1 Problem formulation

The indices, sets, parameters and variables applied throughout this article, are first defined to better describe the problem.

Indices	
i, j	Jobs
k	Vehicles
Sets	
I	Set of jobs
K	Set of vehicles
N	I union {O}
Parameters	
Q_k	Capacity of each vehicle k
F_k	Cost of each vehicle k
P_i	Processing time associated to job i
t_{ij}	Travel time between customer of job i and customer of job j
d_i	Due date associated to job i
Decision variables	
C_i	Completion time of job i
X_{ij}^k	Binary decision variable which indicates with 1 that a vehicle k went to customer of job i to customer
Y_k	Binary variable which represents with 1 if a vehicle k is used, and 0 otherwise
A_{ij}	Binary variable which represents if a job i is processed before job j with value 1,and 0 otherwise
T_i	Delivery delay of job i
T_i	Delivery time of job i
D_i	Delivery time of job i
S_k	Time of a vehicle k starts to be used

2 Mathematical Model

$$\min \sum_{i,j=0}^{N} \sum_{k=0}^{K} X_{ij}^k * t_{ij} + \sum_{k=0}^{K} F_k * Y_k + \sum_{i=1}^{N} w_i * T_i \tag{1}$$

st.

$$\sum_{k=0}^{K} \sum_{j=0, j \neq i}^{N} X_{ij}^k = 1, \forall i = 1, ..., N \tag{2}$$

$$\sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} s_i * X_{ij}^k \leq Q_k * Y_k, \forall k = 0, ..., K; \tag{3}$$

$$\sum_{j=0}^{N} X_{0j}^k = Y_k, \forall k = 0, ..., K; \tag{4}$$

$$\sum_{i=0}^{N} X_{ih}^{k} = \sum_{j=0}^{N} X_{hj}^{k}; \forall h = 0, ..., N; \forall k = 0, ..., K \tag{5}$$

$$A_{ij} + A_{ji} = 1; \forall i, j = 0, ..., N; i \neq j; \tag{6}$$

$$A_{ij} + A_{jr} + A_{ri} \geq 1; \forall i, j, r = 0, ..., N; i \neq j \neq r; (7)$$

$$S_k \geq C_j - M * (1 - \sum_{i=1, i \neq j}^{N} X_{ij}^{k}); \forall j = 1, ..., N; \forall k = 0, ..., K; \tag{8}$$

$$D_j - S_k \geq t_{0j} - M * (1 - X_{0j}^{k}); \forall k = 0, ..., K; j = 1, ..., N \tag{9}$$

$$D_j - D_i \geq t_{ij} - M * (1 - \sum_{i=1}^{N} X_{ij}^{k}); \forall i, j = 1, ..., N; \tag{10}$$

$$T_i \geq D_i - d_i; \forall i = 1, ..., N; \tag{11}$$

$$C_j = \sum_{i=0, i \neq j}^{N} (P_i * A_{ij}) + P_j; \forall j = 0, ..., N; \tag{12}$$

$$C_0 = 0; \tag{13}$$

$$X_{ijk} \in \{0, 1\}; \forall i, j = 0, ..., N, k = 0, ..., K; \tag{14}$$

$$A_{ij} \in \{0, 1\}; \forall i, j = 1, ..., N; \tag{15}$$

$$Y_k \in \{0, 1\}; \forall k = 0, ..., N; \tag{16}$$

$$S_k \geq 0; \forall k = 0, ..., K; \tag{17}$$

$$T_i \geq 0; \forall i = 1, ..., N; \tag{18}$$

$$D_i \geq 0; \forall i = 1, ..., N; \tag{19}$$

$$D_0 = 0; \tag{20}$$

$$T_0 = 0; \tag{21}$$

3 Generation of instances.

In this section of the article, the parameters are described as follows:

The processing time of a job i is generated as:
$P_i \sim rand[1, \rho] \forall j = 1, ..., N$

The due date calculation is based in a time window which is defined by:
$d_i \in [w_i, \overline{w}_i]$
And the time window parameters are:
$w_i = P_i + t_{0i} + \pi \sim \pi \in [0, (\delta_2 * \rho * N)/(1 + K)];$
$\overline{w}_i = w_i + \kappa \sim \kappa \in [0, \rho * \delta_2];$

Following it is described the size capacity of each job i:
$s_i \sim [1, P_i]$;

To evaluate penalty of delivered delayed time, the penalty weight is:
$w_i \sim [0, 10.0]$;
(21)To calculate the capacity of each vehicle, the job sizes were used as follows:
$Q_i = max(s_i) + \tau \sim \tau \in [\frac{\sum_{i=0}^{N} s_i}{N}, \varDelta]$;

Having delta equals to: $\varDelta = \rho * C$;

Finally, the price of each vehicle is based on the capacity of that vehicle:
$F_i = Q_i * N$;

The group of deltas mentioned below is the following:
$\delta_1 \in \{0.5\}$;
$\delta_2 \in \{0.5, 0.8, 0.1, 1.5, 2, 2.5\}$;

Variables rho and mu used:
$\rho \in \{100\}$;
$\mu \in \{1, 1.5, 2\}$;

In small instances, the defined number of jobs were:
$N \in \{8, 10, 15, 20\}$;

Also for small instances, the number of cars used were:
$K \in \{2, 4, 5\}$;