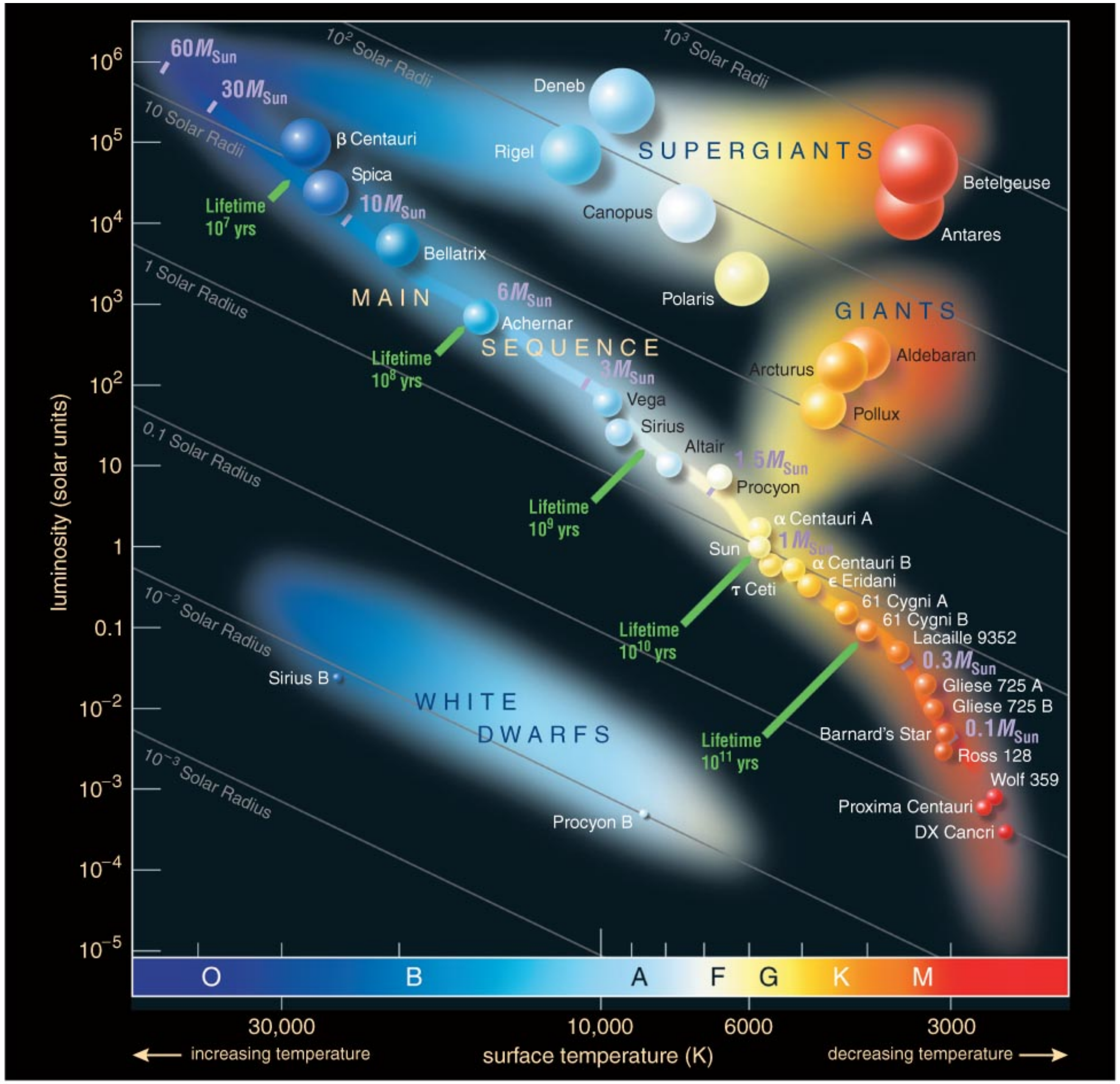
Lecture 2: Observations of Stars



http://www.physics.rutgers.edu/ugrad/441

http://www.physics.rutgers.edu/grad/541

Planck spectrum in wavelength units

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

e.g., erg cm^{-2} s^{-1} sr^{-1} Å^{-1}

peak: Wien displacement law

set $dB_\lambda/d\lambda = 0$

$$\lambda_{\rm max} T = 0.290 \text{ cm K}$$

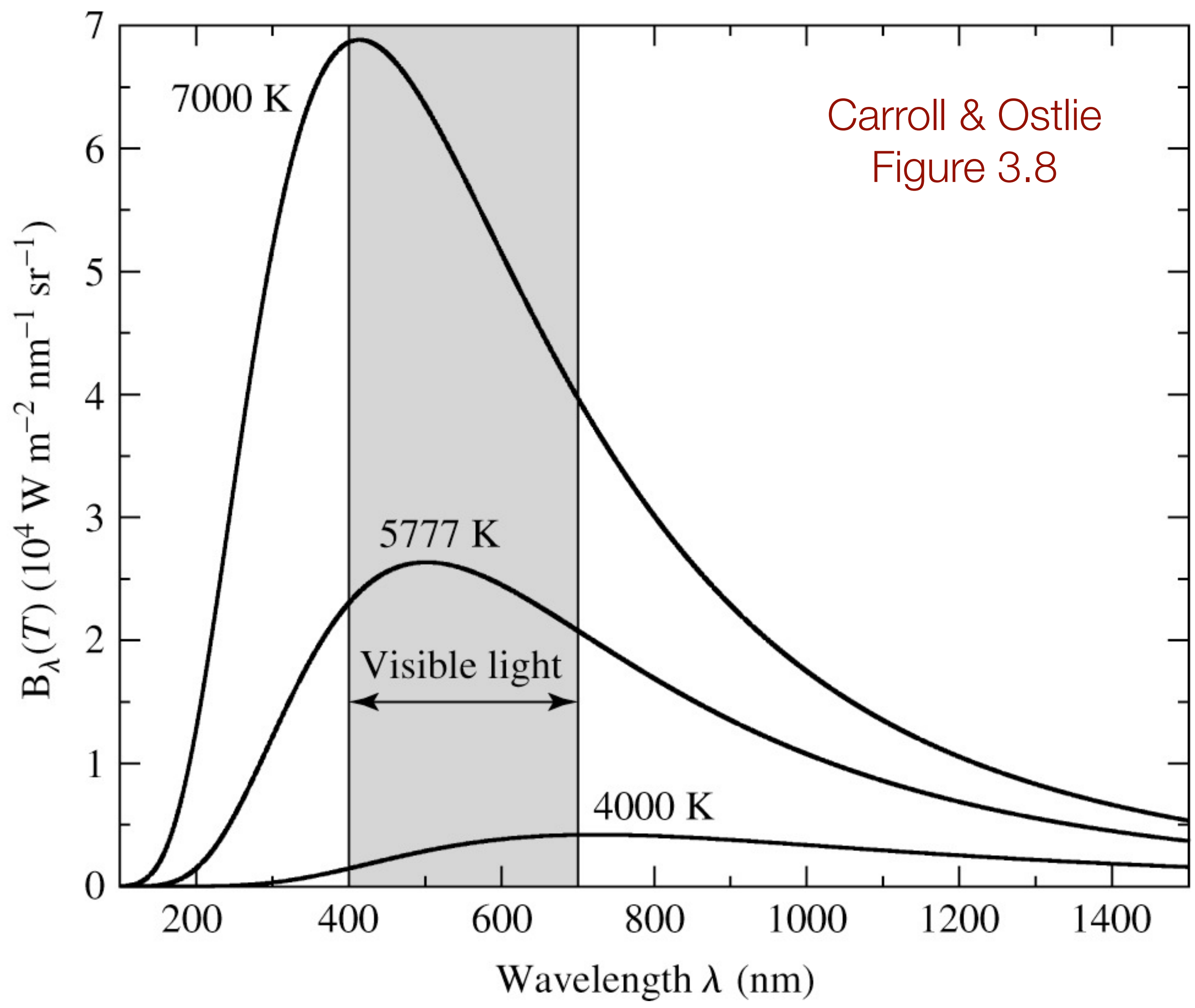
integrating over frequency/wavelength

$$\int_0^\infty B_\lambda \, d\lambda = \int_0^\infty B_\nu \, d\nu = \frac{ac}{4\pi} T^4 = \frac{\sigma}{\pi} T^4$$

$$\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3}, \qquad a = \frac{8\pi^5 k^4}{15 c^3 h^3}.$$

$$a = 7.5657 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$\sigma = ac/4 = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$



Carroll & Ostlie
Figure 3.8

Stefan-Boltzmann constant

Rybicki & Lightman Figure 1.6

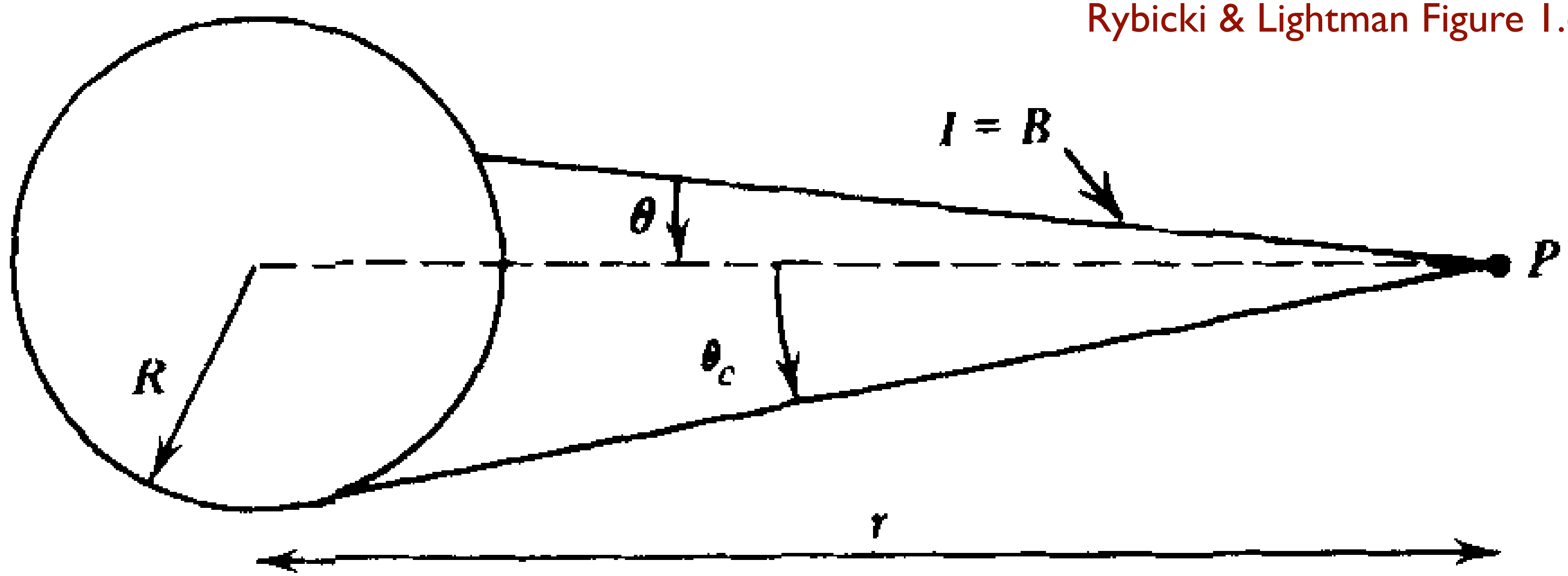


Figure 1.6 Flux from a uniformly bright sphere.

radiant flux density at surface
uniformly bright spherical blackbody

$$F_\lambda = \pi B_\lambda$$

that π is really π sterradians
so units for flux density:
erg cm^{-2} s^{-1} Å^{-1}

bolometric flux at surface

$$F = \int F_\lambda \, d\lambda = \sigma T^4$$

units for flux:
erg cm^{-2} s^{-1}

bolometric means total energy
over all wavelengths of light

bolometric luminosity

$$L = FA = 4\pi R^2 \sigma T^4$$

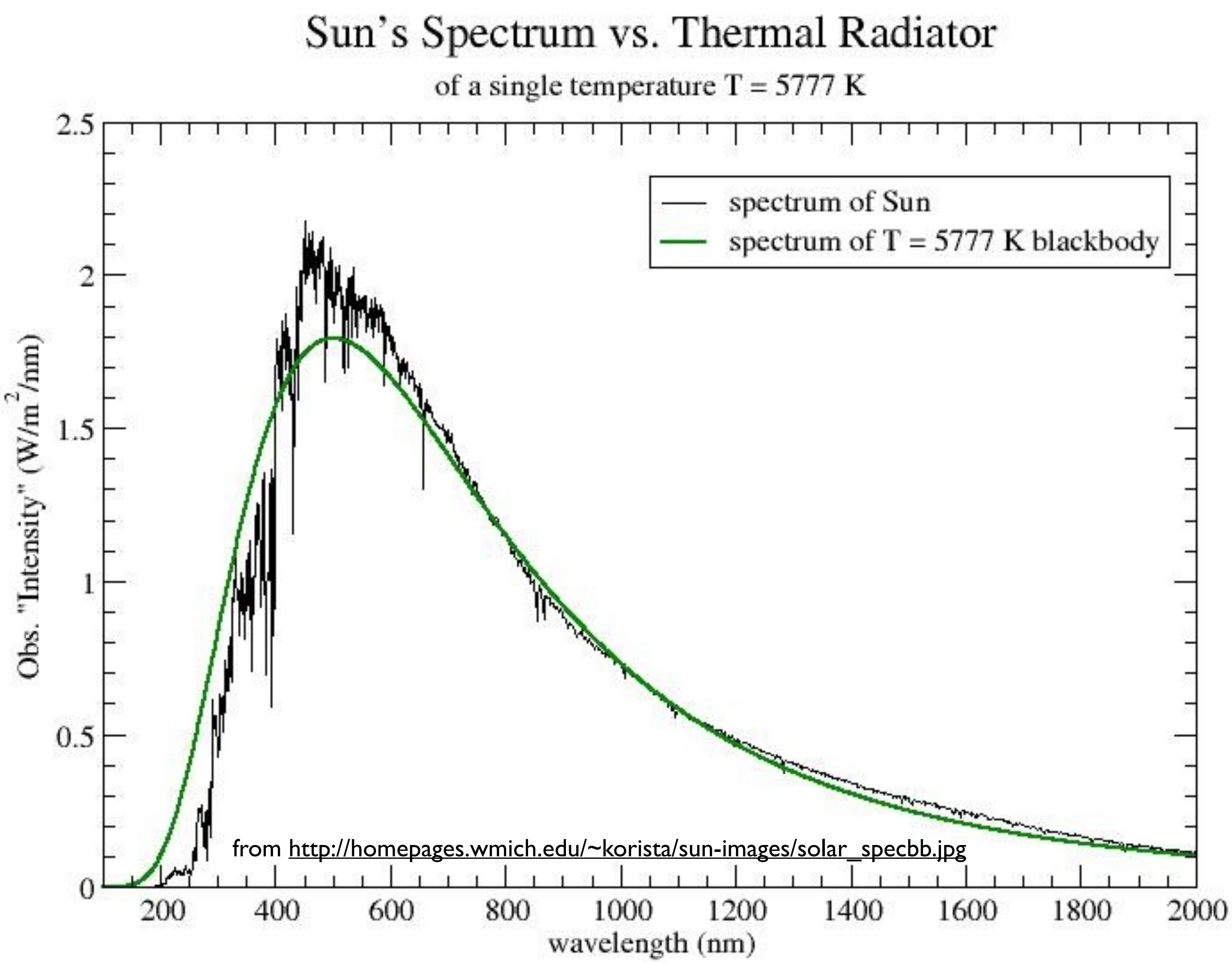
units for luminosity:
erg s^{-1}

Stefan-Boltzmann law

effective temperature

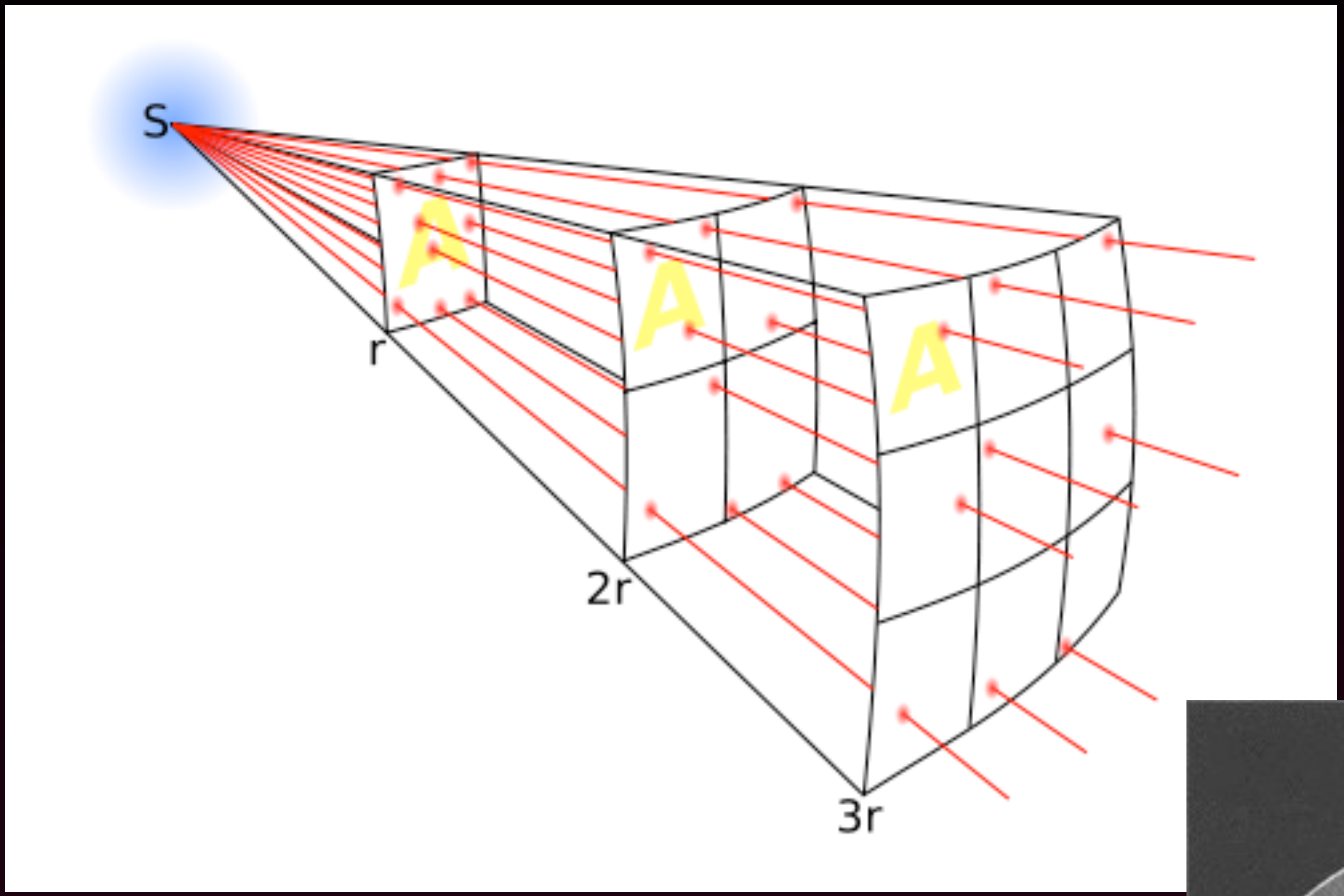
real stars are not perfect blackbodies (absorption lines, “limb darkening”)
you can still measure the bolometric luminosity (total power emitted)
and use that to **define** the *effective temperature*:

$$L = 4\pi R^2 \, \sigma T_{\rm eff}^4$$



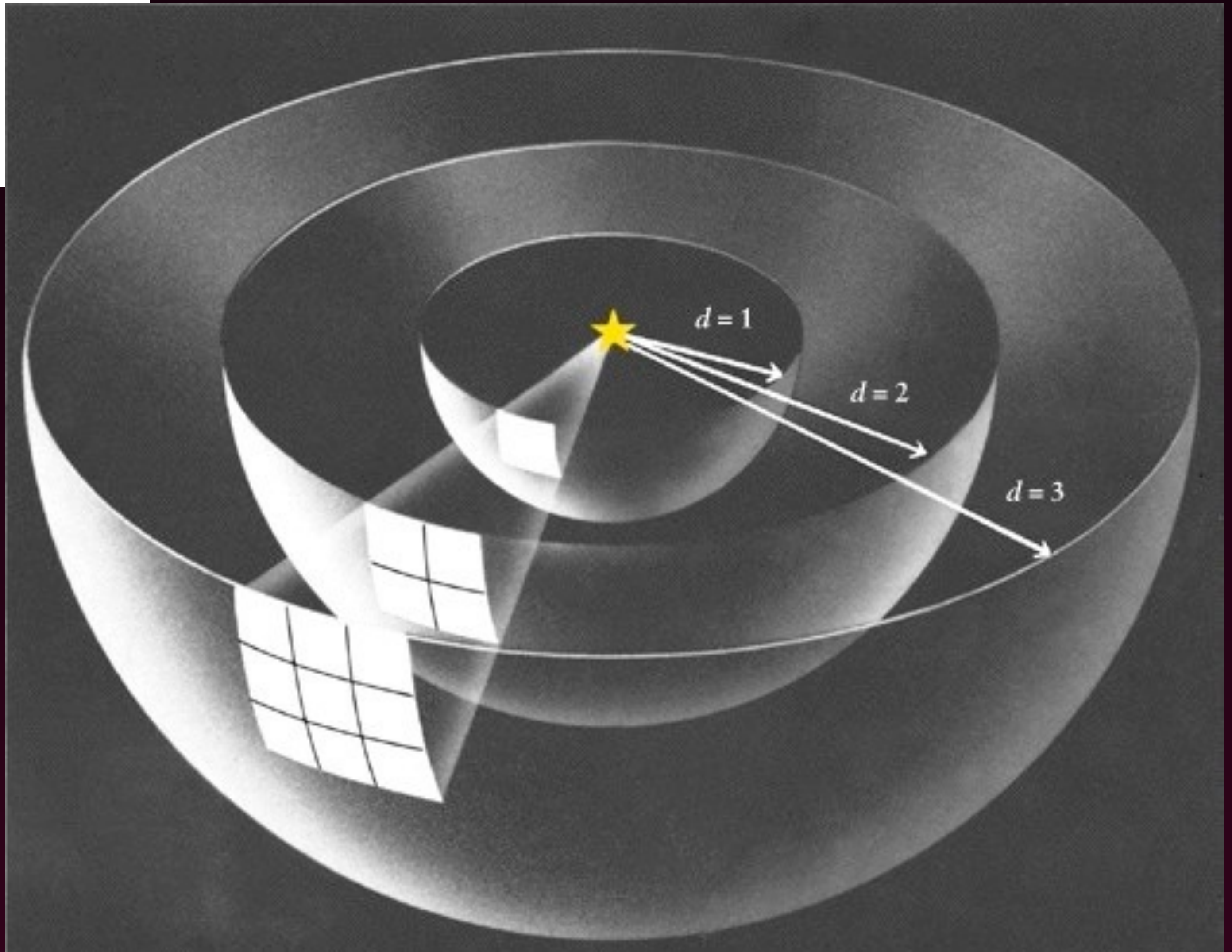
$$T_{\rm eff,\odot} = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma}\right)^{1/4} = \left[\frac{3.83\times 10^{33}\ {\rm erg\ s^{-1}}}{4\pi(6.96\times 10^{10}\ {\rm cm})^2(5.67\times 10^{-5}\ {\rm erg\ s^{-1}\ cm^{-2}\ K^{-4}})}\right]^{1/4} = 5770\ {\rm K}$$

radiation flux: inverse square law



from http://gain11.files.wordpress.com/2008/06/inverse-square-law.png

from http://www.math.lsa.umich.edu/mmss/coursesONLINE/Astro/Ex1.2/

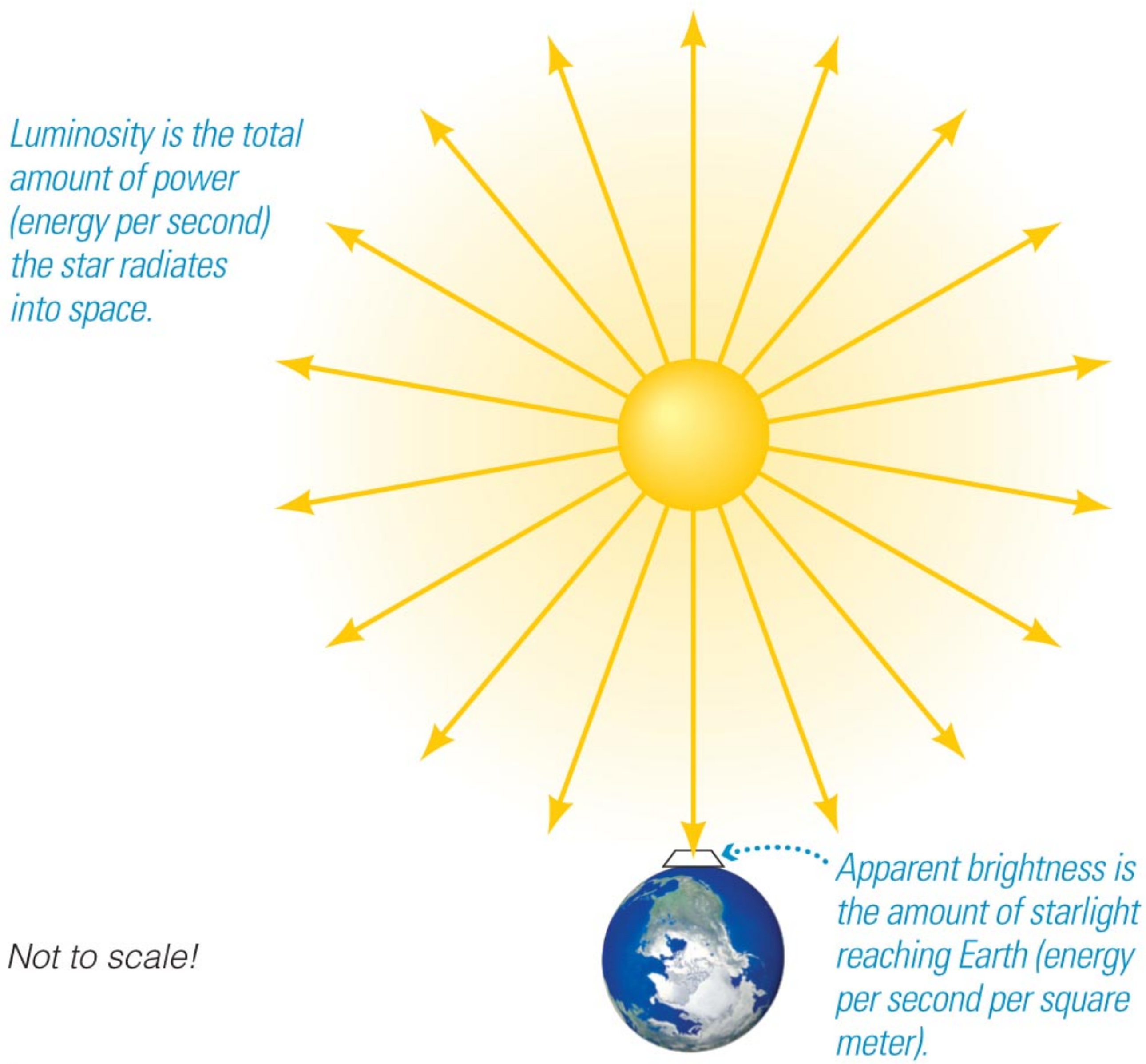


bolometric flux $$f = \frac{L}{4\pi d^2}$$

apparent brightness

flux density $$f_\lambda = \frac{L_\lambda}{4\pi d^2}$$

Luminosity and apparent brightness



Luminosity:

Amount of power a star radiates (energy per second = watts)

Sun's luminosity (L_{Sun}) = 3.8×10^{26} watts

$$f = \frac{L}{4\pi d^2} \qquad L = 4\pi d^2 f$$

Apparent brightness:

Amount of starlight reaching Earth (energy per second per square meter)

$$f_\odot = \frac{L_\odot}{4\pi (1\ \mathrm{AU})^2}$$

Sun's apparent brightness = 1360 watts per square meter

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flux density from a uniform isotropic blackbody

relate the flux density at a distance *d*
to the flux density at the surface of the blackbody

$$f_\lambda = \frac{L_\lambda}{4\pi d^2} = \frac{4\pi R^2 F_\lambda}{4\pi d^2} = F_\lambda \left(\frac{R}{d}\right)^2 = \pi B_\lambda \left(\frac{R}{d}\right)^2$$

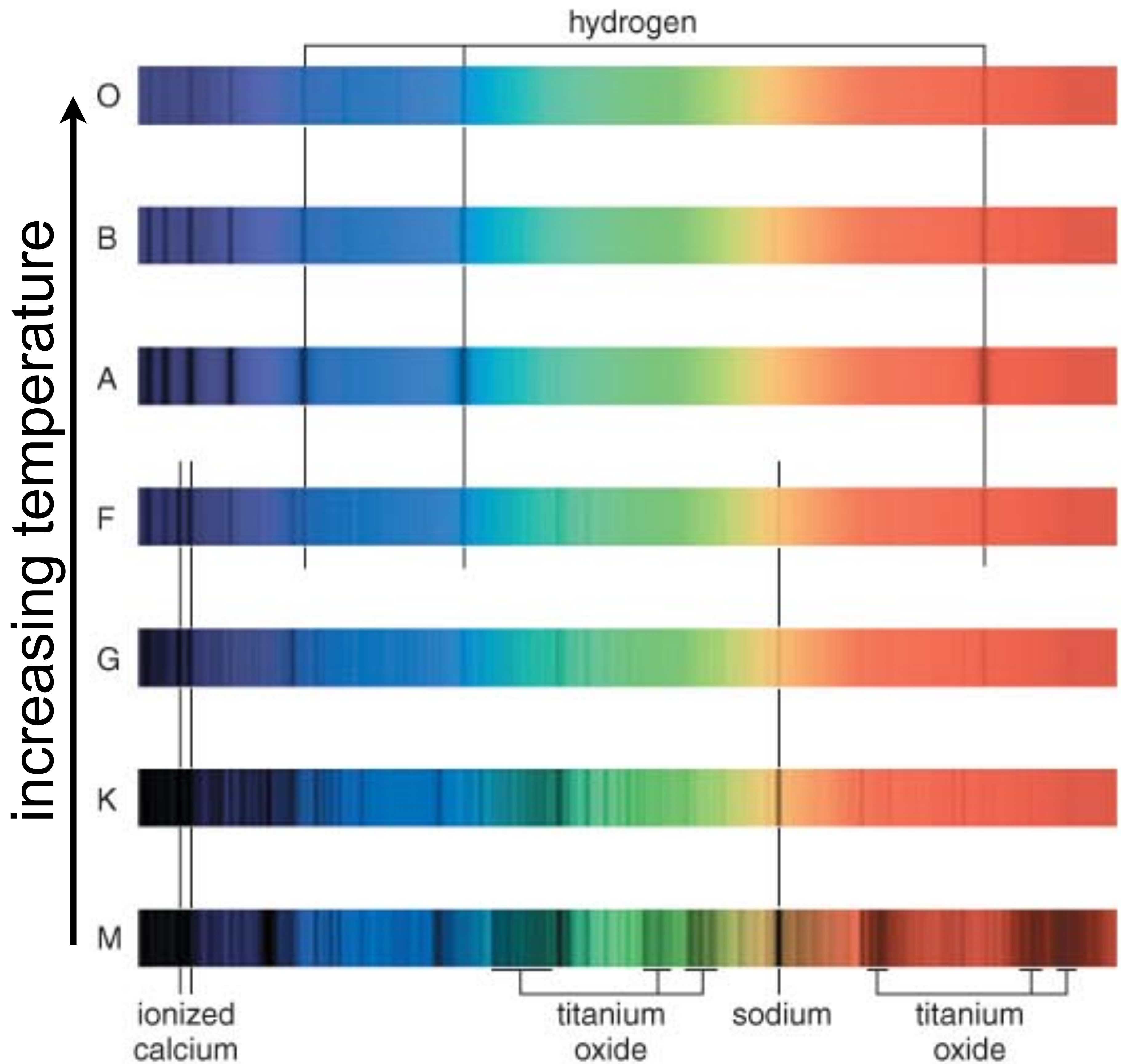
use this on Problem Set #1

Stellar spectral types

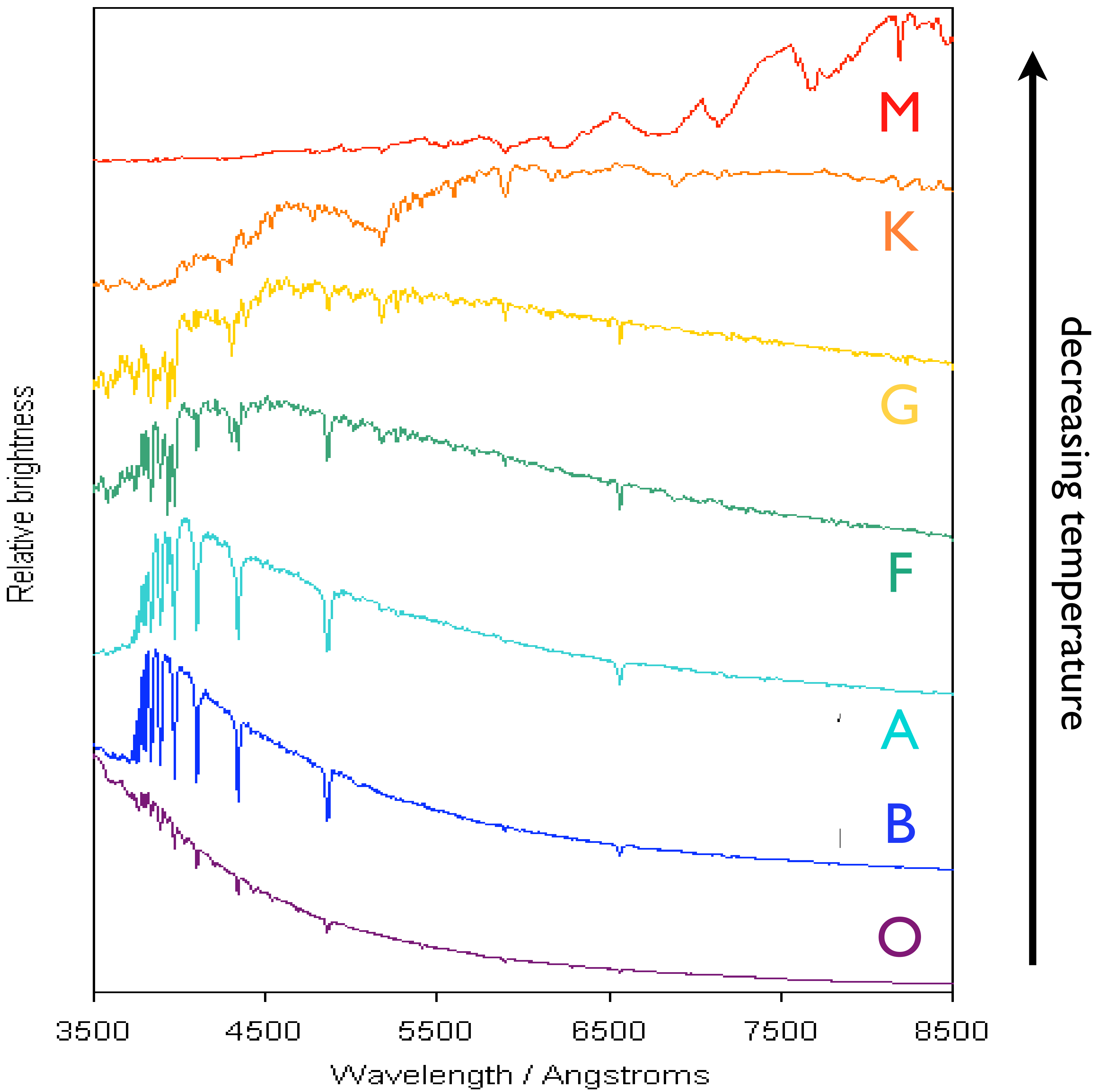
Lines in a star's spectrum correspond to a ***spectral type*** that reveals its temperature.

O B A F G K M

(from hottest to coldest)



Pioneers of Stellar Classification



from http://www.jb.man.ac.uk/distance/life/sample/stars/spectra.gif

Annie Jump Cannon
from http://www.twu.edu/dsc/

astronomical magnitudes

Pogson (1856) quantifying the classification of Hipparchos defined *apparent magnitude:*

$$m_\lambda = -2.5 \log_{10} \left(\frac{f_\lambda}{f_0} \right)$$

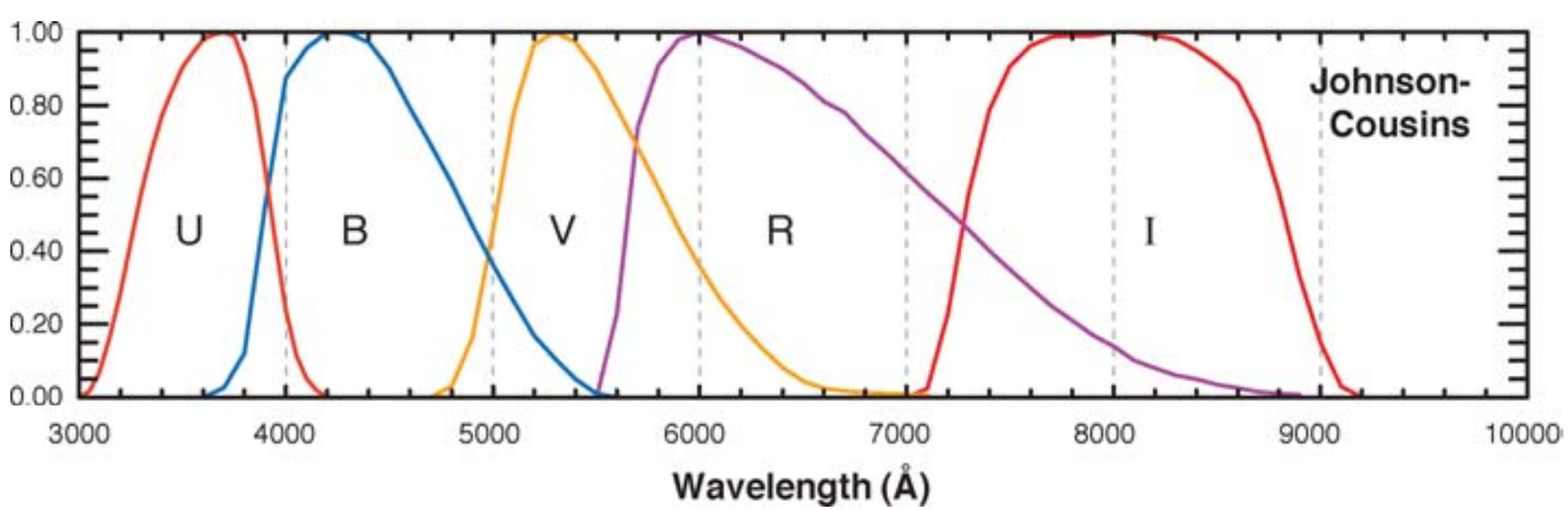
f_0 is the *flux zeropoint*

note the minus sign: lower magnitudes are brighter

the formula is exact: 100 times the flux is 5 magnitudes

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{f_1}{f_2} \right) \iff \frac{f_1}{f_2} = 100^{(m_2 - m_1)/5} = 10^{-0.4(m_1 - m_2)}$$

from now on "log" always means $\log_{10}$
I'll use "ln" for natural log



apparent magnitudes in a "filter" or "passband":
m_B, m_V, …
or just B, V, …

Main table of the brightest stars [edit]

The source of magnitudes cited in this list is the linked Wikipedia articles—this basic list is simply a catalog of what Wikipedia itself documents. References can be found in the individual articles.

V Mag. (m_V)	Proper name	Bayer designation	Distance (ly)	Spectral class
−26.74	Sun		0.000 015 813	G2 V
−1.46	Sirius	α CMa	8.6	A1 V, DA2
−0.74	Canopus	α Car	310	A9 II
−0.27 (0.01 + 1.33)	Alpha Centauri (Rigil Kentaurus)	α Cen	4.4	G2 V, K1 V
−0.05	Arcturus	α Boo	37	K0 III
0.03 (−0.02 - 0.07var)	Vega	α Lyr	25	A0 Va
0.08 (0.03 - 0.16var)	Capella	α Aur	42	K0 III, G1 III
0.13 (0.05 - 0.18var)	Rigel	β Ori	860	B8 Ia
0.34	Procyon	α CMi	11	F5 IV-V
0.46 (0.40 - 0.46var)	Achernar	α Eri	140	B6 Vep
0.50 (0.2 - 1.2var)	Betelgeuse	α Ori	640[6]	M2 Iab
0.61	Hadar	β Cen	350	B1 III
0.76	Altair	α Aql	17	A7 V
0.76 (1.33 + 1.73)	Acrux	α Cru	320	B0.5 IV, B1 V
0.86 (0.75 - 0.95var)	Aldebaran	α Tau	65	K5 III
0.96 (0.6 - 1.6var)	Antares	α Sco	600	M1.5 Iab, B3 V
0.97 (0.97 - 1.04var)	Spica	α Vir	260	B1 III-IV, B2 V
1.14	Pollux	β Gem	34	K0 III
1.16	Fomalhaut	α PsA	25	A3 V
1.25 (1.21 - 1.29var)	Deneb	α Cyg	2,600	A2 Ia
1.25 (1.23 - 1.31var)	Mimosa	β Cru	350	B0.5 II, B2 V
1.39	Regulus	α Leo	77	B7 V
1.50	Adhara	ε CMa	430	B2 Iab:
1.62	Shaula	λ Sco	700	B2 IV
1.62 (1.98 + 2.97)	Castor	α Gem	52	Am, A1 V

Visible to typical human eye[1]	Apparent magnitude	Bright-ness relative to Vega	Number of stars brighter than apparent magnitude[2] in the night sky
Yes	−1.0	250%	1 (Sirius)
	0.0	100%	4
	1.0	40%	15
	2.0	16%	48
	3.0	6.3%	171
	4.0	2.5%	513
	5.0	1.0%	1602
	6.0	0.4%	4800
	6.5	0.25%	9096[3]
No	7.0	0.16%	14 000
	8.0	0.063%	42 000
	9.0	0.025%	121 000
	10.0	0.010%	340 000

from https://en.wikipedia.org/wiki/List_of_brightest_stars and https://en.wikipedia.org/wiki/Apparent_magnitude

photometric *colors*

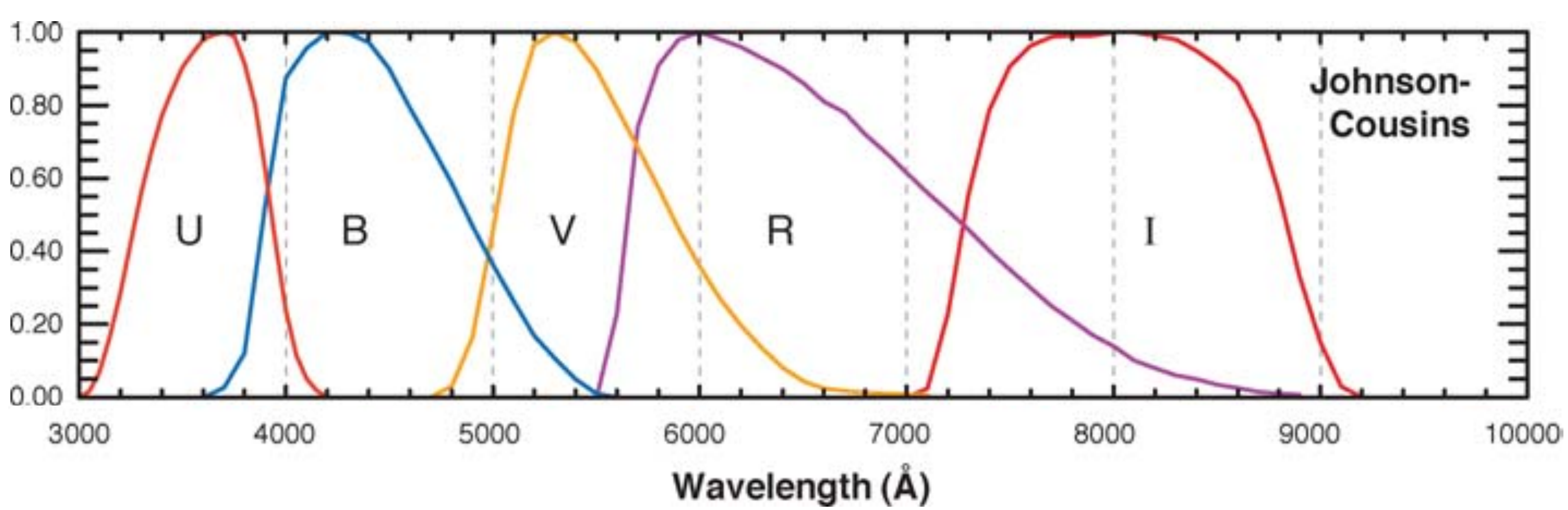
a photometric color corresponds to the flux ratio in different wavelength regions, e.g.,

$$B - V = m_B - m_V = -2.5 \log \left(\frac{f_B / f_{0B}}{f_V / f_{0V}} \right) = -2.5 \log \left(\frac{f_B}{f_V} \right) + \mathrm{const}$$

by convention the shorter wavelength goes first: *U-B, B-V, R-I, …*
so that larger colors are redder (more flux at longer wavelength)

In the Johnson-Cousins *UBVRI* system, the flux zeropoints are set by the star *Vega* which has zero apparent magnitude in all filters (and thus zero colors)

the Sun has *B-V* = +0.63; redder than Vega



apparent magnitudes in a "filter" or "passband": m_B, m_V, …
or just B, V, …

https://commons.wikimedia.org/wiki/
File:Lyra_constellation_detail_long_exposure.jpg

Bessell (2005)

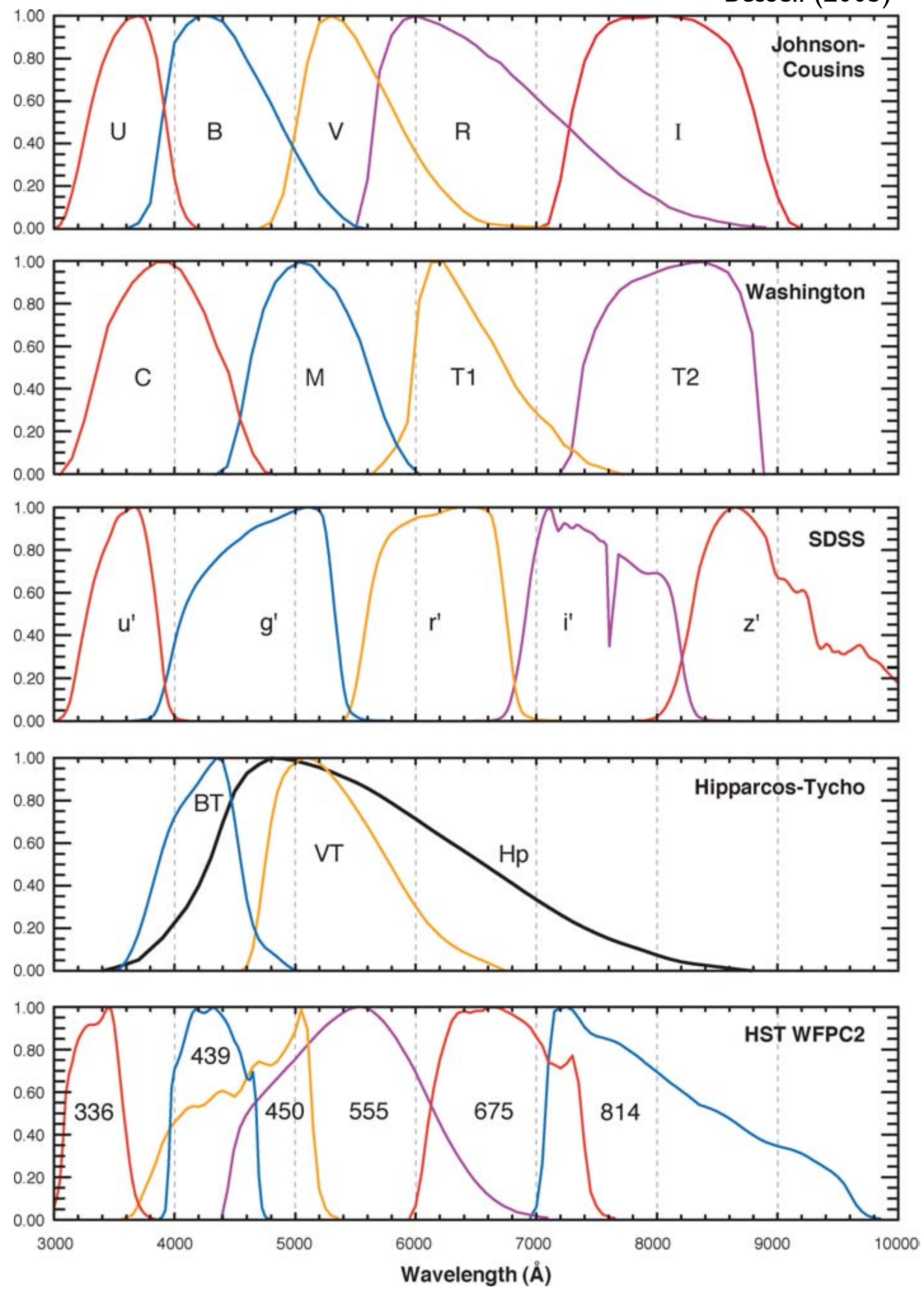


Figure 1 Schematic passbands of broad-band systems.

absolute magnitude & distance modulus

the absolute magnitude *M* of an object is the apparent magnitude it would have if it were at a distance of 10 parsecs

$$M = m - 5\log\left(\frac{d}{10\ \mathrm{pc}}\right) \quad \Longrightarrow \quad m = M + 5\log\left(\frac{d}{10\ \mathrm{pc}}\right)$$

why does this formula have a + sign? why 5?

the Sun has $M_V = +4.86$, $M_B = +5.52$

the *distance modulus* μ of an object is a logarithmic distance:

$$\mu = m - M = 5\log\left(\frac{d}{10\ \mathrm{pc}}\right)$$

bolometric magnitudes

magnitude version of the bolometric flux (all wavelengths combined)

$$m_{\rm bol} = -2.5\log\left(\frac{f_{\rm bol}}{f_0}\right) = -2.5\log\left(\int_0^\infty f_\lambda\, d\lambda\right) + \rm const$$

can define a *bolometric correction BC* to go
from a filtered magnitude (usually *V*) to the bolometric magnitude

$$BC = m_{\rm bol} - m_V \;\;\Leftrightarrow\;\; m_{\rm bol} = m_V + BC$$

can also define an absolute bolometric magnitude:

$$BC = m_{\rm bol} - m_V = M_{\rm bol} - M_V \;\;\Leftrightarrow\;\; M_{\rm bol} = M_V + BC$$

the Sun has $M_{\rm bol} = +4.74$ so we can also relate the bolometric magnitude to the luminosity *L*

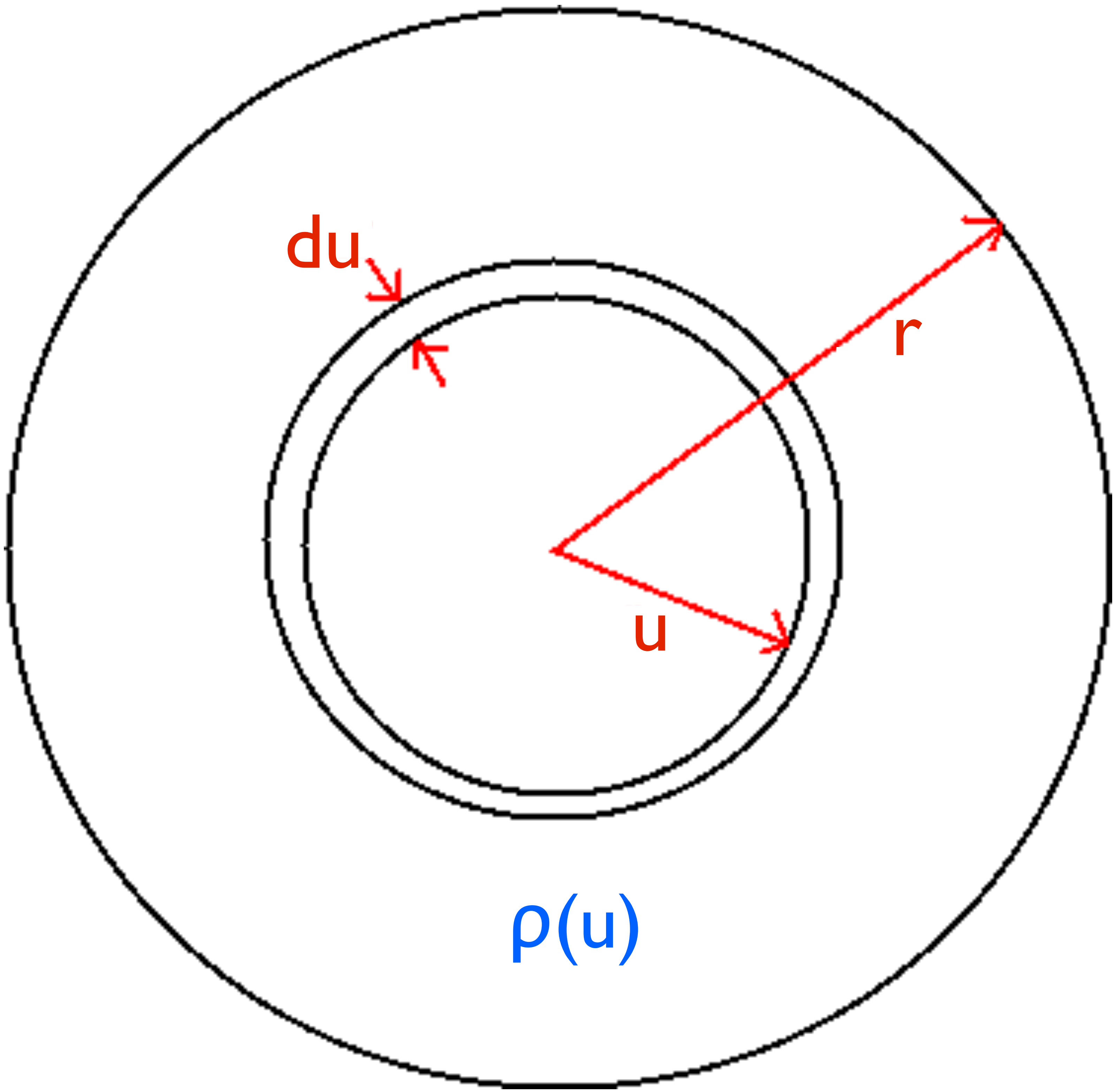
$$M_{\rm bol} = -2.5\log\left(\frac{L}{L_\odot}\right) + 4.74$$

Table 2.1. The Temperature, Spectral Type, Luminosity, Absolute Bolometric Magnitude, Absolute Visual Magnitude, Bolometric Correction, and Colors of Main-sequence Stars (Cox 2000).

T_{eff} (K)	Type	log $L/L_\odot$	M_{bol}	M_V	BC	U-B	B-V
42,000	O5 V	5.94	−10.1	−5.7	−4.40	−1.19	−0.33
30,000	B0 V	4.78	−7.2	−4.0	−3.16	−1.08	−0.30
20,900	B2 V	2.39	−4.8	−2.45	−2.35	−0.84	−0.24
11,400	B8 V	2.34	−1.1	−0.25	−0.80	−0.34	−0.11
8180	A5 V	1.17	+1.8	+1.95	−0.15	+0.10	+0.15
6650	F5 V	0.54	+3.4	+3.5	−0.14	−0.02	+0.44
5790	G2 V	−0.10	+4.5	+4.7	−0.20	+0.12	+0.63
4830	K2 V	−0.50	+6.0	+6.4	−0.42	+0.64	+0.91
3840	M0 V	−1.06	+7.4	+8.8	−1.38	+1.22	+1.40
3170	M5 V	−1.94	+9.6	+12.3	−2.73	+1.24	+1.64

Cox (2000), Lamers & Levesque Table 2.1

spherical systems: integrating over shells



the mass enclosed out
to a radius r is called the

enclosed mass:

$$m(r) = \int_{u=0}^{u=r} \rho(u)\, 4\pi u^2\, du$$

$$= M(r) = M(<r) = M_r$$

varied notation

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

from http://burro.cwru.edu/Academics/Astr221/StarPhys/gravity.html

hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)g(r) = -\frac{Gm(r)\rho(r)}{r^2}$$

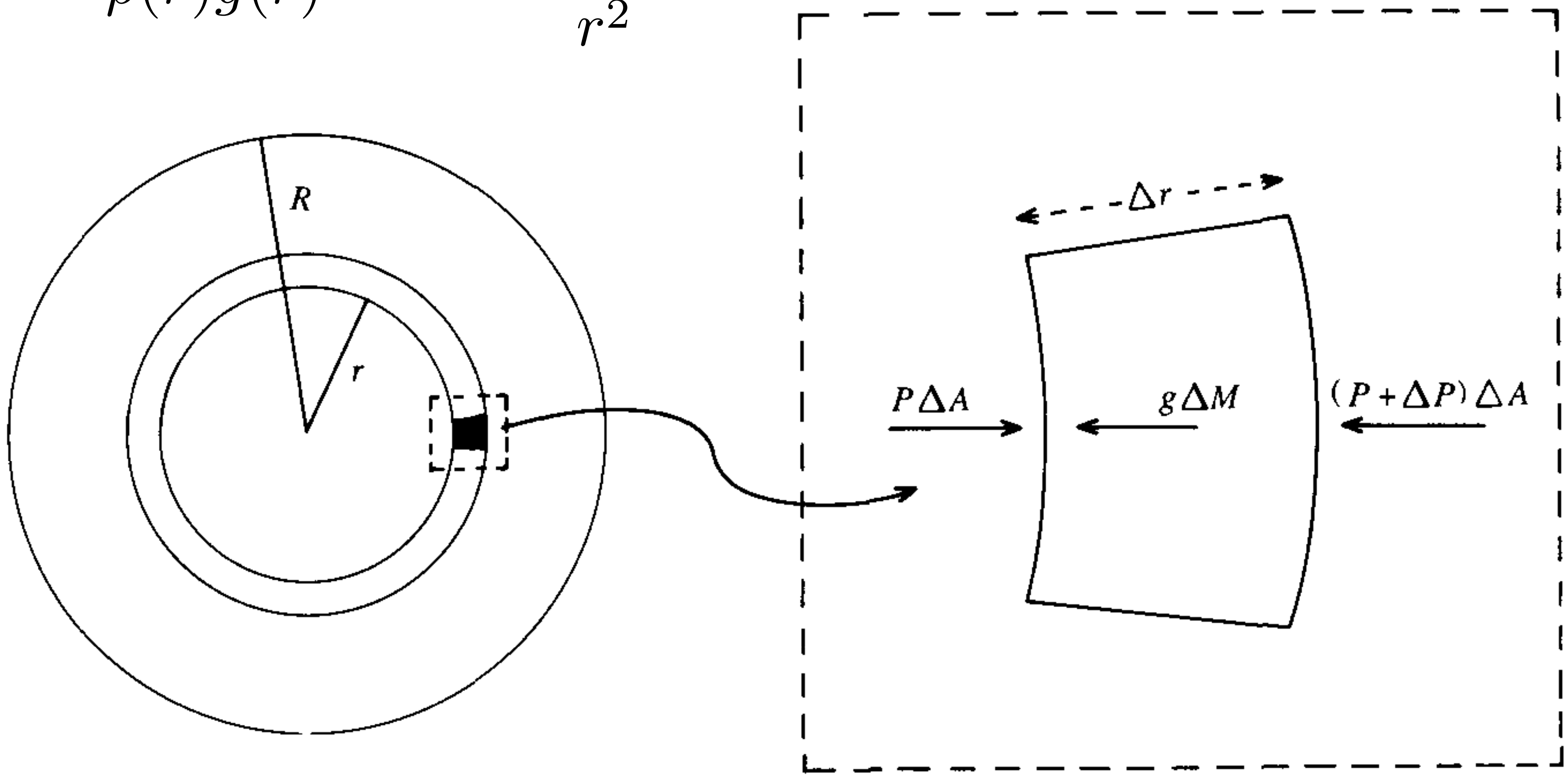


Fig. 1.1 A spherical system of mass M and radius R. The forces acting on a small element with volume $\Delta r\ \Delta A$ at distance r from the centre due to gravity and pressure are indicated. The gravitational attraction of the mass $m(r)$ within r produces an inward force which is equal to $g(r)\ \rho(r)\ \Delta r\ \Delta A = g(r)\ \Delta M$. If there is a non-zero pressure gradient at r, the difference in pressure on the inner and outer surfaces leads to an additional force which can oppose gravity.

Phillips Figure 1.1

spherical system: gravitational potential energy

$$E_{GR} = -\int_{m=0}^{m=M} \frac{Gm(r)}{r}\,\mathrm{d}m, \quad = -\int_0^R \frac{Gm(r)\rho(r)4\pi r^2}{r}\,\mathrm{d}r.$$

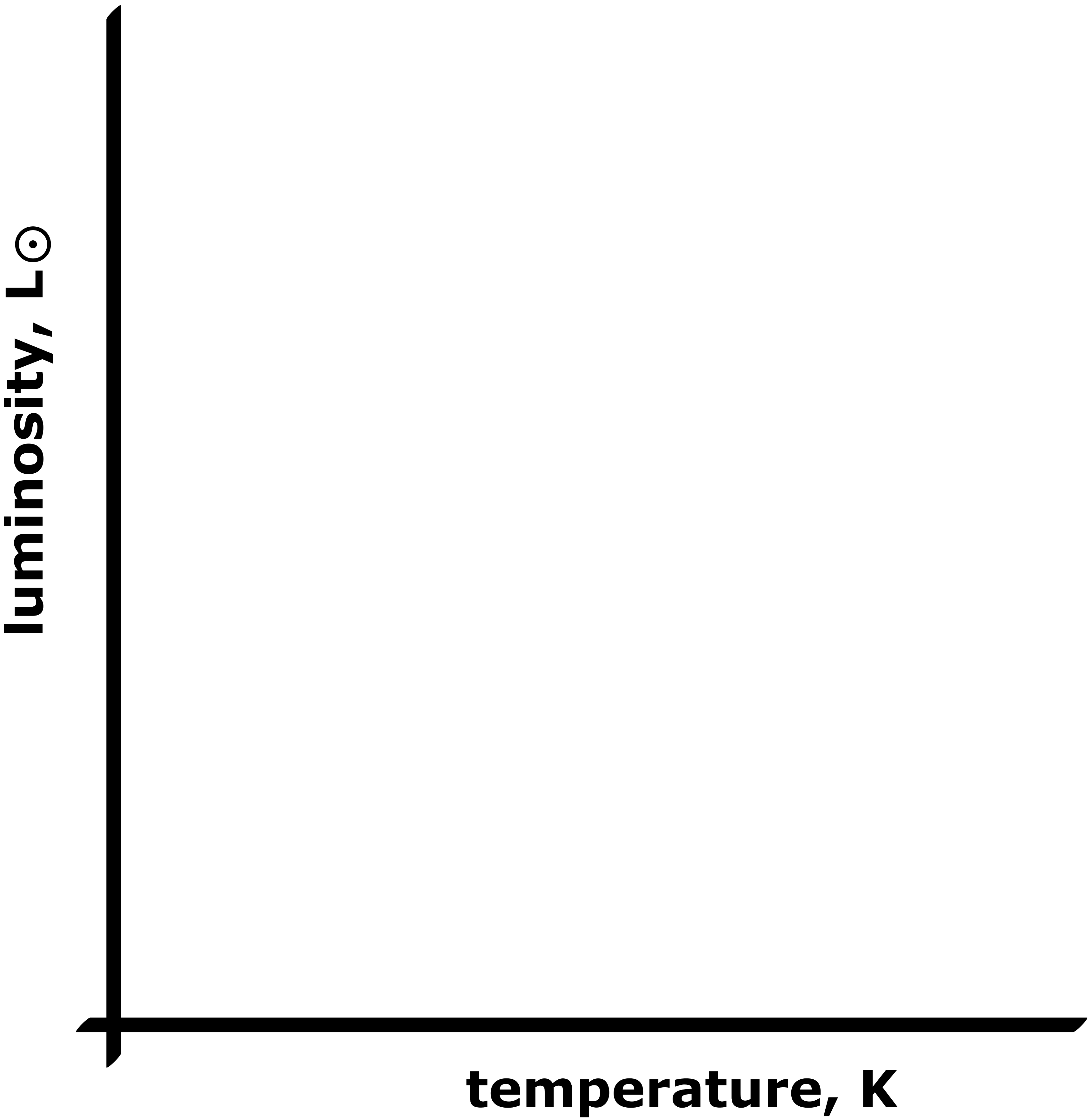
$$= -4\pi G \int_0^R m(r)\,\rho(r)\,r\,dr$$

the average pressure of a spherical system is directly related
to the gravitational potential energy density; this is the *stellar virial theorem:*

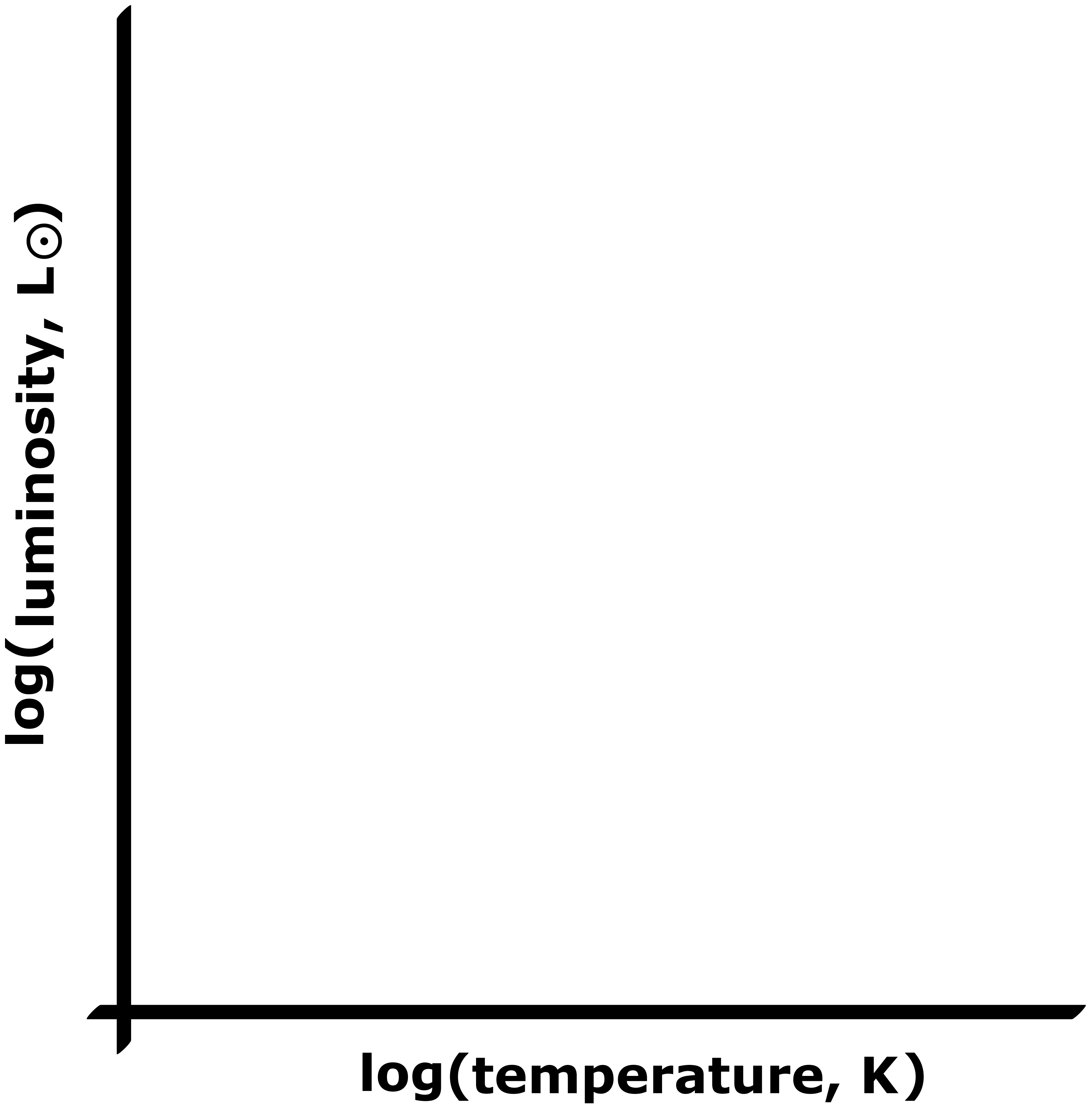
$$\langle P \rangle = -\frac{1}{3}\frac{E_{GR}}{V}.$$

V here is the volume
of the star

The Hertzsprung-Russell Diagram



The Hertzsprung-Russell Diagram



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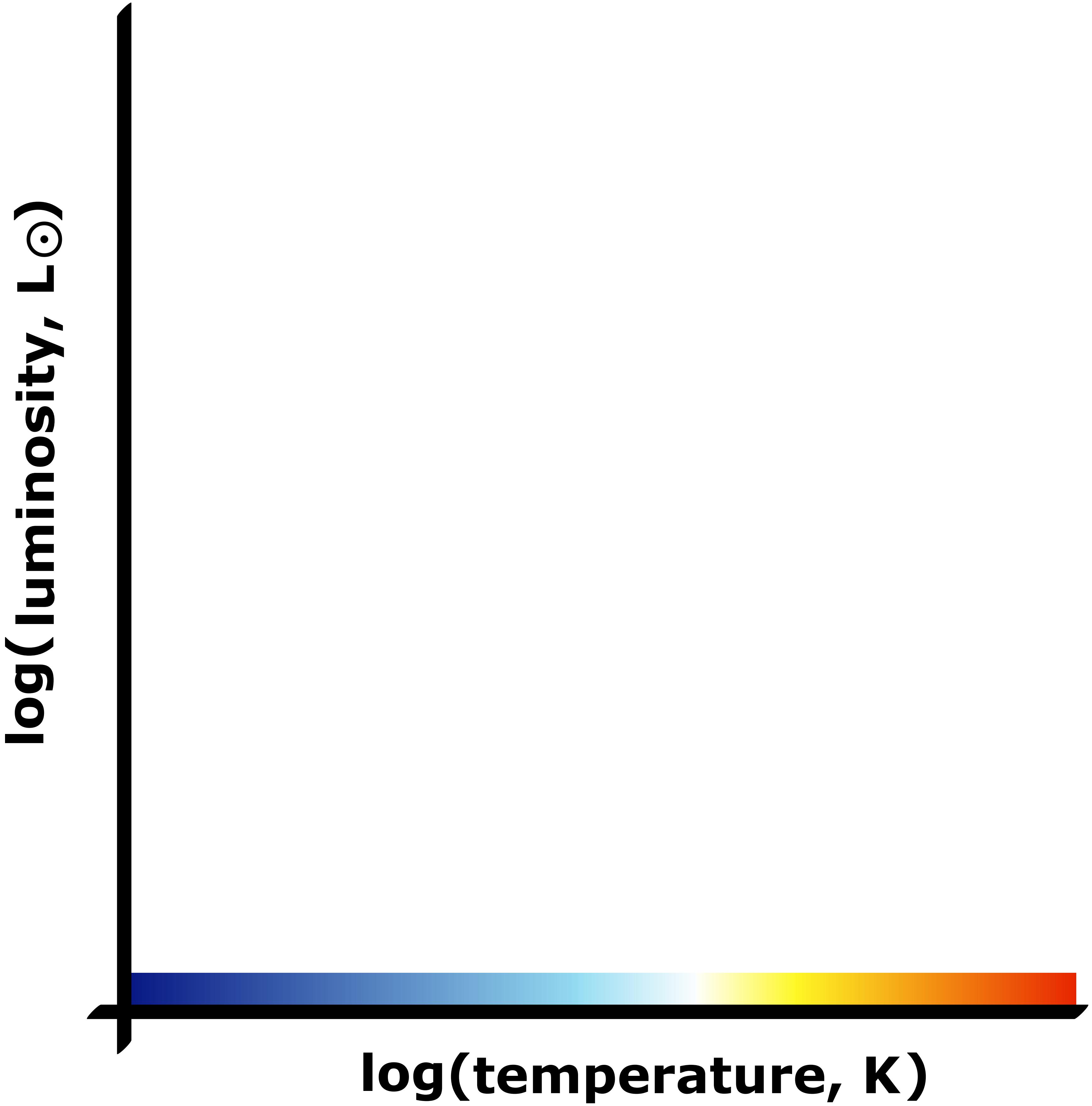

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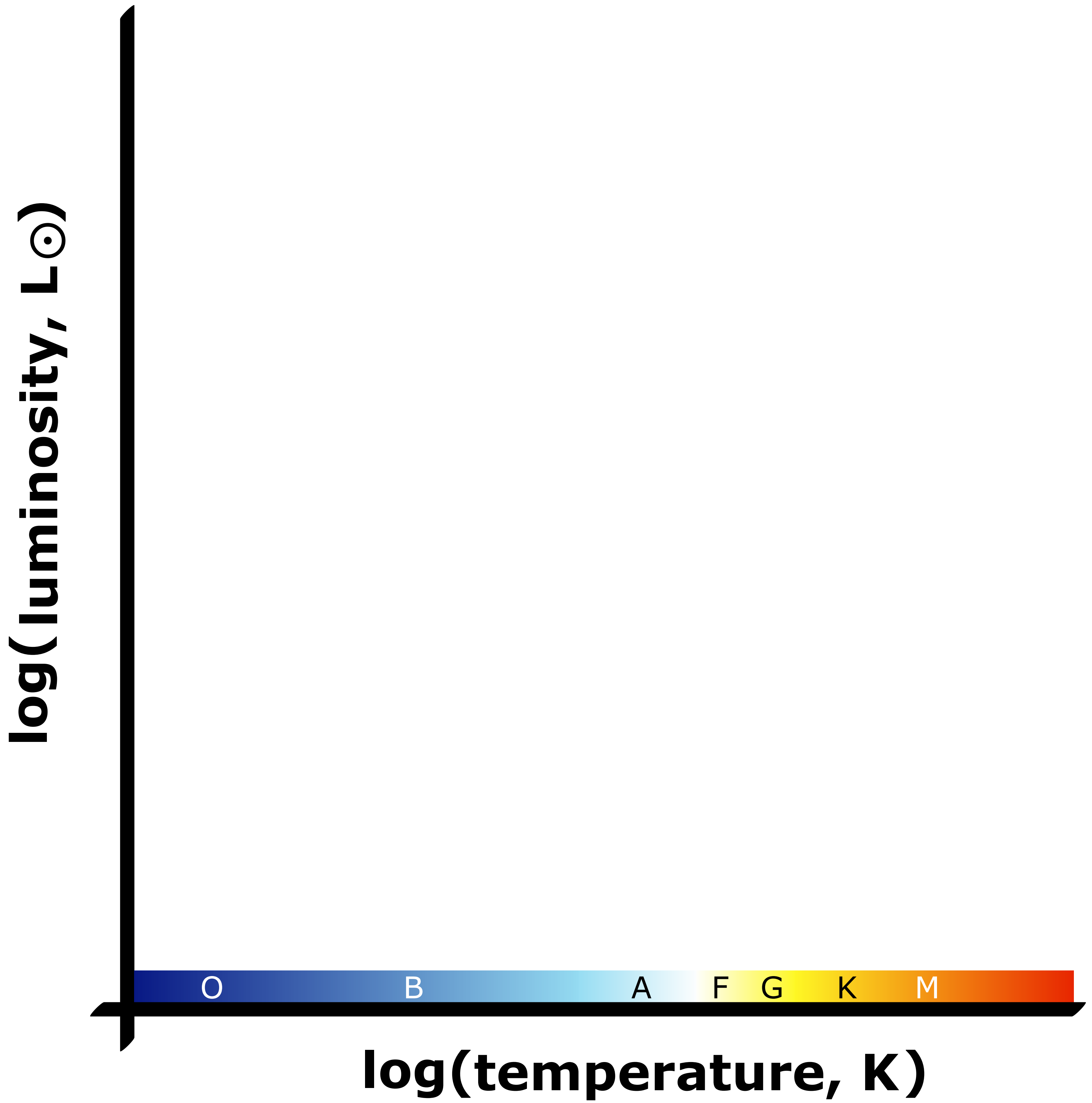

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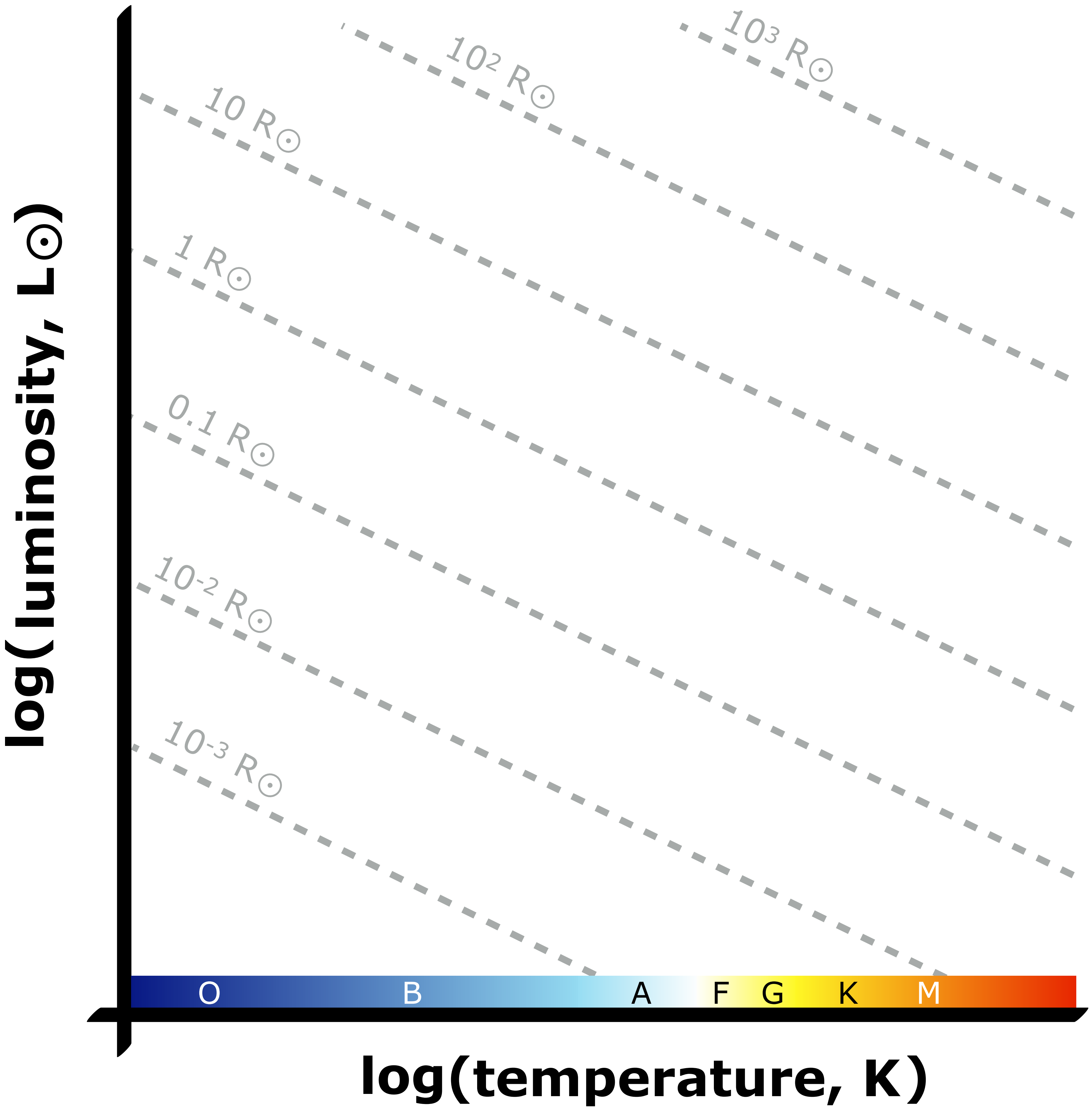
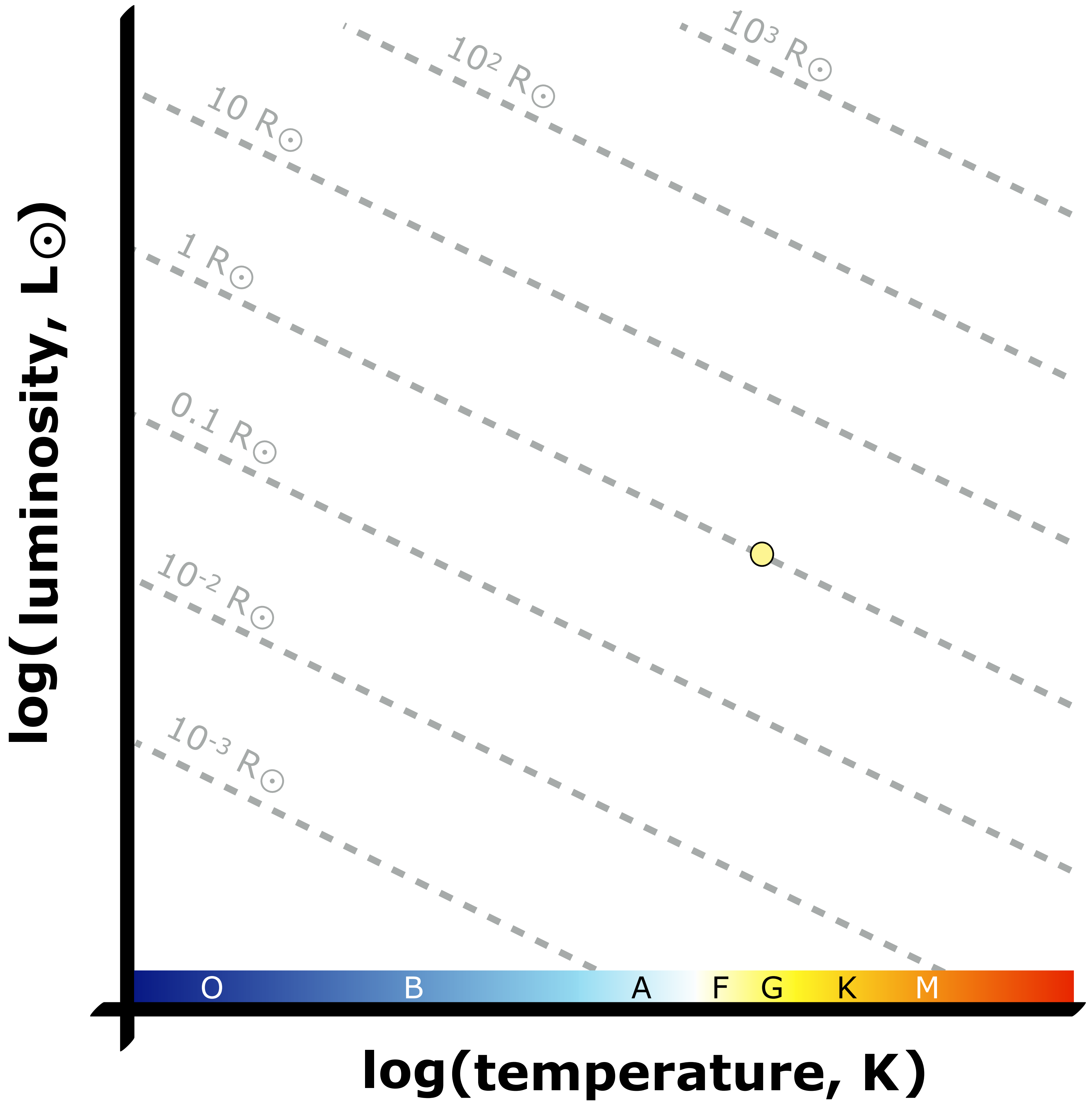


lines of constant radius

The Hertzsprung-Russell Diagram



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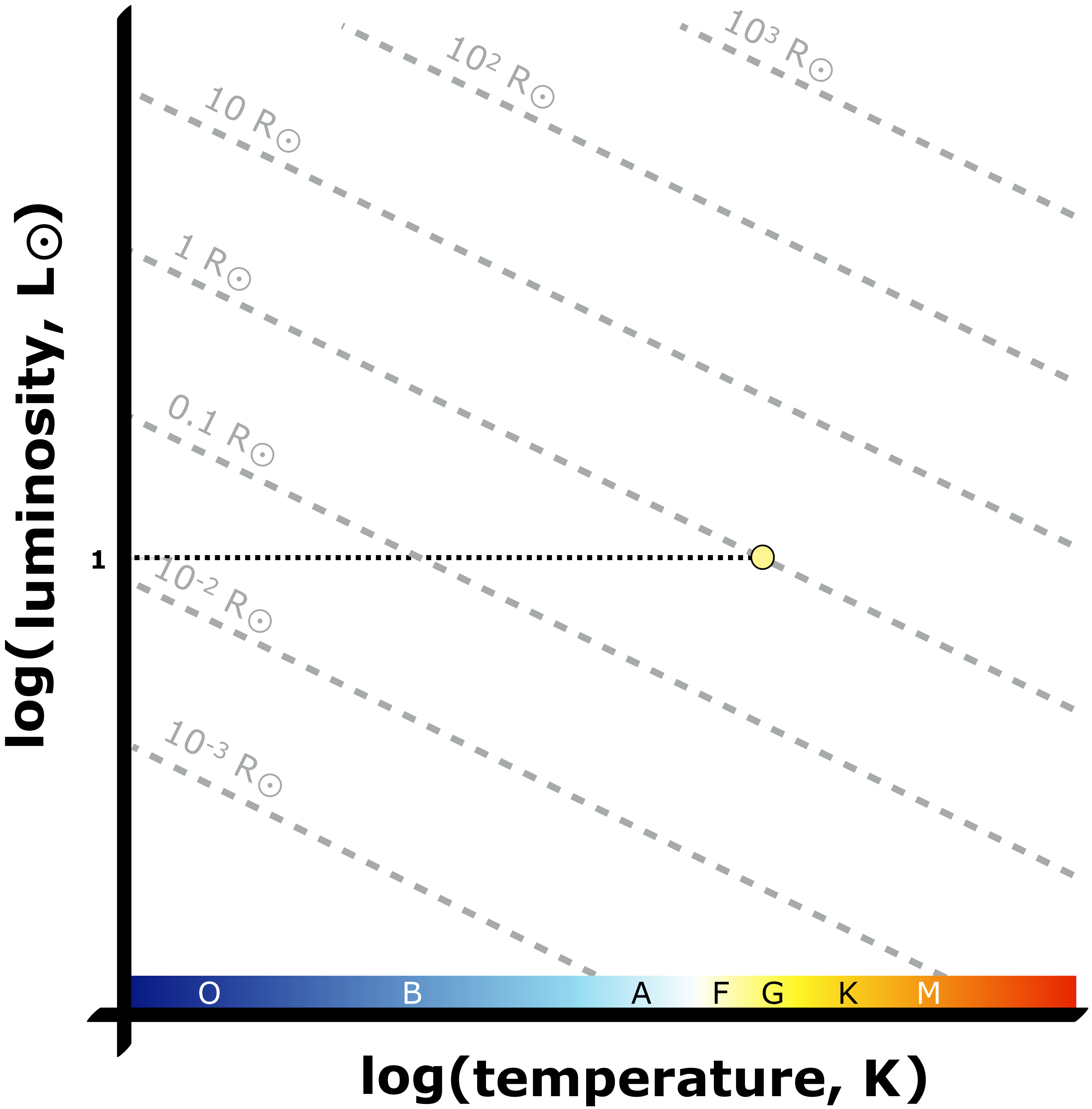

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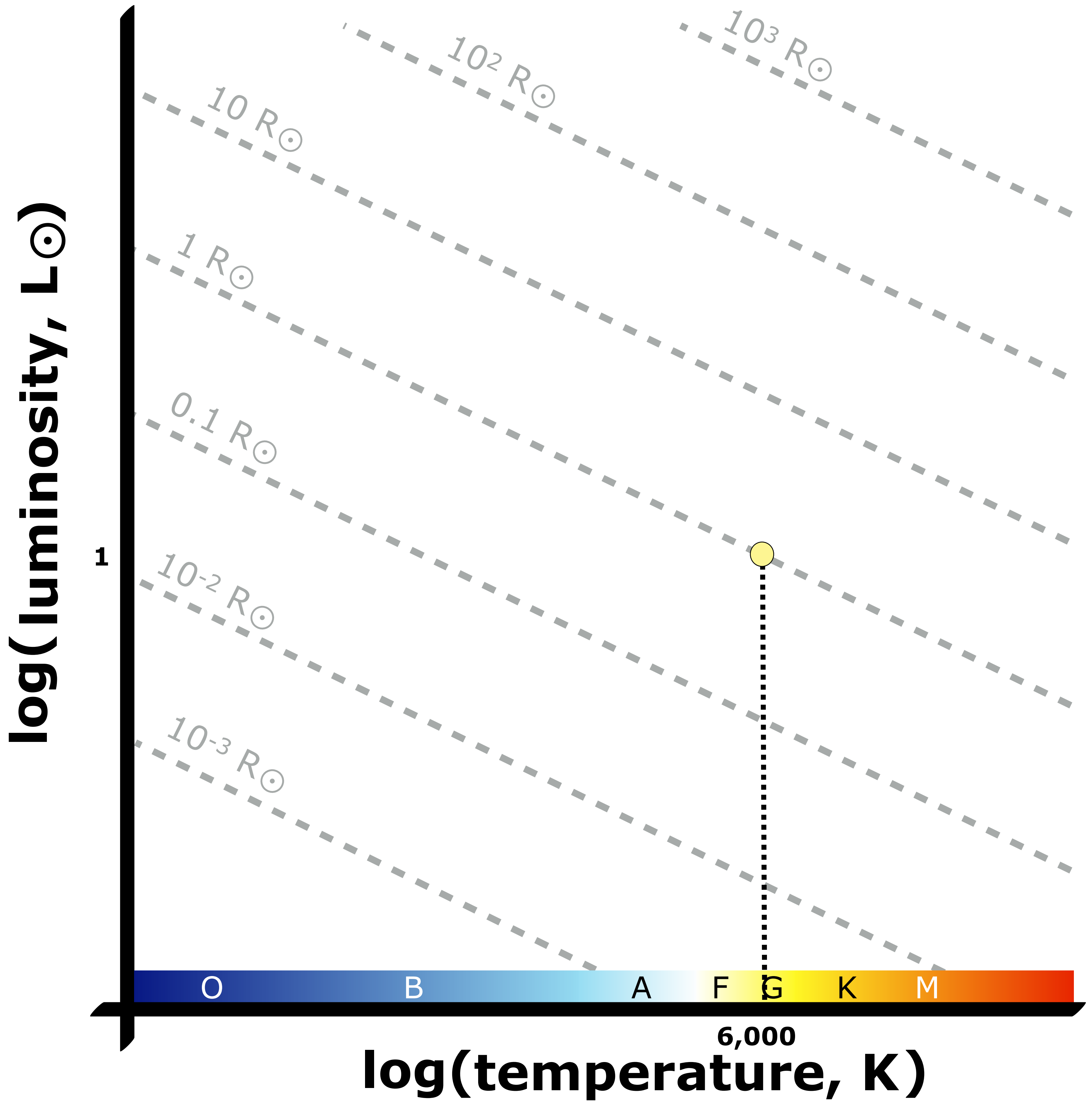

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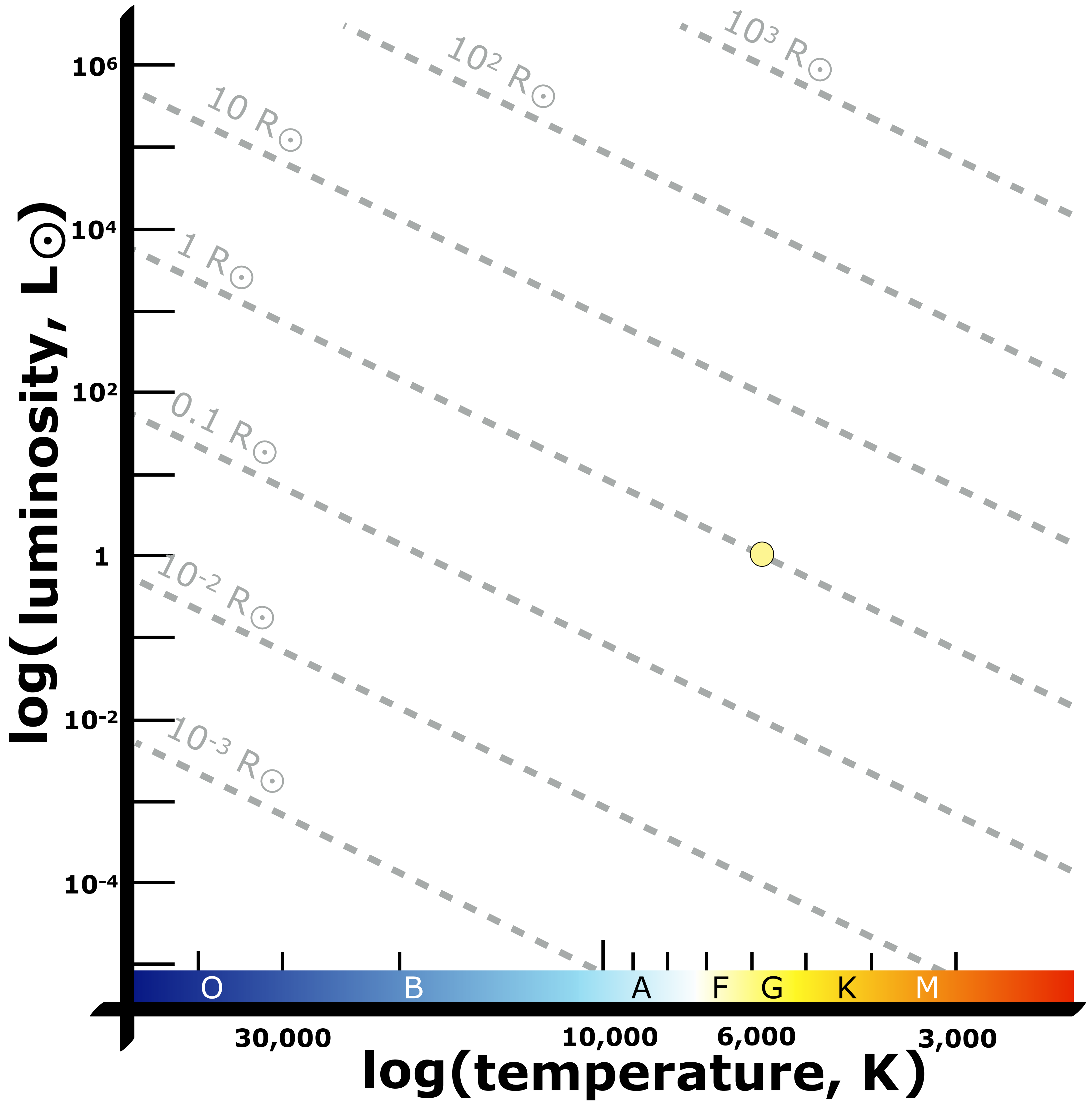

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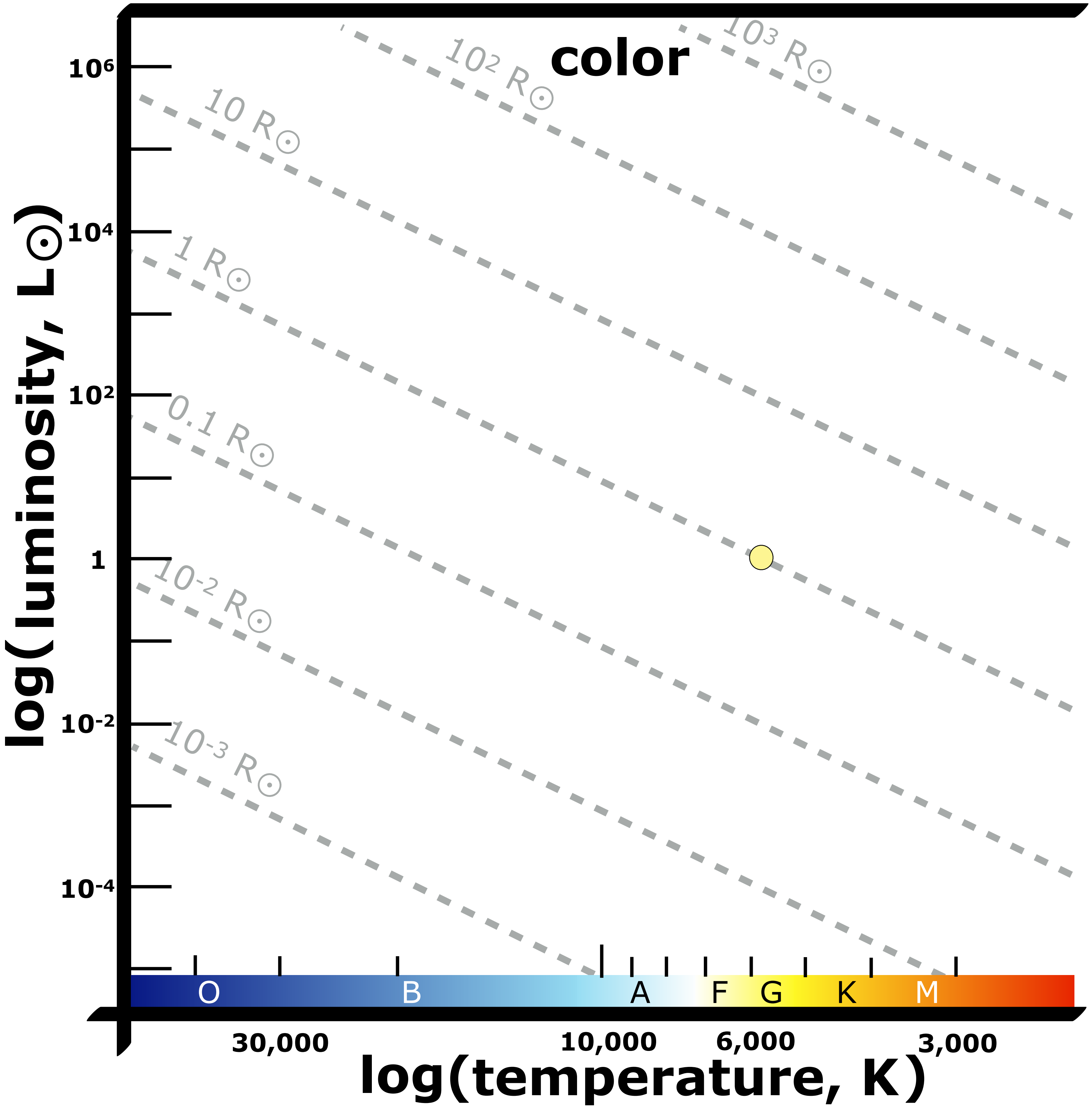
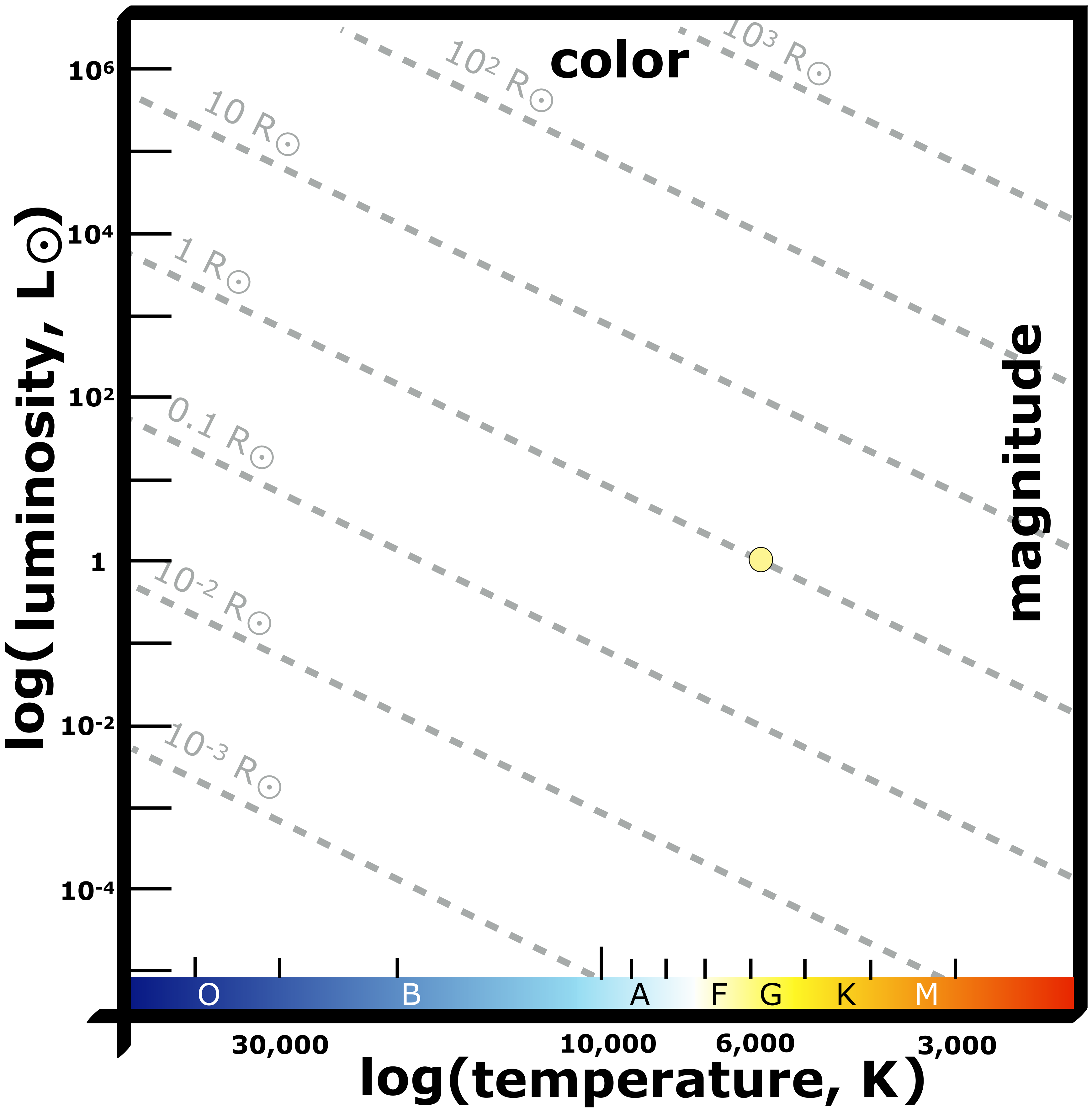
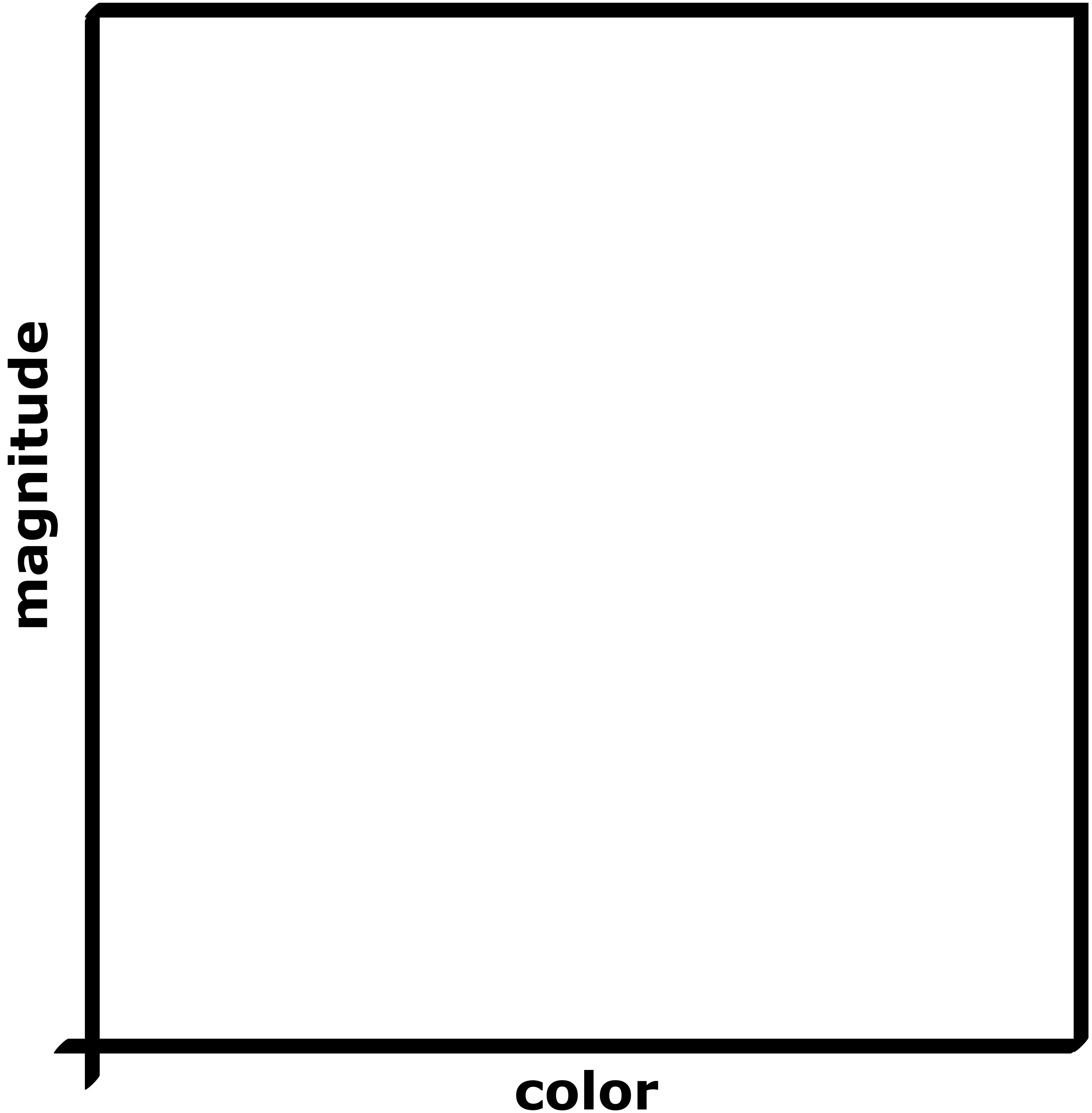


The Hertzsprung-Russell Diagram



The Color-Magnitude Diagram



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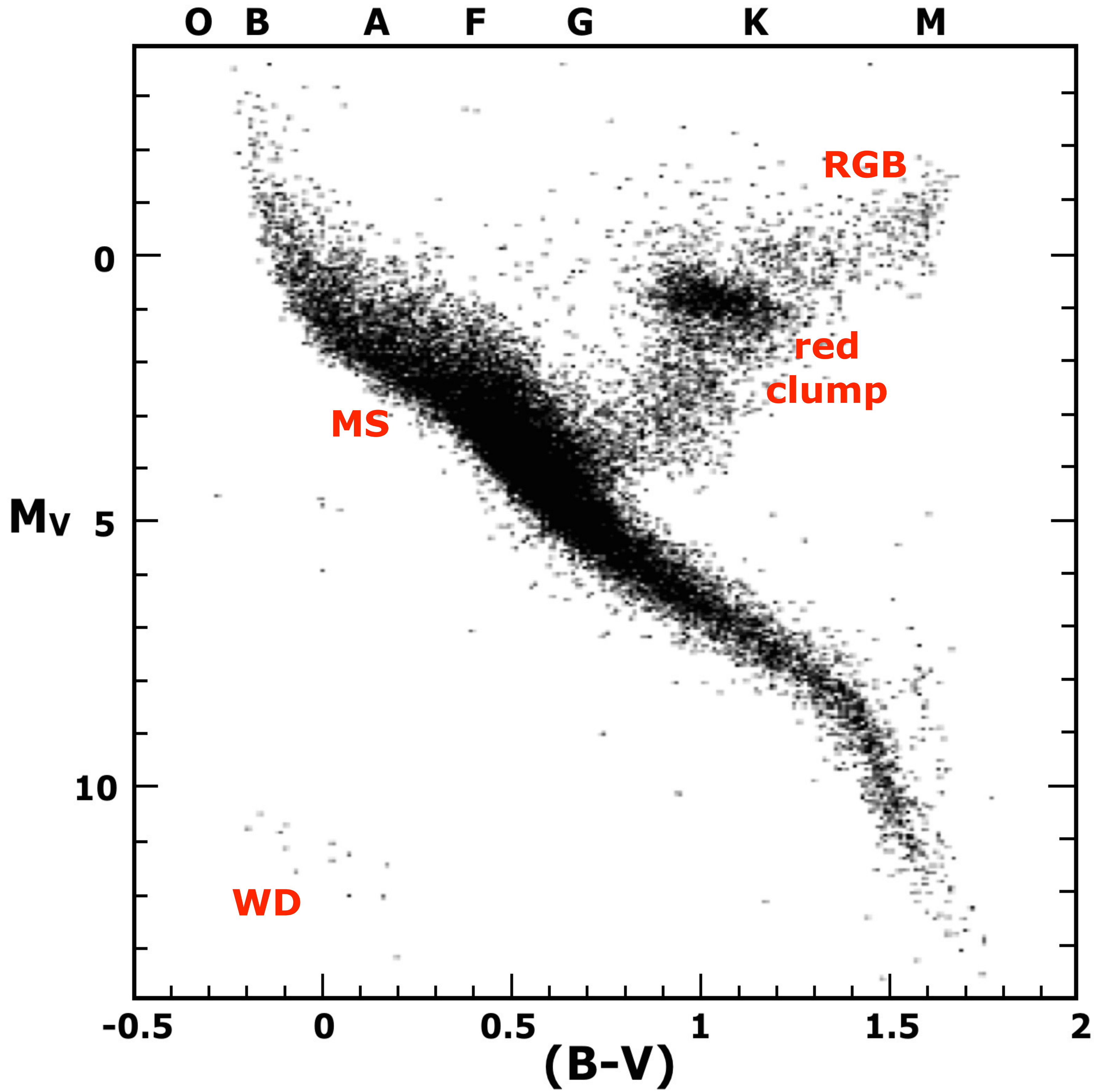
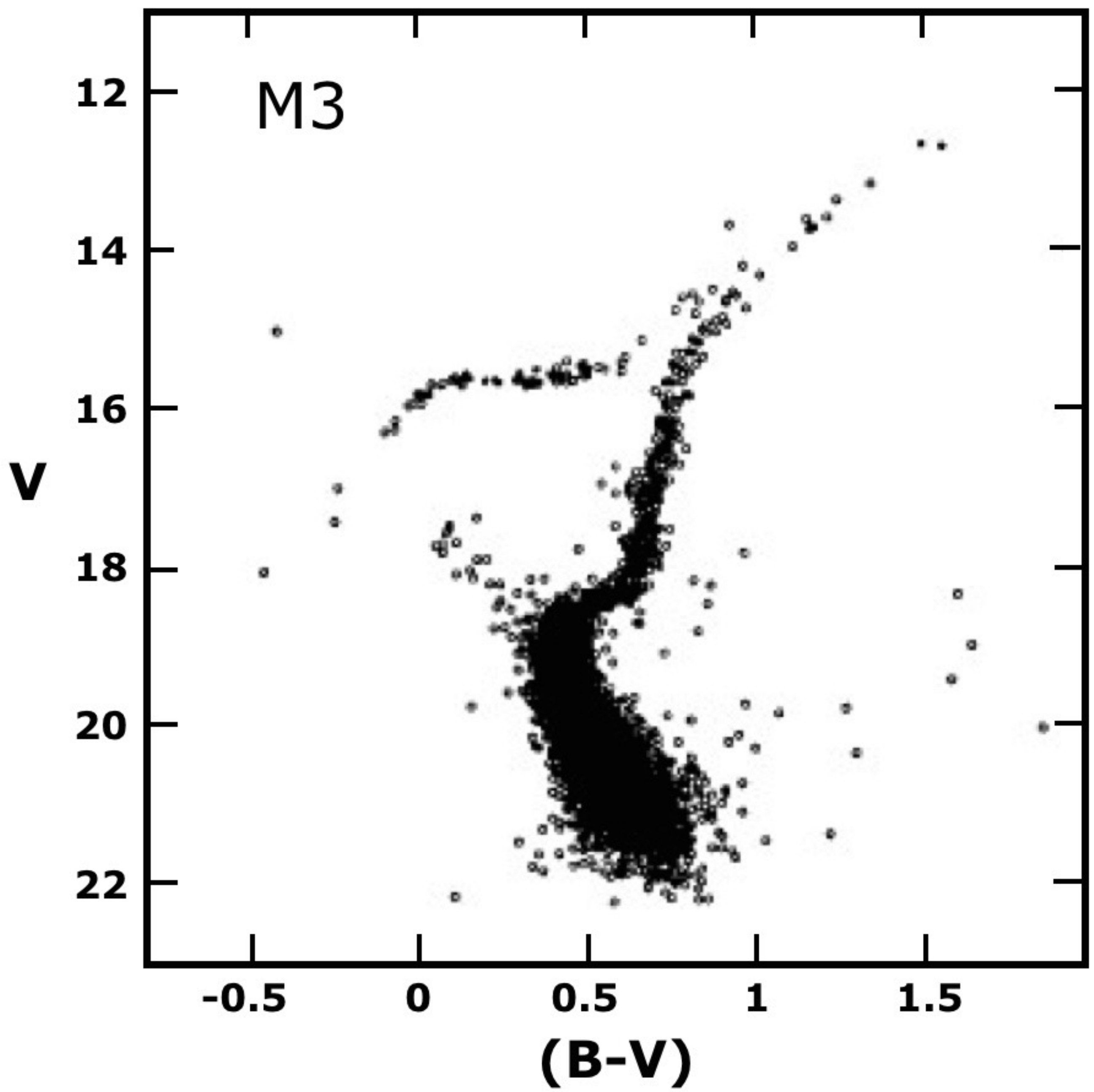
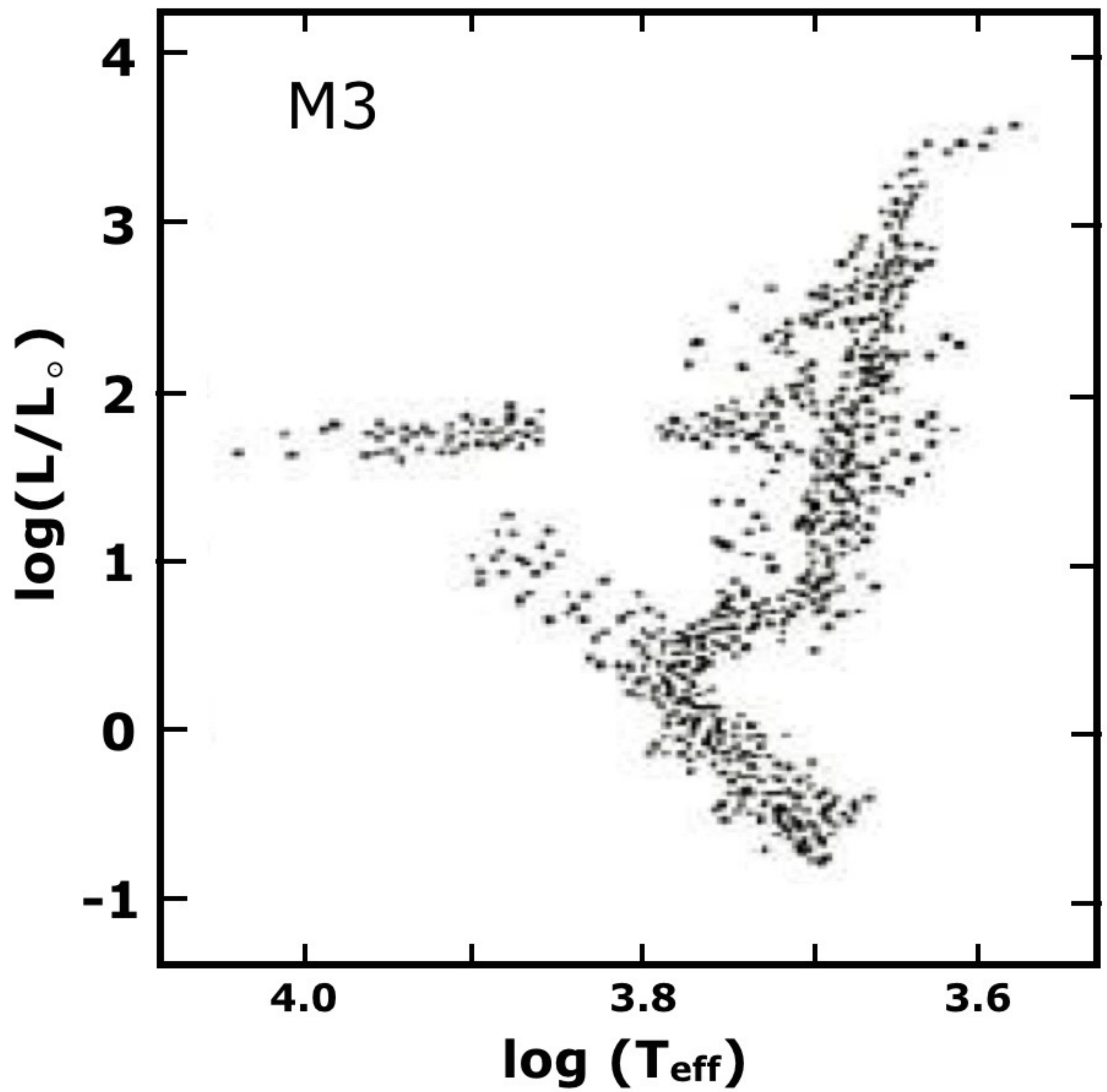


Figure courtesy of ESA.

The Color-Magnitude Diagram





1. on the blue/hot side stars cover a small (B-V), large T_{eff} range
2. on the red/cool side, stars cover a small T_{eff}, large (B-V) range
3. horizontal branch curves down strongly in the CMD

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Right: Reproduced from Johnson, H. L.and Sandage, A. R., 'Three-Color Photometry in the Globular Cluster M3', Astrophysical Journal, vol. 124, p.379,1956.