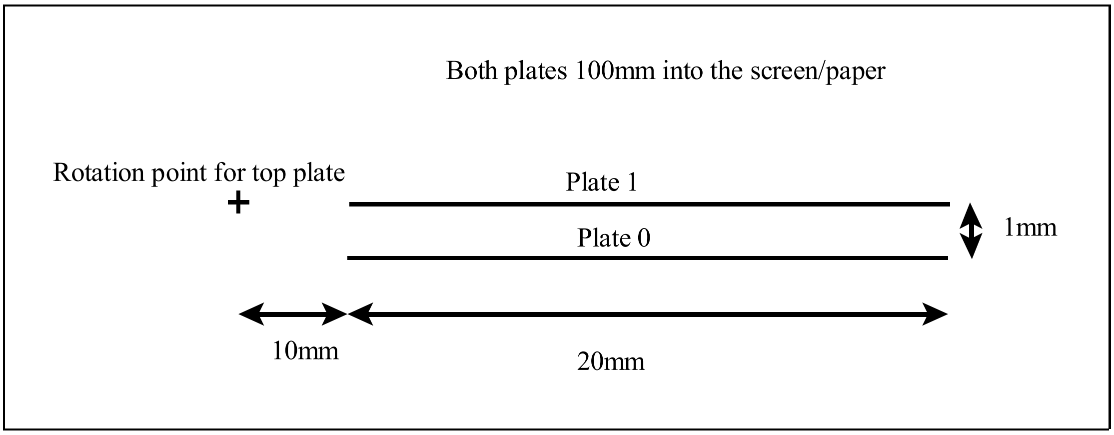
Some Comments on Capacitance Calculations.

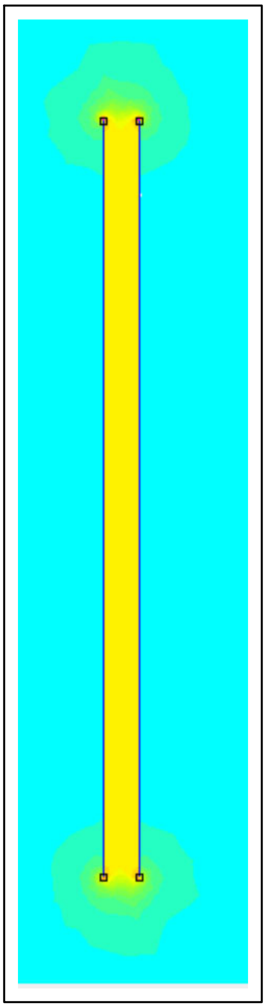
The diagram below shows the structure that I modeled. There are two parallel plate separated by 1mm. Plates are 20mm x 100mm. The top plate is rotated around a point 10mm to the left of top plate.



If we rotate around a point to the left of the top plate the rotating plate start point moves both up and to the left as it remains 10mm from the rotation point. It is following the circumference of a circle. For FEMM these need to be calculated in advance using a bit of trigonometry.

CST manages this by selecting the rotation axis properly. I may check this by redoing the simulations if I have time, but I think it is unlikely to be a problem

The diagram on the right shows FEMM Fields. The plates are 20mm in the vertical and 100mm into paper. Note the high fringing fields. FEMM was plotting electric field magnitude here.

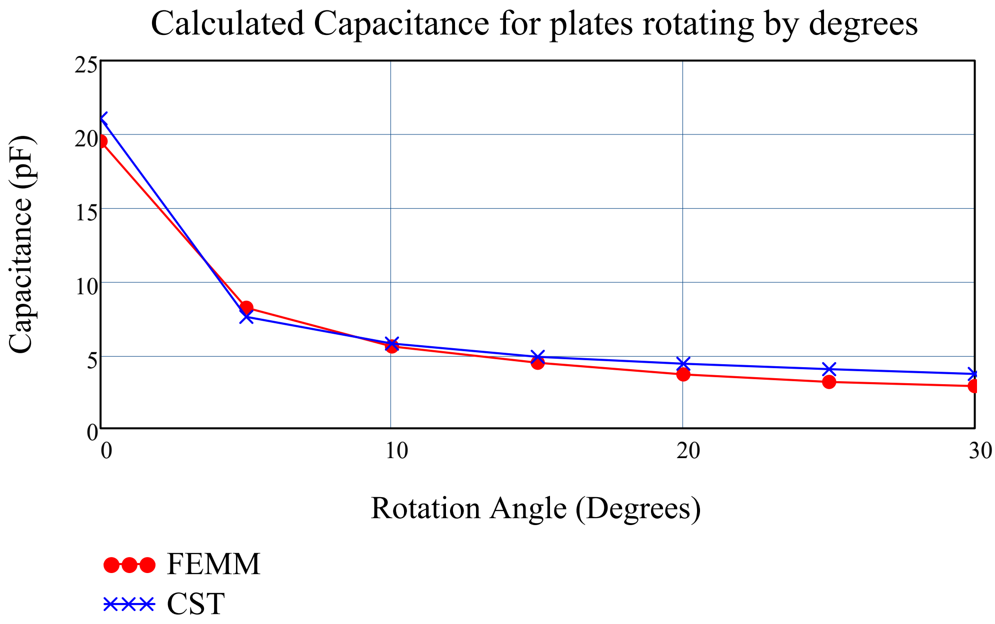


FEMM and CST rotating plate results: First column in degrees and second and third in pF

$$D := \begin{pmatrix} 0 & 19.5 & 21.05 \\ 5 & 8.19 & 7.56 \\ 10 & 5.55 & 5.75 \\ 15 & 4.45 & 4.85 \\ 20 & 3.65 & 4.38 \\ 25 & 3.14 & 4.01 \\ 30 & 2.86 & 3.68 \end{pmatrix}$$

Zero degrees calculation for 1mm separation. Top plate rotated about a point at the same height as it but 10mm to the left of the beginning of the plate.

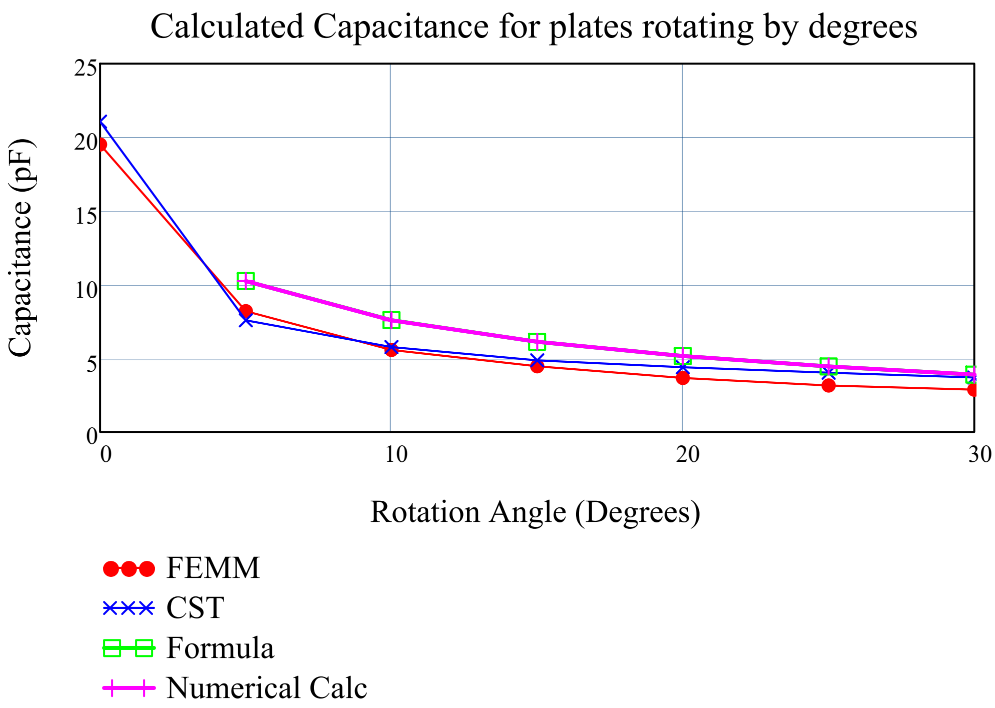
$$k := 0 .. \text{rows}(D) - 1$$



Using the equation based on a top plate rotating about its start point we get the following data added to the graph above. I have shared this equation with you, I think. If not, I will do so. I also ran a numerical equivalent of the derivation of this equation, adding the small capacitance values obtained by subdividing the distance along one axis of the plate. This, as you might expect, agrees with the equation.

$$\theta := 5\cdot\text{deg}, 10\cdot\text{deg} .. 30\cdot\text{deg}$$

$$d := 1\cdot 10^{-3} \qquad x := 20\cdot 10^{-3} \qquad y := 100\cdot 10^{-3} \qquad C(\theta) := \frac{\varepsilon_0\cdot y}{\tan(\theta)}\cdot\ln\left(\frac{x\cdot\tan(\theta) + d}{d}\right)$$



$C(\theta) =$

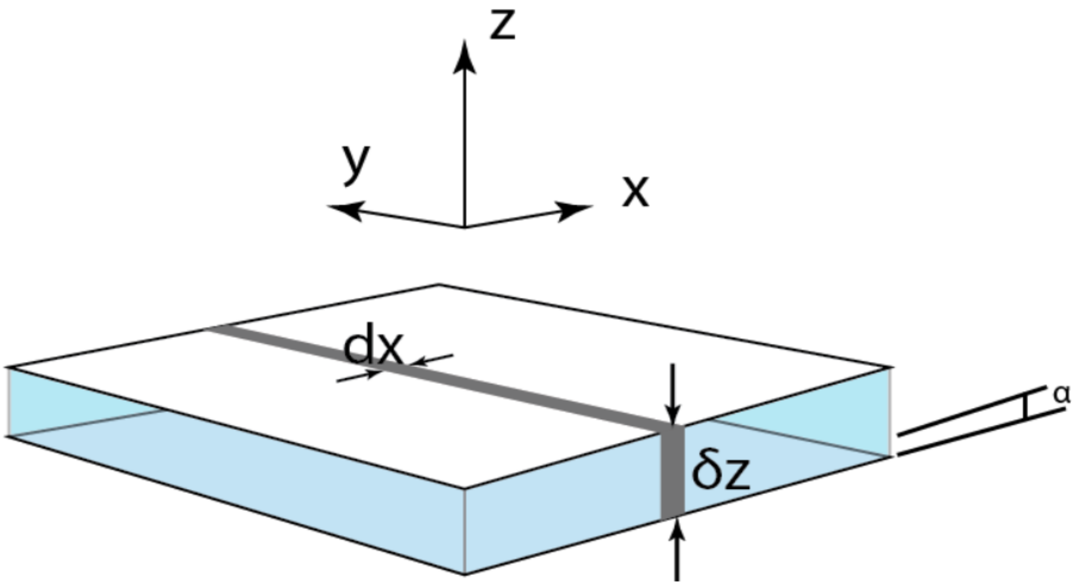
$1.023 \cdot 10^{-11}$
$7.579 \cdot 10^{-12}$
$6.11 \cdot 10^{-12}$
$5.14 \cdot 10^{-12}$
$4.431 \cdot 10^{-12}$
$3.877 \cdot 10^{-12}$

Note that the model is slightly different as this assumes that the pivot point remains at exactly 1mm above the bottom plate and does not move laterally. However, it is still in the right area in terms of results.

I think that we will have more error from variations in the linearity of the textile and the exact dimensions of the plates as well as the effect of the textile parameters.

Equation derivation from
https://physics.stackexchange.com/questions/148283/capacitance-of-two-non-parallel-plates

Suppose you have two plates, almost parallel (off by an angle α). The plates lie in the XY plane, from $(0,0)$ to (x_1, y_1). At $x = 0$, the plates are separated by a distance z_0, and at $x = x_1$, the plates are separated by a distance z_1.



We'll now consider an infinitesimally small element of both plates. (Since parallel capacitances add, and all the infinitesimal pairs are in a parallel configuration, we can use integration)

$$
\begin{aligned}
\tan\alpha &= \frac{z_1 - z_0}{x_1} \\
\mathrm{d}C &= \varepsilon \frac{\mathrm{d}A}{\delta z} \\
\mathrm{d}A &= y_1 \,\mathrm{d}x \\
\delta z &= z_0 + x\tan\alpha \\
\therefore C &= \int \mathrm{d}C \\
&= \int_A \varepsilon \frac{\mathrm{d}A}{\delta z} \\
&= \int_0^{x_1} \varepsilon \frac{y_1 \,\mathrm{d}x}{z_0 + x\tan\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&= \varepsilon\, y[\cot\alpha \ln(z_0 \cos\alpha + x \sin\alpha)]_0^{x_1} \\
&= \varepsilon\, y_1 \left(\frac{\ln(z_0 \cos\alpha + x_1 \sin\alpha)}{\tan\alpha} - \frac{\ln(z_0 \cos\alpha)}{\tan\alpha} \right) \\
&= \varepsilon\, y_1 \left(\frac{\ln(1 + (x_1/z_0) \tan\alpha)}{\tan\alpha} \right) \\
&= \frac{\varepsilon\, y_1}{\tan\alpha} \ln\left(1 + \frac{x_1}{z_0} \frac{z_1 - z_0}{x_1} \right) \\
&= \frac{\varepsilon\, y_1}{\tan\alpha} \ln\left(\frac{z_1}{z_0} \right)
\end{aligned}
$$

If you assume α is small, then $\tan\alpha \approx \alpha$, which gives

$$C = \frac{\varepsilon\, y_1}{\alpha} \ln\left(1 + \frac{x_1}{z_0} \right)$$

I have now found a paper that derives this formula from first principles in two different ways. It also sets out to calculatet the fringing capacitance.

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