Basic Electrical Engineering (TEE 101)

Lecture 19: Phasor representation

By: Dr. Parvesh Saini

Content

This lecture covers:

- Introduction to Phasors
- Representation of AC quantities with Phasors
- Phasor Algebra

Representation of Alternating Voltages and Currents

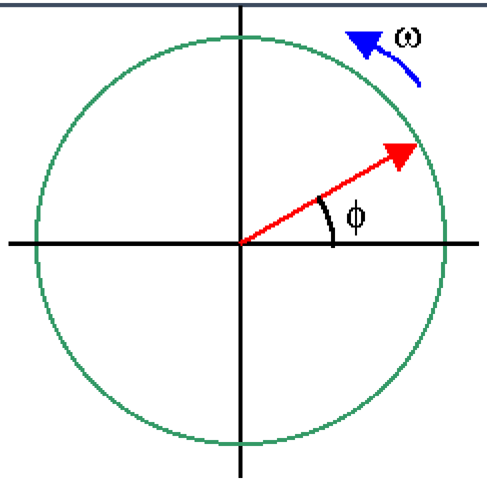
So far we have discussed that an alternating voltage or current may be represented in the form of (*i*) waves and (*ii*) equations.

The waveform presents to the eye a very definite picture of what is happening at every instant. But it is difficult to draw the wave accurately.

No doubt the current flowing at any instant can be determined from the equation form $i = I_m \sin \omega t$ but this equation presents no picture to the eye of what is happening in the circuit.

The above difficulty has been overcome by representing sinusoidal alternating voltage or current by a line of definite length rotating in *anticlockwise direction at a constant angular velocity (ω). Such a rotating line is called a **phasor**.

The length of the phasor is taken equal to the maximum value (on a suitable scale) of the alternating quantity and angular velocity equal to the angular velocity of the alternating quantity.



Phasor

Phasor is a line used to represent a complex electrical quantity as a vector. Phasor also represents an AC quantity with magnitude and direction

Phasor is basically a rotating vector, or Phasor is a line rotating at certain angular velocity in anticlockwise direction

Phasors generate waveforms with sinusoidal nature

Phasor Diagram of a Sinusoidal Waveform

As the single vector rotates in an anti-clockwise direction, its tip at point A will rotate one complete revolution of 360º or 2π representing one complete cycle. If the length of its moving tip is transferred at different angular intervals in time to a graph as shown above, a sinusoidal waveform would be drawn starting at the left with zero time.

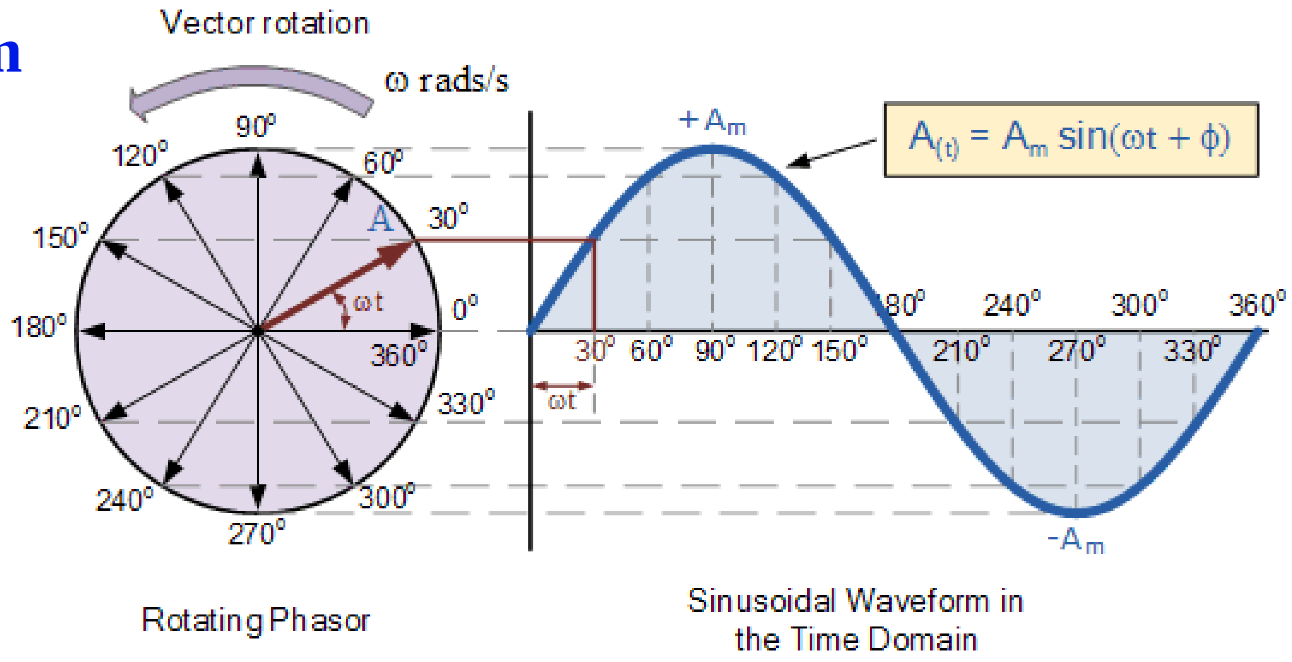


Fig. 1

By: Dr. Parvesh Saini

The following points are worth noting :

(*i*) The length of the phasor represents the maximum value and the angle with axis of reference (*i.e.*, *X*-axis) indicates the phase of the alternating quantity *i.e*. current in this case.

(*ii*) The phasor representation enables us to quickly obtain the numerical values and, at the same time, have a picture before the eye of the events taking place in the circuit.

(*iii*) A phasor diagram permits addition and subtraction of alternating voltages or currents with a fair degree of ease.

(*iv*) Phasor of an ac quantity is represented with a line with an arrow head. The arrow of the phasor represents its direction and length of the line represents the magnitude of the ac quantity.

(*v*) We can conveniently represent the multiple ac quantities with the help of phasors

We can easily use Phasor Diagrams to represent the magnitude and directional relationship between two or more alternating quantities graphically

By: Dr. Parvesh Saini

Phasor Representation of Sinusoidal Quantities

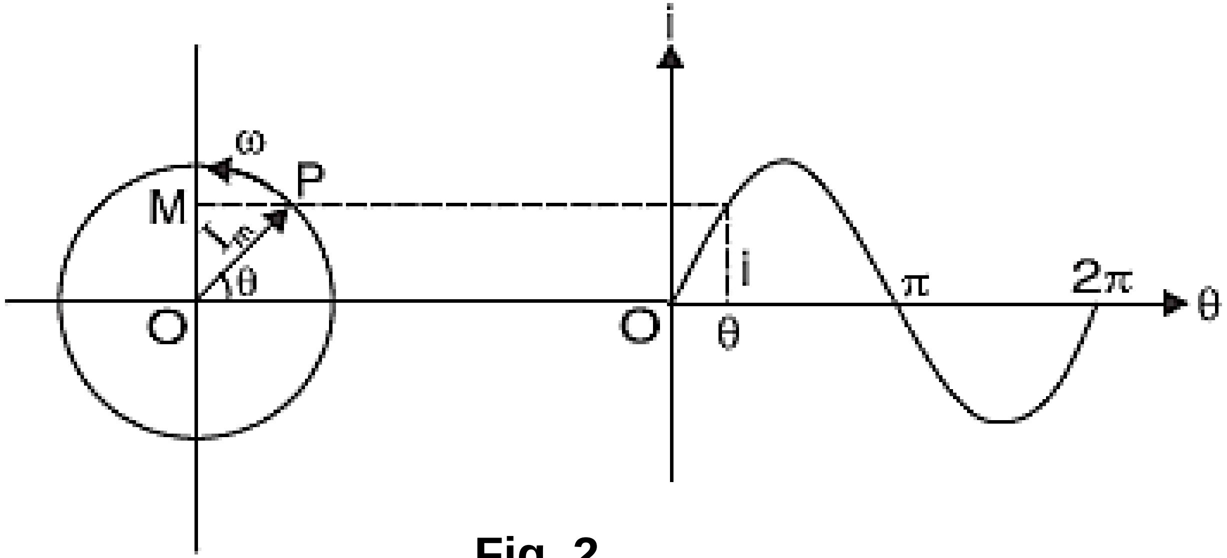


Fig. 2

Phasor Diagram of Sine Waves of Same Frequency

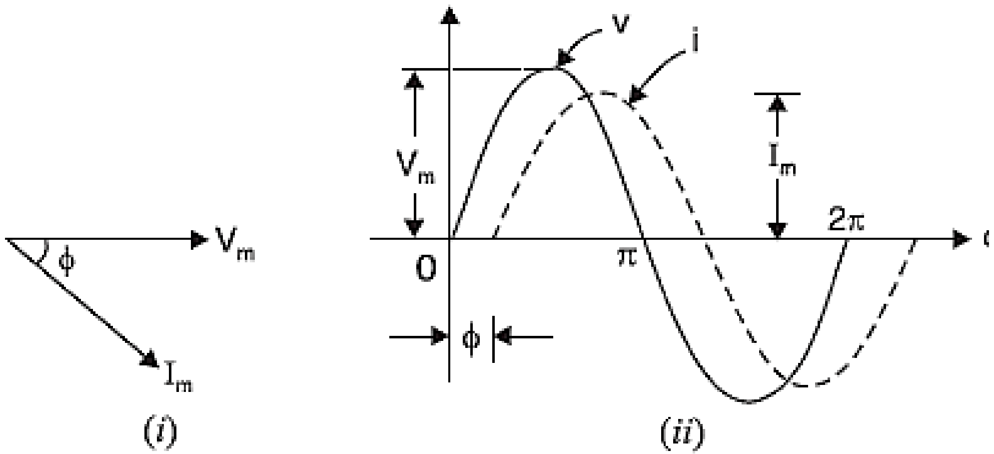


Fig. 3

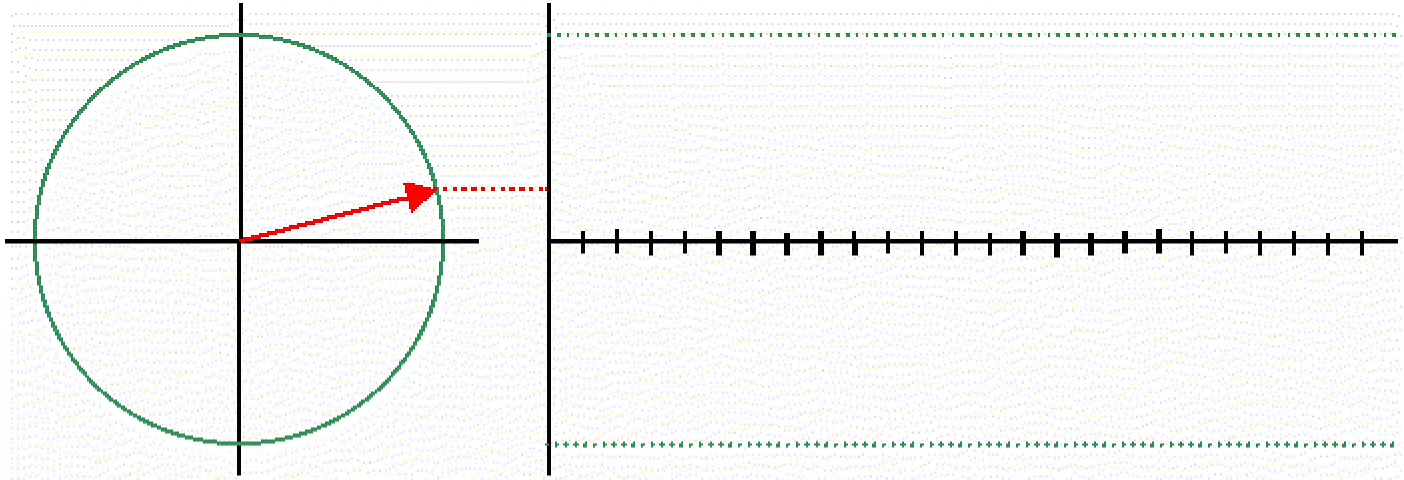
Consider a sinusoidal voltage wave *v* and sinusoidal current wave *i* of the same frequency.
Suppose the current lags behind the voltage by φº. The two alternating quantities can be represented
on the same phasor diagram because the phasors *Vm* and *Im* rotate at the same (as shown in figure 3) angular velocity ω** and hence phase difference φ between them remains the same at all times. When each phasor completes one revolution, it generates the corresponding cycle.
The equations of the two waves can be represented as :

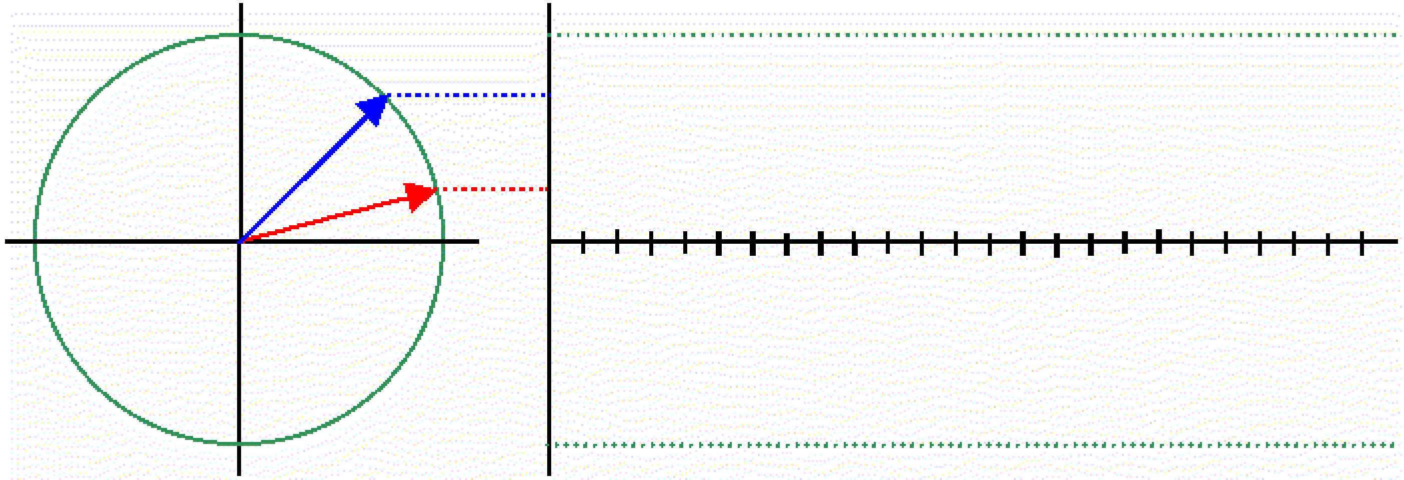
$$\boldsymbol{v = V_m \sin \omega t}$$
$$\boldsymbol{i = I_m \sin (\omega t - \varphi)}$$

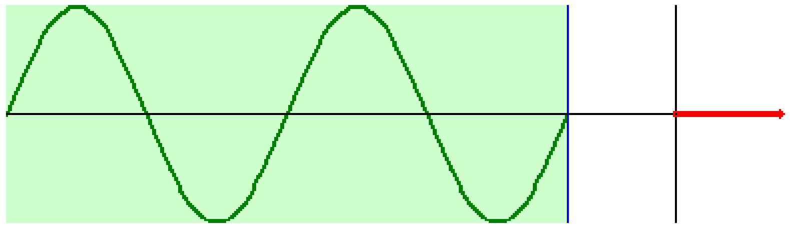
The following points may be noted carefully :

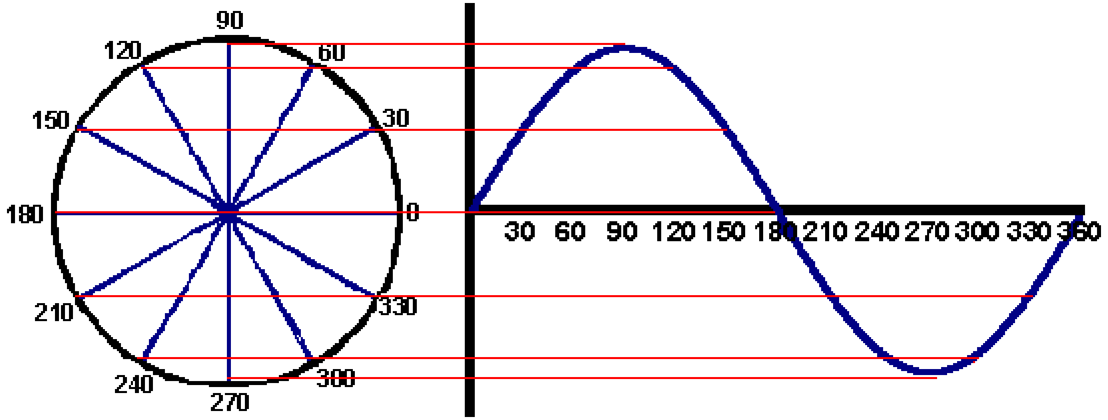
(*i*) The wave diagram and the phasor diagram convey the same information. However, it is more difficult to draw the waves than to sketch the phasor diagram.

(*ii*) Since the two phasors have the same angular velocity (ω) and there is no relative motion between them, they can be displayed in a stationary diagram, the common angular rotation (*ω*t*) being disregarded.





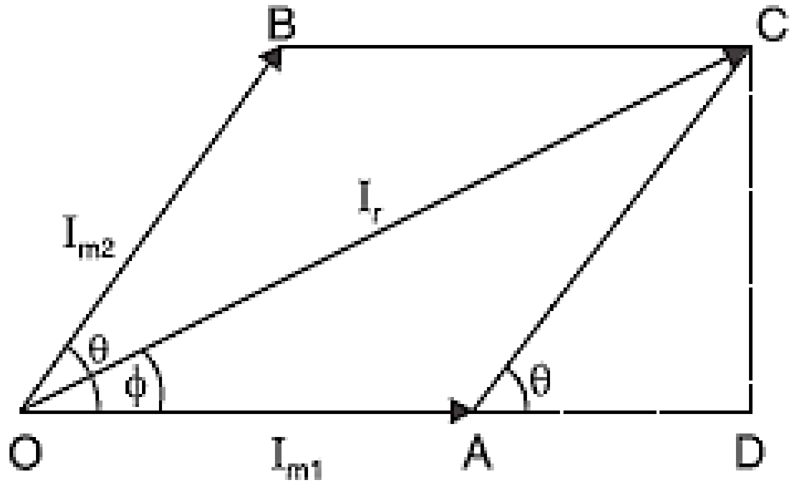




By: Dr. Parvesh Saini

Addition of Alternating Quantities

Alternating voltages and currents are phasors. They are added in the same manner as forces are added. Only phasors of the same kind may be added. Addition of two alternating currents or voltages can be accomplished by one of the famous methods : i.e. **Parallelogram method (or Parallelogram law of vector addition)**



The two phasors *Im*1 and *Im*2 are represented by the adjacent sides *OA* and *OB* respectively of the parallelogram *OACB* [as shown].

The phase difference between the phasors is θ ; *Im*2 leading *Im*1. The maximum value of resultant is *Ir*. It is represented by the diagonal *OC* and leads the phasor *Im*1 by φ.

$$OC = \sqrt{(OD)^2 + (CD)^2} = \sqrt{(I_{m1} + I_{m2}\cos\theta)^2 + (I_{m2}\sin\theta)^2}$$

$$I_r = \sqrt{I^2_{m1} + I^2_{m2} + 2I_{m1} I_{m2} \cos\theta}$$

Also $\tan\phi = \dfrac{CD}{OD} = \dfrac{CD}{OA + AD} = \dfrac{I_{m2}\sin\theta}{I_{m1} + I_{m2}\cos\theta}$

The instantaneous value of resultant current is given by ;

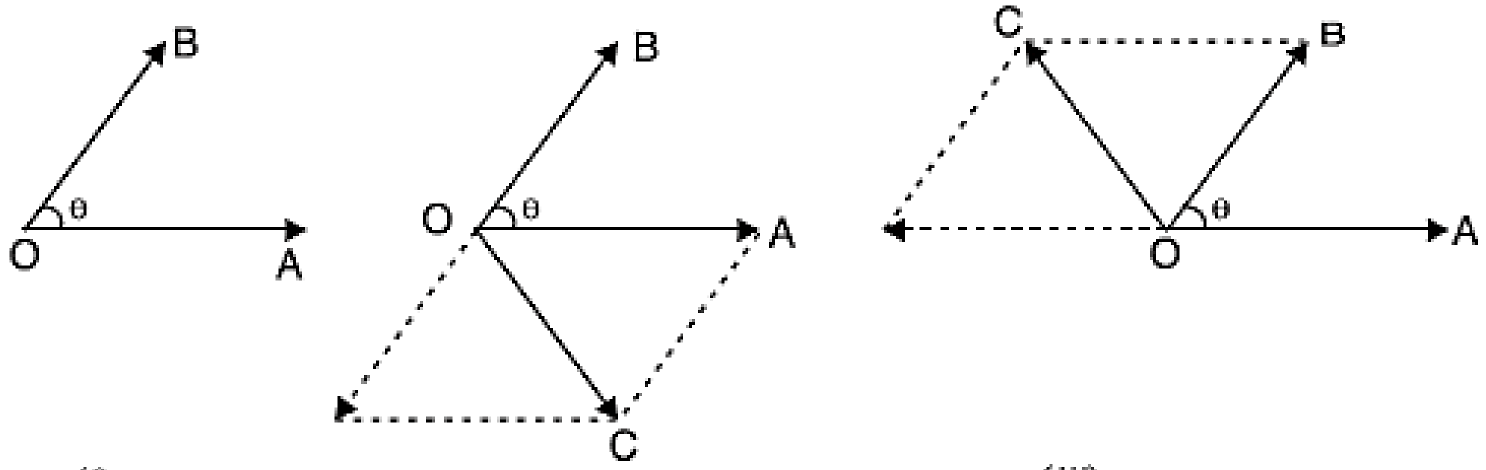
$$ir = Ir \sin(\omega t + \varphi)$$

By: Dr. Parvesh Saini

Subtraction of Alternating Quantities

If difference of two phasors is required, then one of the phasors is reversed and this reversed phasor is then compounded with the other phasor using parallelogram method or method of components.

Consider two phasors *OA* and *OB* representing two alternating quantities of the same kind [See Fig. (*i*)]. The phasor *OB* leads the phasor *OA* by θ. If it is required to subtract the phasor *OB* from *OA*, then *OB* is reversed and is compounded with phasor *OA* as shown in Fig. (*ii*). The phasor difference *OA*–*OB* is given by the phasor *OC*. In Fig. (*iii*), phasor *OC* represents the phasor difference ***OB–OA***.



(*i*) (*ii*) (*iii*)

By: Dr. Parvesh Saini

Thank You