

1 Probability Basics

1.1 Social Groups

Suppose there is a group of 5 people and every pair of them is equally likely to be friends or non-friends. Find the probability of the event that there are at least three people who are mutual friends.

1.2 Counting Cards

I have 10 red and 10 blue cards. I shuffle the cards (all possibilities equally likely) and then label the cards based on their orders: I write the number one on the first card, the number two on the second card, and so on. What is the probability that

- a.) All red cards are assigned numbers less than or equal to 15? (2)
- b.) Exactly 8 red cards are assigned numbers less than or equal to 15? (1.5)
- c.) Exactly 5 red cards are assigned numbers greater than 12 and exactly 4 green cards are assigned numbers less than 7. (1.5)

1.3 Random Particle

Suppose you have a particle on a 2D plane starting at the origin $(0,0)$. After every second its position changes either in the x-coordinate or in the y-coordinate by exactly 1 (change cannot happen in both together). All the four possibilities are equally likely. What is the probability that the particle comes back to the origin in 6 secs? What is the probability that the particle comes back to the origin in 8 secs by remaining only in the first quadrant (ie $x, y \geq 0$ for all time)?
