The Beauty of Embracing English

My English Note

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Contents

Basics of Linear Algebra

1. 对向量的介绍

1.1 Vector

Definition 1.1.1 — Vector. 一个有序的数字列表.

$$\begin{bmatrix} -1.1 \\ 0.0 \\ 3.6 \\ -7.2 \end{bmatrix} \text{或者} \begin{pmatrix} -1.1 \\ 0.0 \\ 3.6 \\ -7.2 \end{pmatrix} \text{或者} \quad (-1.1, 0, 3.6, -7.2)$$

表中的数字是*元素*(*项、系数、分量*)。元素的数量是向量的*大小*(*维数，长度*)。大小为 n 的向量称为*n 维向量*。向量中的数字通常被称作*标量*。

用符号来表示向量，比如 α ，b，一般小写字母表示. 其它表示形式 $\boldsymbol{g}, \vec{a}$

Definition 1.1.2 — n 维向量 a 的第 i 元素. n 维向量 a 的第 i i 元素表示为 a_i.
有时 i 指的是向量列表中的第 i 个向量.

Definition 1.1.3 — $a = b$. 对于所有 i，如果有 $a_i = b_i$，则称两个相同大小的向量 a 和 b 是相等的，可写成 $a = b$

Definition 1.1.4 — stacked vector. 假设 b、c、d 是大小为 m、n、p 的向量

$$a = \begin{bmatrix} b \\ C \\ d \end{bmatrix}$$

$$a = (b_1, b_2, \ldots, b_m, c_1, c_2, \ldots, c_n, d_1, d_2, \ldots, d_p)$$

Definition 1.1.5 — 零向量. 所有项为 0 的 n 维向量表示为 0_n 或者 0

Definition 1.1.6 — 全一向量. 所有项为 1 的 n 维向量表示为 $\square_n$ 或者 1

Definition 1.1.7 — 单位向量. 当第 i 项为 1，其余项为 0 时表示为 e_i

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Definition 1.1.8 — 稀疏向量. 如果一个向量的许多项都是 0，该向量为稀疏 (Sparse) 的。稀疏向量能在计算机上高效地存储和操作。

$\mathrm{nnz}(x)$ 是指向量 x 中非零的项数 (number of non-zeros)，有时用 ℓ_0 表示。

向量 $x = (x_1, x_2)$ 可以在二维中表示一个位置或一个位移、图像、单词统计等。

1.2 Vector Space

Definition 1.2.1 — 向量空间 V. 设 V 是非空子集, P 是一数域, 向量空间 V 满足:

1. 向量加法: $V + V \to V$, 记作 $\forall x, y \in V$, 则 $x + y \in V$(加法封闭)
2. 标量乘法: $F \times V \to V$, 记作 $\forall x \in V, \lambda \in P$, 则 $\lambda x \in V$(乘法封闭)

上述两个运算满足下列八条规则 $(\forall x, y, z \in V, \lambda, \mu \in P)$

1. $x + y = y + x$ (交换律)
2. $x + (y + z) = (x + y) + z$ (结合律)
3. V 存在一个零元素, 记作 0, $x + 0 = x$
4. 存在 x 的负元素，记作 $-x$, 满足 $x + (-x) = 0$
5. $\forall x \in V$, 都有 $1x = x, 1 \in P$
6. $\lambda(\mu x) = (\lambda\mu)x$
7. $(\lambda + \mu)x = \lambda x + \mu x$
8. $\lambda(x + y) = \lambda x + \lambda y$

Corollary 1.2.1 向量空间也称为线性空间.

Corollary 1.2.2 如果 $x, y \in \mathbb{R}^2$, 则 $x + y \in \mathbb{R}^2, \lambda x \in \mathbb{R}^2 (\lambda \in \mathbb{R})$

1.3 向量运算

Definition 1.3.1 — 向量加法. n 维向量 a 和 b 可以相加，求和形式表示为 $a + b$

设向量 a, b, C 是向量空间 V 的元素，即 $a, b, c \in V$。

1. 交换律：$a + b = b + a$
2. 结合律：$(a + b) + c = a + (b + c)$ (因此可写成 $a + b + c$)
3. $a + 0 = 0 + a = a$
4. $a - a = 0$

Corollary 1.3.1 — 向量位移相加. 如果二维向量 a 和 b 都表示位移，则它们的位移之和为 $a + b$

Definition 1.3.2 — 标量与向量的乘法.

$$\beta a = \begin{bmatrix} \beta a_1 \\ \vdots \\ \beta a_n \end{bmatrix}$$

标量 β, γ 与向量 a、b

1. 结合律：$(\beta\gamma)a = \beta(\gamma a)$
2. 左分配律：$(\beta + \gamma)a = \beta a + \gamma a$
3. 右分配律：$\beta(a + b) = \beta a + \beta b$

Definition 1.3.3 — 线性组合. 对于向量 $a_1, \ldots, a_m$ 和标量 $\beta_1, \ldots, \beta_m$,

$$\beta_1 a_1 + \cdots + \beta_m a_m$$

是向量的线性组合。$\beta_1, \ldots, \beta_m$ 是该向量的系数。

■ **Example 1.1** 对于任何向量 $b \in \mathbb{R}^n$, 有如下等式

$$b = b_1 e_1 + \cdots + b_n e_n, b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

■

1.4 内积

Definition 1.4.1 — 内积. 在数域 $\mathbb{R}$ 上的向量空间 V, 定义函数 $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$, 满足:

1. $\langle a, a \rangle \geq 0, \forall a \in V$, 当且仅当 $a = 0$ 时 $\langle a, a \rangle = 0$
2. $\langle \alpha a + \beta b, c \rangle = \alpha \langle a, c \rangle + \beta \langle b, c \rangle, \forall \alpha, \beta \in \mathbb{R}$, 且 $a, b, c \in V$
3. $\langle a, b \rangle = \langle b, a \rangle, \forall a, b \in V$

函数 $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$ 成为内积。

■ **Example 1.2** 在向量空间 $\mathbb{R}^n$ 上，计算两个向量对应项相乘之后求和函数

$$\langle a, b \rangle = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n = a_b^T$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in \mathbb{R}^n$$

■

Proof. $\langle a, a \rangle = a_1 a_1 + a_2 a_2 + \cdots + a_n a_n = \sum_{i=1}^n a_i^2 \geq 0, \langle a, a \rangle = 0$, 则 $a = 0$

$$\begin{aligned} \langle \alpha a + \beta b, c \rangle &= (\alpha a_1 + \beta b_1) c_1 + (\alpha a_2 + \beta b_2) c_2 + \cdots + (\alpha a_n + \beta b_n) c_n \\ &= \alpha \sum_{i=1}^n a_i c_i + \beta \sum_{i=1}^n b_i c_i \\ &= \alpha \langle a, c \rangle + \beta \langle b, c \rangle \end{aligned}$$

$$\langle a, b \rangle = a^T b = b^T a = \langle b, a \rangle$$

■

内积的性质：交换律、结合律、分配律。
交换律：$a^T b = b^T a$
结合律：$(\gamma a)^T b = \gamma \left(a^T b\right)$
分配律：$(a + b)^T c = a^T c + b^T c$

1.4.1 常用的内积等式

Corollary 1.4.1 — 选出第 i 项.

$$e_i^T a = a_i$$

Corollary 1.4.2 — 向量每一项之和.

$$\mathbf{1}^T a = a_1 + \cdots + a_n$$

Corollary 1.4.3 — 向量每一项的平方和.

$$a^T a = a_1^2 + \cdots + a_n^2$$

1.5 Cauchy-Schwartz Inequality

Theorem 1.5.1 — Cauchy-Schwartz Inequality. 设 $\langle\cdot,\cdot\rangle$ 是向量空间 V 上的内积, $\forall x, y \in V$, 则有

$$|\langle x, y\rangle|^2 \le \langle x, x\rangle\langle y, y\rangle$$

Proof. 令 $\lambda \in \mathbb{R}$, 则有 $0 \le \langle x+\lambda y, x+\lambda y\rangle = \langle x, x\rangle + \lambda\langle y, x\rangle + \lambda\langle x, y\rangle + \lambda^2\langle y, y\rangle = \langle x, x\rangle + 2\lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle$

则有 $\lambda^2\langle y, y\rangle + 2\lambda\langle y, x\rangle + \langle x, x\rangle \ge 0, \forall\lambda \in \mathbb{R}$.

$\nabla = (2\langle y, x\rangle)^2 - 4\langle y, y\rangle\langle x, x\rangle \le 0$

$|\langle x, y\rangle|^2 \le \langle x, x\rangle\langle y, y\rangle$

当 $|\langle x, y\rangle|^2 = \langle x, x\rangle\langle y, y\rangle$ 时，有 $\langle x, x\rangle^2 + 2\lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle = 0$

也即 $\langle x+\lambda y, x+\lambda y\rangle = 0$, 因此 $x + \lambda y = 0$, 即 $x = -\lambda y$. ■

1.6 浮点运算

计算机以浮点格式存储 (实) 数值。

基本的算术运算 (加法，乘法等) 被称为浮点运算 (flop)。

算法或操作的时间复杂度：作为输入维数的函数所需要的浮点运算总数。

算法复杂度通常以非常粗略地近似估算。

（程序）执行时间的粗略估计：计算机速度/flops

目前的计算机大约是 1Gflops/秒 (10^9flops/秒)

Corollary 1.6.1 假设有 n 维向量 x 和 y:

- $x + y$ 需要 n 次加法，所以时间复杂度为 (n)flops。
- $x^T y$ 需要 n 次乘法和 $n - 1$ 次加法，所以时间复杂度为 $(2n - 1)$flops。
- 对于 $x^T y$，通常将其时间复杂度简化为 $2n$，甚至为 n。
- 当 x 或 y 是稀疏的时候，算法的实际运算时间会比理论时间更少。

2. Linear Function

2.1 Linear Function

Definition 2.1.1 — Linear Function. f 是一个将 n 维向量映射成数的函数。

$$f : \mathbb{R}^n \to \mathbb{R}$$

线性函数 f 满足以下两个性质 $(k \in \mathbb{R}, x, y \in \mathbb{R}^n)$：

- 齐次性 (homogeneity): $f(kx) = kf(x)$
- 叠加性 (Additivity): $f(x+y) = f(x) + f(y)$

■ **Example 2.1** 求平均值: $f(x) = \frac{1}{n}\sum_{i=1}^{n} x_i$ 为线性函数。 ■

■ **Example 2.2** 求最大值: $f(x) = \max\{x_1, x_2, \ldots, x_n\}$ 并不是线性函数。 ■

Proof. 令 $x = (1, -1), y = (-1, 1), \alpha = 0.5, \beta = 0.5$, 有 $f(\alpha x + \beta y) = 0 \neq \alpha f(x) + \beta f(y) = 1$

$$\begin{aligned} f(x+y) &= \max\{x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n\} \\ &\leq \max\{x_1, x_2, \ldots, x_n\} + \max\{y_1, y_2, \ldots, y_n\} \\ &\leq f(x) + f(y) \end{aligned}$$

■

Theorem 2.1.1 设 $\alpha_{1,\ldots,}\alpha_m \in \mathbb{R}, u_1, \ldots, u_m \in \mathbb{R}^n$, 则线性函数 f 满足

$$\begin{aligned} f(\alpha_1 u_1 + \alpha_2 u_2 + \ldots + \alpha_m u_m) &= f(\alpha_1 u_1) + f(\alpha_2 u_2 + \ldots + \alpha_m u_m) \\ &= \alpha_1 f(u_1) + f(\alpha_2 u_2 + \ldots + \alpha_m u_m) \\ &= \alpha_1 f(u_1) + \alpha_2 f(u_2) + \ldots + \alpha_m f(u_m) \end{aligned}$$

Definition 2.1.2 — 内积函数 (inner product function). 对于 n 维向量 a, 满足以下形式的函数被称为内积函数

$$f(x)=a^{T}x=a_1x_1+a_2x_2+\ldots+a_nx_n$$

上述 $f(x)$ 可以看作是每项 x_{i} 的加权之和。

Corollary 2.1.2 内积函数都是线性的.

Proof.

$$\begin{aligned}f(\alpha x+\beta y)&=a^{T}(\alpha x+\beta y)\\&=a^{T}(\alpha x)+a^{T}(\beta y)\\&=\alpha\left(a^{T}x\right)+\beta\left(a^{T}y\right)\\&=\alpha f(x)+\beta f(y)\end{aligned}$$

■

Definition 2.1.3 — 仿射函数 (affine function). 其一般形式为 $f(x)=a^{T}x+\mathrm{b}$, 其中 $a\in\mathbb{R}^{n}$, $b\in\mathbb{R}$ 为标量。

Theorem 2.1.3 函数 $f:\mathbb{R}^{n}\to\mathbb{R}$ 为仿射函数需要满足
$f(\alpha x+\beta y)=\alpha f(x)+\beta f(y),\alpha+\beta=1,\alpha,\beta\in\mathbb{R},x,y\in\mathbb{R}^{n}$

2.2 泰勒展开

Definition 2.2.1 — 函数 f 第 i 个分量的一阶偏导数.

$$\begin{aligned}\frac{\partial f}{\partial z_i}(z)&=\lim_{t\to 0}\frac{f\left(z_1,\cdots,z_{i-1},z_i+t,z_{i+1},\cdots,z_n\right)-f(z)}{t}\\&=\lim_{t\to 0}\frac{f\left(z+te_i\right)-f(z)}{t}\end{aligned}$$

Definition 2.2.2 — f 在点 z 的梯度.

$$\nabla f(z)=\begin{bmatrix}\frac{\partial f}{\partial z_1}(z)\\ \vdots\\ \frac{\partial f}{\partial z_n}(z)\end{bmatrix}$$

Definition 2.2.3 — Taylor's Approximation.

$$\begin{aligned}f(x)=f(z)&+\frac{\partial f}{\partial x_1}(z)\left(x_1-z_1\right)+\frac{\partial f}{\partial x_2}(z)\left(x_2-z_2\right)+\cdots+\frac{\partial f}{\partial x_n}(z)\left(x_n-z_n\right)\\&+\frac{1}{2!}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial f^2}{\partial x_i\partial x_j}(z)\left(x_i-z_i\right)\left(x_{\mathrm{j}}-z_j\right)+\cdots\end{aligned}$$

■ **Example 2.3** 泰勒公式利用多项式在一点附近逼近函数

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^{k-1}\frac{x^{2k-1}}{(2k-1)!}+\frac{\sin\left[\xi+(2k+1)\frac{\pi}{2}\right]}{(2k+1)!}x^{2k+1}$$

一次逼近: $\sin x \approx x$
三次逼近: $\sin x \approx x - \frac{x^3}{3!}$ ■

Proof.

$$f(x) = P_n(x) + R_n(x)$$

$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots + a_n(x - x_0)^n$
$R_n(x) = o(x - x_0)^n$
$f(x) \approx P_n(x)$
$\therefore P_n(x_0) = f(x_0), P_n'(x_0) = f'(x_0), P_n''(x_0) = f''(x_0), \cdots, P_n^{(n)}(x_0) = f^{(n)}(x_0)$
要求 $P_n(x_0) = f(x_0) \Rightarrow a_0 = f(x_0)$

$$P_n'(x) = a_1 + 2a_2(x - x_0) + \cdots + na_n(x - x_0)^{n-1} \Rightarrow a_1 = f'(x_0)$$

依此类推. $a_n = \frac{f^{(n)}(x_0)}{n!}$ ■

Corollary 2.2.1 — n 阶泰勒多项式.

$$\begin{aligned} P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \end{aligned}$$

where $a_n = \frac{f^{(n)}(x_0)}{n!}$

Corollary 2.2.2 — 对于高阶余项的公式，麦克劳林 (Maclaurin) 公式. 带拉格朗日余项

$$\begin{aligned} f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \\ + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)^{n+1} \end{aligned}$$

$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)^{n+1}$ (ξ 在 x_0 与 x 之间)
在零点展开——麦克劳林 (Maclaurin) 公式

$$\begin{aligned} f(x) =& f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n \\ &+ \frac{f^{(n+1)}(\theta x)}{(n+1)!}x^{n+1} \qquad (0 < \theta < 1) \end{aligned}$$

Definition 2.2.4 — 一阶泰勒公式. 假设 $f : \mathbb{R}^n \to \mathbb{R}$, 函数 f 在 z 点可导

$$\hat{f}(x) = f(z) + \frac{\partial f}{\partial x_1}(z)(x_1 - z_1) + \ldots + \frac{\partial f}{\partial x_1}(z)(x_n - z_n)$$

当 x 非常接近 z 时，$\hat{f}(x)$ 也非常接近 $f(z)$。$\hat{f}(x)$ 是关于 x 的一个仿射函数。

Corollary 2.2.3 — 一阶泰勒公式的内积形式.

$$\hat{f}(x) = f(z) + \nabla f(z)^T (x - z) \quad \nabla f(z) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(z) \\ \vdots \\ \frac{\partial f}{\partial x_n}(z) \end{bmatrix}$$

yi'wei

II Example Codes

3. Text Chapter

3.1 Paragraphs of Text

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3.2 Citation

This statement requires citation [1]; this one is more specific [2, page 162].

3.3 Lists

Lists are useful to present information in a concise and/or ordered way[1].

3.3.1 Numbered List

1. The first item
2. The second item
3. The third item

3.3.2 Bullet Points

- The first item

[1] Footnote example...

- The second item
- The third item

3.3.3 Descriptions and Definitions

Name Description
Word Definition
Comment Elaboration

4. In-text Elements

4.1 Theorems

This is an example of theorems.

4.1.1 Several equations

This is a theorem consisting of several equations.

Theorem 4.1.1 — Name of the theorem. In $E = \mathbb{R}^n$ all norms are equivalent. It has the properties:

$$\big| ||\mathbf{x}|| - ||\mathbf{y}|| \big| \leq ||\mathbf{x} - \mathbf{y}|| \tag{4.1}$$

$$||\sum_{i=1}^{n} \mathbf{x}_i|| \leq \sum_{i=1}^{n} ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{4.2}$$

4.1.2 Single Line

This is a theorem consisting of just one line.

Theorem 4.1.2 A set $\mathcal{D}(G)$ in dense in $L^2(G)$, $|\cdot|_0$.

4.2 Definitions

This is an example of a definition. A definition could be mathematical or it could define a concept.

Definition 4.2.1 — Definition name. Given a vector space E, a norm on E is an application, denoted $||\cdot||$, E in $\mathbb{R}^+ = [0, +\infty[$ such that:

$$||\mathbf{x}|| = 0 \Rightarrow \mathbf{x} = \mathbf{0} \tag{4.3}$$
$$||\lambda\mathbf{x}|| = |\lambda| \cdot ||\mathbf{x}|| \tag{4.4}$$
$$||\mathbf{x} + \mathbf{y}|| \leq ||\mathbf{x}|| + ||\mathbf{y}|| \tag{4.5}$$

4.3 Notations

Notation 4.1. *Given an open subset G of $\mathbb{R}^n$, the set of functions φ are:*

1. *Bounded support G;*
2. *Infinitely differentiable;*

a vector space is denoted by $\mathcal{D}(G)$.

4.4 Remarks

This is an example of a remark.

!

The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

4.5 Corollaries

This is an example of a corollary.

Corollary 4.5.1 — Corollary name. The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

4.6 Propositions

This is an example of propositions.

4.6.1 Several equations

Proposition 4.6.1 — Proposition name. It has the properties:

$$\big| ||\mathbf{x}|| - ||\mathbf{y}|| \big| \leq ||\mathbf{x} - \mathbf{y}|| \tag{4.6}$$
$$||\sum_{i=1}^{n} \mathbf{x}_i|| \leq \sum_{i=1}^{n} ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \tag{4.7}$$

4.6.2 Single Line

Proposition 4.6.2 Let $f, g \in L^2(G)$; if $\forall \varphi \in \mathcal{D}(G)$, $(f, \varphi)_0 = (g, \varphi)_0$ then $f = g$.

4.7 Examples

This is an example of examples.

4.7.1 Equation and Text

■ **Example 4.1** Let $G = \{x \in \mathbb{R}^2 : |x| < 3\}$ and denoted by: $x^0 = (1,1)$; consider the function:

$$f(x) = \begin{cases} \mathrm{e}^{|x|} & \text{si } |x - x^0| \leq 1/2 \\ 0 & \text{si } |x - x^0| > 1/2 \end{cases} \tag{4.8}$$

The function f has bounded support, we can take $A = \{x \in \mathbb{R}^2 : |x - x^0| \leq 1/2 + \epsilon\}$ for all $\epsilon \in]0\,;5/2 - \sqrt{2}[$. ■

4.7.2 Paragraph of Text

■ **Example 4.2 — Example name.** Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris. ■

4.8 Exercises

This is an example of an exercise.

Exercise 4.1 This is a good place to ask a question to test learning progress or further cement ideas into students' minds. ■

4.9 Problems

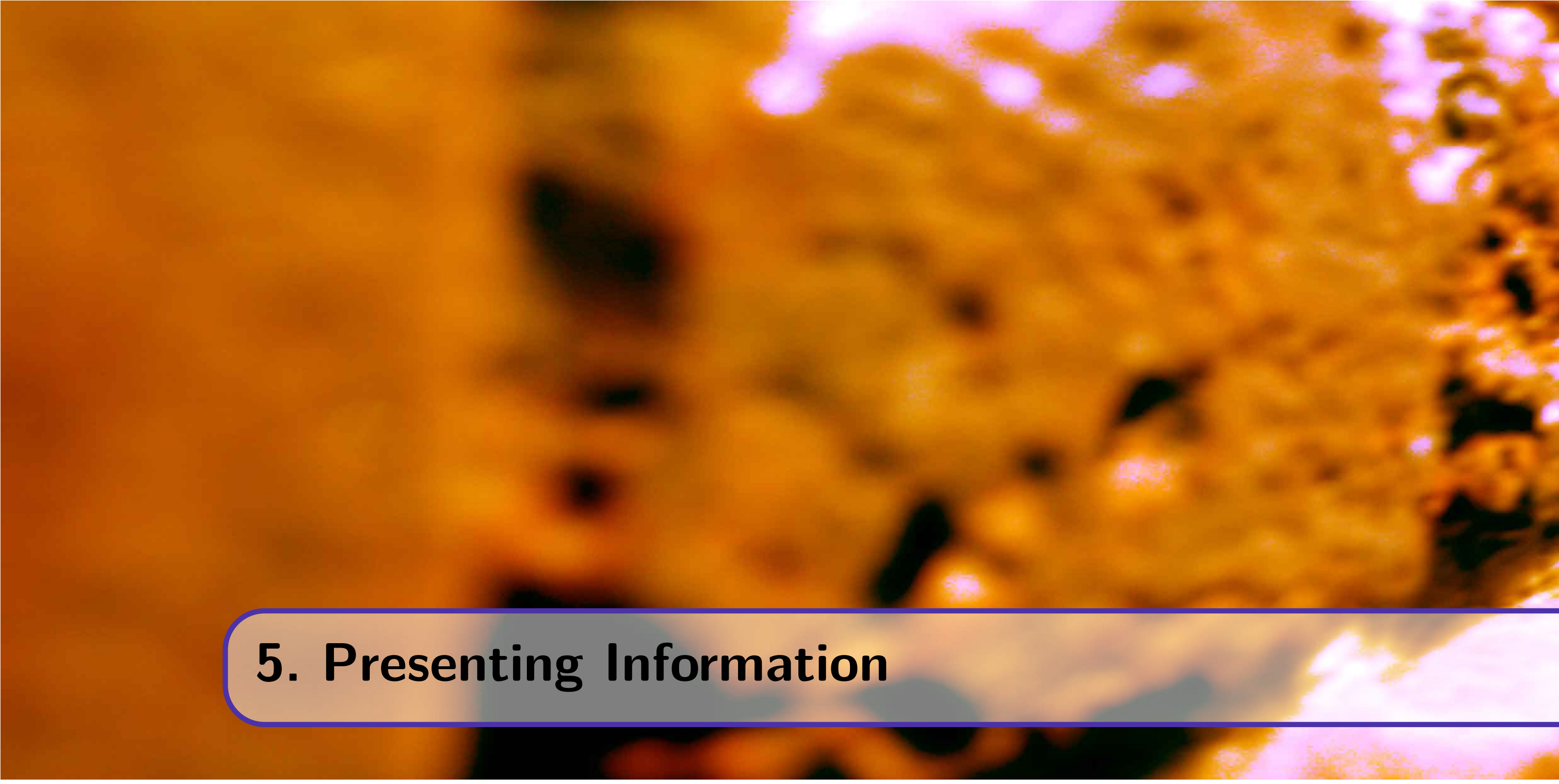
Problem 4.1 What is the average airspeed velocity of an unladen swallow?

4.10 Vocabulary

Define a word to improve a students' vocabulary.

Vocabulary 4.1 — Word. Definition of word.

Part Two



5. Presenting Information

5.1 Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 5.1: Table caption

Referencing Table 5.1 in-text automatically.

5.2 Figure

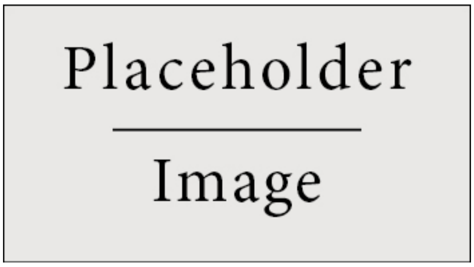
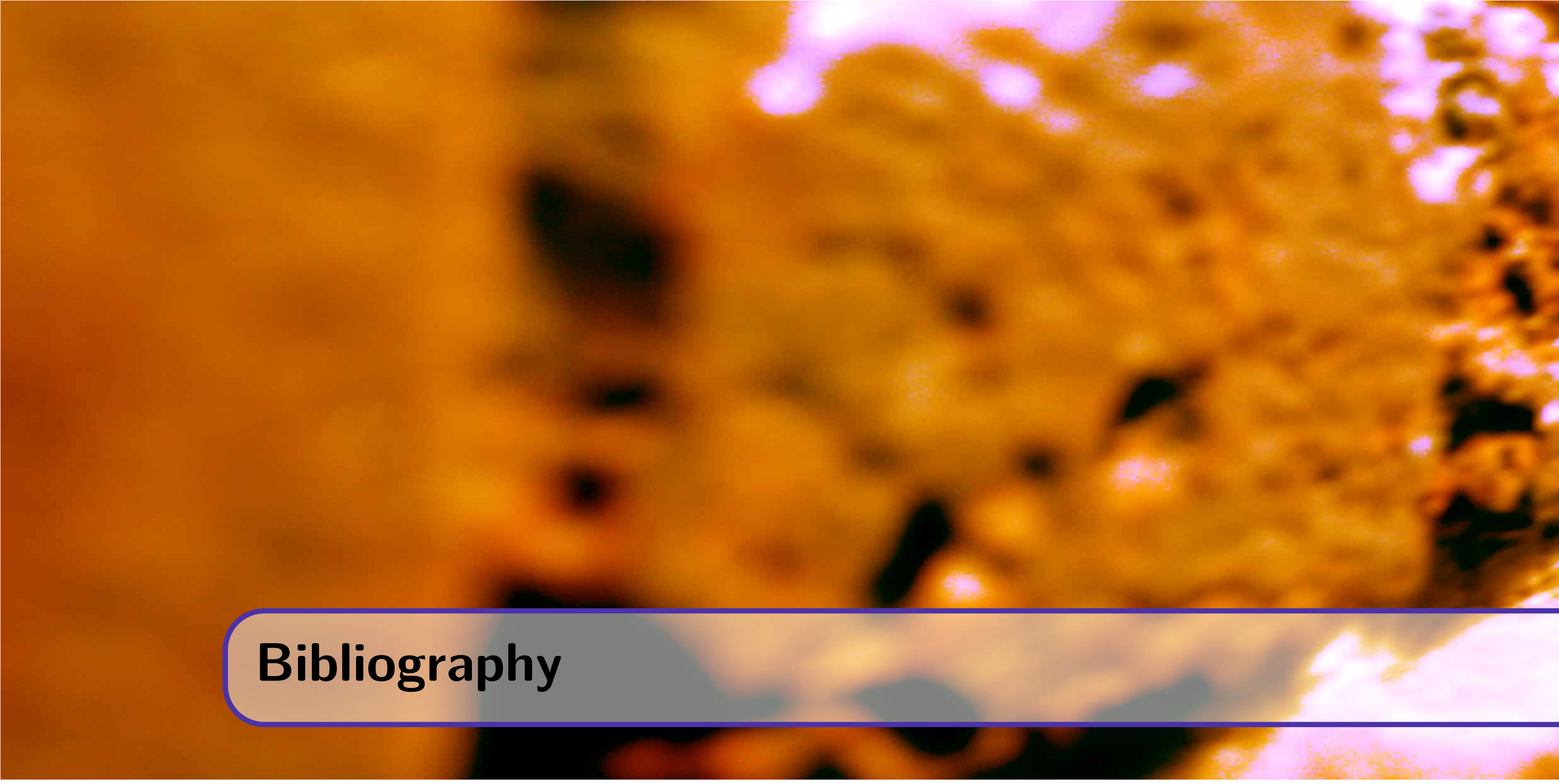


Figure 5.1: Figure caption

Referencing Figure 5.1 in-text automatically.



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