

# Stack

William Fiset

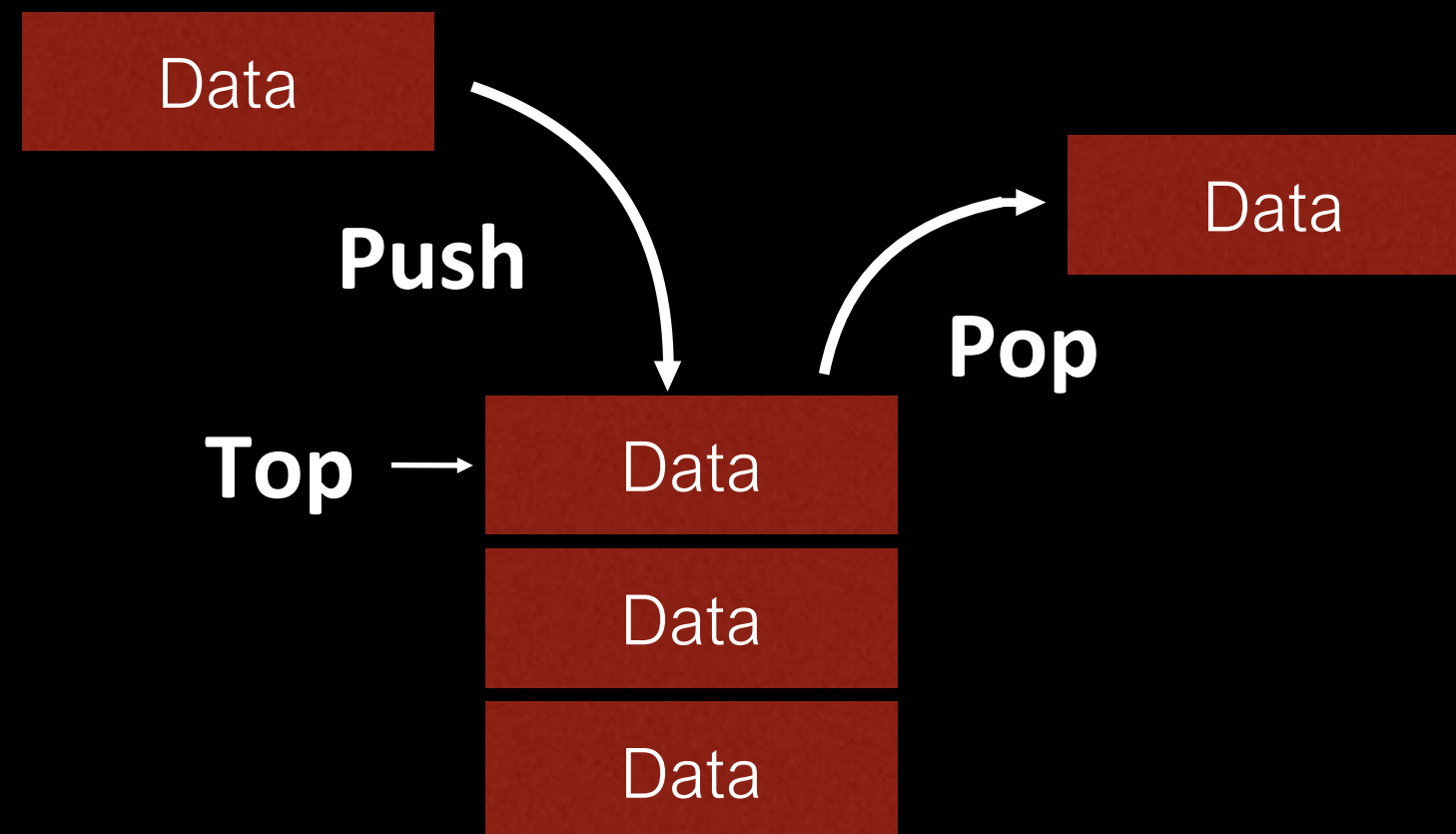
# Outline

- Discussion about Stacks
  - What is a Stack?
  - When and where is a Stack used?
  - Complexity Analysis
  - Stack usage examples
- Implementation details
  - Pushing elements on stack
  - Popping elements from stack
- Code Implementation

# Discussion

# What is a Stack?

A stack is a one-ended linear data structure which models a real world stack by having two primary operations, namely **push** and **pop**.



# What is a Stack?

## Instructions

```
pop()  
push('Onion')  
push('Celery')  
push('Watermelon')  
pop()  
pop()  
push('Lettuce')
```

Apple

Potato

Cabbage

Garlic

# What is a Stack?

## Instructions

→ pop()  
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pop()  
pop()  
push('Lettuce')

Apple

Potato

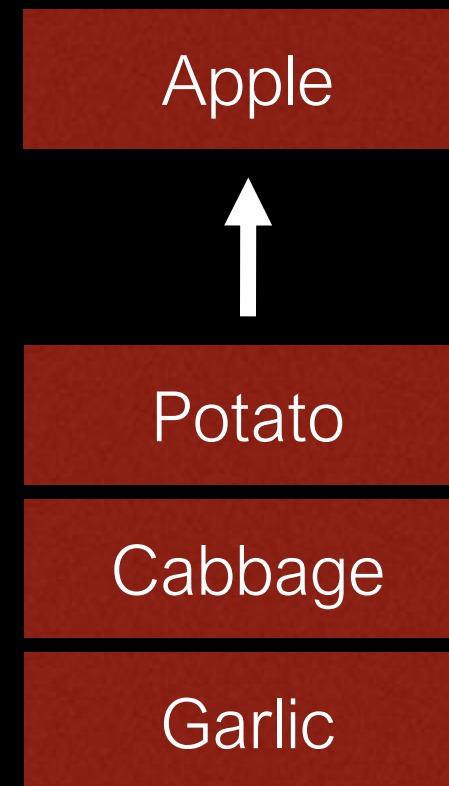
Cabbage

Garlic

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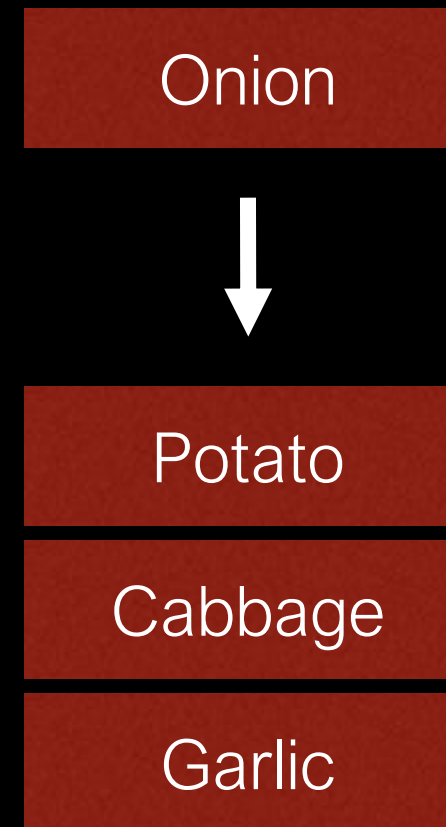
Garlic



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## Instructions

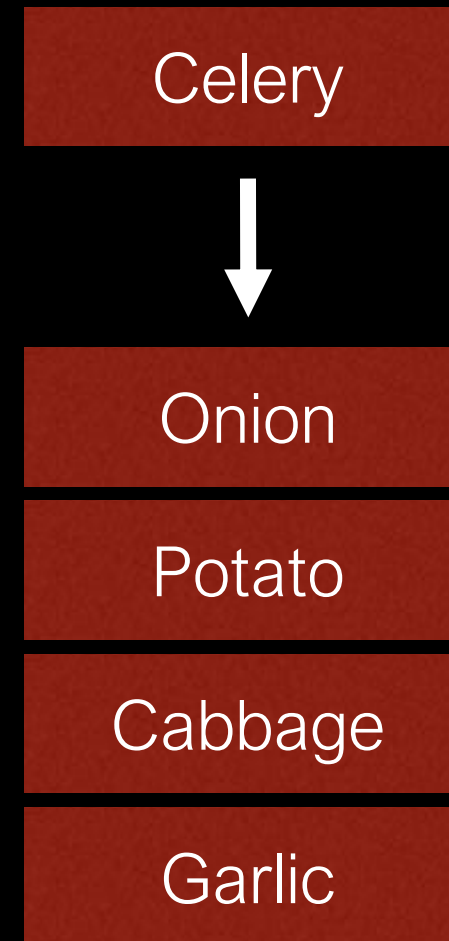
→ pop()  
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|         |
|---------|
| Onion   |
| Potato  |
| Cabbage |
| Garlic  |

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|         |
|---------|
| Celery  |
| Onion   |
| Potato  |
| Cabbage |
| Garlic  |

# What is a Stack?

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pop()  
push('Onion')  
push('Celery')  
→ push('Watermelon')  
pop()  
pop()  
push('Lettuce')

Watermelon



Celery

Onion

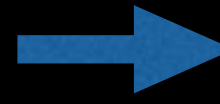
Potato

Cabbage

Garlic

# What is a Stack?

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Watermelon

Celery

Onion

Potato

Cabbage

Garlic

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Watermelon

Celery

Onion

Potato

Cabbage

Garlic

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Watermelon



Celery

Onion

Potato

Cabbage

Garlic



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```

Celery

Onion

Potato

Cabbage

Garlic

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```
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```

Celery

Onion

Potato

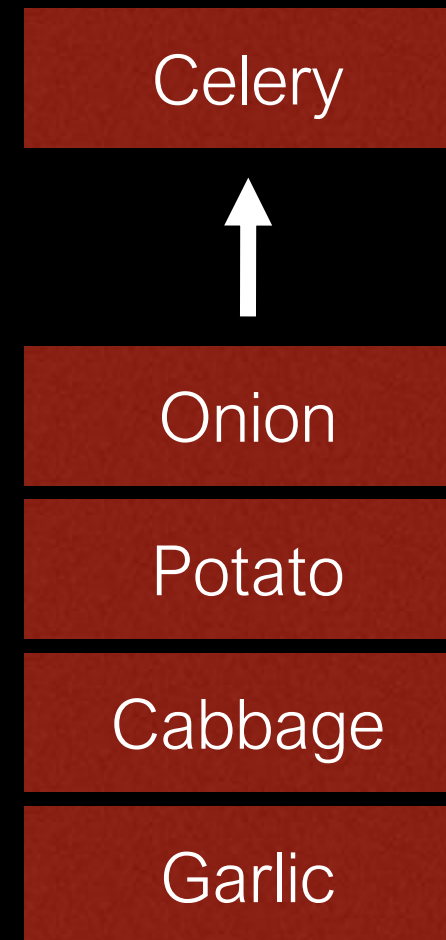
Cabbage

Garlic

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pop()  
→ pop()  
push('Lettuce')
```

Onion

Potato

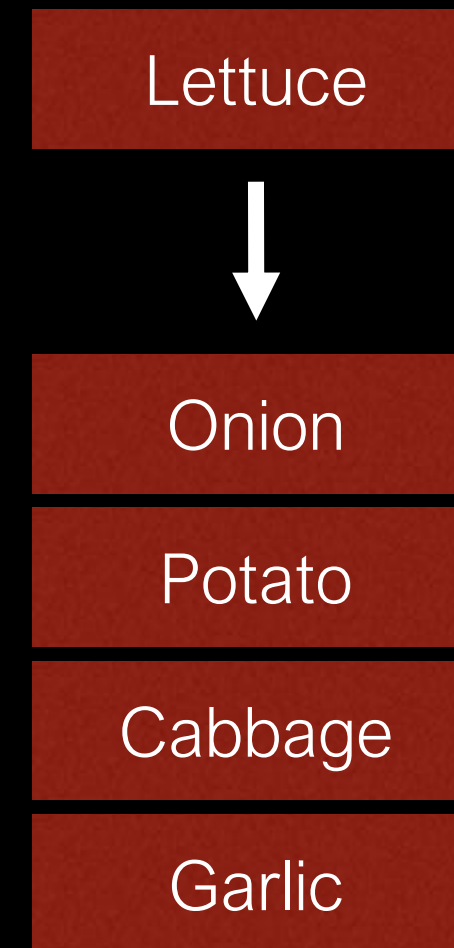
Cabbage

Garlic

# What is a Stack?

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pop()  
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pop()  
pop()  
→ push('Lettuce')
```

|         |
|---------|
| Lettuce |
| Onion   |
| Potato  |
| Cabbage |
| Garlic  |

# When and where is a Stack used?

- Used by undo mechanisms in text editors.
- Used in compiler syntax checking for matching brackets and braces.
- Can be used to model a pile of books or plates.
- Used behind the scenes to support recursion by keeping track of previous function calls.
- Can be used to do a Depth First Search (DFS) on a graph.

# Complexity Analysis



# Complexity

|                  |        |
|------------------|--------|
| <b>Pushing</b>   | $O(1)$ |
| <b>Popping</b>   | $O(1)$ |
| <b>Peeking</b>   | $O(1)$ |
| <b>Searching</b> | $O(n)$ |
| <b>Size</b>      | $O(1)$ |

# Example - Brackets

**Problem:** Given a string made up of the following brackets: `()[]{}` , determine whether the brackets properly match.

`[{}]`



**Valid**

`((()))`



**Valid**

`{}`



**Invalid**

`[()]]()`



**Invalid**

`[]{}({})`



**Valid**

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket:  $\emptyset$

Reversed Bracket:  $\emptyset$

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: [

Reversed Bracket: ]

A red rectangular box containing a white opening square bracket '['.

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: [

Reversed Bracket: ]

[

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: {

Reversed Bracket: }

}

[

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: }

Reversed Bracket: {

}

[

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: }

Reversed Bracket: {

[

[



# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: ]

Reversed Bracket: [

[

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: ]

Reversed Bracket: [



# Example - Brackets

Bracket Sequence:

[[{}]()]

Current Bracket: (

Reversed Bracket: )

(

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: )

Reversed Bracket: (

(

[

# Example - Brackets

Bracket Sequence:

`[[{}]()]`

Current Bracket: )

Reversed Bracket: (



[

# Example - Brackets

Bracket Sequence:

[[{}]()]

Current Bracket: ]

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A red rectangular box containing a white opening square bracket character '['.

# Example - Brackets

Bracket Sequence:

[[{}]()]

Current Bracket: ]

Reversed Bracket: [

# Example - Brackets

Bracket Sequence:

[[{}]() ] → Valid

Current Bracket: ]

Reversed Bracket: [



# Example - Brackets

Bracket Sequence:

[{ })[]

Current Bracket:  $\emptyset$

Reversed Bracket:  $\emptyset$

# Example - Brackets

Bracket Sequence:

[{}][ ]

Current Bracket: [

Reversed Bracket: ]

A red rectangular box containing a white opening square bracket '['.

# Example - Brackets

Bracket Sequence:

[{ })[]

Current Bracket: {

Reversed Bracket: }

}

[

# Example - Brackets

Bracket Sequence:

[{}][ ]

Current Bracket: }

Reversed Bracket: {

}

[

# Example - Brackets

Bracket Sequence:

[{}][

Current Bracket: }

Reversed Bracket: {

[

# Example - Brackets

Bracket Sequence:

[{}][

Current Bracket: )

Reversed Bracket: (

[

# Example - Brackets

Bracket Sequence:

[{}][



Invalid

Current Bracket: )

Reversed Bracket: (

[

# Example - Brackets

Let S be a stack

**For** bracket **in** bracket\_string:

    rev = getReversedBracket(bracket)

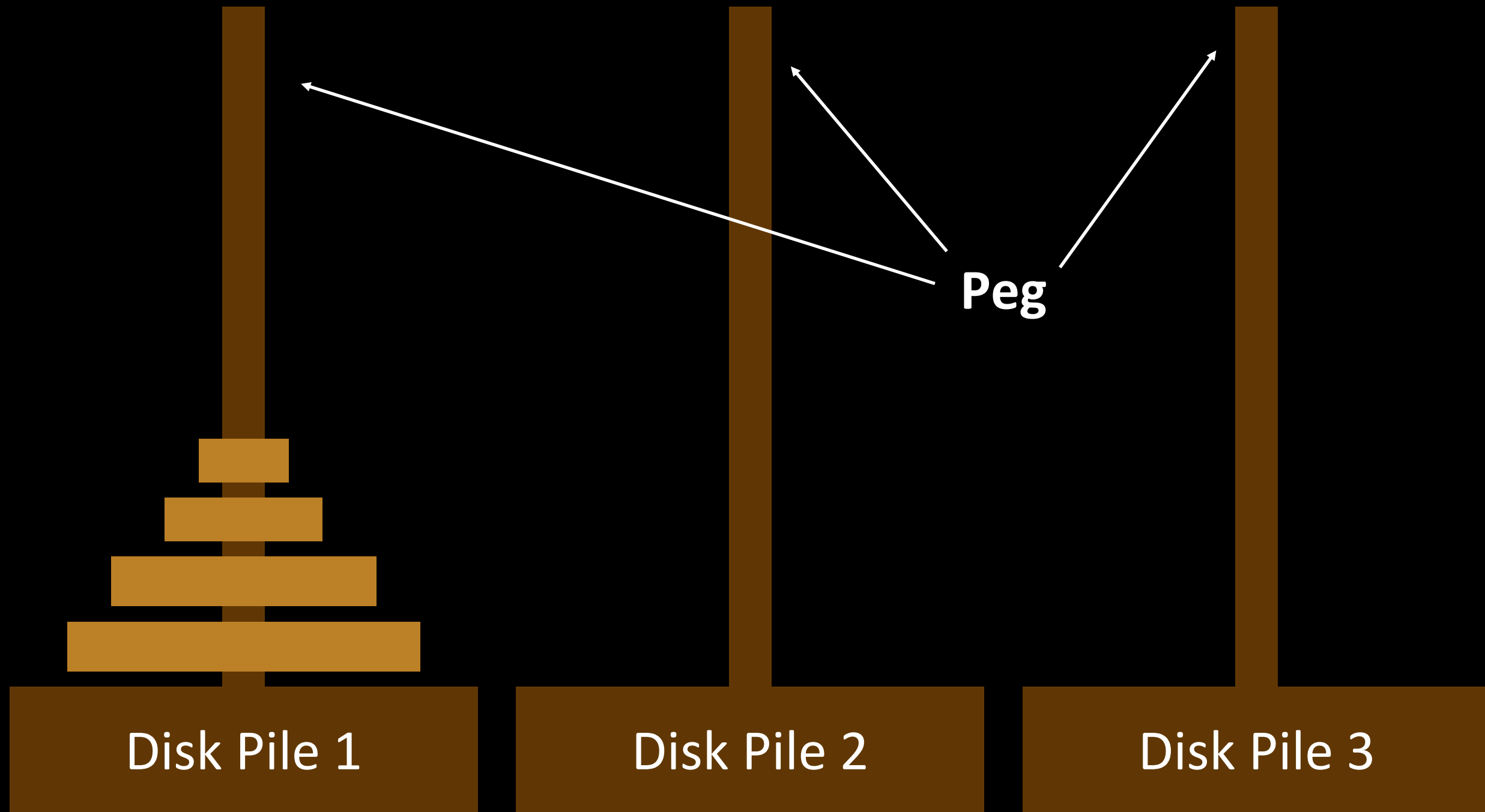
**If** isLeftBracket(bracket):  
        S.push(bracket)

**Else If** S.isEmpty() or S.pop() != rev:  
        **return** false *// Invalid*

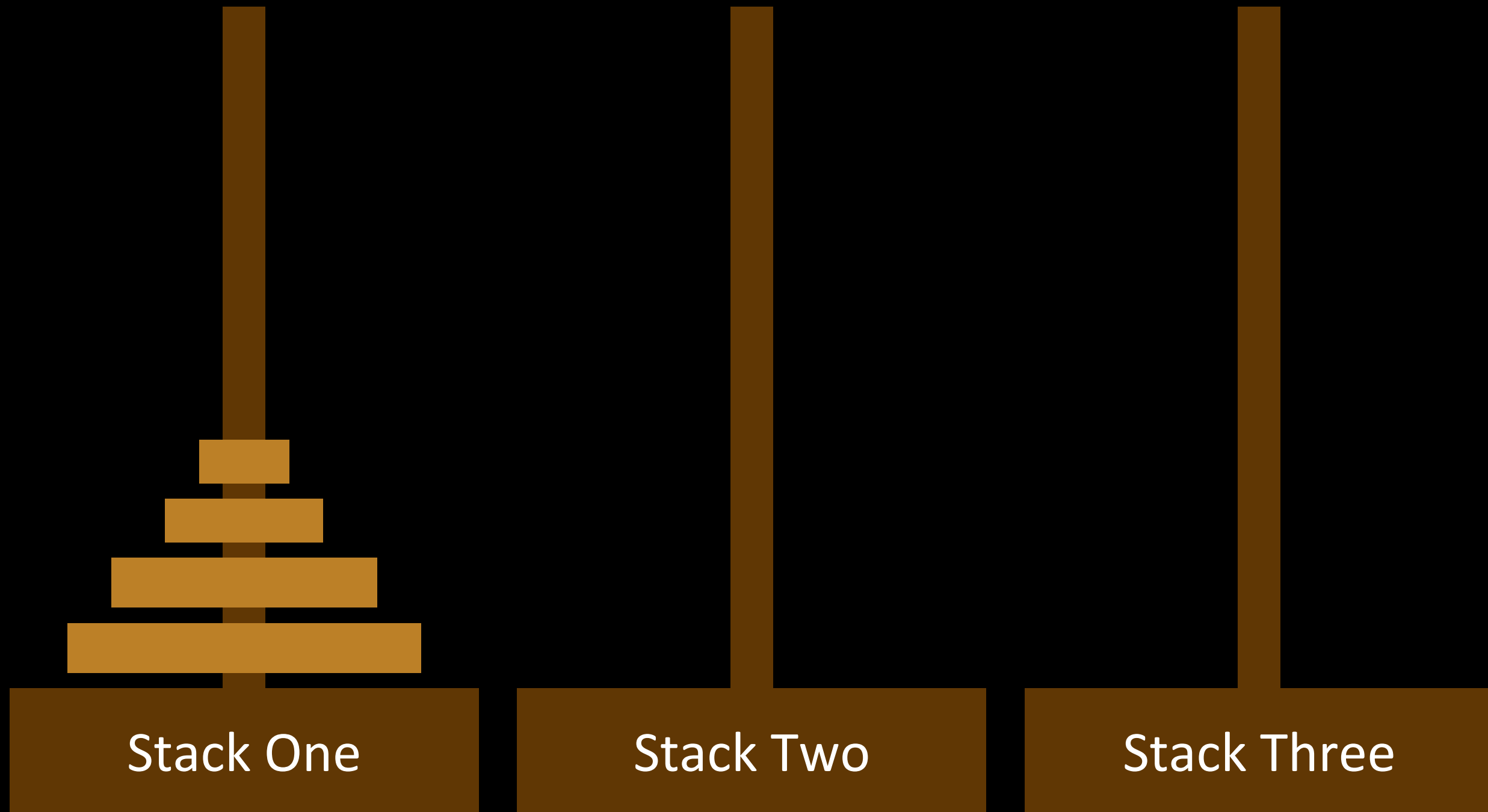
**return** S.isEmpty() *// Valid if S is empty*



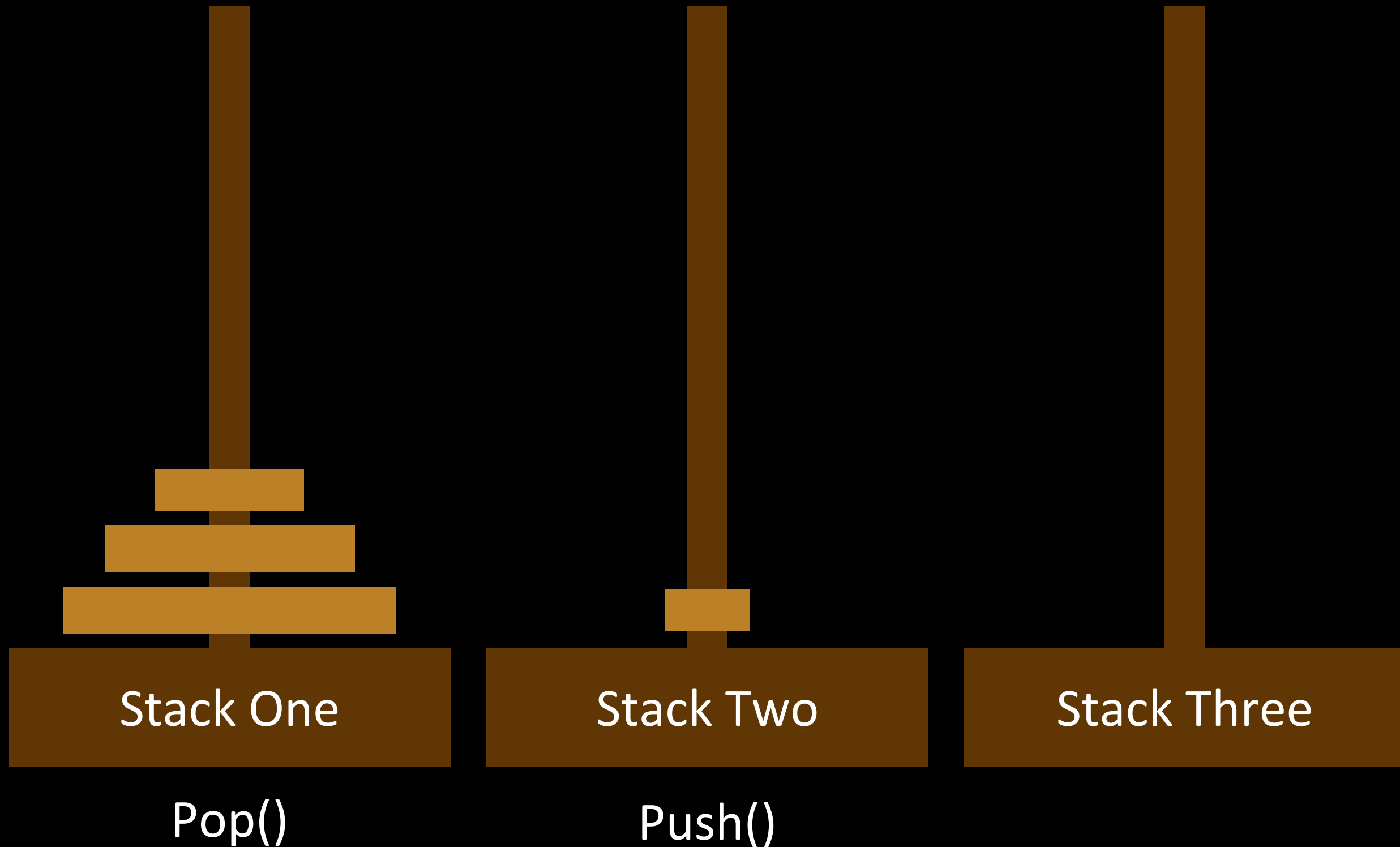
# Tower of Hanoi



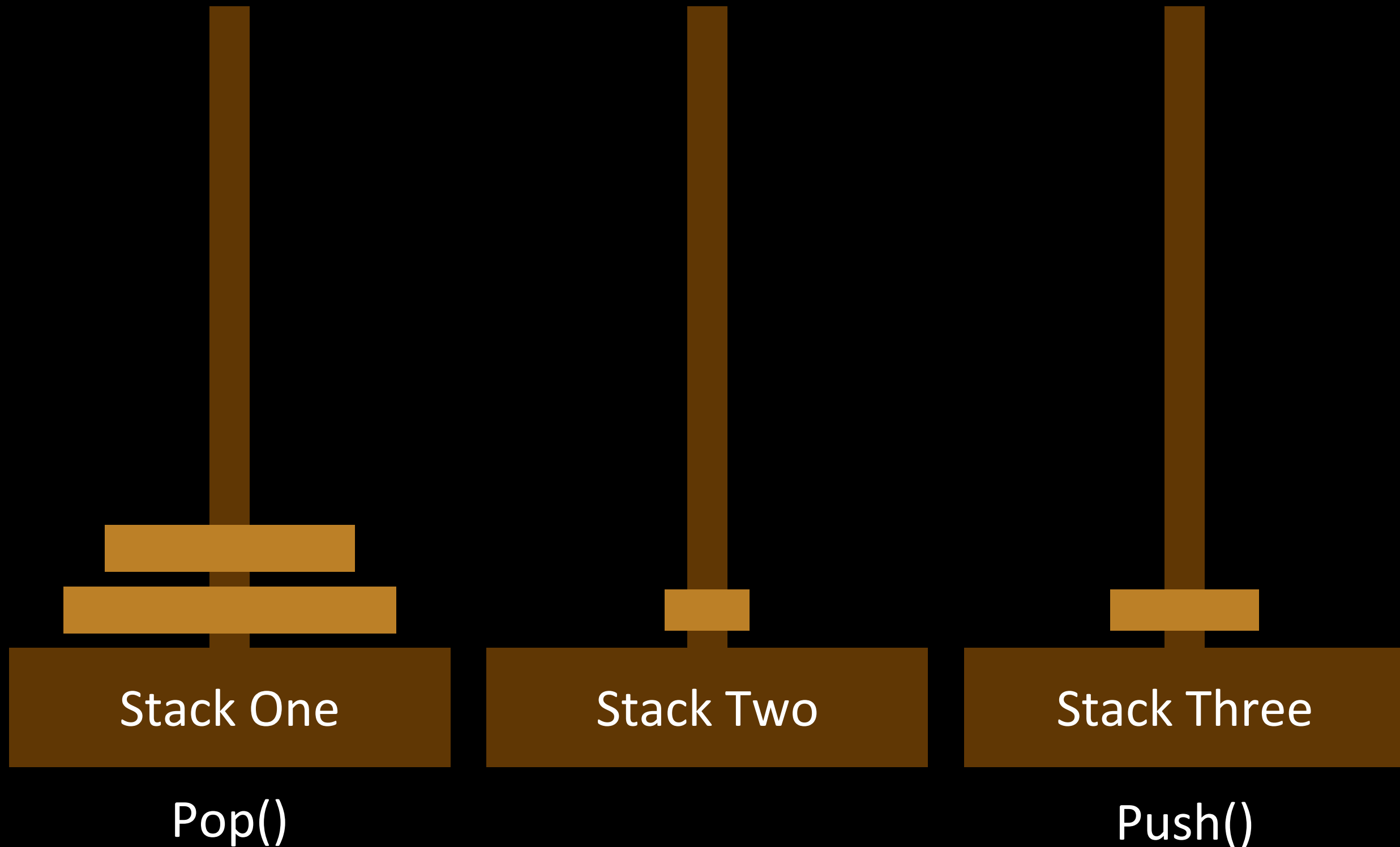
# Tower of Hanoi



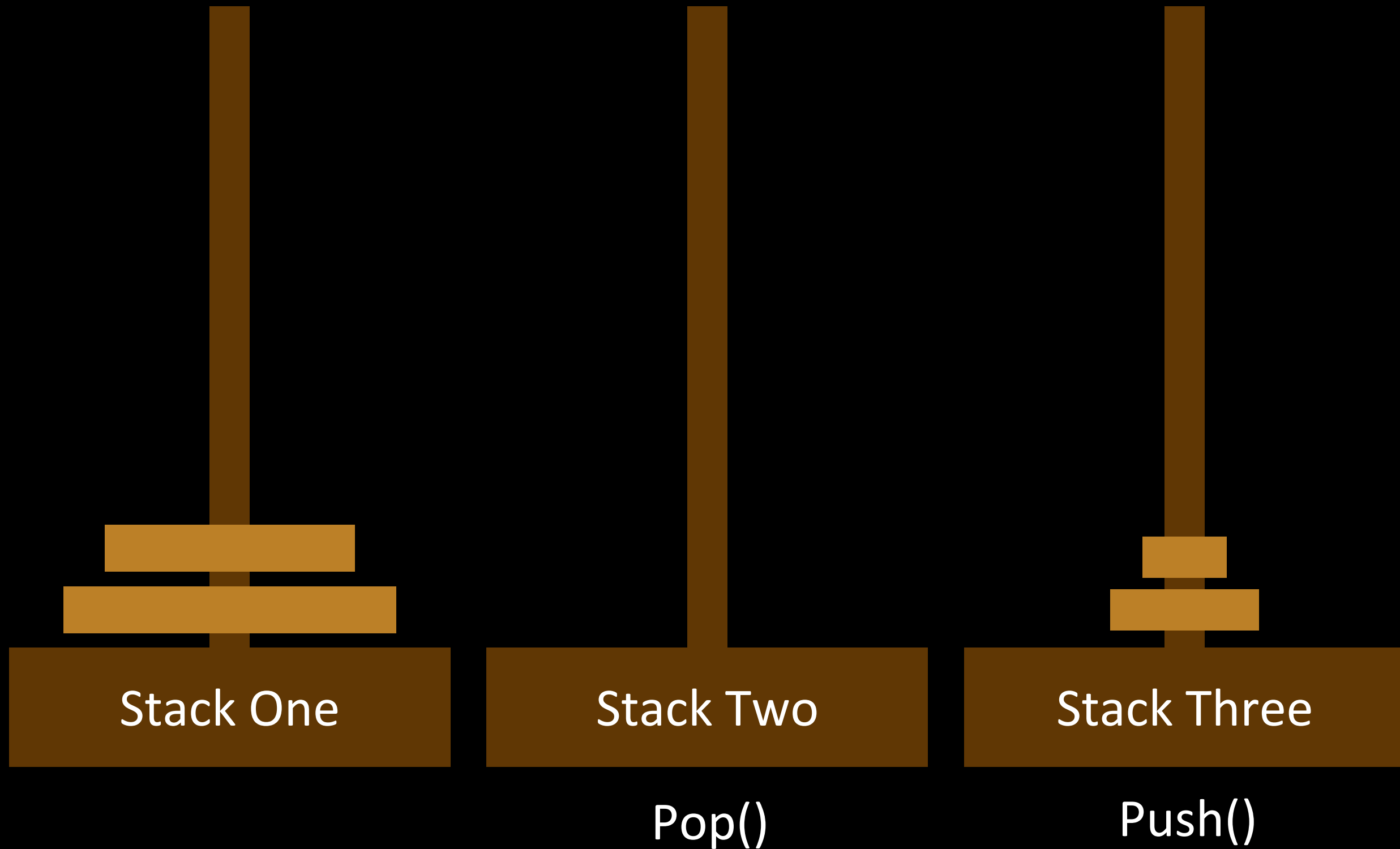
# Tower of Hanoi



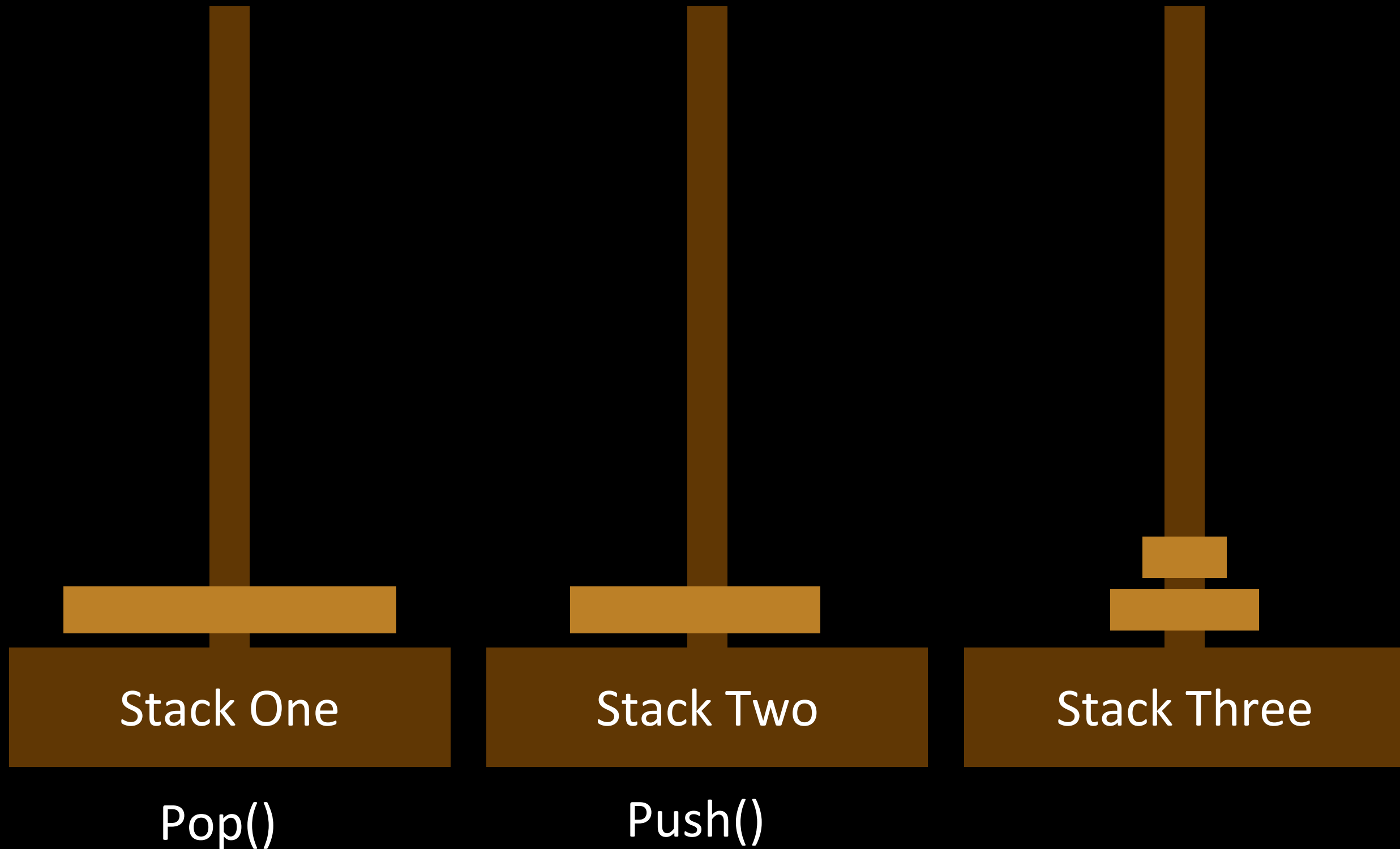
# Tower of Hanoi



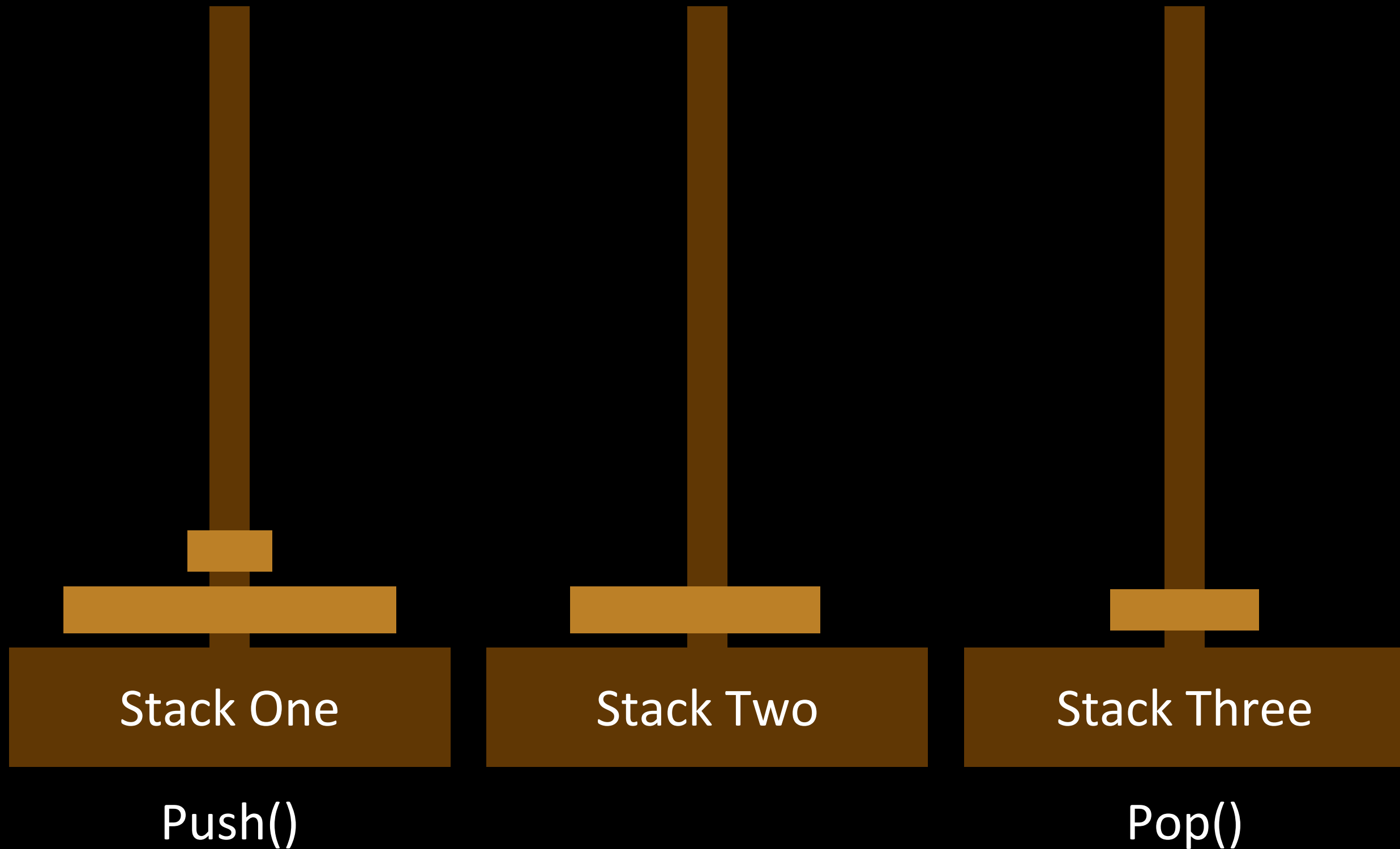
# Tower of Hanoi



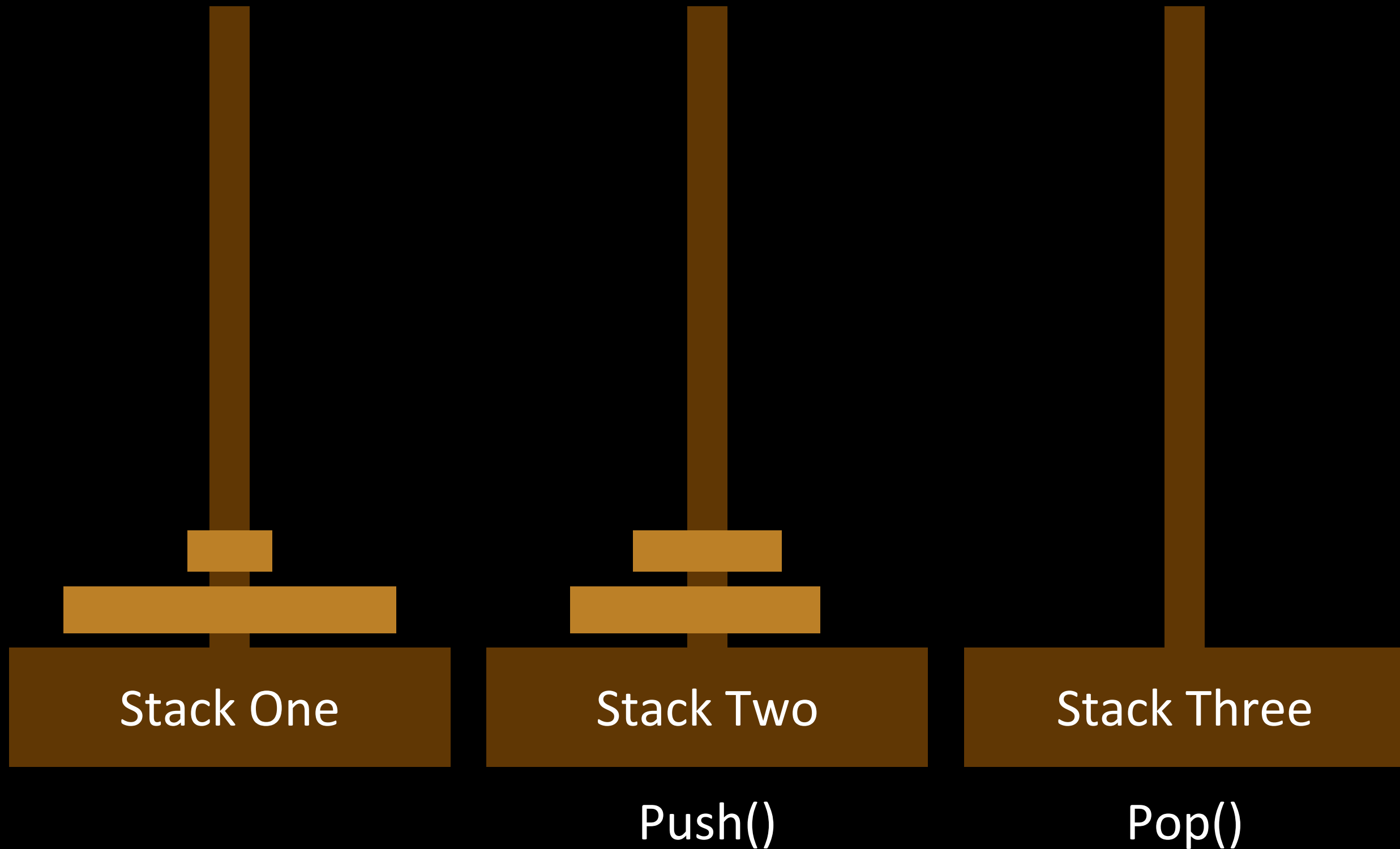
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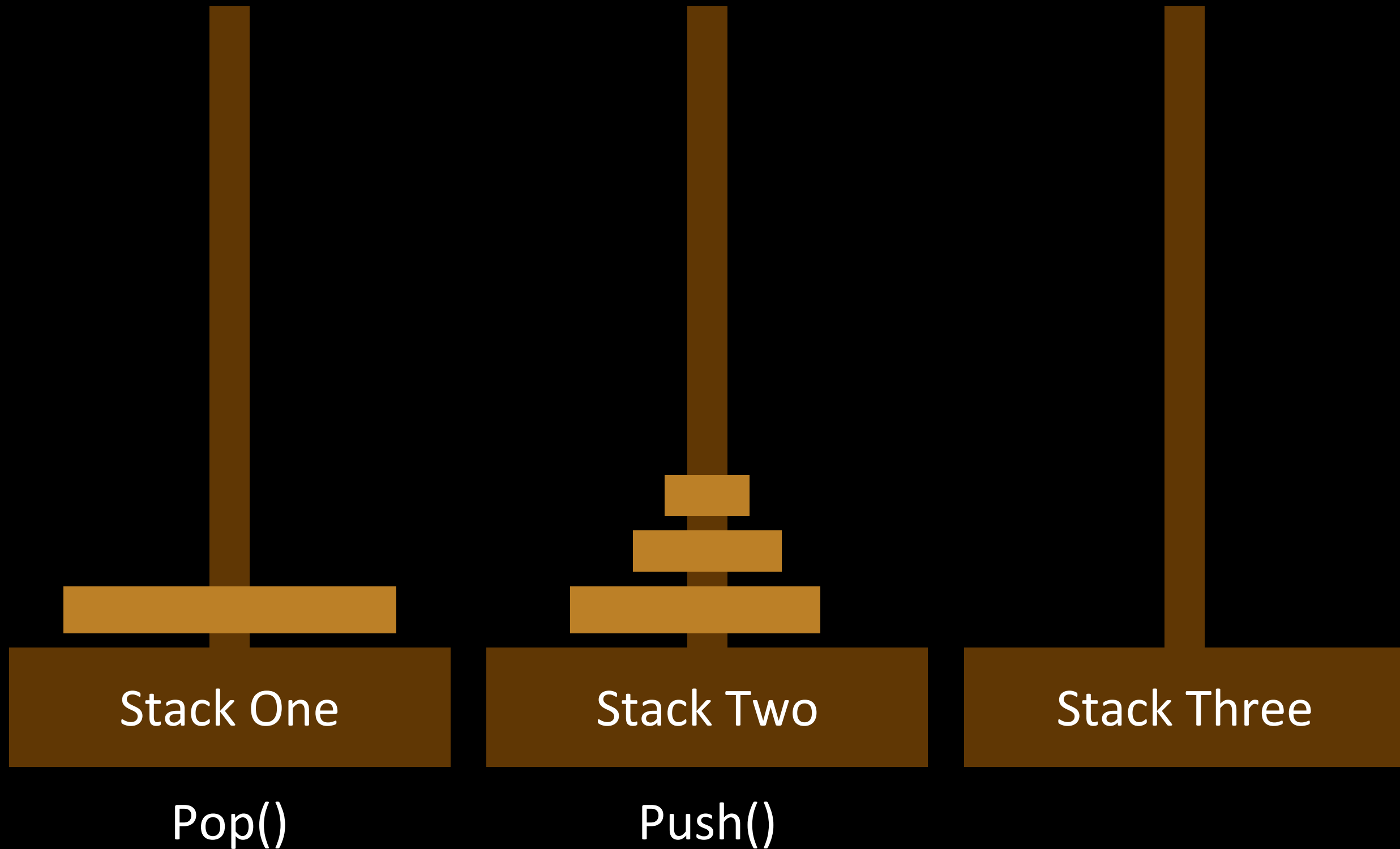


# Tower of Hanoi

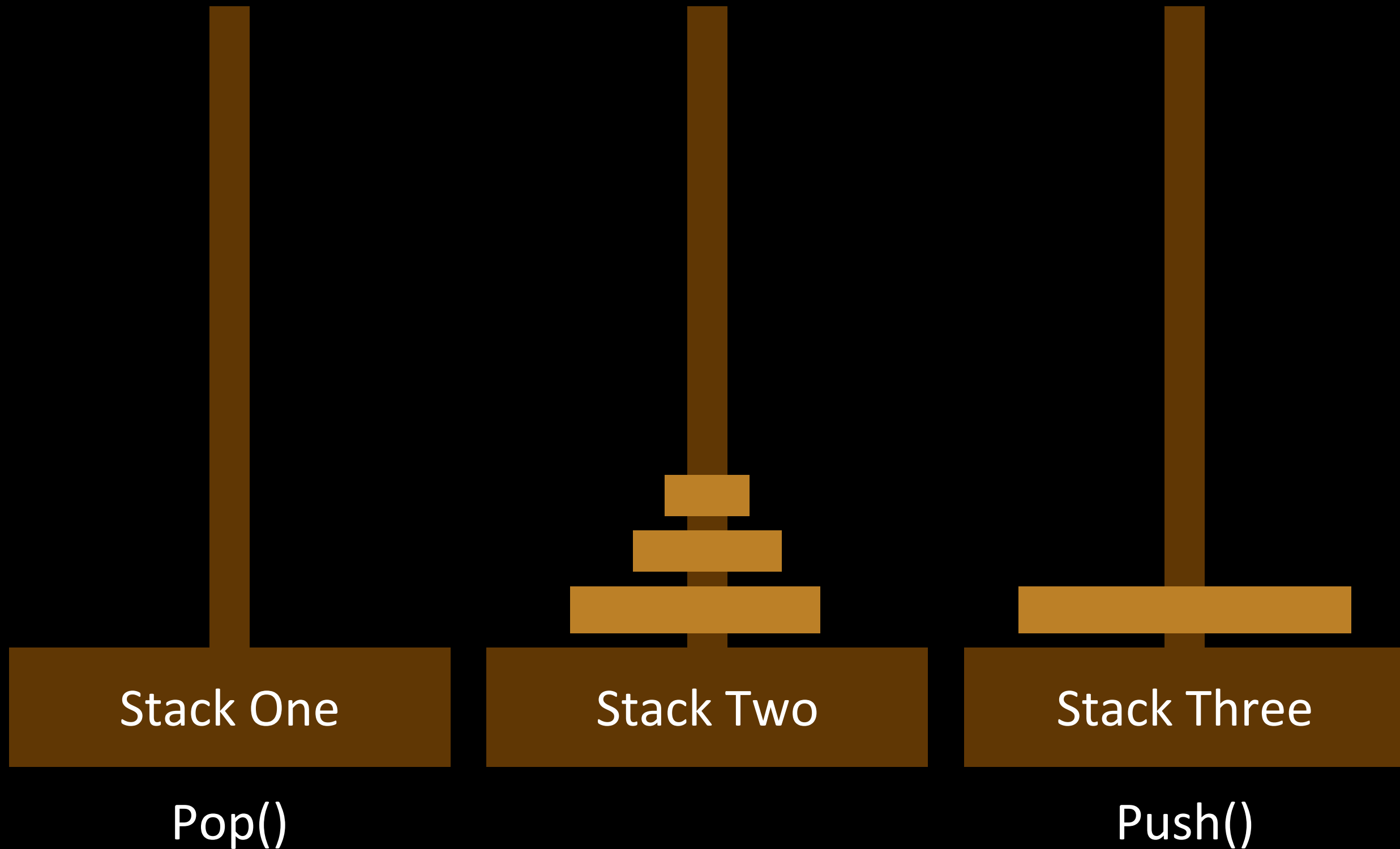




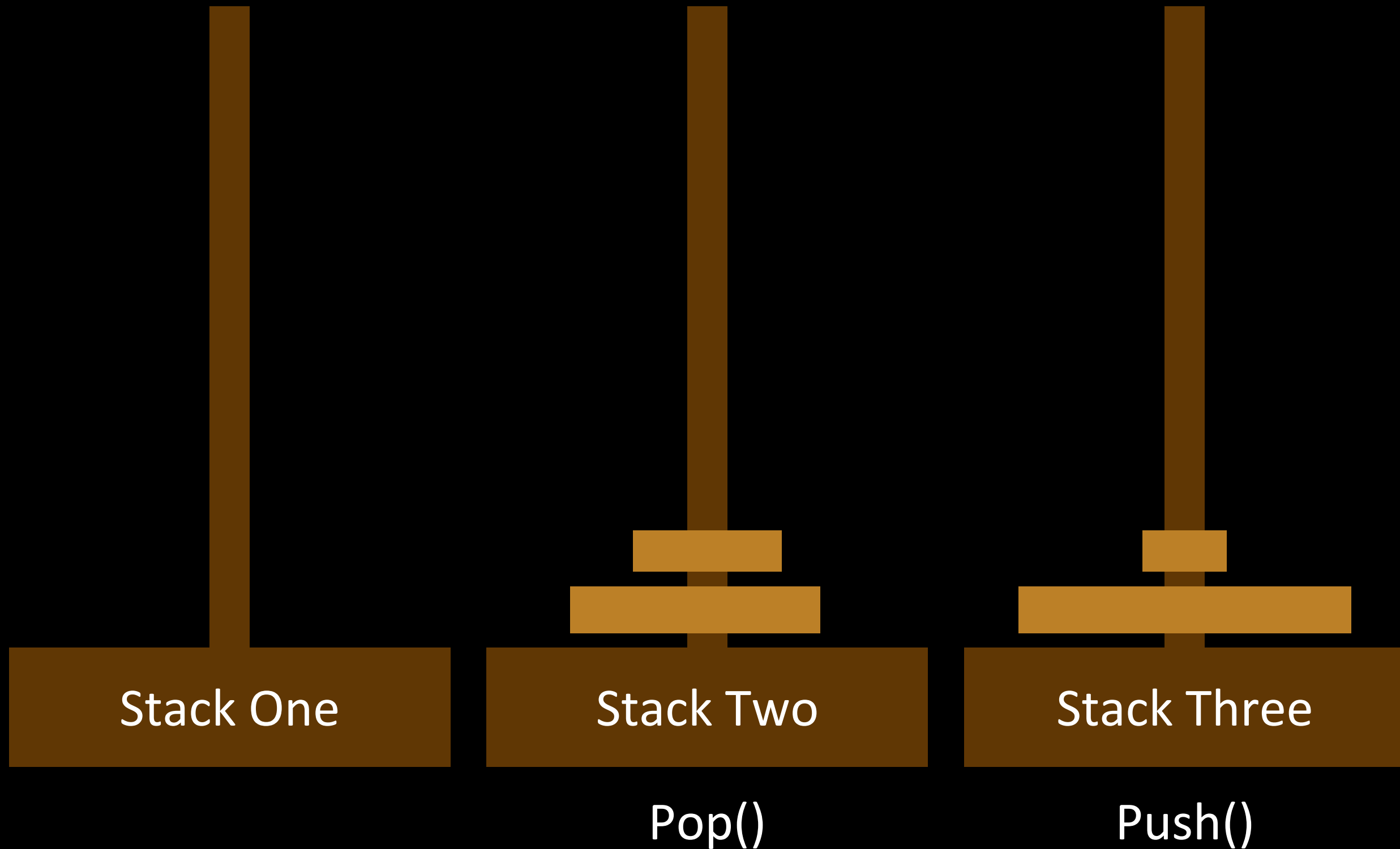
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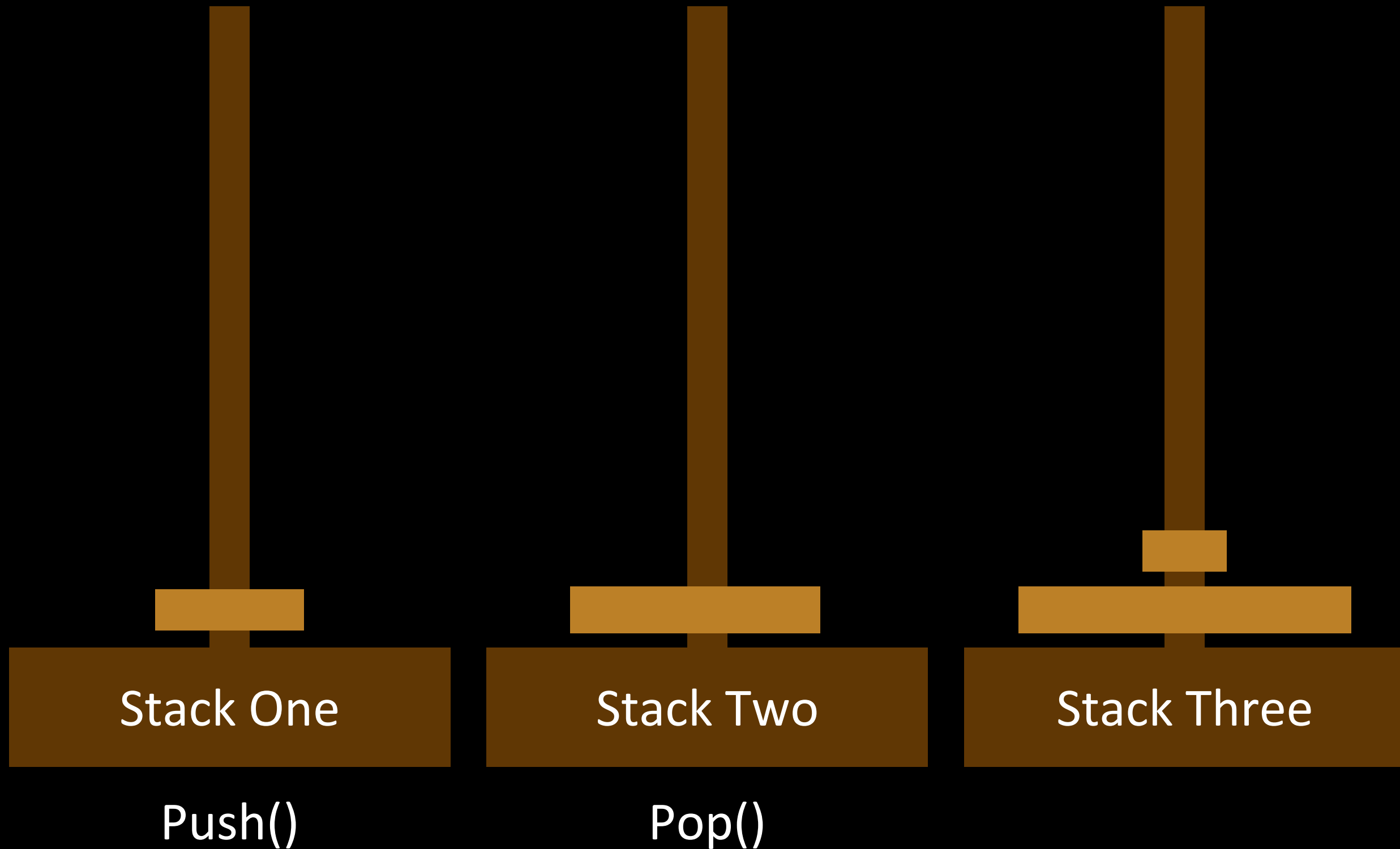
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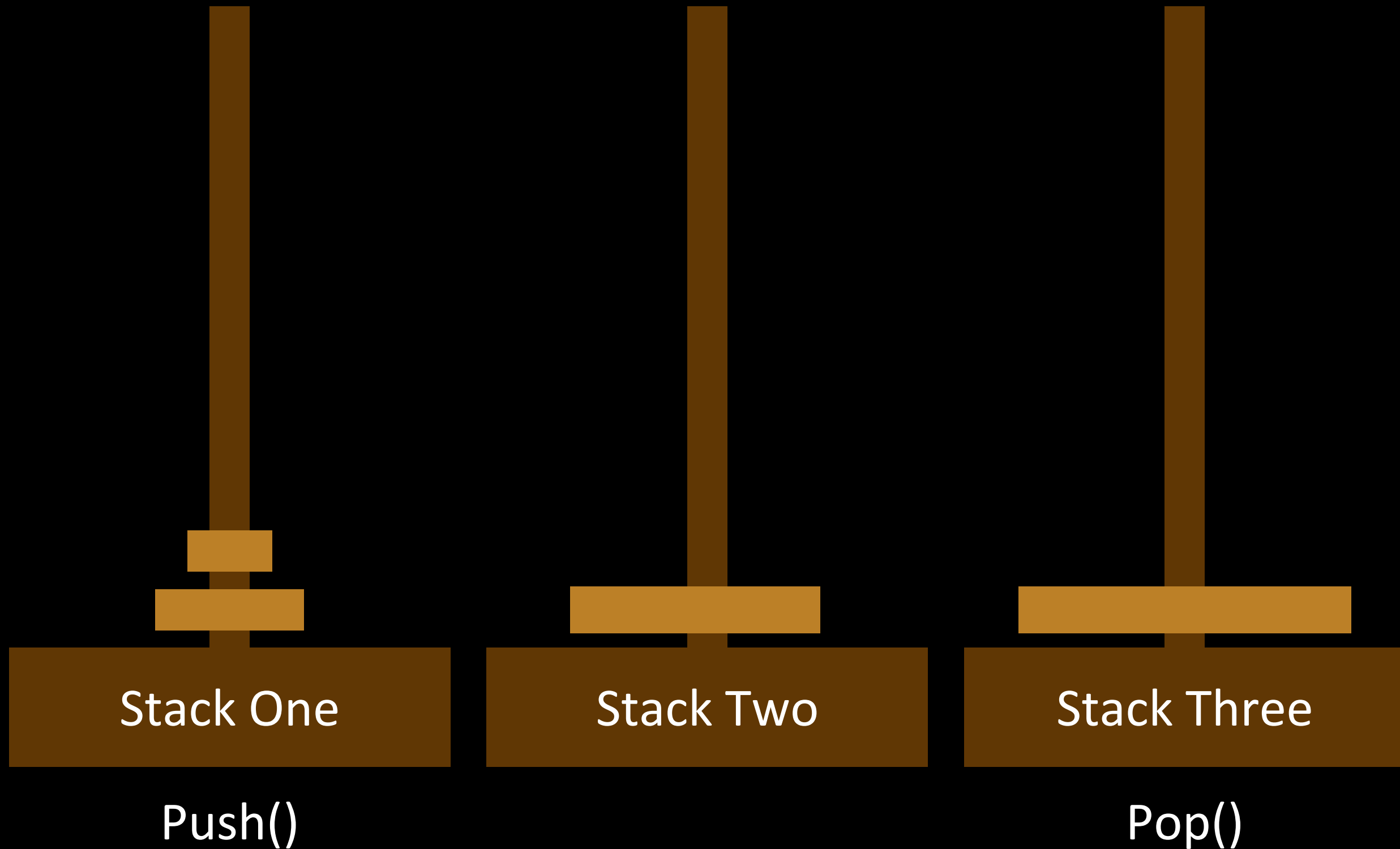
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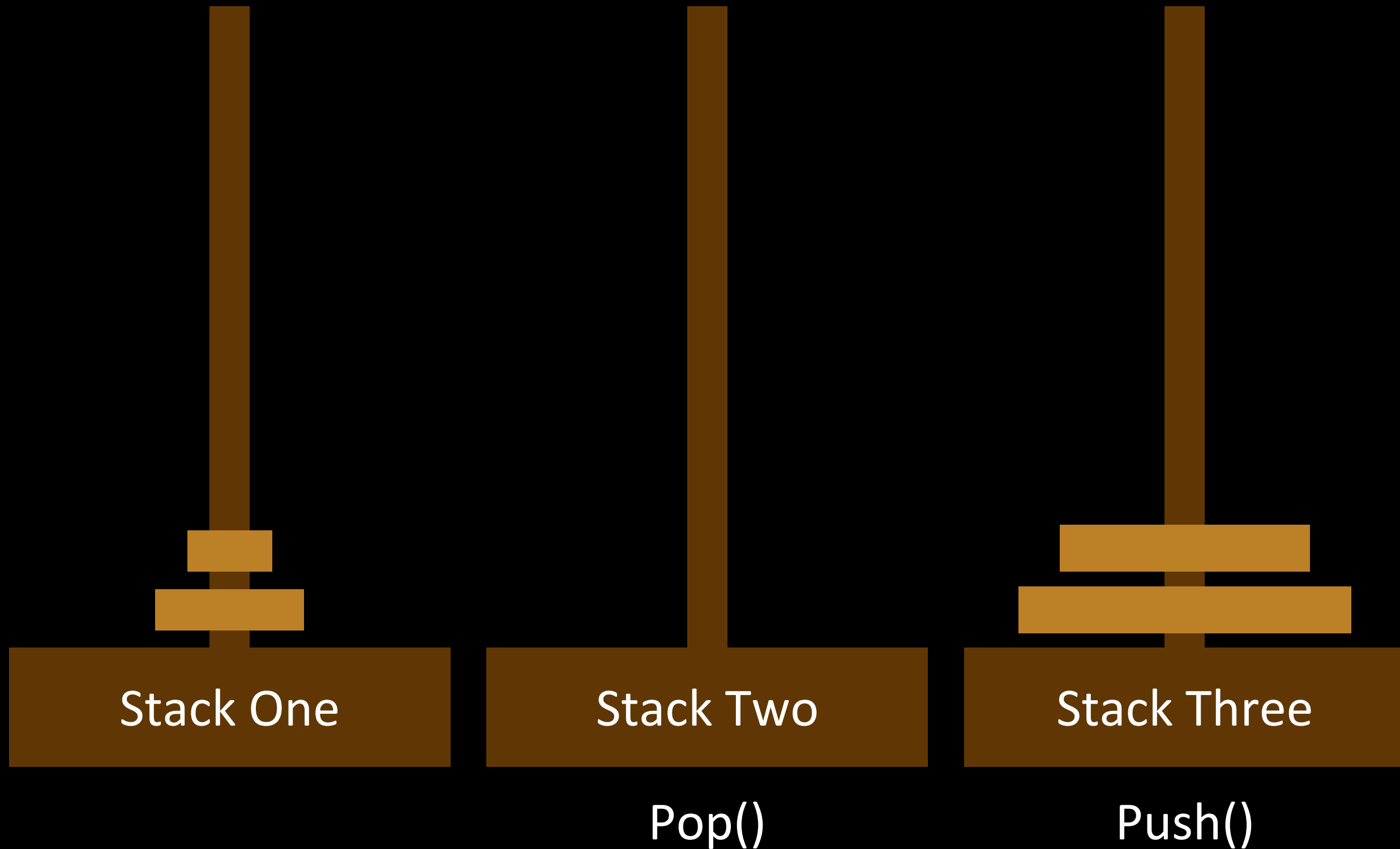
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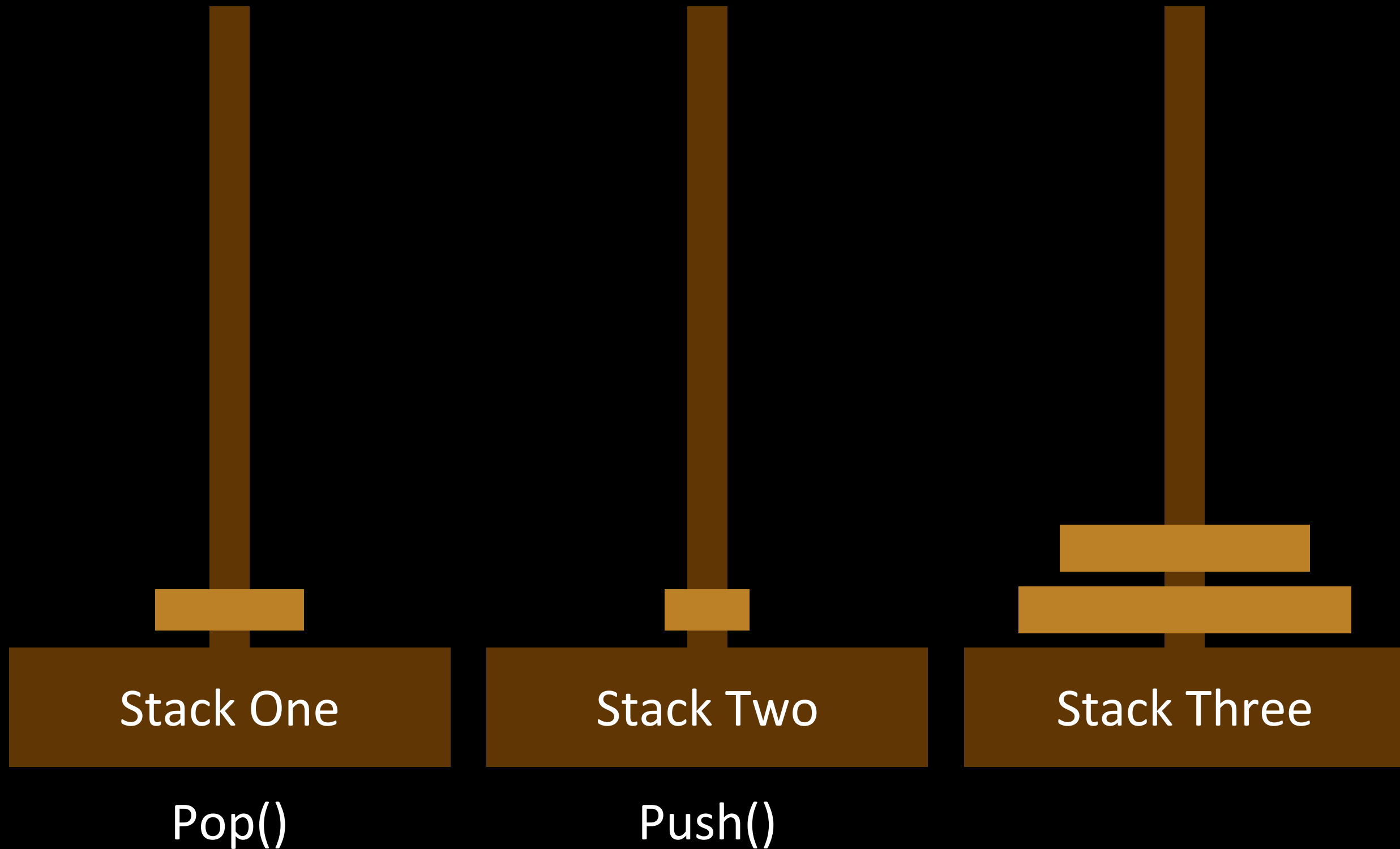
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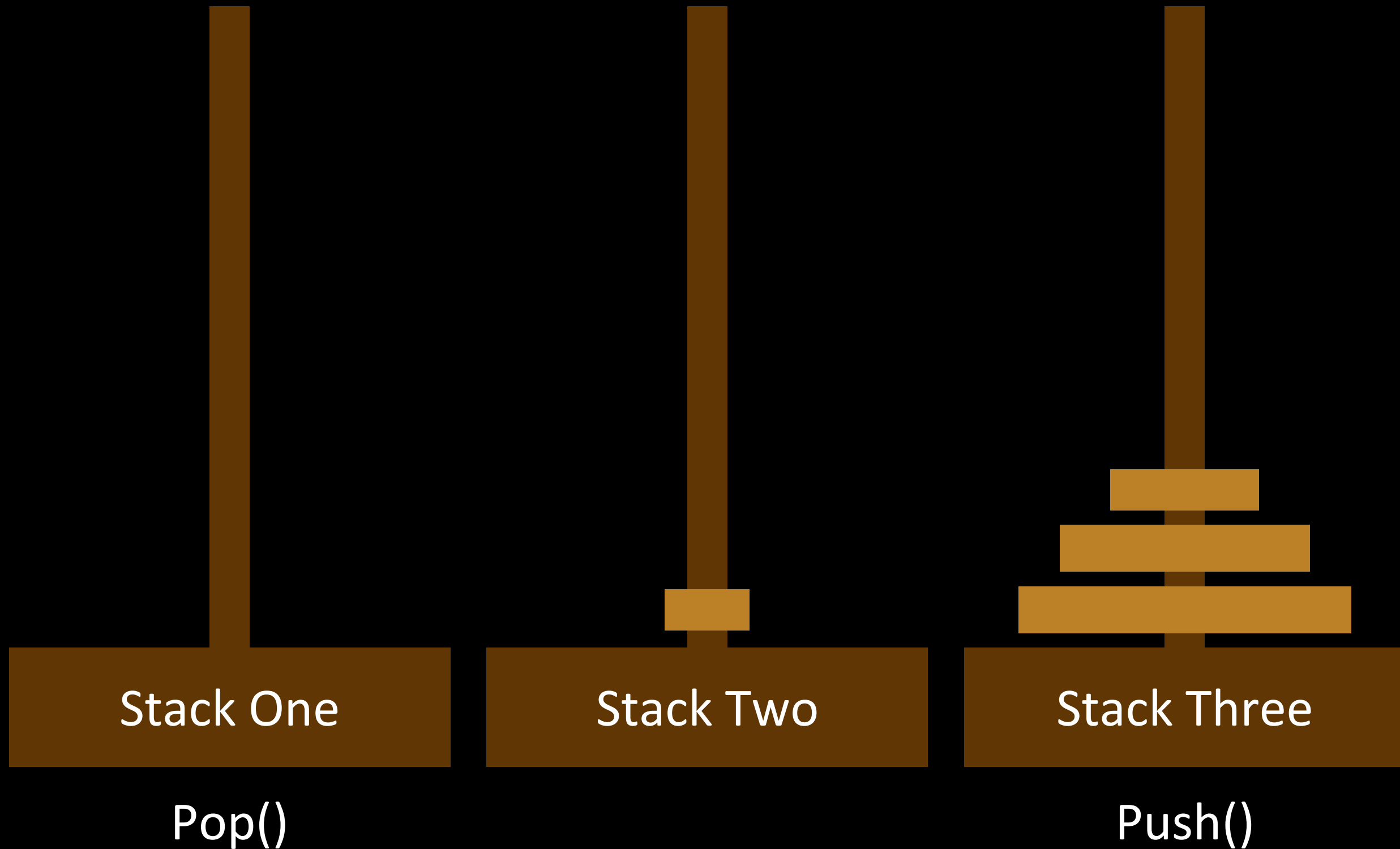
# Tower of Hanoi



# Tower of Hanoi

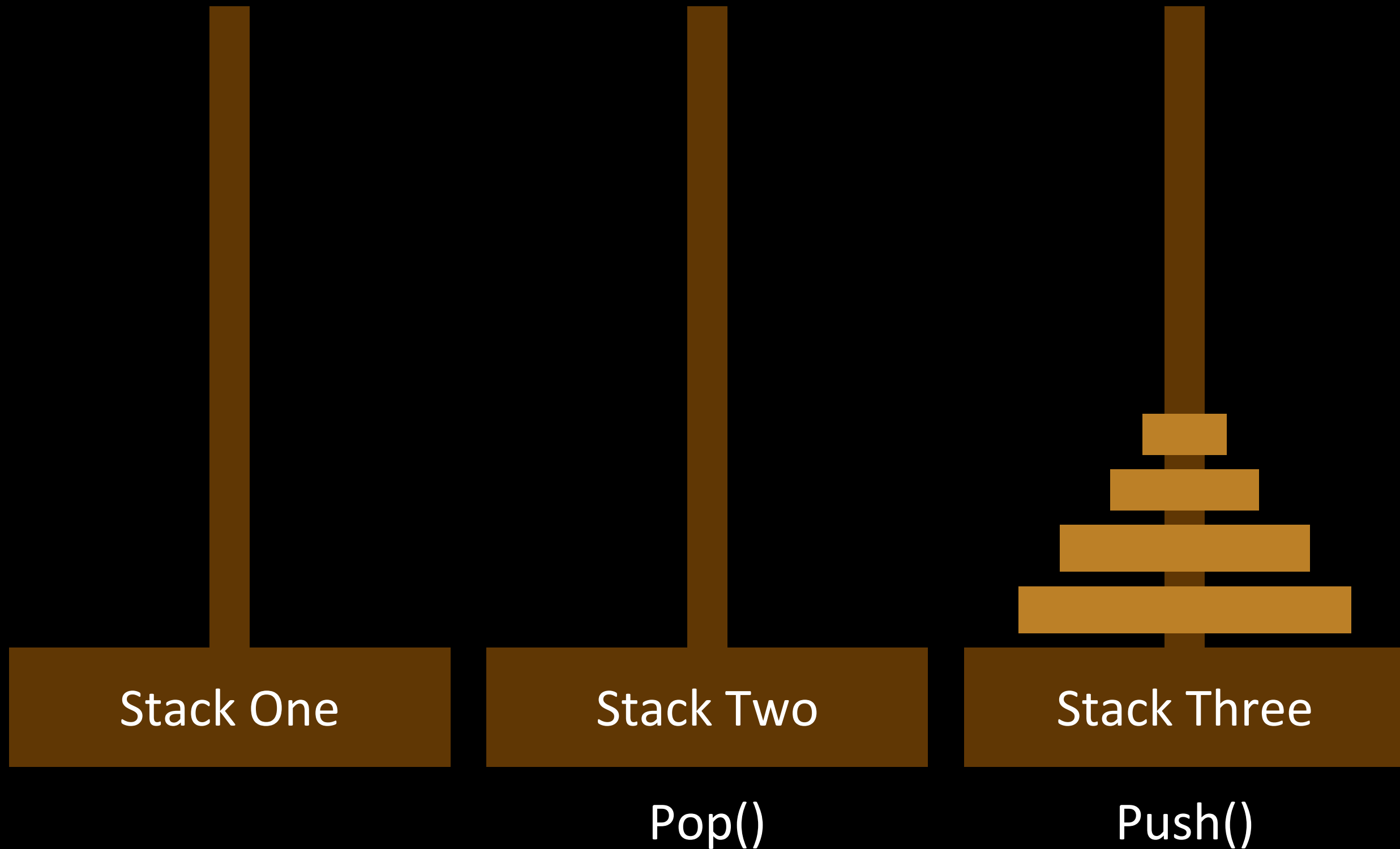


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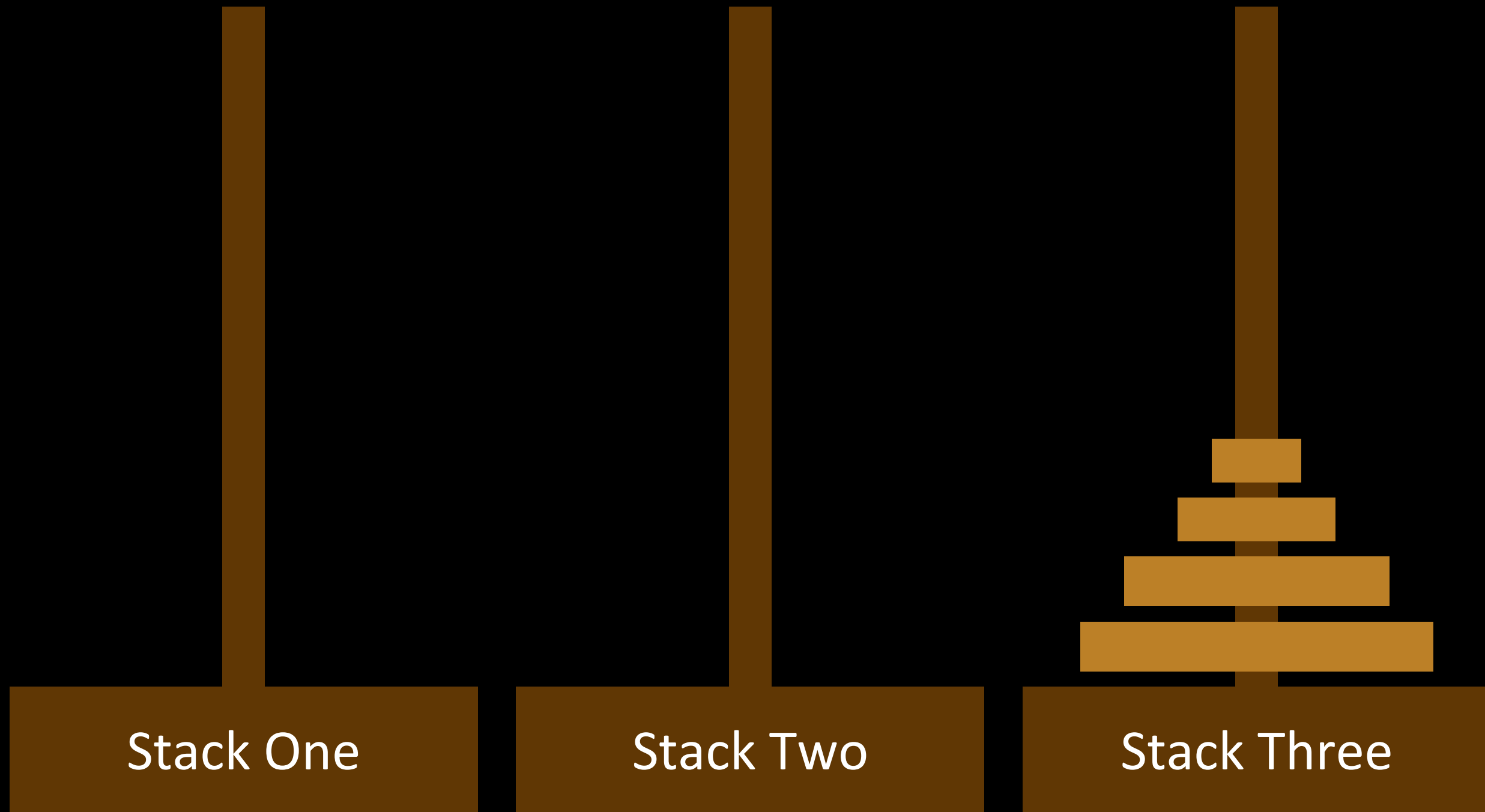




# Tower of Hanoi



# Tower of Hanoi



# Stack Operations

William Fiset

# Pushing

## Instructions

Push(4)

Push(2)

Push(5)

Push(13)

# Pushing

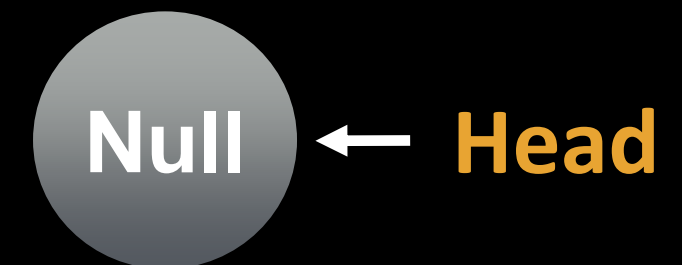
## Instructions

Push(4)

Push(2)

Push(5)

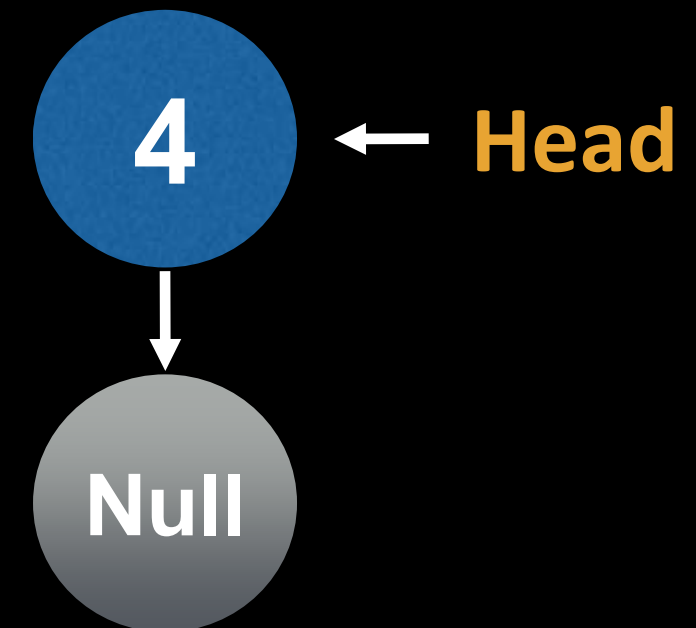
Push(13)



# Pushing

## Instructions

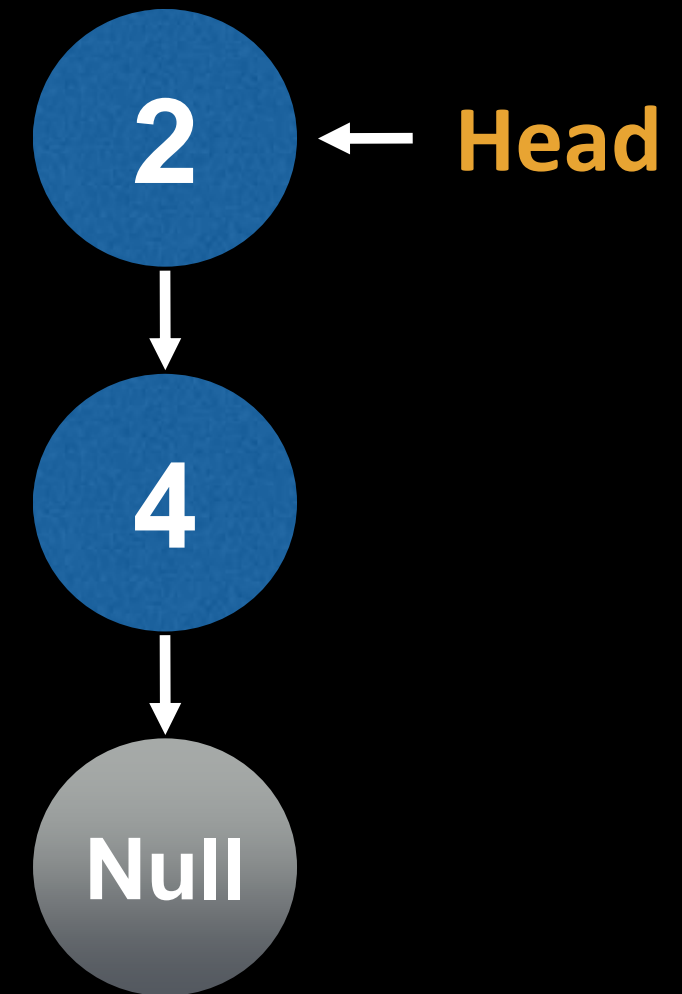
→ Push(4)  
Push(2)  
Push(5)  
Push(13)



# Pushing

## Instructions

→ Push(4)  
Push(2)  
Push(5)  
Push(13)



# Pushing

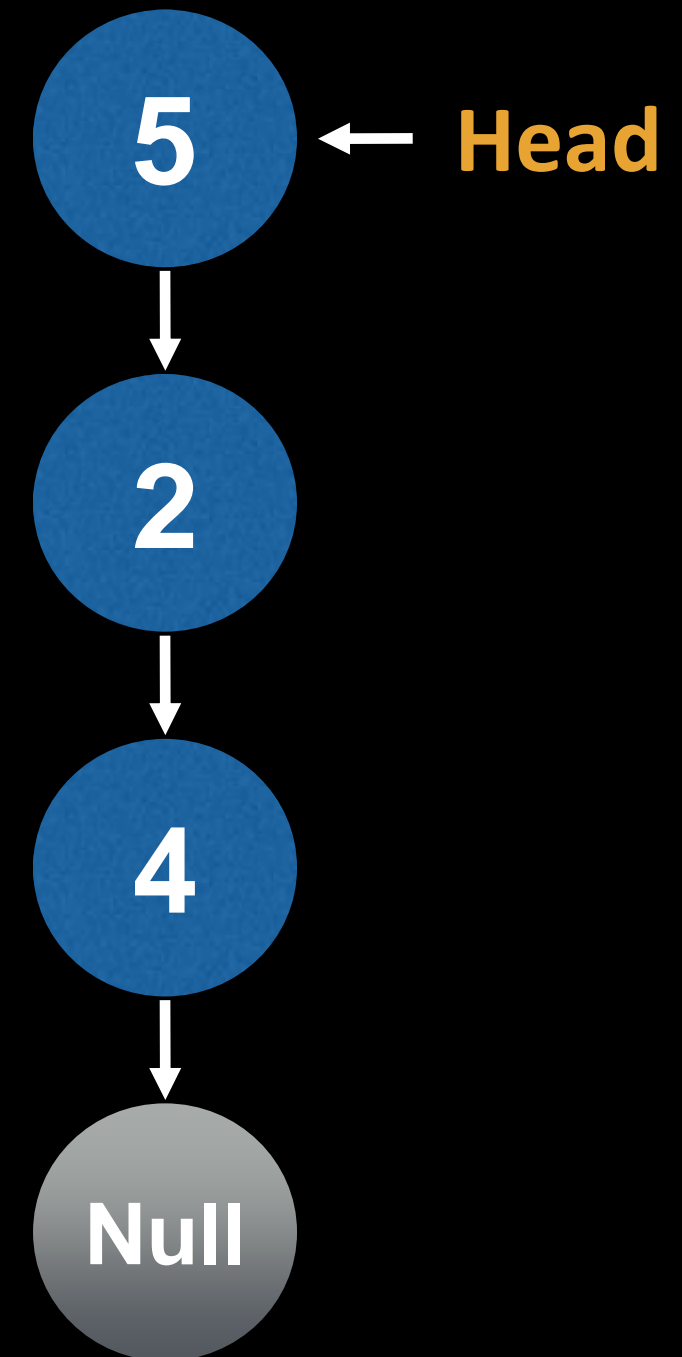
## Instructions

Push(4)

Push(2)

→ Push(5)

Push(13)





# Pushing

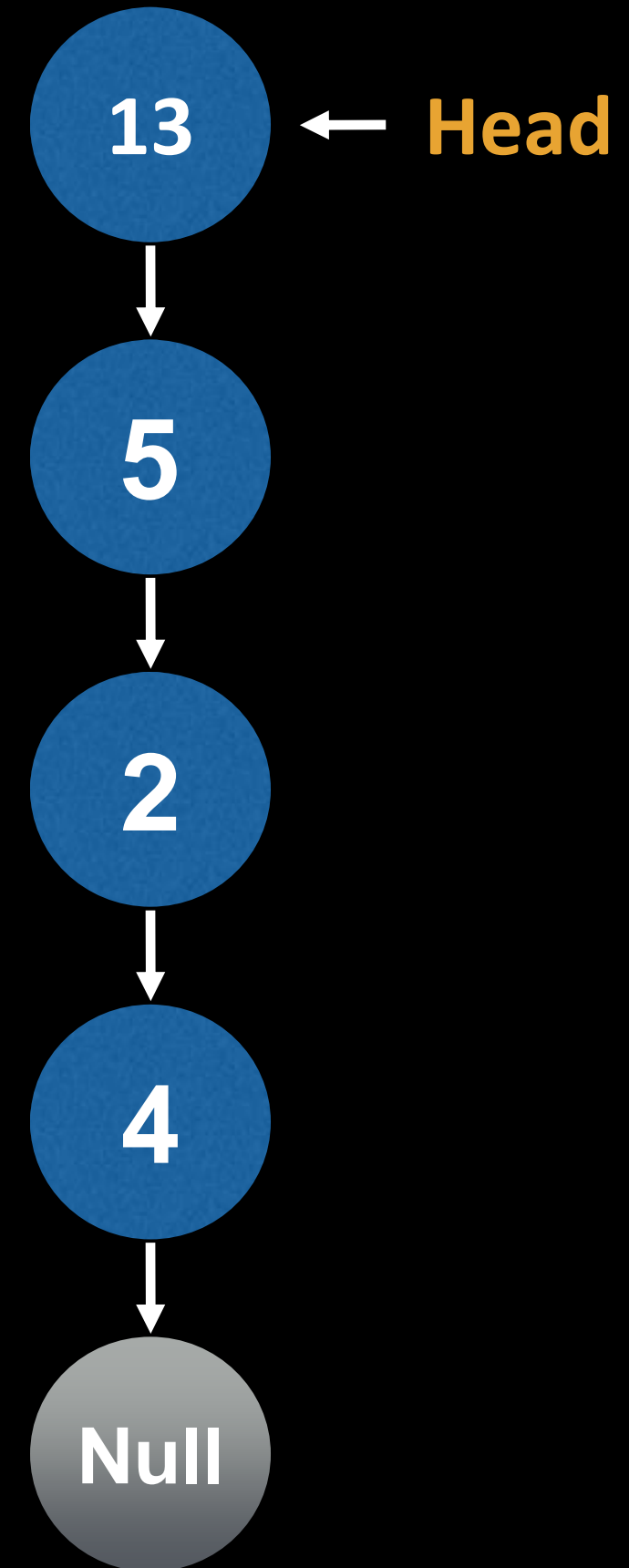
## Instructions

Push(4)

Push(2)

Push(5)

→ Push(13)



# Popping

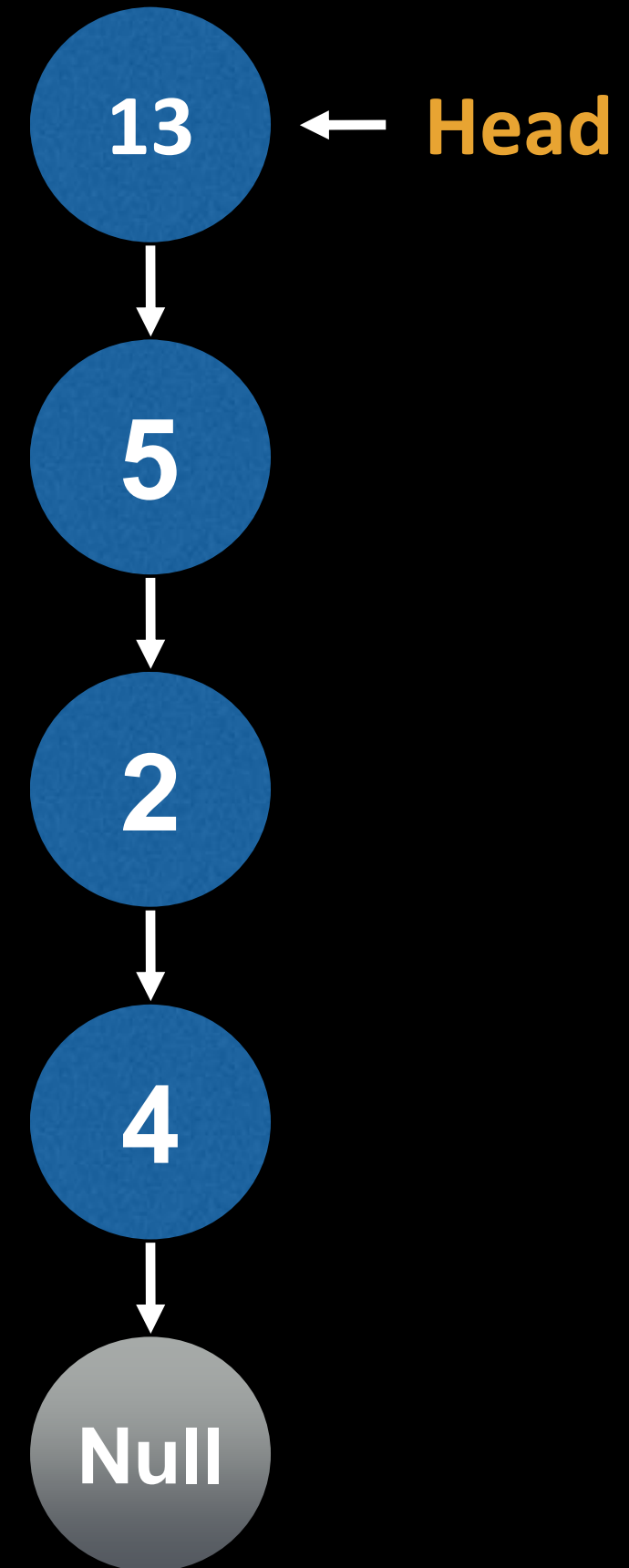
## Instructions

Pop()

Pop()

Pop()

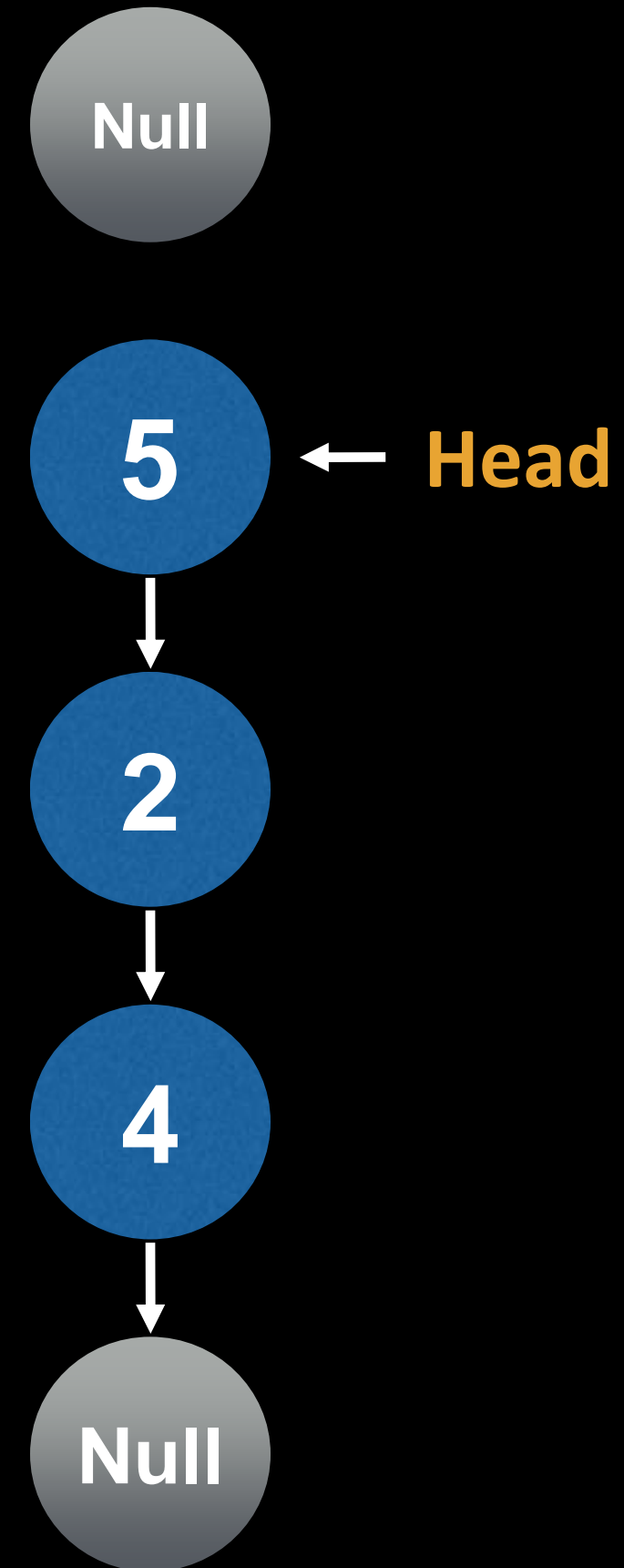
Pop()



# Popping

## Instructions

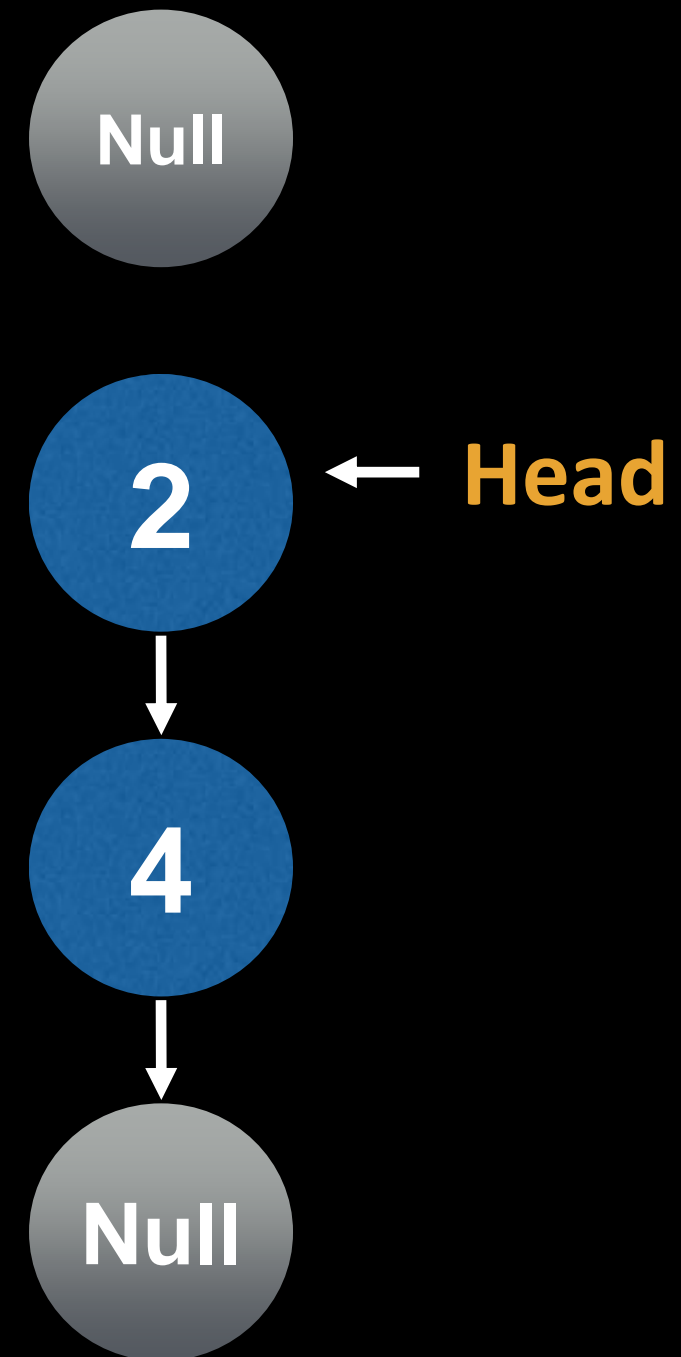
→ Pop()  
Pop()  
Pop()  
Pop()



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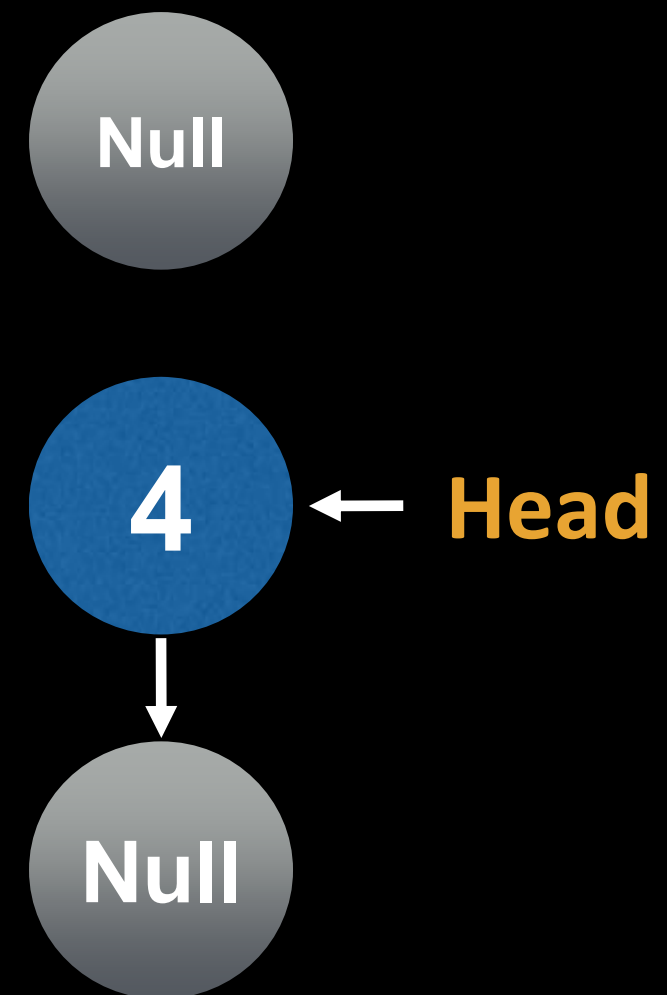
→ Pop()  
Pop()  
Pop()  
Pop()



# Popping

## Instructions

Pop()  
Pop()  
→ Pop()  
Pop()



# Popping

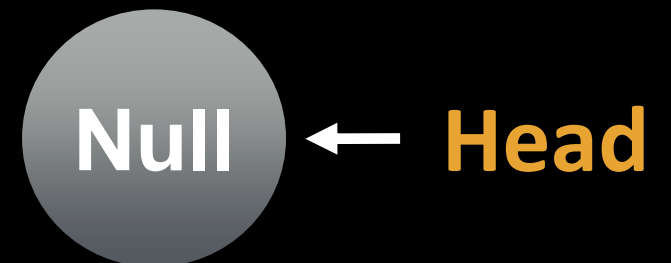
## Instructions

Pop()

Pop()

Pop()

→ Pop()



# Popping

## Instructions

Pop()

Pop()

Pop()

Pop()

