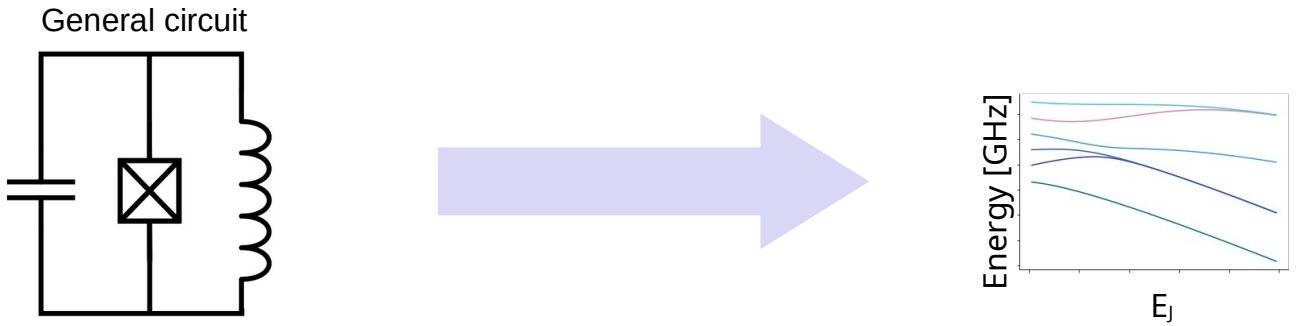
Circuit Quantization and Diagonalization

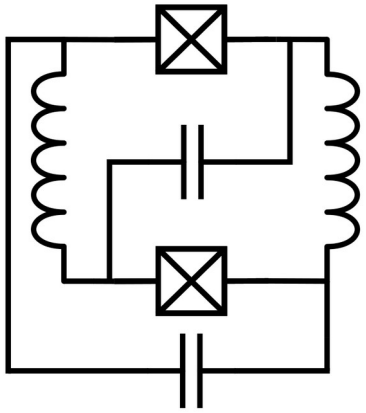
with scqubits



Sai Pavan Chitta

Northwestern University

Outline



$\mathcal{L}(\{\theta_i, \dot{\theta}_i\}; \Phi_{\text{ext}}, n_g)$

Canonical quantization

$H(\{Q_i, \theta_i\})$

- Challenges:
  - Identification of the boundary conditions
  - Appropriate variable choice

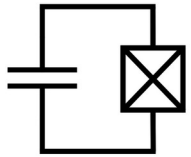
M. F. Gely et al., New J. Phys. 22: 013025 (2020)
P. Aumann et al., arXiv:2106.05342 (2021)

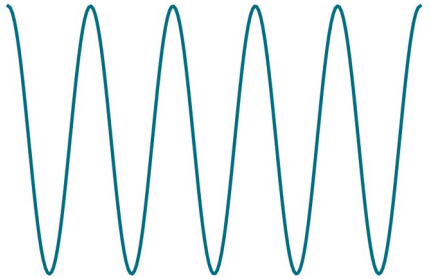
M. H. Devoret, Les Houches LXIII (1995)
B. Yurke et al., Phys. Rev. A 29, 1419 (1984)
G. Burkard et al., Phys. Rev. B 69, 064503 (2004)
S.E. Rasmussen et al., PRX Quantum 2 (4): 040204 (2021)

Boundary Conditions and Variable Classification

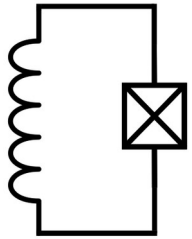
- Boundary conditions:
  - affect the nature of Hilbert space
  - influence the choice of basis states
- How do we get to the boundary conditions? - Potential
- Degree of Freedom (DoF): periodic and extended

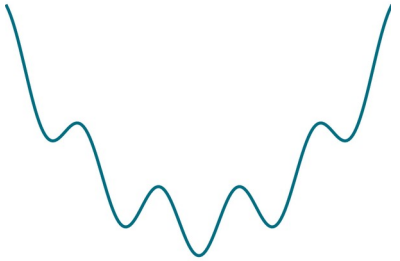
$$E_J \cos(\varphi)$$



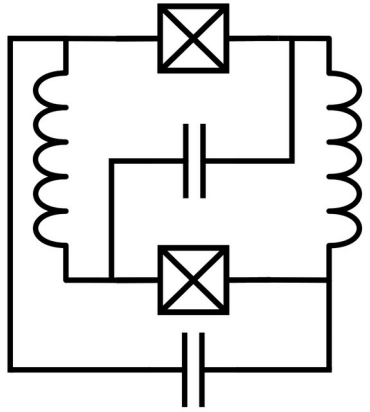


$$E_J \cos(\varphi) + \frac{E_L}{2}\varphi^2$$





$$- E_J \cos(\varphi_3) - E_J \cos(\varphi_2 - \varphi_1)$$
$$+ \frac{1}{2} E_L (\varphi_2 - \varphi_3)^2 + \frac{1}{2} E_L (\varphi_1)^2$$



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- ‘good’ vs ‘bad’ choice?

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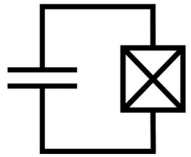

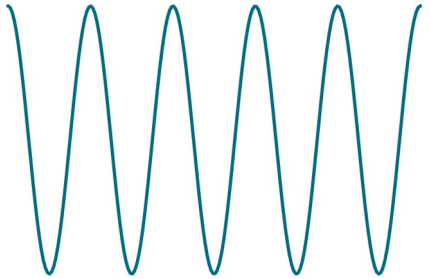

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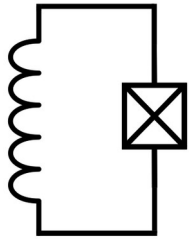

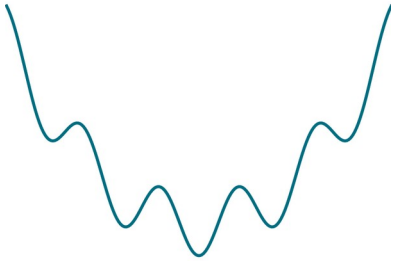

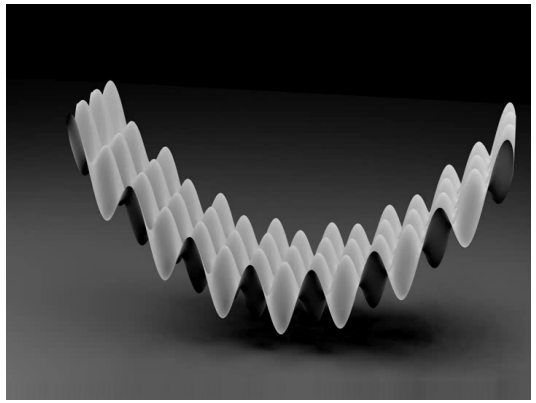

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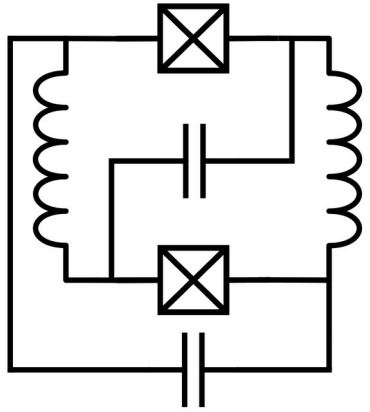

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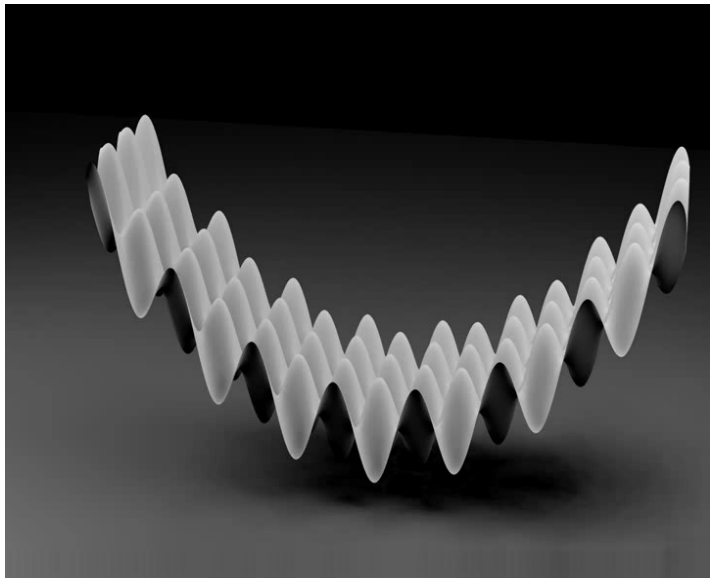
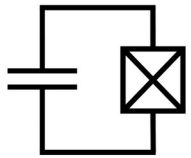
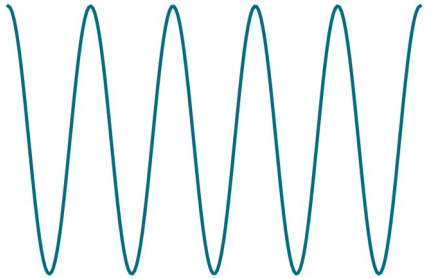
- ‘good’ vs ‘bad’ choice?
- Special cases: free and frozen

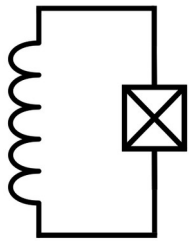


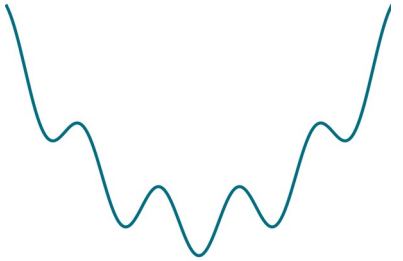
$E_J \cos(\varphi)$

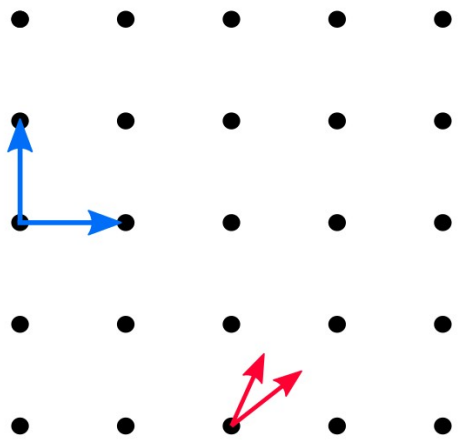




$E_J \cos(\varphi) + \frac{E_L}{2}\varphi^2$





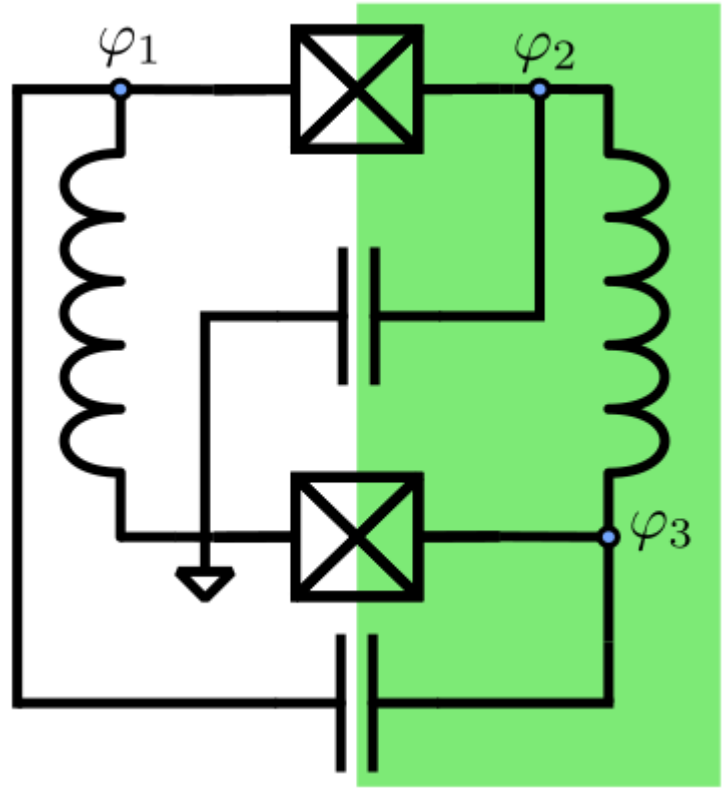


Construction of Variables

- Ho do we identify such variables? - Inspect circuit topology
  - Inductors generate extended DoFs
  - “Islands” generate periodic DoFs
- Eliminate non-dynamical variables
  - Frozen – missing capacitive connections
  - Cyclic – associated with “capacitive islands”

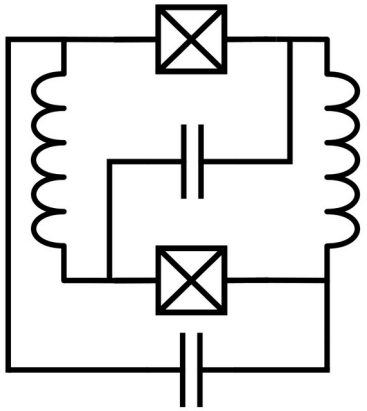
$$\begin{aligned} \varphi_1 &= \alpha \\ \varphi_2 &= \frac{1}{2}\beta + \theta \\ \varphi_3 &= -\frac{1}{2}\beta + \theta \end{aligned}$$

extended periodic



P. Groszkowski et al., New J. Phys. 20 (4): 043053 (2018)
D. Ding *et al.,* Phys. Rev. B,103:174501 (2021)

Automating Circuit Quantization



periodic, extended variables $\mathcal{L}(\{\theta_i, \dot{\theta}_i\})$

- Eliminate cyclic, frozen DoF
- Legendre transform

$H(\{\hat{Q}_i, \hat{\theta}_i\})$

graph description of circuit

↓

spanning tree & closure branches
resolve constraints / magn. flux dependence

↓

symbolic circuit Lagrangian
(node variables)

↓

find **subspaces in configuration space**:

i) **C** - no flux differences across JJ, L
ii) **P** - no flux differences across L
iii) **F** - no flux differences across C, JJ

↓

construct **transformation matrix** using basis vectors of **F, C, P** (in this order)

↓

eliminate cyclic and frozen modes

↓

Legendre transform

↓

symbolic Hamiltonian

↓

basis choice, operator construction

↓ *or* ↓

numerical Hamiltonian, diagonalization

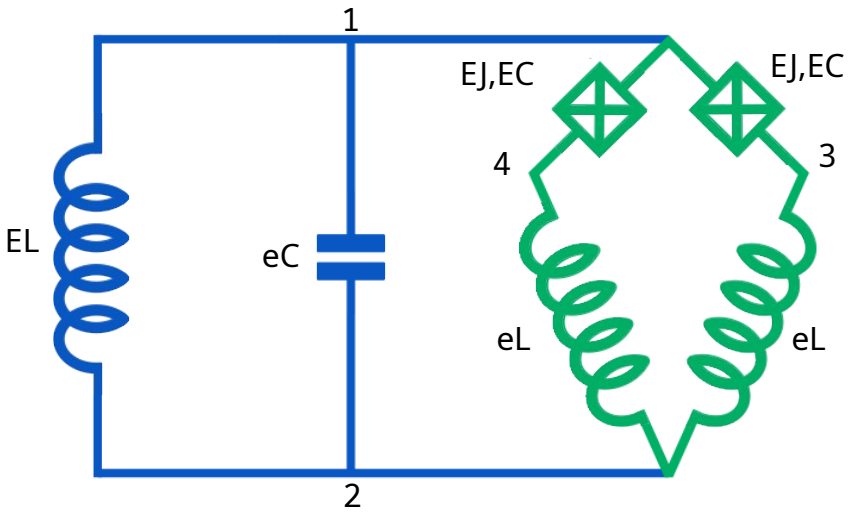
subsystem hierarchy [user input]

↓

hierarchical diagonalization

Algorithm and Implementation

New module to be published in next major release of scqubits




```
nodes: 4
branches:
JJ    1,3   EJ    EC
JJ    1,4   EJ    EC
L     1,2   EL
C     1,2   eC
L     2,3   eL
L     2,4   eL
```

W. C. Smith et al., arXiv:2010.15488 (2020)

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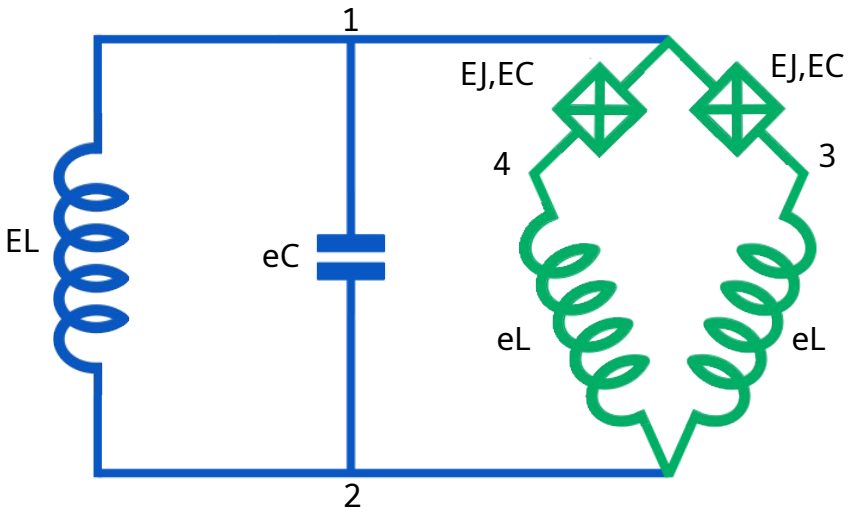


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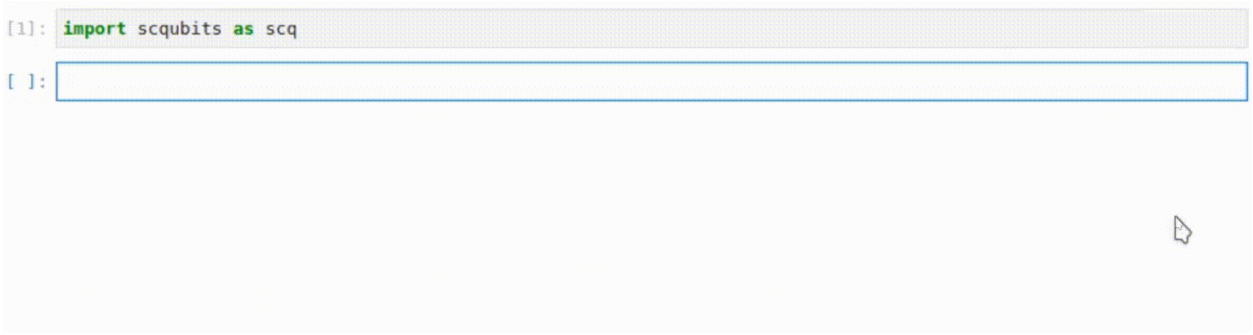

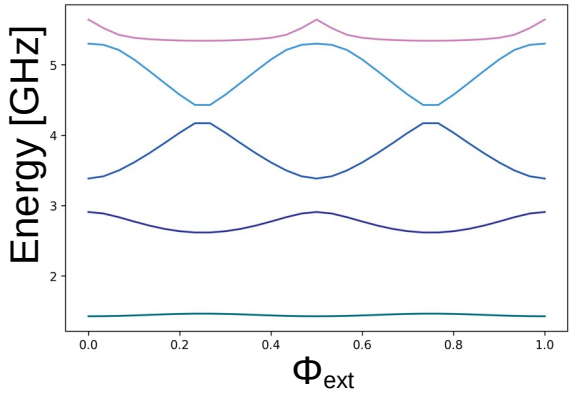

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