Basic Electrical Engineering (TEE 101)

Lecture 26: Series AC circuits ($R-L$ and $R-C$) and Impedance

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Content

This lecture covers:

- **Series R – L circuit**
- **Series R – C circuit**
- **Concept of Impedance (Z)**

Series R – L
AC circuit

This is the most general case met in practice as nearly all a.c. circuits contain both resistance and inductance. Fig. 1 shows a pure resistance of R ohms connected in series with a coil of pure inductance L henry.

Let V = r.m.s. value of the applied voltage

It is measured in I = r.m.s. value of the circuit current

$V_R = I R$ .... where V_R is in phase with I

$V_L = I X_L$ .... where V_L leads I by 90°

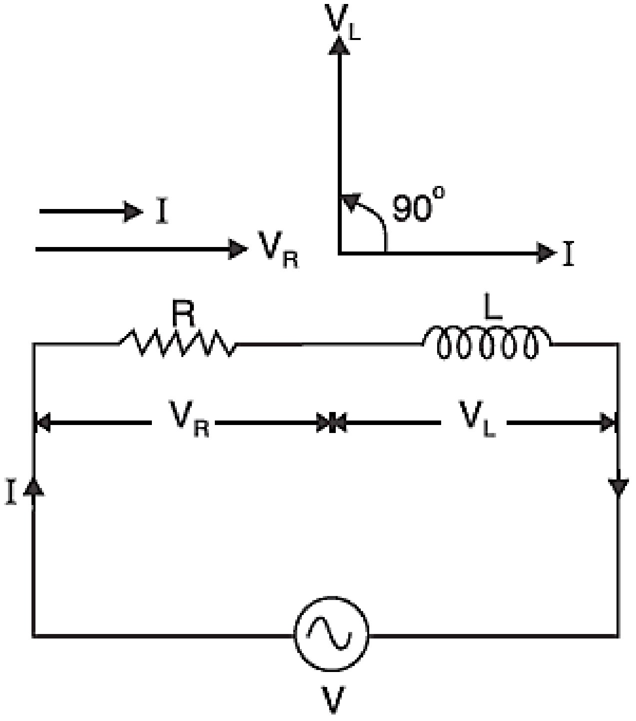

Fig. 1 circuit diagram

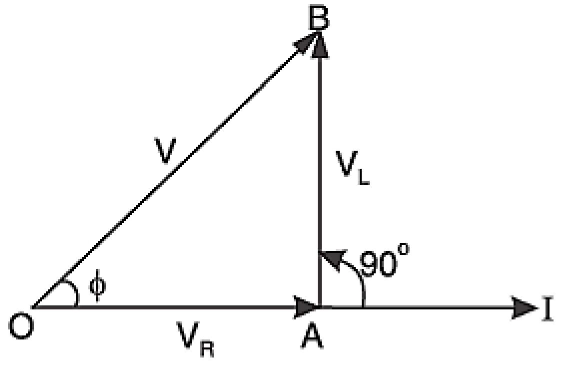

Fig. 2 Phasor Diagram

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Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.

The voltage drop V_R (= I R) is in phase with current and is represented in magnitude and direction by the phasor *OA*.

The voltage drop V_L (= I X_L) leads the current by 90° and is represented in magnitude and direction by the phasor *AB*. The applied voltage *V* is the phasor sum of these two drops *i.e.*

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called **impedance** of the circuit. It is represented by *Z* and is measured in ohms (Ω).

$$I = \frac{V}{Z} \quad \text{where } Z = \sqrt{R^2 + X_L^2}$$

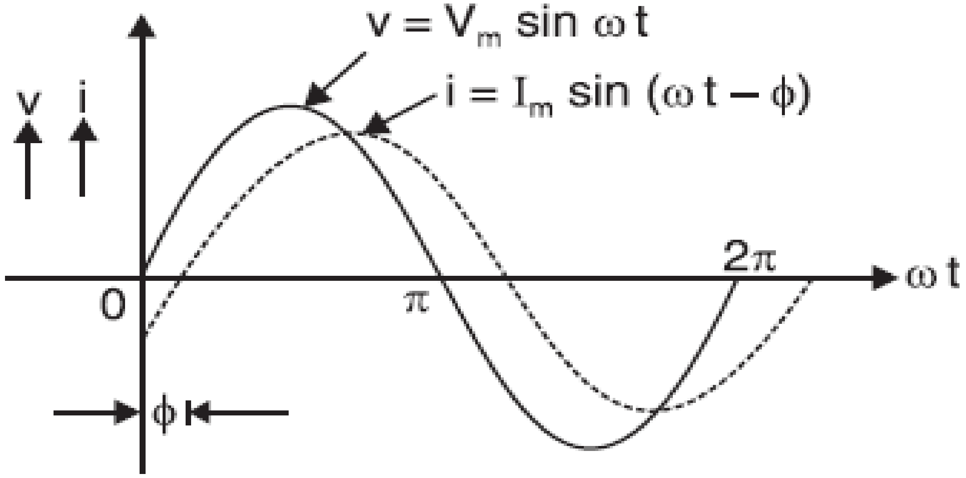


Fig. 3 Wave Diagram of series RL circuit

It is clear from the phasor diagram (fig. 2) and wave diagram (fig. 3) that circuit current *I* lags the appliedvoltage *V* by φ°

Series R – C AC circuit

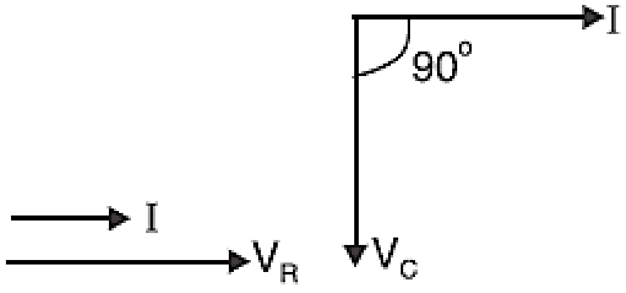
Fig. 4 shows a resistance of R ohms connected in series with a capacitor of C farad.

Let V = r.m.s. value of the applied voltage

It is measured in I = r.m.s. value of the circuit current.

$V_R = I\,R$ .... where V_R is in phase with I

$V_C = I\,X_C$ .... where V_C lags I by 90°



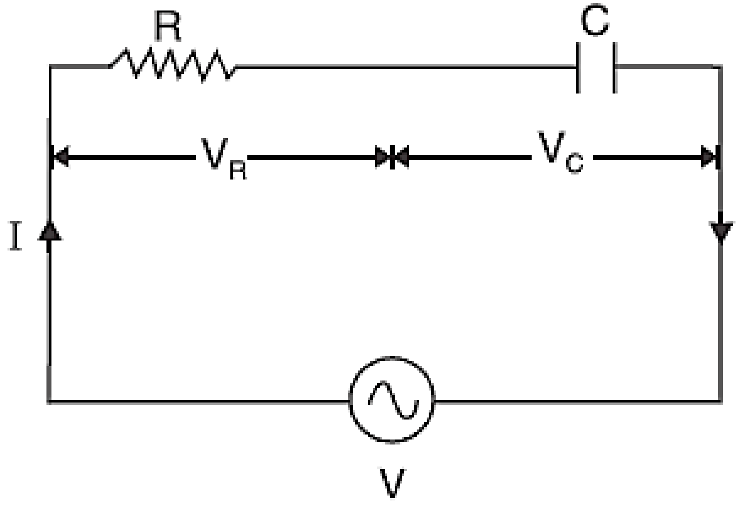


Fig. 4 circuit diagram

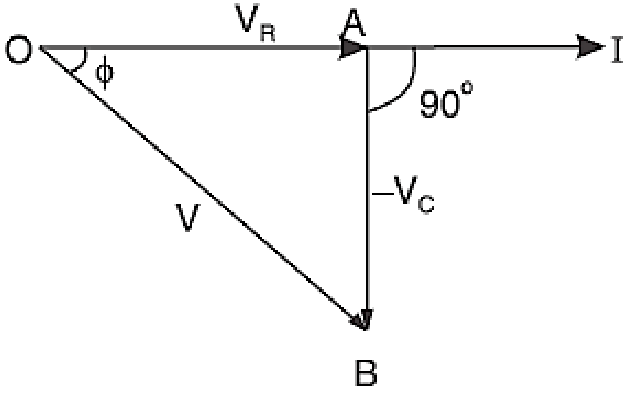


Fig. 5 Phasor Diagram

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Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 5

The voltage drop V_R (= IR) is in phase with current and is represented in magnitude and direction by the phasor OA.

The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drops.

$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2+X_C^2}$ offers opposition to current flow and is called **impedance** of the circuit. It is represented by *Z* and is measured in ohms (Ω).

$$I = \frac{V}{Z} \quad \text{where, } Z=\sqrt{R^2+X_C^2}$$

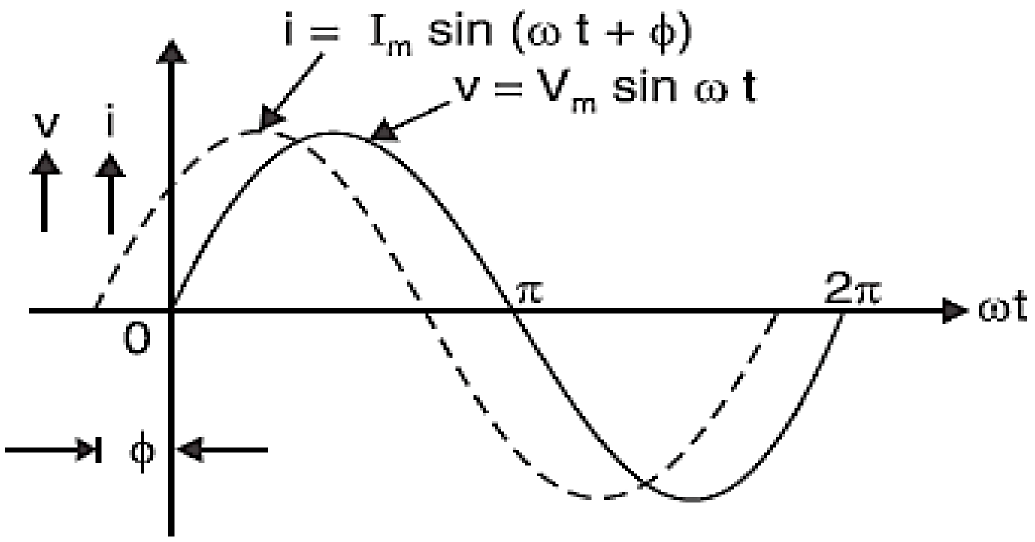

Fig. 6 Wave Diagram of series RC circuit

It is clear from the phasor diagram (fig. 5) and wave diagram (fig. 6) that circuit current *I* leads the appliedvoltage *V* by φ°

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IMPEDANCE (Z)

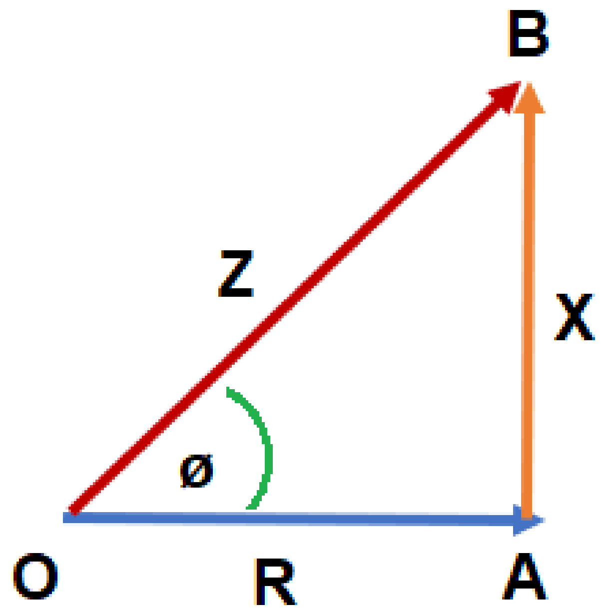
Impedance. *The total opposition offered to the flow of alternating current by a circuit is called* **impedance** *Z of the circuit.*

Impedance consists of two components viz. resistance and reactance. Therefore, it can be expressed in these two components.

Let the impedance of an alternating current circuit is **Z=R+jX** **where, R and X are known as resistance and reactance respectively**

It is clear from the above expression of Z is that, it is a complex number and hence can be geometrically represented in the same manner as a complex number. The geometrical representation is shown below.

The magnitude of impedance is given by, $|Z| = \sqrt{R^2 + X^2}$



For series R – L AC circuit: Impedance, $Z = \sqrt{R^2 + X_L^2}$ where $X_L = 2\pi f L$

The magnitude of impedance in *R-L* series circuit depends upon the values of *R, L* and the supply frequency *f*.

For series R – C AC circuit: Impedance, $Z=\sqrt{R^2+X_C^2}$, where $X_C = \dfrac{1}{2\pi fC}$

The magnitude of impedance in *R-C* series circuit depends upon the values of *R, C* and the supply frequency *f*.

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IMPEDANCE TRIANGLE

- The phasor diagram of a *R-L* series circuit is shown in Fig. 7.
- Dividing each side of the phasor diagram by the same factor *I*, we get a triangle whose sides represent *R*, X_L and *Z*.
- Such a triangle is known as *impedance triangle* (See Fig. 7 and 8).
- Just as in Fig. 7, the impedance triangle is also a right-angled triangle.
- Similarly, for R-C series circuit, the impedance triangle is shown in figure 8.

Impedance triangle is a useful concept in a.c. circuits as it enables us to calculate :

(i) the impedance of the circuit *i.e.*,

(ii) power factor of the circuit *i.e.*, cos $\varphi = R/Z$

(iii) phase angle f *i.e.*, tan $\varphi = X_L/R$

(iv) whether current leads or lags the voltage.

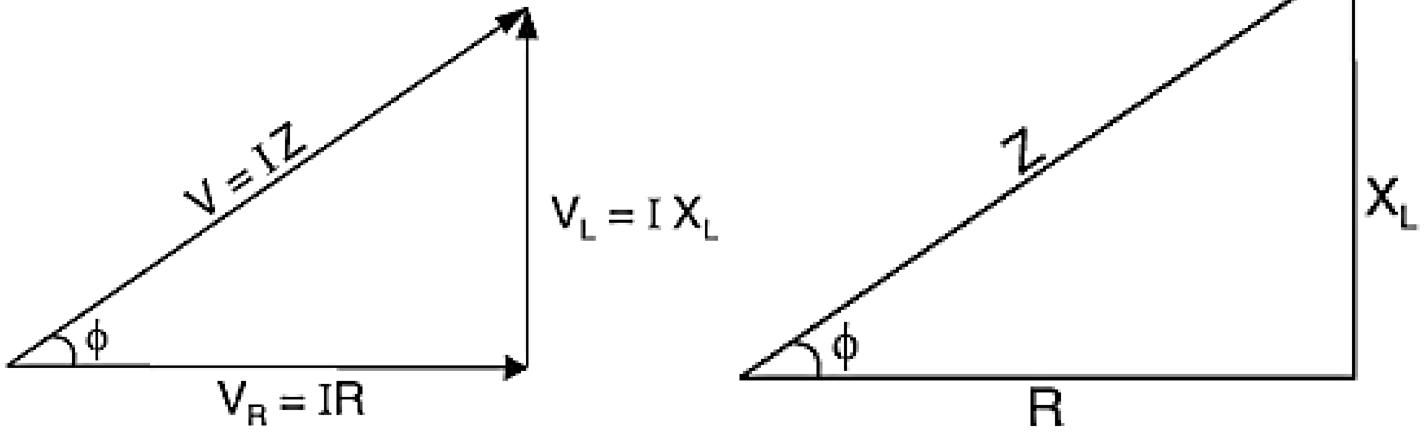

Fig. 7 Impedance Triangle of RL series circuit

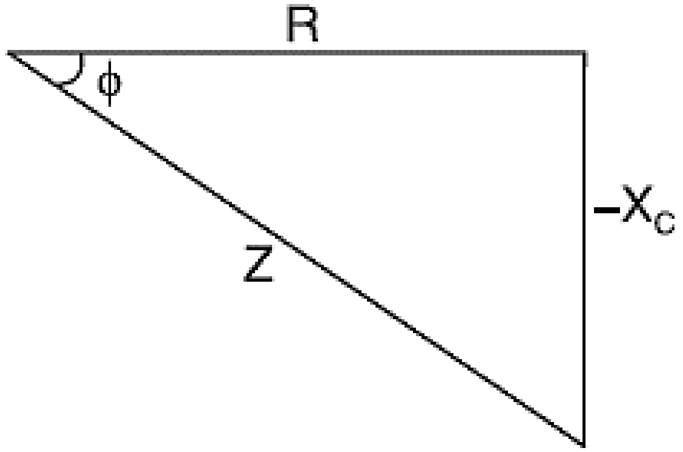

Fig. 8 Impedance Triangle of RC series circuit

Thank You