Basic Electrical Engineering (TEE 101)

Lecture 27: Power in AC circuits and Series RLC Circuit

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Content

This lecture covers:

- **Power in general AC circuit**
- **R-L-C Series A.C. Circuit**
- **Impedance Triangle of series RLC circuit**

Power in general AC circuit

In this, we will derive the expression of active power in any general AC circuit.

Generally, most of our loads are inductive in nature, and hence, assuming the circuits to be inductive, we will derive the expression of power.

Consider the general case when the phase difference between voltage and current is φ. If the current lags the voltage by φ, the voltage and current may be written as:

$$\upsilon = V_m \sin\theta \qquad i = I_m \sin(\theta - \phi)$$

Where, Vm and Im are the peak values of the applied voltage and current

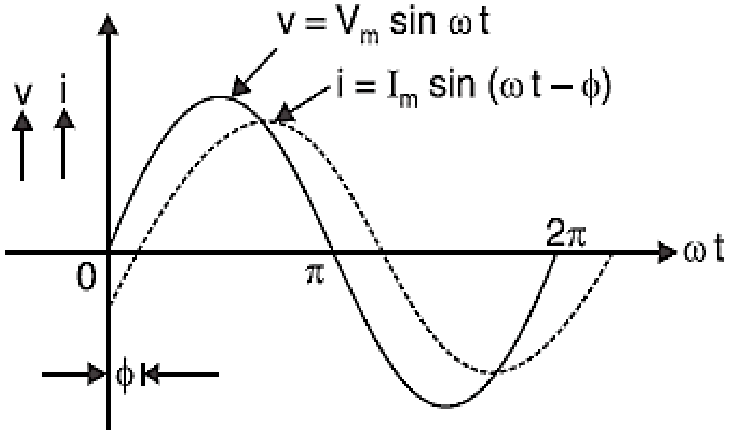

Fig. 3 Wave Diagram of a general AC circuit (Inductive)

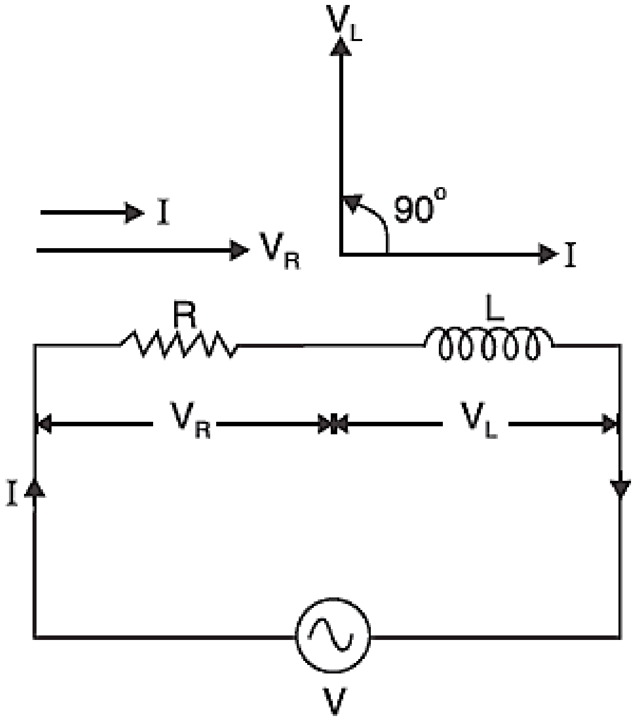

Fig. 1 circuit diagram

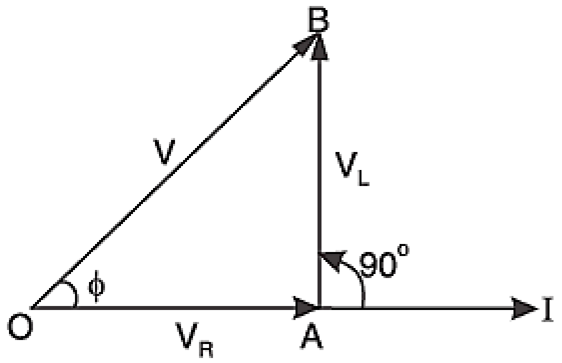

Fig. 2 Phasor Diagram

The instantaneous power is given by:

$p = vi$

$p = (V_m \sin\theta)(I_m \sin(\theta - \varphi))$

$p = V_m I_m (\sin\theta \sin(\theta - \varphi))$ -------- (1)

$$p = V_m I_m \left[\frac{\cos\phi - \cos(2\theta - \phi)}{2} \right] \qquad \text{-------- (2)}$$

$$\left(\because \sin\alpha \sin\beta = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta) \right)$$

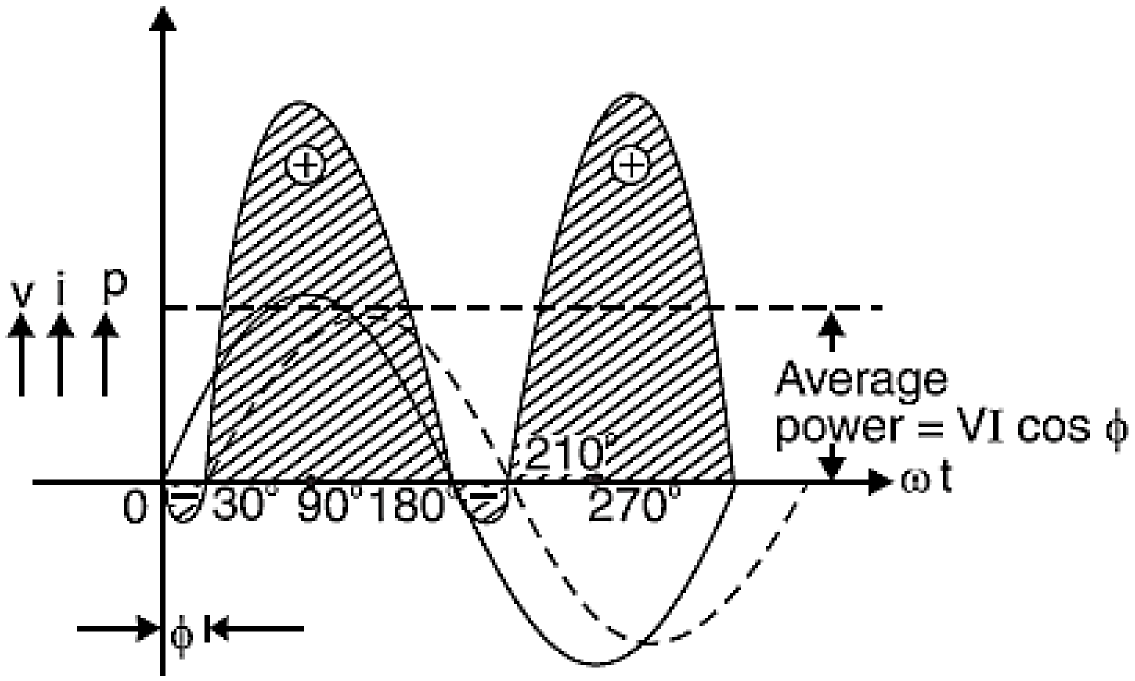

Fig.4 Power curve for a general AC circuit

The active power can be calculated by using the following expression:

$$P = \frac{1}{2\pi} \int_0^{2\pi} p\, d\theta$$

$$P = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \left[\frac{\cos\phi - \cos(2\theta - \phi)}{2} \right] d\theta$$

$$P = \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos\phi\, d\theta - \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos(2\theta - \phi)\, d\theta$$

$$P = \frac{V_m I_m}{4\pi} \cos\phi \left| \theta \right|_0^{2\pi} - \frac{V_m I_m}{4\pi} \left| \frac{1}{2} \sin(2\theta - \phi) \right|_0^{2\pi}$$

$$P = \frac{V_m I_m}{4\pi}(2\pi)(\cos\phi) - \frac{V_m I_m}{8\pi}\left[\sin(4\pi - \phi) - \sin(-\phi)\right]$$

$$P = \frac{V_m I_m}{2}(\cos\phi) - \frac{V_m I_m}{8\pi}\left[-\sin(\phi) + \sin(\phi)\right] \qquad \left(\because \sin(4\pi - \phi) = -\sin\phi\right)$$

$$P = \frac{V_m I_m}{2}(\cos\phi)$$

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)(\cos\phi) \qquad \left(\because \frac{V_m}{\sqrt{2}} = V\,(\text{r.m.s voltage})\right) \qquad \left(\because \frac{I_m}{\sqrt{2}} = I\,(\text{r.m.s current})\right)$$

$$P = VI\cos\phi \qquad \text{-------- (3)}$$

Equation (3) gives the expression for active power in general AC circuit

Even, if the AC circuit is capacitive, then also the expression for active power would be same as given by equation (3)

In, equation (3), cosφ is known as Power Factor.

From figure (4), shaded negative area represents the energy returned from the circuit to the source. The shaded positive area during this interval represents the energy supplied from the source to the load. Hence, it is observed that during each current (or voltage) cycle a part of the energy called active energy is consumed, while the other part called the reactive energy is interchanged between the source and the load. The difference between the total positive and total negative areas during a cycle of current (or voltage) gives the net active energy of the circuit.

R-L-C series AC circuit

This is a general series a.c. circuit. Figure 5 shows R, L and C connected in series across a supply voltage V (r.m.s.). The resulting circuit current is I (r.m.s.).

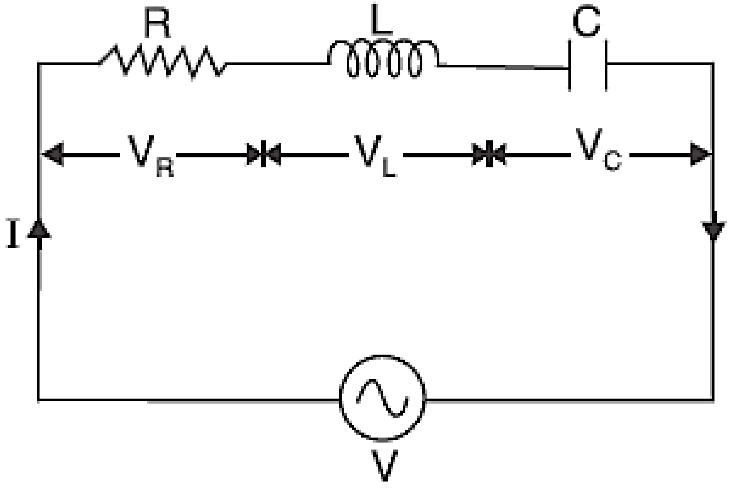

Fig. 5 circuit diagram

$V_R = I\,R$ .... where V_R is in phase with I

$V_L = I\,X_L$ .... where V_L leads I by 90°

$V_C = I\,X_C$ .... where V_C lags I by 90°

Instantaneous voltage is $v = V_m \sin \omega t$,

Instantaneous current is $i = I_m \sin(\omega t \pm \varphi)$

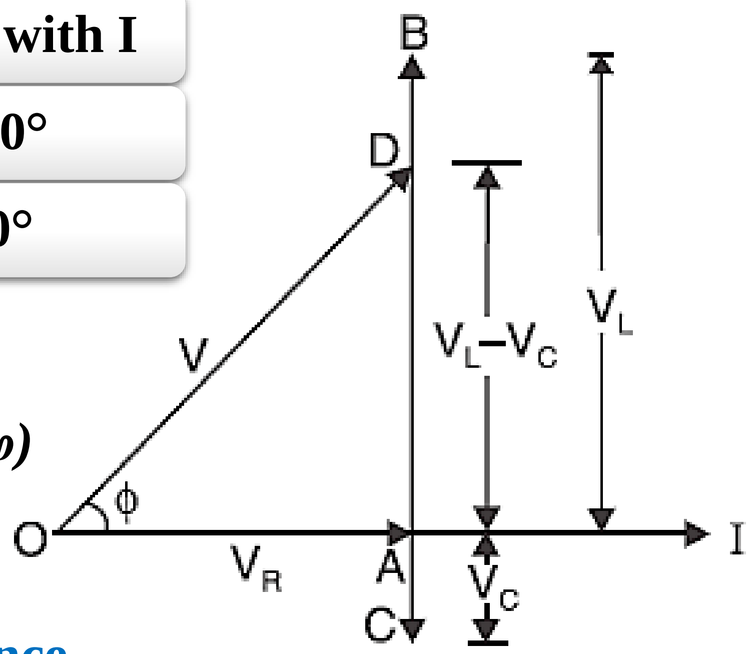

Fig. 6 Phasor Diagram

As before, the phasor diagram is drawn taking current as the reference phasor. In the phasor diagram (See Figure 6), *OA* represents V_R, *AB* represents V_L and *AC* represents V_C.

It may be seen that V_L is in phase opposition to V_C.

It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. For the case considered, $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ and is represented by AD. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and is represented by OD.

$$V = V_R + j\left(V_L - V_C\right)$$

$$magnitude,\ |V| = \sqrt{\left(V_R\right)^2 + \left(V_L - V_C\right)^2} = \sqrt{\left(IR\right)^2 + \left(IX_L - IX_C\right)^2}$$

$$|V| = I\sqrt{\left(R\right)^2 + \left(X_L - I_C\right)^2}$$

$$I = \frac{V}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

The quantity $\sqrt{R^2 + \left(X_L - X_C\right)^2}$ offers opposition to current flow and is called **impedance (Z)** of the circuit. Measured in ohms

Circuit power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$ -------- ***(4)***

Also, $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{X_L - X_C}{R}$ -------- ***(5)***

$$\left(\text{Impedance}\right) Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Since X_L, X_C and R are known, phase angle ϕ of the circuit can be determined.

Power consumed, $P = V\,I\cos\phi$

$cos\varphi = \dfrac{R}{Z}$ and $V = IZ$

Hence, power consumed is: $P = (IZ)\,I\left(\dfrac{R}{Z}\right)$

$\boldsymbol{P = (I^2R)}$ **(This is expected because there is no power loss in L or C)**

Three cases of *R-L-C* series circuit. We have seen that the impedance of a *R-L-C* series circuit is given by ;

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

(*i*) When $X_L - X_C$ is positive (*i.e.* $X_L > X_C$), phase angle φ is positive and the circuit will be inductive. In other words, in such a case, the circuit current *I* will lag behind the applied voltage *V* by φ ; the value of φ being given by eq. (5) above.

(*ii*) When $X_L - X_C$ is negative (*i.e.* $X_C > X_L$), phase angle φ is negative and the circuit is capacitive. That is to say the circuit current *I* leads the applied voltage *V* by φ ; the value of φ being given by eq. (5) above.

(*iii*) When $X_L - X_C$ is zero (*i.e.* $X_L = X_C$), the circuit is purely resistive. In other words, circuit current *I* and applied voltage *V* will be in phase *i.e.* φ = 0°. The circuit will then have unity power factor.

The value of φ will be positive or negative depending upon which reactance (X_L or X_C) predominates

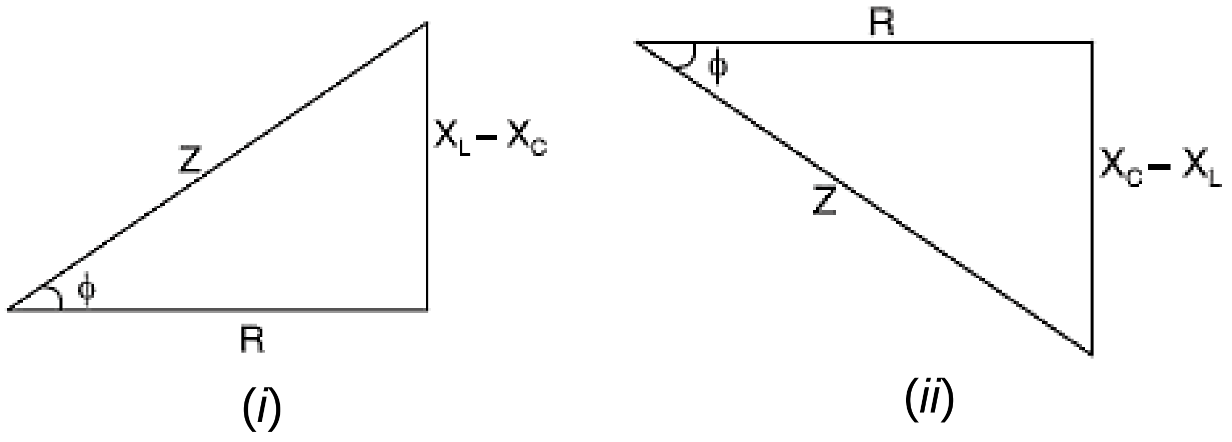

Figure 7 (*i*) shows the impedance triangle of the circuit for the case when $X_L > X_C$
Whereas figure 7 (ii) represents the impedance triangle for the case when $X_C > X_L$.

Fig. 7 Impedance Triangles

Thank You