Physics 444: Midterm
October 31, 2019

Name:

You have 80 minutes for this exam. You may use a calculator and 1 pre-prepared sheet of equations. Each question is worth 10 points. Partial credit is given, so be sure to explain your reasoning. Unless explicitly stated, do not assume the parameters of the Universe assumed in a problem are those of the Benchmark Universe.

1. Suppose you lived in a spatial-flat universe filled only with photons. The photons are in a Maxwell-Boltzmann distribution with a temperature T that changes with time and the scale factor. What is the Hubble parameter in terms of fundamental constants and the temperature T?

2. In general, we assume that energy densities must have $|w| \leq 1$, but let us consider for a moment a spatially-flat universe filled with energy density that has $w = +2$.

   (a) If you had a box full of this material, with dimensions of the box that changed with the scale factor a, how much energy would be in the box when $a = 1/3$ as compared to when $a = 1$?

   (b) From the Friedmann Equation, determine the time-dependence of a for this universe. (You must prove this time-dependence, not just assert an answer from your equation sheet)

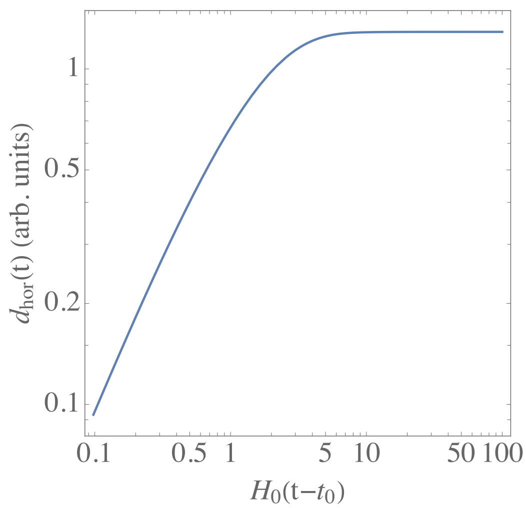
3. Consider a spatially-flat universe filled with matter, with a present-day Hubble parameter of $H_0 = 70$ km/s/Mpc. In this universe, there is a galaxy located at a comoving coordinate of $r = 1$ Gpc from you.

   (a) A supernova went off in this galaxy, which is seen by you today (time t_0). At what time t_e did the supernova occur?

   (b) What is the luminosity distance of this supernova?

4. In the **Benchmark Universe**, what was the Hubble parameter at matter-radiation equality?

5. Let us consider a universe which is **not** the Benchmark Universe. Below is a plot of the distance today ($t = t_0$) to the furthest object that can be seen at a future time t in that universe. Does this universe contain dark energy (i.e., does $\Omega_\Lambda \neq 0$)? Explain your reasoning.



Cheat Sheet

Physical constants and useful conversions:

$$\begin{aligned}
c &= 3\times 10^{8}\ \mathrm{m/s}\\
G &= 6.67\times 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}\\
k &= 8.62\times 10^{-5}\ \mathrm{eV/K}\\
h &= 6.63\times 10^{-34}\ \mathrm{Js} = 4.14\times 10^{-15}\ \mathrm{eVs}\\
1\ \mathrm{Mpc} &= 3.09\times 10^{22}\ \mathrm{m}\\
1\ \mathrm{J} &= 6.24\times 10^{18}\ \mathrm{eV}\\
1\ \mathrm{km/s/Mpc} &= 3.24\times 10^{-20}\ \mathrm{s^{-1}}\\
1\ \mathrm{year} &= 3.15\times 10^{7}\ \mathrm{s}\\
1\ \mathrm{parsec} &= 3.26\ \mathrm{ly}\\
1\,M_\odot &= 2\times 10^{30}\ \mathrm{kg}\\
1\,L_\odot &= 3.83\times 10^{26}\ \mathrm{W} = 3.83\times 10^{33}\ \mathrm{erg/s}
\end{aligned}$$

Benchmark Universe:

$$T_0 = 2.7255\ \mathrm{K},\ H_0 = 67.8\ \mathrm{km/s/Mpc},\ t_0 = 13.8\ \mathrm{Gyr}, \epsilon_{c,0} = 4870\ \mathrm{MeV/m^3},$$
$$\Omega_{\Lambda,0} = 0.692,\ \Omega_{m,0} = 0.306,\ \Omega_{r,0} = 9.03\times 10^{-5}$$

Equations:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm emit}}{\lambda_{\rm emit}}, \qquad z = \frac{H}{c} r \text{ (Hubble's Law)}, \ 1 + z = \frac{1}{a}$$

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa^2 d\Omega^2\right], \qquad S_\kappa(r) = \begin{cases} R_0 \sin(r/R_0) & \kappa = +1 \\ r & \kappa = 0 \\ R_0 \sinh(r/R_0) & \kappa = -1 \end{cases}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}, \qquad 1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$$

$$\dot{\epsilon} + \frac{3\dot{a}}{a}(\epsilon + P) = 0, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^3}(\epsilon + 3P)$$

$$P = w\epsilon, \qquad \epsilon_w = \epsilon_{w,0} a^{-3(1+w)}$$

$$\epsilon_\gamma(T) = \frac{\pi^2}{15}\frac{(kT)^4}{\hbar^3 c^3} \qquad n_\gamma(T) = \frac{2.4041}{\pi^2}\frac{(kT)^3}{\hbar^3 c^3}$$

$$\epsilon_c = \frac{3c^2}{8\pi G}H^2 \qquad \Omega_w = \frac{\epsilon_w}{\epsilon_c}$$

$$d_p(t_0, t_e) = c\int_{t_e}^{t_0} \frac{dt}{a(t)} \qquad d_p(t_e) = ca(t_e)\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$H_0 t = \int_0^a \frac{da}{\sqrt{\Omega_{w,0} a^{-(1+3w)} + (1 - \Omega_0)}}, \qquad a(t) = \begin{cases} \frac{t}{t_0} & \Omega_0 = 0, \kappa = -1 \\ (t/t_0)^{2/3(1+w)} & \Omega_{w,0} = 1, \kappa = 0 \\ e^{H_0(t-t_0)} & \Omega_\Lambda = 1, \kappa = 0 \end{cases}$$

$$t_0 = \frac{2}{3(1+w)}H_0^{-1} \qquad (\Omega_{w,0} = 1, \kappa = 0, w > -1)$$

$$d_A = \frac{\ell}{\delta\theta} = \frac{S_\kappa(r)}{1+z} \qquad d_L = (1+z)S_\kappa(r)$$

$$d_L(z) \approx \frac{cz}{H_0}\left[1 + \left(\frac{1 - q_0}{2}\right) z\right] \qquad q_0 = \frac{1}{2}\sum_w \Omega_{w,0}(1 + 3w)$$