Engineering Physics
Course Code : TPH 201

By Dr. Vishal Chauhan

<u>Frames of reference</u>: A frame of reference is a set of coordinates that can be used to determine positions and velocities of objects in that frame; different frames of reference move relative to one another.

<u>POSTULATES OF SPECIAL RELATIVITY:</u>

1. Principle of Relativity – All the laws of physics are the same in all inertial reference frames.

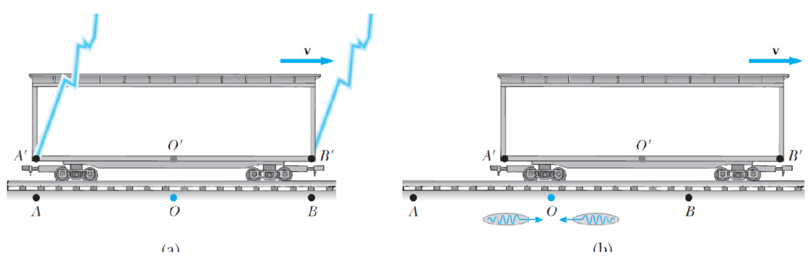
2. The speed of light in a vacuum is the same (3.0 x108 m/s) in all inertial reference frames regardless of the motion of the observer or source.

<u>CONSEQUENCES OF SPECIAL RELATIVITY:</u>

a) There is no such thing as absolute length or absolute time in relativity.

b) A time interval (or length) measurement depends on the reference frame in which it is made.

Consider the following thought experiment devised by Einstein: A trainbox is moving with constant velocity when two lightning bolts strike the end of the trainbox leaving marks on the boxcar and ground. Two observers are located midway between the ends of trainbox. One at O' and another fixed on the ground at O.



a) Relative to observer at O the two light signals reach him at the same time. Since the light signals traveled at the same speed over equal distances, he concludes that the two events at A and B occurred simultaneous.

b) Relative to observer at O' the light signals reach him at different times. Since he is at the midpoint he observes that the light signals must take the same time to reach him because both light signals travel at the same speed over equal distances. Since the light signals do not reach him at the same time, he concludes that they were **NOT** simultaneous.

IN GERERAL, TWO EVENTS THAT ARE SIMULTANEOUNS IN ONE INERTIAL REFERENCE FRAME ARE NOT SIMULTANEOUS IN A SECOND INERTIAL RF MOVING AT CONSTANT VELOCITY RELATIVE TO THE FIRST!!!

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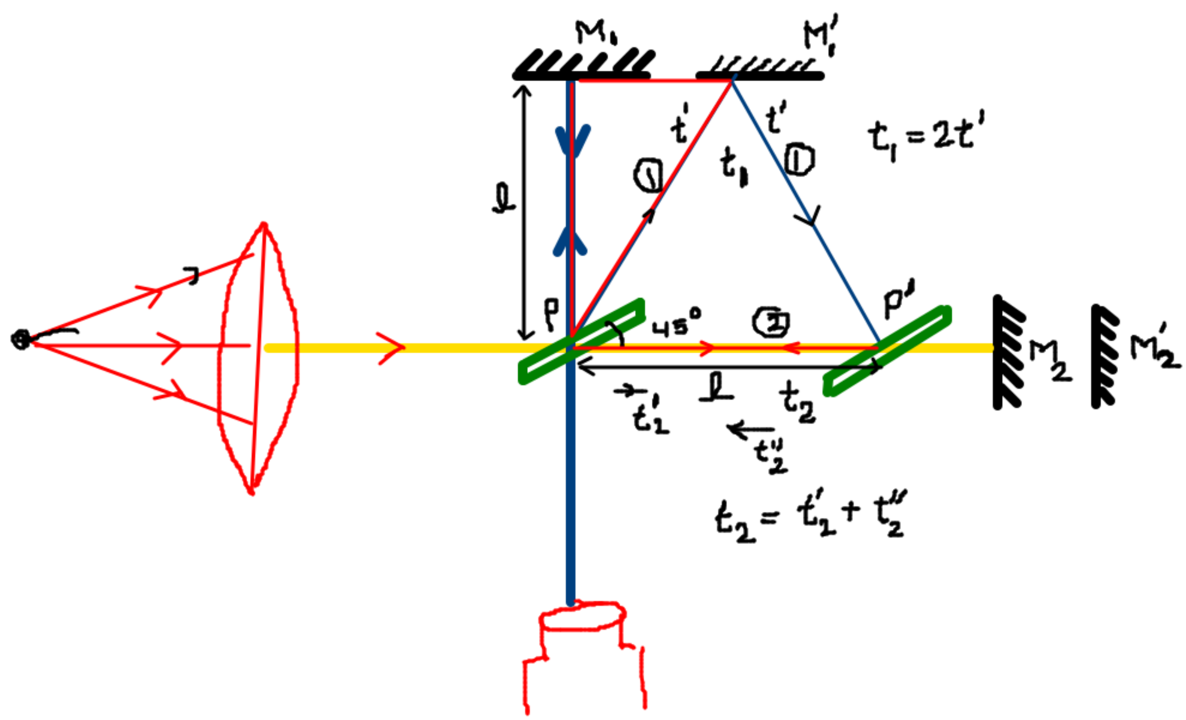
Michelson Morley Experiment

Michelson and Morley built a Michelson interferometer, which essentially consists of a light source, a half-silvered glass plate, two mirrors, and a telescope. The mirrors are placed at right angles to each other and at equal distance from the glass plate, which is obliquely oriented at an angle of 45° relative to the two mirrors. In the original device, the mirrors were mounted on a rigid base that rotates freely on a basin filled with liquid mercury in order to reduce friction.
Prevailing theories held that ether formed an absolute reference frame with respect to which the rest of the universe was stationary. It would therefore follow that it should appear to be moving from the perspective of an observer on the sun-orbiting Earth. As a result, light would sometimes travel in the same direction of the ether, and others times in the opposite direction. Thus, the idea was to measure the speed of light in different directions in order to measure speed of the ether relative to Earth, thus establishing its existence.

Michelson and Morley were able to measure the speed of light by looking for interference fringes between the light which had passed through the two perpendicular arms of their apparatus. These would occur since the light would travel faster along an arm if oriented in the "same" direction as the ether was moving, and slower if oriented in the opposite direction. Since the two arms were perpendicular, the only way that light would travel at the same speed in both arms and therefore arrive simultaneous at the telescope would be if the instrument were motionless with respect to the ether. If not, the crests and troughs of the light waves in the two arms would arrive and interfere slightly out of synchronization, producing a diminution of intensity. (Of course, the same effect would be achieved if the arms of the interferometer were not of the same length, but these could be adjusted accurately by looking for the intensity peak as one arm was moved. Changing the orientation of the instrument should then show fringes.)

Although Michelson and Morley were expecting measuring different speeds of light in each direction, they found no discernible fringes indicating a different speed in any orientation or at any position of the Earth in its annual orbit around the Sun.



(i) Time taken by light to reach from P to M_2

$$t_2' = \frac{l}{c-v} = \frac{\text{distance}}{\text{speed}}$$

$\longrightarrow c$

$\longrightarrow v$

Relative speed $= c - v$

(ii) Time taken by light to reach M_2 to P

$$t_2'' = \frac{l}{c+v}$$

$\longleftarrow c$

$\longrightarrow v$

(iii) Total time $\Rightarrow$ $t_2 = t_2' + t_2''$

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{l(c+v) + l(c-v)}{(c-v)(c+v)}$$

$$t_2 = \frac{lc + \cancel{lv} + lc - \cancel{lv}}{c^2 - v^2}$$

$$t_2 = \frac{2lc}{c^2 - v^2}$$

$$t_2 = \frac{2lc/c^2}{c^2 - v^2/c^2}$$

$$\boxed{t_2 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1}}$$

Now time taken by light ray from P to M_1' and M_1' to P'.

In $\Delta PM_1M_1'$

$$\Rightarrow (PM_1')^2 = (PM_1)^2 + (M_1M_1')^2$$

$$(ct')^2 = l^2 + (v.t')^2$$

$$c^2t'^2 = l^2 + v^2t'^2$$

$$c^2t'^2 - v^2t'^2 = l^2$$

$$t'^2(c^2 - v^2) = l^2$$

$$t' = \sqrt{\frac{l^2}{(c^2 - v^2)}}, \quad \because t_1 = 2t'$$

$$= \frac{2l}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t_1 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}}$$

time Diff $(\Delta t) = t_2 - t_1$

$$\Rightarrow \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1} - \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\Rightarrow \Delta t = \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1} - \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$\because (1+x)^n = 1 + nx + \cdots$$

where $x << 1$ $\quad \therefore \frac{v^2}{c^2} << 1$

$$\therefore \left(1-\frac{v^2}{c^2}\right)^{-1} = 1 + \frac{v^2}{c^2}$$

$$\& \left(1-\frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$\Delta t = \frac{2l}{c}\left[1 + \frac{v^2}{c^2} - 1 - \frac{v^2}{2c^2}\right] \Rightarrow \frac{2l}{c}\left[\frac{v^2}{c^2} - \frac{v^2}{2c^2}\right]$$

$$\Delta t = \frac{\cancel{2}l}{c}\left(\frac{v^2}{\cancel{2}c^2}\right) \Rightarrow \Delta t = \frac{v^2 l}{c^3}$$

Path Diff. $\Delta P = c \cdot \Delta t$

$$= \cancel{c} \cdot \frac{v^2 l}{c^{\cancel{3}}}$$

$$\boxed{\Delta P = \frac{v^2 l}{c^2}}$$

Then whole experiment shifted (rotated) at 90 degree.

then $\Delta P = -\Delta P$

Effective Path diff $\Delta P' = \Delta P - (-\Delta P)$

$$= 2\Delta P$$

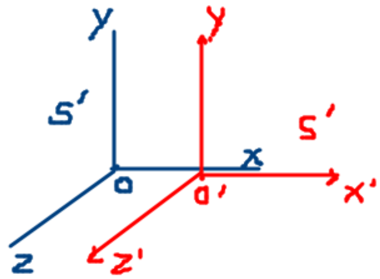
$$\boxed{\Delta P' = \frac{2v^2 l}{c^2}}$$

No. of fringe shifted $= \dfrac{2v^2 l}{c^2 \lambda}$

$$\Delta N = \frac{2 \times (3\times10^4)^2 \times 11}{(3\times10^8)^2 \times 5500 \times 10^{-10}}$$

$\Delta N = 0.4 \longrightarrow$ It remained same ever

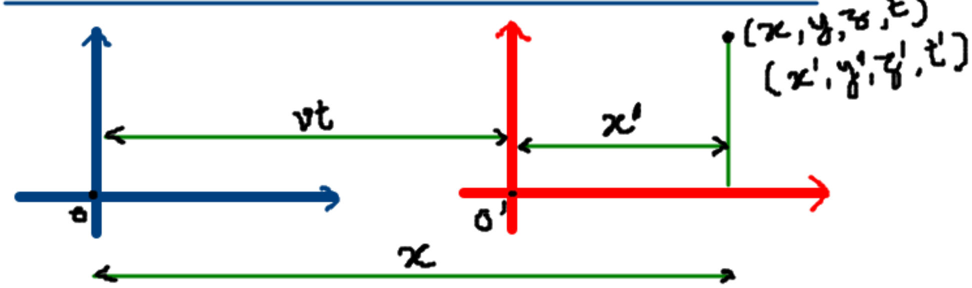
Galilean Transformation



(i) If S is an inertial frame then all frames moving with uniform velocity relative to S are inertial frames.
(ii) Here, S′ is moving with uniform velocity w.r.to S therefore S′ is also inertial.

① S & S' both are inertial
② x & x' are on the same line
y & y' are parallel.
z & z' are parallel.
③ When origin of S' is on the origin of S, $t = t' = 0$
④ Coordinates of an event in S at time t (x, y, z)
⑤ Coordinates of an that event in S' frame (x', y', z')
If x, y, z & t are known x', y', z' & t' can be calculated.

Frame S is moving with v-velocity in (+ve) x-direction.



$$x = vt + x'$$

$$\begin{cases} x' = x - vt & \text{———— (1)} \\ y' = y & \text{———— (2)} \\ z' = z & \text{———— (3)} \\ t' = t & \text{———— (4)} \end{cases}$$

or

$$x = x' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

These are Galilean Transformation Equations.

From these equations we can derive velocity and acceleration transformation equations.

Differentiate eq^n ①, ② & ③ w.r. to time t →

$$\frac{d(x')}{dt} = \frac{d}{dt}(x - vt)$$

$$V'_x = \frac{dx}{dt} - v$$

$$\boxed{\begin{aligned} V'_x &= V_x - V \\ V'_y &= V_y \\ V'_z &= V_z \end{aligned}}$$

$$\because V_x = \frac{dx}{dt}, \quad V_y = \frac{dy}{dt}, \quad V_z = \frac{dz}{dt}$$

By differentiating velocity transformation equations we can obtain the transformation equations for acceleartion.

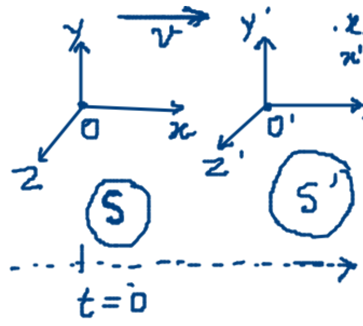
$$a'_x = \frac{d(V'_x)}{dt}$$

$$a'_x = \frac{d}{dt}(V_x - V)$$

$$a'_x = \frac{dV_x}{dt}$$

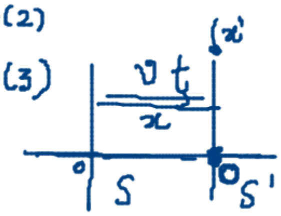
$$\boxed{\begin{aligned} a'_x &= a_x \\ a'_y &= a_y \\ a'_z &= a_z \end{aligned}}$$

Hence, this shows that particles acceleration is same in all inertial frames.



Lorentz transformation Eqns

$$\left[\begin{array}{ll} x' = ax + bt & \text{——— (1)} \\ y' = y & \text{——— (2)} \\ z' = z & \text{——— (3)} \\ t' = dx + ft & \end{array}\right.$$



O, O'

t = 0

when S' is crossing origin of S at t = 0

$$\begin{cases} S : x = vt,\ y = 0,\ z = 0 \\ S' : x' = 0,\ y' = 0,\ z' = 0,\ t' \end{cases}$$

$$x' = ax + bt \quad \text{——— (1)}$$

$$x' = a(vt) + bt$$

$$0 = avt + bt$$

$$0 = (av + b)t$$

$$\Rightarrow av + b = 0, \quad \boxed{b = -av}$$

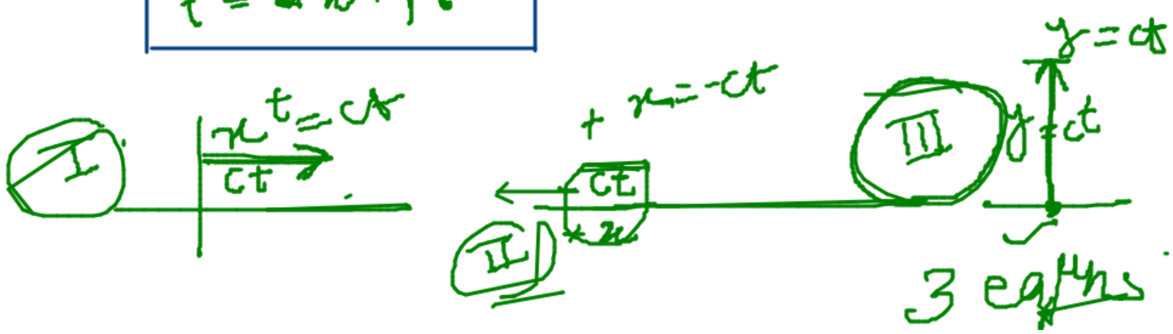
$$x' = ax + bt$$

$$x' = ax - avt$$

$$\boxed{x' = a(x - vt)}$$

$$\boxed{\begin{array}{l} y' = y \\ z' = z \\ t' = dx + ft \end{array}}$$

C



(I) $x = ct, \; y = 0, \; z = 0, \; t$

$$x' = a(x - vt)$$
$$= a(ct - vt)$$
$$x = at(c - v)$$

$$t' = d \times ct + ft$$
$$t' = t(dc + f)$$
$$\frac{ct'}{t'} = \frac{a\cancel{t}(c - v)}{\cancel{t}(dc + f)}$$
$$c = \frac{a(c - v)}{dc + f}$$
$$\boxed{dc^2 + fc = a(c - v)} \quad \text{———} (1)$$

(II) $x = -ct, \; y = 0, \; z = 0$

$$x' = a(x - vt)$$
$$= a(-ct - vt)$$
$$x = -at(c + v)$$
$$t' = dx + ft$$
$$= -dct + ft$$
$$t' = (-dc + f)t$$

$$x' = -ct'$$
$$\cancel{-}\frac{c\cancel{t'}}{\cancel{t'}} = \cancel{-}\frac{a\cancel{t}(c + v)}{(-dc + f)\cancel{t}}$$

$$c = \frac{a(c + v)}{-dc + f}$$
$$\boxed{-dc^2 + fc = a(c + v)} \quad \text{———} (2)$$

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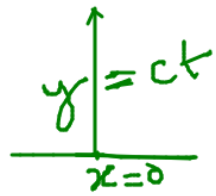
III



$x = 0 \quad y = ct, \ z = 0, \ t$

$$\left[\begin{array}{l} x' = a(x - vt) = -avt \\ y' = ct \\ z' = 0 \\ t' = x + ft = ft \end{array}\right.$$

$$d^2 = x'^2 + y'^2 + z'^2$$
$$= a^2v^2t^2 + c^2t^2 + 0$$
$$c^2f^2\cancel{t^2} = a^2v^2t^2 + c^2t^2$$
$$= \cancel{t^2}(a^2v^2 + c^2)$$

$d = ct'$
$d^2 = c^2t'^2$
$t'^2 = f^2t^2$
$d^2 = c^2f^2t^2$

$$\boxed{c^2f^2 = a^2v^2 + c^2} \longrightarrow (3)$$

$$dc^2 + fc = a(c - v) \longrightarrow (1)$$
$$-dc^2 + fc = a(c + v) \longrightarrow (2)$$
$$a^2v^2 + c^2 = c^2f^2 \longrightarrow (3)$$

1 + 2

$$\cancel{2}f\cancel{c} = \cancel{2}a\cancel{c}$$
$$\boxed{f = a}$$

$$a^2v^2 + c^2 = c^2a^2$$
$$c^2 = a^2c^2 - a^2v^2$$
$$c^2 = a^2(c^2 - v^2)$$

$$a^2 = \frac{c^2}{(c^2 - v^2)}$$
$$= \frac{\cancel{c^2}\,1}{\cancel{c^2}\left(1 - \frac{v^2}{c^2}\right)}$$

$$\boxed{a = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

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$$dc^2 + fc = a(c - v)$$

$$dc^2 + ac = ac - av$$

$$dc^2 = \cancel{ac} - av - \cancel{ac}$$

$$dc^2 = -av$$

$$d = \frac{-av}{c^2}$$

$\because f = a,$

$$\boxed{d = \frac{-v/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\left[\begin{aligned} x' &= a(x - vt) \\ y' &= y \\ z' &= z \\ t' &= dx + ft \end{aligned}\right.$$

$$x' = \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{-v/c^2\, x}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t' = \frac{t - vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Lorentz Transformation Equations

$$\boxed{\begin{aligned} x' &= \frac{(x - vt)}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \end{aligned}}$$

$$\boxed{t' = \frac{t - vx/c^2}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Q: $x^2 + y^2 + z^2 - c^2t^2$ —— (1)

$x'^2 + y'^2 + z'^2 - c^2t'^2$ —— (2)

Show that these two quantities are equal in both frames or invariant under Lorentz Transformation.

Sol: $\because \; x' = \left(\frac{x - vt}{\sqrt{1 - v^2/c^2}}\right), \; y' = y, \; z' = z$

from eqn (2)

$$= \left[\frac{x - vt}{\sqrt{1 - v^2/c^2}}\right]^2 + y^2 + z^2 - c^2\left[\frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}\right]^2$$

$$= y^2 + z^2 + \frac{(x - vt)^2 - c^2\left[t - \frac{xv}{c^2}\right]^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= y^2 + z^2 + \frac{x^2 + v^2t^2 - 2xvt - c^2\left(t^2 - \frac{2xvt}{c^2} + \frac{x^2v^2}{c^4}\right)}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\Rightarrow y^2+z^2+\frac{x^2+v^2t^2-\cancel{2xvt}-c^2t^2+\cancel{2xvt}-\frac{x^2v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\right)}$$

$$= y^2+z^2+\frac{x^2\left(1-\frac{v^2}{c^2}\right)-t^2\left(c^2-v^2\right)}{\left(1-\frac{v^2}{c^2}\right)}$$

$$= y^2+z^2+\frac{x^2\cancel{\left(1-\frac{v^2}{c^2}\right)}}{\cancel{\left(1-\frac{v^2}{c^2}\right)}}-\frac{t^2\cancel{(c^2-v^2)}}{\frac{\cancel{c^2-v^2}}{c^2}}$$

$$= \boxed{y^2+z^2+x^2-c^2t^2}$$

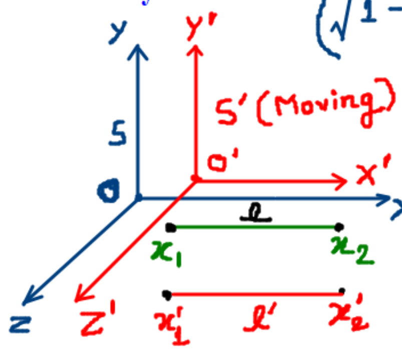
Found same as equation (1)
Hence found invariant under
Lorentz Transformation.

Length Contraction: When an object moves with a velocity (v) (comparable to the velocity of light) relative to a stationary observer, its measured length appears to be contracted and the length contracted by a factor of $\left(\sqrt{1-\frac{v^2}{c^2}}\right)$.

$$l = x_2 - x_1 \qquad (1)$$

$$l' = x_2' - x_1' \qquad (2)$$

$$\because x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$



From eqn (2) →

$$l' = \frac{x_2-vt}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_1-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$l' = \frac{x_2-\cancel{vt}-x_1+\cancel{vt}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{x_2-x_1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{l}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{l = l'\sqrt{1-\frac{v^2}{c^2}}}$$

Q: What will be the length of a meter rod appear to be for a person travelling parallel to the length of the road at the speed of 0.8c relative to the rod ?

Sol:

$$\because \; l = l'\sqrt{1-\frac{v^2}{c^2}} \quad , \quad l' = 1m, \; v = 0.8c$$

$$l = 1\sqrt{1-\frac{(0.8c)^2}{c^2}}$$

$$= \sqrt{1-\frac{(0.64)\cdot \cancel{c^2}}{\cancel{c^2}}}$$

$$= \sqrt{0.36}$$

$$= 0.6\,m$$

Time Dilation: A clock moving with a velocity (v) relative to an observer appears to go slow when at rest relative to him by a factor of $\sqrt{1-\frac{v^2}{c^2}}$

Let initially S and S′ frames are at rest relative to each other later S remains stationary and S′ is moving with a velocity (v) which is comparable to the speed of light. Then let the clock in S frame is situated at a position x and given out signals at two instants of time t1 and t2 as measured by an observed in S.

$$t_0 = t_2 - t_1$$

Let the time measured by an observer O′ in moving frame S′ between the same two events be t_1' and t_2' Thus $t = t_2' - t_1'$

$$t = \frac{t_2 - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1 - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t_2 - \cancel{\frac{xv}{c^2}} - t_1 + \cancel{\frac{xv}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t_0 = t\sqrt{1 - \frac{v^2}{c^2}}}$$

Q: The mean life of a meson is 2×10^{-8} s. Calculate the mean life of a meson moving with a velocity of 0.8c.

Sol:

$t_0 = 2\times10^{-8}$ sec.

$v = 0.8c$

$t = ?$

$$\because \quad t_0 = t\sqrt{1 - \frac{v^2}{c^2}}$$

$$2\times10^{-8} = t\sqrt{1 - \frac{(0.8c)^2}{c^2}}$$

$$2\times10^{-8} = t\sqrt{1 - \frac{(0.64)\cancel{c^2}}{\cancel{c^2}}}$$

$$t = \frac{2\times10^{-8}}{\sqrt{0.36}} = \frac{2\times10^{-8}}{0.6} = 3.33\times10^{-8}\text{ sec}$$

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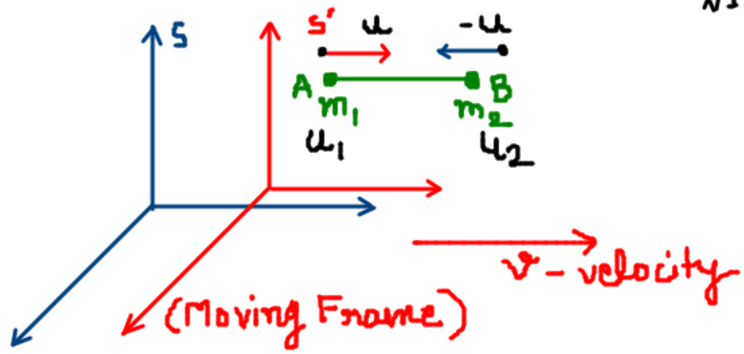
Derivation of mass with velocity

In relativistic mechanics the mass of a body varies with its velocity. The mass of a body moving at very high speed (comparable to speed of light) relative to an observer is larger than its mass when it is at rest by a factor of $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$



By Lorentz transformation, following equation for the variation of mass with velocity is obtained;

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

As per relativistic velcoity addition theorem;

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \quad (1) \quad , \quad u_2 = \frac{-u+v}{1-\frac{uv}{c^2}} \quad (2)$$

(i) Let is consider two identical bodies A and B of masses m1 and m2 are moving with velocities u and -u in the moving frame along a straight line. Then consider the collision between the two w.r. to the stationary frame.

(ii) At the time of collision, the two bodies are momentarily at rest relative to moving frame but still moving w. r. to stationary frame.

(iii) If it is an elastic collision then according to the principle of conservation of momentum.

Momentum before impact = Momentum after impact

$$m_1 u_1 + m_2 u_2 = m_1 v + m_2 v$$

$$\Rightarrow \quad m_1\left(\frac{u+v}{1+\frac{uv}{c^2}}\right) + m_2\left(\frac{-u+v}{1-\frac{uv}{c^2}}\right) = m_1 v + m_2 v$$

$$\Rightarrow \quad m_1\left(\frac{u+v}{1+\frac{uv}{c^2}} - v\right) = m_2\left[v - \left(\frac{-u+v}{1-\frac{uv}{c^2}}\right)\right]$$

$$\Rightarrow m_1\left(\frac{u+\cancel{v}-\cancel{v}-\cancel{\frac{uv^2}{c^2}}}{1+\frac{uv}{c^2}}\right) = m_2\left[\frac{\cancel{v}+u-\cancel{v}-\cancel{\frac{uv^2}{c^2}}}{1-\frac{uv}{c^2}}\right]$$

$$\frac{m_1}{1+\frac{uv}{c^2}} = \frac{m_2}{1-\frac{uv}{c^2}} \qquad \boxed{\frac{m_1}{m_2} = \frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}}} \quad \text{---} (3)$$

$$\left(1-\frac{u_1^2}{c^2}\right) \Rightarrow 1 - \frac{1}{c^2}\left(\frac{u+v}{1+\frac{uv}{c^2}}\right)^2$$

$$\Rightarrow 1 - \frac{\left(\frac{u+v}{c}\right)^2}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$= \frac{\left(1+\frac{uv}{c^2}\right)^2 - \left(\frac{u+v}{c}\right)^2}{\left(1+\frac{uv}{c^2}\right)^2} \Rightarrow \frac{\left(1-\frac{u^2}{c^2}\right) - \frac{v^2}{c^2}\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\frac{\left(1-\frac{u^2}{c^2}\right)\left[1-\frac{v^2}{c^2}\right]}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\left(1-\frac{u_1^2}{c^2}\right) = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$u_1 \xrightarrow{\;u\;} \qquad \xleftarrow{\;-u\;} u_2$$

$$\left(1+\frac{uv}{c^2}\right)^2 = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}$$

$$1+\frac{uv}{c^2} = \sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}}$$

$$1-\frac{uv}{c^2} = \sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_2^2}{c^2}\right)}}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{\frac{\cancel{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}}{\left(1-\frac{u_1^2}{c^2}\right)}}}{\sqrt{\frac{\cancel{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}}{\left(1-\frac{u_2^2}{c^2}\right)}}}$$

$$\frac{m_1}{m_2} = \sqrt{\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}}}$$

$$m_2 = m_0, \quad m_1 = m$$
$$u_2 = 0 \qquad u_1 = u$$

$$\frac{m}{m_0} = \sqrt{\frac{1}{1-\frac{u^2}{c^2}}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}}$$

if $u \lll c$
$u^2 \lll c^2$
then
$m = m_0$

if
$u = c$
then
$m = \infty$

Relation between relativistic mass and momentum

$$E = mc^2$$

$$\because \quad m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{m^2 v^2}{m^2 c^2}}}$$ Divide & Multiply by m^2

$$= \frac{m_0 c^2}{\sqrt{1 - \frac{p^2}{m^2 c^2} \times \frac{c^2}{c^2}}}$$ Divide & Multiply by c^2

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{p^2 c^2}{m^2 c^4}}}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{p^2 c^2}{E^2}}}$$

Square both sides

$$E^2 = \frac{m_0^2 c^4}{1 - \frac{p^2 c^2}{E^2}}$$

$$E^2\left(1 - \frac{p^2 c^2}{E^2}\right) = m_0^2 c^4$$

$$E^2 - \frac{\cancel{E^2} p^2 c^2}{\cancel{E^2}} = m_0^2 c^4$$

$$E^2 - p^2 c^2 = m_0^2 c^4$$

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$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$\boxed{E = \sqrt{m_0^2 c^4 + p^2 c^2}}$$

It is the relationship between energy and momentum and known as relativistic energy.