Physics 444: Problem Set #2
due September 22, 2021

1. Imagine you are living in a 2-dimensional space. You look at a small object which is oriented so that its physical extent perpendicular to your line of sight is $d\ell$.

   (a) If the space you are living in is flat, what is the angular size $d\theta$ that the small object takes up in your field of view as a function of the distance $r \gg d\ell$ between you and the object?

   (b) If instead you are living on a sphere of radius R, what is the angular size $d\theta$ that the small object takes up in your field of view, as a function of the distance *measured on the surface of the sphere* r? (You can imagine that the rays of light travel along the surface of the sphere.) Explain the behavior of $d\theta$ as $r \to \pi R$.

2. Imagine we lived in a *static* (that is, not expanding or collapsing) universe that was topologically a 3-sphere and had a radius of curvature of $R = 5$ Gpc.

   (a) We will eventually learn that the Cosmic Microwave Background has a special scale length that is particularly important. This length (in our Universe) is 250 kpc. Assuming the CMB is 4.2 Gpc away, what is the angular size of this length in our imagined curved, non-expanding universe? How would that compare to the angular size in a flat universe?

   (b) Assume that you were really living in the curved universe, but *thought* you were living in the flat one. How far away would you think the CMB was?

   (c) Assuming you could see it, what would the apparent angular size of an object at a distance of πR be? Explain what your result means, physically.

3. Suppose you are a two-dimensional being, living on a sphere of radius R. Show that if you draw a circle of radius r, the circle's circumference will be

$$C = 2\pi R \sin(r/R).$$

Idealize the Earth as a perfect circle of radius $R = 6371$ km. If you could measure distances with an error of ± 1 meter, how large a circle would you have to draw on the Earth's surface to convince yourself that the Earth is spherical rather than flat?