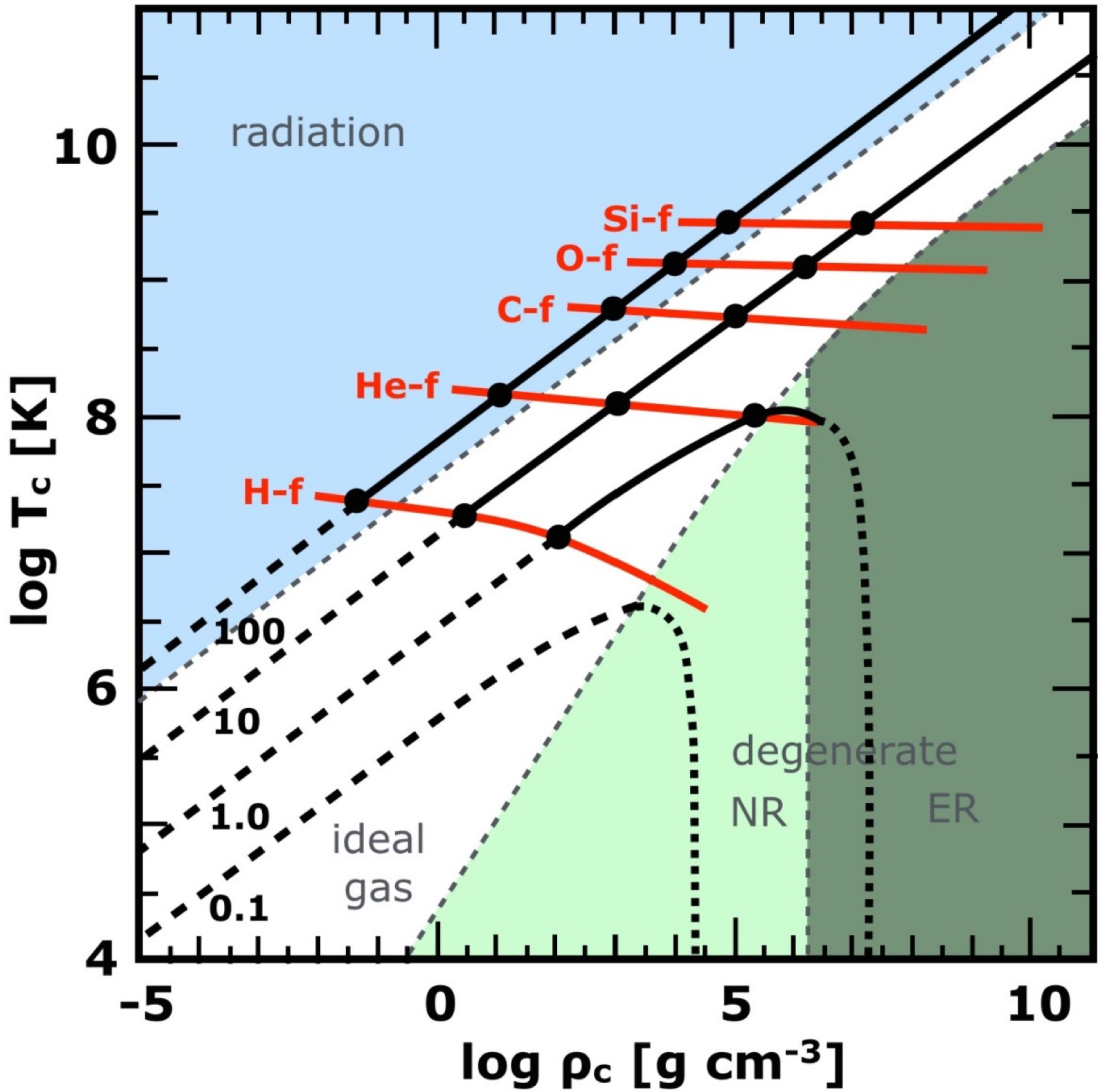
Lecture 11: Stellar Models & Evolution

Lamers & Levesque Ch. 10, 13



Advanced burning: massive stars

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{16}_{8}\mathrm{O} + \gamma \\
{}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma
\end{aligned}
$$

T > 10^8 K

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} &\rightarrow {}^{16}_{8}\mathrm{O} + 2\,{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{23}_{11}\mathrm{Na} + p \\
&\rightarrow {}^{23}_{12}\mathrm{Mg} + n \\
&\rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma
\end{aligned}
$$

carbon burning,
T > 6 × 10^8 K

oxygen burning,
T > 10^9 K

$$
\begin{aligned}
{}^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} &\rightarrow {}^{24}_{12}\mathrm{Mg} + 2\,{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{31}_{15}\mathrm{P} + p \\
&\rightarrow {}^{31}_{16}\mathrm{S} + n \\
&\rightarrow {}^{32}_{16}\mathrm{S} + \gamma
\end{aligned}
$$

silicon burning,
T > 3 × 10^9 K

$$
\begin{aligned}
{}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{32}_{16}\mathrm{S} + \gamma \\
{}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{36}_{18}\mathrm{Ar} + \gamma \\
&\vdots \\
{}^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{56}_{28}\mathrm{Ni} + \gamma
\end{aligned}
$$

photodisintegration!

Fusion temperatures/timescales

Phillips Table 4.2

TABLE 4.2 The time scale for the nuclear burning stages for a star of mass 25 $M_\odot$, and the central temperature and density at which they take place. This data is based on the calculations of Weaver, cited by Rolfs and Rodney (1988).

Stage	Time scale	Temperature (10^9 K)	Density (kg m^{-3})
Hydrogen burning	7×10^6 years	0.06	5×10^4
Helium burning	5×10^5 years	0.23	7×10^5
Carbon burning	600 years	0.93	2×10^8
Neon burning	1 year	1.7	4×10^9
Oxygen burning	6 months	2.3	1×10^{10}
Silicon burning	1 day	4.1	3×10^{10}

25 M$_\odot$ star

Lamers & Levesque Table 8.4

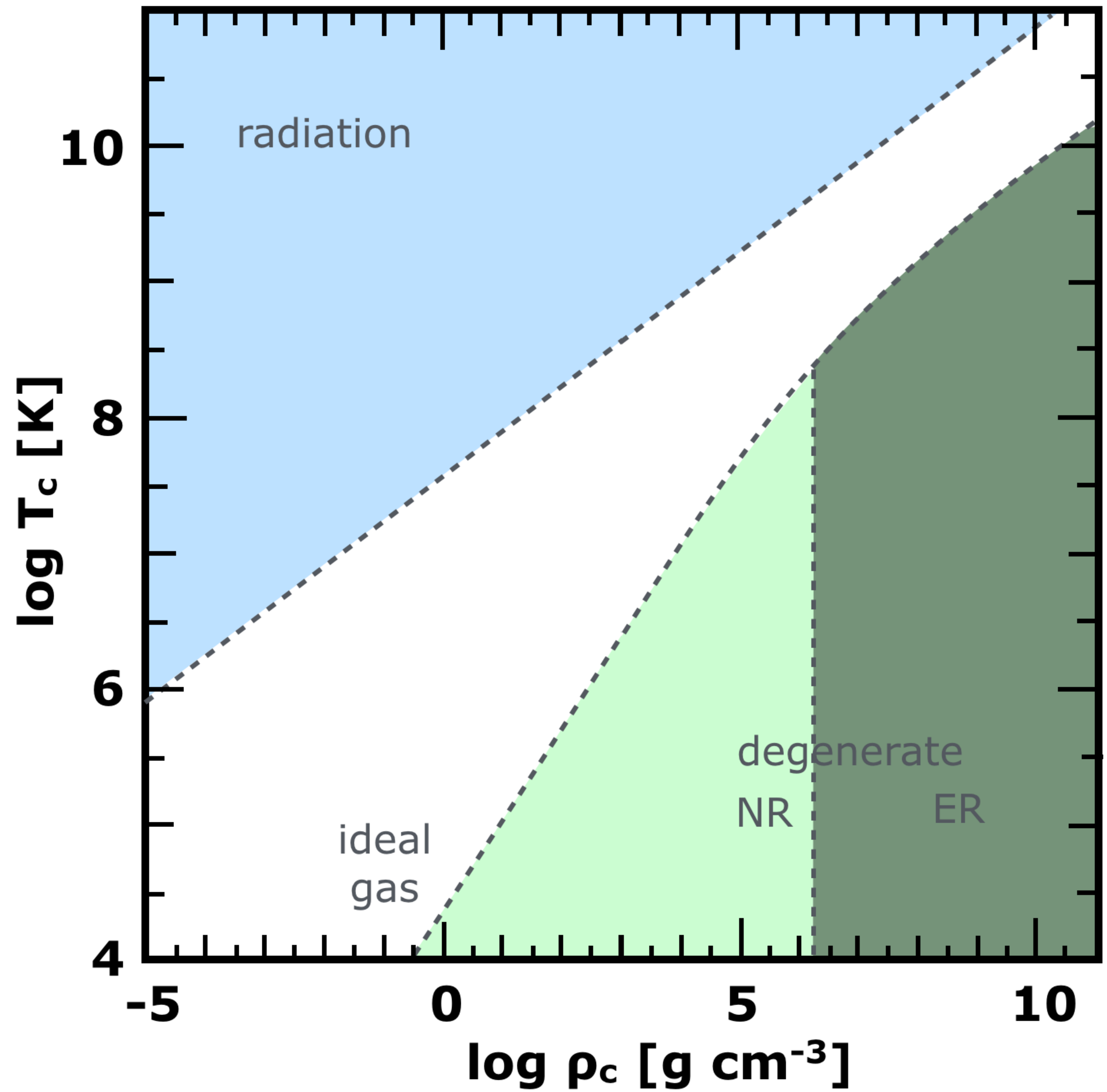
Table 8.4. Summary of the Most Important Reaction Rates in Stars

Fuel (1)	Process (2)	$T_{\rm thresh}$ 10^6 K (3)	Product (4)	$E_{\rm net}$ MeV/nucl (5)	T_c 10^6 K (6)	$L_{\rm net}/L$ (7)	Duration yr (8)
H	p-p chain	4	He	6.55	–	–	–
H	CNO cycle	15	He	6.25	35	0.94	1.1×10^7
He	3-α fusion	100	C,O	0.61	180	0.96	2.0×10^6
C	C-fusion	600	Ne,Mg,Na,O	0.54	810	0.16	2.0×10^3
Ne	Ne photdis	900	O,Mg,Si		1600	5.3×10^{-4}	0.7
O	O-fusion	1000	S,Si,P,Mg	0.30	1900	8.2×10^{-5}	2.6
Si	Si nucl equil.	3000	Fe,Ni,Cr,Ti	<0.18	3300	5.8×10^{-7}	0.05

Notes. photdis = photodisintegration. **nucl equil** = nuclear equilibrium = photodisintegration + capture of p, n, and He. Column (5) = energy generated per nucleon (He has 4 nucleons, C has 12 nucleons etc.). Columns (6), (7), and (8) refer to the evolution of a star of 15$M_\odot$ (Based on Woosley & Janka 2005, and Maeder 2009).

15 M$_\odot$ star

Fusion and Stellar Evolution



Adapted from Onno Pols, 2011.

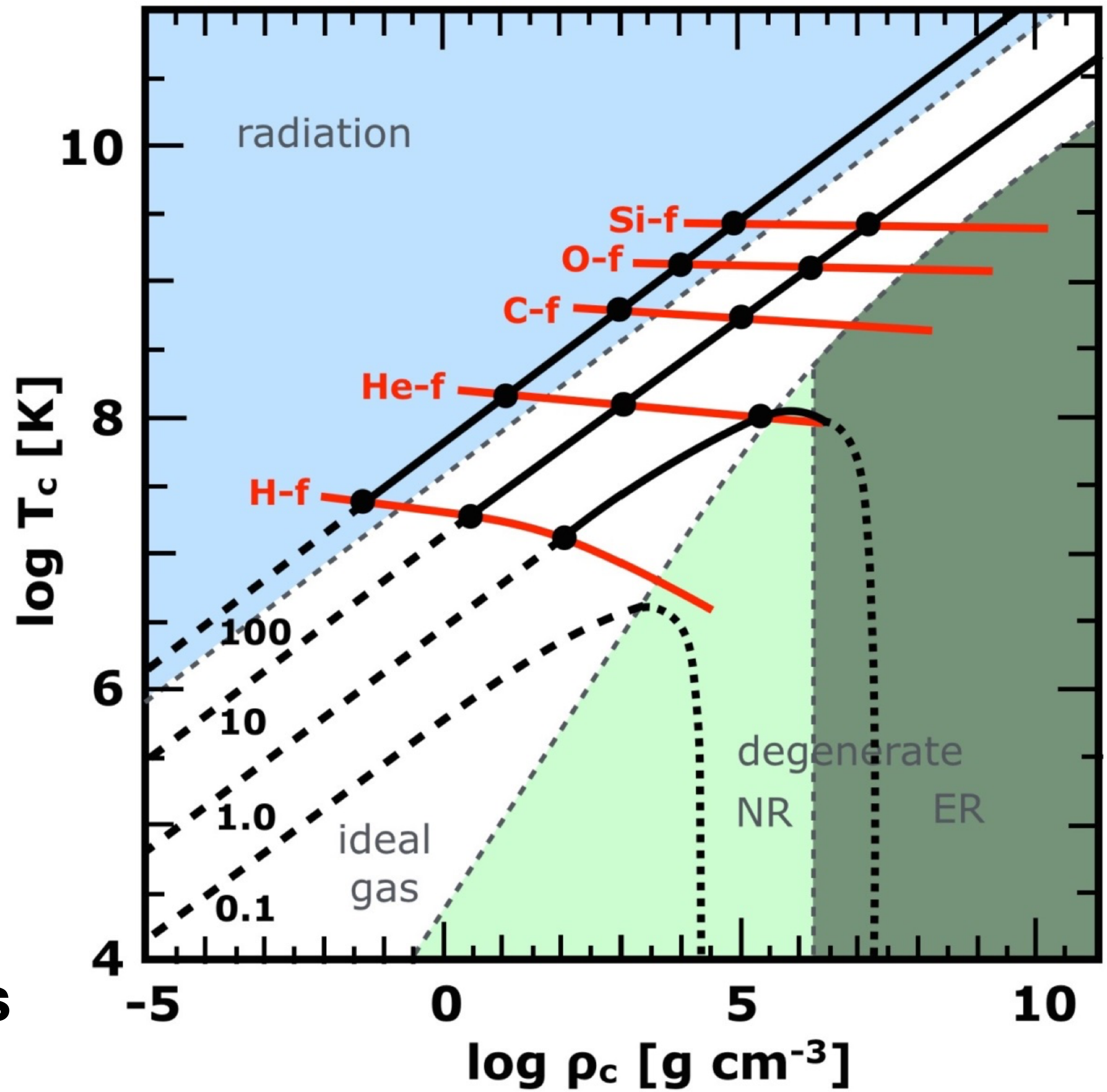
Fusion and Stellar Evolution

stars evolve up and to the right (hotter & denser center) until WD

lowest mass stars only fuse H→He

“low-mass stars” fuse H→He→C,O

high mass (> 8 $M_{\odot}$) stars fuse to iron & do not make WDs



Adapted from Onno Pols, 2011.

10.1 Assumptions for Computing Stellar Evolution

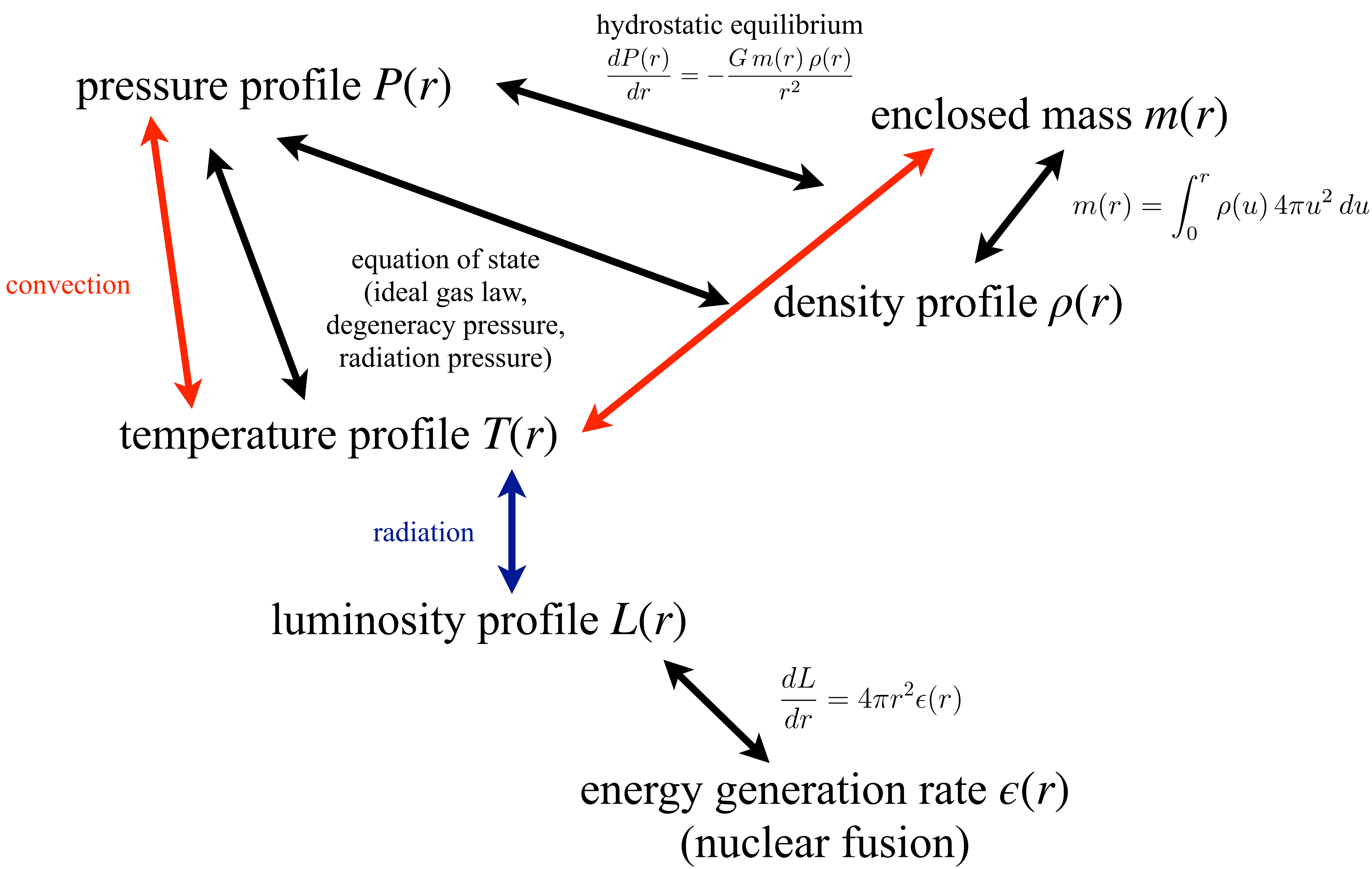
The stellar models that we describe here are simple and valid for most evolutionary phases. They are based on the following assumptions.

1. The star is **spherically symmetric**, which implies that
   - the physical quantities vary only in the radial direction: $P(r)$, $\rho(r)$, $T(r)$, etc., and
   - the effects of rotation and magnetic fields are ignored. (Rotation will be discussed later in Section 25.)
2. The star is in **hydrostatic equilibrium**: at each depth, the layers are stable.
3. The star is in **thermal equilibrium**: the energy generated in the star equals the energy radiated outward.
4. The energy sources are
   - thermonuclear energy,
   - gravitational energy, which is important for contracting or expanding stars, and
   - internal energy, which is important for cooling white dwarfs.
5. The **energy transport** is by
   - radiation,
   - convection, and
   - conduction, which is important for degenerate stars.

6. The **chemical composition is changing.**
   - Newly formed stars have a homogenous initial composition described by
     X = mass fraction of H,
     Y = mass fraction of He,
     Z = mass fraction of the rest, mainly C, N, and O.
   - As the star evolves, its chemical composition changes as a function of time and location due to nuclear fusion in the core or in shells.
   - The chemical composition in certain layers may also change due to mixing by convection or convective overshooting.

Under these assumptions, the calculation of stellar evolution consists of
(a) computing a series of subsequent hydrostatic equilibrium models,
(b) each with the chemical structure calculated using the nuclear reaction rates of the previous model,
(c) extrapolating it with a small time step for calculating the new chemical distribution in the star, and
(d) computing the next model in hydrostatic equilibrium with the new chemical structure.

stellar structure



10.2 The Equations of Stellar Structure

	in r Euler coordinates	
Mass continuity	$\frac{dm}{dr} = 4\pi r^2 \rho$	(10.1)
Hydrost equilibrium	$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$	(10.2)
Energy generation*	$\frac{dL}{dr} = 4\pi r^2 \rho(\epsilon - \epsilon_v - \frac{Tds}{dt})$	(10.3)
Energy transport		
by radiation	$\frac{dT}{dr} = -\frac{3}{4ac} \cdot \frac{\kappa\rho}{T^3} \frac{L}{4\pi r^2}$	(10.4a)
or		
by convection	$\frac{dT}{dr} = \frac{\gamma_{\rm ad}-1}{\gamma_{\rm ad}} \cdot \frac{T}{P} \cdot \frac{dP}{dr}$	(10.4b)

These structure equations describe either T, P, L, ρ, m as a function of r (Euler-coordinates)

Equation of state $P = P_{\rm gas} + P_{\rm rad} = P_{\rm rad} + P_e + P_{\rm ion}$ (10.5)

$$P_{\rm rad} = \frac{a}{3} T^4$$
$$P_i = \rho \mathcal{R} T / \mu_i$$
$$P_e = \rho \mathcal{R} T / \mu_e$$

or $P_e = K_1\ (\rho/\mu_e)^{5/3}$ (nonrelativ. electr. degen.)

or $P_e = K_1\ (\rho/\mu_e)^{5/3}$ (electr. degen.)

or $P_e =$ equation (4.23) (partial electr. degen.)

Absorption coefficient $\kappa = \kappa_{ff} + \kappa_{\rm bf} + \kappa_e$

$$\kappa_{ff} + \kappa_{\rm bf} \sim \rho T^{-7/2} \qquad (10.6)$$
$$\kappa_e \sim \rho/\mu_e \sim 1 + X$$

Nuclear energy production $\epsilon = \epsilon_0 \rho^m T^n - \epsilon_v$ (10.7)

ϵ_0, m, n depend on the reaction and the abundance

$-\epsilon_v$ is the loss of energy by escaping neutrinos

Composition $X(m)$, $Y(m)$, $Z(m)$ or $X_i(m)$ with $i = 1 \ldots$ all isotopes

Lamers & Levesque Ch. 10.2

10.2 The Equations of Stellar Structure

	in r Euler coordinates	in $m = M(r)$ Lagrange coordinates	
Mass continuity	$\frac{dm}{dr} = 4\pi r^2 \rho$	$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$	(10.1)
Hydrost equilibrium	$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$	$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$	(10.2)
Energy generation*	$\frac{dL}{dr} = 4\pi r^2 \rho(\epsilon - \epsilon_v - \frac{T ds}{dt})$	$\frac{dL}{dm} = \epsilon - \epsilon_v - \frac{T\,ds}{dt}$	(10.3)
Energy transport **by radiation**	$\frac{dT}{dr} = -\frac{3}{4ac} \cdot \frac{\kappa\rho}{T^3} \frac{L}{4\pi r^2}$	$\frac{dT}{dm} = -\frac{3}{4ac} \cdot \frac{\kappa}{T^3} \frac{L}{(4\pi r^2)^2}$	(10.4a)
or **by convection**	$\frac{dT}{dr} = \frac{\gamma_{\rm ad}-1}{\gamma_{\rm ad}} \cdot \frac{T}{P} \cdot \frac{dP}{dr}$	$\frac{dT}{dm} = \frac{\gamma_{\rm ad}-1}{\gamma_{\rm ad}} \cdot \frac{T}{P} \cdot \frac{dP}{dm}$	(10.4b)

Equation of state $P = P_{\rm gas} + P_{\rm rad} = P_{\rm rad} + P_e + P_{\rm ion}$ (10.5)

$P_{\rm rad} = \frac{a}{3} T^4$

$P_i = \rho \mathcal{R} T / \mu_i$

$P_e = \rho \mathcal{R} T / \mu_e$

or $P_e = K_1\ (\rho/\mu_e)^{5/3}$ (nonrelativ. electr. degen.)

or $P_e = K_1\ (\rho/\mu_e)^{5/3}$ (electr. degen.)

or $P_e =$ equation (4.23) (partial electr. degen.)

Absorption coefficient $\kappa = \kappa_{ff} + \kappa_{\rm bf} + \kappa_e$

$\kappa_{ff} + \kappa_{\rm bf} \sim \rho T^{-7/2}$ (10.6)

$\kappa_e \sim \rho/\mu_e \sim 1 + X$

Nuclear energy production $\epsilon = \epsilon_0 \rho^m T^n - \epsilon_v$ (10.7)

ϵ_0, m, n depend on the reaction and the abundance

$-\epsilon_v$ is the loss of energy by escaping neutrinos

Composition $X(m)$, $Y(m)$, $Z(m)$ or $X_i(m)$ with $i = 1 \ldots$ all isotopes

These structure equations describe either
$T,\ P,\ L,\ \rho,\ m$ as a function of r (Euler-coordinates)
or
$T,\ P,\ L,\ \rho,\ r$ as a function of m (Lagrange-coordinates)

can convert from Euler coordinates to Langrange coordinates using

$$\frac{dx}{dm} = \frac{dx}{dr} \cdot \frac{dr}{dm} \text{ with } \frac{dr}{dm} = \frac{1}{4\pi\rho r^2}.$$

Lamers & Levesque Ch. 10.2

10.3 Boundary Conditions

$$\text{surface: } m = M \;\Leftrightarrow\; r = R$$

The equations of stellar structure in Lagrangian coordinates consist of four differential equations for $r(m)$, $P(m)$, $L(m)$, and $T(m)$. Solving these equations for a star of mass M requires four boundary conditions:

$$r(m = 0) = 0, \tag{10.8a}$$

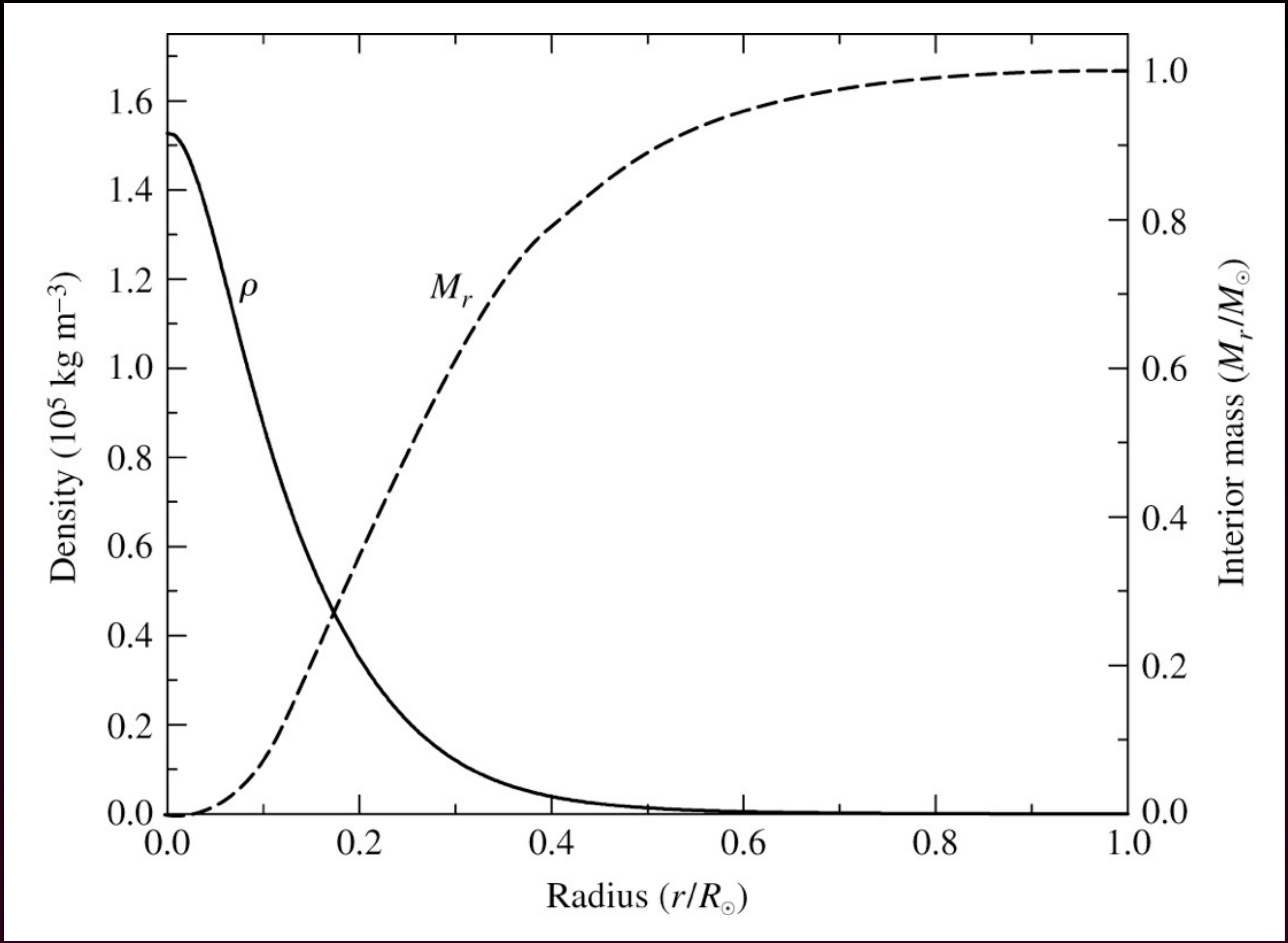
$$L(m = 0) = 0, \tag{10.8b}$$

$$P(m = M) = 0, \tag{10.8c}$$

$$T(m = M) = T_{\rm eff} \equiv \{L(M)/4\pi\sigma r(m = M)^2\}^{1/4}. \tag{10.8d}$$

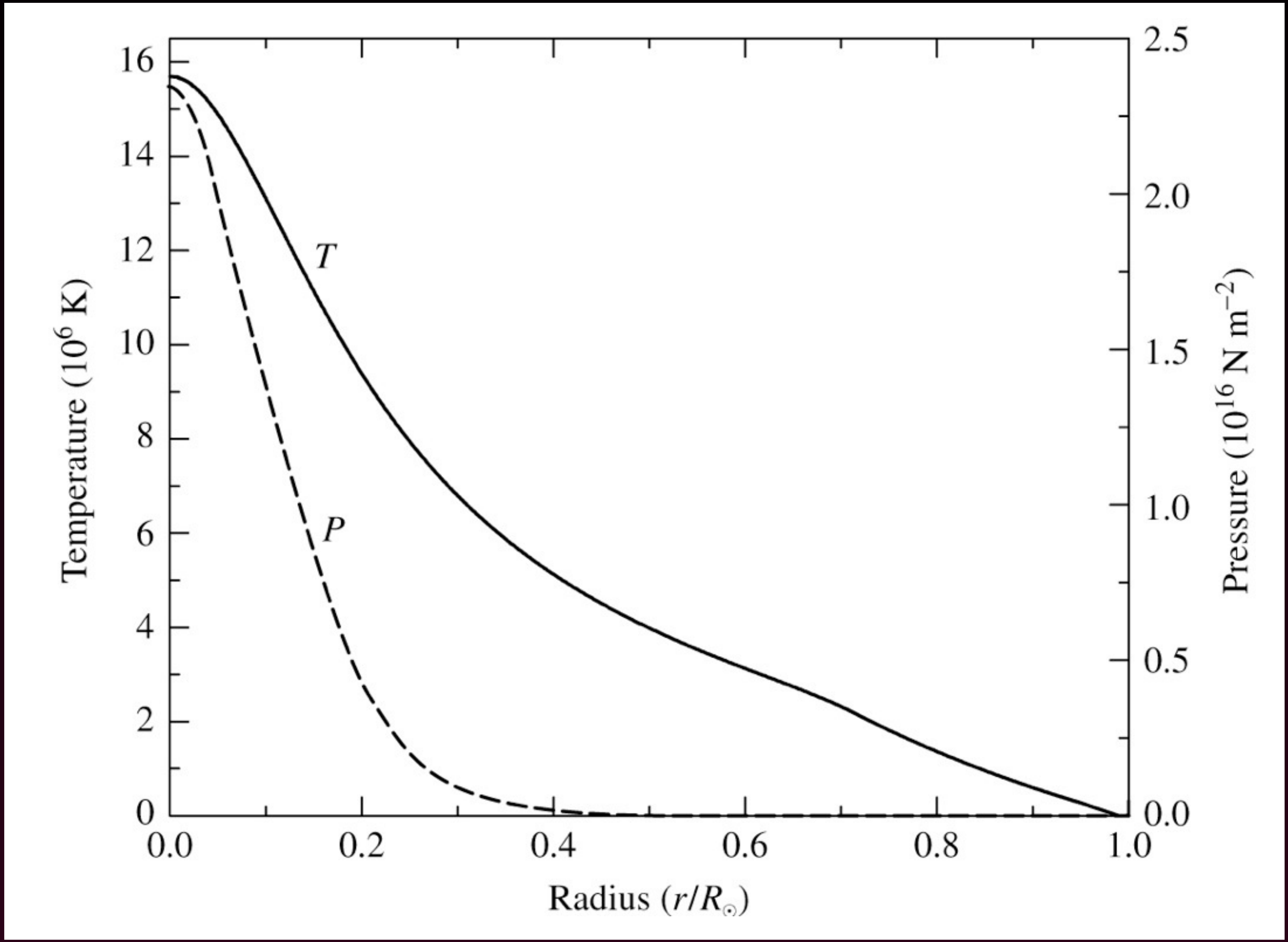
The stellar radius is defined by $P(M) = 0$, which implies $R = r(P = 0)$.

the Sun: density and enclosed mass



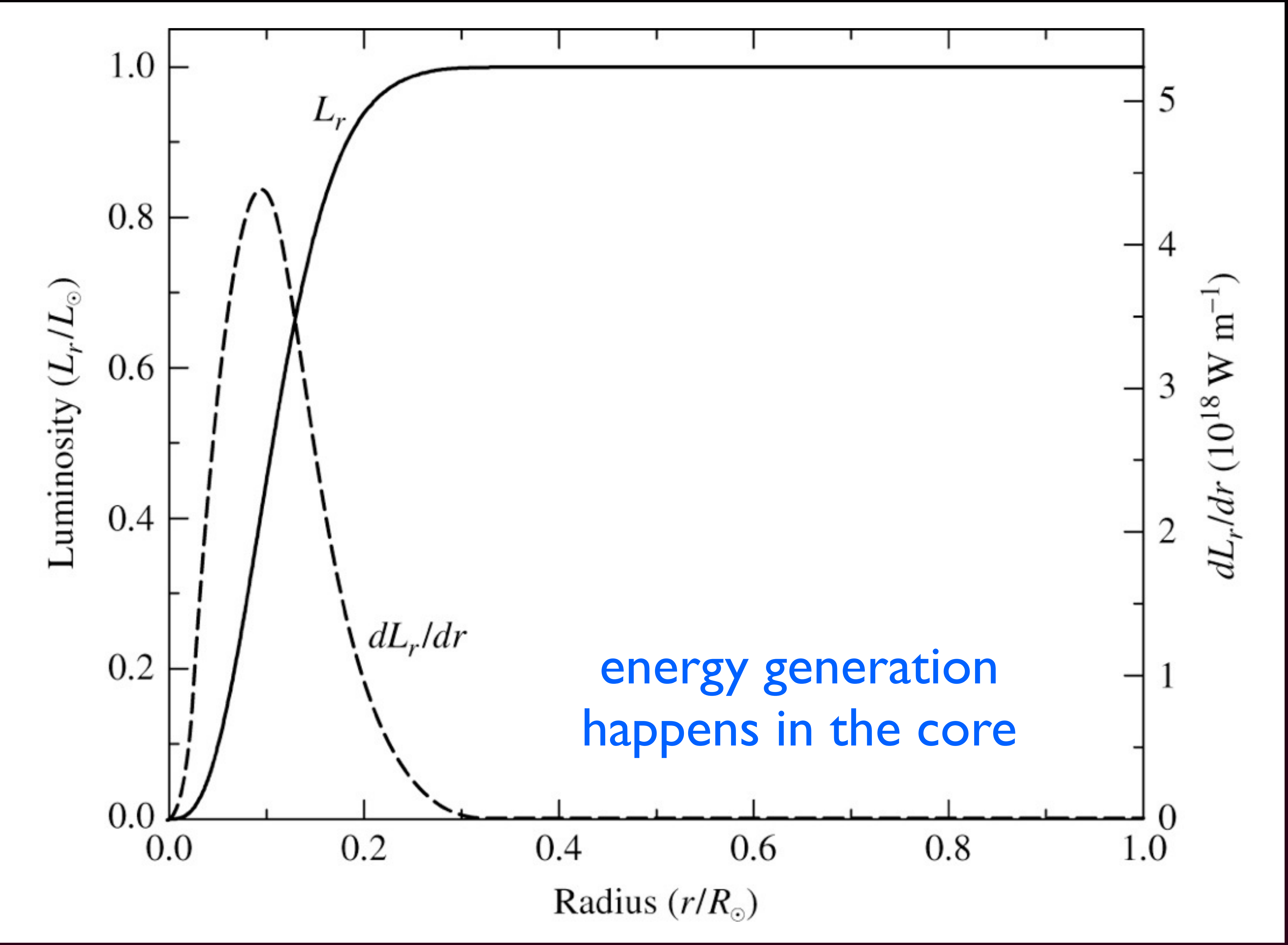
from Caroll & Ostlie, Figure 11.6

the Sun: temperature and pressure



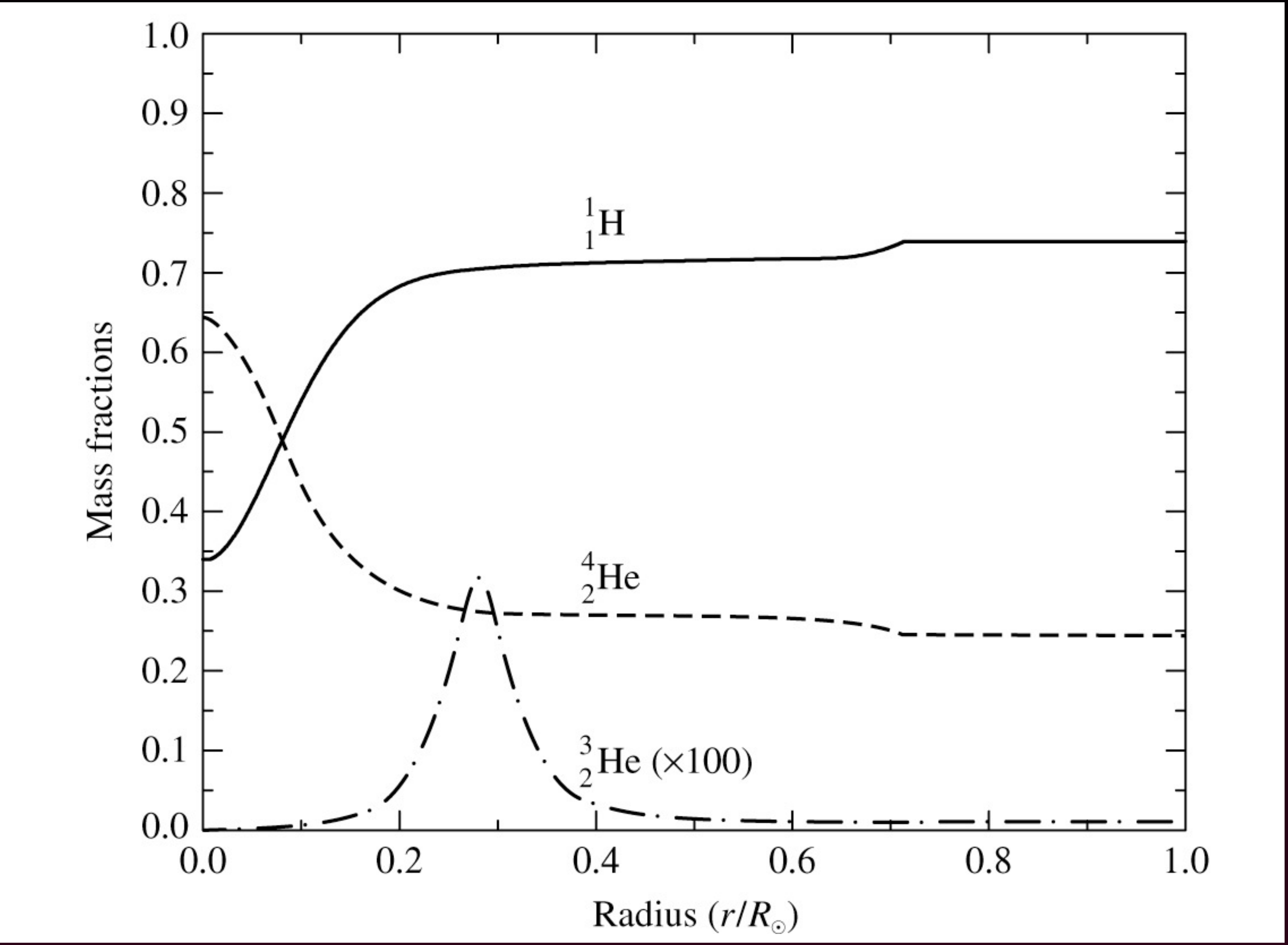
from Caroll & Ostlie, Figure 11.4

the Sun: energy generation and luminosity



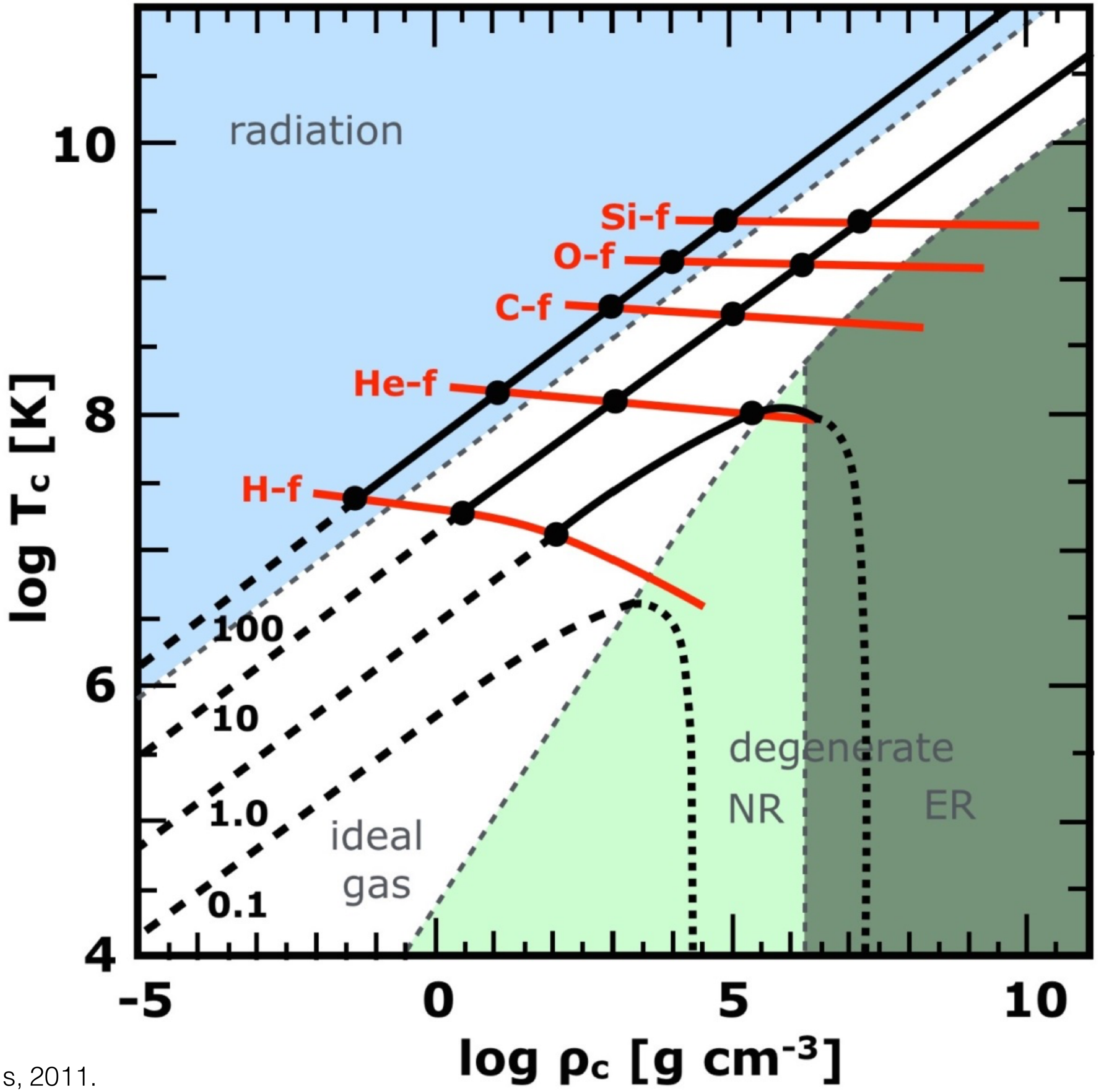
from Caroll & Ostlie, Figure 11.5

the Sun: composition



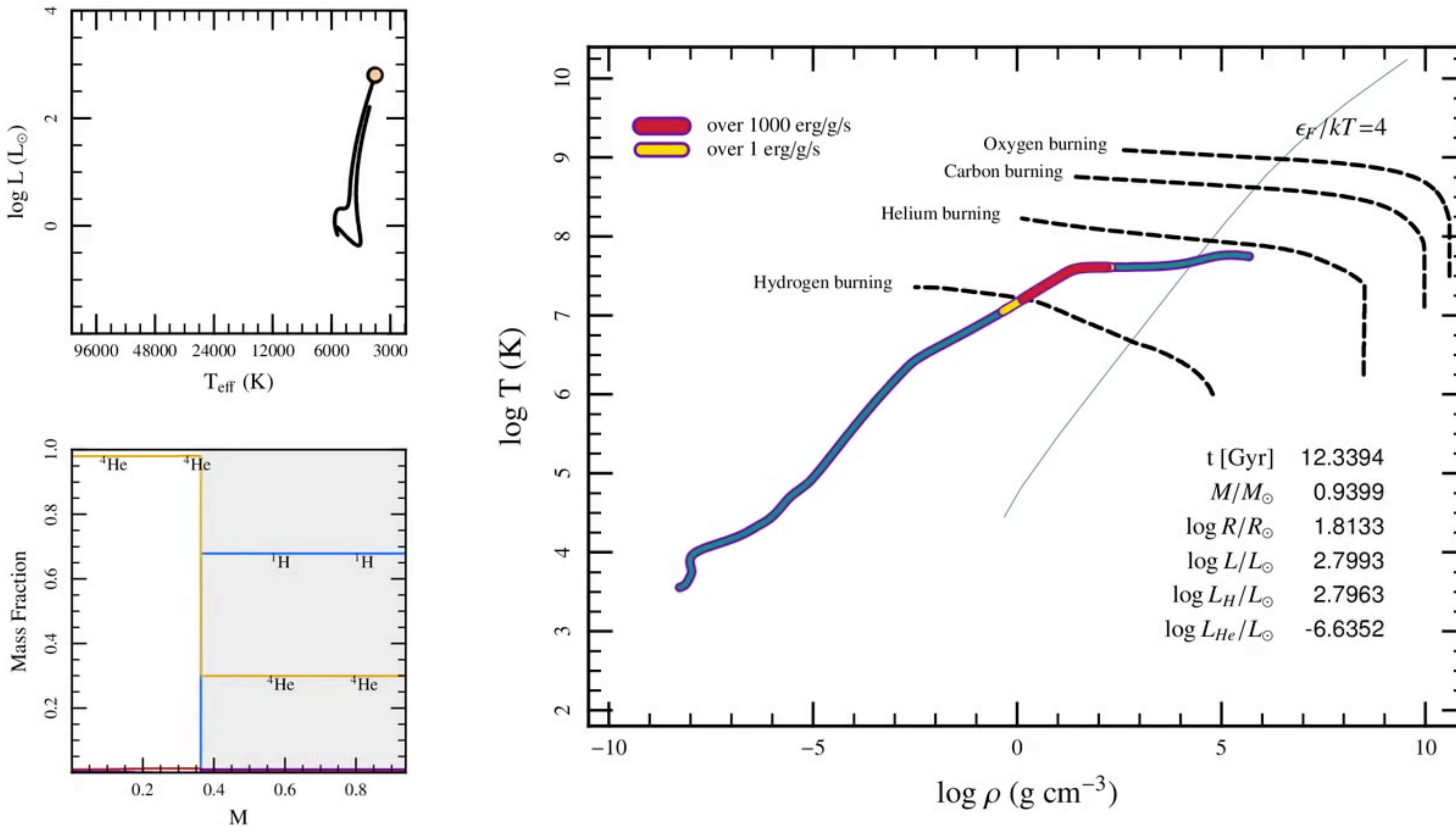
from Caroll & Ostlie, Figure 11.3

Fusion and Stellar Evolution



Adapted from Onno Pols, 2011.

MESA: open stellar modeling code



from Josiah Schwab, MESA model of a 1 solar mass star
https://www.youtube.com/watch?v=oZY3TtA63sE