Basic Electrical Engineering (TEE 101)

Lecture 22: Analysis of single phase AC Circuits

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Content

This lecture covers:

- Analysis of pure resistive AC circuit
- Analysis of pure inductive AC circuit
- Analysis of pure capacitive AC circuit

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Analysis of pure resistive AC circuit

When an alternating voltage is applied across pure resistance, then free electrons flow (*i.e.* current) in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

Consider a circuit containing a pure resistance of $R\ \Omega$ connected across an alternating voltage source [See Fig. 1]. Let the alternating voltage be given by the equation :

$$v = V_m \sin \omega t \qquad ...(i)$$

As a result of this voltage, an alternating current i will flow in the circuit. The applied voltage has to overcome the drop in the resistance only *i.e.*

$$v = i R$$

$$i = v/R$$

Substituting the value of v, we get,

$$i = \frac{V_m}{R} \sin \omega t \qquad ...(ii)$$

The value of i will be maximum (*i.e.* Im) when $\sin \omega t = 1$.

$$I_m = V_m/R$$

Eq. (*ii*) becomes : $i = I_m \sin \omega t$ ...(*iii*)

In terms of r.m.s. values,

$$V_m = I_m R$$

Divide with $\sqrt{2}$

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} R$$

$$V = IR \qquad ...(iv)$$

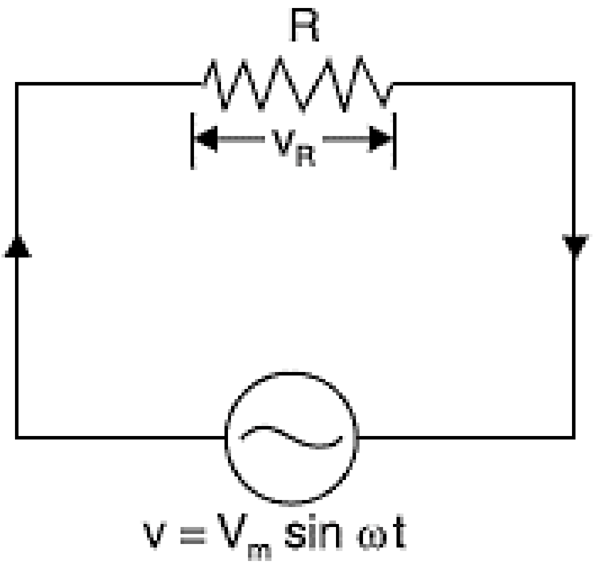

Fig. 1

Where, V and I are the RMS values of Alternating voltage and current

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Phase angle.

(i) It is clear from eqs. (*i*) and (*iii*) that the applied voltage and the circuit current are in phase with each other *i.e*. they pass through their zero values at the same instant and attain their positive and negative peaks at the same instant.

(ii) This is also indicated by the phasor diagram shown in Fig. 2 .

(iii) Note that r.m.s. values have been used in drawing the phasor diagram.

(iv) The wave diagram shown in Fig. 3 also depicts that current is in phase with the applied voltage.

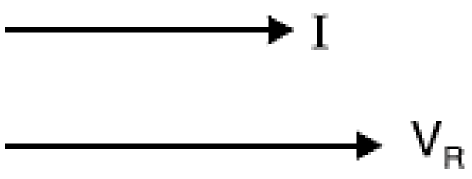


Fig. 2: phasor diagram

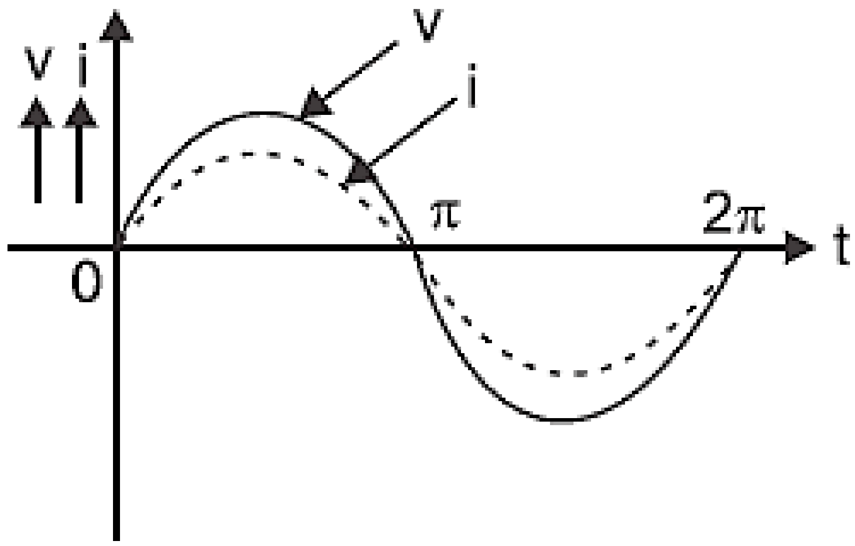


Fig. 3: wave diagram

Analysis of pure Inductive AC circuit

When an alternating current flows through a pure inductive coil, a back e.m.f. (= $L\,di/dt$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. Since there is no ohmic drop, the applied voltage has to overcome the back e.m.f. only.

∴ **Applied alternating voltage = Back e.m.f.**

Consider an alternating voltage applied to a pure inductance of L henry as shown in Fig.4.

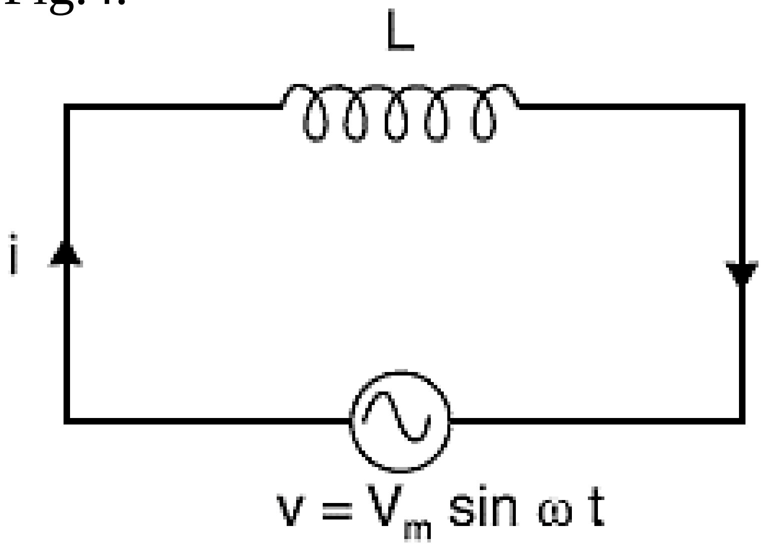


Fig.4

Let the equation of the applied alternating voltage be :

$$v = Vm \sin \omega t \qquad ...(i)$$

Clearly, $$V_m \sin(\omega t) = L\frac{di}{dt}$$

or $$di = \frac{V_m}{L}\sin(\omega t)\,dt$$

Integrating both sides, we get,

$$\int di = \frac{V_m}{L}\int \sin(\omega t)\,dt$$

or $$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

or $$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(ii)$$

The value of i will be maximum (*i.e.* Im) when $\sin(\omega t - \pi/2)$ is unity

$$I_m = \frac{V_m}{\omega L}$$

Substituting the value of $Vm/\omega L = Im$ in eq. (*ii*), we get,

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(iii)$$

By: Dr. Parvesh Saini

Phase angle.

(i) It is clear from eqs. (*i*) and (*iii*) that current lags behind the voltage by $\pi/2$radians or 90^o.

(ii) Hence in a pure inductance, current lags the voltage by 90^o.

(iii) This is also indicated by the phasor diagram shown in Fig. 5. Note that r.m.s. values have been used in drawing the phasor diagram.

(iv) The wave diagram shown in Fig. 6 also depicts that current lags the voltage by 90^o.

(v) There is also physical explanation for the lagging of current behind voltage in an inductive coil.

(vi) Inductance opposes the change in current and serves to delay the increase or decrease of current in the circuit.

(vii) This causes the current to lag behind the applied voltage.

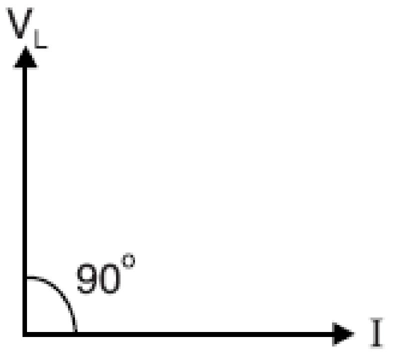


Fig. 5: PHASOR diagram

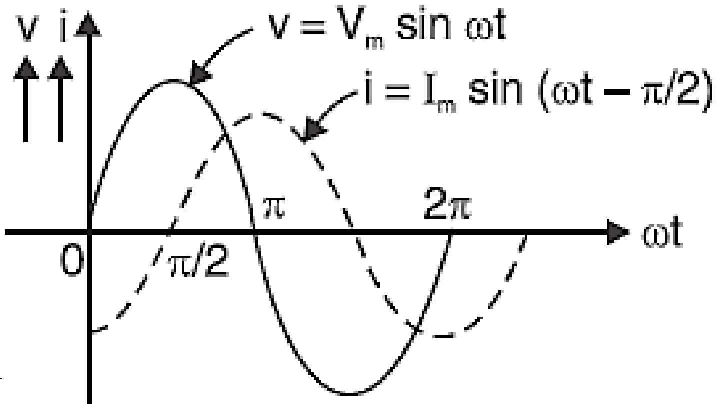


Fig. 6: wave diagram

Inductive reactance. Inductance not only causes the current to lag behind the voltage but it also limits the magnitude of current in the circuit. We have seen above that :

$$I_m = V_m/\omega L$$

$$\frac{V_m}{I_m} = \omega L$$

Clearly, the opposition offered by inductance to current flow is ωL. This quantity ωL is called the *inductive reactance* X_L of the coil. It has the same *dimensions as resistance and is, therefore, measured in Ω.

inductive reactance $X_L = \omega L = 2\pi f L$

Note that X_L will be in Ω if L is in henry and f in Hz.

Analysis of pure capacitive AC circuit

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses.

The result is that electrons move to and fro around the circuit, connecting the plates, thus constituting alternating current.

Consider an alternating voltage applied to a capacitor of capacitance *C* farad as shown in Fig. 7.

Let the equation of the applied alternating voltage be :

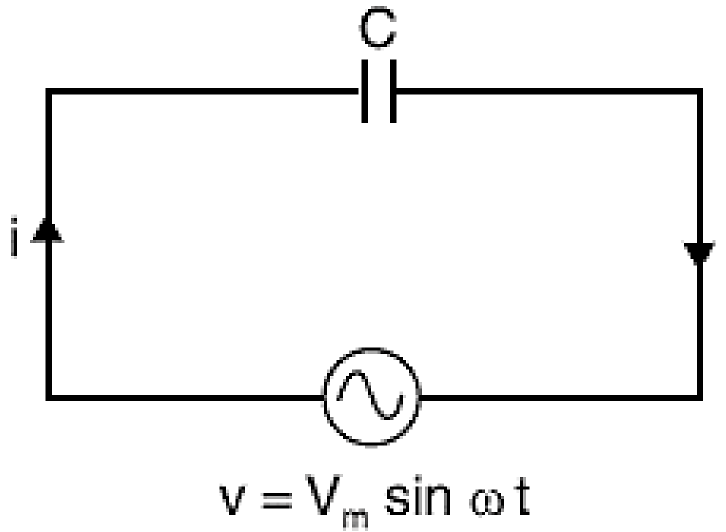


Fig. 7

$$v = V_m \sin \omega t \qquad \ldots(i)$$

As a result of this alternating voltage, alternating current will flow through the circuit. Let at any instant *i* be the current and *q* be the charge on the plates.

Charge on capacitor, $q = C\,v = C\,V_m \sin \omega t$

$\therefore$ Circuit current, $i = \frac{d}{dt}(q) = \frac{d}{dt}(C V_m \sin \omega t) = \omega C V_m \cos \omega t$

$$\therefore \quad i = \omega C V_m \sin(\omega t + \pi/2) \quad \ldots(ii)$$

The value of i will be maximum ($i..e.$ I_m) when $\sin(\omega t + \pi/2)$ is unity.

$$\therefore \quad I_m = \omega C V_m$$

Substituting the value $\omega C V_m = I_m$ in eq. (ii), we get,

$$i = I_m \sin(\omega t + \pi/2) \quad \ldots(iii)$$

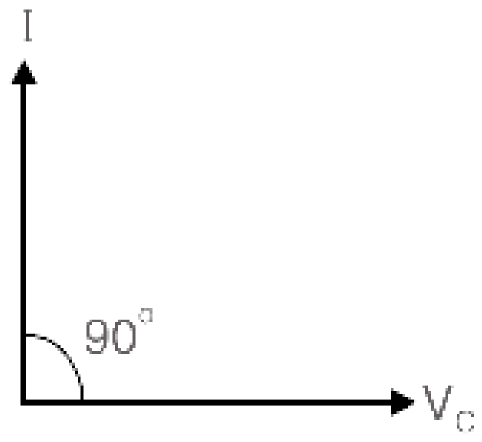


Fig. 8: Phasor Diagram

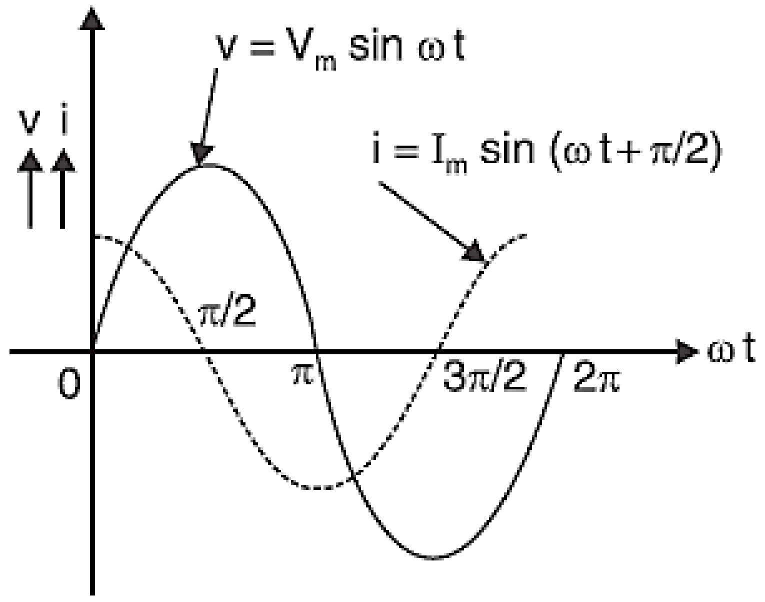


Fig. 9: Wave Diagram

Phase angle.

(i) It is clear from eqs. (*i*) and (*iii*) that current leads the voltage by π/2 radians or 90º. *Hence in a pure capacitance, current leads the voltage by 90º.*

(ii) This is also indicated in the phasor diagram shown in Fig.8.

(iii) The wave diagram shown in Fig. 9 also reveals the same fact.

(iv) There is also physical explanation for the lagging of voltage behind the current in a capacitor.

(v) Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor. This causes the voltage to lag behind the current.

(*ii*) **Capacitive reactance.** Capacitance not only causes the voltage to lag behind current but it also limits the magnitude of current in the circuit. We have seen above that :

$$I_m = \omega C V_m$$

or

$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

If V_C and I are the r.m.s. values, then,

$$\frac{V_m}{I_m} = \frac{V_C}{I} = \frac{1}{\omega C} \qquad (V = V_C)$$

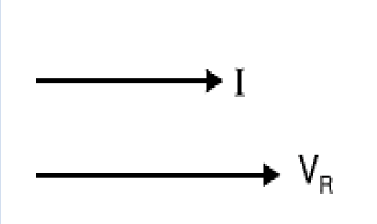
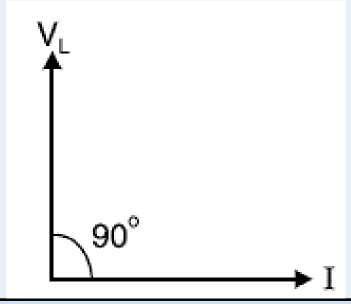
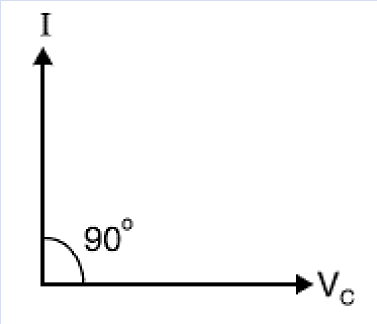
Clearly, the opposition offered by capacitance to current flow is $1/\omega C$. This quantity $1/\omega C$ is called the *capacitive reactance* X_C of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

$$\therefore \qquad I = V_C/X_C$$

where capacitive reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

Note that X_C will be in Ω if C is in farad and f in Hz.

Summary

Component	Voltage	Current	Phase Angle	Remark	Phasor Diagram
Resistor	$v=V_m\sin(\omega t)$	$i=I_m\sin(\omega t)$ Where, $I_m = V_m/R$	Zero	Voltage across a resistor and current through it are always in same phase	
Inductor	$v=V_m\sin(\omega t)$	$i=I_m\sin(\omega t - 90°)$ Where, $I_m = V_m/\omega L$ $X_L = \omega L$ Inductive Reactance (Ω)	90°	Voltage across an inductor always leads its current by 90°	
Capacitor	$v=V_m\sin(\omega t - 90°)$	$i=I_m\sin(\omega t)$ Where, $I_m = \omega C V_m$ $X_C = 1/\omega C$ Capacitive Reactance (Ω)	90°	Voltage across a capacitor always lags its current by 90°	

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Thank You