Lecture 12: Stars on the Main Sequence

Lamers & Levesque Ch. 13

H-R diagram from *Gaia* DR2; https://sci.esa.int/web/gaia/-/60240-the-hertzsprung-russell-diagram

Midterm: in class Tuesday March 8

will cover course material through March 1st Lecture 13

things to review (roughly in order of importance):

- **problem sets 1–3 (& solutions)**
- **lecture slides**
- **Phillips Ch. 1–5 (esp. end of chapter summaries)**
- **Lamers & Levesque Ch. 1–11, 13, 14, 16–19**

exam will be 80 minutes; 441/541 have same exam

mix of conceptual questions (short answer), derivation, and some short calculations (bring a calculator)

you are allowed to bring a formula sheet:

<u>*one side only*</u> of a 8.5” x 11” (letter size) sheet of paper

Stellar Timescales

1) Dynamical:

free-fall: what if the pressure in a star suddenly vanished?

$$\left.\begin{aligned} \Delta r \sim g t^2 \rightarrow t^2 \sim \Delta r / g \\ \Delta r \sim R \\ g \sim GM/R^2 \end{aligned}\right\} t_{\mathrm{ff}} \simeq \sqrt{\frac{R}{GM/R^2}} \sim \sqrt{\frac{R^3}{GM}} \rightarrow \boxed{t_{\mathrm{ff}} \simeq \frac{1}{\sqrt{G\bar{\rho}}}}$$

sound speed crossing time: what if the *gravity* of a star vanished?

$$\left. t_{\mathrm{sound}} \sim R/c_s \right\} \quad t_{\mathrm{sound}} \sim \frac{R}{\sqrt{GM/R}} \sim \sqrt{\frac{R^3}{GM}} \sim \sqrt{\frac{1}{g\rho}} \rightarrow \boxed{t_{\mathrm{sound}} \simeq \frac{1}{\sqrt{G\bar{\rho}}}}$$

($t_{\mathrm{ff}} \simeq t_{\mathrm{sound}}$! why?)

This is called the **dynamical timescale**: $\boxed{\tau_{\mathrm{dyn}} = \frac{1}{\sqrt{G\bar{\rho}}}}$

Stellar Timescales

1) Dynamical: $\boxed{\tau_{\rm dyn} = \frac{1}{\sqrt{G\overline{\rho}}}}$

Sun: $\tau_{\rm dyn} \sim$ hours

2) Thermal/Kelvin-Helmholtz:

How long can a star live if nuclear fusion shuts off?
(i.e., thermal energy is the only energy source left)

$$\left.\begin{aligned} \tau_{KH} &\simeq E_{th}/L \\ \text{Virial } E_{\rm th} &= -\frac{1}{2}E_{\rm pot} \end{aligned}\right\} \tau_{KH} \simeq -\frac{E_{\rm pot}}{L} \simeq \frac{GM^2}{R}/L$$

$$\boxed{\tau_{KH} = \frac{GM^2}{RL}}$$

Stellar Timescales

1) Dynamical: $\boxed{\tau_{\rm dyn} = \frac{1}{\sqrt{G\bar{\rho}}}}$

Sun: $\tau_{\rm dyn} \sim$ hours

Sun: $\tau_{\rm KH} \sim 10^7$ years

2) Thermal/Kelvin-Helmholtz: $\boxed{\tau_{KH} = \frac{GM^2}{RL}}$

3) Nuclear: $\boxed{\tau_{\rm nucl} \sim \frac{E_{\rm nucl}}{L} \sim f_M\, Mc^2\, \varepsilon_n\, /L}$

Sun:
$\tau_{\rm nucl} \sim 10^{10}$ years

ε_n=efficiency=fraction of mass converted into energy ($\varepsilon_n = \frac{\Delta m}{m}$)
(H=0.007, He = 0.0007)

f_M=fraction of stellar mass that takes part in nuclear fusion
(Sun: $f_M \sim 0.1$; for MS stars, $f_M \cdot \varepsilon_H \simeq 10^{-3}$)

So $\tau_{\rm nucl} \sim 10^{-3}\, Mc^2/L$ (H-fusion), $\tau_{\rm nucl} \sim 10^{-4} Mc^2/L$ (He-fusion)

Stellar Timescales

1) Dynamical: $\boxed{\tau_{\rm dyn} = \frac{1}{\sqrt{G\bar{\rho}}}}$

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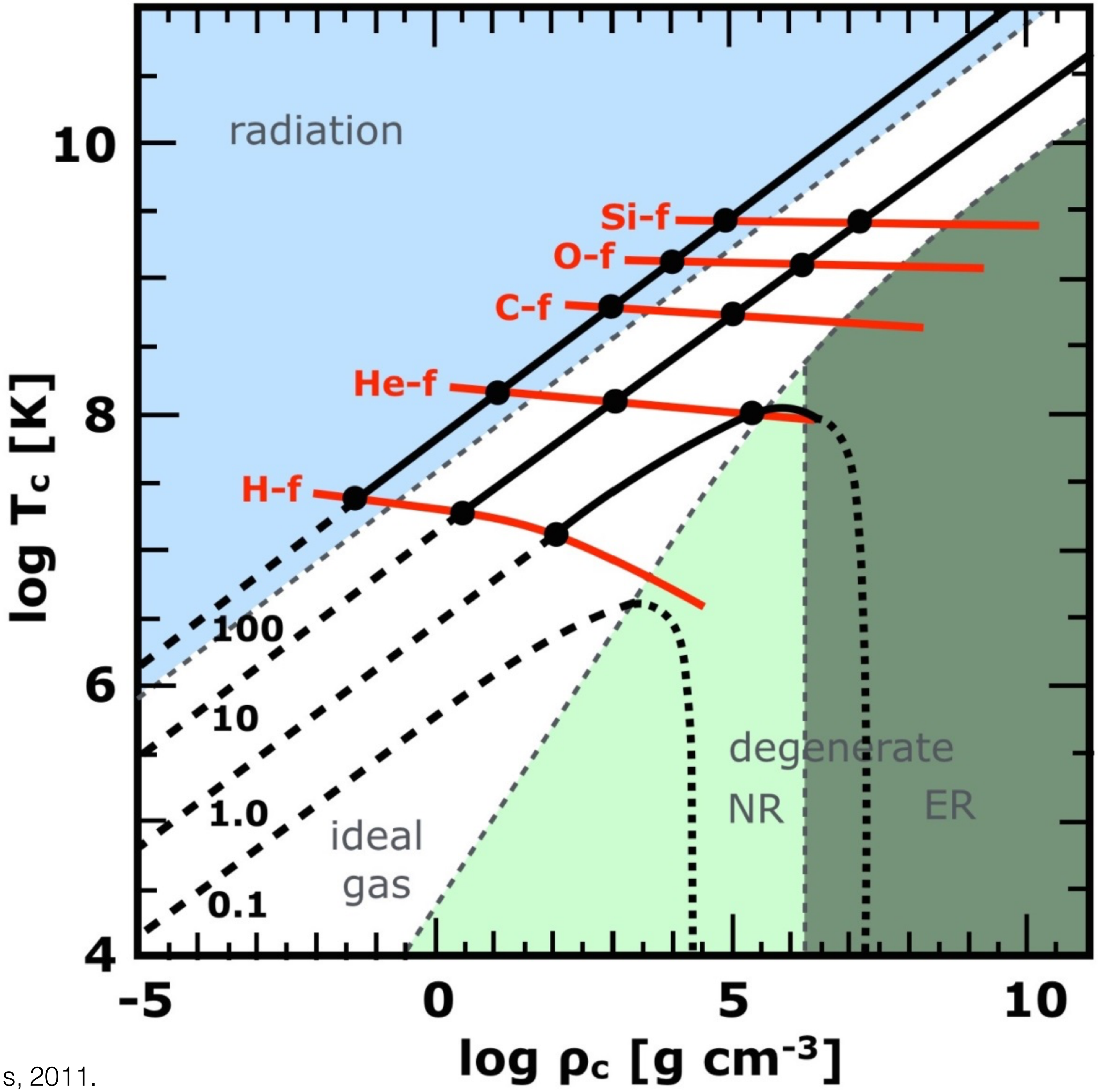
$\tau_{\rm nucl} \sim 10^{10}$ years

4) Convective: ~week (Sun) to ~month (AGB star)

$$\boxed{\tau_{dyn} \ll \tau_{\rm conv} \ll \tau_{\rm KH} \ll \tau_{nucl}}$$

∴ …except for explosive phases, stars are always in quasi-H.E.
…contraction phases last about 1% as long as nuclear phases

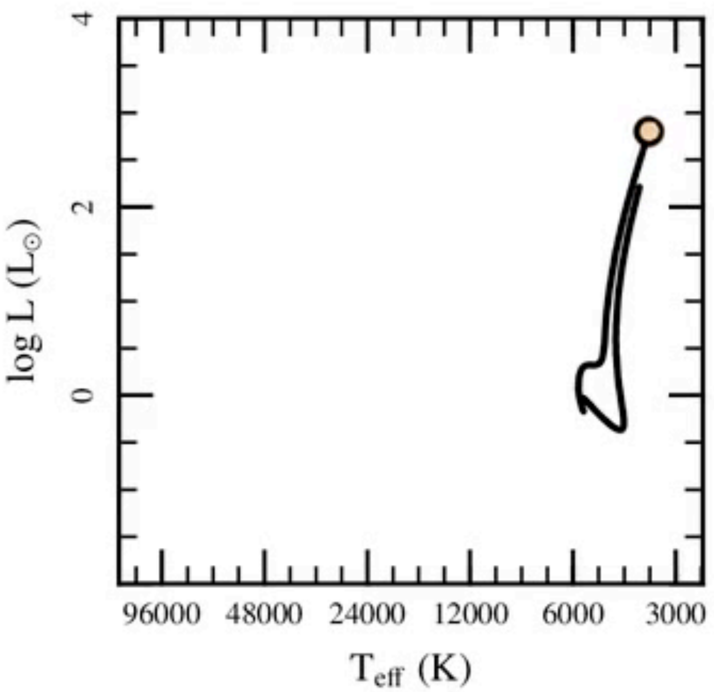
Fusion and Stellar Evolution

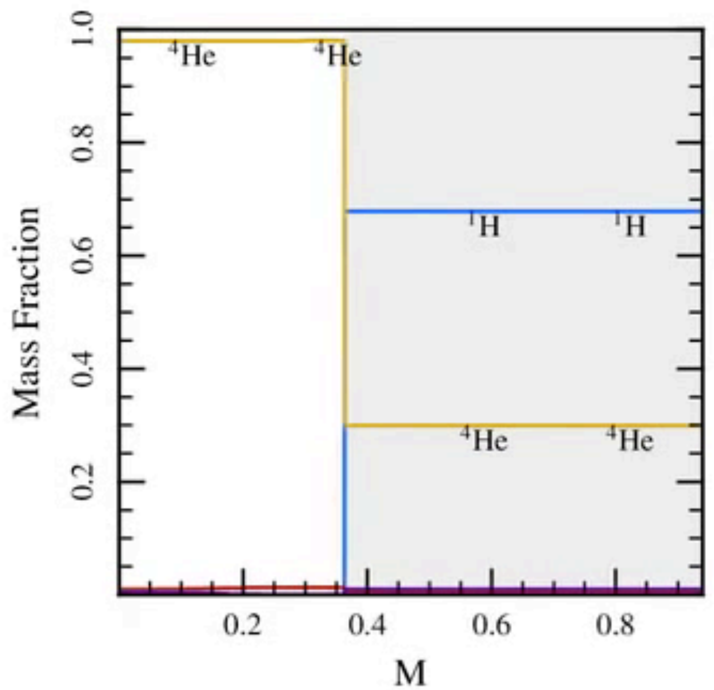


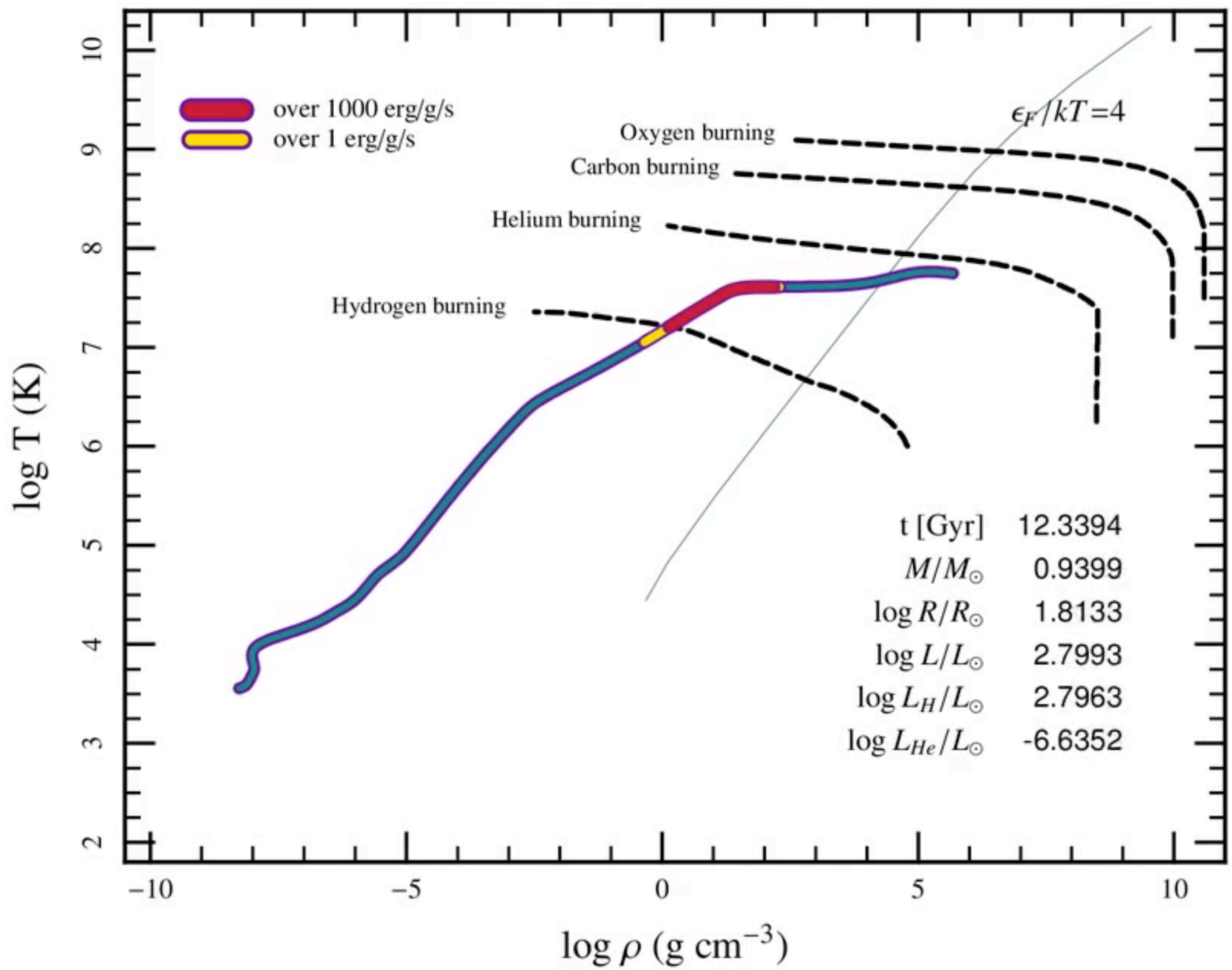
Adapted from Onno Pols, 2011.

MESA: open stellar modeling code

http://mesa.sourceforge.net







from Josiah Schwab, MESA model of a 1 solar mass star
https://www.youtube.com/watch?v=oZY3TtA63sE

MESA: open stellar modeling code

MESA Stellar Evolution Movies

In the past, I've taught the Astro 310 course (Introduction to Stellar Astrophysics) with the help of movies that animate the evolution of stars of various masses. These movies have been very successful in conveying key aspects of main-sequence and RGB stellar evolution; however, they are based on models created using *EZ-Web*, and are therefore increasingly unreliable beyond the onset of helium burning. (The helium flash is fudged in low-mass stars, and in high-mass stars the evolution has to be stopped during carbon burning).

Given that MESA is now my preferred way to calculate stellar models, *and* given that I've finally weaned myself off IDL and onto Python, I thought it time to update the movies.

Download Movies

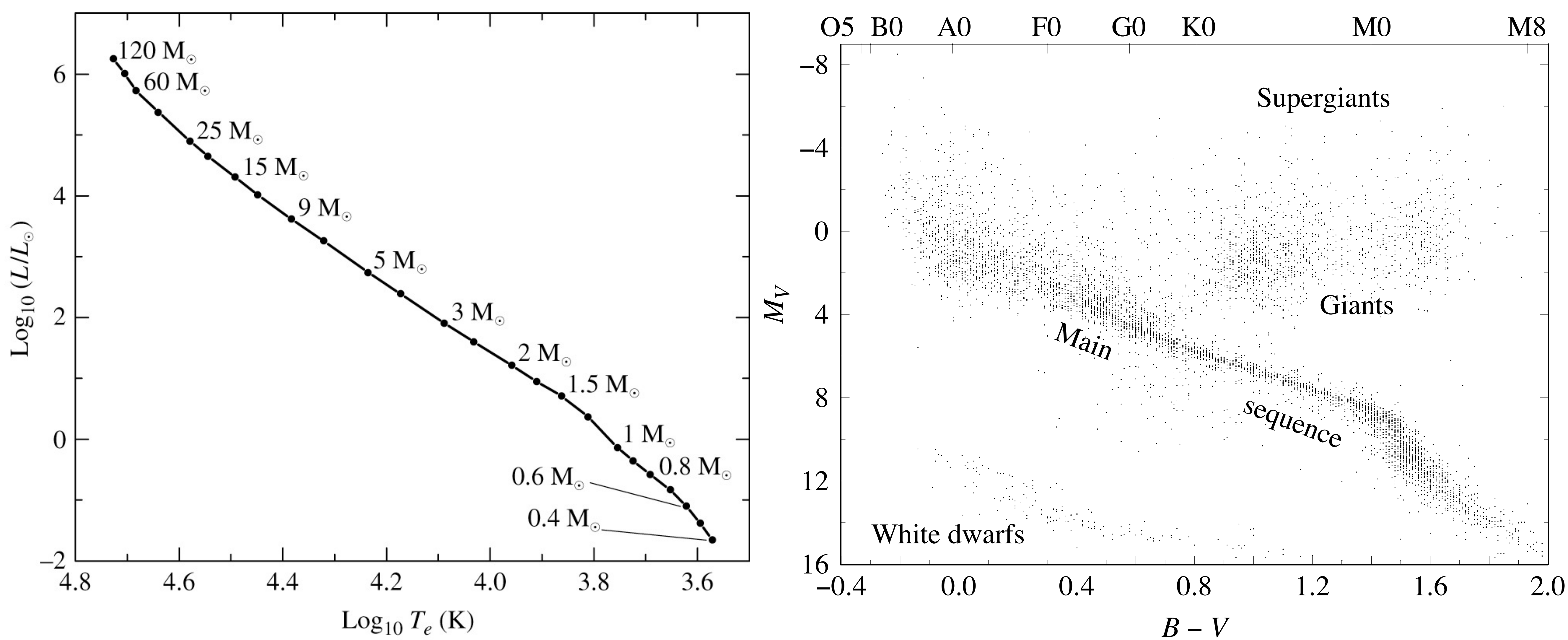
The table below contains links to the "best-so-far" set of MESA-based movies I've created. Blanks in the table are partly deliberate (e.g., massive stars don't undergo helium flashes) and partly due to the fact that I'm still generating movies (which take a surprisingly long time to render). Please note that the already-extant movies are themselves a work-in-progress, and will change as I update movies with additional info, snazzier graphics, better axis scaling, etc.

	Evolutionary Phase				
Mass ($M_\odot$)	Main Sequence	Red-Giant Branch	Helium Flash	Helium Burning	Asypmtotic Giant Branch
0.3	EOS Composition Energy				
1.0	EOS Composition Energy	EOS Composition Energy	EOS Composition Energy	EOS Composition Energy	EOS Composition Energy
3.0	EOS Composition Energy	EOS Composition Energy		EOS Composition Energy	
10.0	EOS Composition Energy	EOS Composition Energy		EOS Composition Energy	

The 'EOS' links are to movies showing the equation-of-state variables (P, ρ, T and μ); the 'Composition' links are to movies showing the composition (X, Y and CNO mass fractions); and the 'Energy' links are to movies showing the energy generation (PP, CNO, triple α).

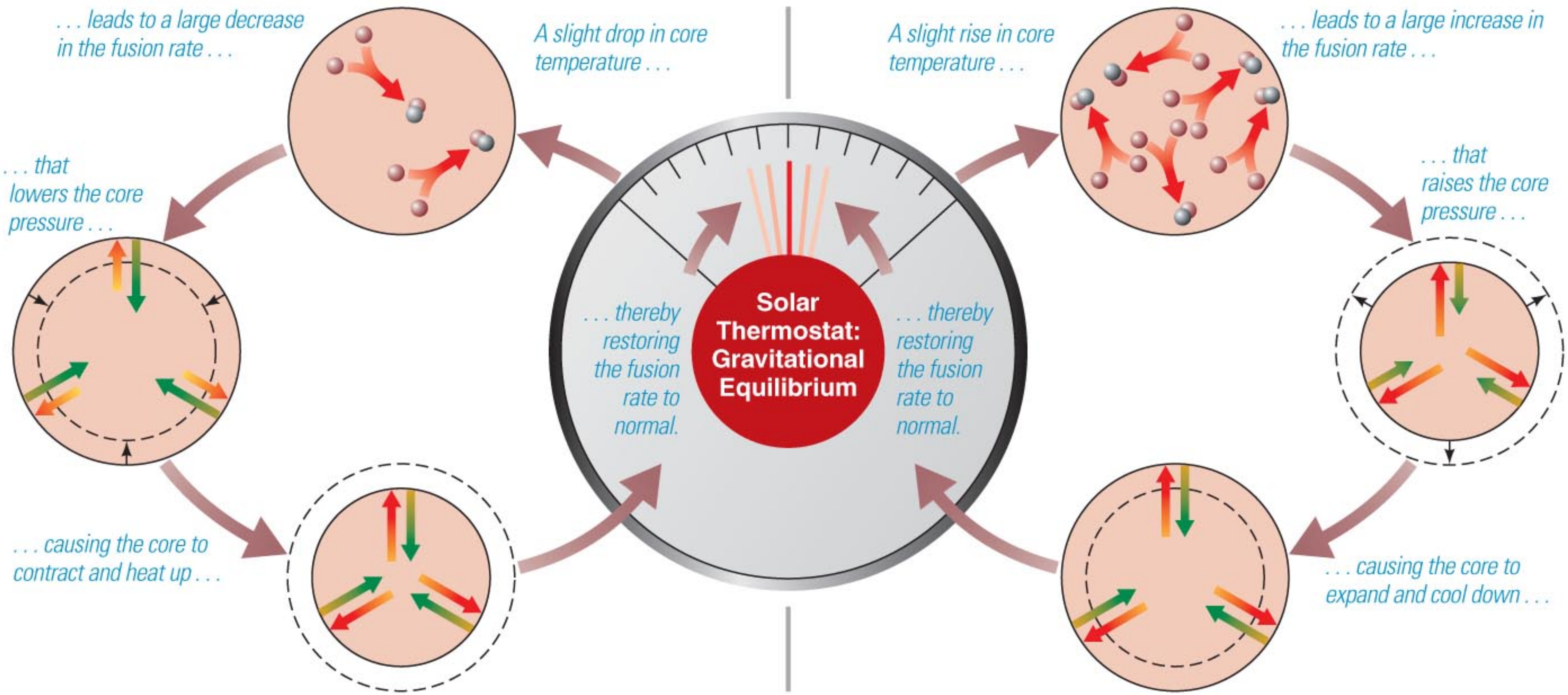
check out the movies at
http://www.astro.wisc.edu/~townsend/static.php?ref=mesa-movies
http://www.astro.wisc.edu/~townsend/static.php?ref=ez-web-movies

HR diagram: theory and observations



the main sequence is a *mass* sequence; Carroll & Ostlie Figures 10.13 and 8.13

Stellar Thermostat



Decline in core temperature causes fusion rate to drop, so core contracts and heats up.

Rise in core temperature causes fusion rate to rise, so core expands and cools down.

essential point: temperature is regulated (stable)
when there is fusion in the core (center of star)

Main Sequence Evolution

Changes during H fusion

<u>T_C</u>: very little; **why?**

Main Sequence Evolution

Changes during H fusion

$\underline{T_c}$: very little; **why?** (fusion rate sensitivity)

Main Sequence Evolution

Changes during H fusion

T_c: very little; **why?** (fusion rate sensitivity)

μ: increases (H $\rightarrow$ He)

(from $P_c/\rho_c \sim T_c/\mu_c$, ρ_c increases more than P_c)

Main Sequence Evolution

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(from $P_c/\rho_c \sim T_c/\mu_c$, ρ_c increases more than P_c)

R: increases (denser core → envelope expands; virial eq.)

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<u>R</u>: increases (denser core → envelope expands; virial eq.)

<u>L</u>: increases ("μ-effect"; higher ρ increases fusion rate)

Main Sequence Evolution

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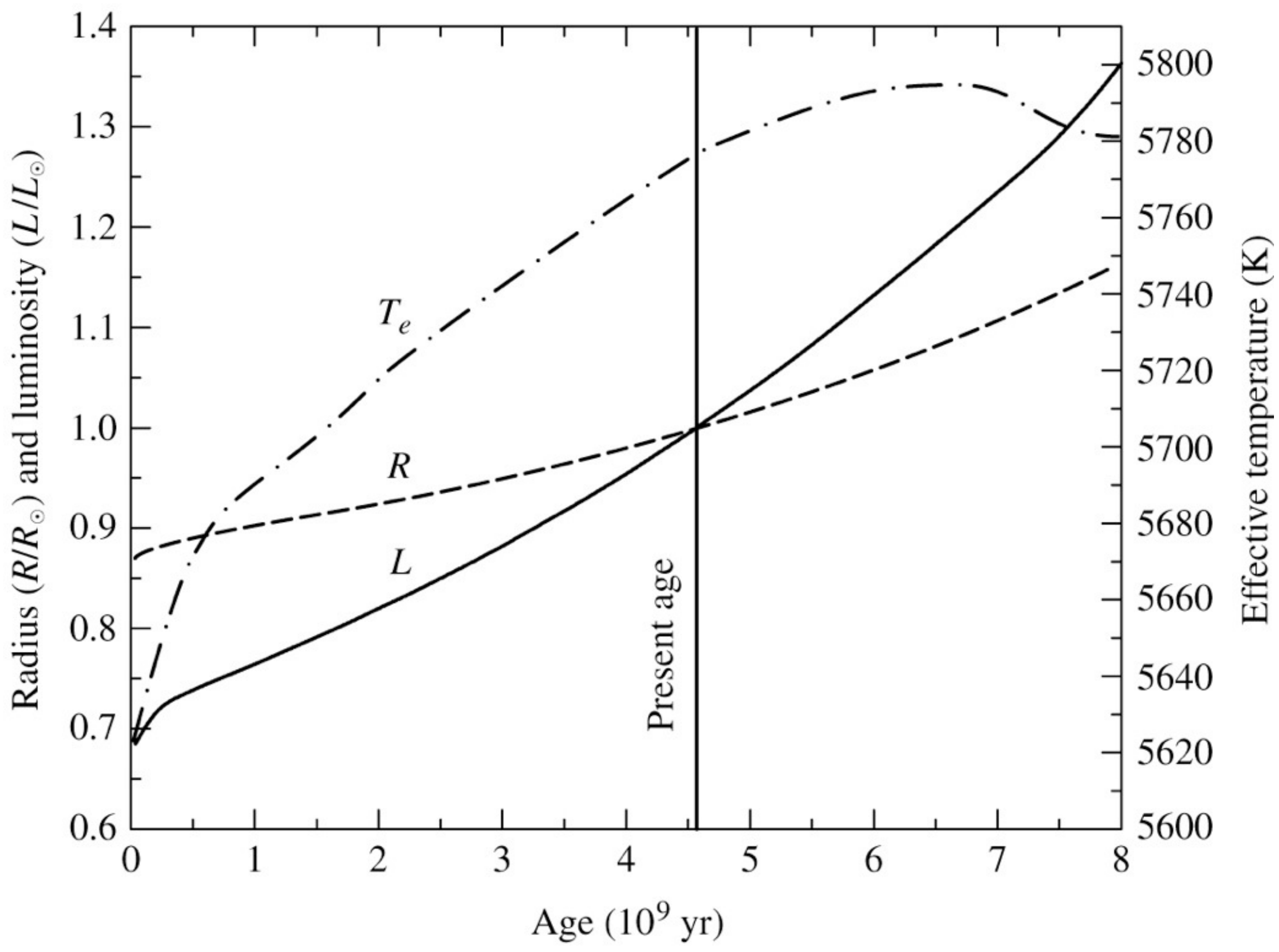
L: increases ("μ-effect"; higher ρ increases fusion rate)

T_{eff}: from $T_{\rm eff} \sim (L/R^2)^{1/4}$

(R² increases more than L), T_{eff} *decreases*

width of main-sequence: evolution



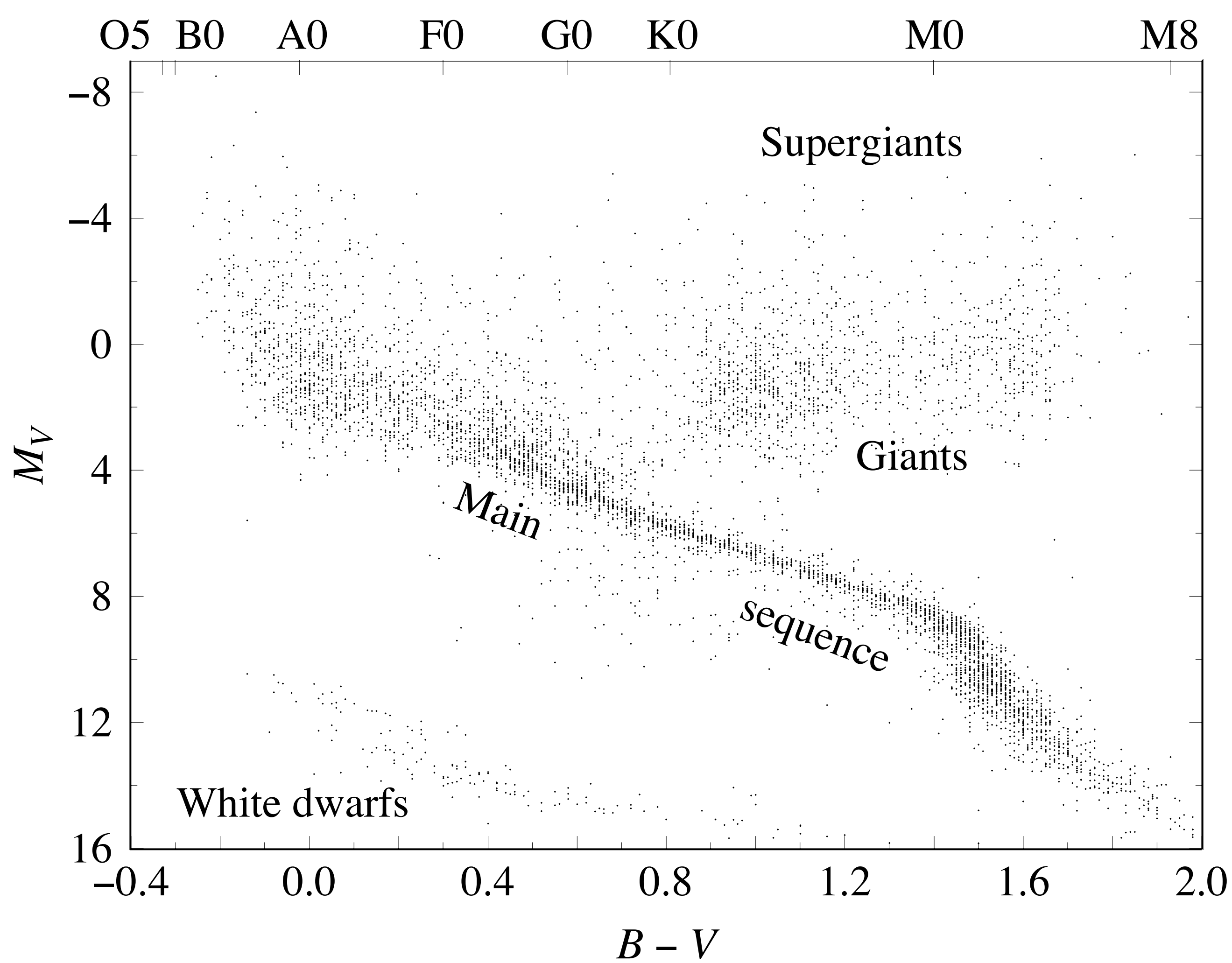


Table 13.1. The Parameters of a 1 $M_\odot$ Model at Three Different MS Ages (Data are from Schaller et al. 1992)

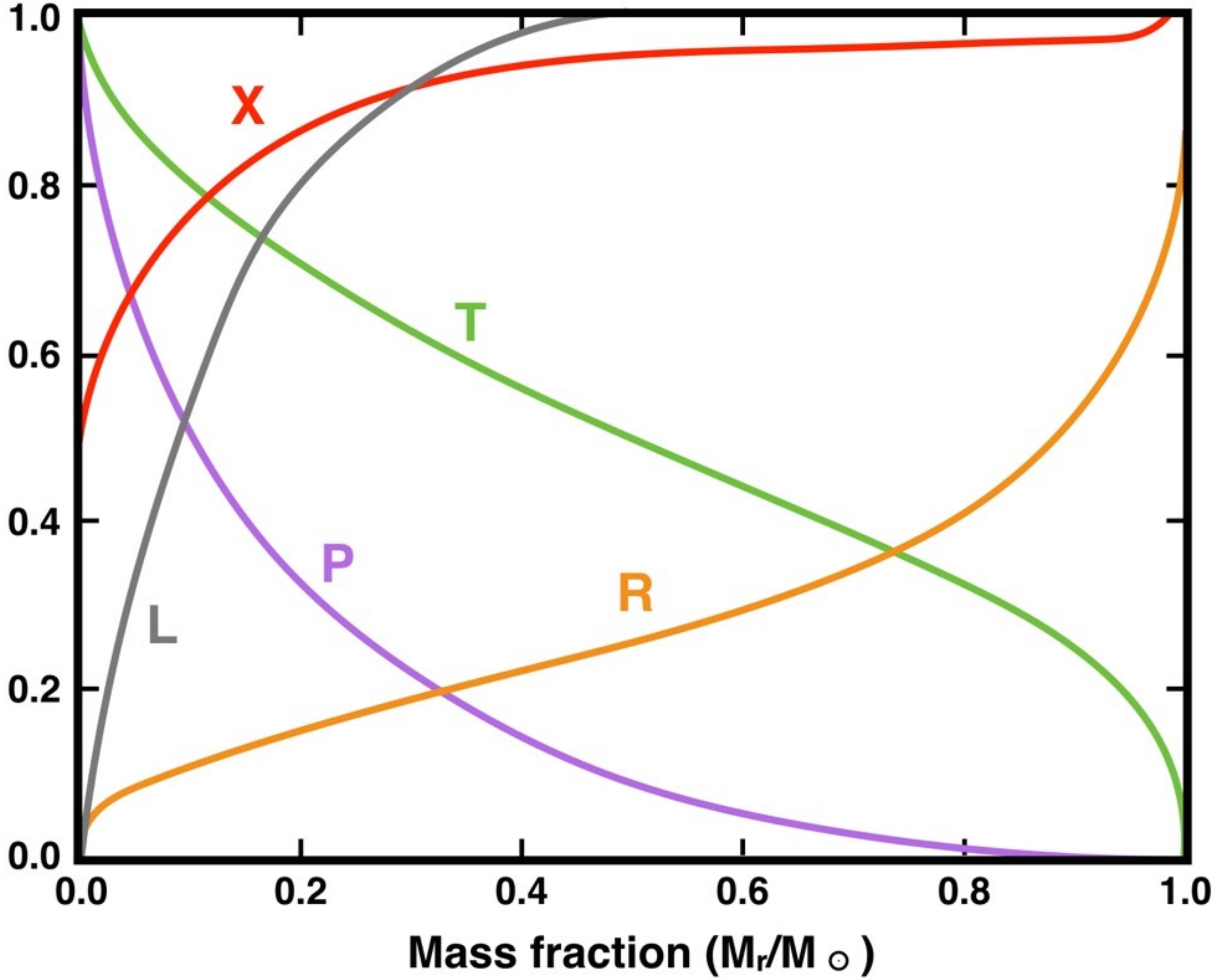
Age Phase	$L/L_\odot$	$R/R_\odot$	T_{eff}	T_c	P_c	ρ_c	X_c	Y_c	Z_c	μ_c
0 Gyr ZAMS	0.69	0.66	6540	13.6	1.24	78	0.680	0.301	0.02	0.71
4.5 Gyr Now	1.00	1.00	5820	15.9	2.36	157	0.302	0.677	0.02	0.88
9.4 Gyr TAMS	1.55	1.25	5820	18.7	6.73	572	0.000	0.980	0.02	1.32

Note. Units of T_{eff} are in K, T_c are in MK, P_c are in 10^{17} dyn cm^{-2}, and ρ_c are in g cm^{-3}.

Main Sequence Evolution

Changes during H fusion

Example: Sun-like star…



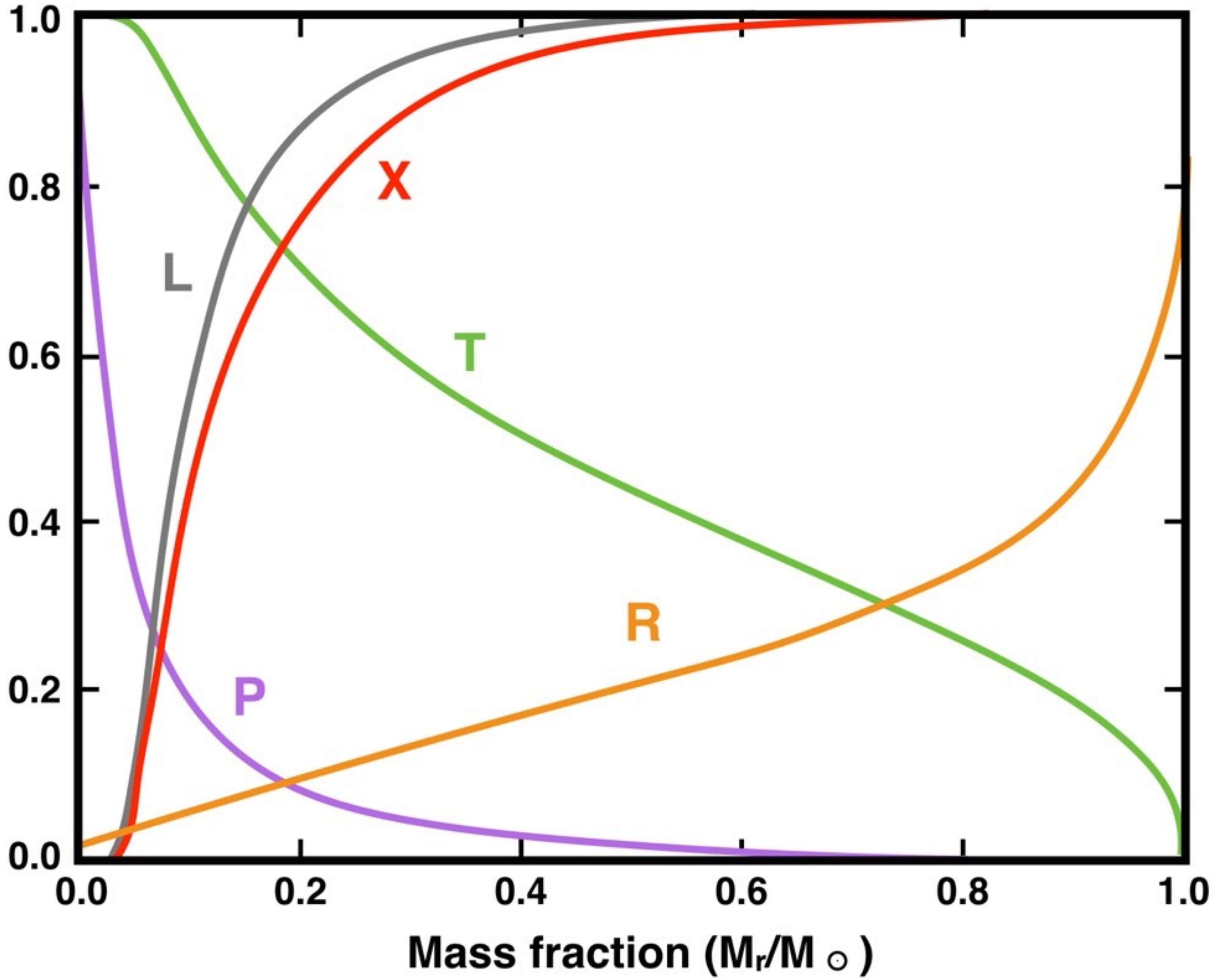
at 4.5 Gyr (sun now)

Reproduced from Iben J, 'Stellar Evolution.VI. Evolution from the Main Sequence to the Red-Giant Branch for Stars of Mass 1 M_{sun}, 1.25 M_{sun}, and 1.5 M_{sun}', Astrophysical Journal, vol. 147, p.624, 1967."

Main Sequence Evolution

Changes during H fusion

Example: Sun-like star…

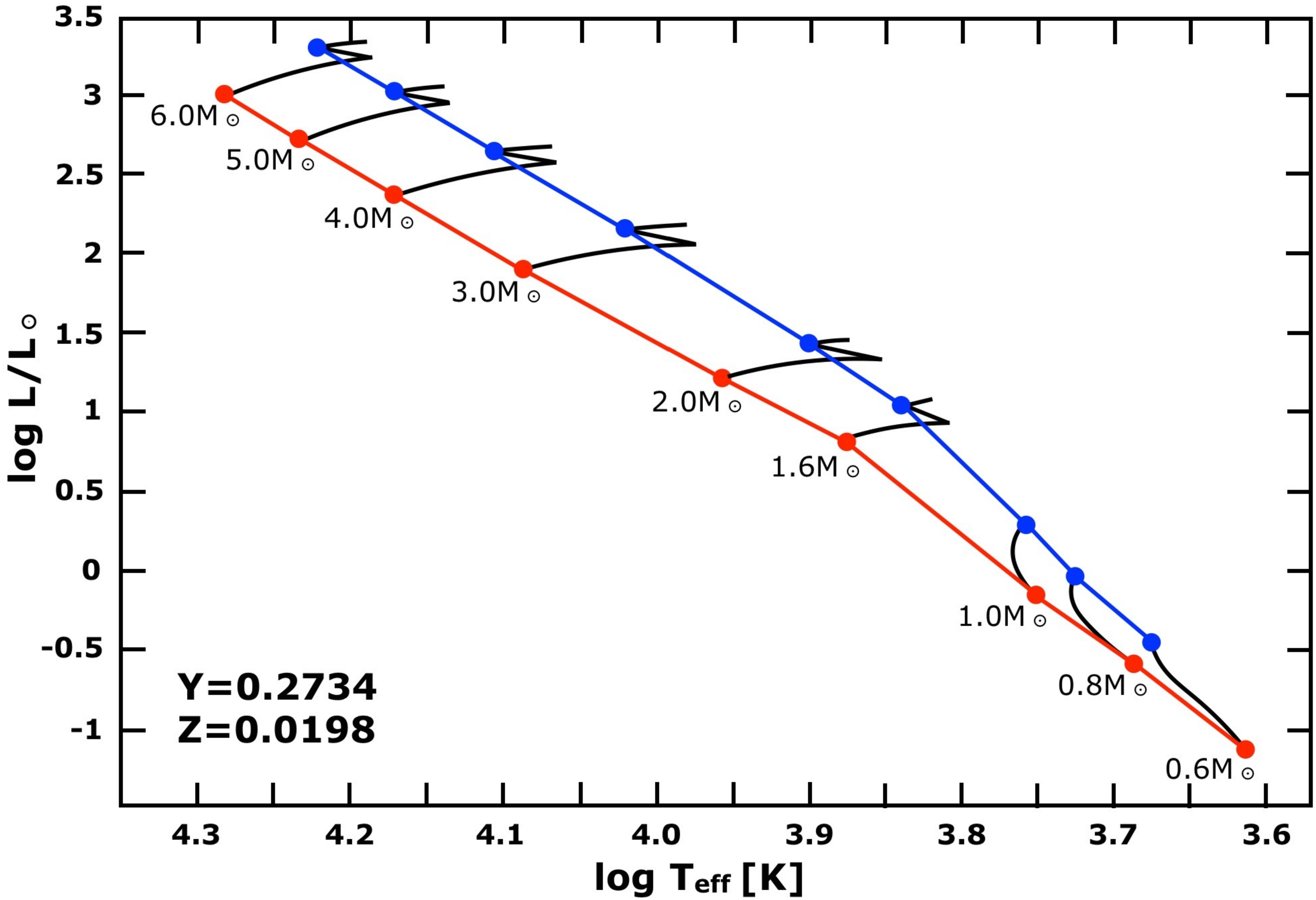


at 9.2 Gyr (TAMS)

Reproduced from Iben J, 'Stellar Evolution.VI. Evolution from the Main Sequence to the Red-Giant Branch for Stars of Mass 1 M_{sun}, 1.25 M_{sun}, and 1.5 M_{sun}', Astrophysical Journal, vol. 147, p.624, 1967."

Main Sequence Evolution

Changes during H fusion



Main Sequence Evolution

Changes during H fusion

T_c: very little; **why?** (fusion rate sensitivity)

μ: increases (H → He)

(from $P_c/\rho_c \sim T_c/\mu_c$, ρ_c increases more than P_c)

R: increases (denser core → envelope expands; virial eq.)

L: increases ("μ-effect"; higher ρ increases fusion rate)

T_{eff}: from $T_{\text{eff}} \sim (L/R^2)^{1/4}$

(R^2 increases more than L), T_{eff} *decreases*

Main Sequence Evolution

Changes during H fusion

$\underline{T_c}$: very little; **why?** (fusion rate sensitivity)

$\underline{\mu}$: increases (H → He)

(from $P_c/\rho_c \sim T_c/\mu_c$, ρ_c increases more than P_c)

$\underline{R}$: increases (denser core → envelope expands; virial eq.)

$\underline{L}$: increases ("μ-effect"; higher ρ increases fusion rate)

$\underline{T_{eff}}$: from $T_{\rm eff} \sim (L/R^2)^{1/4}$

(R^2 increases more than L), T_{eff} *decreases*

Chemical profile (for $M \gtrsim 1.2 M_\odot$): H fusion happens via T-sensitive CNO cycle and in a convective core…

Main Sequence Evolution

Changes during H fusion

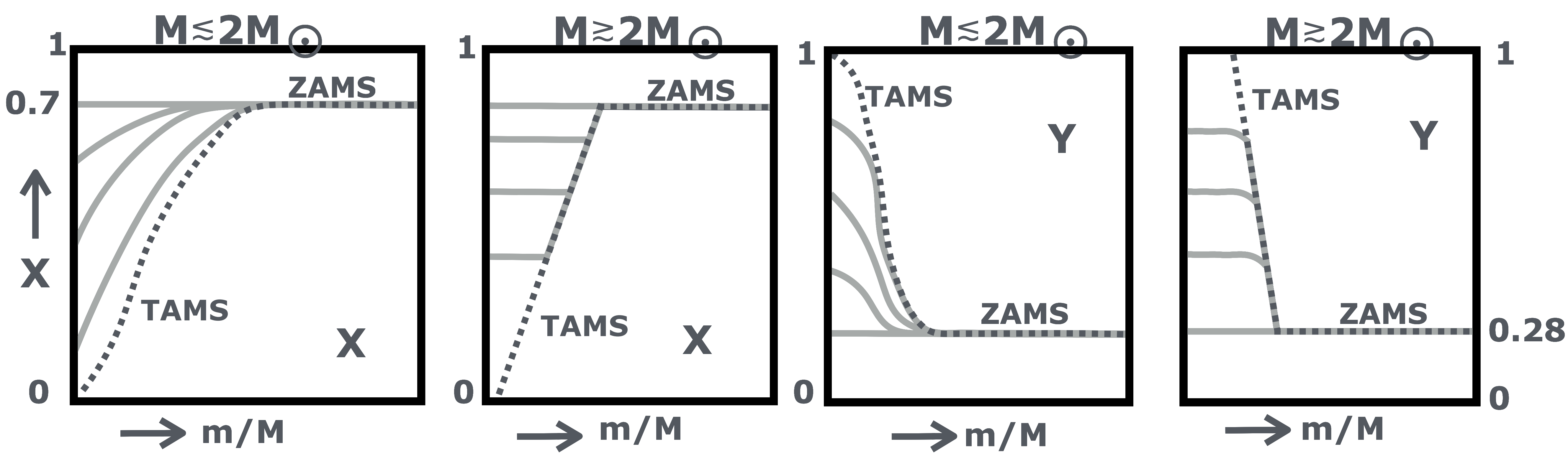
Chemical profile (for $M \gtrsim 1.2M_\odot$): H fusion happens via

T-sensitive CNO cycle and in a convective core…

i. convection refuels core with fresh H, extends MS lifetime

ii. chemical profile is flat in the center (as opposed to peaked for lower-mass stars)

iii. as core H-fraction decreases, convection zone shrinks in mass



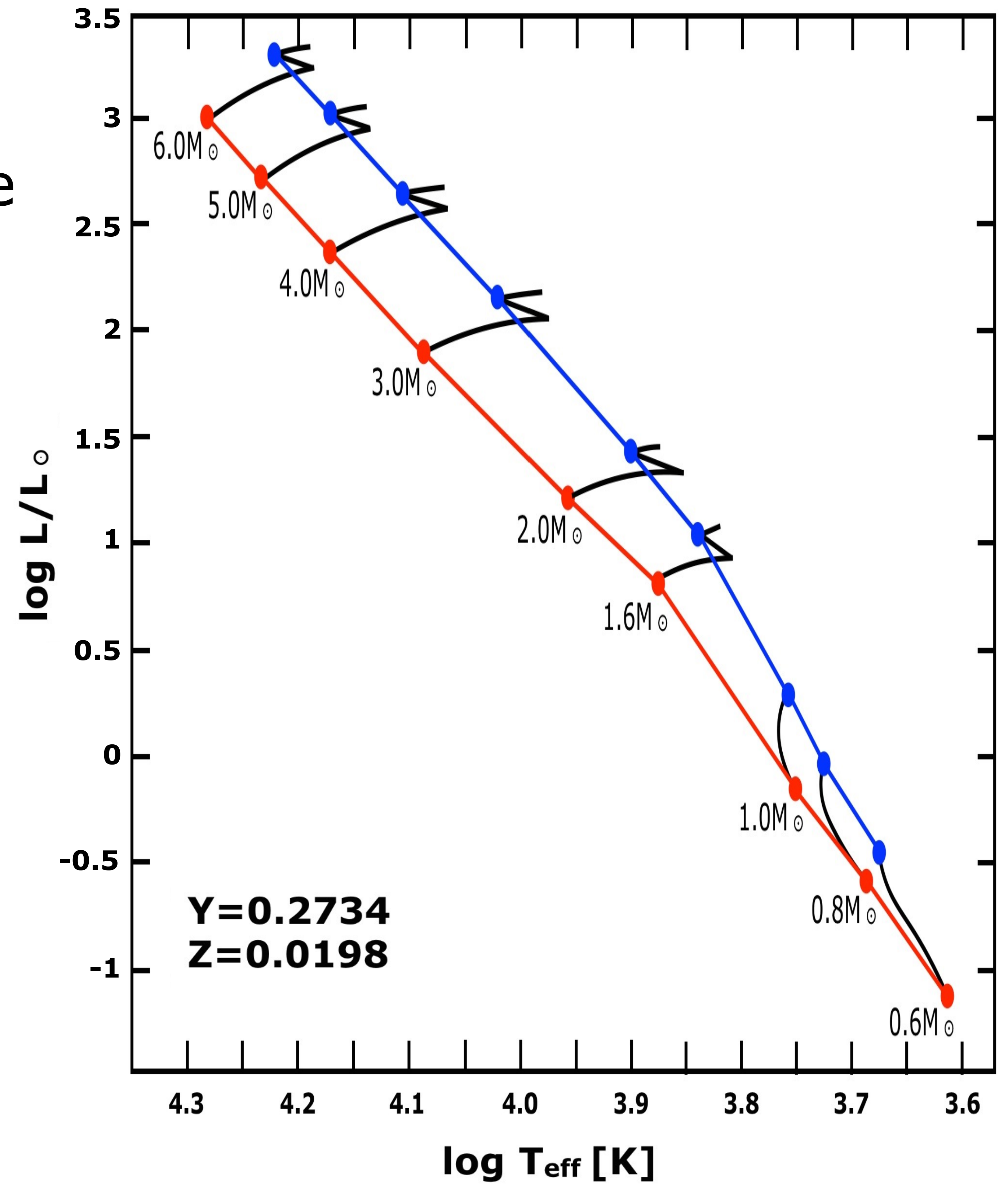
Main Sequence Evolution

End of the main sequence

When H fusion stops, the core contracts to compensate for radiative losses.

M < 1.2M$_\odot$: core contracts gradually as H-fusion slowly peters out

M ≳ 1.2M$_\odot$: contraction happens suddenly; H is exhausted all at once due to convective mixing



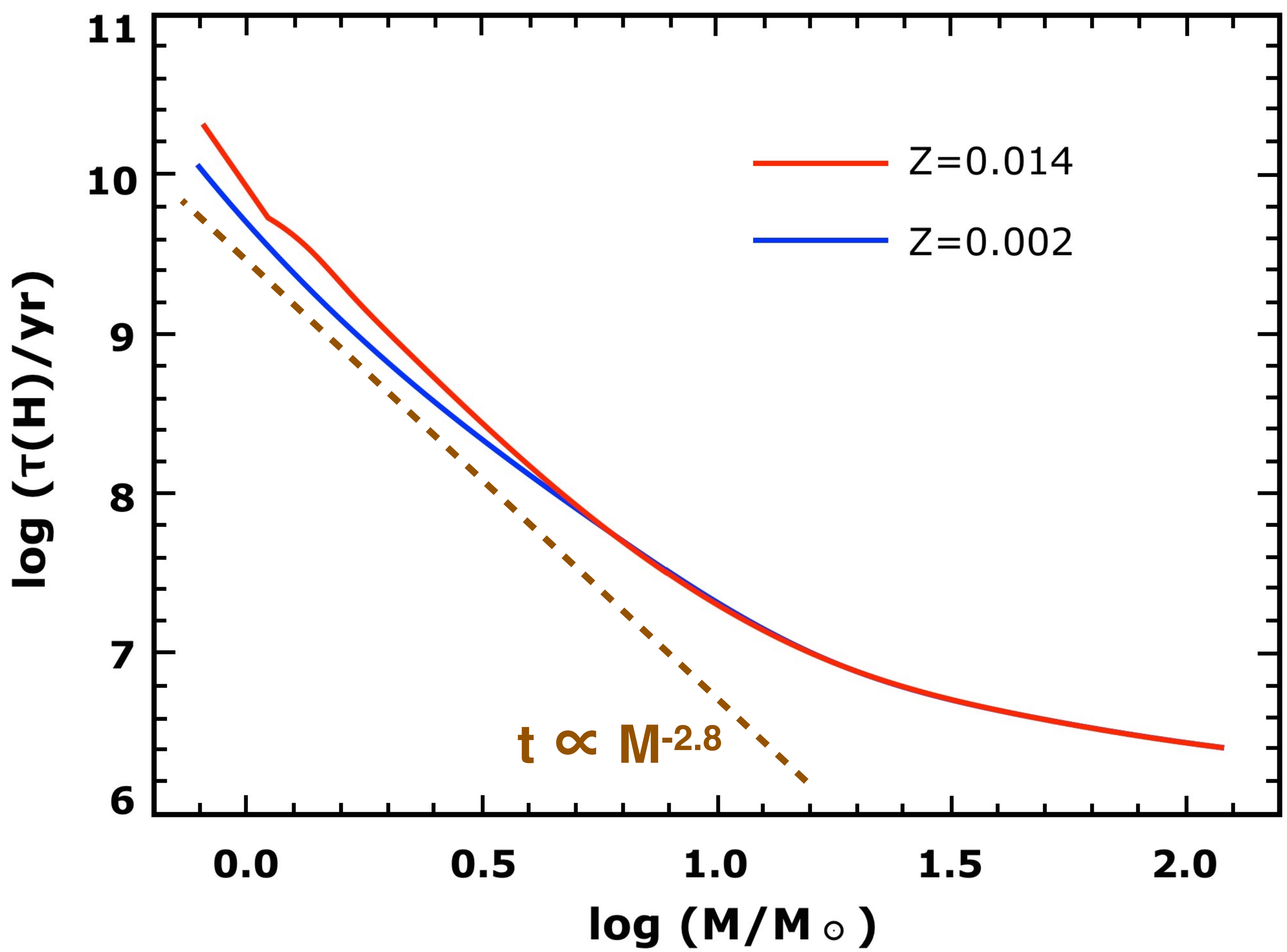
Main Sequence Evolution

The main sequence lifetime

Duration of the MS phase depends on M and L. MS lifetimes would be proportional to M/L (with $L \sim M^{3.8}$ so $t \sim M^{-2.8}$) if the same mass fraction of all stars was used for core H fusion.

It is not quite so steep because of two effects:

1) T_c increases with M, so H fusion is possible in a larger mass fraction.
2) If $M_i > 1.2 M_\odot$, core convection brings material into the core fusion region.



For z=0.014

$$\log \tau(H)/\mathrm{yr} = 9.9551 - 3.3674\,x + 0.4794\,x^2 + 0.3676\,x^3 - 0.1013\,x^4, \;\; x = \log(M/M_\odot)$$