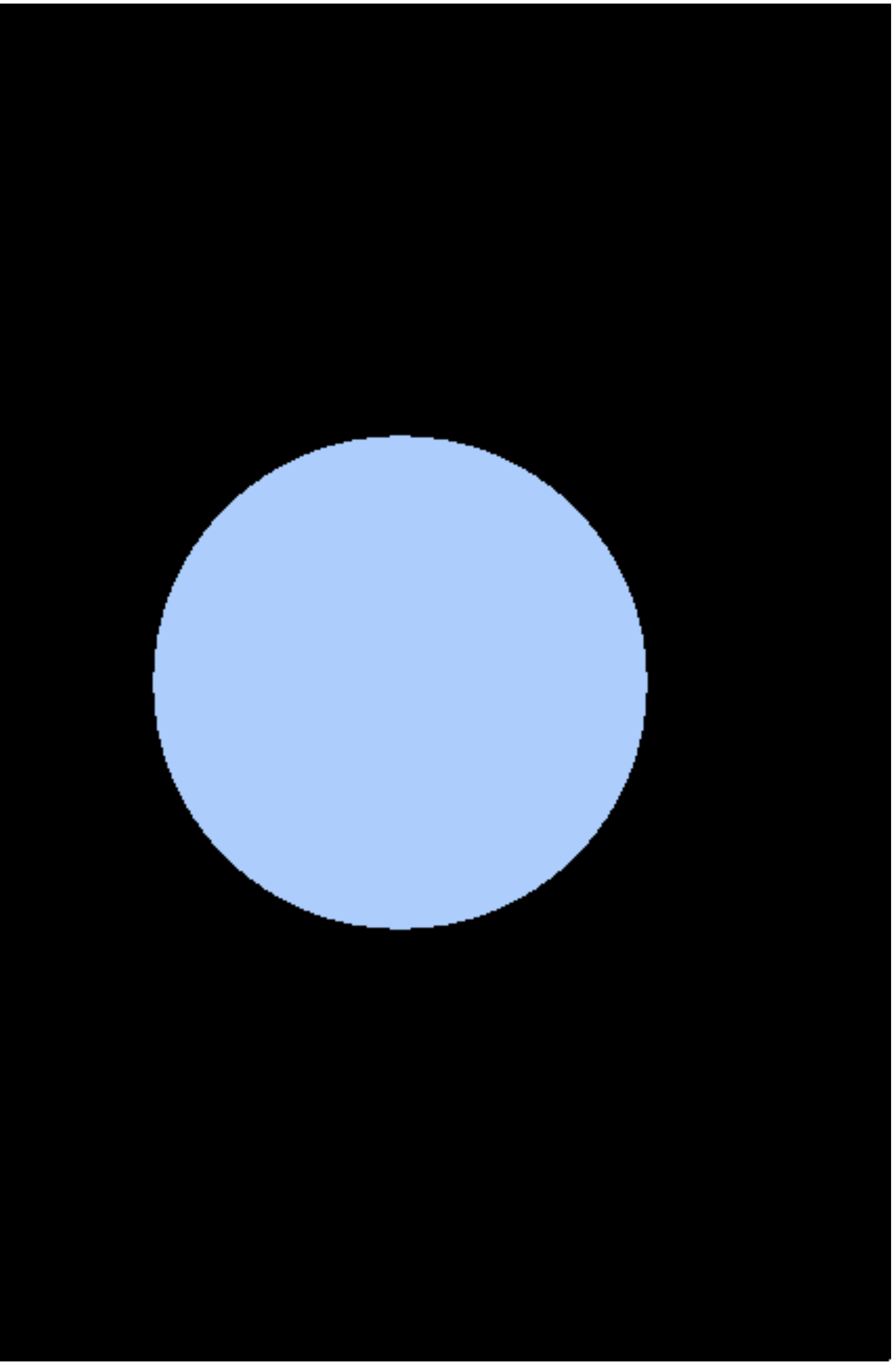
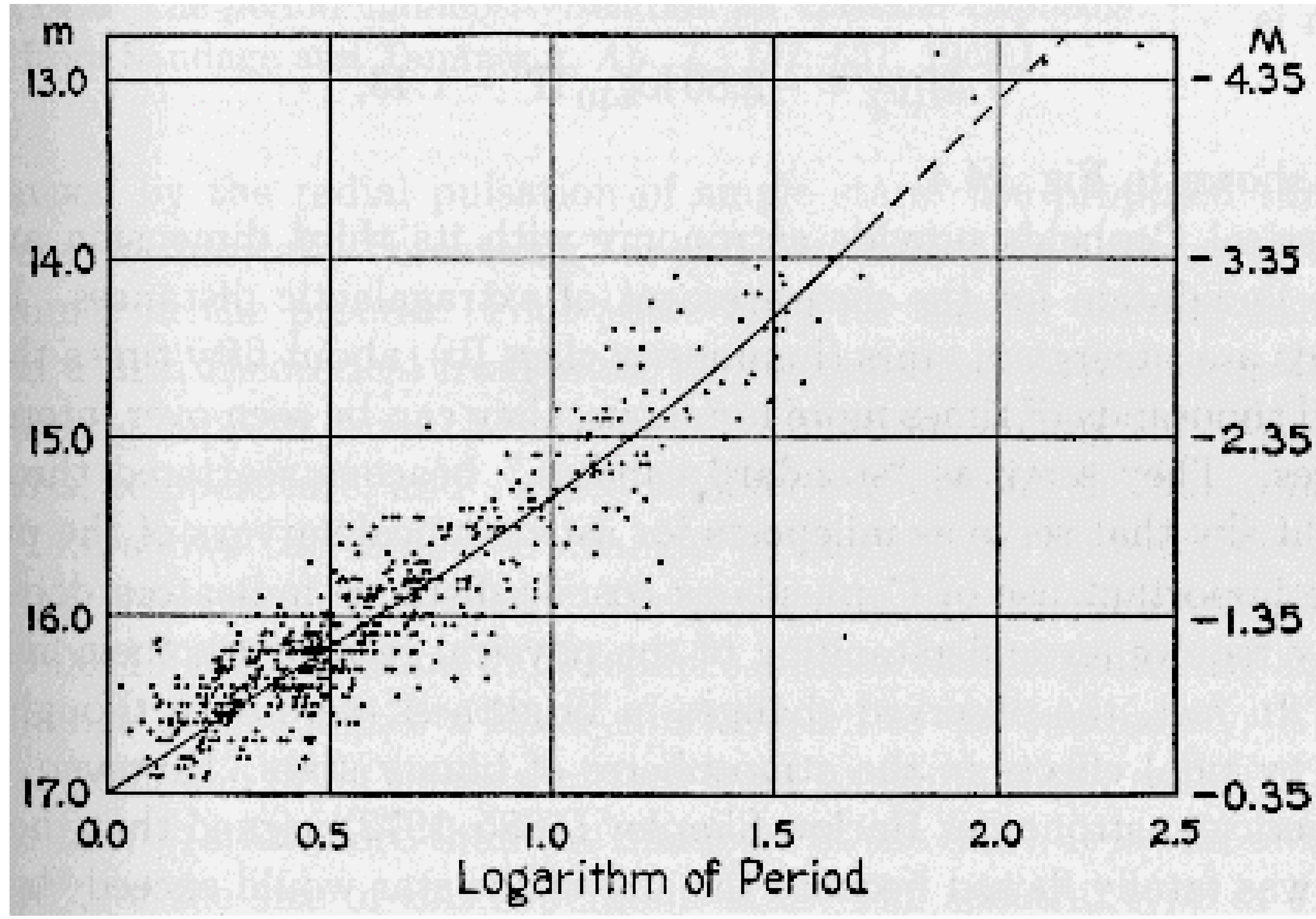
Lecture 14: Stellar Pulsation

Lamers & Levesque Ch. 21





Leavitt (1912) Law: Cepheid Period-Luminosity relationship

Midterm: in class Tuesday March 8

will cover course material through March 1st Lecture 13

things to review (roughly in order of importance):

- **problem sets 1–3 (& solutions)**
- **lecture slides**
- **Phillips Ch. 1–5 (esp. end of chapter summaries)**
- **Lamers & Levesque Ch. 1–11, 13, 14, 16–19**

exam will be 80 minutes; 441/541 have same exam

mix of conceptual questions (short answer), derivation, and some short calculations (bring a calculator)

you are allowed to bring a formula sheet:

one side only of a 8.5” x 11” (letter size) sheet of paper

Midterm: in class Tuesday March 8

some possibly useful formulas

(for which I am on purpose not providing more details)

$$\frac{dP}{dr} = -\rho(r)\,g(r) = -\frac{Gm(r)\rho(r)}{r^2} \qquad \langle P\rangle = -\frac{1}{3}\frac{E_{\rm pot}}{V} \qquad T_c \sim \frac{GM\mu m_p}{kR} \qquad P_c \sim \frac{GM^2}{R^4}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda kT) - 1} \qquad F = \sigma T^4 \qquad L = 4\pi R^2 \sigma T_{\rm eff}^4 \qquad f = L/4\pi d^2 \qquad d = 1/p$$

$$m = -2.5\log(f/f_0) \qquad \mu = m - M = 5\log(d/10\ {\rm pc}) \qquad M_{\rm bol} = -2.5\log(L/L_\odot) + 4.74$$

$$P = K\rho^\gamma \qquad \gamma = 1 + 1/n \qquad R \propto M^{(1-n)/(3-n)} \qquad E_{\rm pot} = -\frac{3}{5-n}\frac{GM^2}{R}$$

$$P = \frac{1}{3}aT^4 \qquad P = nkT = \frac{\rho kT}{\mu m_p} \qquad P = \frac{h^2}{5m_e}\left[\frac{3}{8\pi}\right]^{2/3} n_e^{5/3} \qquad P = \frac{hc}{4}\left[\frac{3}{8\pi}\right]^{1/3} n_e^{4/3}$$

$$E_n = -13.6\ {\rm eV}\left(\frac{Z^2}{n^2}\right) \qquad g_n = 2n^2 \qquad \frac{n_m}{n_n} = \frac{g_m}{g_n}\exp\left(-\frac{E_m - E_n}{kT}\right)$$

$$\frac{n_{II}}{n_I} = \frac{2Z_{II}}{n_e Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\exp\left(-\frac{\chi_I}{kT}\right) \qquad \left|\frac{dT}{dr}\right| > \frac{\gamma - 1}{\gamma}\frac{T}{P}\left|\frac{dP}{dr}\right| = \frac{\gamma - 1}{\gamma}\frac{\mu m_p}{k}g$$

$$\ell = \frac{1}{n\sigma} = \frac{1}{\rho\kappa} \qquad \frac{dT}{dr} = -\frac{3}{4ac}\frac{\rho\kappa}{T^3}\frac{L}{4\pi r^2} \qquad E = \Delta mc^2 = \epsilon Mc^2 \qquad E_G = (\pi\alpha Z_A Z_B)^2\, 2m_r c^2$$

$$\text{probability} \approx \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right] \qquad E_0 = \left[\frac{E_G(kT)^2}{4}\right]^{1/3} \qquad \alpha = \left(\frac{E_G}{4kT}\right)^{1/3} - \frac{2}{3}$$

$$R_{AB} = n_A n_B\left[\frac{8}{\pi m_r}\right]^{1/2}\left[\frac{1}{kT}\right]^{3/2}\int_0^\infty S(E)\exp\left[-\frac{E}{kT} - \left(\frac{E_G}{E}\right)^{1/2}\right]dE \qquad R_{AB} \propto T^\alpha$$

$$\frac{dm}{dr} = 4\pi r^2\rho \qquad \frac{dL}{dr} = 4\pi r^2\rho\epsilon \qquad \kappa_{\rm es} = \frac{n_e\sigma_T}{\rho} \approx 0.2(1+X)\ {\rm cm^2\ g^{-1}} \qquad L_{\rm Edd} = \frac{4\pi GMm_p c}{\sigma_T}$$

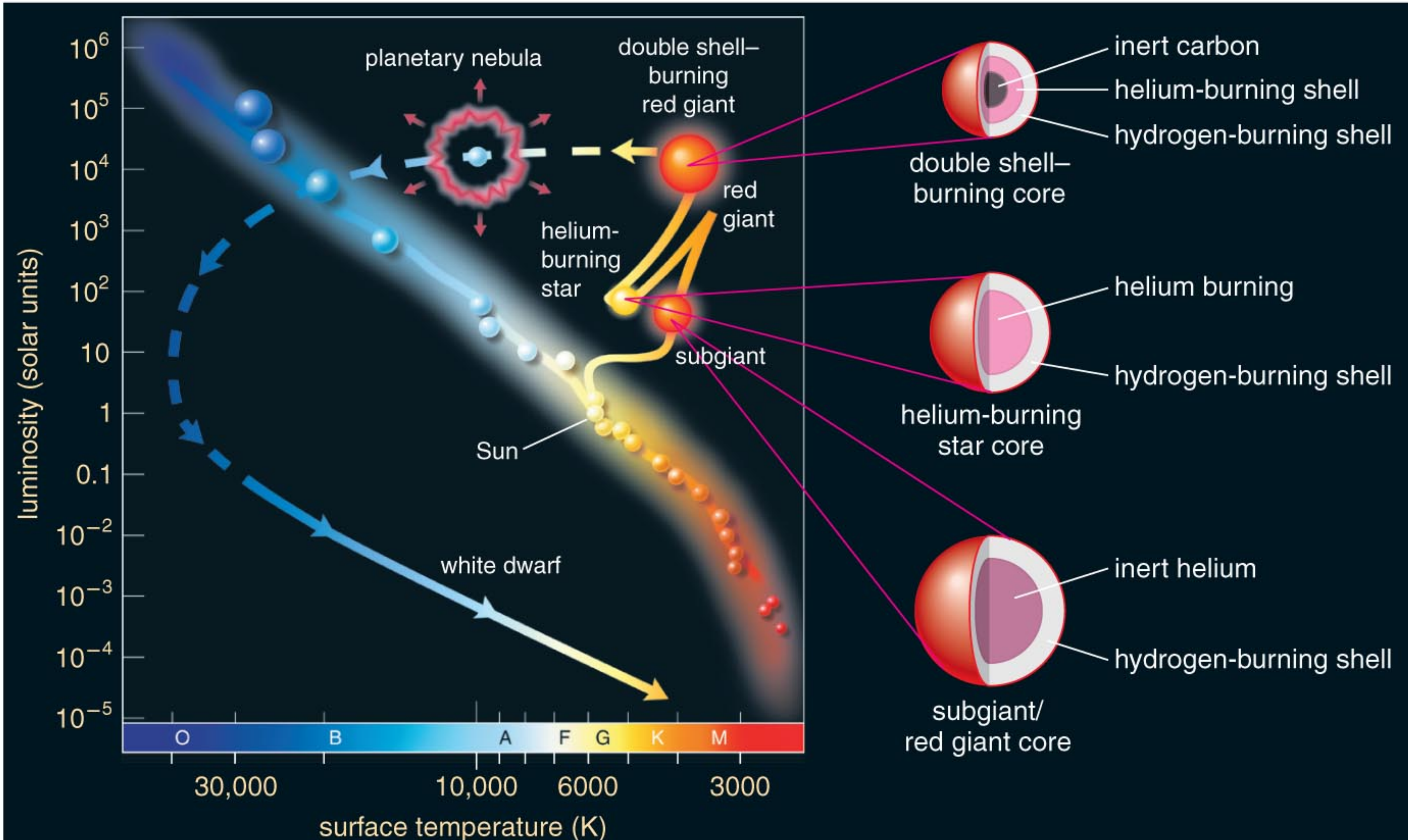
$$t_{\rm dyn} = \frac{1}{\sqrt{G\rho}} \qquad t_{\rm KH} = \frac{GM^2}{RL} \qquad t_{\rm nuc} = \frac{E_{\rm nuc}}{L} \approx \frac{f_M\epsilon Mc^2}{L} \qquad t_{\rm dyn} \ll t_{\rm conv} \ll t_{\rm KH} \ll t_{\rm nuc}$$

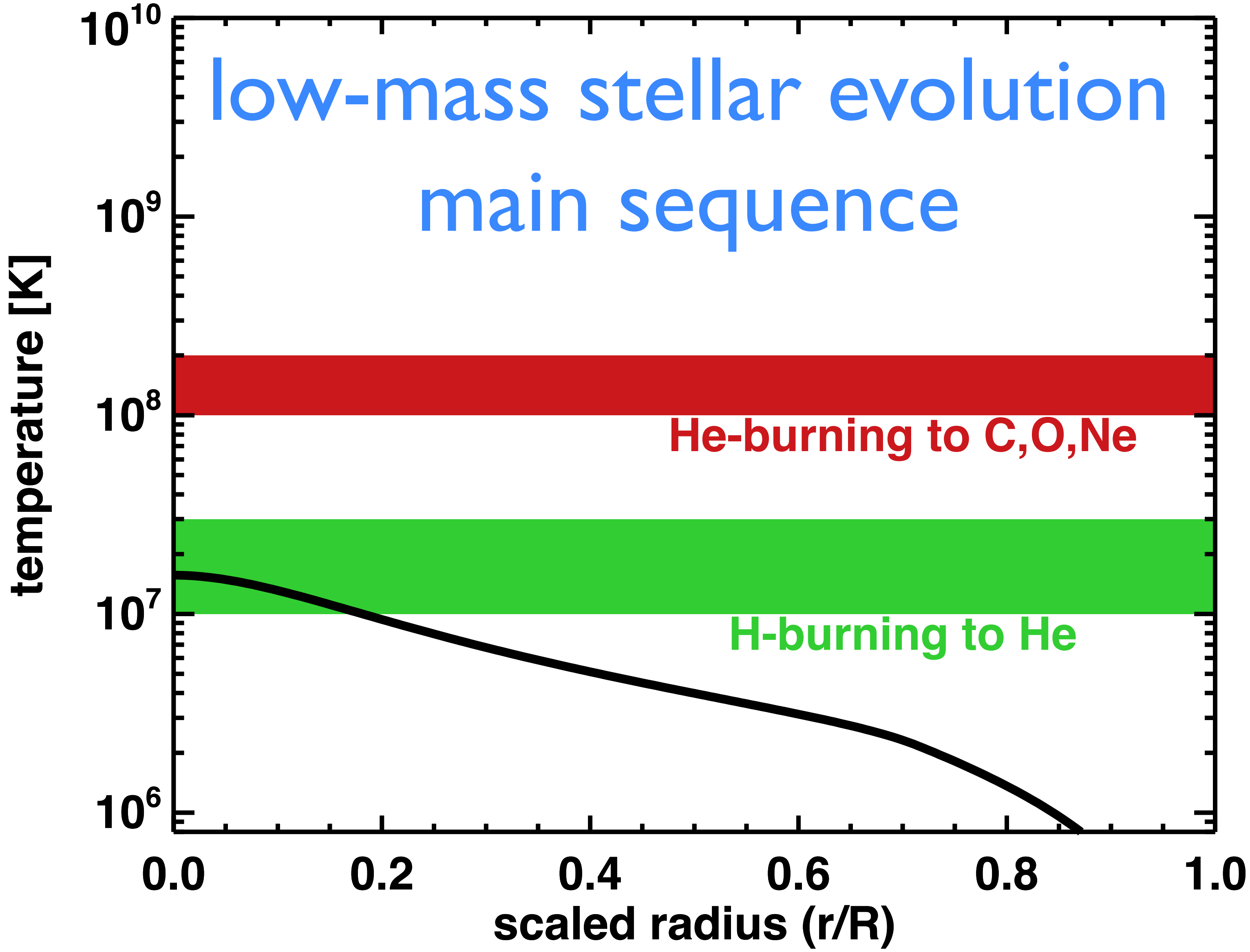
These are the formulas I will provide.

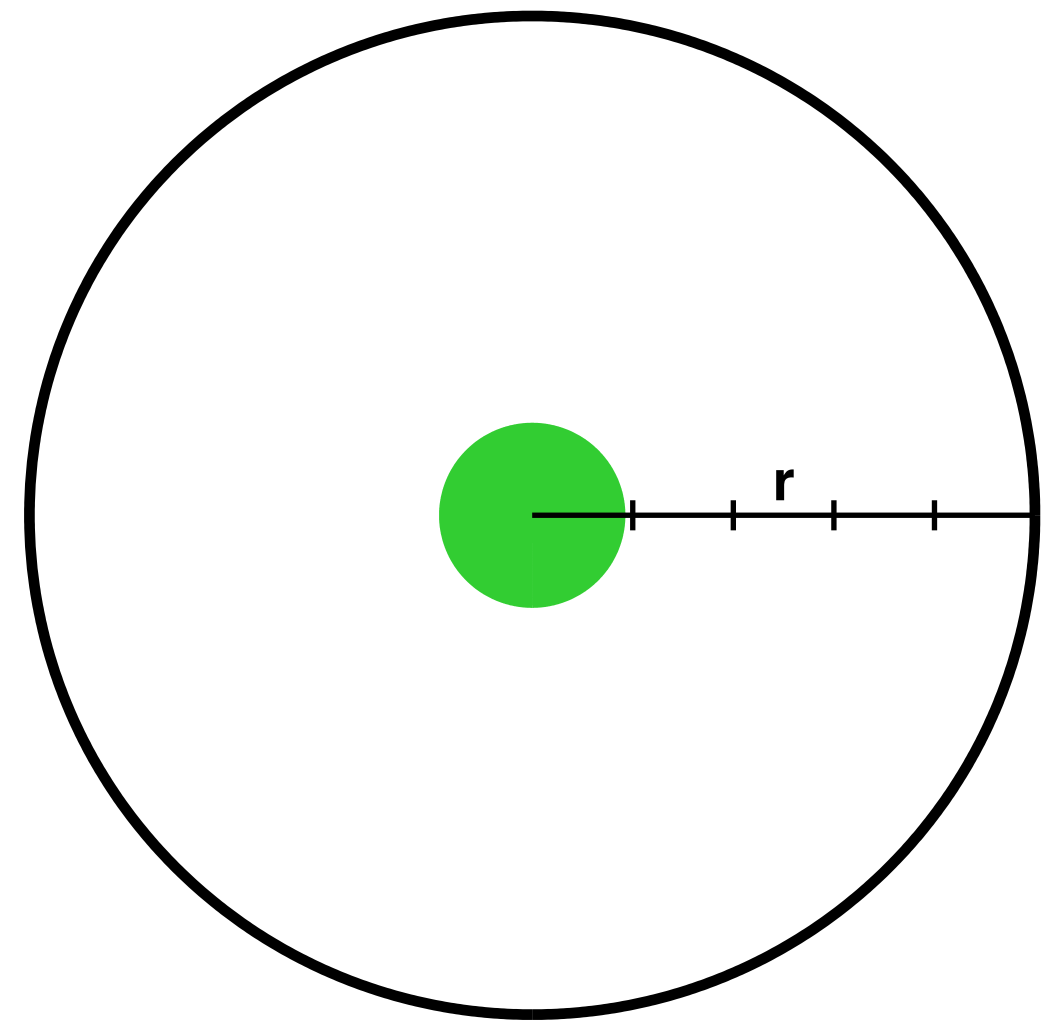
In addition to these, you can bring one side of one letter-sized sheet of paper with your own notes or formulas.

This file is uploaded on Canvas under Files → Exam Resources

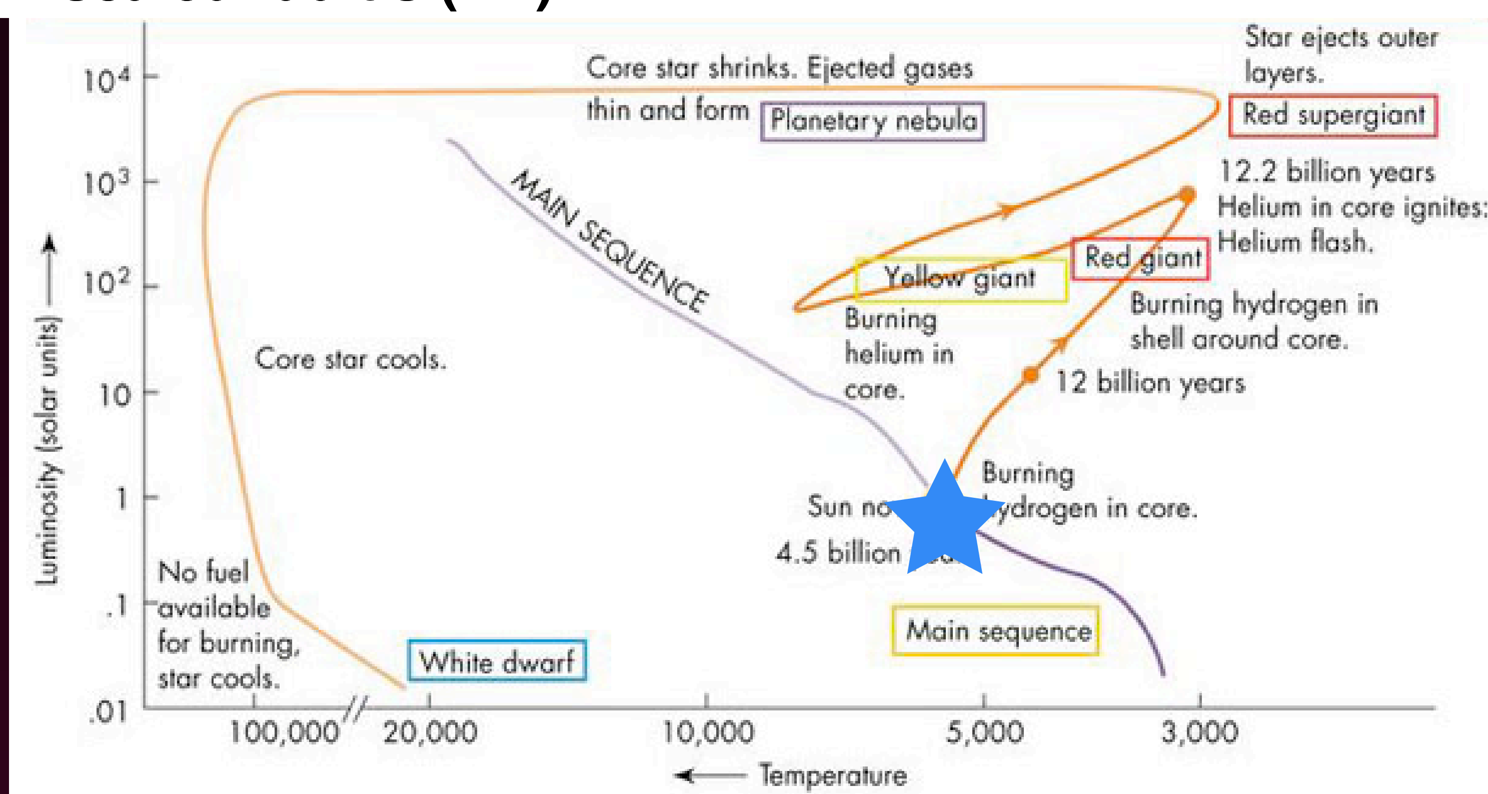
Life Track of a Sun-like Star

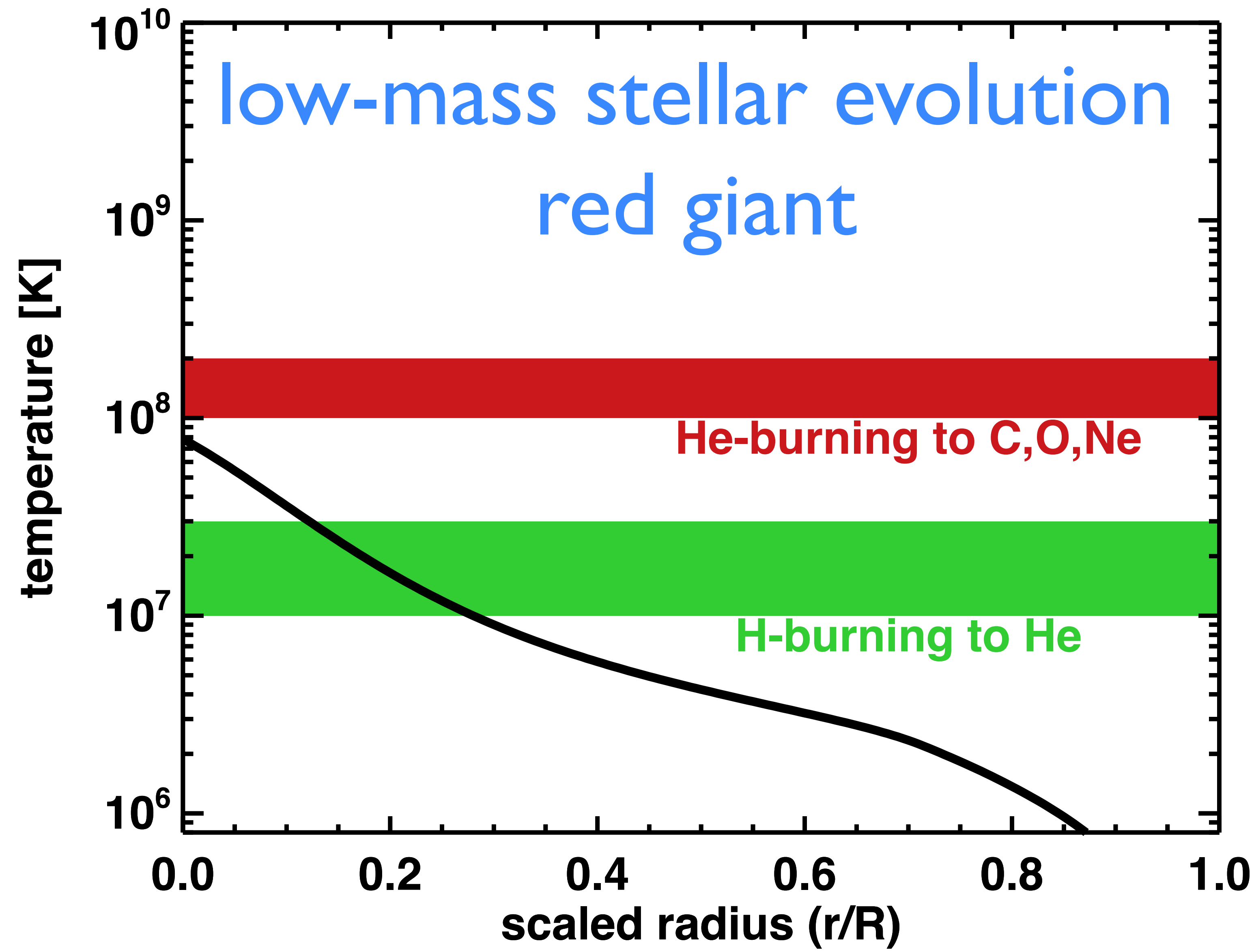


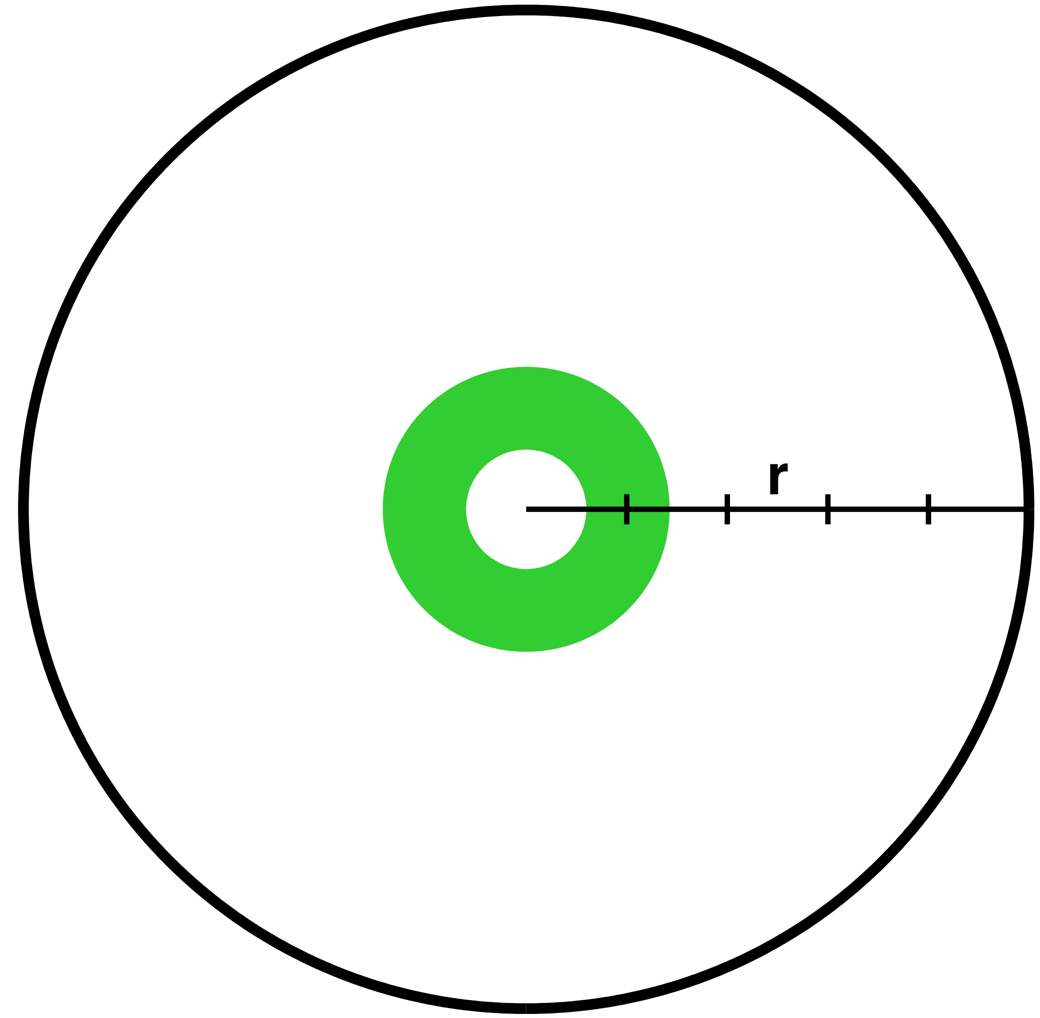




schematic, not to scale!







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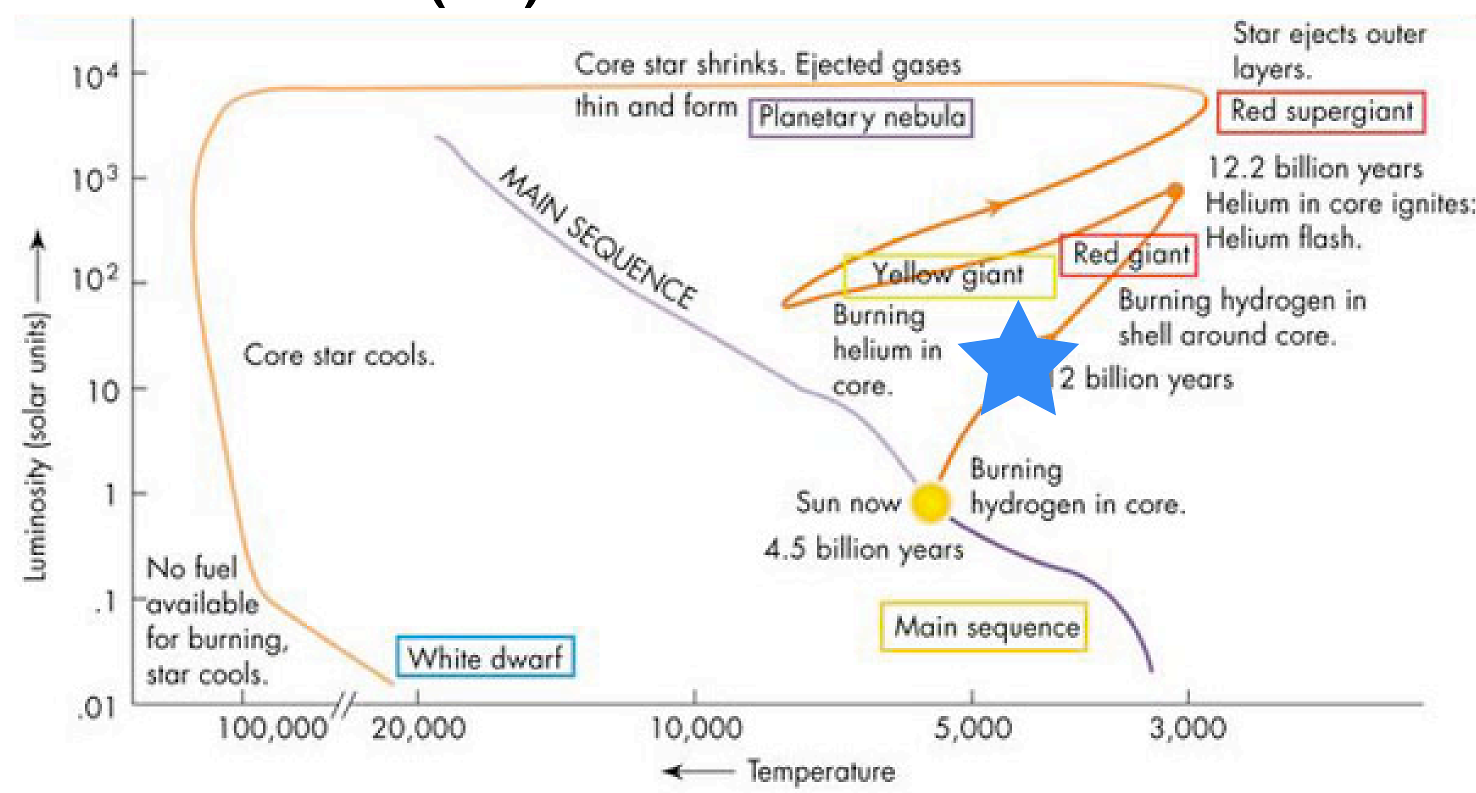

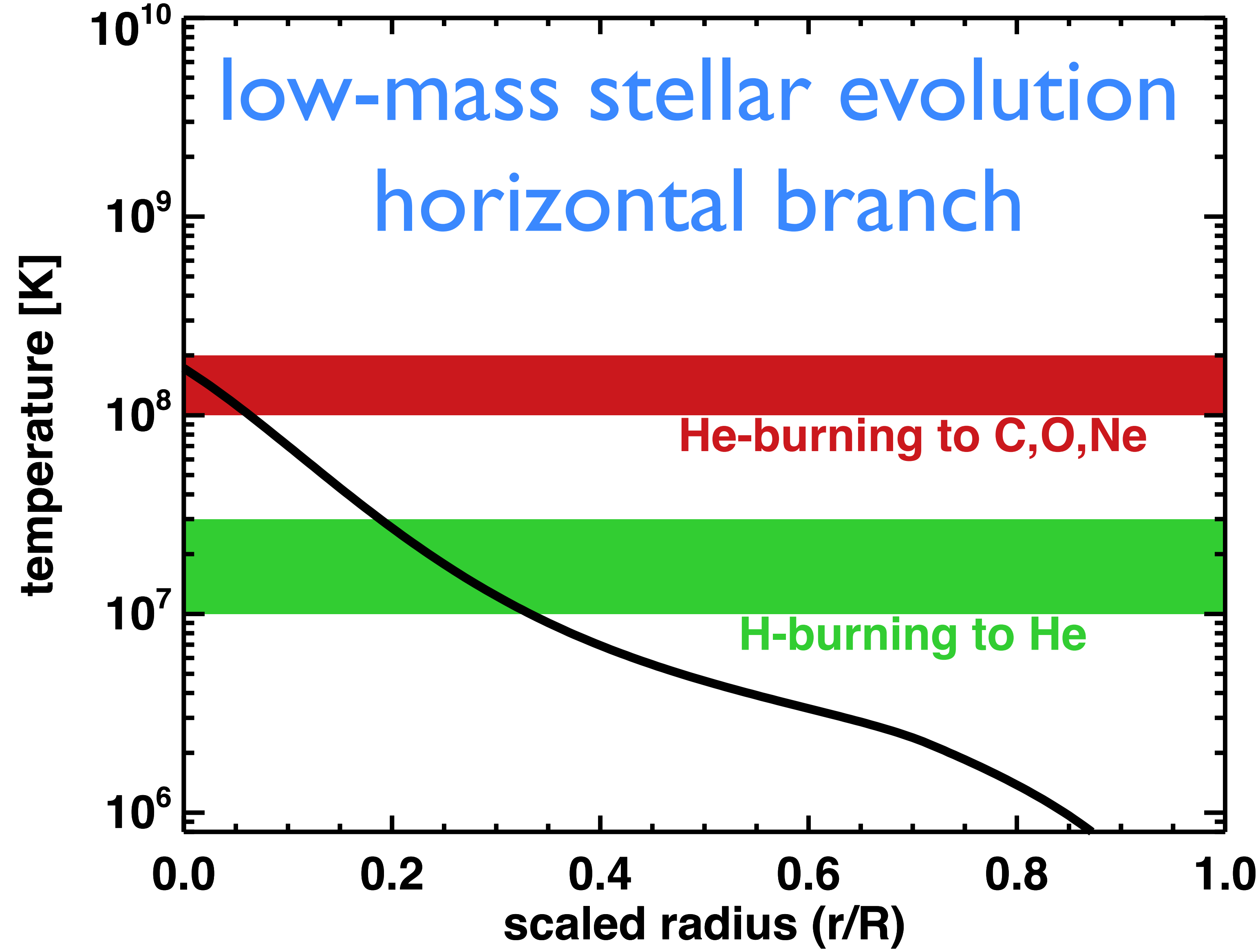

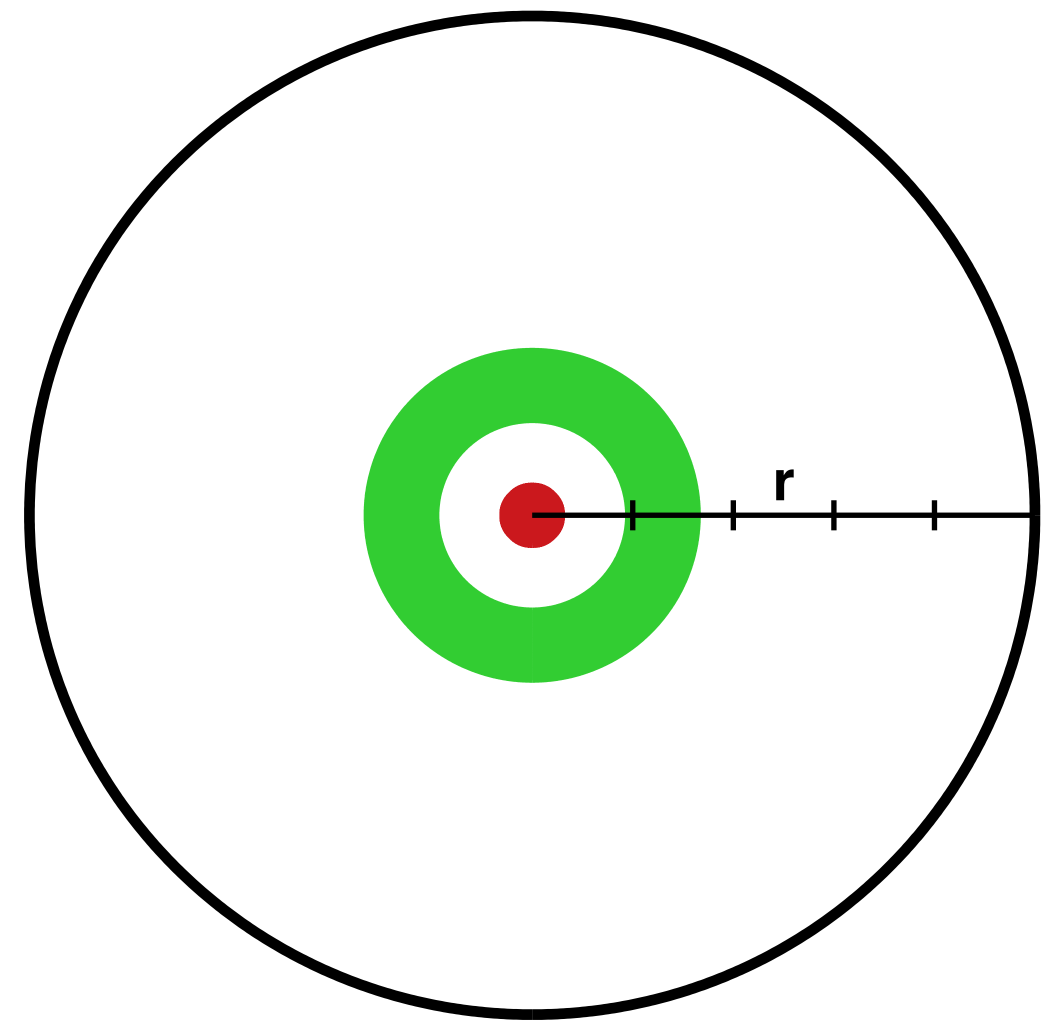

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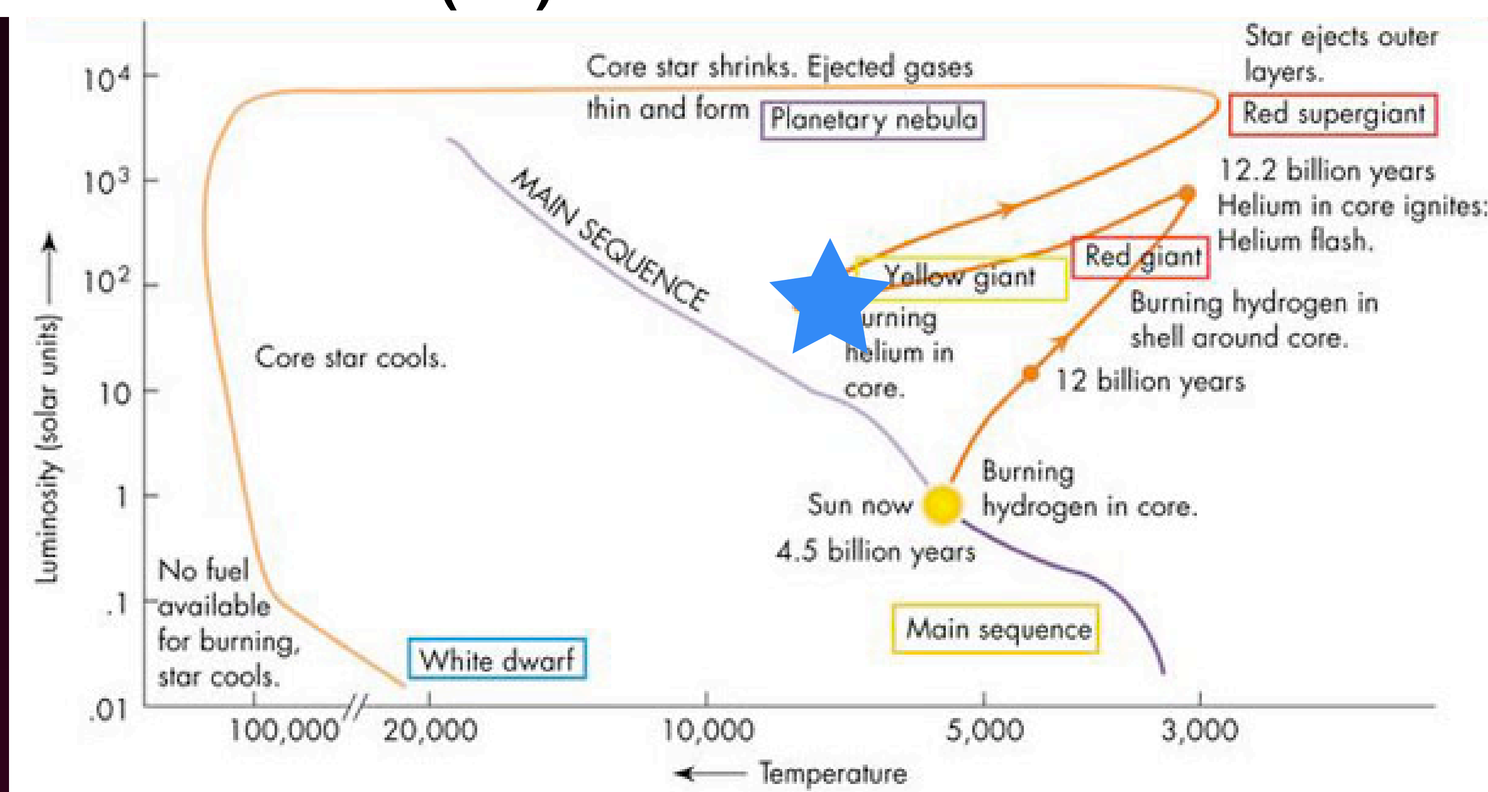

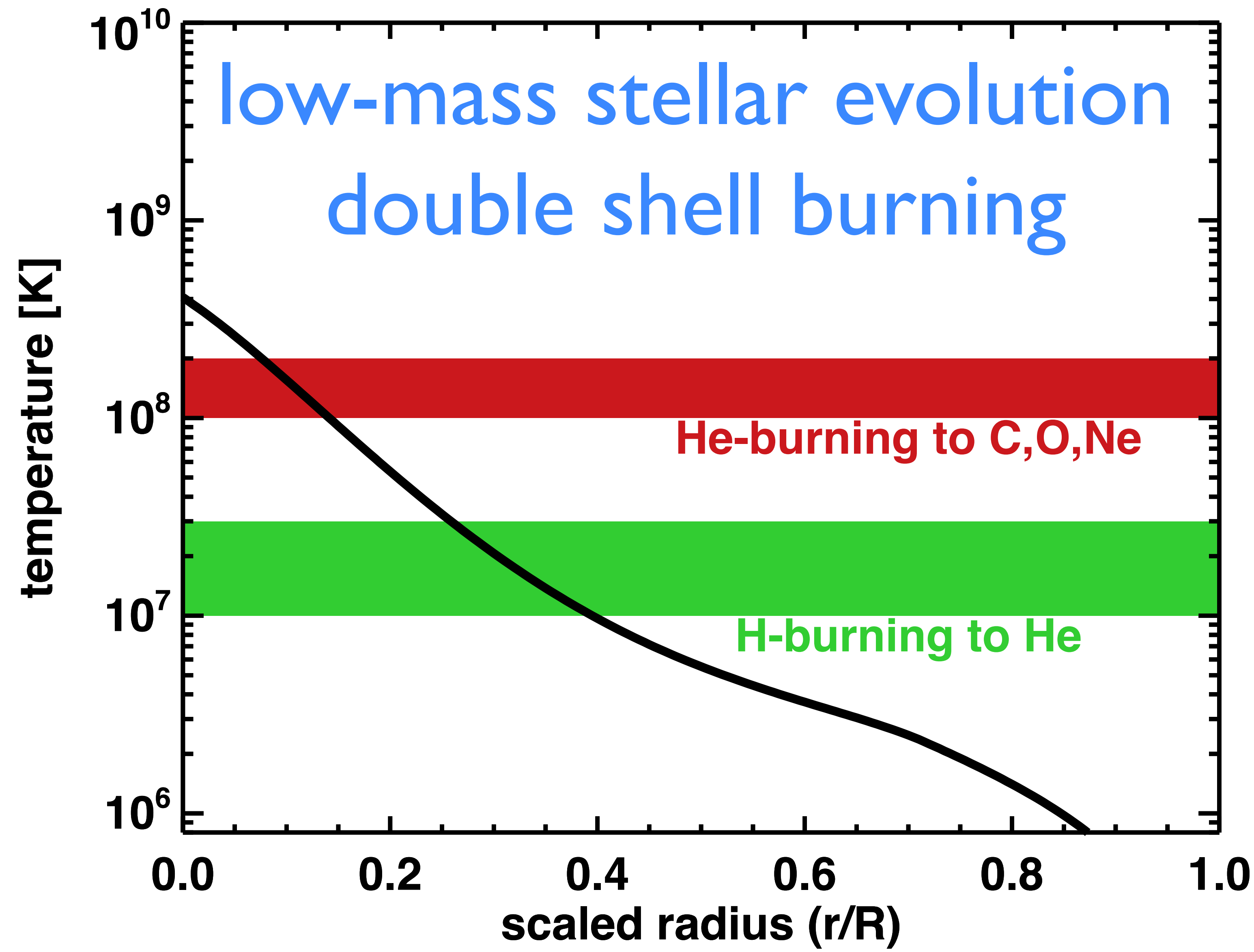

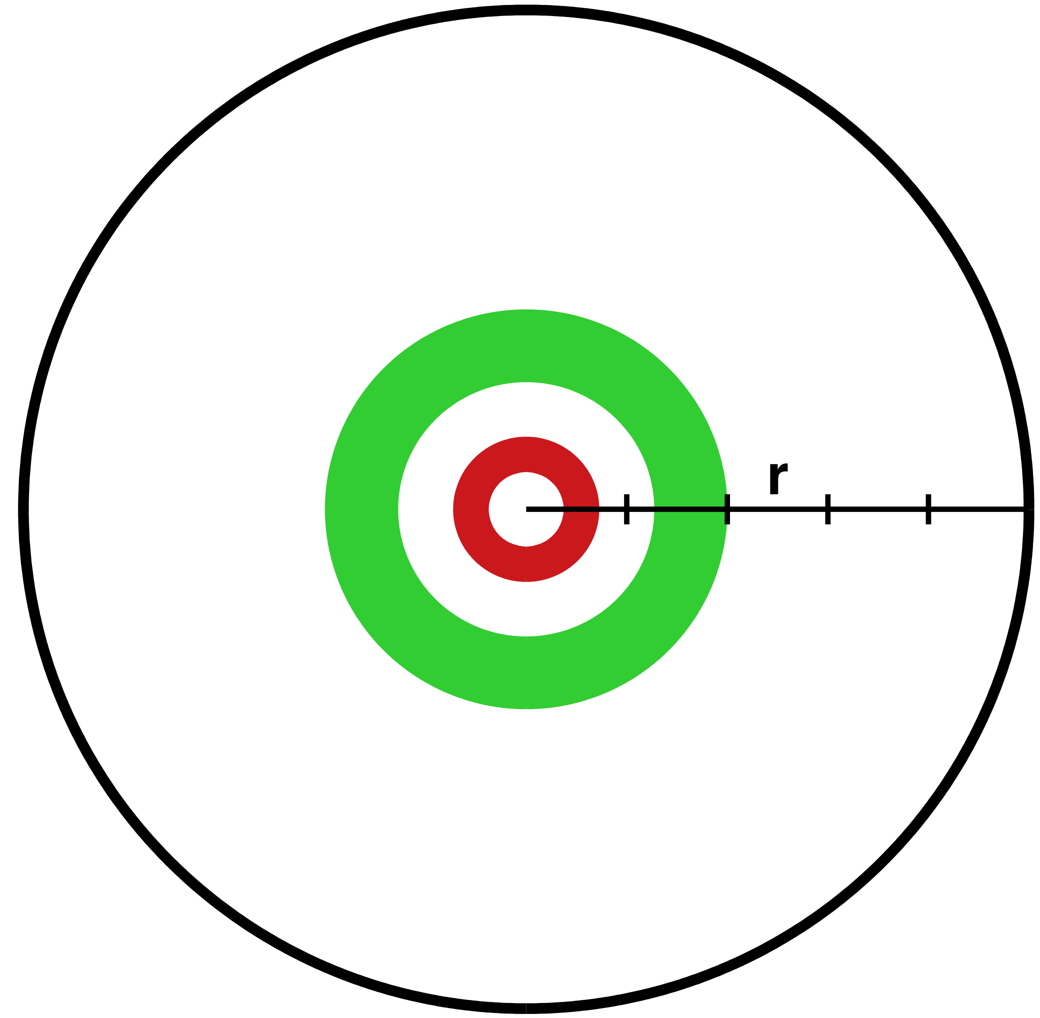

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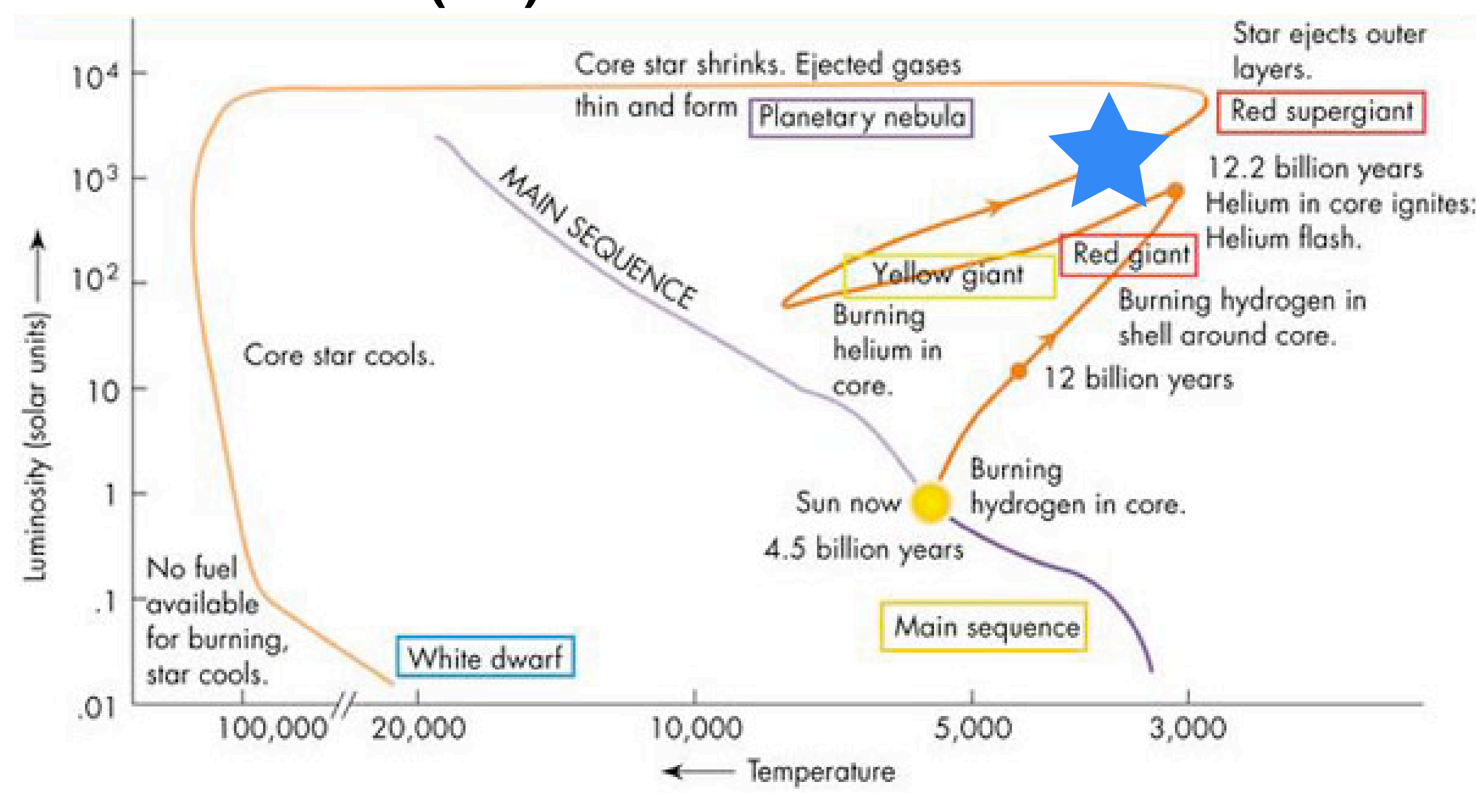
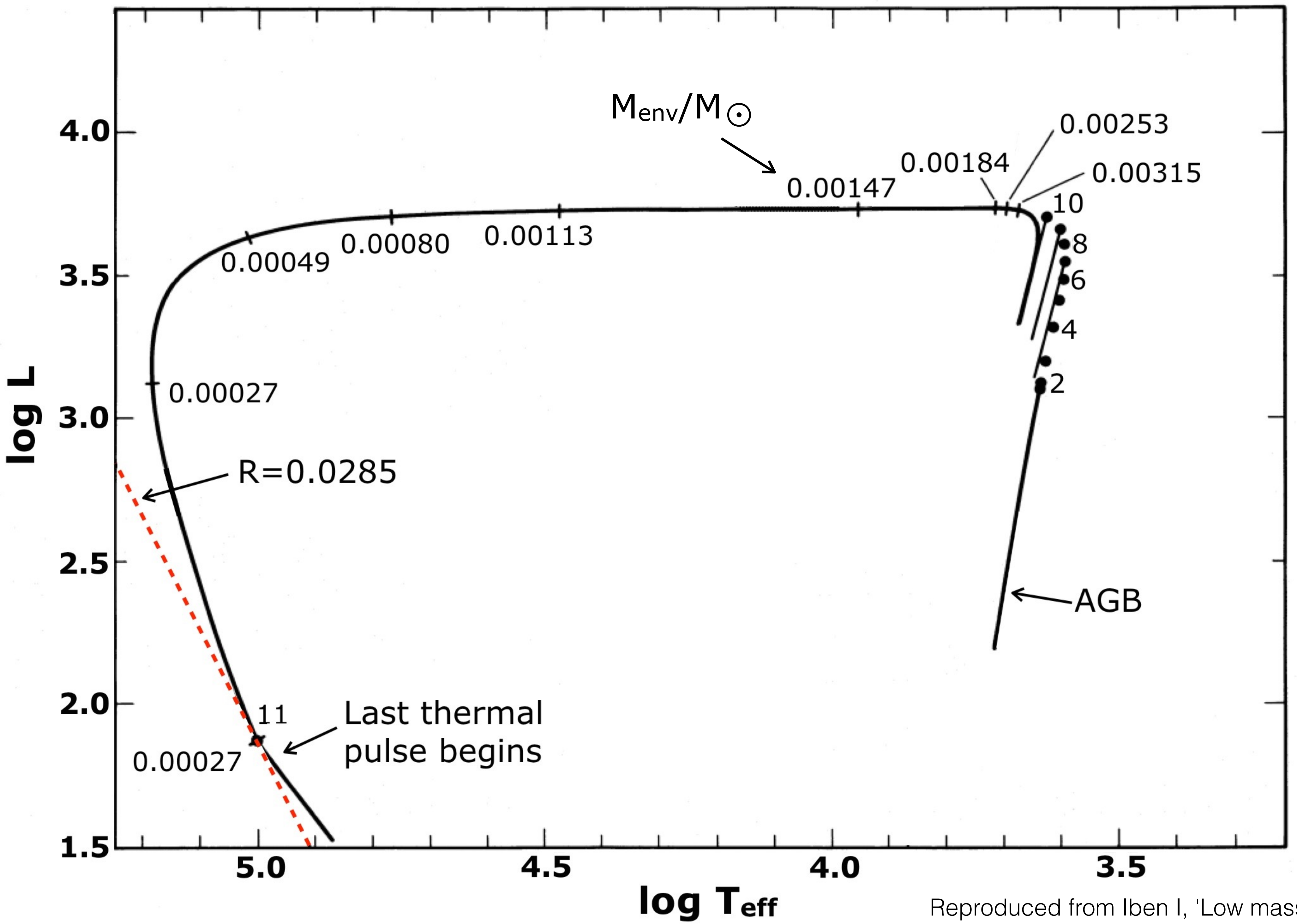


Post-AGB Evolution

A star leaves the AGB when the H-envelope mass has decreased to ~10^{-2}-10^{-3} M⊙. The envelope shrinks…



Reproduced from Iben I, 'Low mass asymptotic giant branch evolution. I', Astrophysical Journal, Part 1, vol. 260, p. 821-837.1982.

t=0

T_{eff}~3,000 K

R~3R⊙

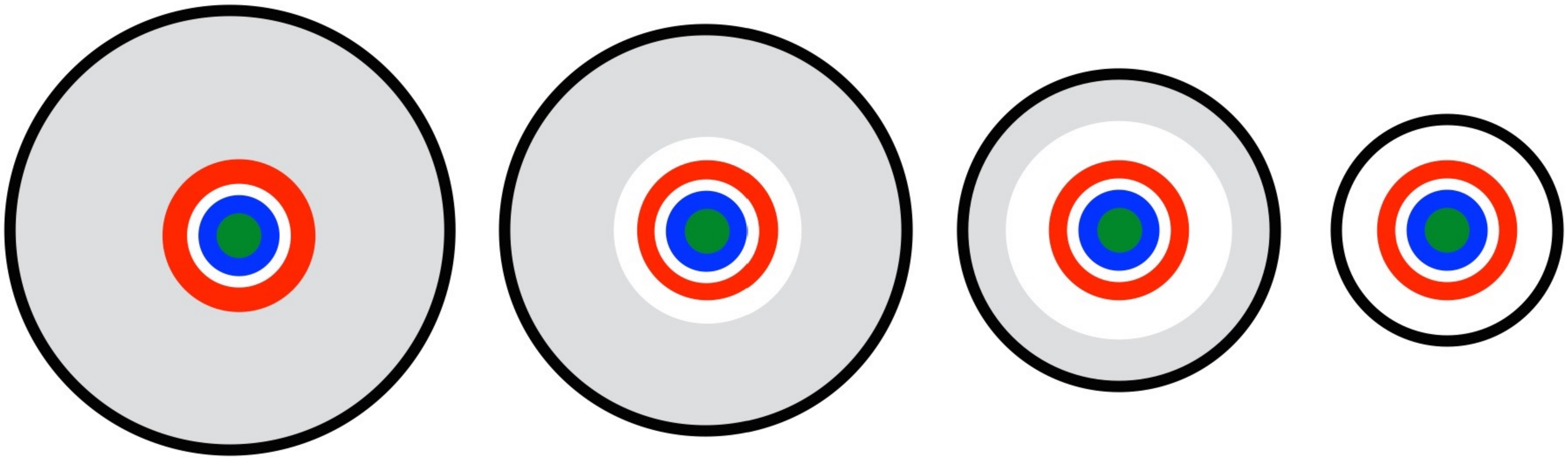
M_{env}=0.003M⊙

The star now moves from the AGB phase to the central star of a PN…

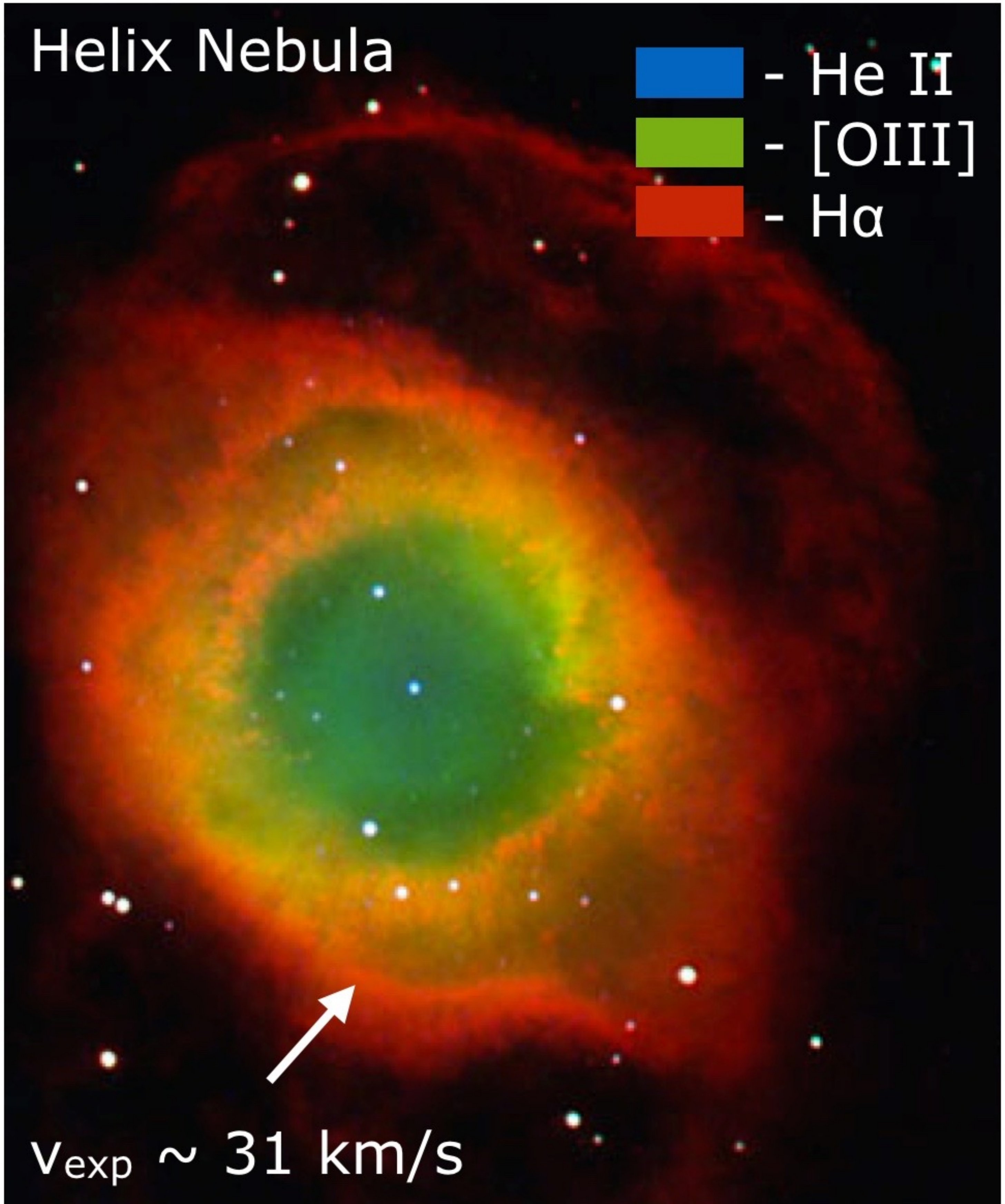
Post-AGB Evolution

A star leaves the AGB when the H-envelope mass has decreased to ~10^{-2}-10^{-3} M$_\odot$. The envelope shrinks…

Phase	AGB	post-AGB	CSPN	end of HRD crossing
$M_{env}/M_\odot$	$\sim 3\times10^{-3}$	2×10^{-3}	10^{-3}	3×10^{-4}
$R/R_\odot$	150	25	2.7	0.25
t (yr)	0.0	1×10^{3}	2×10^{3}	7×10^{3}



Planetary Nebulae



The newly-UV-bright central star ionizes the planetary nebula?

Expansion speed of PN is much faster than the velocity of ejected AGB winds: *shocks* from CS wind

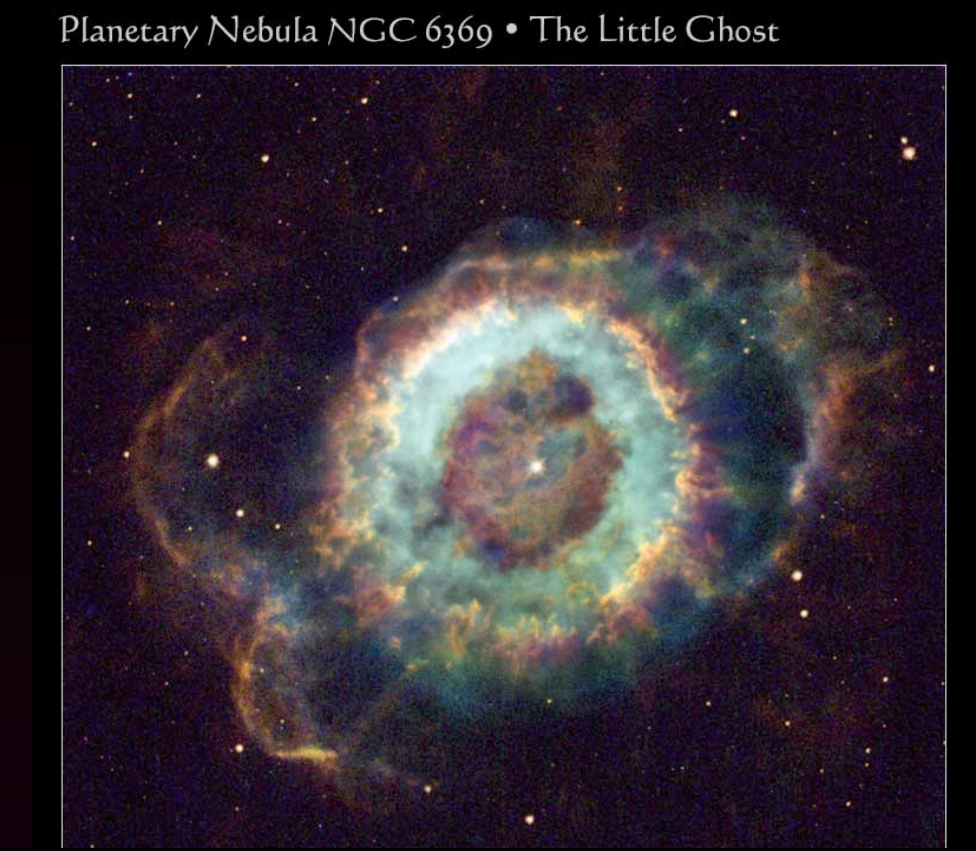
— mass of the PN after

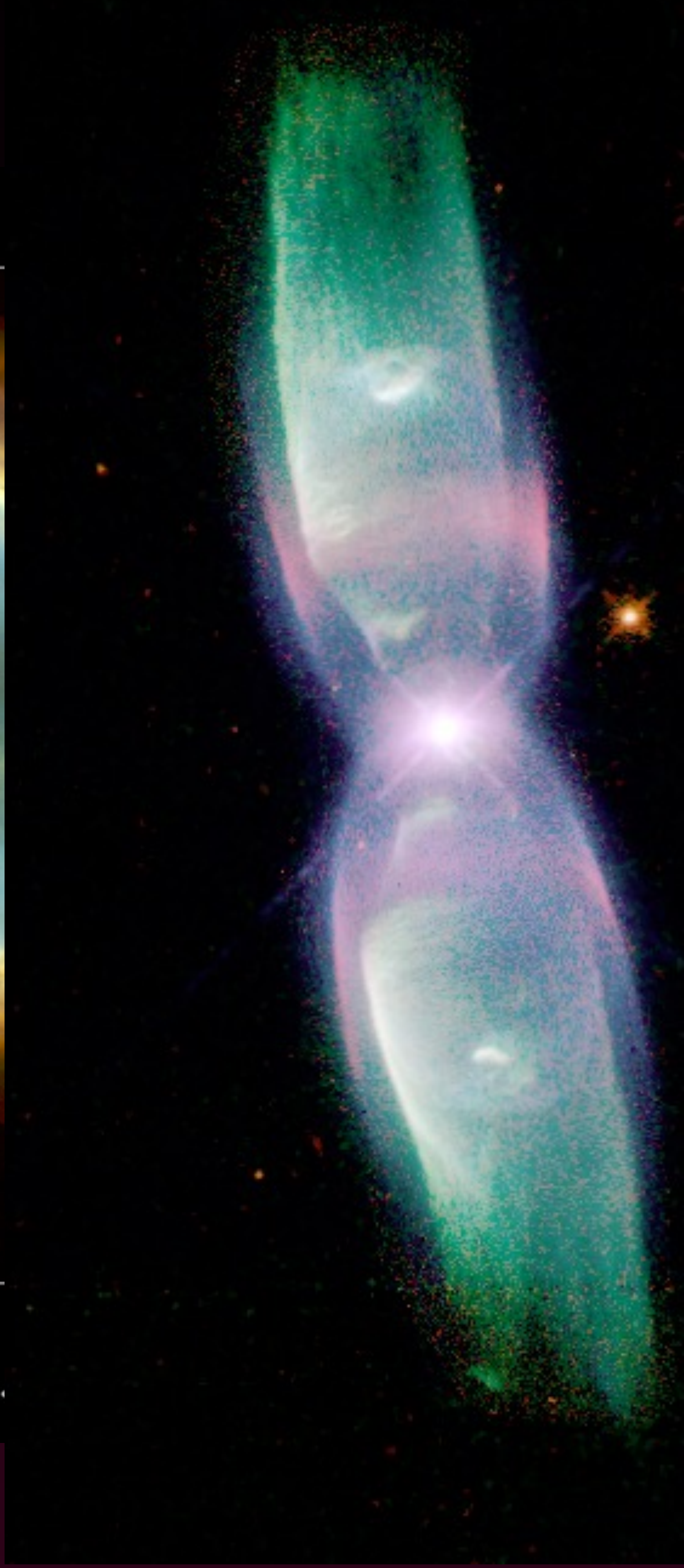
10,000y is only ~0.2$M_\odot$
— expansion velocity of PN is ~30-50 km/s
— the size after 10,000y is ~0.2-0.5pc

(morphology impacted by rotation, non-spherical winds, binarity, B fields...)

Courtesy of NASA.

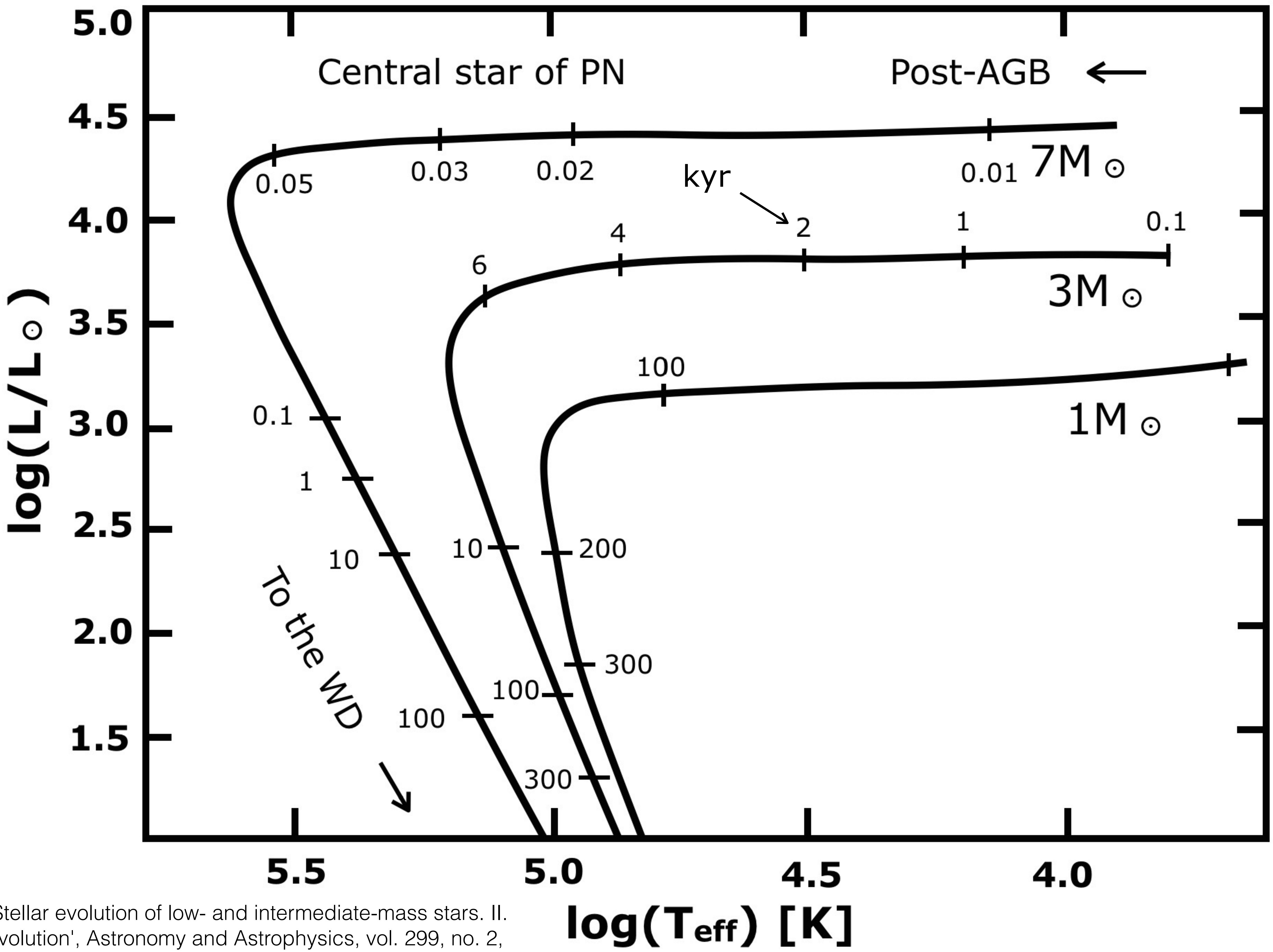
planetary nebulae





planetary nebulae observed by the Hubble Space Telescope

Planetary Nebulae



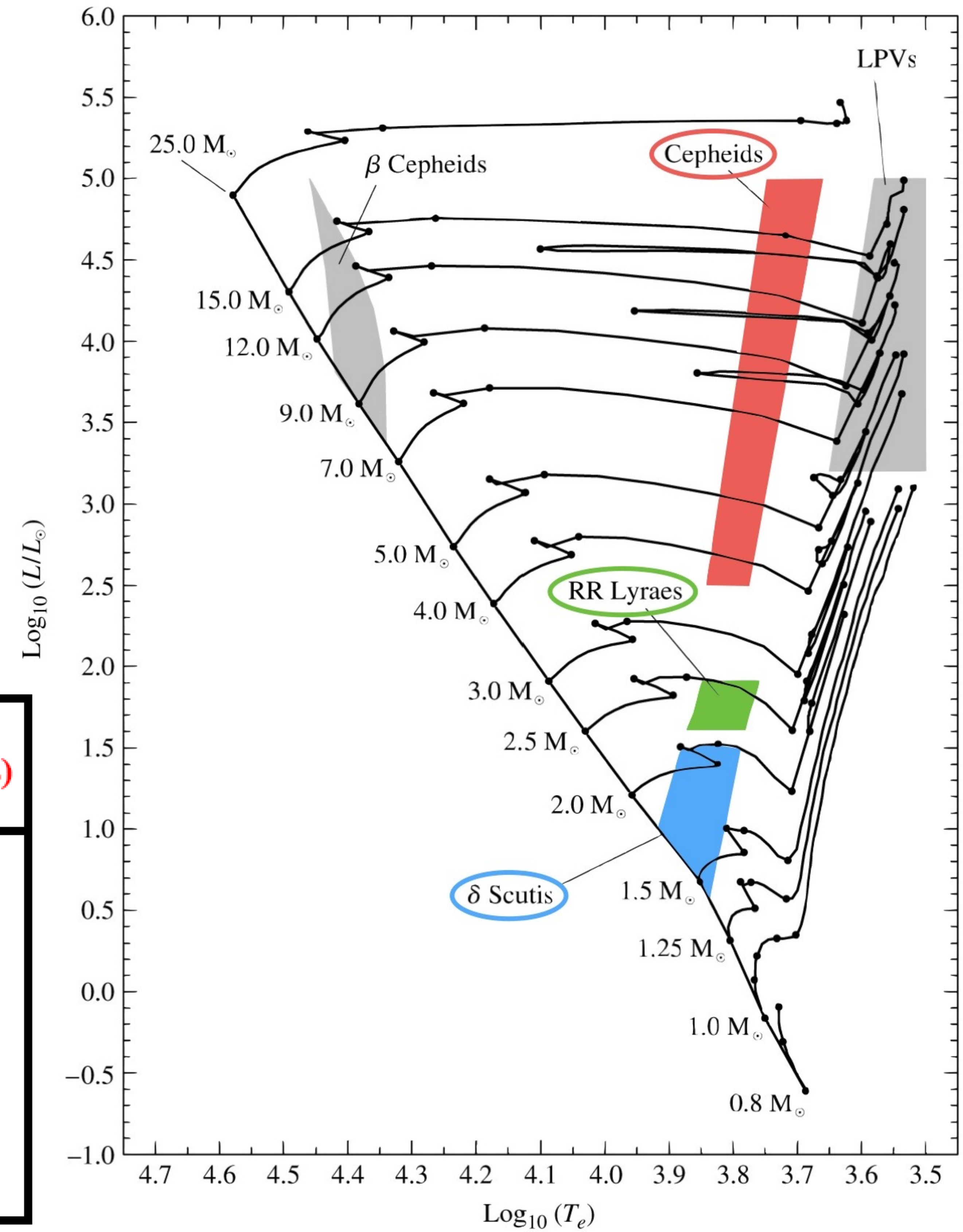
Blöcker T, 'Stellar evolution of low- and intermediate-mass stars. II. Post-AGB evolution', Astronomy and Astrophysics, vol. 299, no. 2, pp.755. 1995. Reproduced with permission. © ESO.

Pulsating Stars

A large fraction of stars pulsate, and these pulsations can occur at a variety of different evolutionary stages.

Classical radial pulsators mostly occupy the **instability strip** in between the main sequence and Hayashi limit on the HRD.

Type	Pop	Period (days)	log $(L/L_\odot)$	Spectral type	$\bar{Q}$ (days)
δ Cepheids	I	2 - 60	2.8 – 4.6	F – G	0.04
δ Scuti	I	0.04 - 0.2	0.9 – 1.6	A – F	0.04
β Cepheids	I	0.1 – 0.2	4.0 – 4.7	B1 – B2	0.03
W Virginis	II	1 - 20	2.0 – 2.9	F – G	0.06
RR Lyrae	II	0.3 – 1	1.5 – 1.6	A – F	0.04



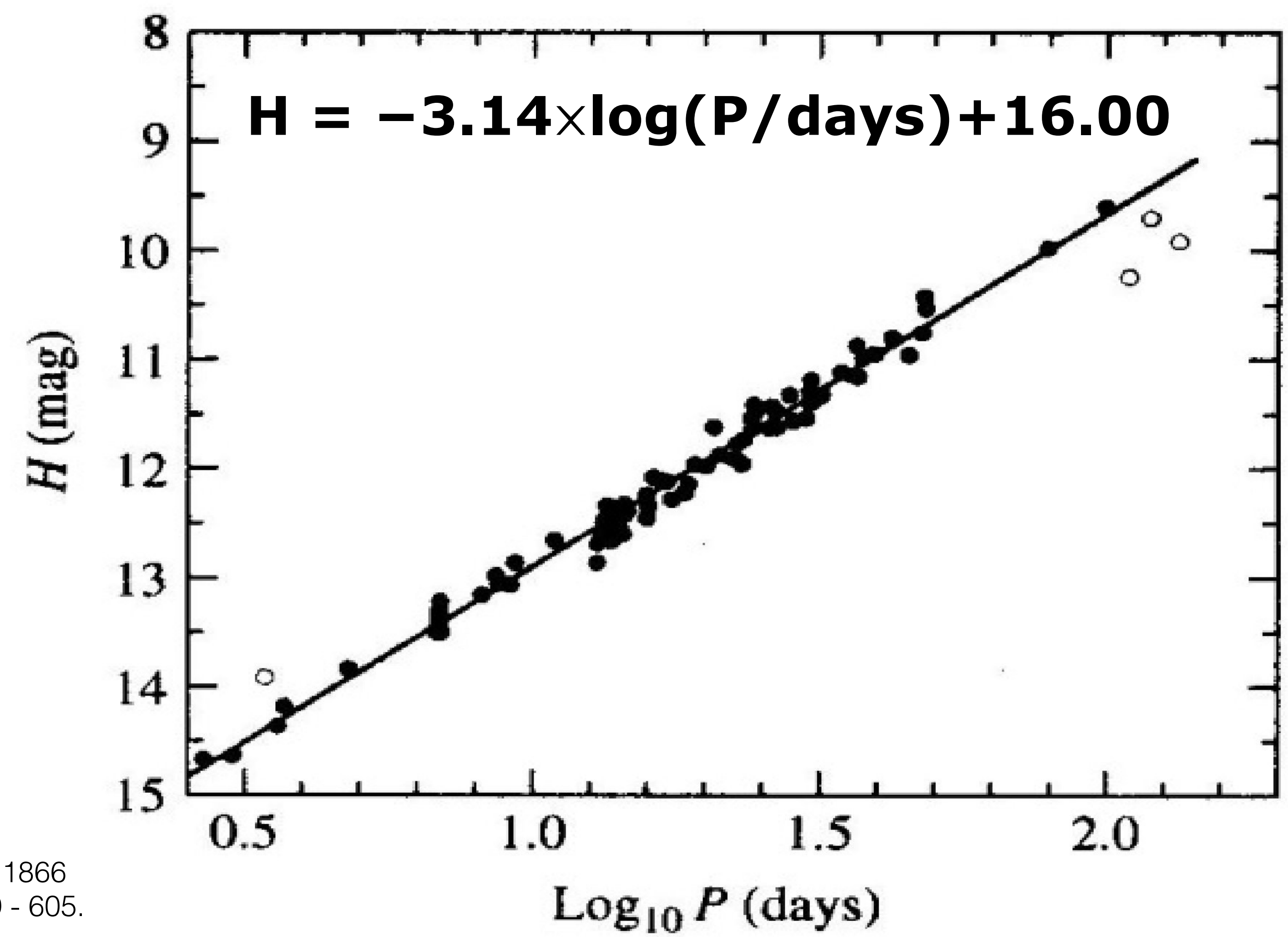
Pulsating Stars

For classical radial pulsators the pulsation period (P) depends on the timescale for restoring equilibrium; the restoring force here is the gas pressure.

$$P = C \times \tau_{\mathrm{dyn}} = \frac{C}{\sqrt{G\bar{\rho}}} = Q \times (\overline{\rho_{\odot}}/\bar{\rho})^{1/2}$$

Q is the **pulsation constant** (and is, in fact, actually slightly dependent on P)

Radial pulsators also have strong period-luminosity relations, with a higher L corresponding to a longer P. Observed P-L relations vary depending on the chosen photometric band.

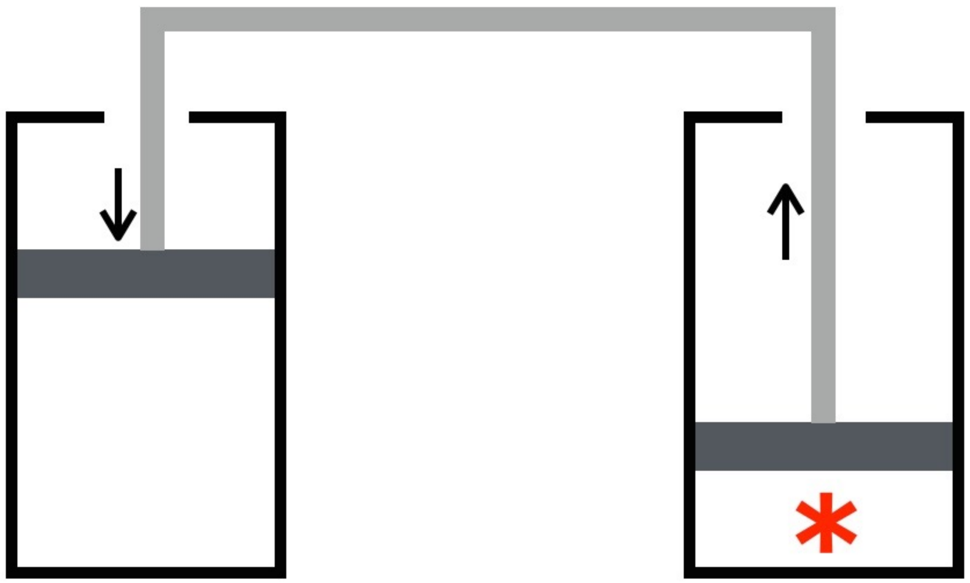


Pulsating Stars

The κ-mechanism of classical radial pulsators

The κ-mechanism (κ = opacity) excites a radial pulsation mode in the partially ionized layers of the H and He ionization zones in these stars.

A good analogy is the combustion motor of a car. Producing a spark during compression in a cylinder heats the gas and drives the piston upward; this is done in alternating cylinders to keep the motor running.



spark=heating=expansion

Pulsating Stars

The κ-mechanism of classical radial pulsators

In pulsating stars the heat input during compression is provided by an increase in opacity, which blocks the transfer of radiation and heats that layer. The inverse happens during expansion.

In a fully ionized or fully neutral layer:

Compression : $\rho\uparrow \rightarrow T$ increases adiabatically (equation 4.33) $T \sim \rho^{2/3}\uparrow \rightarrow \kappa \sim \rho^{-4/3}\downarrow$
so radiation escapes more easily if a layer is compressed.

In a partly ionized layer:

Compression : $\rho\uparrow \rightarrow T$ rises marginally because the heat goes into ionizing the gas:
$P \sim \rho^{\gamma}$ and $T \sim \rho^{\gamma-1} \rightarrow \kappa \sim \rho T^{-7/2} \sim \rho^{(9-7\gamma)/2} \rightarrow \boldsymbol{\kappa\uparrow}$ **if $\boldsymbol{\gamma < 9/7}$.**
$\rightarrow \kappa\uparrow$ and so radiation flow is blocked (trapped) in the compressed layer
$\rightarrow$ energy input during compression (like in an engine).

Expansion : $\rho\downarrow$ but T is about constant as gas recombines and releases energy:
$P \sim \rho^{\gamma}$ and $T \sim \rho^{\gamma-1}$ with $\gamma < 9/7 \rightarrow \kappa \sim \rho T^{-7/2} \sim \rho^{(9-7\gamma)/2}\downarrow \rightarrow \kappa\downarrow$
and the energy escapes by radiation.

Pulsating Stars

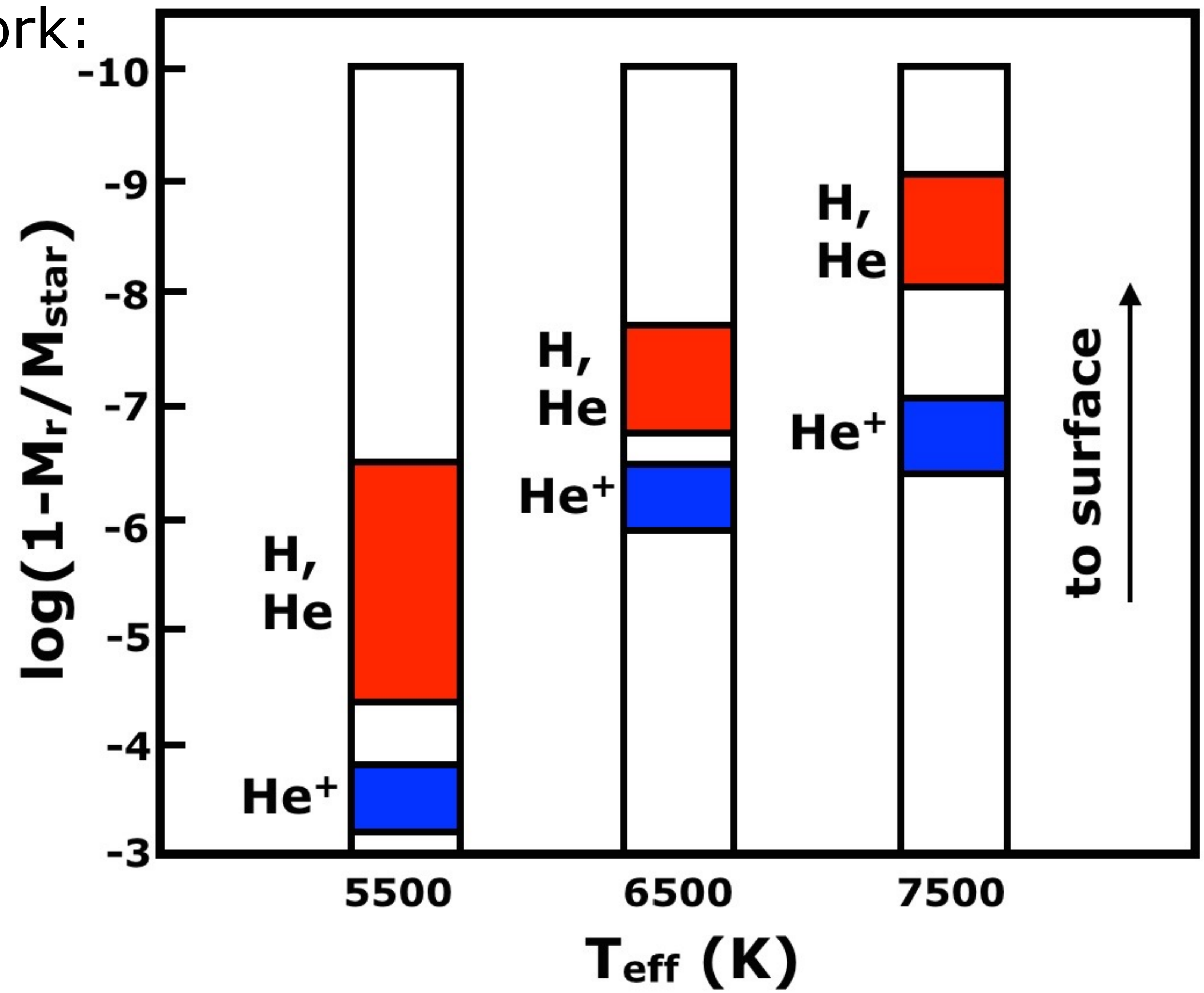
The κ-mechanism of classical radial pulsators

There are three ionization zones in a star starting from the outside in: $H \rightarrow H^+$, $He \rightarrow He^+$, and $He^+ \rightarrow He^{++}$.

For the κ-mechanism to work:

1) Ionization layer cannot be too deep; otherwise the layers above it can dampen pulsation (happens in cool stars)

2) Ionization layer cannot be too close to the surface, where there is not enough mass on top of it to produce efficient pulsation (happens in hot stars)

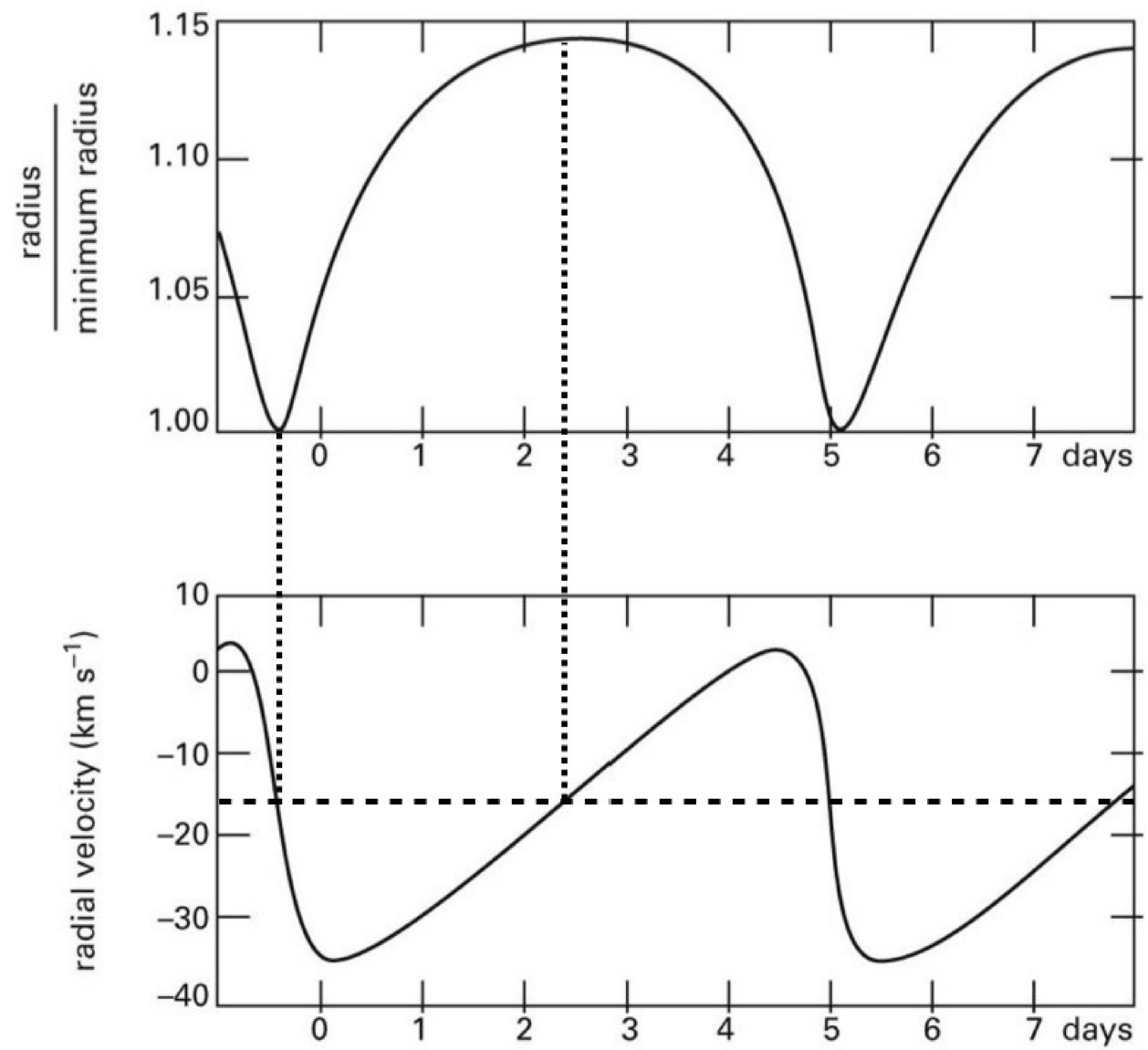


Pulsating Stars

Example: δ Cephei

On the right is the radial variation of δ Cep, the prototype Cepheid variable.

The radial pulsation of the star changes in L (observe as variations in the V band lightcurve…), T_{eff}, and radial velocities (negative for expansion, positive for contraction).

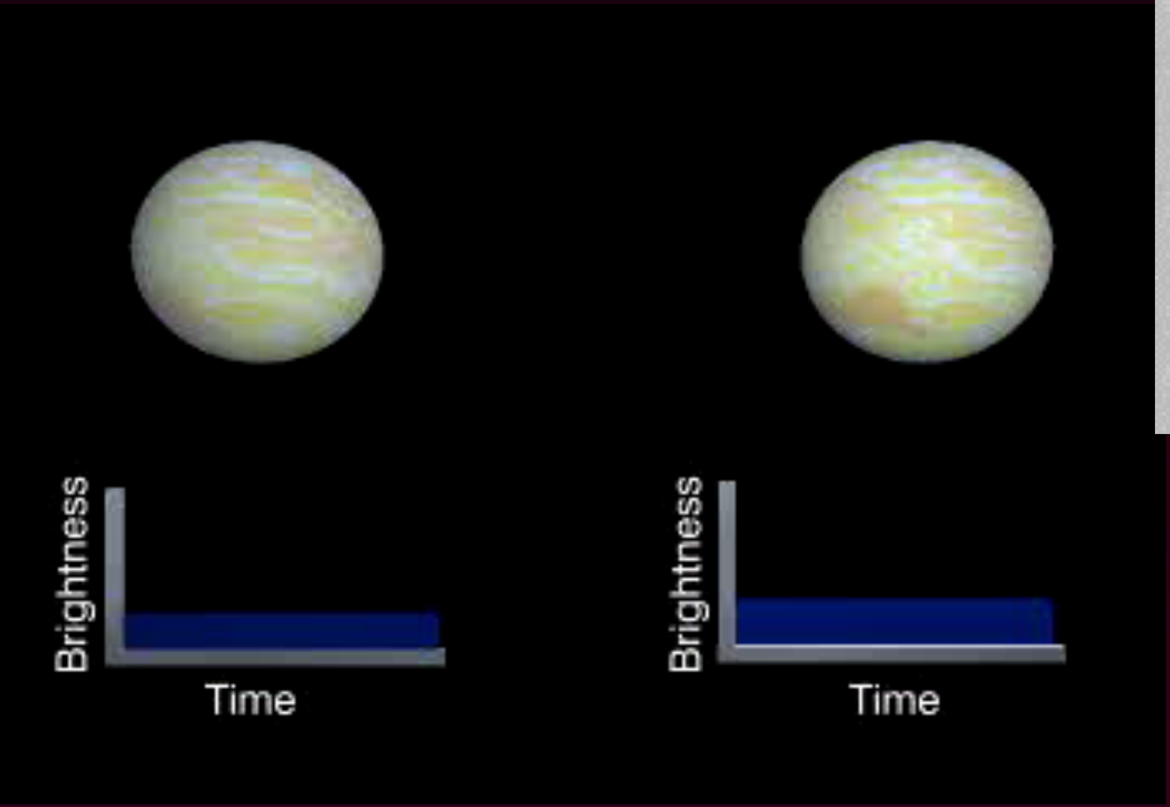


Most radial pulsators vary in the fundamental mode; the gas moves in the same direction throughout the star, alternating in and out. Stars can also pulsate in radial overtones when there are n>0 nodes in the stellar interior; the period of an overtone pulsation is shorter than the fundamental pulsation period by ~1/n.

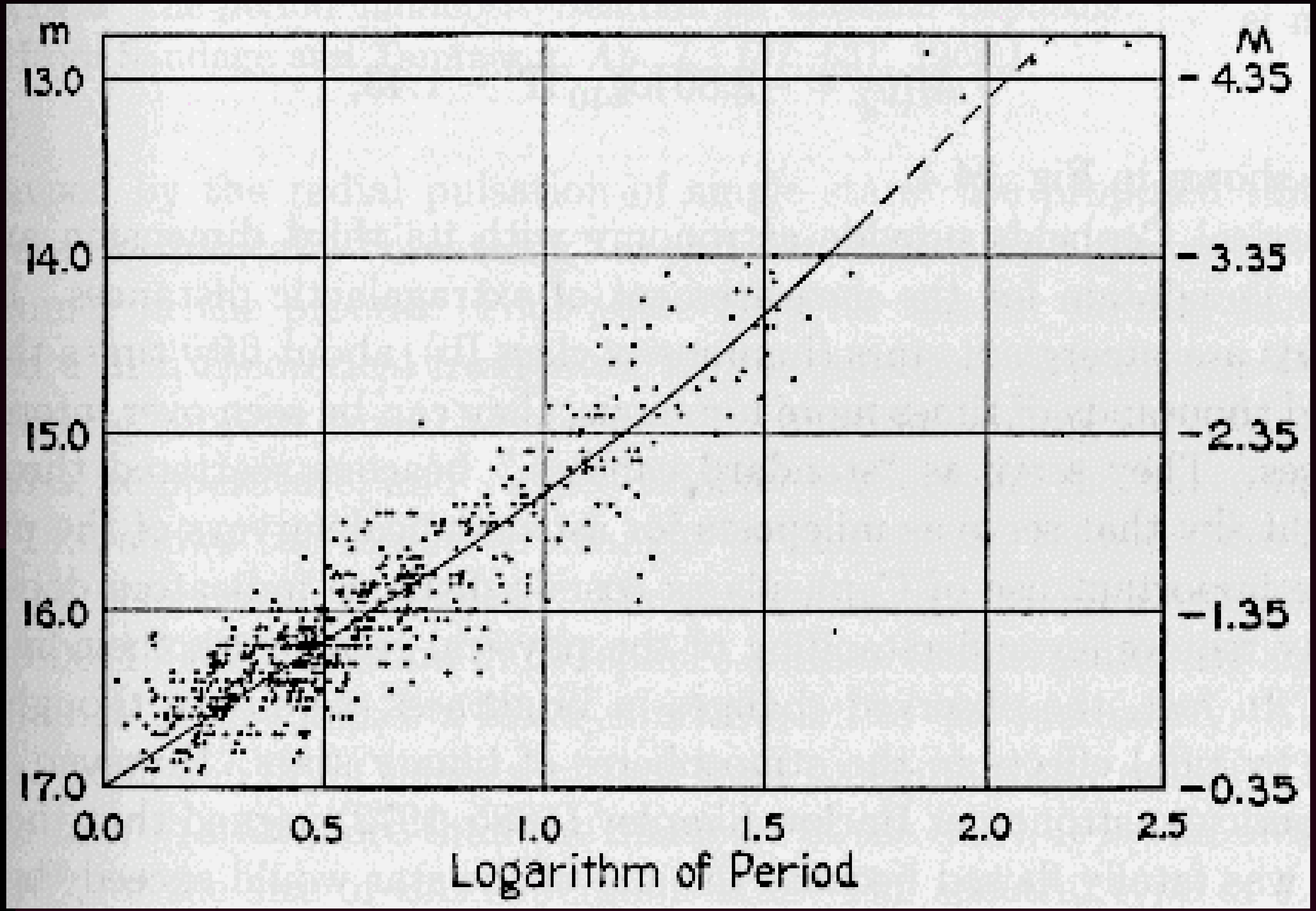
the Harvard “Computers”

Harvard College Observatory, 1891
from http://ocp.hul.harvard.edu/ww/people_fleming.html

the Cepheid period-luminosity relationship



Henrietta Swan Leavitt and a period-luminosity diagram of Cepheids in the Small Magellanic Cloud

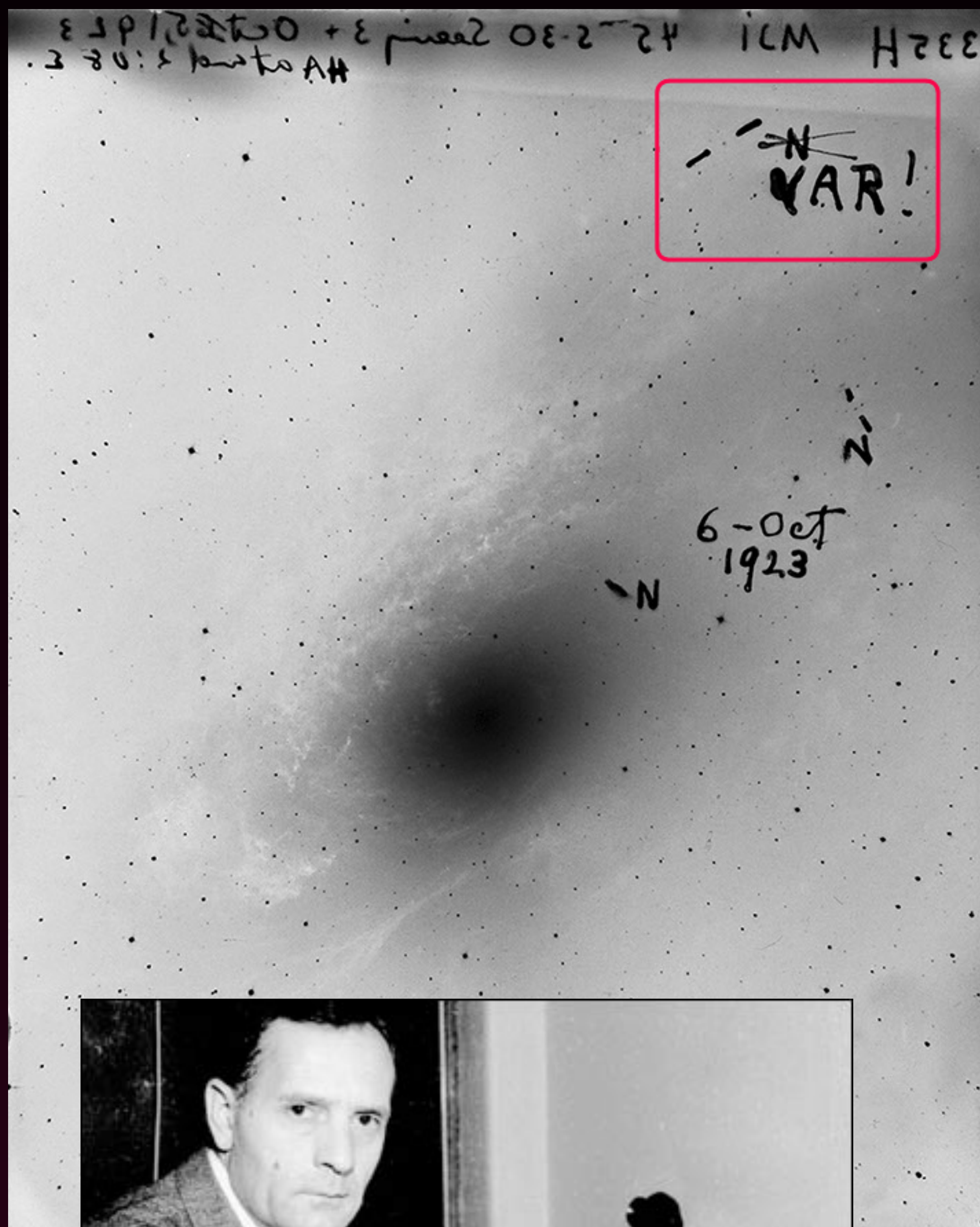


from http://en.wikipedia.org/wiki/File:Leavitt_henrietta_b1.jpg, http://www.aavso.org/vstar/vsots/0302.shtml and http://hubblesite.org/newscenter/archive/releases/1994/49/video/

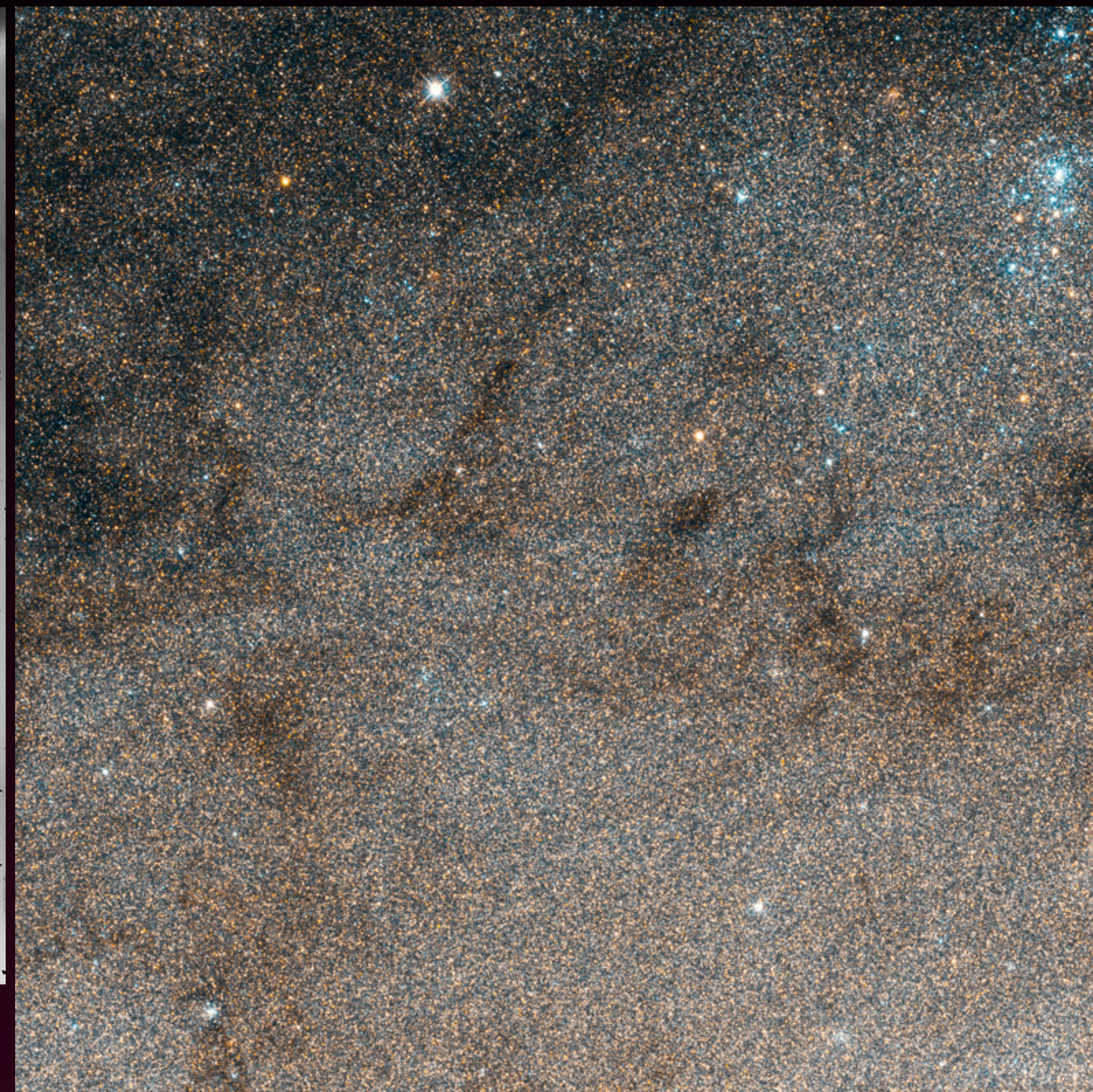
The Puzzle of "Spiral Nebulae"

- Before Hubble, some scientists argued that "spiral nebulae" were entire galaxies like our Milky Way, while others maintained they were smaller collections of stars within the Milky Way.
- The debate remained unsettled until Edwin Hubble finally measured the distance to the Andromeda Galaxy using Cepheid variables as standard candles.

The Andromeda ~~Nebula~~ Galaxy

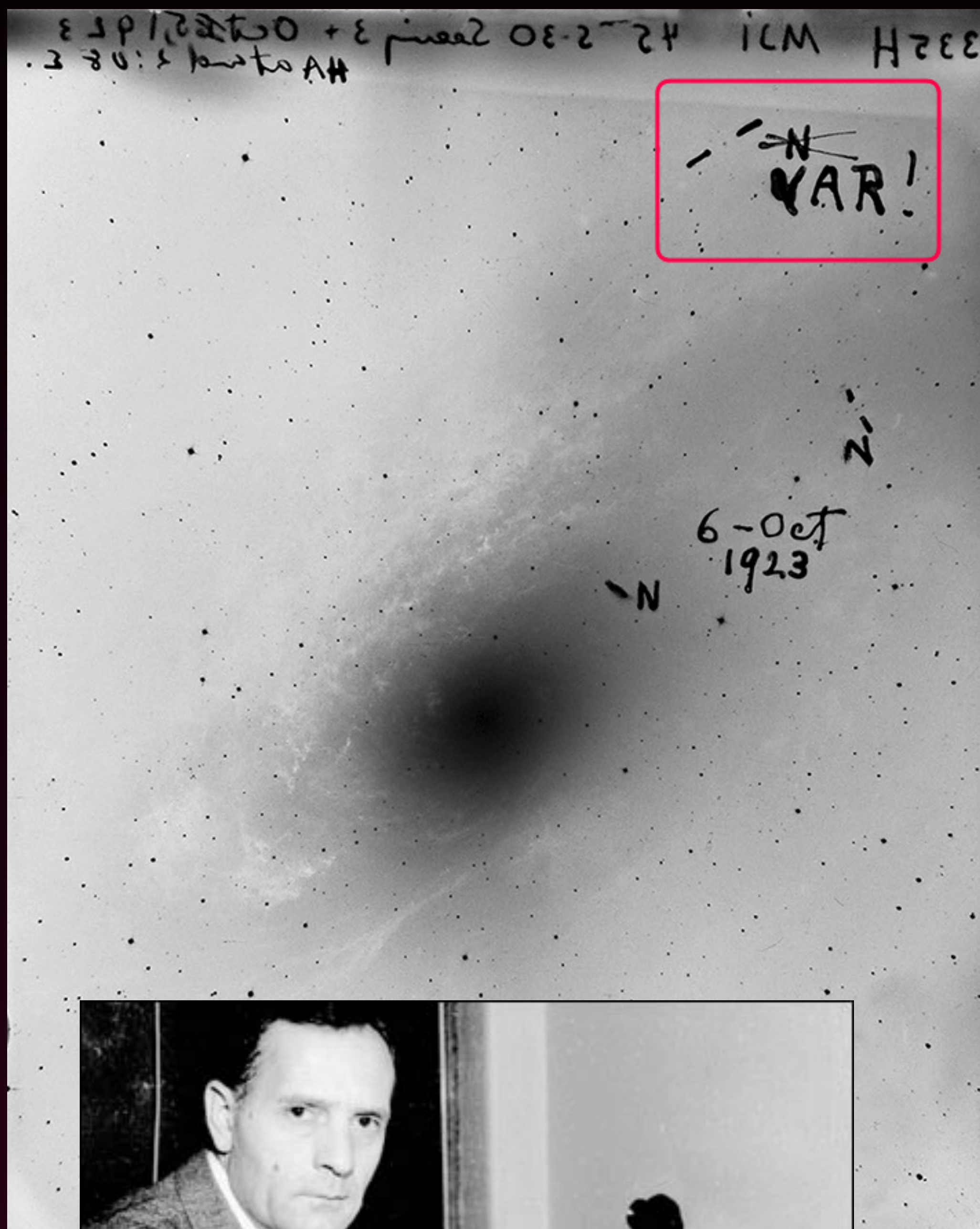


Edwin Hubble



find Hubble's Cepheid variable star
in this Hubble Space Telescope image!

The Andromeda ~~Nebula~~ Galaxy



Edwin Hubble

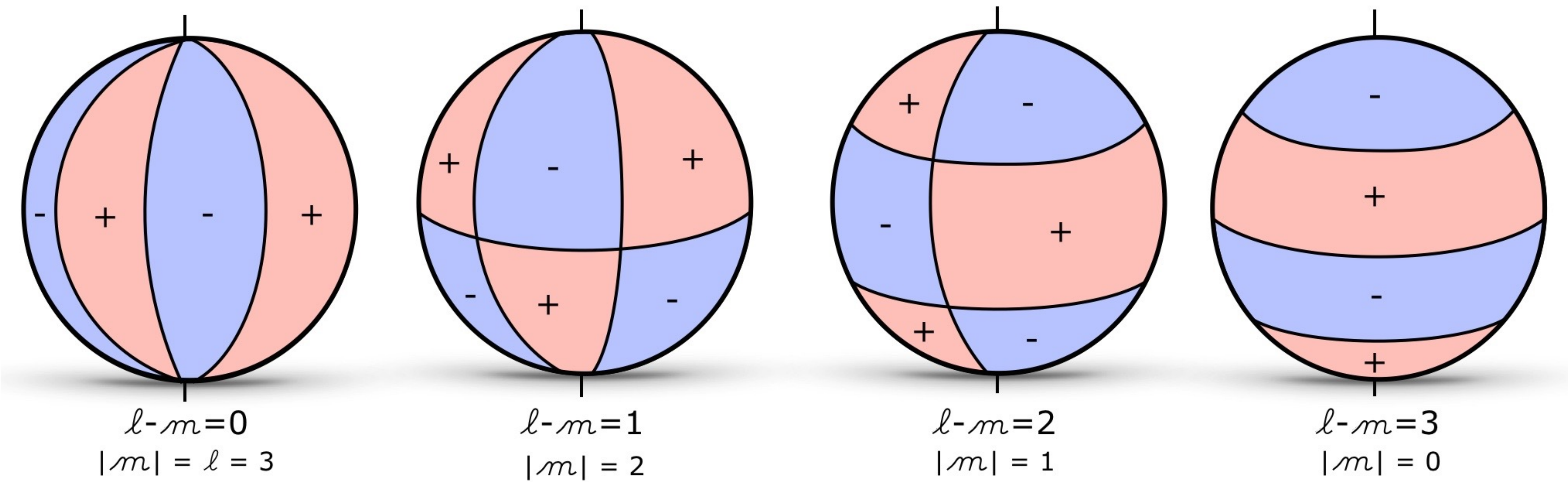
find Hubble’s Cepheid variable star in this Hubble Space Telescope image!

Pulsating Stars

Nonradial pulsations and asteroseismology

Some stars can also pulsate nonradially (changing its shape and deviating from spherical symmetry). Also known as oscillations, they can be described by spherical harmonic functions and are characterized by two quantum numbers:

m = # of meridional nodes (0 = rotational symmetry)
l-m = number of latitude nodes (if l=m there are no latitude nodes)



Observationally detecting radial pulsations can be difficult, and the brightness variations decrease with increasing quantum numbers.

Pulsating Stars

Nonradial pulsations and asteroseismology

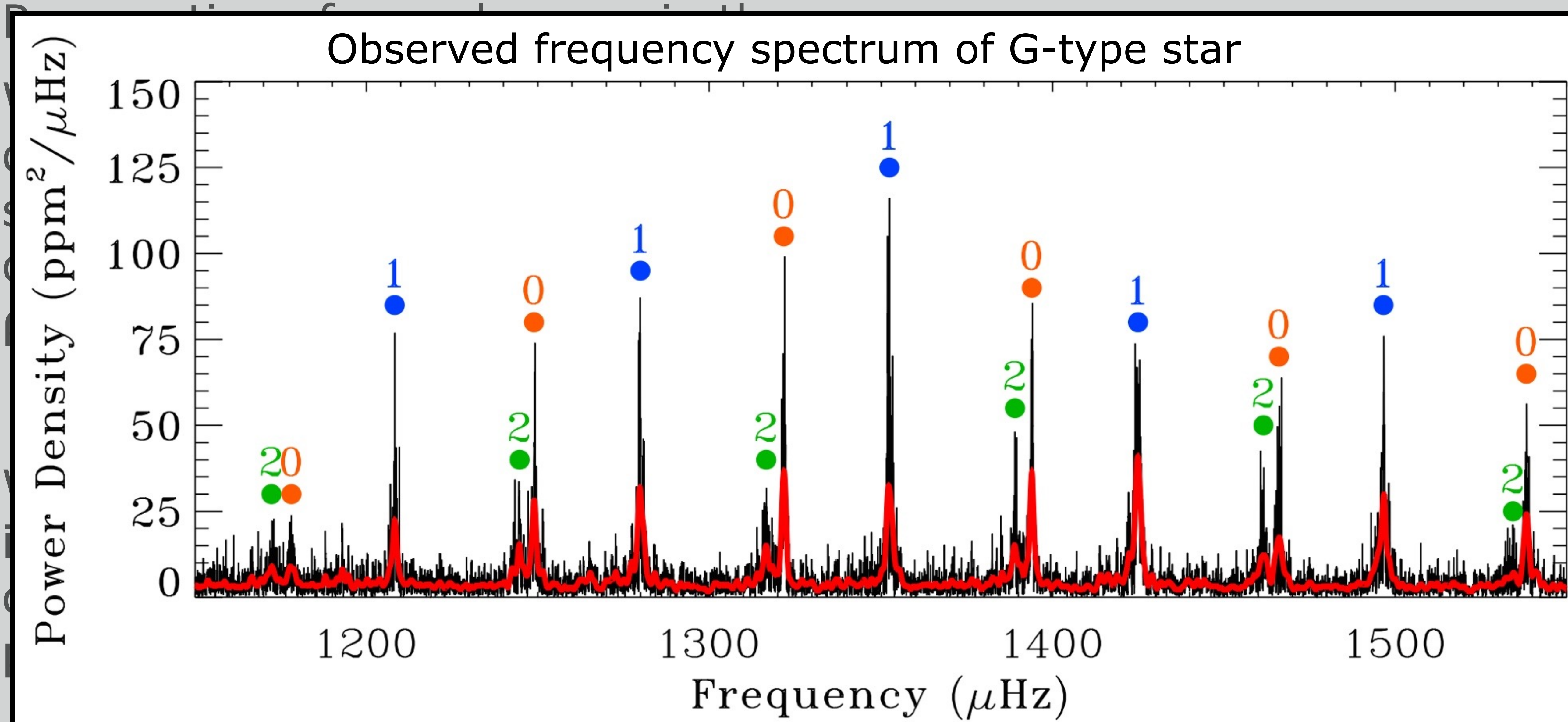
Nonradial pulsations can be due to:

—**p-modes (pressure)**: pressure force is the dominant restoring force, causing both horizontal and vertical motion. Occur in upper layers of star; p-mode pulsation frequencies give info about T, ρ, and chemical structure of the outer envelope.

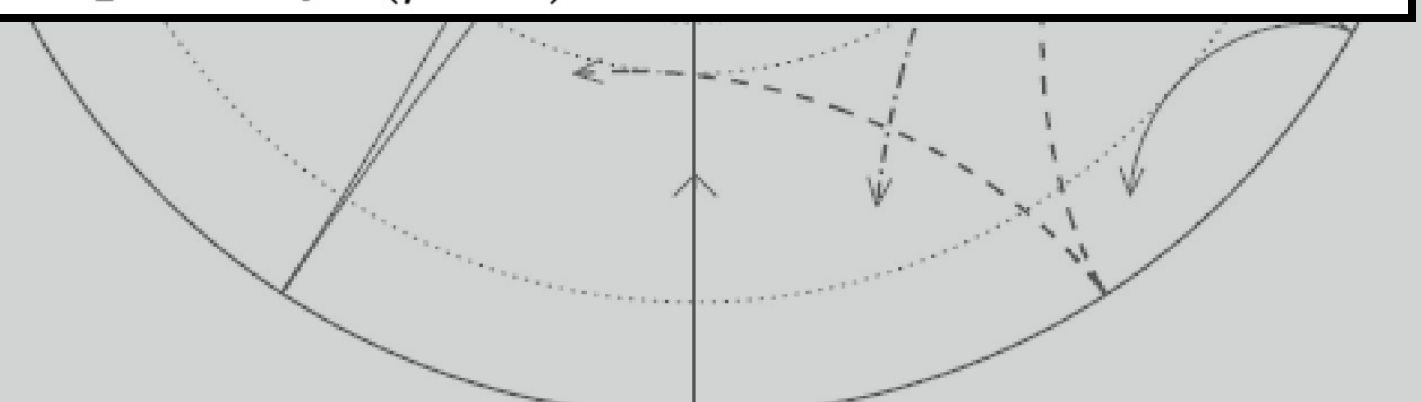
—**g-modes (gravity)**: gravity is the dominant restoring force. g-modes represent internal gravity waves traveling deep into the stellar interior; the frequencies give info about T, ρ, and chemical structure deep in the star, offering a means of studying internal mixing by convection, overshooting, and meridional circulation.

Pulsating Stars

Nonradial pulsations and asteroseismology



$$\lambda = 2\pi R/\sqrt{(l(l+1))}$$



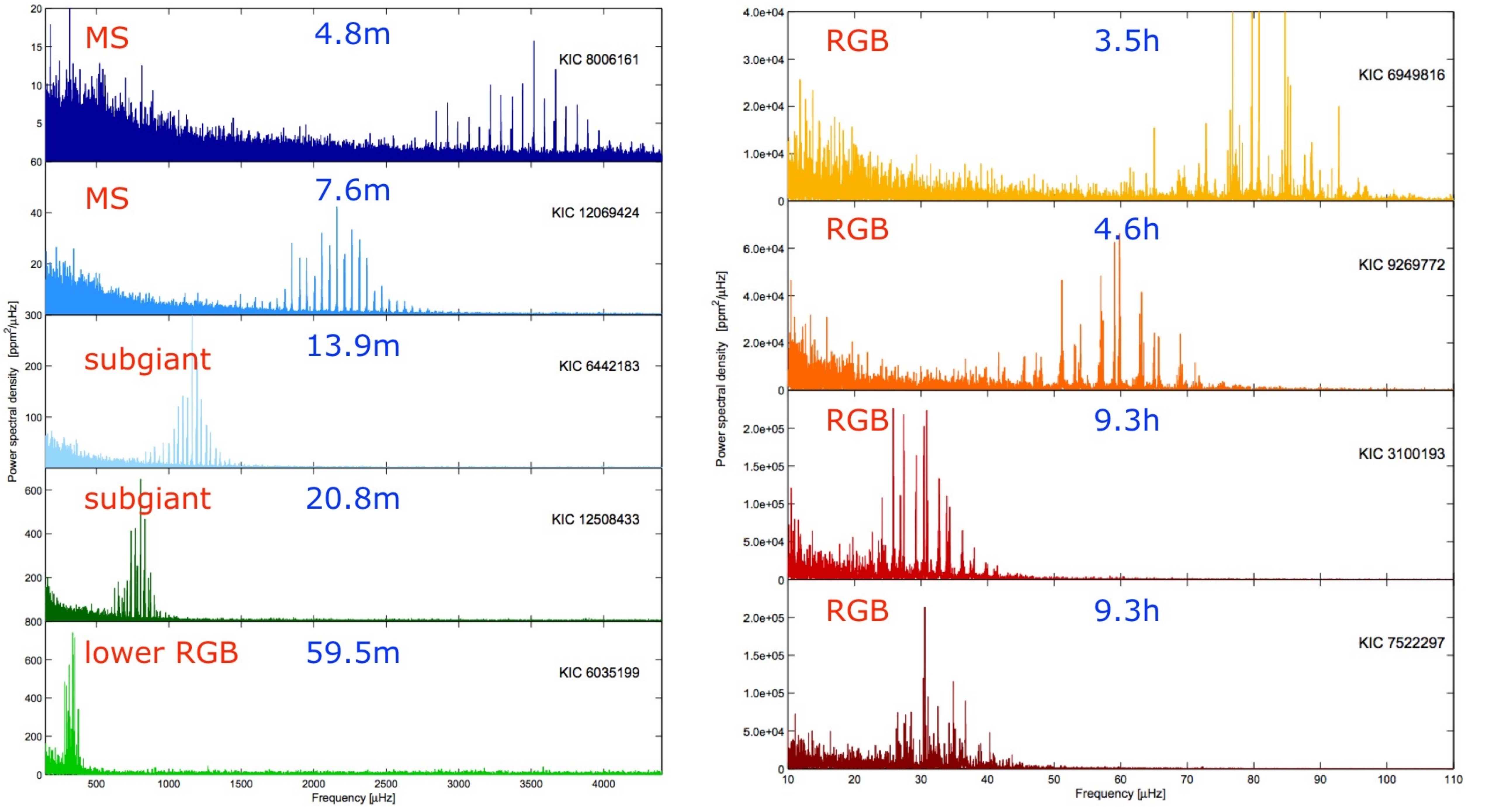
Reproduced from White et al, 'Solving the Mode Identification Problem in Asteroseismology of F Stars Observed with Kepler', Astrophysical Journal, 751 L36, 2012.

Figure with permission from Jørgen Christensen-Dalsgaard, 'Helioseismology', Rev. Mod. Phys. 74, 1073, 2002, Copyright 2002 by the American Physical Society.

Pulsating Stars

Nonradial pulsations and asteroseismology

Comparison of frequency spectra for 9 stars shows clear trend toward lower frequency oscillations at lower surface gravities.



Reproduced from Chaplin W J and Miglio A, ''New grids of stellar models from 0.8 to 120 solar masses at Z = 0.020 and Z = 0.001', 'Asteroseismology of Solar-Type and Red-Giant Stars', Annual Review of Astronomy and Astrophysics, vol. 51, issue 1, pp. 353-392, 2013. Reproduced with permission.