Quantum Metrology in Superconducting Transmons

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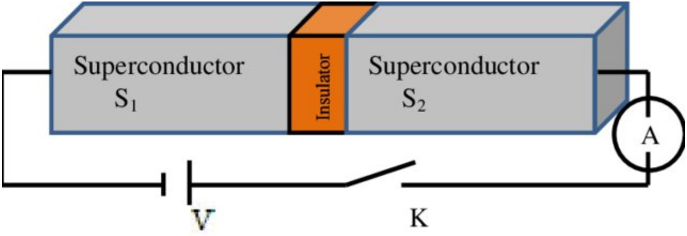
Motivation: Superconductivity



Schematic of a Josephson junction. Source: Maruf et al. (2018)

- Superconducting quantum circuits with Josephson junctions provide low-dissipation and long-coherence platforms for quantum technologies.
- Transmons are superconducting qubits based on the Josephson cosine potential $V(\hat{\phi}) = -E_J \cos \hat{\phi}$, which introduces nonlinearity into their dynamics and create distinct quantum states (e.g., —0⟩ and —1⟩) for reliable computation.

Motivation: Metrology

- By expanding the cosine potential $V(\hat{\phi})$, we can obtain a Hamiltonian that contains non-linear higher-order terms.
- Nonlinear terms can increase sensitivity to parameters, but the resulting states are more fragile to decoherence, so the practical advantage can vanish if noise dominates.
- An optimal measurement is needed to saturate F_Q (so $F_C = F_Q$) and extract the maximum information available from the noisy quantum state and retain the advantage of non-linear dynamics.

Objective

Achieve $F_C = F_Q$ to maximize metrology precision in transmons.

Quantum and Classical Fisher Information

Quantum Fisher Information (F_Q)

The maximum amount of information about a parameter g obtainable from a quantum state ψ_g through an optimal measurement. Defined as:

$$F_Q = 4\left(\langle \partial_g \psi_g | \partial_g \psi_g \rangle - \left| \langle \psi_g | \partial_g \psi_g \rangle \right|^2\right) \tag{1}$$

Classical Fisher Information (F_C)

The amount of information about a parameter g obtainable from the probability distribution $p(x|g)$ of a measurement outcome x. Defined as:

$$F_C = \int dx \, \frac{1}{p(x|g)} \left(\frac{\partial p(x|g)}{\partial g}\right)^2 \tag{2}$$

$F_C \leq F_Q$, equality requires optimal measurement.

Research Questions and Method

To saturate CFI to QFI, we address these questions:

1. What is the optimal initial state? $\longrightarrow$ *Achieves higher* F_Q
2. What is the optimal measurement? $\longrightarrow$ *Achieves* $\mathrm{F}_C = F_Q$

Method

1. Expand the cosine potential to obtain H_{eff}.
2. Apply the evolution operator $U(t) = e^{-iH_{\text{eff}}t/\hbar}$ to an initial state.
3. Compute F_Q.
4. Apply measurement and compute the probability distribution.
5. Compute F_C and see how it compares to F_Q for this measurement.

The Effective Hamiltonian

Upon expanding the potential up to the sixth order give:

$$-E_J \cos\hat{\phi} \;=\; -E_J \;+\; \frac{E_J}{2}\hat{\phi}^2 \;-\; \frac{E_J}{24}\hat{\phi}^4 \;+\; \frac{E_J}{720}\hat{\phi}^6 \;-\; .. \tag{3}$$

We obtain:

$$H_{eff} = \hbar\omega_p\,\hat{n} - \frac{3E_C}{4}\,\hat{n} - \frac{E_C}{4}\,\hat{n}(\hat{n}-1) + \frac{E_C}{18}\sqrt{\frac{2E_C}{E_J}}\,\hat{n}(\hat{n}-1)(\hat{n}-2) \tag{4}$$

Where:

$$\hbar\omega_p = \sqrt{8E_J E_C}, \qquad \hat{n} = a^\dagger a, \tag{5}$$

$\hbar\omega_p\, n$ is the linear energy term, and the remaining terms account for charging energy E_C and Josephson energy E_J (non-linear terms)

Evolution of initial states

- Cat states had a lower performance (i.e. lower F_Q) compared to a superposition of Fock states $|0\rangle + |n\rangle$ when n is large.
- H_{eff} is diagonal in Fock basis n and has eigenvalues

$$E_n = \hbar\omega_p\, n - \frac{3E_C}{4}\, n - \frac{E_C}{4}\, n(n-1) + \frac{E_C}{18}\sqrt{\frac{2E_C}{E_J}}\, n(n-1)(n-2), \quad (6)$$

with $E_0 = 0$.

- The time-evolved state thus picks a phase:

$$|\psi(t)\rangle = e^{-iH_{\text{eff}}t/\hbar}|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_0t/\hbar}|0\rangle + e^{-iE_nt/\hbar}|n\rangle\right). \quad (7)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i\phi}|n\rangle\right), \qquad \phi = E_nt/\hbar \quad (8)$$

To calculate F_Q for ω_p, we calculate the partial derivative:

$$\partial_g|\psi\rangle = -itn|\psi\rangle \longrightarrow F_Q(\omega_p) = n^2\, t^2 \quad (9)$$

Measurements: Fock VS Homodyne

Fock Measurement

- Counts photon number, $F_C = 0$, not optimal.

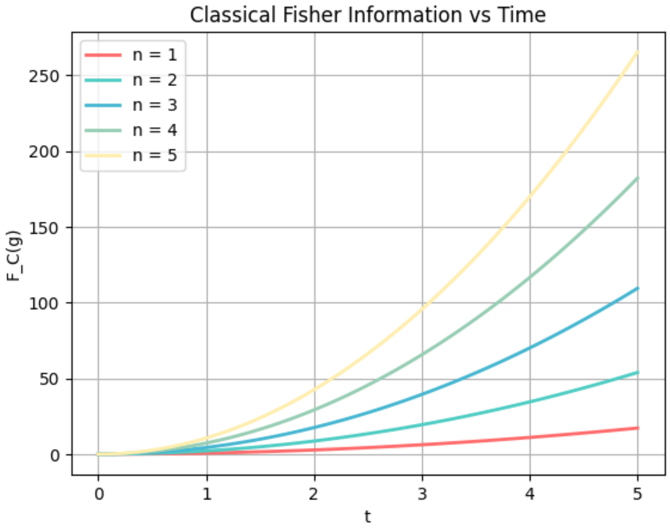
Homodyne Measurement

- Measures quadrature (amplitude).
- $$F^{\max}_{C_{|0\rangle,|n\rangle}} = n^2\, t^2 \left[2 - \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{1 + \frac{H_n^2(x)}{2^n n!}}\, dx \right], \tag{10}$$

  where $H_n(x)$ are Hermite polynomials.

For $n = 1$ the integral evaluates to 1.15, $n = 2 : 1.30$, $n = 3 : 1.34$.. Thus, $F_C < F_Q$, suboptimal.

Visual Results



Future work

- Try other measurements more optimal for non-linear dynamics, such as the projection into $|+>, |->$

Acknowledgments

- We thank Dr. Andrea Maiani for inspiring the work on transmon potential.