GRAPHIC ERA HILL UNIVERSITY

Department of Mathematics

TMA-316 : Discrete Mathematical Structures and Combinatorics

(Assignment No: 2)

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1. Let U be the set of real numbers, $A = \{x : x^2 - 1 = 0\}$ and $B = \{-1, 4\}$. Compute :-
   a) $\overline{A}$, b) $\overline{B}$, c) $\overline{A \cup B}$, d) $\overline{A \cap B}$.

2. Let A, B and C are finite sets with $|A| = 6$, $|B| = 8$, $|C| = 6$, $|A \cup B \cup C| = 11$, $|A \cap B| = 3$, $|A \cap C| = 2$ and $|B \cap C| = 5$. Find $|A \cap B \cap C|$.

3. Suppose that $A \oplus B = A \oplus C$. Does this guarantee that $B = C$? Justify your conclusion.

4. Let $A = \{x : x \text{ is an integer and } x^2 < 16\}$. Identify each of the following as true or false :-
   a) $\{0, 1, 2, 3\} \subseteq A$, b) $\{\} \subseteq A$, c) $\{-3, -2, -1\} \subseteq A$, d)$A \subseteq \{-3, -2, -1, 0, 1, 2, 3\}$.

5. Consider the theorem :-

   $$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|.$$

   Verify it for the sets $A = \{a, b, c, d, e\}$, $B = \{a, b, e, g, h\}$ and $C = \{b, d, e, g, h, k, m, n\}$.

6. Draw a Venn diagram to represent the situation $A \subseteq C$ and $B \subseteq C$. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(1, 2), (1, 4), (2, 3), (2, 5), (3, 6), (4, 7)\}$. Give the matrix representation and digraph of relation R.

7. Define the followings with suitable examples:-
   a) Relation, c) Reflexive Relation, c) Irreflexive Relation,
   d) Symmetric Relation e) Antisymmetric Relation f) Asymmetric Relation,
   g) Transitive Relation.

   If the cardinality of a set is n, determine - i) least number of elements, ii) most number of elements, and iii) possible numbers of relations for each case mentioned in Question-7.

8. Let $A = \mathbb{Z}$, the set of integers, and let $R = \{(a, b) \in A \times A \mid a < b\}$ so that R is a relation '**less than**'. Is R Symmetric, Asymmetric or/and Antisymmetric ?

9. Let $A = \mathbb{Z}^+$, the set of positive integers, and let $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$. Is R Symmetric, Asymmetric or/and Antisymmetric ?

10. Let $A = \{1, 2, 3, 4\}$. Give a relation R on A which is:-
    a) neither symmetric nor antisymmetric,
    b) ant-symmetric and reflexive but not transitive,
    c) transitive and reflexive but not antisymmetric.

11. Let R be a binary relation defined as : $R = \{(a, b) \in R \ : \ a - b \leq 3\}$. Determine whether R Reflexive, Irreflexive, Symmetric, Asymmetric, Antisymmetric and/or Transitive.

12. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) \in R \ : \ x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation.

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13. Show that the relation '$\subseteq$' defined on the power set $P(A)$ of a set A is a partial order relation.

14. Show that the relation '$\leq$' defined on the the set of positive integers (I_+) is a partial order relation.

15. Let $\mathbb{N}$ be the set of all natural numbers. The relation R on the set $N \times N$ of ordered pairs of natural numbers is defined as : $(a, b)\ R\ (c, d)$ if and only if $ad = bc$. Prove that R is an equivalence relation.

16. Let R be an equivalence relation on a set A, prove that R^{-1} is also an equivalence relation.

17. Let R be an equivalence relation on a non-empty set A. Let a and b be arbitrary elements in A, prove that:-
    a) $a \in [a]$ i.e. $[a]$ is non-empty, b) $b \in [a] \Leftrightarrow [b] = [a]$, c) $[a] = [b] \Leftrightarrow (a, b) \in R$,
    d) equivalence class of a and b are either disjoint or identical, i.e. either $[a] = [b]$ or $[a] \cup [b] = \phi$.

18. Define reflexive, symmetric and transitive closures of a relation with suitable example. Find these closure of the relation R, defined on set $A = \{1, 2, 3, 4\}$ such that $R = \{(1, 1), (1, 2), (1, 4), (2, 4), (3, 1), (3, 2), (4, 2), (4, 3), (4, 4)\}$.

19. Define the function and types of functions with suitable examples.

20. List all possible functions $X = \{a, b, c\}$ to $Y = \{0, 1\}$ and indicate in each case whether the function is one to one, is onto and is one-one onto.

21. Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find fog ang gof, where $f(x) = x^2$ and $g(x) = x + 4$. State whether functions are injective, surjective or bijective.

22. Let $A = \{x : x \in R, \text{ and } -\pi/2 \leq x \leq \pi/2\}$, and $B = \{y : y \in R, \text{ and } -1 \leq x \leq 1\}$. Show that the function $f : A \to B$ such that $f(x) = \sin x$, for all $x \in A$ is one one onto. Also find the inverse function f^{-1}.

23. Let $f : X \to Y$ be an everywhere defined invertible function and A and B be arbitrary non-empty subsets of Y. Show that :-
    a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

24. Draw a Venn diagram to represent the situation $A \subseteq C$ and $B \subseteq C$.

25. Determine the Hasse Diagram of the relation R
    (a) on set $A = \{1, 2, 3, 4\}$ such that $R = \{(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)\}$,
    (b) on set $A = \{1, 2, 3, 4, 5\}$ such that $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 4), (3, 5), (2, 2), (3, 3), (4, 4), (5, 5)\}$,

26. Find the transitive closure R by Warshall's Algorithm $A = \{\text{Set of positive integers} \leq 10\}$ and $R = \{(a, b) \mid a \text{ divide by } b\}$.

27. Let R be the relation on the set $A = \{5, 6, 8, 10, 28, 36, 48\}$ and $R = \{(a, b) \mid a \text{ is a divisor of } b\}$. Draw Hasse diagram. Compare with digraph. Determine, whether R is equivalence relation.

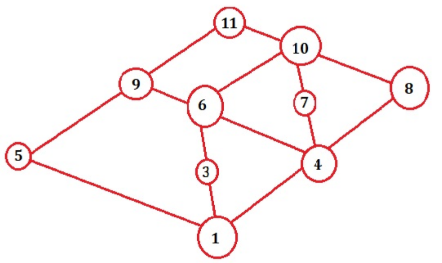
28. For any $a, b \in L$, show that $\wedge(a \vee b) = a$ and $a \vee (a \wedge b) = a$.

29. If n be a positive integer and S_n be a set of all divisors of n. Let D denotes the relation of 'division'. Draw the diagrams of the lattices for :
    (a) $n = 24$ (b) $n = 30$ (c) $n = 6$.

30. Elaborate the term isomorphic poset ? Let $A = \{1, 2, 3, 4\}$ and $\leq$ (Relation) be partial order of divisibility on A.

31. Show that whether the relation $(x, y) \in R$, if $x \geq y$ defined on the set of positive integer is a partial order relation.

32. Let $A = \{1, 2, 3, 4, ..., 11\}$, be the poset whose Hasse diagram shown in the figure. Find the LUB and GLB of $B = \{6, 7, 10\}$ if they exist.



33. A partition of a positive integer m is a set of positive integers whose sum is m. Draw the diagram of the partitions of m, where $m = 4, 5$ and 6.

34. Prove that the set $P(S), \subseteq)$ for any set S is a lattice.

35. If $(L, \leq)$ is a lattice, then $(L, \geq)$ is also a lattice.

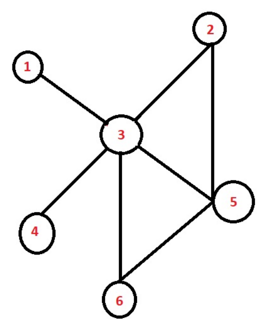
36. Show that dual of a lattice is a lattice.

37. Show that every chain is a distributed lattice.

38. If $L(\cap, \cup)$ is a complimented distributed lattice, then the compliment of $a \in L$ is unique.

39. Show that if $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ denote the set of the divisors of 36 ordered of divisibility, then $(D_{36},' l')$ is lattice.

40. Let $A = \{1, 2, 3, 4, 5, 6\}$ be ordered as mentioned in the figure :
    (a) Find all the minimal and maximal elements of A.
    (b) Does A have a first or last element ?



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End of the assignment..