



Figure 1: .

# 1. The Baseline Mathematical Model (without control)

The proposed model describes the transmission dynamics of malaria between humans and mosquitoes, incorporating awareness dynamics ( $A_h$ ) and the larval stage of mosquitoes ( $L_v$ ).

$$\frac{dS_h}{dt} = \Lambda_h + \alpha_1 R_h - \lambda_h(t) S_h - (\mu_h + m) S_h + A_h \quad (1)$$

$$\frac{dA_h}{dt} = m S_h - \sigma \lambda_h(t) A_h - (\mu_h + \gamma) A_h \quad (2)$$

$$\frac{dE_h}{dt} = \lambda_h(t)(S_h + \sigma A_h) - (\nu_h + \mu_h) E_h \quad (3)$$

$$\frac{dI_h}{dt} = \nu_h E_h - (\gamma + \mu_h + \delta_h) I_h \quad (4)$$

$$\frac{dR_h}{dt} = \gamma I_h - (\mu_h + \alpha_1) R_h \quad (5)$$

$$\frac{dL_v}{dt} = \psi(t) N_v - (\nu_L + \mu_L) L_v \quad (6)$$

$$\frac{dS_v}{dt} = \nu_L L_v - \lambda_v(t) S_v - \mu_v(t) S_v \quad (7)$$

$$\frac{dE_v}{dt} = \lambda_v(t) S_v - (\nu_v(T) + \mu_v(t)) E_v \quad (8)$$

$$\frac{dI_v}{dt} = \nu_v(T) E_v - \mu_v(t) I_v \quad (9)$$

## 1.1. Force of Infection Functions

The transition rates between susceptible and exposed classes are determined by the force of infection. In this model, the transmission is frequency-dependent, defined as follows:

$$\lambda_h(t) = \frac{a(T)\beta_h I_v}{N_h} \quad (10)$$

$$\lambda_v(t) = \frac{a(T)\beta_v I_h}{N_v} \quad (11)$$

Where the total populations for humans and adult mosquitoes are defined respectively as:

- $N_h = S_h + A_h + E_h + I_h + R_h$
- $N_v = S_v + E_v + I_v$

Here,  $N_v$  represents the total population of adult mosquitoes (vectors) at time  $t$ , excluding the aquatic larval stage ( $L_v$ ).

Note:  $N_v$  includes only adult mosquitoes ( $S_v, E_v, I_v$ ) as they are the only stages involved in the transmission cycle, while  $L_v$  is the source of recruitment into  $N_v$ .

## 2. The Optimal Control Model

Incorporating four time-dependent control measures  $u_1, u_2, u_3, u_4$ :

$$\dot{S}_h = \Lambda_h + \alpha_1 R_h - (1 - u_1) \lambda_h(t) S_h - (\mu_h + m) S_h + A_h \quad (12)$$

$$\dot{A}_h = m S_h - \sigma(1 - u_1) \lambda_h(t) A_h - (\mu_h + m) A_h \quad (13)$$

$$\dot{E}_h = (1 - u_1) \lambda_h(t) (S_h + \sigma A_h) - (\nu_h + \mu_h) E_h \quad (14)$$

$$\dot{I}_h = \nu_h E_h - (\gamma + u_2 + \mu_h + \delta_h) I_h \quad (15)$$

$$\dot{R}_h = (\gamma + u_2) I_h - (\mu_h + \alpha_1) R_h \quad (16)$$

$$\dot{L}_v = \psi(t) N_v - (\nu_L + \mu_L + u_3) L_v \quad (17)$$

$$\dot{S}_v = \nu_L L_v - (1 - u_1) \lambda_v(t) S_v - (\mu_v(t) + u_4) S_v \quad (18)$$

$$\dot{E}_v = (1 - u_1) \lambda_v(t) S_v - (\nu_v(T) + \mu_v(t) + u_4) E_v \quad (19)$$

$$\dot{I}_v = \nu_v(T) E_v - (\mu_v(t) + u_4) I_v \quad (20)$$

### 2.1 Description of Control Variables

The optimal control variables  $u_i(t) \in [0, 1]$  represent the following interventions:

- $u_1(t)$ : Personal protection measures (ITNs and repellents).
- $u_2(t)$ : Medical treatment control (anti-malarial drugs).
- $u_3(t)$ : Larviciding control (aquatic stage treatment).
- $u_4(t)$ : Adulticide control (IRS).

## 3. Comprehensive Table of Model Parameters

The following table summarizes all parameters used in the SEIRS-LSEI malaria model, classified by their biological and functional roles.

Table 1: Description of all parameters in the human-mosquito transmission model.

Symbol	Description	Category
<i>Human Population Parameters</i>		
$\Lambda_h$	Recruitment rate of humans (birth/immigration)	Demographic
$\mu_h$	Natural human mortality rate	Demographic
$\nu_h$	Rate of progression from exposed ( $E_h$ ) to infected ( $I_h$ )	Clinical
$\gamma$	Natural recovery rate of infected humans	Clinical
$\alpha_1$	Rate of immunity loss (moving from $R_h$ back to $S_h$ )	Clinical
$\delta_h$	Malaria-induced death rate	Clinical
<i>Awareness and Behavioral Parameters</i>		
$m$	Rate at which susceptible humans acquire awareness	Behavioral
	Rate at which aware humans lose awareness (forgetting rate)	Behavioral
$\sigma$	Factor reducing the risk of infection for aware individuals ( $0 < \sigma < 1$ )	Behavioral
<i>Mosquito Population Parameters</i>		
$\psi(t)$	Seasonal oviposition rate (eggs laid per adult female)	Aquatic
$\nu_L$	Maturation rate from larval stage ( $L_v$ ) to adult mosquito ( $S_v$ )	Aquatic
$\mu_L$	Natural mortality rate of larvae	Aquatic
$\mu_v(t)$	Temperature-dependent mortality rate of adult mosquitoes	Vector
$\nu_v(T)$	Temperature-dependent extrinsic incubation period (EIP)	Vector
<i>Transmission and Seasonality Parameters</i>		
$a(T)$	Temperature-dependent mosquito biting rate	Transmission
$\beta_h$	Probability of transmission from mosquito to human	Transmission
$\beta_v$	Probability of transmission from human to mosquito	Transmission
$\eta$	Amplitude of seasonal forcing (intensity of rainfall effect)	Seasonality
$\phi$	Phase shift (time of peak mosquito recruitment)	Seasonality