# Basic Electrical Engineering (TEE 101)

## Lecture 14: Maximum Power Transfer Theorem

# Content

This lecture covers:

- Introduction to Maximum Power Transfer Theorem
- Proof of Maximum Power Transfer Theorem
- Steps to solve a network using Maximum Power Transfer Theorem

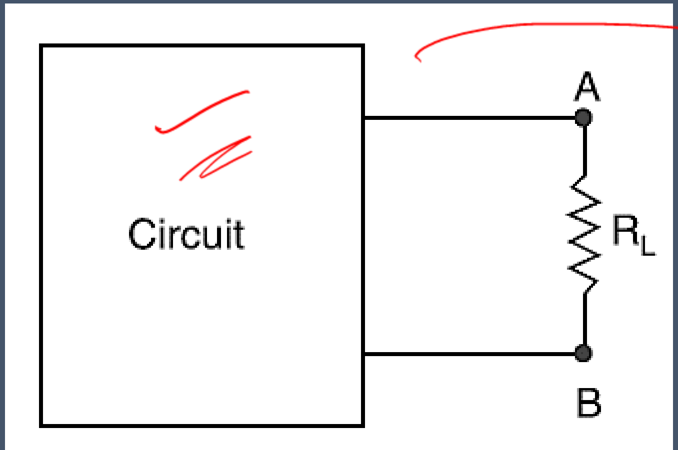
# Maximum Power Transfer Theorem - Introduction

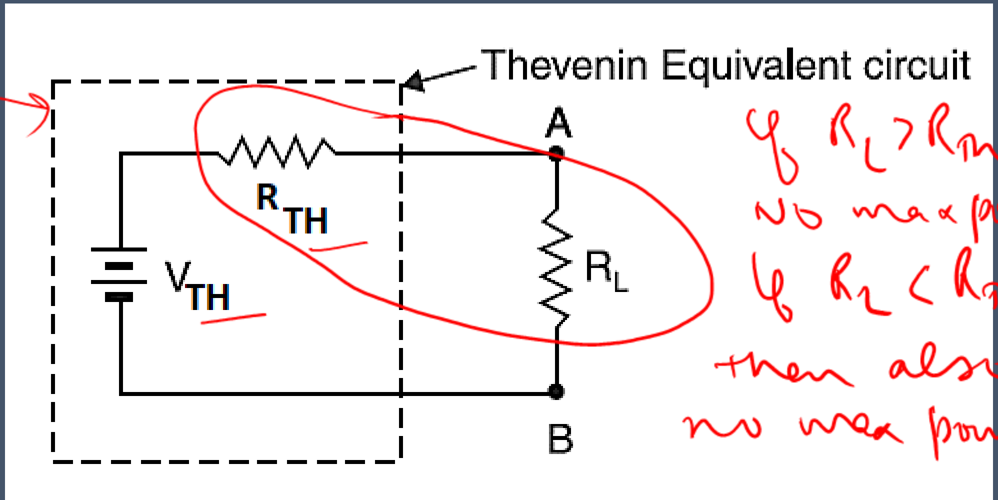
This theorem deals with transfer of maximum power from a source to load and may be stated as under :

*In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the Thevenin's Resistance of the Network"*

(Internal Resistance)

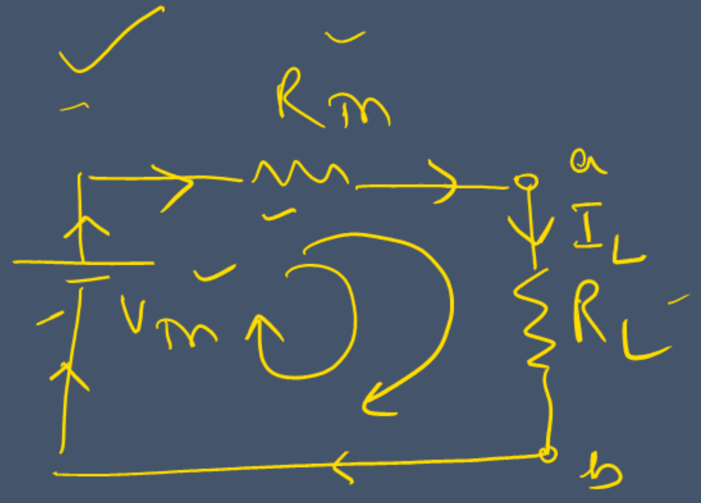
**i.e. $R_L = R_{TH}$**




If $R_L > R_{TH}$
No max power
If $R_L < R_{TH}$
then also
no max power

# Proof of Maximum Power Transfer Theorem




$R_{Th}$ is fixed

$R_L$ is variable

The condition of $R_L = R_{Th}$ is now proved here:

The power dissipated by load is

$$P_L = I_L^2 R_L \quad \text{--- (1)} \qquad (\because P = I^2 R)$$

The load current, $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \quad \text{--- (2)}$$

Condition for maxima; $\boxed{\dfrac{dP_L}{dR_L} = 0}$

$$\frac{dP_L}{dR_L} = (V_{Th})^2 \left[ \frac{(R_{Th}+R_L)^2 \times 1 - R_L \times 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4} \right] = 0$$

or, $(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L) = 0$

or, $(R_{Th}+R_L)\left[R_{Th}+R_L-2R_L\right] = 0$

or, $\underbrace{(R_{Th}+R_L)}(R_{Th}-R_L) = 0$

$R_{Th}+R_L \neq 0$

hence;

$R_{Th} - R_L = 0$

or $\boxed{R_L = R_{Th}}$

This proves that the max. power is dissipated by the load when $\underline{R_L = R_{Th}}$

## Efficiency of the N/W

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{P_L}{P_S}$$

$P_L = I_L^2 R_L$ and $P_S = I_L^2 (R_{Th} + R_L)$

$$\eta = \frac{\cancel{I_L^2} R_L}{\cancel{I_L^2} (R_{Th} + R_L)} = \frac{R_L}{R_{Th} + R_L}$$

at $R_L = R_{Th}$

$$\eta = \frac{R_L}{R_L + R_L} \left[ \text{or } \frac{R_{Th}}{R_{Th} + R_{Th}} \right] = \frac{R_L}{2R_L} = \frac{1}{2} \; (= 50\,\%)$$

Max. power does not mean Max. efficiency

$P_L = I_L^2 R_L$

a) if $R_L = 0$

then, $P_L = 0$

b) if $R_L = \infty$

then, $I_L = 0$

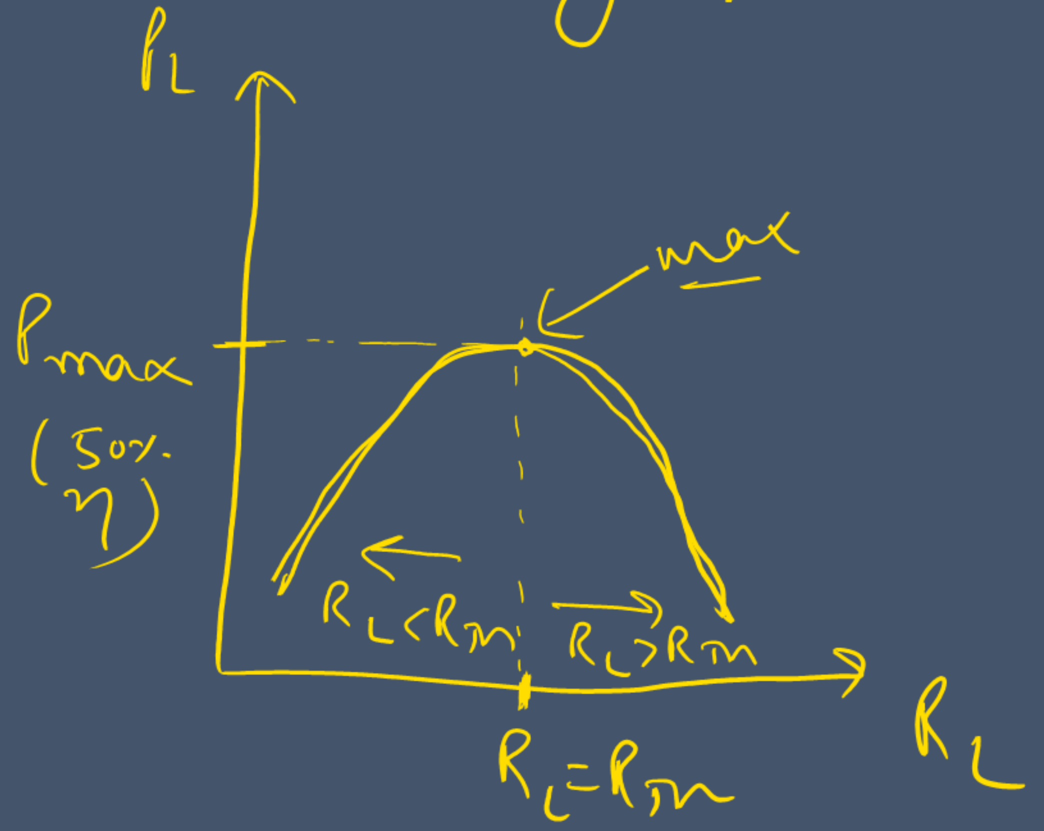
hence, $P_L = 0$

From this we have seen that the power is minimum for the two extreme cases of $R_L$. Hence, to justify the statement of MPT theorem, we can say that the max. power is transferred by the source to load (or the max power is dissipated by the load) is when we have some value of $R_L$ in between $0$ & $\infty$.

if $R_L \uparrow$ $P_L \uparrow$ But $I_L \downarrow$ which affects $P_L$

if $R_L \downarrow$ $P_L \downarrow$ but $I_L \uparrow$ " " "

Let us plot the graph between $P_L$ and $R_L$.




what is the max. power dissipated by the load?

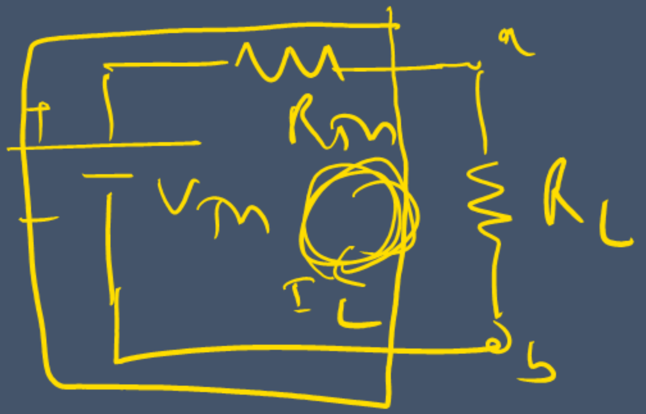
$$P_L = I_L^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

at $R_L = R_{Th}$

$$P_{L\,max} \left(\text{or } P_{max}\right) = \frac{V_{Th}^2}{(R_L+R_L)^2} R_L$$

$$\boxed{P_{L\,max} = \frac{V_{Th}^2}{4R_L}}$$

Now, let us see the max power delivered by the source to load.




$$P_s = I_L^2 R_{Th}$$

$$= \frac{V_{Th}^2}{(R_{Th}+R_L)^2} \times R_{Th}$$

at $R_L = R_{Th}$

$$P_{max}(\text{delivered}) = \frac{V_{Th}^2}{(R_{Th}+R_{Th})^2} \times R_{Th}$$

$$P_{max}(\text{del.}) = \frac{V_{Th}^2}{4R_{Th}}$$

How to apply MPT theorem?

1. Determine the Thevenin's Equivalent of the given circuit.

2. at $R_L = R_{th}$, Calculate the load current

3. Calculate the value $P_{L\,max} \left( = \frac{V_{th}^2}{4R_L} \right)$ or also the power delivered by source at $R_L = R_{th}$ i.e $P_{max} = \frac{V_{th}^2}{4R_{th}}$

# Thank You