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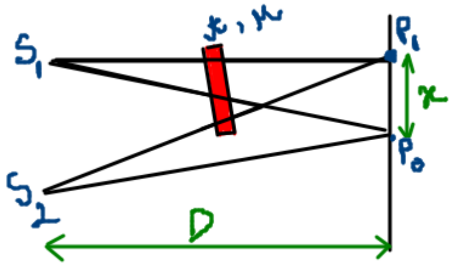
Applications of Fresnel's Biprism

(i) Determination of wavelength of monochromatic light.

(ii) Determination of thickness of thin film.



t = thickness of film

μ = refractive index of the medium

Path difference due to thin film

$$\Delta = \mu t - t$$

$$\Delta = t(\mu - 1) \quad \text{———— (1)}$$

$$\Delta = S_2P - S_1P$$

$\because$ For bright fringe, path diff $= n\lambda$ ———— (2)

from (1) & (2) $\Rightarrow$

$$n\lambda = t(\mu - 1)$$

$$\boxed{t = \frac{n\lambda}{\mu - 1}} \quad \text{———— (A)}$$

$\because$ we know for dark fringe

$$\Delta = (2n-1)\frac{\lambda}{2}$$

using eq^n (1)

$$(2n-1)\frac{\lambda}{2} = t(\mu - 1)$$

$$\boxed{t = \frac{(2n-1)\lambda}{2(\mu - 1)}} \quad \text{———— (B)}$$

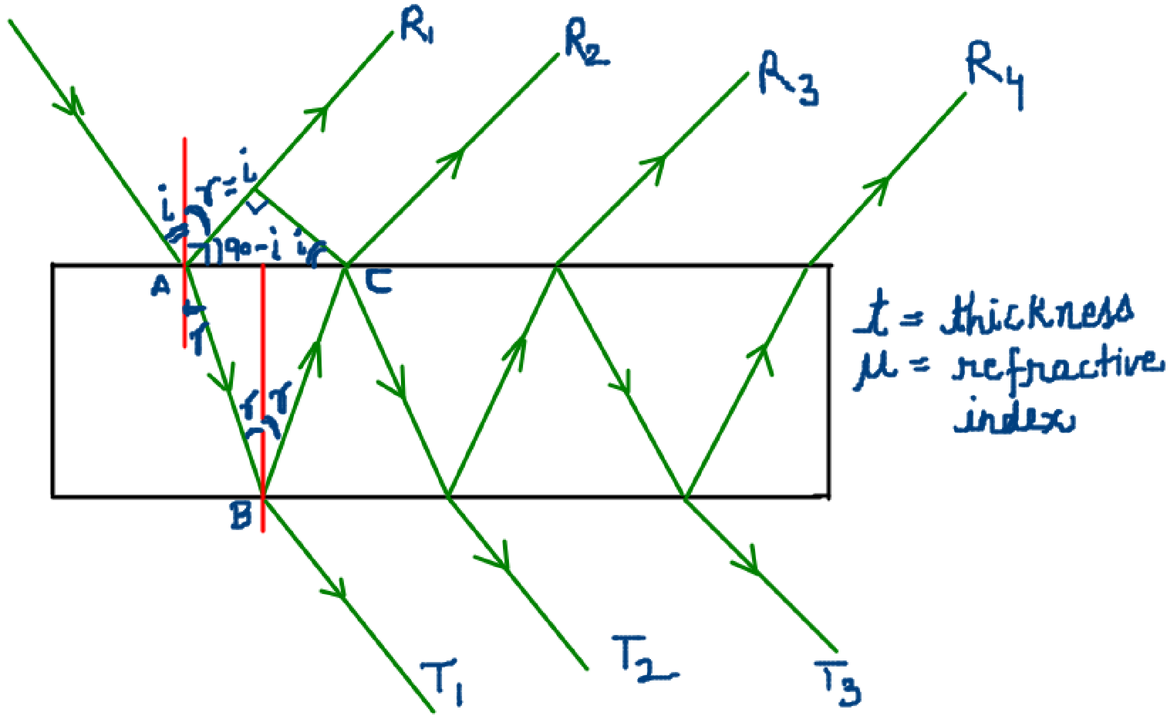
we also know that

$$\Delta = \frac{xd}{D}$$

then $$\frac{xd}{D} = t(\mu - 1)$$

$$\boxed{t = \frac{xd}{D(\mu - 1)}} \quad \text{———— (C)}$$

Interference due to uniform thin film



(i) For reflected rays of light

$$\boxed{\text{Path diff. } (\Delta) = 2\mu t \cos r - \frac{\lambda}{2}}$$

(1) Condition for constructive interference

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t = (2n+1)\frac{\lambda}{2}} \quad \text{if } \cos r = 1$$

(2) Condition for destructive interference

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t = n\lambda} \quad \text{if } \cos r = 1$$

(i) For transmitted rays of light

$$\boxed{\Delta = 2\mu t \cos r}$$

(1) Condition for constructive interference

$$2\mu t \cos r = n\lambda$$

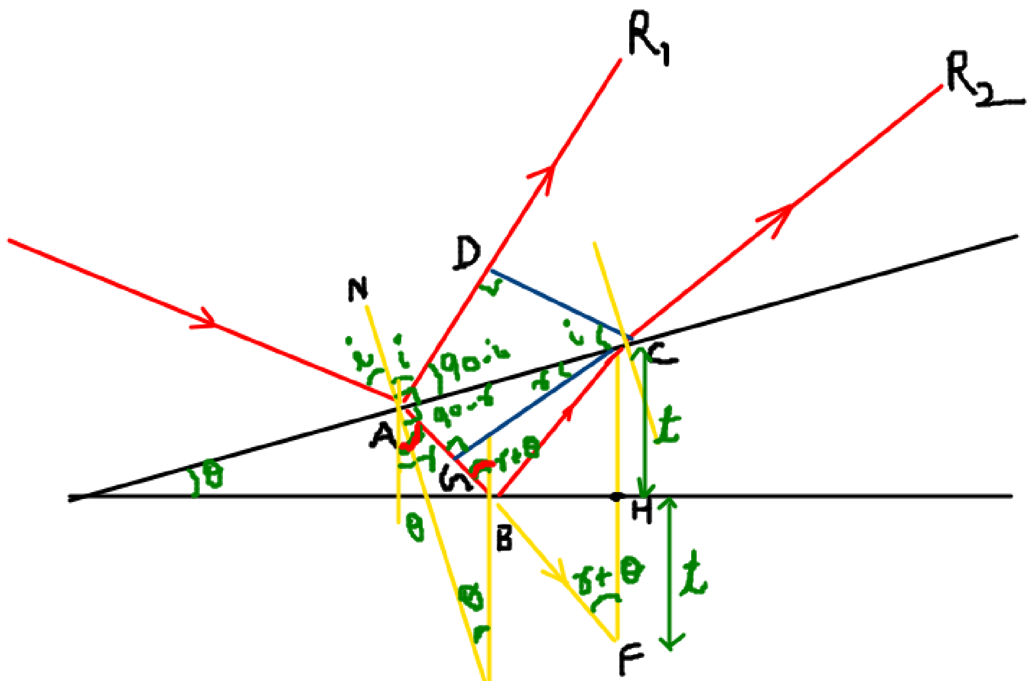
$$\text{or} \quad \boxed{2\mu t = n\lambda} \quad \text{if } \cos r = 1$$

(2) Condition for destructive interference

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

$$\text{or} \quad \boxed{2\mu t = (2n \pm 1)\frac{\lambda}{2}} \quad \text{if } \cos r = 1$$

Interference due to non-uniform thin film or wedge shaped films



Path Difference = (travelling wave in medium - wave travelling in air)

$$\Delta = (AB+BC)\mu - AD(\mu_{air})$$

$$\Delta = (AB+BC)\mu - AD \quad \text{——— (1)} \qquad [\because \mu_{air}=1]$$

$\because$ $BC = BF$

$CH = HF = t$

$$\Delta = (AB+BC)\mu - AD$$

$$\Delta = (AB+BF)\mu - AD \qquad [\because AB+BF = AF]$$

$$\Delta = (AF)\mu - AD \quad \text{——— (2)}$$

In $\Delta ACD \rightarrow \quad \sin i = \dfrac{AD}{AC}$ ——— (3), In $\Delta ACG \rightarrow \quad \sin r = \dfrac{AG}{AC}$ ——— (4)

$(3) \div (4) \Rightarrow$

$$\frac{\sin i}{\sin r} = \frac{AD}{\cancel{AC}} \times \frac{\cancel{AC}}{AG} = \frac{AD}{AG}$$

$\because$ Snell's law $\dfrac{\sin i}{\sin r} = \mu$

$$\therefore \frac{AD}{AG} = \mu \Rightarrow AD = \mu AG$$

From eqn (2) $\Delta = \mu AF - AD$

$$\Delta = \mu AF - \mu AG$$

$$\Delta = \mu (AF - AG)$$

$$\Delta = \mu (GF) \quad \text{———} (5)$$

In $\Delta GFC \rightarrow$

$$\cos(r+\theta) = \frac{GF}{CF} \qquad \because CF = 2t$$

$$GF = 2t \cos(r+\theta)$$

From eqn (5)

$$\Delta = \mu [2t \cos(r+\theta)]$$

$$\Delta = 2\mu t \cos(r+\theta)$$

By stoke's law $\boxed{\Delta = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}}$ ——— (X)

General eqn for path diff.

(i) Condition for bright fringes

$\because \Delta = n\lambda$ (for cons. Interference)

$$n\lambda = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}$$

$$n\lambda + \frac{\lambda}{2} = 2\mu t \cos(r+\theta)$$

$$\boxed{(2n+1)\frac{\lambda}{2} = 2\mu t \cos(r+\theta)}$$

(i) Condition for dark fringes

$$\because \Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$(2n-1)\frac{\lambda}{2} = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}$$

$$(2n-1)\frac{\lambda}{2} + \frac{\lambda}{2} = 2\mu t \cos(r+\theta)$$

$$\frac{\cancel{2}n\lambda}{\cancel{2}} = 2\mu t \cos(r+\theta)$$

$$\boxed{n\lambda = 2\mu t \cos(r+\theta)}$$

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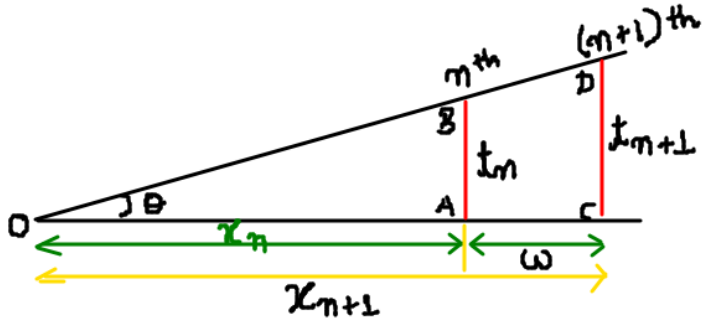
Fringe width for thin wedge shaped film



$$\because \omega = x_{n+1} - x_n \quad \text{---------(1)}$$

$$\text{In } \Delta OAB \rightarrow \tan\theta = \frac{AB}{OA} = \frac{t_n}{x_n}, \quad t_n = x_n \tan\theta \quad \text{---------(2)}$$

$$\text{In } \Delta OCD \rightarrow \tan\theta = \frac{DC}{OC} = \frac{t_{n+1}}{x_{n+1}}, \quad t_{n+1} = x_{n+1}\tan\theta \quad \text{---------(3)}$$

$$(3)-(2) \Rightarrow \quad t_{n+1} - t_n = (x_{n+1} - x_n)\tan\theta$$

$$t_{n+1} - t_n = \omega \tan\theta \quad \text{---------(4)}$$

For dark fringes

$$n\lambda = 2\mu t \cos(r+\theta) \quad , \text{if } i=0, \text{then } r=0$$

$$n\lambda = 2\mu t \cos\theta \quad \text{or} \quad n\lambda = 2\mu t_n \cos\theta \quad \text{---------(5)}$$

$$(n+1)\lambda = 2\mu t_{n+1} \cos\theta \quad \text{---------(6)}$$

$$(6)-(5) \Rightarrow \quad 2\mu(t_{n+1} - t_n)\cos\theta = (n+1)\lambda - n\lambda$$

$$(t_{n+1} - t_n) = \frac{\lambda}{2\mu\cos\theta} \quad \text{---------(7)}$$

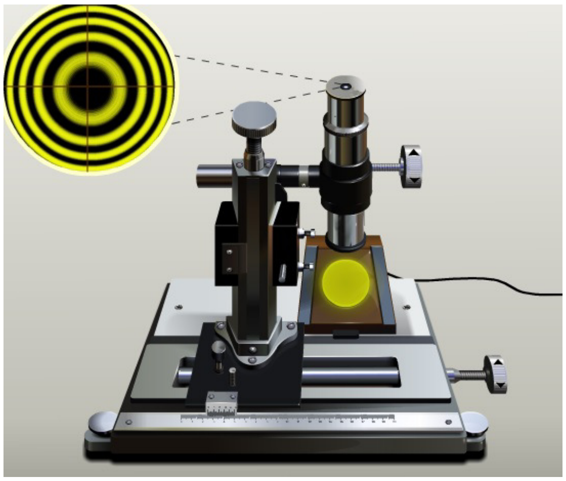
Put this value in(4)

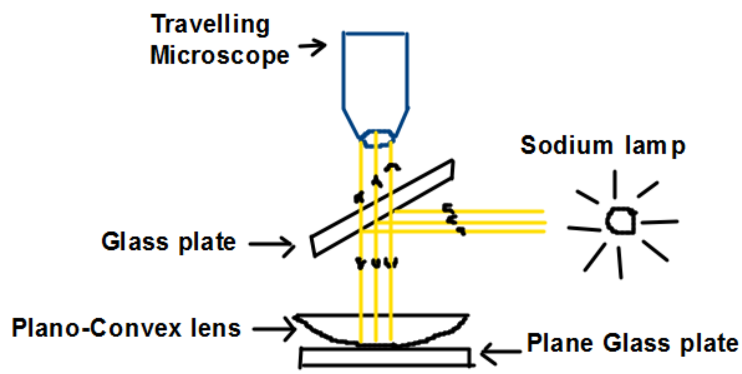
$$\frac{\lambda}{2\mu\cos\theta} = \omega\tan\theta$$

$$\frac{\lambda}{2\mu\cancel{\cos\theta}} = \omega\frac{\sin\theta}{\cancel{\cos\theta}} \Rightarrow \quad \omega = \frac{\lambda}{2\mu\sin\theta}, \quad \text{if } \theta \simeq 0 \text{ (very small)}$$

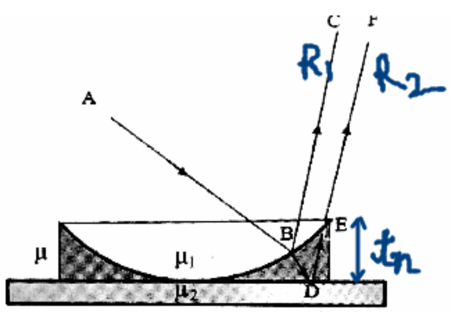
$$\boxed{\omega = \frac{\lambda}{2\mu\theta}}, \quad \left\{ \begin{array}{l} \omega \propto \lambda \\ \propto \frac{1}{\theta} \end{array} \right\}$$

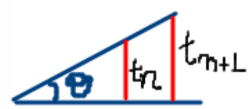
Newton's Rings





Newton's Rings due to reflected light





- Thickness of lens (Plano-Convex lens) will be the thickness of thin wedge shaped film.
- We know that interference pattern always depends upon the path difference between two interfering light rays.
- We also know that for a wedge shaped thin film and in the case of reflected light the value of path difference (Δ) is $\rightarrow$

$$\boxed{\Delta = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}} \qquad (1)$$

if θ is very less, $\theta \simeq 0$

then $\cos(r+\theta) = \cos(r)$

for the value of $r = 0^\circ$, $\cos(0^\circ) = 1$

$$\boxed{\Delta = 2\mu t_n - \frac{\lambda}{2}} \quad \text{---} (2)$$

- **For Dark fringes (the case of destructive interference):**

$$\left[\Delta = (2n-1)\frac{\lambda}{2}\right] \quad \text{---} (3)$$

From 2 & 3 →

$$2\mu t_n - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t_n = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t_n = \frac{\cancel{2}n\lambda}{\cancel{2}} - \cancel{\frac{\lambda}{2}} + \cancel{\frac{\lambda}{2}}$$

$$\boxed{2\mu t_n = n\lambda} \quad \text{---} (4)$$

$$\boxed{2t_n = n\lambda} \quad (\text{For air } \mu = 1) \quad (5)$$

- **For Bright fringes (the case of constructive interference):**

$$[\Delta = n\lambda] \quad \text{---} (6)$$

From 2 & 3 →

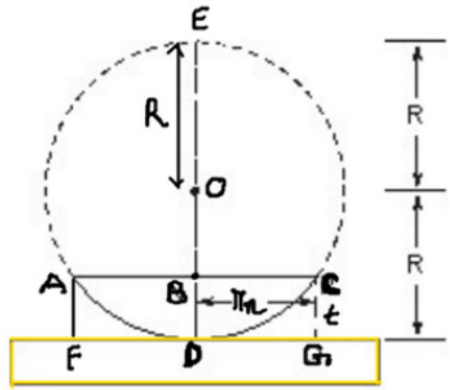
$$2\mu t_n - \frac{\lambda}{2} = n\lambda$$

$$2\mu t_n = n\lambda + \frac{\lambda}{2}$$

$$\boxed{2\mu t_n = (2n+1)\frac{\lambda}{2}} \quad \text{---} (7)$$

$$\boxed{2t_n = (2n+1)\frac{\lambda}{2}} \quad \text{---} (8) \quad (\because \text{for air } \mu = 1)$$

Diameter of Newton's Ring



r_n = radius of n^{th} ring

$DG = FD = AB = BC = r_n$

$BD = t$

$ED = 2R$

$BE = 2R - t$

We know, in a circle

$$\underbrace{AB}_{FD} \times \underbrace{BC}_{DG} = BE \times BD$$

$$FD \times DG = BE \times BD$$

$$\Rightarrow r_n \times r_n = (2R - t) \times t$$

$$\Rightarrow r_n^2 = 2Rt - t^2$$

$\because$ t is very small in comparison to R

$\therefore$ $t^2 \simeq 0$

$$\Rightarrow r_n^2 = 2Rt$$

or $\boxed{2t = \dfrac{r_n^2}{R}}$ —— (1)

r_n = radius of n^{th} ring

D_n = diameter of n^{th} ring

$\because$ $D_n = 2r_n$

$r_n = \dfrac{D_n}{2} \Rightarrow \boxed{r_n^2 = \dfrac{D_n^2}{4}}$ —— (2)

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∵ **For bright Newton rings:** $\boxed{2t_n = (2n-1)\frac{\lambda}{2}}$

From (1) → $\left(\because 2t_n = \frac{r_n^2}{R}\right)$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

From (2) → $\left(\because r_n^2 = \frac{D_n^2}{4}\right)$

$$\frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$$

$$D_n^2 = (2n-1)\frac{\lambda}{2}.4R$$

$$D_n^2 = 2(2n-1)\lambda R$$

$$D_n^2 \propto (2n-1) \quad [\because 2, \lambda, R \text{ are constant}]$$

$$\boxed{D_n \propto \sqrt{2n-1}} \text{ for bright rings}$$

for $n=1$, $D_1 \propto \sqrt{1}$
$n=2$, $D_2 \propto \sqrt{3}$
$n=3$, $D_3 \propto \sqrt{5}$
... so on

Therefore, we can say that the diameters of bright rings are directly proportional to the square root of odd natural numbers.

∵ **For dark Newton's Rings:** $\boxed{2t = n\lambda}$

From (1) $\left(\because 2t = \frac{r_n^2}{R}\right)$

$$\frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = nR\lambda$$

From (2) $\left(\because r_n^2 = \frac{D_n^2}{4}\right)$

$$\frac{D_n^2}{4} = nR\lambda$$

$$D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda}$$

∵ 4, R, λ are constant →

$$D_n \propto \sqrt{n}$$

for $n=1$, $D_1 \propto \sqrt{1}$
$n=2$, $D_2 \propto \sqrt{2}$
$n=3$, $D_3 \propto \sqrt{3}$
$n=4$, $D_4 \propto \sqrt{4}$
... so on

Therefore, we can say that the diameters of dark rings are directly proportional to the square root of all natural numbers.

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Applications of Newton's Rings

1. To determine the wavelength of monochromatic light.

$\because$ We know $\rightarrow$ $\boxed{D_n^2 = 4nR\lambda}$ ——(1)

& $\boxed{D_{n+p}^2 = 4(n+p)R\lambda}$ ——(2)

Here p is the difference in the number of two rings.

From (1) & (2)

$$\Rightarrow D_{n+p}^2 - D_n^2 = 4(n+p)R\lambda - 4nR\lambda$$

$$\Rightarrow D_{n+p}^2 - D_n^2 = \cancel{4nR\lambda} + 4pR\lambda - \cancel{4nR\lambda}$$

*For a given R, the value of λ $\rightarrow$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

*For a given λ, the value of R $\rightarrow$

$$\boxed{R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}}$$

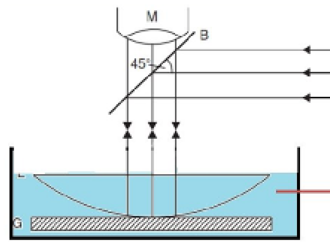
2. To determine the refractive index of transparent medium.

$\because$ we know that for bright rings: $2t = (2n-1)\frac{\lambda}{2}$

$\because 2t = \frac{r_n^2}{R}$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = (2n-1)\frac{\lambda}{2}$$



If a medium is introduced $\rightarrow$

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

$$\mu\left(\frac{r_n^2}{R}\right) = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = \frac{(2n-1)\lambda R}{2\mu}$$

$\because r_n^2 = \frac{D_n^2}{4}$

From 1 & 2:

$$D_n^2 = 2(2n-1)\lambda R/\mu \quad \text{———} (1)$$

$$D_{n+p}^2 = 2[2(n+p)-1]\lambda R/\mu \quad \text{———} (2)$$

$$D_{n+p}^2 - D_n^2 = \frac{2[2(n+p)-1]\lambda R}{\mu} - \frac{2(2n-1)\lambda R}{\mu}$$

$$= \frac{4\cancel{n\lambda R} + 4p\lambda R - \cancel{2\lambda R} - \cancel{4n\lambda R} + \cancel{2\lambda R}}{\mu}$$

$$\boxed{\left(D_{n+p}^2 - D_n^2\right)_{\text{medium}} = \frac{4p\lambda R}{\mu}} \quad \text{———} (3)$$

In case of air $\mu = 1$

$$\boxed{\left(D_{n+p}^2 - D_n^2\right)_{\text{air}} = 4p\lambda R} \quad \text{———} (4)$$

$$(4)\div(3) \Rightarrow \boxed{\frac{\left(D_{n+p}^2 - D_n^2\right)_{\text{medium}}}{\left(D_{n+p}^2 - D_n^2\right)_{\text{air}}} = \mu}$$

Therefore, by this we can determine the refractive index of a transparent medium.

Q. In Newton's ring experiment the diameter of 15th ring was found to be 0.59cm and that of the 5th ring 0.336cm. If the radius of the plano-convex lens is 100 cm. Calculate the wavelength of the light used.

?

Sol:

$D_{15} = 0.59\ cm = 0.59 \times 10^{-2}\ m.$

$D_5 = 0.336\ cm = 0.336 \times 10^{-2}\ m.$

$R = 100\ cm = 1\ m$

$\lambda = ?$

$p = 15 - 5 = 10$

$$\because \boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

$$\lambda = \frac{(0.59 \times 10^{-2})^2 - (0.336 \times 10^{-2})}{4 \times 10 \times 1}$$

$$\boxed{\lambda = 5880\ \text{Å}}$$ Ans:-

Q. Newton's rings are observed in reflected light of wavelength 590 nm. The diameter of the 10th dark ring is 0.5 cm. Find (i) the radius of curvature of the lens and (ii) the thickness of the air film.

Sol:

$\lambda = 590\ nm = 590 \times 10^{-9}\ m$

$n = 10$

$D_{10} = 0.5\ cm = 0.5 \times 10^{-2}\ m$

$R = ?\ ,\ t = ?$

$\because D_n^2 = 4nR\lambda$

(i) $R = \frac{D_n^2}{4n\lambda} = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 590 \times 10^{-9}}$

$R = 1.059\ m$ (Ans

(ii) $\because$ For dark ring $\rightarrow 2t = n\lambda,\ t = \frac{n\lambda}{2}$

$t = \frac{10 \times 590 \times 10^{-9}}{2}$

$t = 2.95 \times 10^{-6}\ m$ (Ans:-