## 4.1 Fixed Points

Let *M* be a matrix given by

$$\begin{pmatrix}
3 & 2 \\
2 & 3
\end{pmatrix}
\begin{pmatrix}
5 & 4 \\
4 & 5
\end{pmatrix}$$

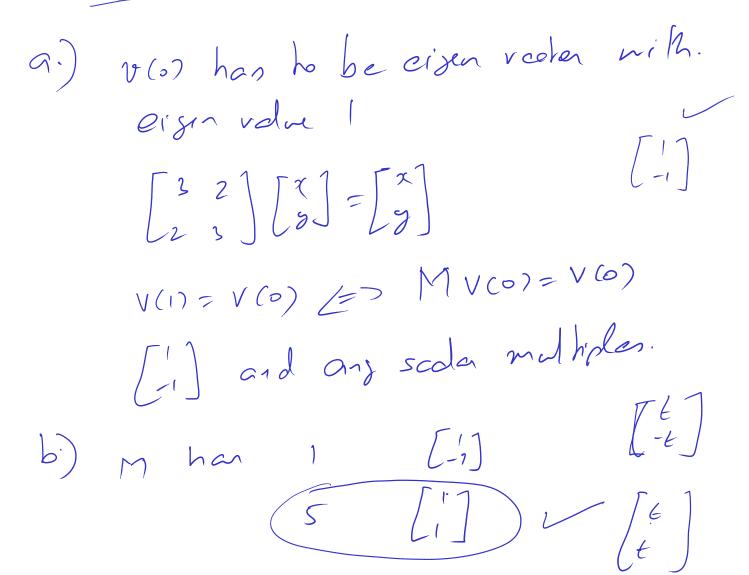
Given any vector  $v(0) = \begin{pmatrix} x(0) \\ y(0) \end{pmatrix}$ , we can create an infinite sequence of vectors  $v(1), v(2), \cdots$  by using the rule

$$v(t+1) = Mv(t)$$
 for all natural numbers  $t$ 

a.) Find all vectors v(0) such that

$$v(0) = v(1) = v(2) = v(3) = \cdots$$

b.) Find all vectors v(0), such that v(0), v(1), v(2), v(3),  $\cdots$ , belongs to a 1 dimensional subspace.



## **Commuting Matrices** 4.2

Simultareado Dicionlinable.

Let A, B be commuting matrices of dimension  $n \times n$  (ie AB = BA) and suppose A is diagonalizable with n distinct eigenvalues.

- a.) Show that of v is an eigenvector of A with eigenvalue  $\lambda$ , then Bv is also an eigenvector of A with eigenvalue  $\lambda$ .
- b.) Show that if v is an eigenvector of A, the v is also an eigenvector of B. Should the eigenvalues be the same?

c.) Explain why the above implies that there is a single change of basis such that A, B are both diagonal in the same basis.

A(Bv) = A(Br) => Br in eigenrealer ville eigenralie A.

Eigen Space (A) = dv: Av=Av)

t. corven panding to different eigenvalues lin. indep => din (Eigen Space (x))=1

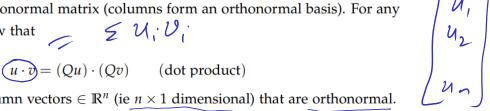
Bre, v E Eisen Space (A)

B10 = 2v

eigen veetor of B nih eigenvolue &.

## Decomposition

a.) Let Q be an  $n \times n$  orthonormal matrix (columns form an orthonormal basis). For any vectors  $\underline{v}, \underline{u} \in \mathbb{R}^n$ , show that  $\mathcal{L} \mathcal{U}_i$ 



b.) Let  $\{u_1, \dots u_n\}$  be column vectors  $\in \mathbb{R}^n$  (ie  $n \times 1$  dimensional) that are orthonormal. Suppose  $M \in \mathbb{R}^{n \times n}$  ( $n \times n$  dimensional matrix) defined by:

$$V \qquad M = \sum_{i=1}^{n} \alpha_{i} u_{i} u_{i}^{T} \quad \text{where } \alpha_{i} \text{ are scalars.} \qquad = U_{1} C_{1} + U_{2} C_{2}$$

Note that  $u_i u_i^T$  are  $n \times n$  dimensional. What are the eigenvectors and eigenvalues of M? (need to explain why)

 $u \cdot v = u^{T} v$   $(Qu) \cdot (Qv) = (Qu)^{T} (Qv)$   $u^{T} Q^{T} Q v$  $\begin{vmatrix} u_1^{\prime} & u_1 u_2 \\ u_1 u_2 & u_2^{\prime} \end{vmatrix}$ 

 $Mu_{i} = \underbrace{\sum_{i=1}^{n} \lambda_{i}}_{i} \underbrace{u_{i} u_{i}^{T} u_{i}}_{i} \underbrace{P_{i} \cdot v_{i} \cdot v_{i}}_{e_{i} \cdot v_{i}} \underbrace{e_{i} \cdot v_{i}}_{e_{i} \cdot v_{i}} \underbrace{u_{i} \cdot u_{i} \cdot v_{i}}_{u_{i} \cdot v_{i}}$   $= \lambda_{i} u_{i} \underbrace{u_{i}^{T} u_{i}}_{i} + \underbrace{\sum_{i=2}^{n} \lambda_{i} \cdot u_{i} \cdot u_{i}^{T} u_{i}}_{e_{i} \cdot v_{i}} \underbrace{v_{i} \cdot v_{i}^{T} u_{i}}_{e_{i} \cdot v_{i}}$  $\left| \mathcal{U}_{1} \right| = a \mathcal{U}_{1}$ 1s M symmetric? ti, Mi inan eigen veda with eigen value di