Basic Electrical Engineering (TEE 101)

Lecture 21: Numerical Practice – II (AC Circuits)

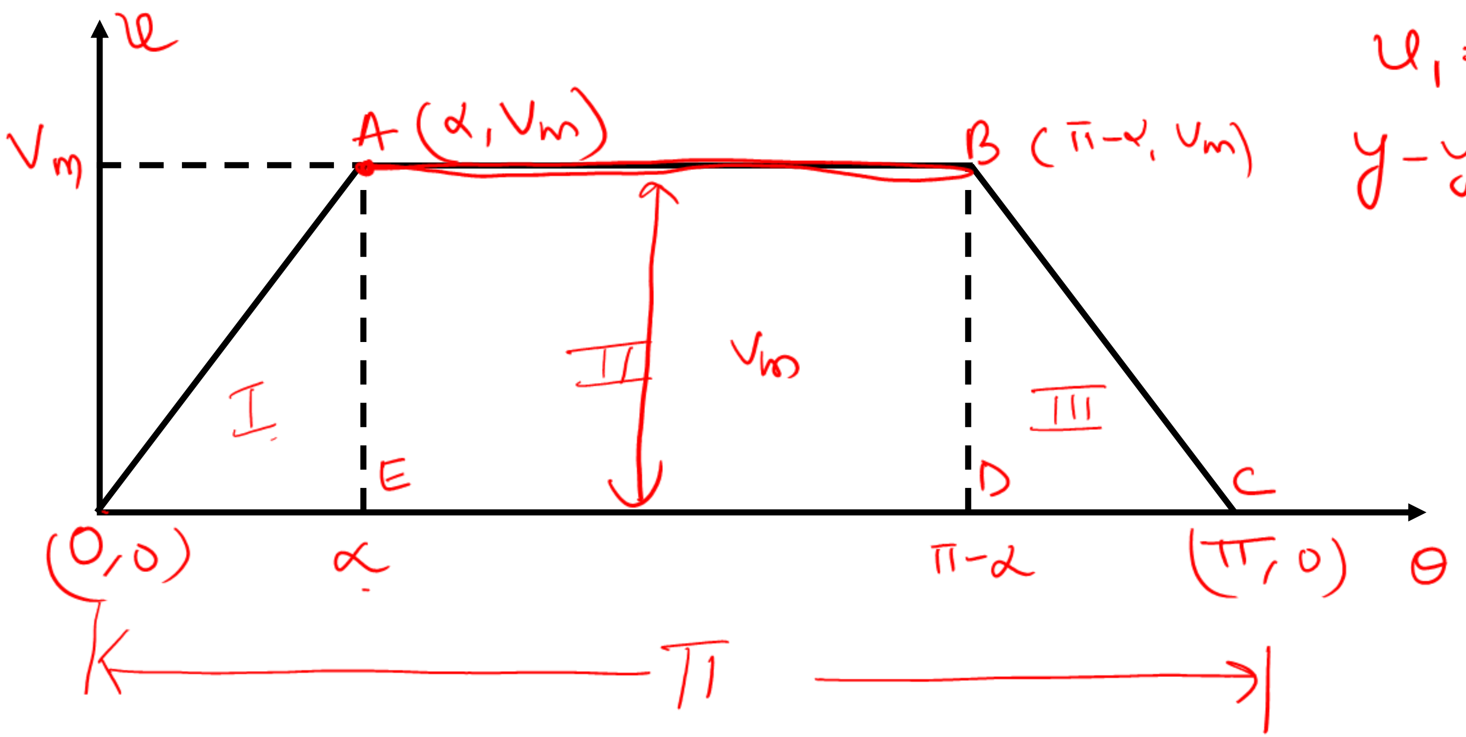
By: Dr. Parvesh Saini

Content

This lecture covers numerical on:

- **RMS and Average values of AC quantities**
- **Solving complex numbers**
- **Converting rectangular form to polar and vice versa**

Q. 1 Determine the average and RMS value of the trapezoidal waveform as shown in figure below:



$v_1 \Rightarrow$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Solution :→ Let v is divided in 3 voltages v_1, v_2 and v_3

v_1 is from $0 \rightarrow \alpha$

v_2 " " $\alpha \rightarrow \pi - \alpha$

v_3 " " $\pi - \alpha \rightarrow \pi$

we can use the eqn of line to determine u_1, u_2 and u_3.

So,

u_1 is

$y_1 = 0 \qquad y_2 = V_m$

$x_1 = 0 \qquad x_2 = \alpha$

$y = u_1$

$x = \theta$

$$u_1 - 0 = \frac{V_m - 0}{\alpha - 0}(\theta - 0)$$

$$\boxed{u_1 = \frac{V_m}{\alpha}\theta} \quad \text{—— (1)}$$

$0 \rightarrow \alpha$

u_2 is

$$\boxed{u_2 = V_m} \quad \text{— (2)}$$

$\alpha \rightarrow \pi - \alpha$

u_3 is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

In this case

$y = u_3$

$x = \theta$

$y_1 = V_m$

$x_1 = \pi - \alpha$

$y_2 = 0$

$x_2 = \pi$

$$u_3 - V_m = \frac{0 - V_m}{\pi - \pi + \alpha}(\theta - \pi + \alpha)$$

$$u_3 - V_m = \frac{-V_m}{\alpha}(\theta - \pi + \alpha)$$

$$\boxed{u_3 = \frac{V_m}{\alpha}(\pi - \theta)} \quad \text{—— (3)}$$

$\pi - \alpha \rightarrow \pi$

The average value of the given waveform can be obtained as

$$V_{av} = \frac{1}{T}\int_0^T u\,dt$$

$$= \frac{1}{\pi}\left[\int_0^{\alpha} u_1\,dt + \int_{\alpha}^{\pi-\alpha} u_2\,dt + \int_{\pi-\alpha}^{\pi} u_3\,dt\right] \quad \text{--- } (4)$$

$$V_{av} = \frac{1}{\pi}\left[\int_0^{\alpha} \frac{V_m}{\alpha}\theta\,d\theta + \int_{\alpha}^{\pi-\alpha} V_m\,d\theta + \int_{\pi-\alpha}^{\pi} \frac{V_m}{\alpha}(\pi-\theta)\,d\theta\right]$$

$$V_{av} = \frac{1}{\pi}\left[\frac{V_m}{\alpha}\left|\frac{\theta^2}{2}\right|_0^{\alpha} + V_m\left|\theta\right|_{\alpha}^{\pi-\alpha} + \frac{V_m}{\alpha}\left|\pi\theta - \frac{\theta^2}{2}\right|_{\pi-\alpha}^{\pi}\right]$$

$$V_{av} = \frac{1}{\pi}\left[\frac{V_m}{2\alpha}(\alpha^2 - 0) + V_m(\pi-\alpha-\alpha) + \frac{V_m}{2\alpha}\left\{\left(2\pi(\pi-\pi+\alpha) - (\pi^2-(\pi-\alpha)^2\right)\right\}\right]$$

$$V_{av} = \frac{1}{\pi}\left[\frac{1}{2}\alpha V_m + (\pi-2\alpha)V_m + \frac{1}{2}\alpha V_m\right] = \frac{1}{\pi}\left[\alpha V_m + \pi V_m - 2\alpha V_m\right]$$

$$\boxed{V_{av} = \left(1 - \frac{\alpha}{\pi}\right)V_m} \quad \text{--- } (5)$$

The RMS value of given waveform can be obtained as:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T u^2\,dt}$$

$$V_{RMS} = \sqrt{\frac{1}{\pi}\left[\int_0^{\alpha} u_1^2\,dt + \int_{\alpha}^{\pi-\alpha} u_2^2\,dt + \int_{\pi-\alpha}^{\pi} u_3^2\,dt\right]} \quad \text{—— (6)}$$

$$V_{RMS}^2 = \frac{1}{\pi}\left[\int_0^{\alpha}\left(\frac{V_m}{\alpha}\theta\right)^2 d\theta + \int_{\alpha}^{\pi-\alpha} V_m^2\,d\theta + \int_{\pi-\alpha}^{\pi}\left[\frac{V_m}{\alpha}(\pi-\theta)\right]^2 d\theta\right] \quad \text{—— (7)}$$

$$V_{RMS}^2 = \frac{1}{\pi}\left[\frac{V_m^2}{\alpha^2}\left|\frac{\theta^3}{3}\right|_0^{\alpha} + V_m^2\left|\theta\right|_{\alpha}^{\pi-\alpha} - \frac{V_m^2}{\alpha^2}\left|\frac{(\pi-\theta)^3}{3}\right|_{\pi-\alpha}^{\pi}\right] \quad \text{—— (8)}$$

$$= \frac{1}{\pi}\left[\frac{V_m^2}{3\alpha^2}(\alpha^3-0) + V_m^2(\pi-\alpha-\alpha) - \frac{V_m^2}{3\alpha^2}(-\alpha^3)\right]$$

$$= \frac{1}{\pi}\left[\frac{2V_m^2\alpha}{3} + V_m^2(\pi-2\alpha)\right] = V_m^2\left[\pi - \frac{4\alpha}{3}\right]$$

So, V_{RMS} is

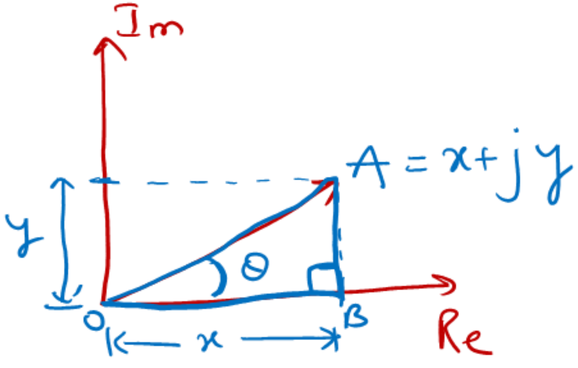
$$V_{RMS} = \sqrt{V_m^2\left(\pi - \frac{4\alpha}{3}\right)}$$

or

$$\boxed{V_{RMS} = V_m\sqrt{\pi - \frac{4\alpha}{3}}} \quad \text{—— (9)}$$

A phasor can be represented as a complex number

$j = \sqrt{-1}$



$$\boxed{A = x + jy}$$ ⇐ Rectangular form

or

⇑

Cartesian form

i) The magnitude of complex number ($|A|$)

ii) its phase (θ) [or phase angle]

If you have a complex number, then, the magnitude can be determined as :

$$|A| = \sqrt{(Re)^2 + (Im)^2}$$

$$|A| = \sqrt{x^2 + y^2} = r \text{ (assume)}$$

The phase angle θ can be obtained as :

$$\tan\theta = \frac{AB}{OB} = \frac{y}{x}$$

$$\boxed{\theta = \tan^{-1}\frac{y}{x}}$$

The given complex number can also be represented as :

$$\boxed{A = |A|\angle\theta = r\angle\theta}$$ Polar representation

In Δ OAB

magnitude

$AB = y = |A| \sin\theta$

$OB = x = |A| \cos\theta$

$\boxed{|A| = r}$

$\boxed{A = x + jy}$

$A = r\cos\theta + j r \sin\theta$

$A = r[\underbrace{\cos\theta + j\sin\theta}_{e^{j\theta}}]$

$\boxed{A = re^{j\theta}}$ $\Leftarrow$ exponential form of the complex number

Rectangular form, $A = x + jy$

Polar form, $A = r\angle\theta$

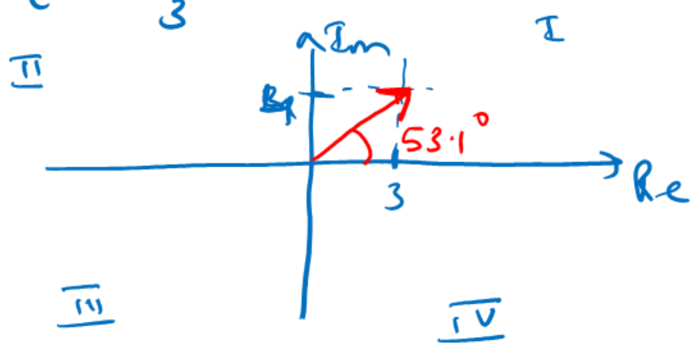
exponential form, $A = re^{j\theta}$

$\boxed{|A| \text{ and } \theta}$

$A = 3 + j4$

$|A| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

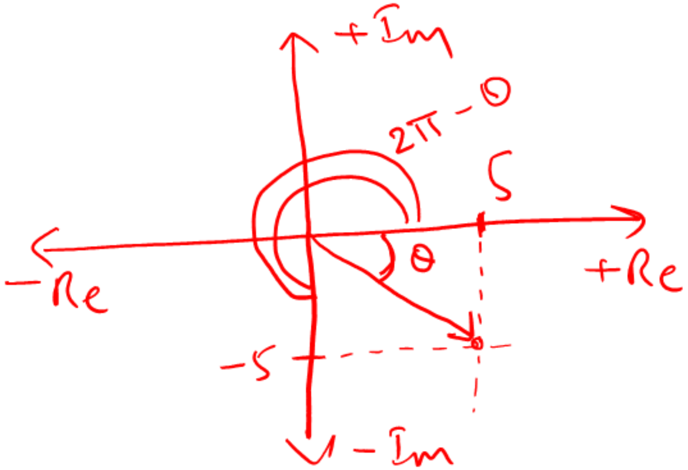
$\theta = \tan^{-1}\frac{4}{3} = 53.1^\circ$



(ii) $A = 5 - j5$ (compare with $x + jy$)

$x = 5$

$y = -5$



A is located in 4th quadrant

$|A| = \sqrt{(5)^2 + (5)^2}$

$|A| = 5\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-5}{5}\right) = -\tan^{-1} 1$

$\theta = -45^\circ$ or 315°

The polar representation of given complex number is

$A = 5\sqrt{2} \angle -45^\circ$

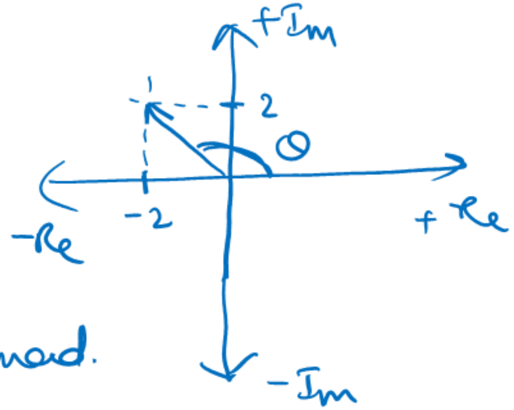
or $A = 5\sqrt{2} \angle 315^\circ$

(iii) $A = -2 + j2$

$A = x + jy$

$x = -2$

$y = 2$



A is located in 2nd Quad.

$|A| = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -\tan^{-1}(1)$

$\theta = 135^\circ$

$A = 2\sqrt{2} \angle 135^\circ$

Thank You