = -\frac{\mu m_p}{k} g$$

and then the Schwarzschild criterion can be written:

$$\left|\frac{dT}{dz}\right| > \frac{\gamma - 1}{\gamma} \frac{T}{P} \left|\frac{dP}{dz}\right| \qquad \left|\frac{dT}{dz}\right| > \frac{\gamma - 1}{\gamma} \frac{\mu m_p}{k} g$$

where $g = g(z) = g(r)$ is the local acceleration due to gravity

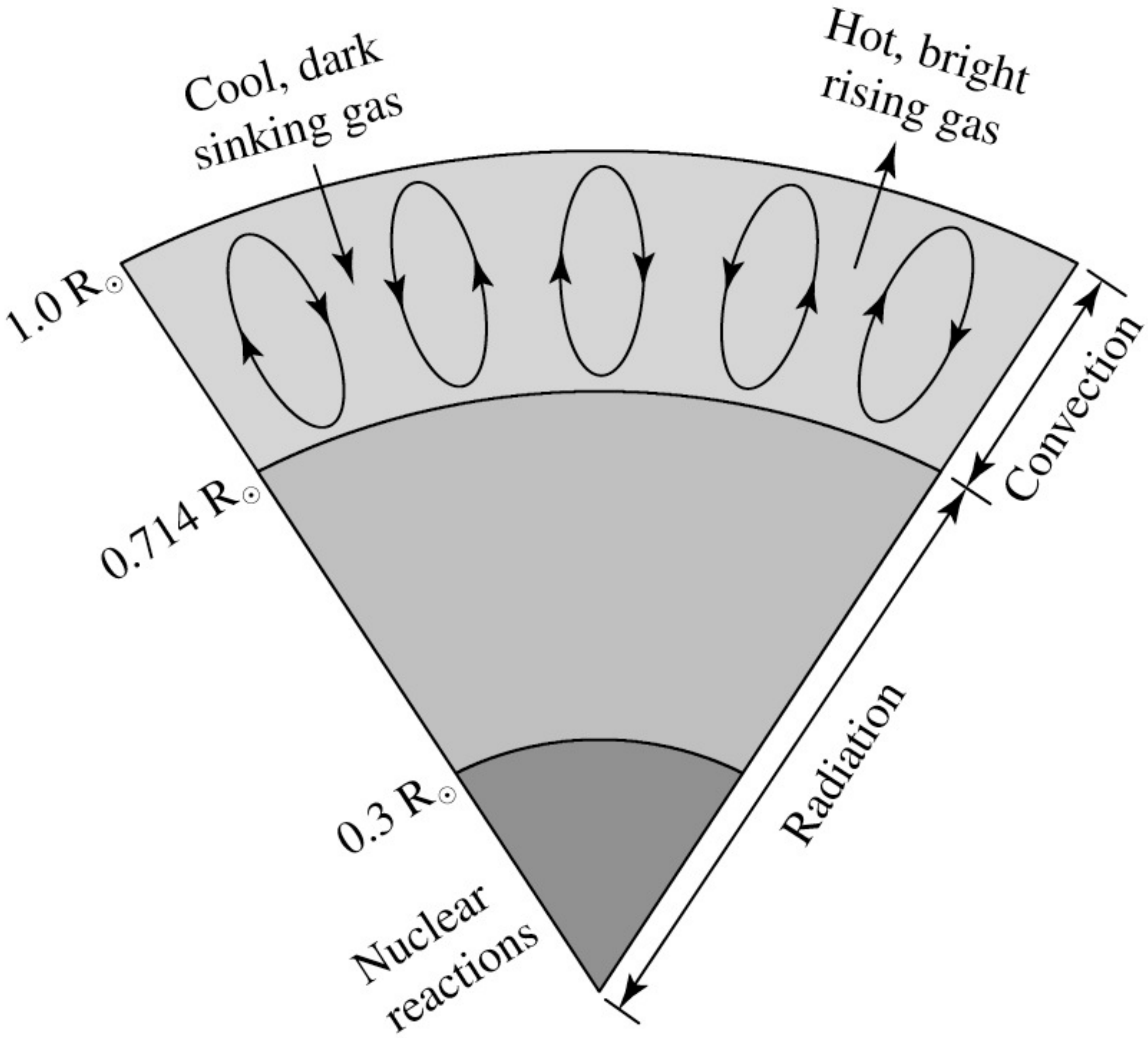
so convection will occur when
temperature gradient is strong
or *acceleration due to gravity is weak*

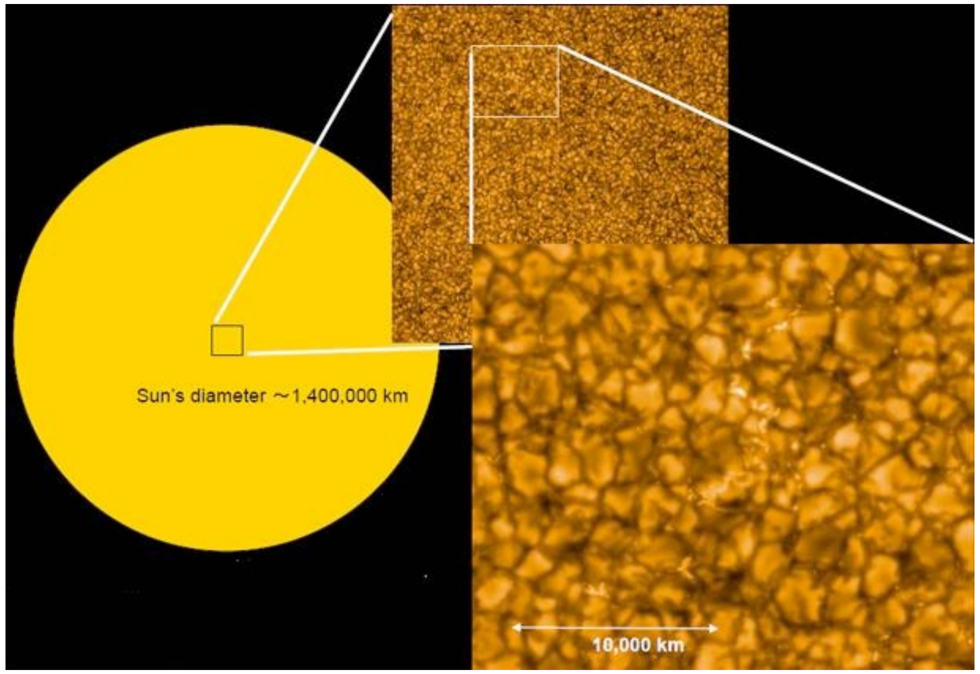
cores of high-mass stars

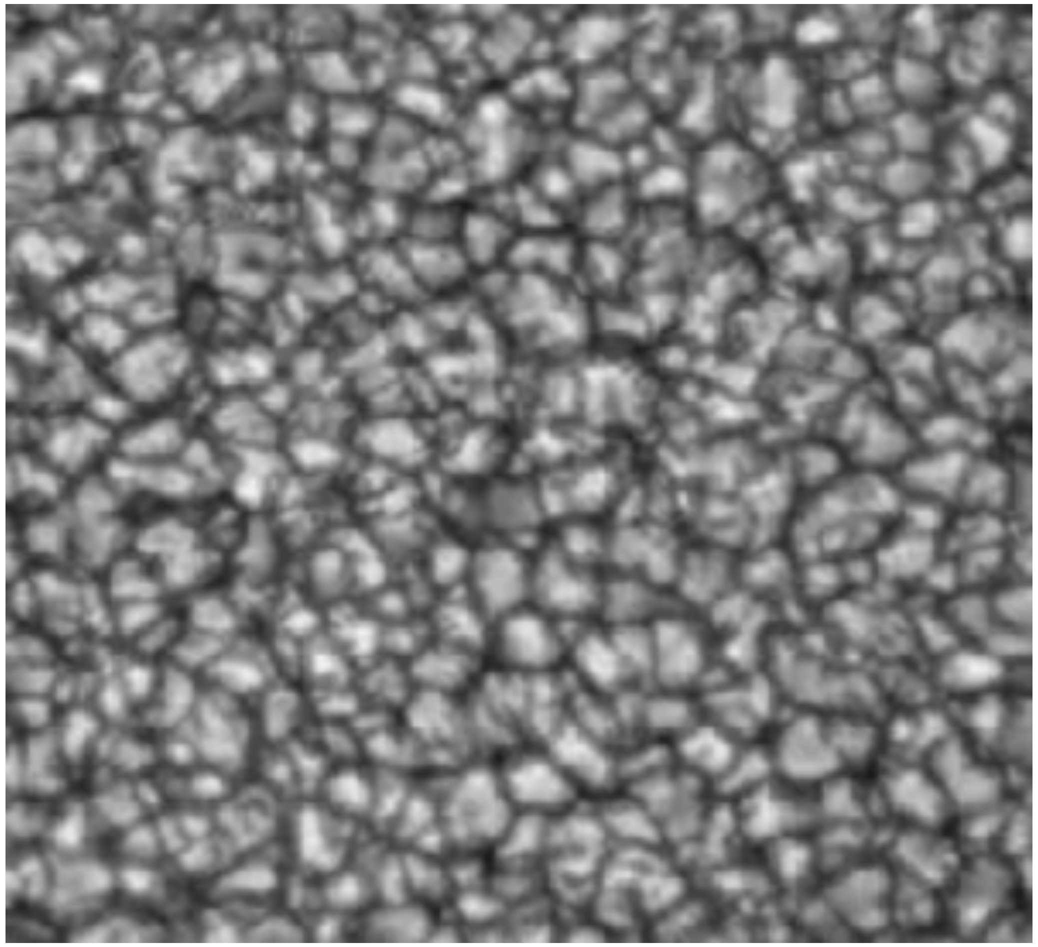
outer parts of low-mass stars

very low-mass stars ($M \lesssim 0.3\ M_\odot$)
are completely convective

Convection in the Sun



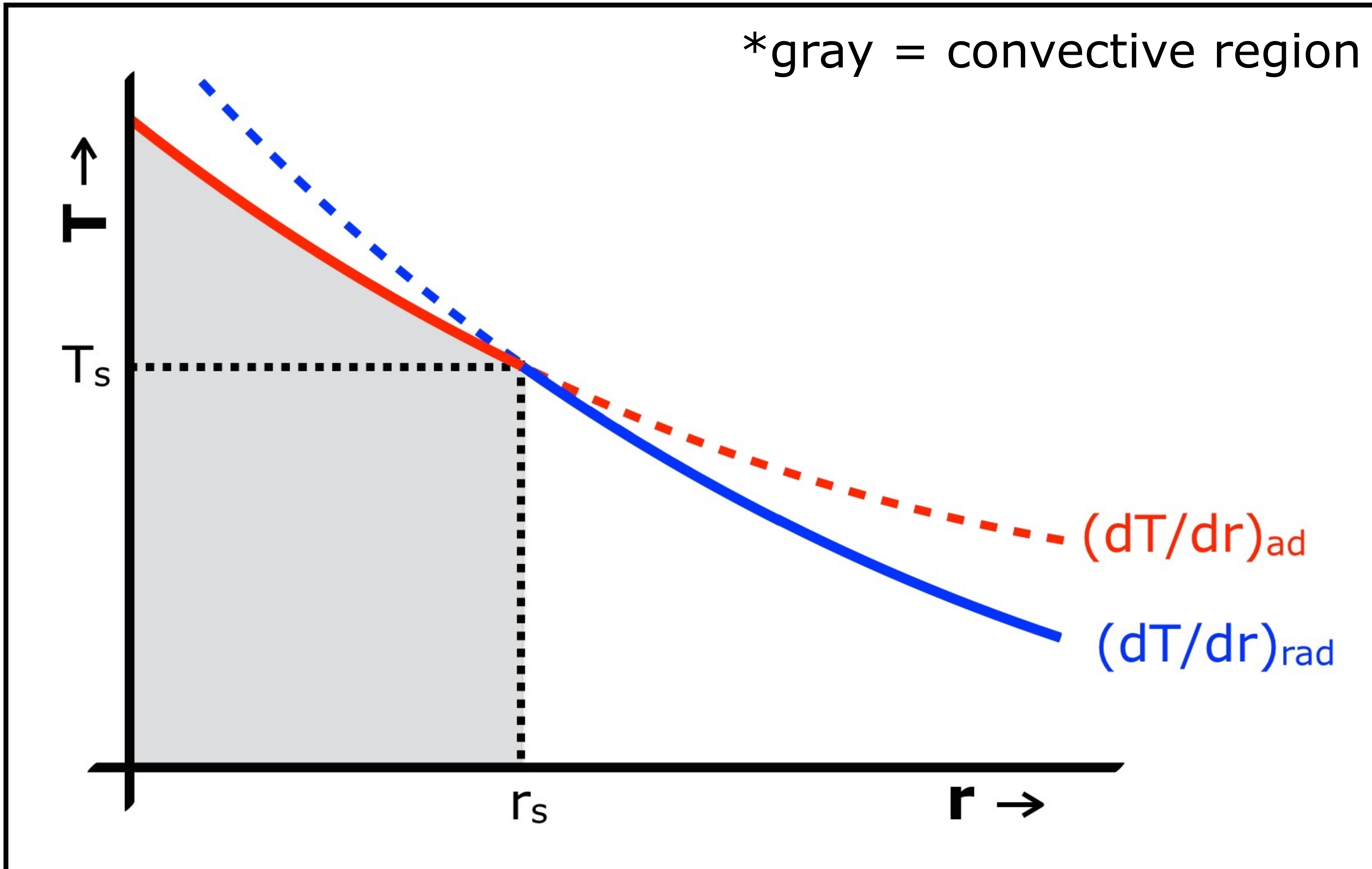




from Carroll & Ostlie Figure 11.2,
http://solarb.msfc.nasa.gov and
http://science.nasa.gov/ssl/pad/solar/images/SVST_granulation.mpg

Convective Energy Transfer

<u>Schwarzschild criterion</u> $\frac{\gamma - 1}{\gamma}\frac{T}{P}\left|\frac{dP}{dr}\right| \equiv |dT/dr|_{\mathrm{ad}} < |dT/dr|_{\mathrm{rad}}$



Convective Energy Transfer

Ledoux criterion

We derived the Schwarzschild criterion for convection by considering a chemically homogenous medium: μ(r) = const

Consider a varying μ(r); should it *increase* or *decrease* w/ r?

$$P = nkT = \frac{\rho k T}{\mu m_p} \quad \Rightarrow \quad \rho = \frac{m_p}{k}\frac{\mu P}{T}$$

$$\ln\rho = \ln m_p - \ln k + \ln\mu + \ln P - \ln T$$

$$\frac{d\rho}{\rho} = \frac{d\mu}{\mu} + \frac{dP}{P} - \frac{dT}{T}$$

$$\text{assume homogeneous composition} \quad \frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} - \frac{\Delta T}{T}$$

$$\left|\frac{d\ln T}{dz}\right| > \frac{\gamma - 1}{\gamma}\left|\frac{d\ln P}{dz}\right|$$

Schwarzschild criterion
for convection

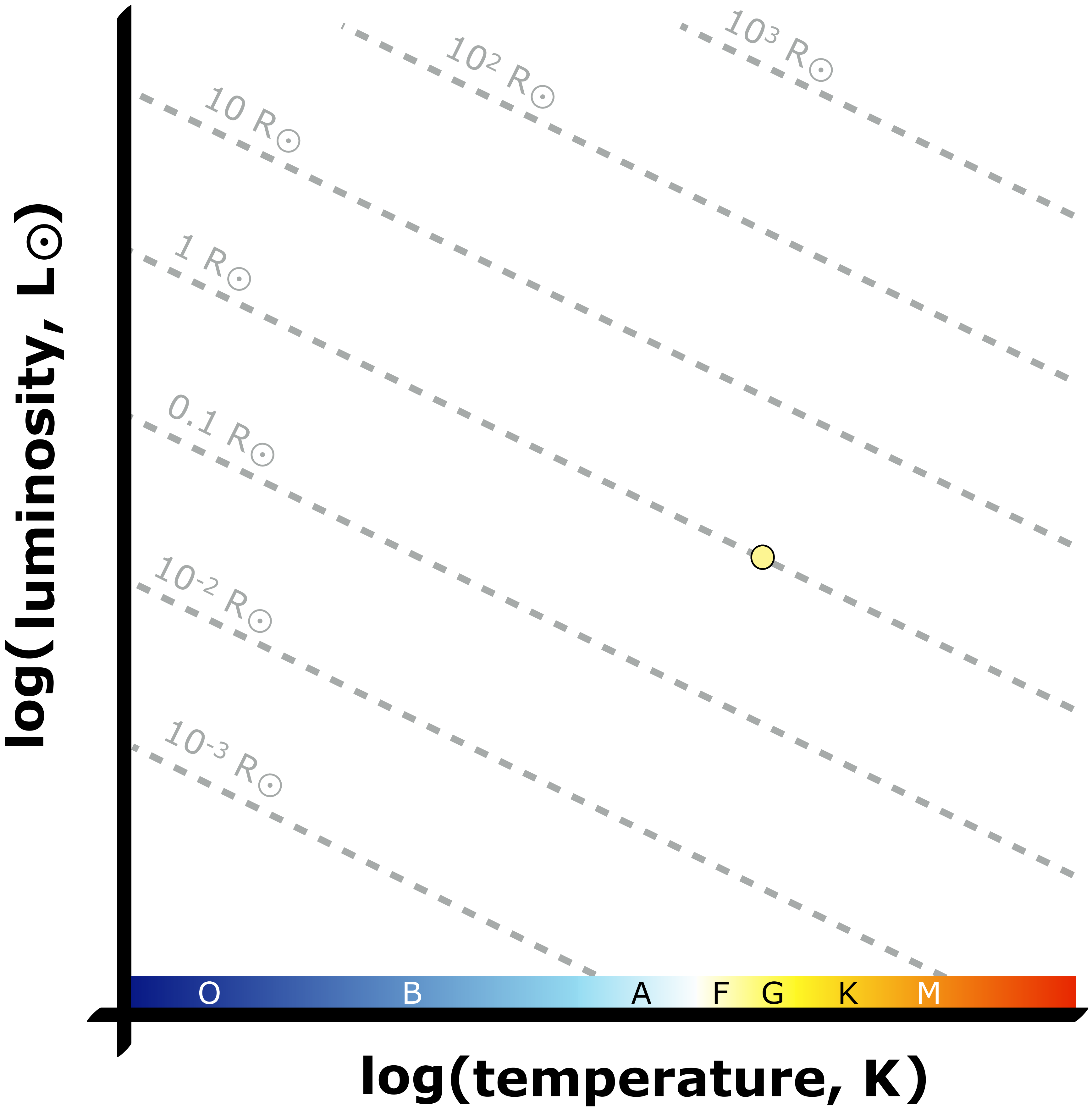
$$\text{else} \quad \frac{\Delta\rho}{\rho} = \frac{\Delta\mu}{\mu} + \frac{\Delta P}{P} - \frac{\Delta T}{T}$$

$$\left|\frac{d\ln T}{dz}\right| > \frac{\gamma - 1}{\gamma}\left|\frac{d\ln P}{dz}\right| + \left|\frac{d\ln\mu}{dz}\right|$$

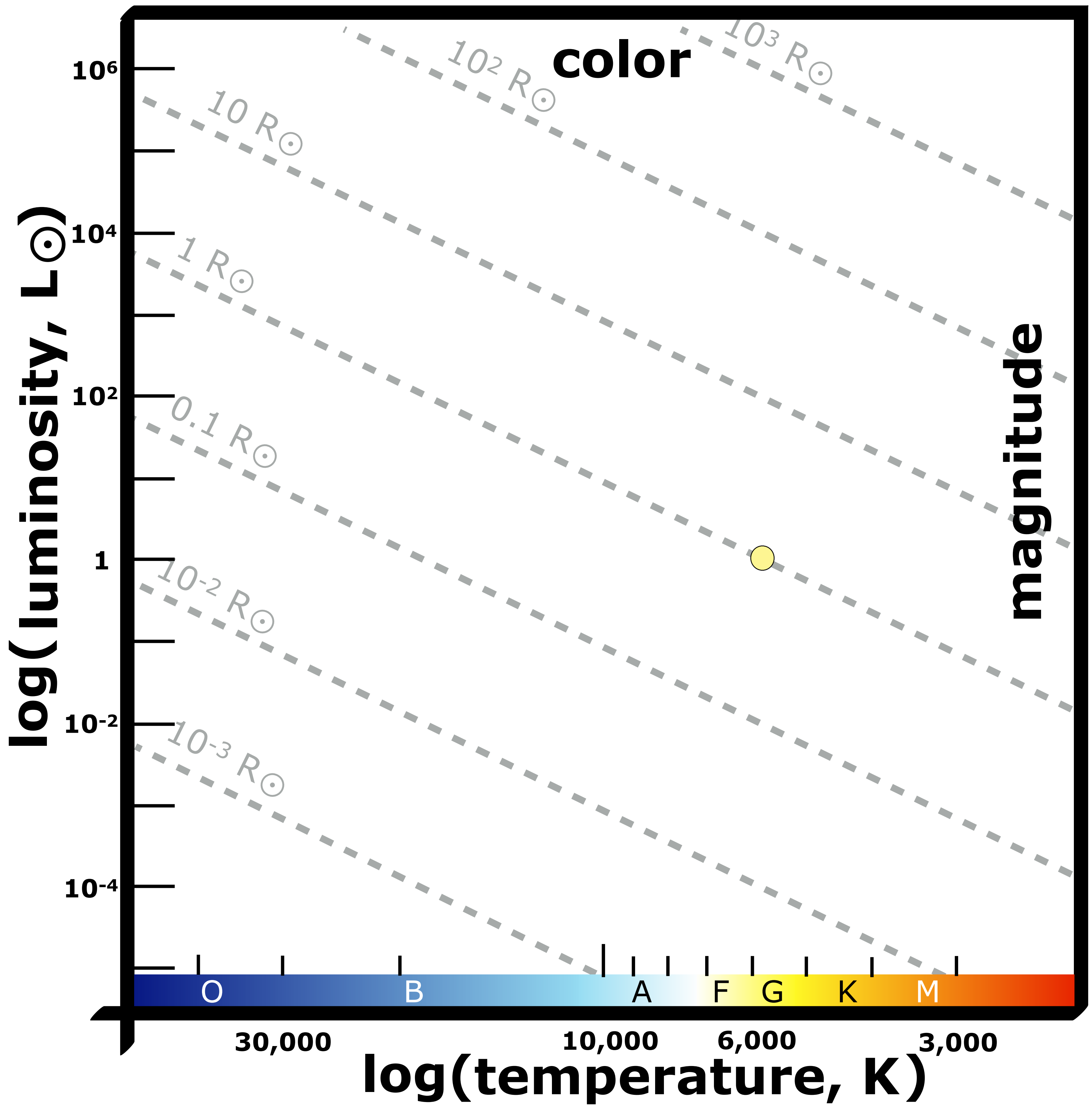
Ledoux criterion
for convection

for typical μ decreasing outwards, this helps stabilize against convection a little

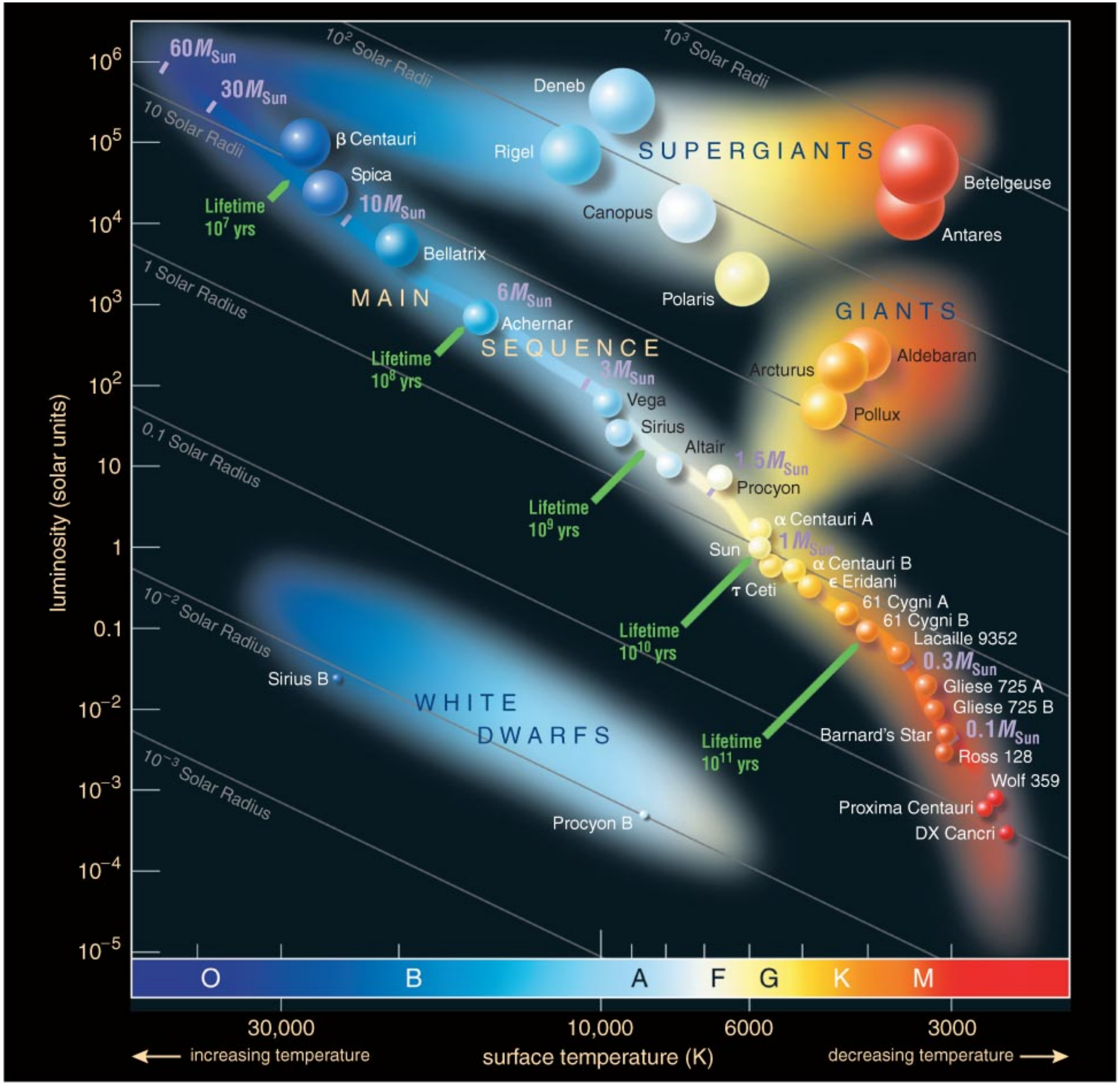
The Hertzsprung-Russell Diagram



The Hertzsprung-Russell Diagram



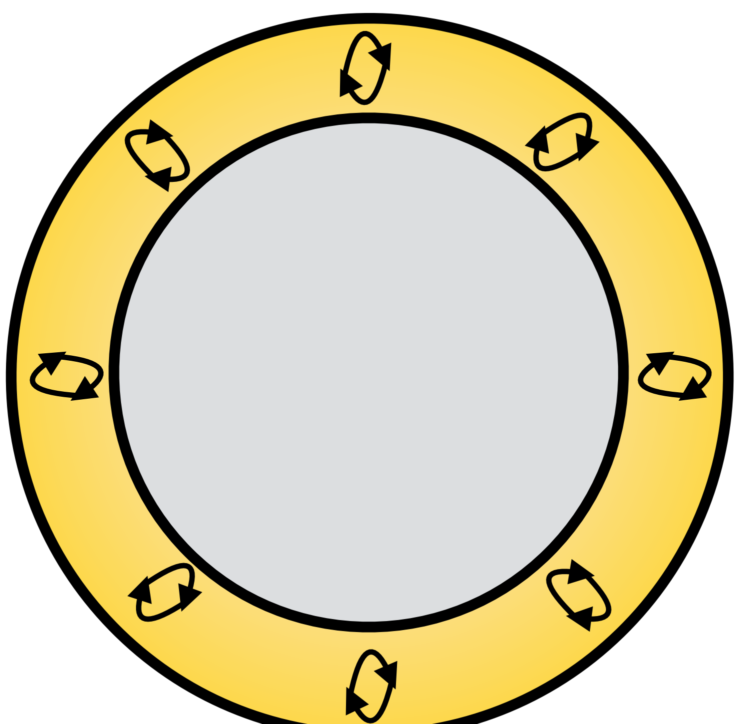
The Hertzsprung-Russell Diagram

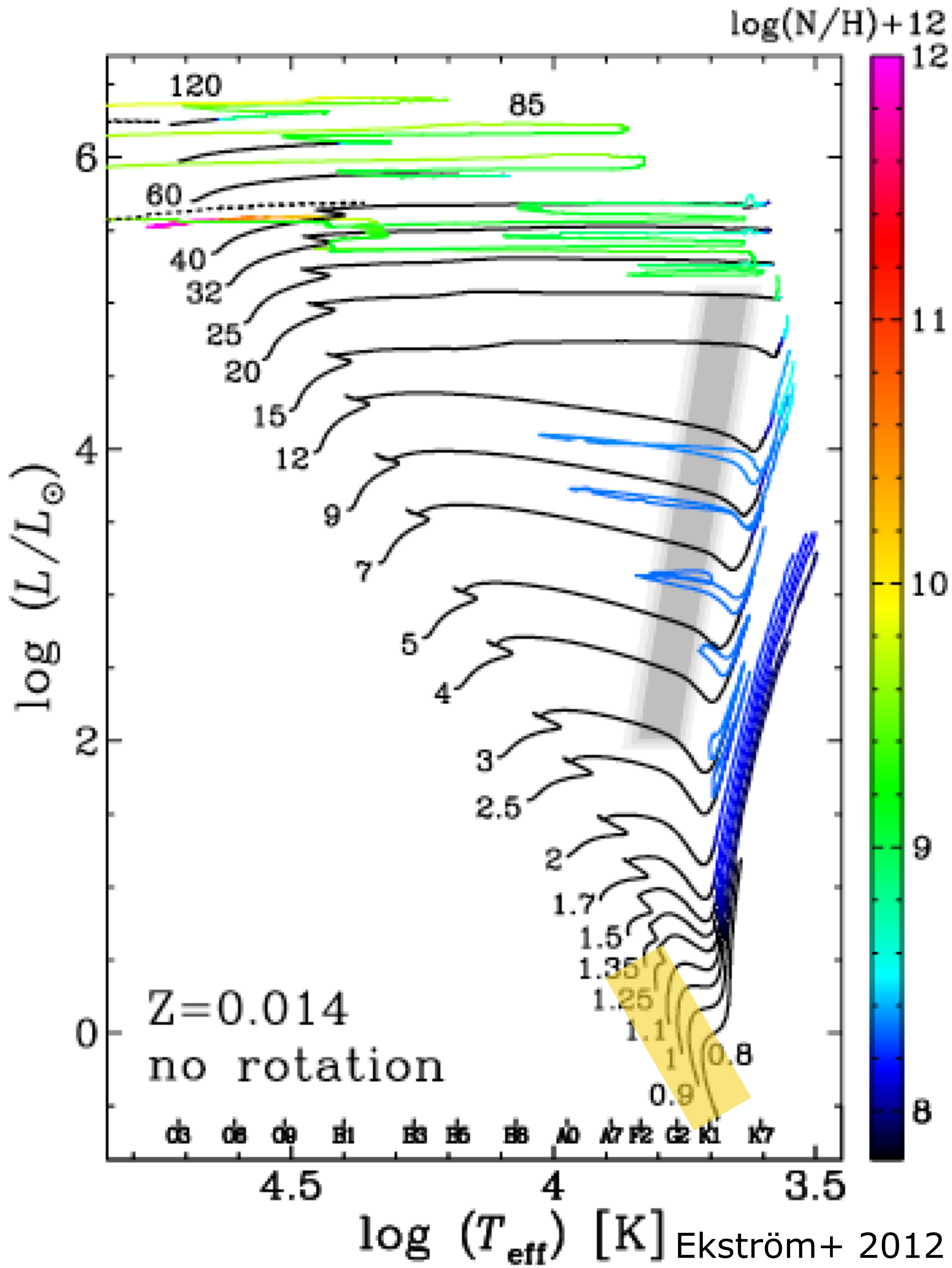


Convection in Stars

Where does convection actually happen?

1. low gravitational acceleration
   —outer layers of cool stars





Ekström+ 2012

Convection in Stars

Where do
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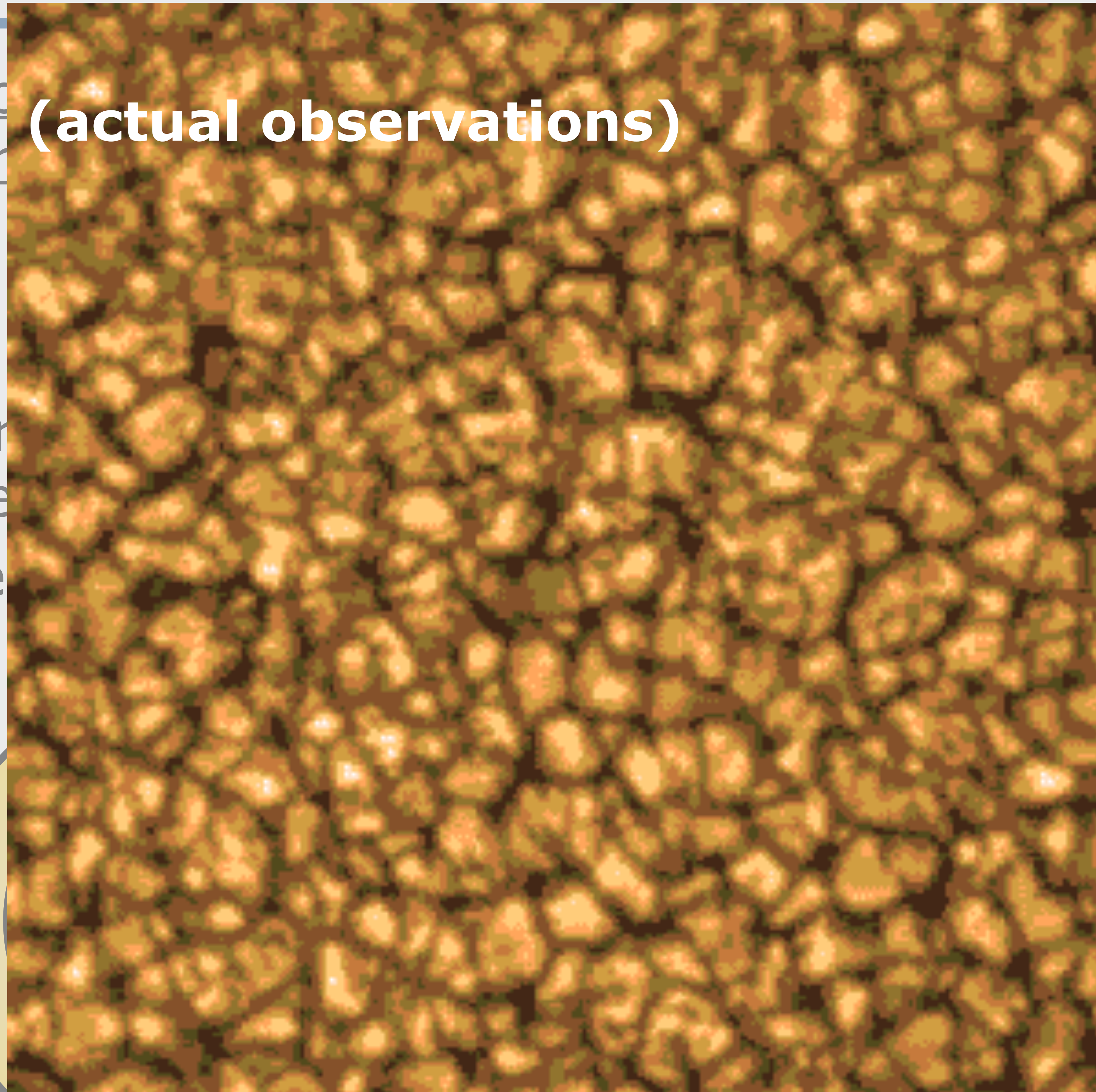

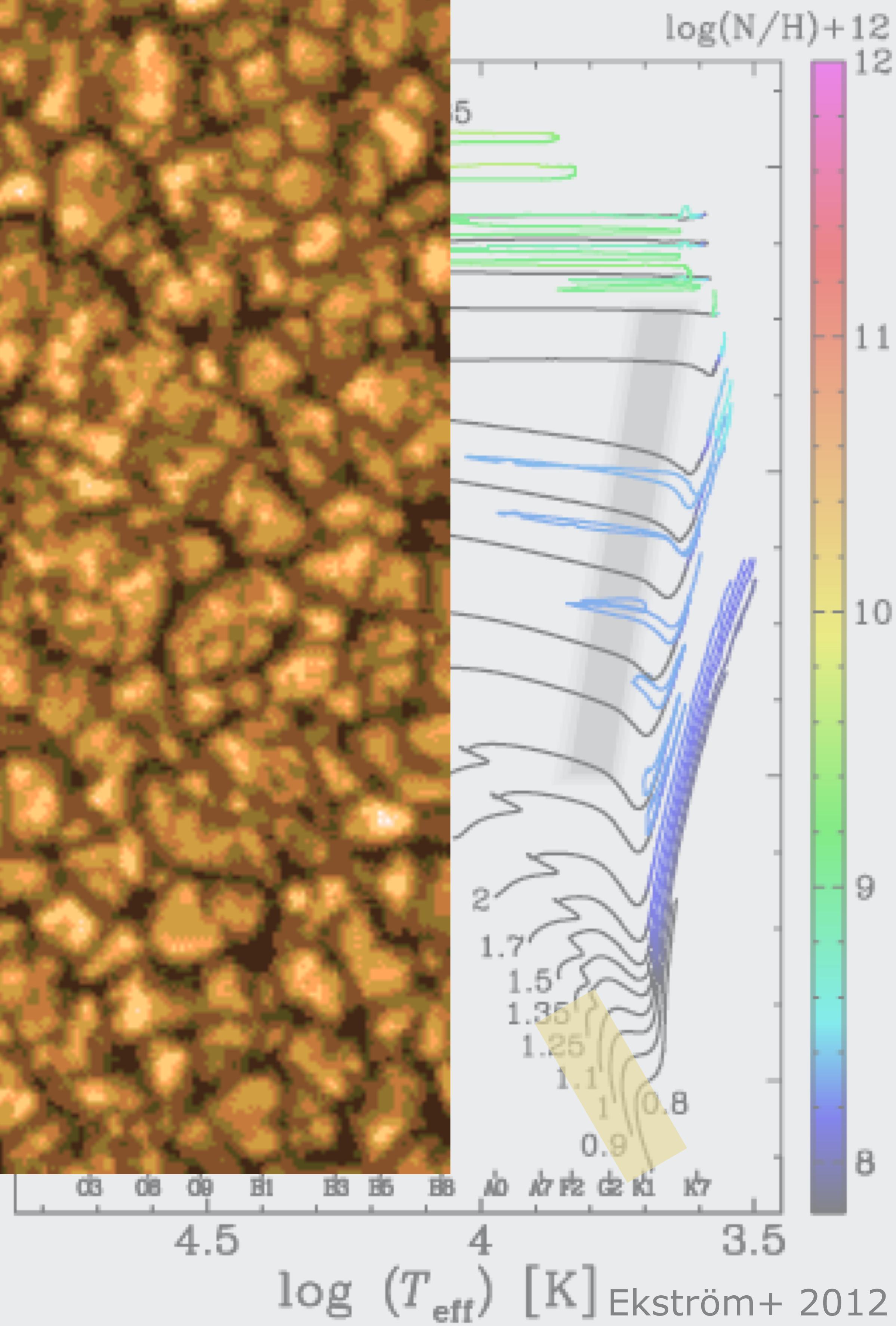

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Convection in Stars

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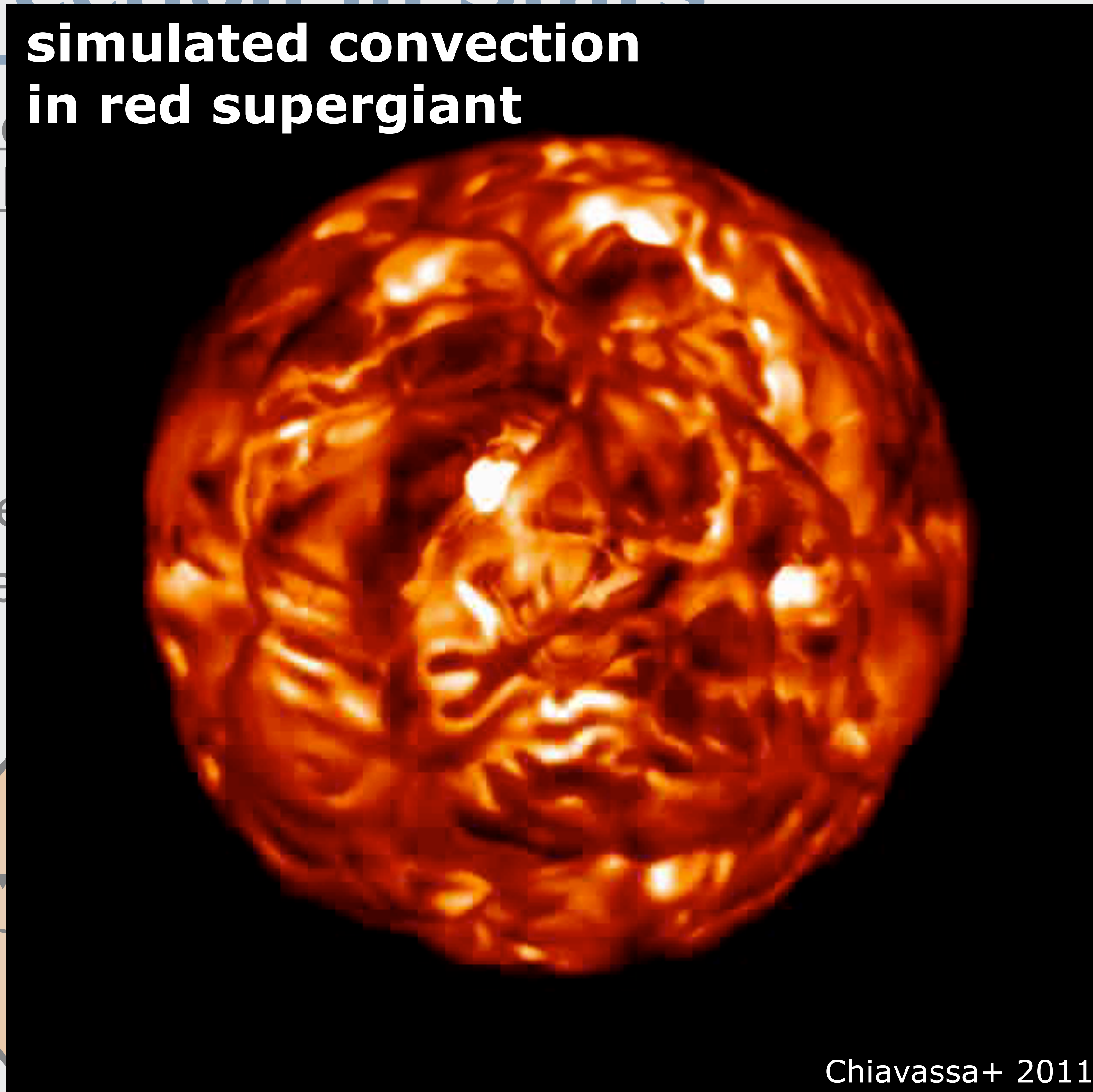

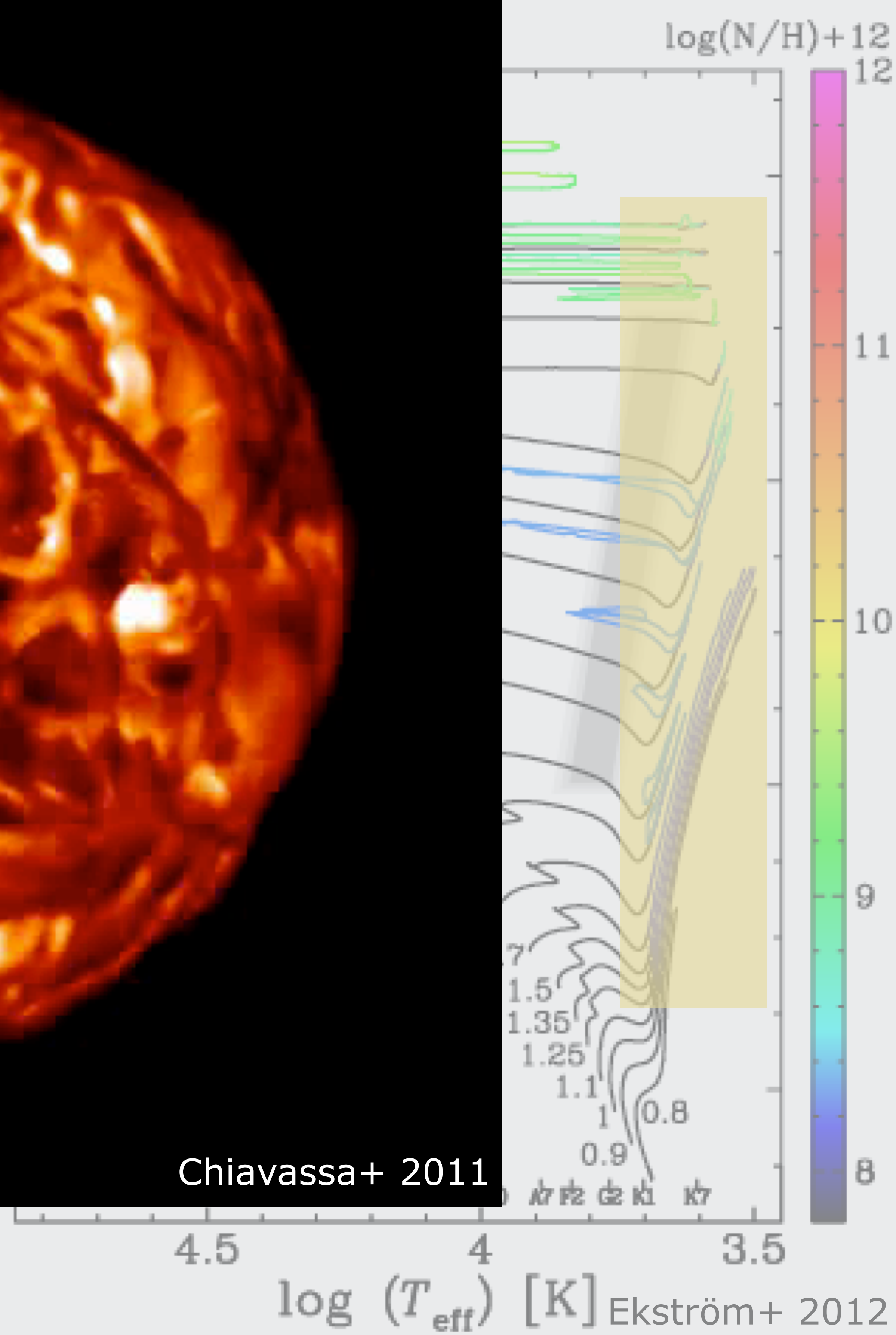

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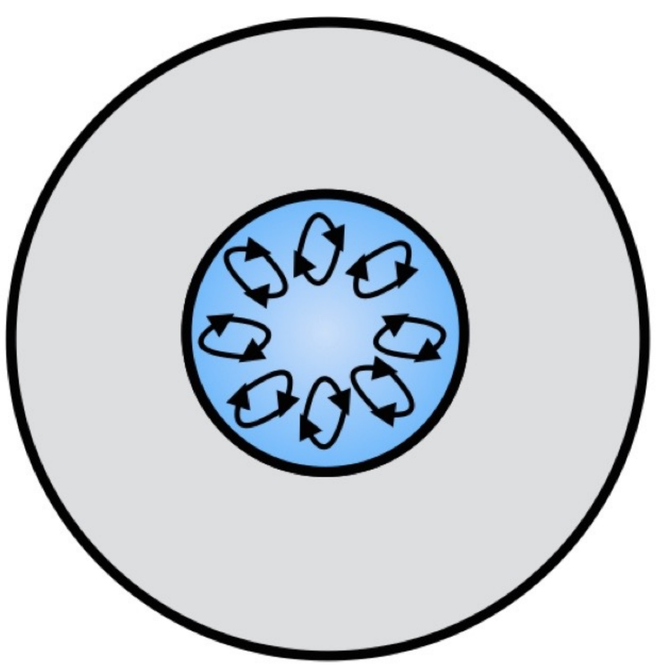
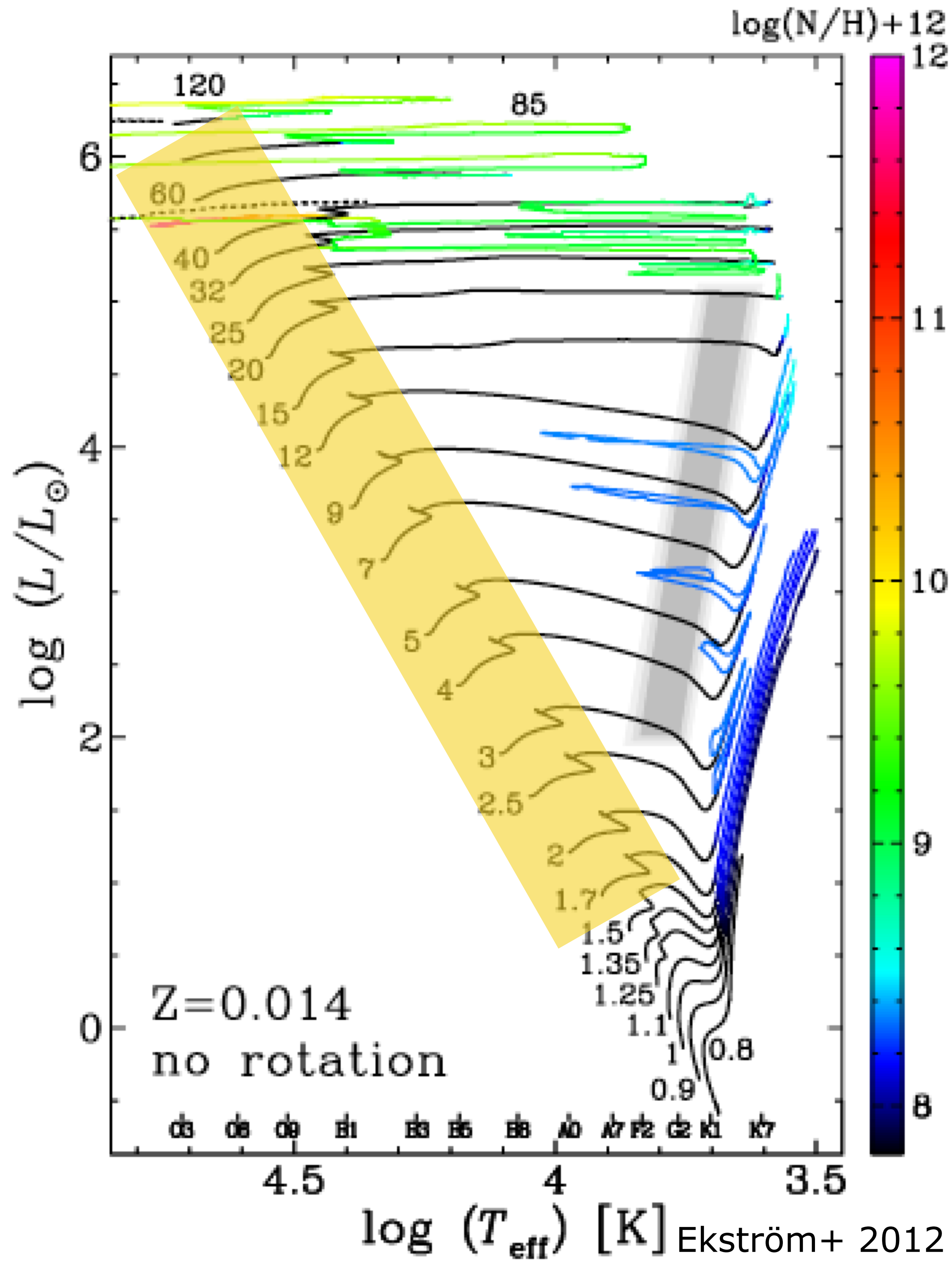
Convection in Stars

Where does convection actually happen?

1. low gravitational accel.
   —outer layers of cool stars
2. large L: temp gradient
   —cores of massive stars





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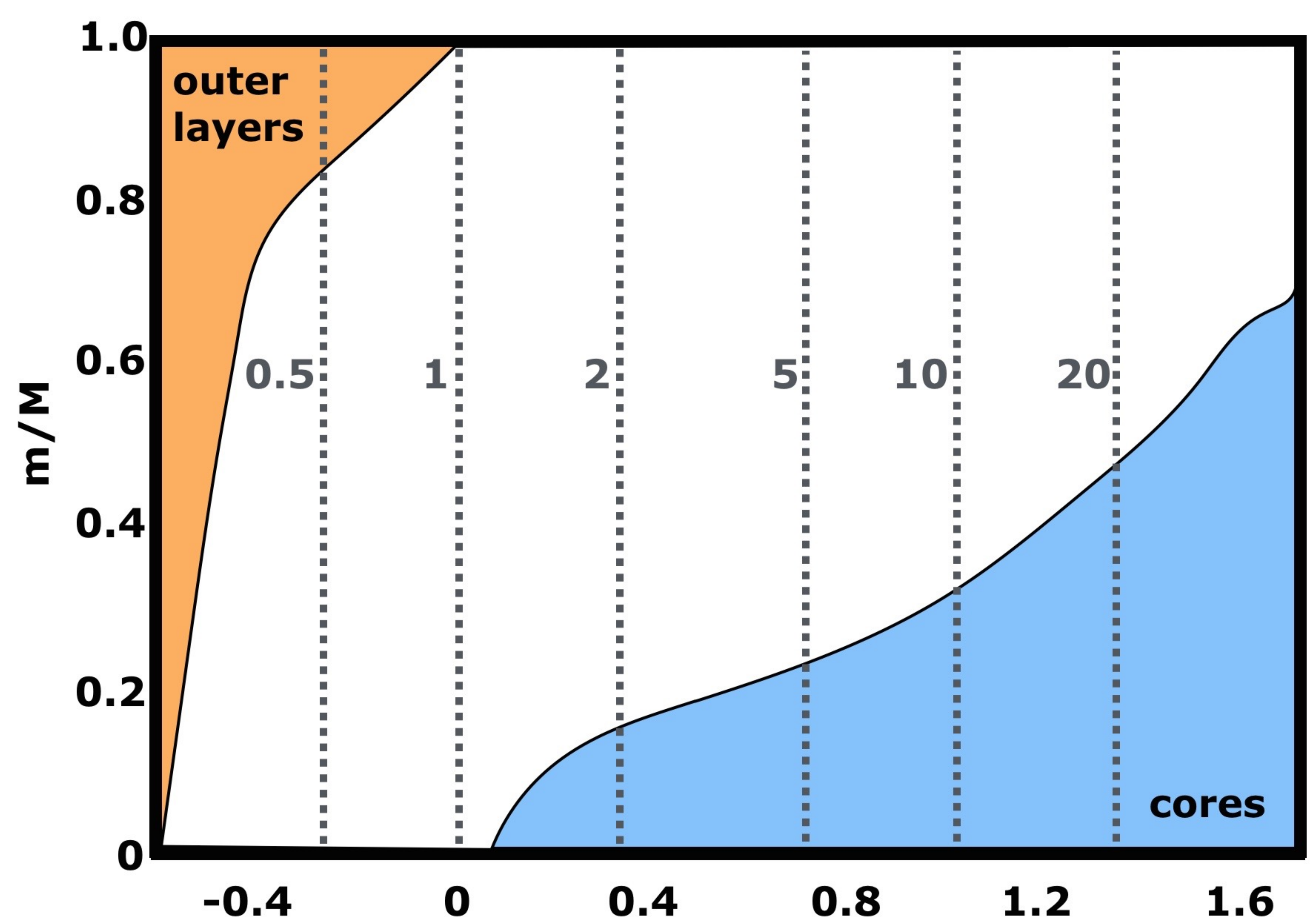
Convection in Stars



Reproduced from Kippenhahn R., Weigert A., Weiss A. (2012) Polytropic Gaseous Spheres. In: Stellar Structure and Evolution. Astronomy and Astrophysics Library. © Springer-Verlag Berlin Heidelberg 2012."

Convection in Stars

Convective overshooting

Convection is based on Archimedes force.

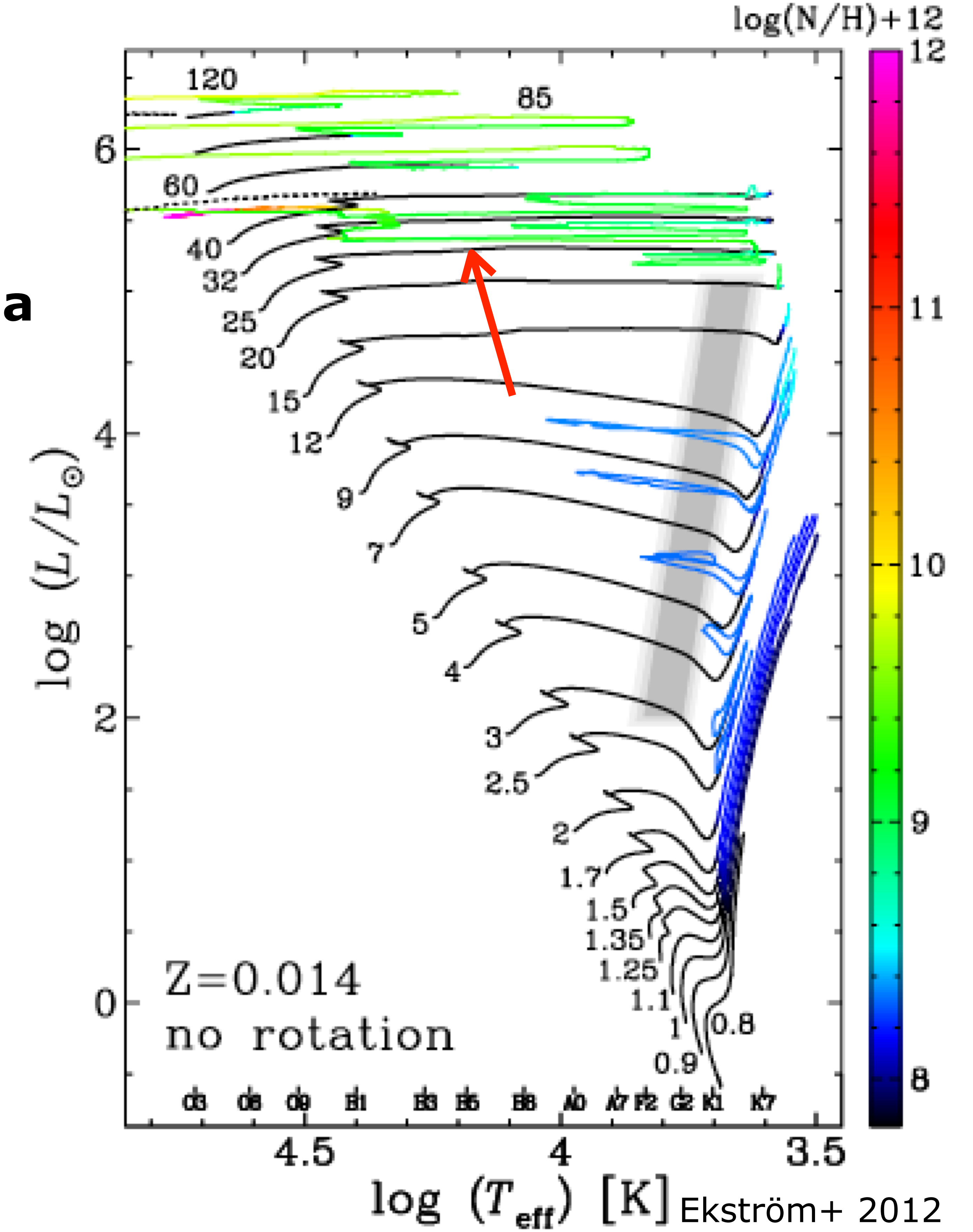
What would happen if you held a football underwater and let go?

Same thing happens in stars…

—nuclear products like He and N appear at massive stars surfaces very early…

—convective mixing extends past the convective core

—difference is that convective overshooting doesn't transport energy…



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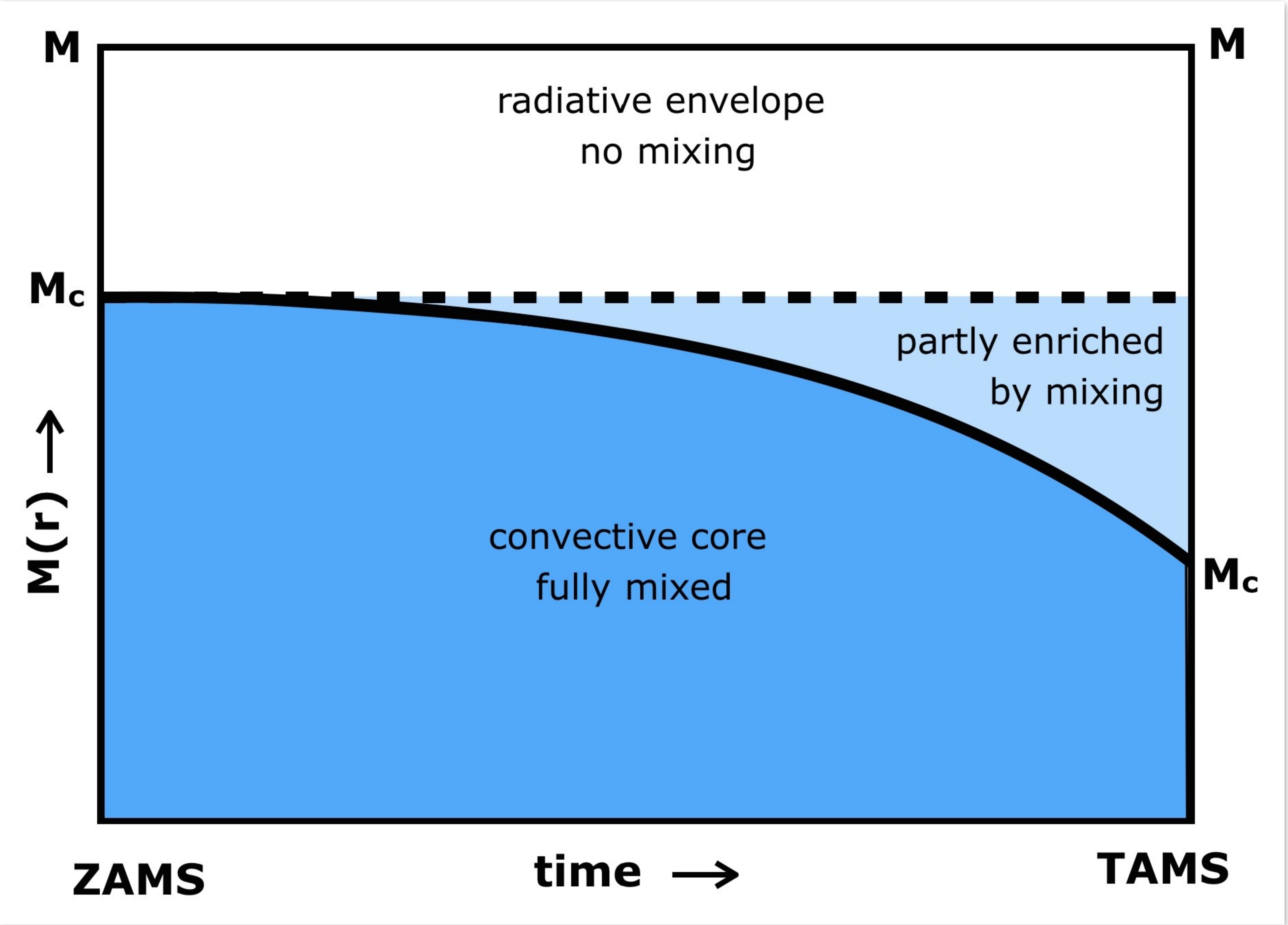
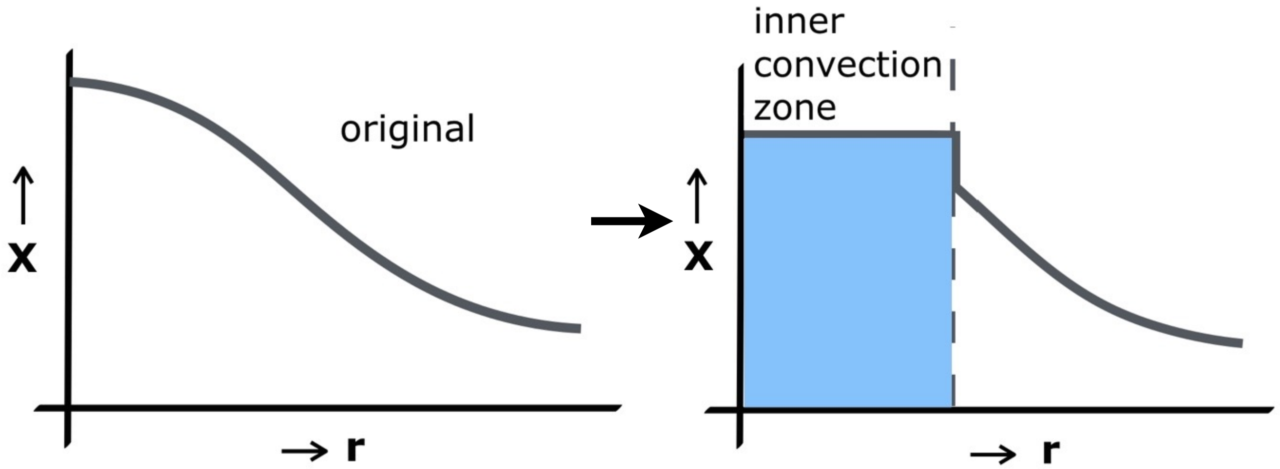
Convection in Stars

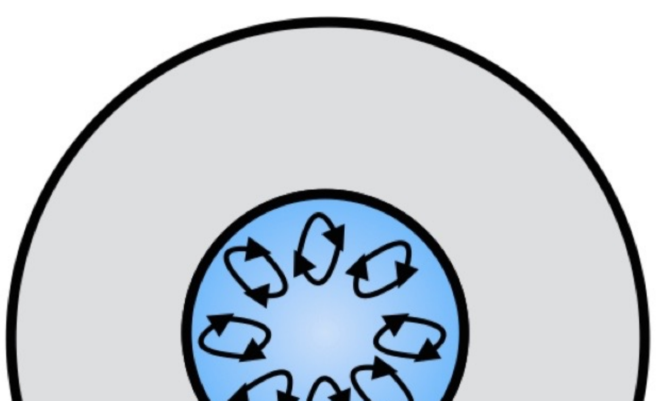


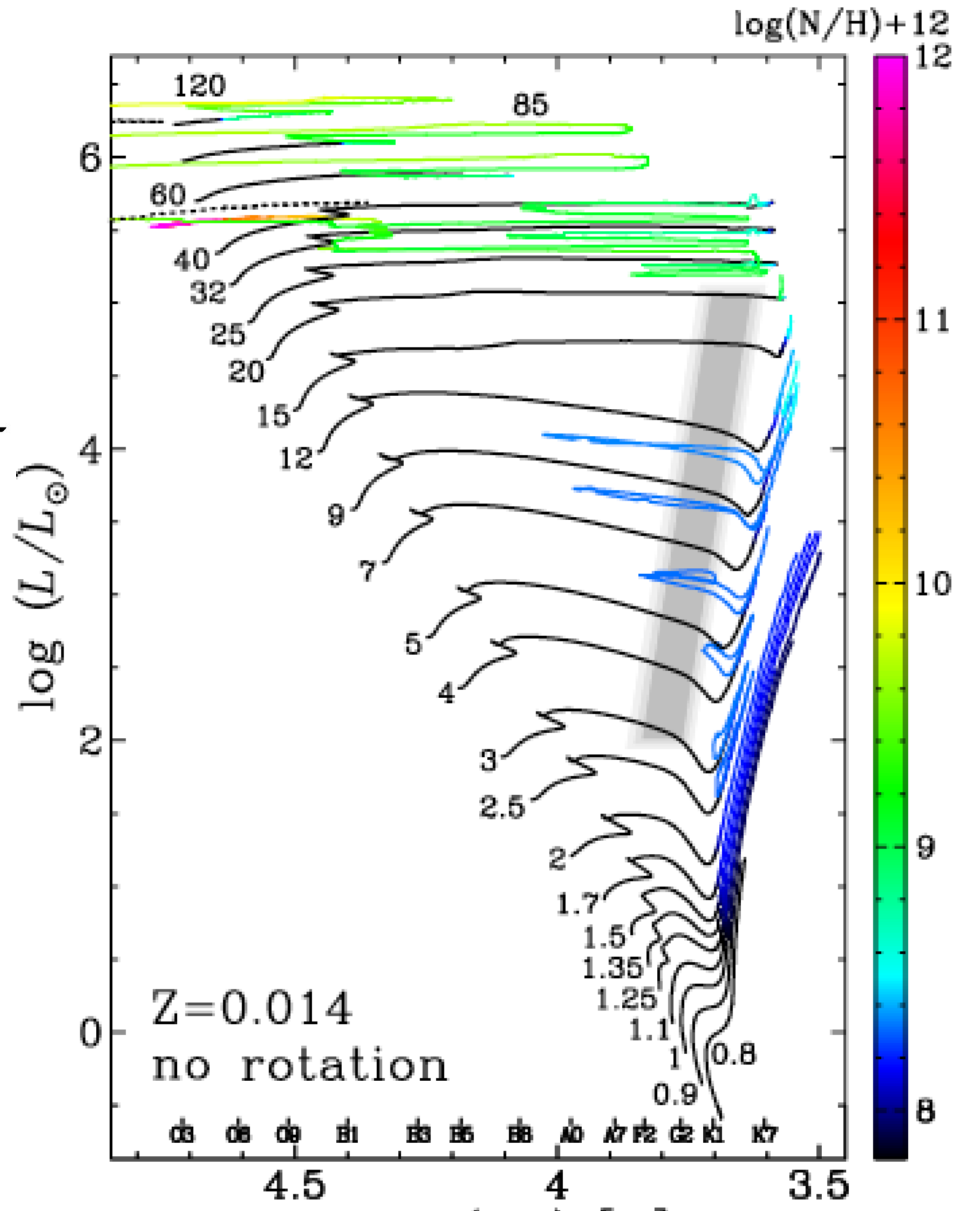
Convection in Stars

Convective mixing



Mixing in massive star cores extends their MS lifetimes…





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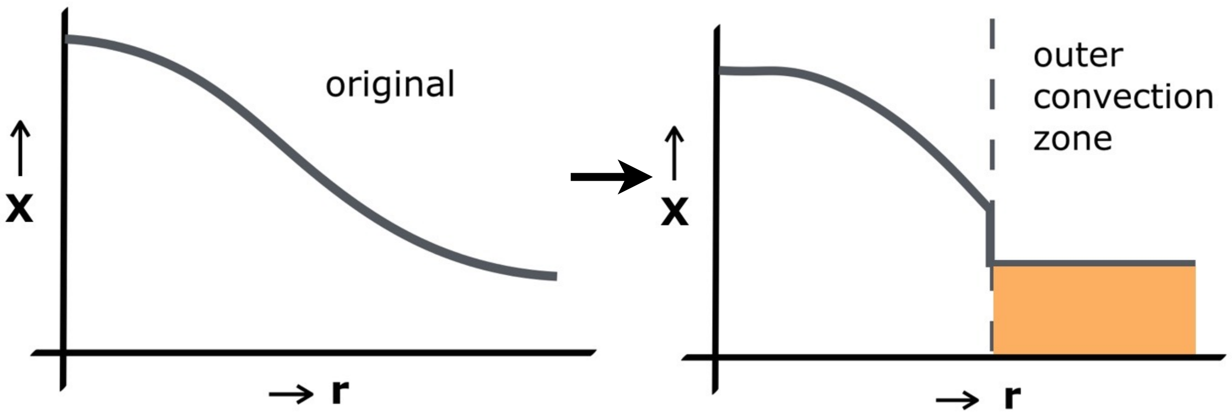
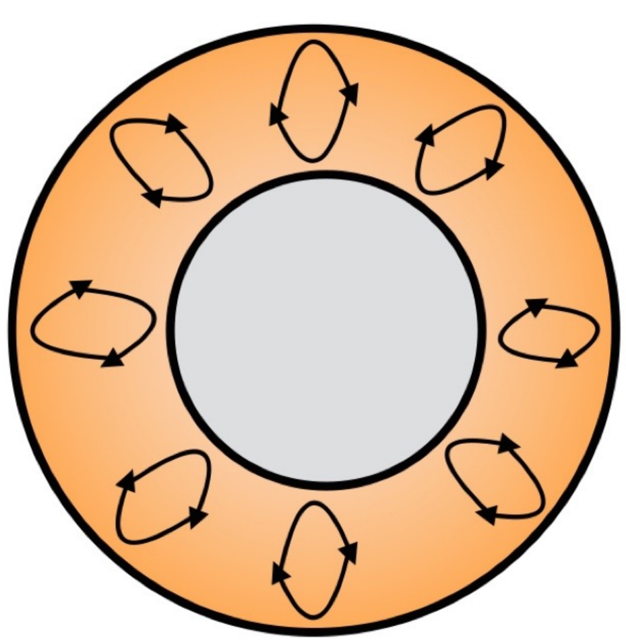
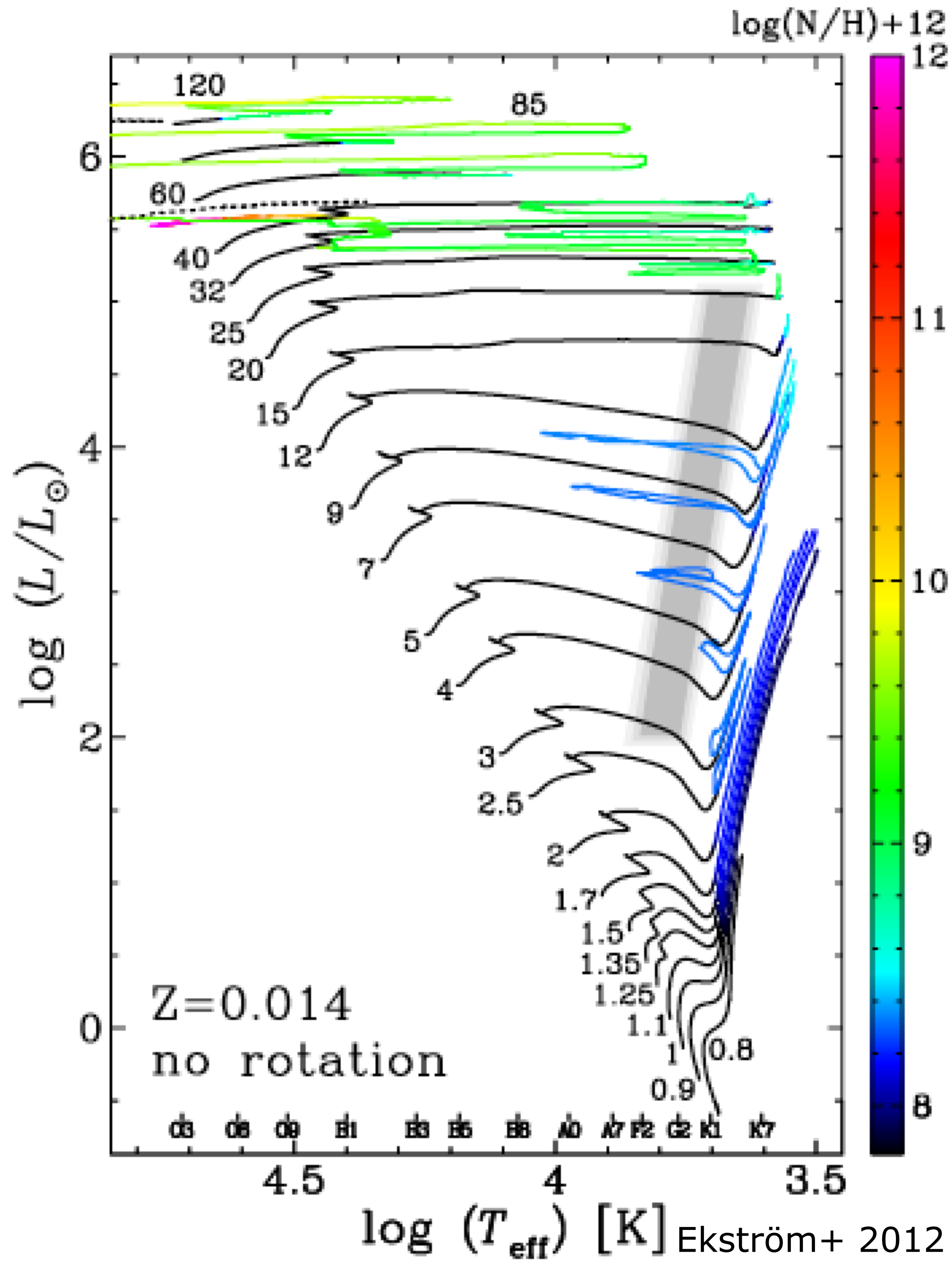
Convection in Stars

<u>Convective mixing</u>



Mixing in stars with large convective *outer* layers causes "dredge-up"…





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