

## Лабораторная работа 6

### Элементы вариационного исчисления

Найти допустимые экстремали функционала  $J(y)$ , удовлетворяющие граничным условиям  $y(a) = A, y(b) = B$ . Что можно сказать об экстремумах функционала?

1.  $J(y) = \int_1^e \left( \frac{2y(x)}{x^2} - (y'(x))^2 \right) dx, y(1) = 0, y(e) = 1.$
2.  $J(y) = \int_0^{\pi/4} \left( (y'(x))^2 - 4y^2(x) \right) dx, y(0) = 0, y(\frac{\pi}{4}) = 1.$
3.  $J(y) = \int_{-1}^0 \left( 12x \cdot y(x) - (y'(x))^2 \right) dx, y(-1) = 1, y(0) = 0.$
4.  $J(y) = \int_{-1}^1 \left( (8 - 24x^2)y(x) - (y'(x))^2 \right) dx, y(-1) = y(1) = -1.$
5.  $J(y) = \int_0^{\pi} \left( \frac{y(x)}{2} \sin \frac{x}{2} - (y'(x))^2 \right) dx, y(0) = 0, y(\pi) = 1.$
6.  $J(y) = \int_0^{\pi/2} \left( 2(\sin x + \cos x)y(x) - (y'(x))^2 \right) dx, y(0) = 1, y(\frac{\pi}{2}) = 1.$
7.  $J(y) = \int_0^1 \left( 8e^{2x}y(x) + (y'(x))^2 \right) dx, y(0) = 1, y(1) = e^2 - 1.$
8.  $J(y) = \int_0^1 \left( \frac{2y(x)}{(x+1)^2} - (y'(x))^2 \right) dx, y(0) = 0, y(1) = \ln 2.$
9.  $J(y) = \int_1^2 \left( \frac{8y(x)}{x^3} + (y'(x))^2 \right) dx, y(1) = 2, y(2) = 1.$
10.  $J(y) = \int_0^{\pi} \left( 6x \cdot y(x) - (y'(x))^2 - 4\cos 2x \cdot y'(x) \right) dx, y(0) = 0, y(\pi) = \pi^3.$
11.  $J(y) = \int_0^3 \left( \frac{y(x)}{2\sqrt{(x+1)^3}} - (y'(x))^2 + 8x^3 \cdot y'(x) \right) dx, y(0) = 1, y(3) = 83.$
12.  $J(y) = \int_0^2 \left( (y'(x))^2 + x \cdot y'(x) \right) dx, y(0) = y(2) = 0.$
13.  $J(y) = \int_0^{\pi} \left( (y'(x))^2 - 4\cos 2x \cdot y'(x) \right) dx, y(0) = 0, y(\pi) = 0.$
14.  $J(y) = \int_0^{\pi} \left( (y'(x))^2 + 6\sin 3x \cdot y'(x) \right) dx, y(0) = 1, y(\pi) = -1.$
15.  $J(y) = \int_1^e \left( (y'(x))^2 - \frac{2y'(x)}{x} \right) dx, y(1) = 0, y(e) = 1.$
16.  $J(y) = \int_1^e \left( (y'(x))^2 - \frac{4x \cdot y'(x)}{x^2 + 1} \right) dx, y(-1) = \ln 2, y(1) = \ln 2.$

$$17. J(y) = \int_2^6 \left( (y'(x))^2 - \frac{y'(x)}{\sqrt{x-2}} \right) dx, y(2) = 0, y(6) = 2.$$

$$18. J(y) = \int_0^2 \left( (y'(x))^4 + (y'(x))^3 \right) dx, y(0) = 0, y(2) = 4.$$

$$19. J(y) = \int_{-1}^0 \left( x \cdot y(x) - (y'(x))^2 \right) dx, y(0) = 1, y(1) = \frac{1}{4}.$$

$$20. J(y) = \int_0^\pi \left( 4 \cos 2x \cdot y'(x) - (y'(x))^2 \right) dx, y(0) = -2, y(\pi) = -2.$$

$$21. J(y) = \int_0^1 \left( \frac{2y'(x)}{x+1} - (y'(x))^2 \right) dx, y(0) = 3, y(1) = 3 + \ln 2.$$

$$22. J(y) = \int_0^3 \left( \frac{y'(x)}{\sqrt{x+1}} + (y'(x))^2 \right) dx, y(0) = 1, y(3) = 0.$$

$$23. J(y) = \int_1^2 \left( \frac{4y'(x)}{x^3} + (y'(x))^2 \right) dx, y(1) = 1, y(2) = \frac{1}{4}.$$

$$24. J(y) = \int_1^2 \left( \left( 4x + \frac{2}{x^2} \right) y'(x) - (y'(x))^2 \right) dx, y(1) = 0, y(2) = 3, 5.$$

$$25. J(y) = \int_0^1 \left( (8x^3 - 4) y'(x) - (y'(x))^2 \right) dx, y(0) = 1, y(1) = 0.$$

$$26. J(y) = \int_0^1 \left( y(x) + 2x \cdot y'(x) + (y'(x))^2 \right) dx, y(0) = y(1) = 0.$$

$$27. J(y) = \int_0^{\ln 2} \left( (y'(x))^2 + 3y^2(x) \right) e^{2x} dx, y(0) = 0, y(\ln 2) = \frac{15}{8}.$$

$$28. J(y) = \int_0^1 x^2 (y'(x))^2 dx, y(0) = 3, y(1) = 1.$$

$$29. J(y) = \int_0^6 \left( 2x \cdot y(x) - (y'(x))^2 \right) dx, y(0) = y(6) = 1.$$

$$30. J(y) = \int_0^1 \left( (y'(x))^4 - 6(y'(x))^2 \right) dx, y(0) = y(1) = 0.$$

$$31. J(y) = \int_0^\pi \left( 4 \cos x \cdot y(x) + (y'(x))^2 - y^2(x) \right) dx, y(0) = y(\pi) = 0.$$

$$32. J(y) = \int_0^1 \left( e^{x+y(x)} y(x) - y'(x) - \sin x \right) dx, y(0) = 2, y(1) = 0.$$