

Simulation of a penning trap

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I. INTRODUCTION

II. METHODS

We model the particles being trapped inside the Penning trap with ordinary differential equations (ODEs) derived from foundational physics equations. Let us quickly go through them here, before using them to argue our model is valid. The first of these equations is Newton's second law (1), which we use to say something about how our particles' velocity is changed in relation to outside forces.

$$m\ddot{\mathbf{r}} = \sum_i \mathbf{F}_i \quad (1)$$

Next comes Coulomb's law (2), that describes how electrically charged particles interact. Here, k_e is Coulomb's constant.

$$\mathbf{E} = k_e \sum_{j=1} nq_j \frac{\mathbf{r} - \mathbf{r}_j}{|\mathbf{r} - \mathbf{r}_j|^3} \quad (2)$$

Electric fields and potential are related through (3).

$$\mathbf{E} = -\nabla V \quad (3)$$

Lastly, the Lorentz force (4) is the force acting on a charged particle moving through an electrical and magnetic field.

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad (4)$$

In addition to these fundamental equations, we have two equations describing how an idealized Penning trap work, that we need to use for our model as well. The electric potential inside the trap is defined by (5), where V_0 is the potential applied at the terminals, and d is the characteristic distance, a dimensionless size.

$$V(x, y, z) = \frac{V_0}{2d^2}(2z^2 - x^2 - y^2) \quad (5)$$

In addition to the electrical field that the last equation described, the magnetic field in the trap is defined as in (6).

$$\mathbf{B} = B_0 \hat{\mathbf{e}}_z = (0, 0, B_0) \quad (6)$$

OPPGAVE 1

$$m\dot{\mathbf{r}} = q\mathbf{E} + q\dot{\mathbf{r}} \times \mathbf{B} \quad (7)$$

$$m\dot{\mathbf{r}} = q(-\nabla V) + q\dot{\mathbf{r}} \times B_0 \hat{\mathbf{e}}_z \quad (8)$$

$$m\dot{\mathbf{r}} = q \frac{V_0}{2d^2} \begin{pmatrix} -2x \\ -2y \\ 4z \end{pmatrix} + q \begin{pmatrix} \dot{y}B_0 \\ -\dot{x}B_0 \\ 0 \end{pmatrix} \quad (9)$$

$$m\dot{\mathbf{r}} = \begin{pmatrix} -q \frac{V_0}{d^2} x + q \dot{y} B_0 \\ -q \frac{V_0}{d^2} y - q \dot{x} B_0 \\ q \frac{V_0}{d^2} 2z \end{pmatrix} \quad (10)$$

OPPGAVE 2

$$\mathbf{f}(t) = \mathbf{x}(t) + i\mathbf{y}(t) \quad (11)$$

$$\dot{\mathbf{f}}(t) = \dot{\mathbf{x}}(t) + i\dot{\mathbf{y}}(t) \quad (12)$$

$$\ddot{\mathbf{f}}(t) = \ddot{\mathbf{x}}(t) + i\ddot{\mathbf{y}}(t) \quad (13)$$

$$\ddot{x} - \omega_0 \dot{y} + \frac{1}{2} \omega_z^2 x = 0 \quad (14)$$

$$\left(\ddot{f} - i\ddot{y} \right) - \omega_0 \left(\frac{\dot{f}\dot{x}}{i} \right) + \frac{1}{2} \omega_z^2 (f - iy) = 0 \quad (15)$$

$$\ddot{f} - i \left(-\omega_0 \dot{x} + \frac{1}{2} \omega_z^2 y \right) + i\omega_0 \dot{f}\dot{x} + \frac{1}{2} \omega_z^2 (f - iy) = 0 \quad (16)$$

$$\ddot{f} + i\omega_0 \dot{x} - \omega_0 \dot{x} + \frac{1}{2} \omega_z^2 iy - \frac{1}{2} \omega_z^2 iy + i\omega_0 \dot{f} - \frac{1}{2} \omega_z^2 f = 0 \quad (17)$$

$$\ddot{f} + i\omega_0 \dot{f} - \frac{1}{2} \omega_z^2 f = 0 \quad (18)$$

OPPGAVE 3

$$\text{Re}(-i\omega_{\pm}) \leq 0$$

III. RESULTS AND DISCUSSION

IV. CONCLUSION
