Physics 444: Problem Set #3
due September 29, 2021

1. The principle of wave-particle duality tells us that a particle with momentum p has an associated de Broglie wavelength $\lambda = h/p$; this wavelength increases as $\lambda \propto a$ as the Universe expands.

   (a) Suppose a particle of mass m was moving relative to the comoving coordinates of the Universe with speed $v = \beta_0 c$ at time t_0. Calculate v as a function of $a(t)$.

   (b) Calculate the acceleration of the particle (relative to the comoving coordinates) as a function of $a(t)$ and the Hubble parameter $H(t) = \dot{a}/a$.

2. Suppose the energy density of the cosmological constant is equal to the present critical density $\epsilon_\Lambda = \epsilon_{c,0} = 4870$ MeV/m^3. What is the total energy of the cosmological constant within a sphere 1 AU in radius? What is the rest energy of the Sun? Comparing these two numbers, do you expect the cosmological constant to have a significant effect on the motion of planets within the Solar System?

   How does the energy in the cosmological constant within a sphere of 1 AU centered on the Sun compare to the energy contained in the sunlight within that radius?

3. The visible Universe consists of about 10^{53} kg of baryonic matter, of which only about 25% is in galaxies. Let us assume that all of the mass in galaxies is in stars, all of which are just like our sun, with a luminosity $L_\odot = 3.8 \times 10^{26}$ W (this assumption massively overestimates the number of stars and their average luminosity). Let us further assume that all of these stars have been burning continuously since the beginning of the Universe, 13.8 billion years ago (this assumption again overestimates the length of time stars have been burning). Finally, let us assume that none of the photons emitted by these stars are absorbed by other matter (again, clearly a ridiculous assumption). What is the energy in photons in the Universe today as a result of stellar fusion? Compare this to the energy in the stars themselves.