3 – Minimization of DFA

3.1 Minimization of Finite Automata:

- How to test two descriptors of two regular languages define the same language (equivalent)?
- We can any DFA and find an equivalent DFA that has minimum number of states. In fact this DFA is unique.
- Given any two minimum-state DFA's that are equivalent, we can rename the states so that the two DFA's become the same.

3.1.1 Testing equivalence of states:

- When two different states 'p' and 'q' be replaced by a single state that behaves like both 'p' and 'q'.
- We say states 'p' and 'q' are equivalent if
  - For all input strings 'w', both $\hat{\delta}$(p, w) and $\hat{\delta}$(q, w) are accepting states or both are non -accepting states.
  - If two states are not equivalent then they are *distinguishable*.

3.1.2 Table Filling Algorithm:

This algorithm is a recursive discovery of distinguishable pairs in a DFA.

Basis:

If 'p' is an accepting state and 'q' is non accepting then the pair {p, q} is *distinguishable*.

Induction:

- Let 'p' and 'q' be some states.
- Let for some input symbol 'a' $r = \delta(p, a)$ and $s = \delta(q, a)$.
- Suppose the states 'r' and 's' are *distinguishable* then {p, q} is a pair of *distinguishable* states.

The Method:

1. Draw a table for all pairs of states {p, q}.
2. Mark all pairs {p, q} where p ∈ F and q ∉ F. (*distinguishable*)
3. If there are any unmarked pairs {p, q} such that [δ (p, a), δ (q, a)] is marked then mark [p, q], where ‘a’ is an input symbol. REPEAT THIS STEP NO MORE PAIRS OF STATES CAN BE MARKED.

Example: Execute table filling algorithm on the DFA of Figure 3.1 .

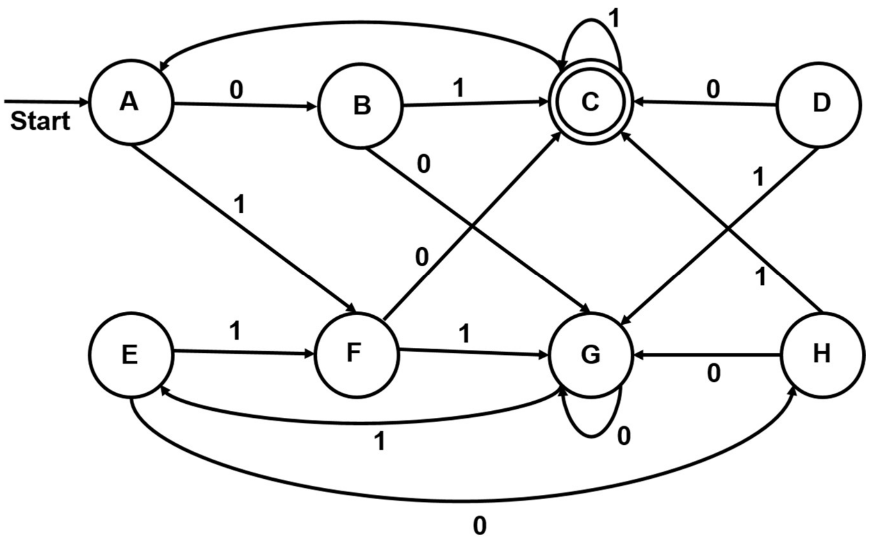

Fig 3.1

1. Initially there are no x’s in the table.
2. For the *basis* since C is the only accepting state, place x’s in each pair that involves C, i.e., the pairs {C, A}, {C, B}, {C, D}, {C, E}, {C, F}, {C, G}, {C, H} are marked as *distinguishable* pairs.

	A	B	C	D	E	F	G
B							
C	x	x					
D			x				
E			x				
F			x				
G			x				
H			x				

3. For the *induction*, now that we know some *distinguishable* pairs, we can discover others.

   Consider the unmarked pairs.

{A, B} δ(A, 0)= B ⎤ The pair {B, G} δ(B, 0)= G ⎦ is unmarked Leave the pair {A, B} unmarked.	δ(A, 1)= F ⎤ The pair {F, C} δ(B, 1)= C ⎦ is marked. Mark the pair {A, B}
{A, D} δ(A, 0)= B ⎤ The pair {B, C} δ(D, 0)= C ⎦ is marked Mark the pair {A, D}	Transition on 1 not required as the symbol 0 distinguishes the states {A, D}.
{A, E} δ(A, 0)= B ⎤ The pair {B, H} δ(E, 0)= H ⎦ is unmarked Leave the pair {A, D} unmarked.	δ(A, 1)= F ⎤ The pair {F, F} δ(E, 1)= F ⎦ is equivalent. Leave the pair {A, D} unmarked.
{A, F} δ(A, 0)= B ⎤ The pair {B, C} δ(F, 0)= C ⎦ is marked Mark the pair {A, F}.	Transition on 1 not required as the symbol 0 distinguishes the states {A, F}.
{A, G} δ(A, 0)= B ⎤ The pair {B, G} δ(G, 0)= G ⎦ is unmarked Leave the pair {A, G} unmarked.	δ(A, 1)= F ⎤ The pair {F, E} δ(G, 1)=E ⎦ is unmarked Leave the pair {A, G} unmarked.
{A, H} δ(A, 0)= B ⎤ The pair {B, G} δ(H, 0)= G ⎦ is unmarked Leave the pair {A, H} unmarked.	δ(A, 1)= F ⎤ The pair {F, C} δ(H, 1)= C ⎦ is marked Mark the pair {A, H}.

Now the table looks like

B	x						
C	x	x					
D	x		x				
E			x				
F	x		x				
G			x				
H	x		x				
	A	B	C	D	E	F	G

{B, D} $\left.\begin{array}{l}\delta(B, 0)= G\\ \delta(D, 0)= C\end{array}\right\}$ The pair {G, C} is marked Mark the pair {B, D}.	Transition on 1 not required as the symbol 0 distinguishes the states {B, D}.
{B, E} $\left.\begin{array}{l}\delta(B, 0)= G\\ \delta(E, 0)= H\end{array}\right\}$ The pair {G, H} is unmarked Leave the pair {B, E} unmarked.	$\left.\begin{array}{l}\delta(B, 1)= C\\ \delta(E, 1)= F\end{array}\right\}$ The pair {C, F} is marked Mark the pair {B, E}
{B, F} $\left.\begin{array}{l}\delta(B, 0)= G\\ \delta(F, 0) = C\end{array}\right\}$ The pair {G, C} is marked. Mark the pair {B, F}.	Transition on 1 not required as the symbol 0 distinguishes the states {B, F}.
{B, G} $\left.\begin{array}{l}\delta(B, 0)= G\\ \delta(G, 0) = G\end{array}\right\}$ The pair {G, G} is equivalent. Leave the pair {B, G} unmarked.	$\left.\begin{array}{l}\delta(B, 1)= C\\ \delta(G, 1) = E\end{array}\right\}$ The pair {C, E} is Marked. Mark the pair {B, G}.
{B, H} $\left.\begin{array}{l}\delta(B, 0)= G\\ \delta(H, 0) = G\end{array}\right\}$ The pair {G, G} is equivalent. Leave the pair {B, H} unmarked.	$\left.\begin{array}{l}\delta(B, 1)= C\\ \delta(H, 1) = C\end{array}\right\}$ The pair {C, C} is equivalent. Leave the pair {B, H} unmarked.

Now the table looks like

B	x						
C	x	x					
D	x	x	x				
E		x	x				
F	x	x	x				
G		x	x				
H	x		x				
	A	B	C	D	E	F	G

{D, E} δ(D, 0)= C The pair {C, H} δ(E, 0)= H is marked. Mark the pair {D, E}.	Transition on 1 not required as the symbol 0 distinguishes the states {D, E}.
{D, F} δ(D, 0)= C The pair {C, C} δ(F, 0)= C is equivalent. Leave the pair {D, F} unmarked.	δ(D, 1)= G The pair {G, G} δ(F, 1)= G is equivalent. Leave the pair {D, F} unmarked
{D, G} δ(D, 0)= C The pair {G, C} δ(G, 0)= G is marked. Mark the pair {D, G}.	Transition on 1 not required as the symbol 0 distinguishes the states {D, G}.
{D, H} δ(D, 0)= C The pair {G, C} δ(H, 0)= G is Marked. Mark the pair {D, H}.	Transition on 1 not required as the symbol 0 distinguishes the states {D, H}.

Now the table looks like

B	x						
C	x	x					
D	x	x	x				
E		x	x	x			
F	x	x	x				
G		x	x	x			
H	x		x	x			
	A	B	C	D	E	F	G

{E, A} δ(E, 0)= H ⎤ The pair {H, B} δ(A, 0)= B ⎦ is unmarked Leave the pair {E, A} unmarked.	δ(E, 1)= F ⎤ The pair {F, F} δ(A, 1)= F ⎦ is equivalent. Leave the pair {E, A} unmarked.
{E, F} δ(E, 0)= H ⎤ The pair {C, H} δ(F, 0)= C ⎦ is marked Mark the pair {E, F}.	Transition on 1 not required as the symbol 0 distinguishes the states {E, F}.
{E, G} δ(E, 0)= H ⎤ The pair {H, G} δ(G, 0) = G⎦ is unmarked. Leave the pair {E, G} unmarked.	δ(E, 1)= F ⎤ The pair {F, E} δ(G, 1) = E⎦ is marked. Mark the pair {E, G}.
{E, H} δ(E, 0)= H ⎤ The pair {H, G} δ(H, 0) = G⎦ is unmarked. Leave the pair {E, H} unmarked.	δ(E, 1)= F ⎤ The pair {F, C} δ(H, 1) = C⎦ is marked. Mark the pair {E, H}.

Now the table looks like

	A	B	C	D	E	F	G
B	x						
C	x	x					
D	x	x	x				
E		x	x	x			
F	x		x		x		
G		x	x	x	x		
H	x		x	x	x		

{F, B} $\left.\begin{array}{l}\delta(F, 0)= C\\ \delta(B, 0)= G\end{array}\right\}$ The pair {C, G} is marked Mark the pair {F, B}.	Transition on 1 not required as the symbol 0 distinguishes the states {F, B}.
{F, G} $\left.\begin{array}{l}\delta(F, 0)= C\\ \delta(G, 0)= G\end{array}\right\}$ The pair {G, C} is marked Mark the pair {F, G}.	Transition on 1 not required as the symbol 0 distinguishes the states {F, G}.
{F, H} $\left.\begin{array}{l}\delta(F, 0)= C\\ \delta(H, 0) = G\end{array}\right\}$ The pair {C, G} is marked. Mark the pair {F, H}.	Transition on 1 not required as the symbol 0 distinguishes the states {F, H}.

Now the table looks like

	A	B	C	D	E	F	G
B	x						
C	x	x					
D	x	x	x				
E		x	x	x			
F	x	x	x		x		
G		x	x	x	x	x	
H	x		x	x	x	x	

{G, A} δ(G, 0)= G ⎫ The pair {G, B} δ(A, 0) = B ⎭ is marked. Mark the pair {G, A}.	Transition on 1 not required as the symbol 0 distinguishes the states {G, A}.
{G, H} δ(G, 0)= G ⎫ The pair {G, G} δ(H, 0) = G ⎭ is equivalent. Leave the pair {H, G} unmarked.	δ(G, 1)= E ⎫ The pair {E, C} δ(H, 1)= C ⎭ is marked. Mark the pair {G, H}.

Now the table looks like

B	x						
C	x	x					
D	x	x	x				
E		x	x	x			
F	x	x	x		x		
G	x	x	x	x	x	x	
H	x		x	x	x	x	x
	A	B	C	D	E	F	G

Try to mark the remaining unmarked pairs {A, E}, {B, H} and {D, F}

{A, E} δ(A, 0)= B ⎫ The pair {B, H} δ(E, 0) = H ⎭ is unmarked. Leave the pair {A, E} unmarked.	δ(A, 1)= F ⎫ The pair {F, F} δ(E, 1)= F ⎭ is equivalent. Leave the pair {A, E} unmarked.
{B, H} δ(B, 0)= G ⎫ The pair {G, G} δ(H, 0)= G ⎭ is equivalent. Leave the pair {B, H} unmarked.	δ(B, 1)= C ⎫ The pair {C, C} δ(H, 1)= C ⎭ is equivalent. Leave the pair {B, H} unmarked.
{D, F} δ(D, 0)= C ⎫ The pair {C, C} δ(F, 0)= C ⎭ is equivalent. Leave the pair {D, F} unmarked.	δ(D, 1)= G ⎫ The pair {G, G} δ(F, 1)= G ⎭ is equivalent. Leave the pair {D, F} unmarked.

Since NO MORE PAIRS OF STATES CAN BE MARKED STOP.

The pairs {A, E}, {B, H}, {D, F} are not marked i.e., the states A and E are equivalent, B and H are equivalent also the states D and F are equivalent.

Hence each of these pairs can be merged into one state.

The minimized DFA is

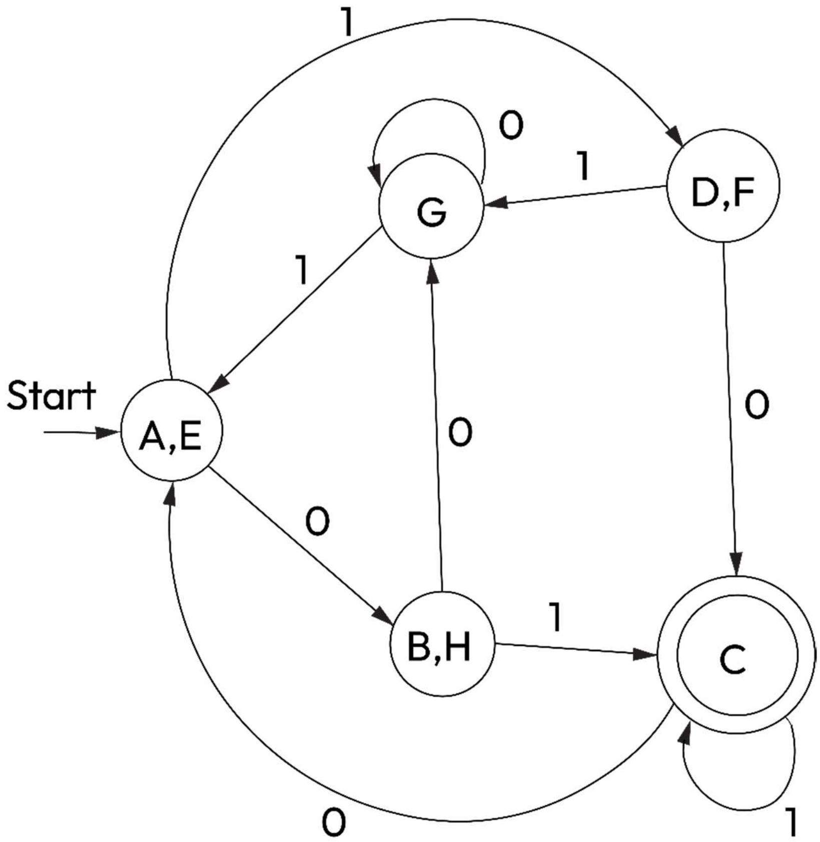

Fig 3.2