Basic Electrical Engineering (TEE 101)

Lecture 16: Values of Alternating Voltage and Current

Content

This lecture covers:

- Instantaneous Value
- Peak Value
- Average Value
- R.M.S. or Effective Value

Values of Alternating Voltage and Current

In a d.c. system, the voltage and current are constant so that there is no problem of specifying their magnitudes.

However, an alternating voltage or current varies from instant to instant.

A natural question arises how to express the magnitude of an alternating voltage or current.

There are five ways of expressing it, namely ;

- **Instantaneous Value**
- **Peak Value (or Maximum Value)**
- **Peak to Peak Value**
- **Average Value**
- **R.M.S Value (or Effective Value)**

Instantaneous Value

The value of alternating quantity (may be Alternating voltage or Alternating Current) at particular instant of time of the cycle is termed as Instantaneous Value.

There are numerous instantaneous values in alternating quantity in a cycle.

This can be understood from the diagram shown in figure 1.

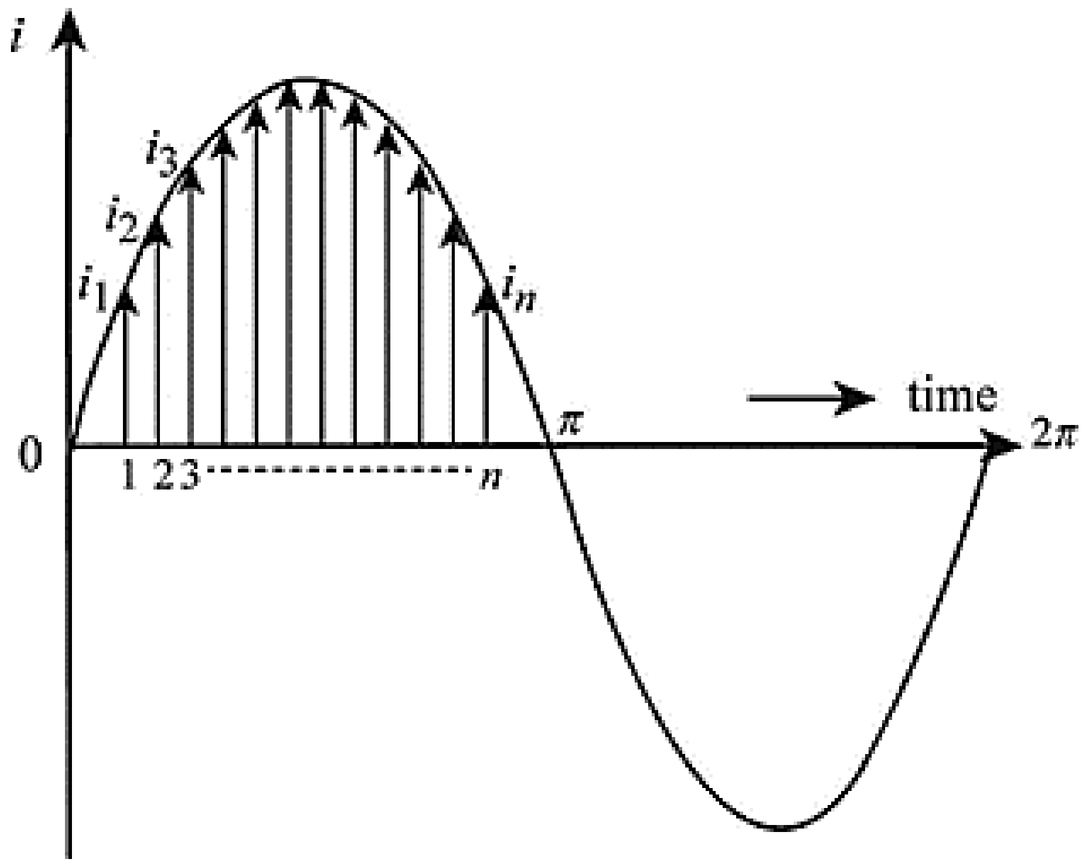


Figure 1

Peak Value

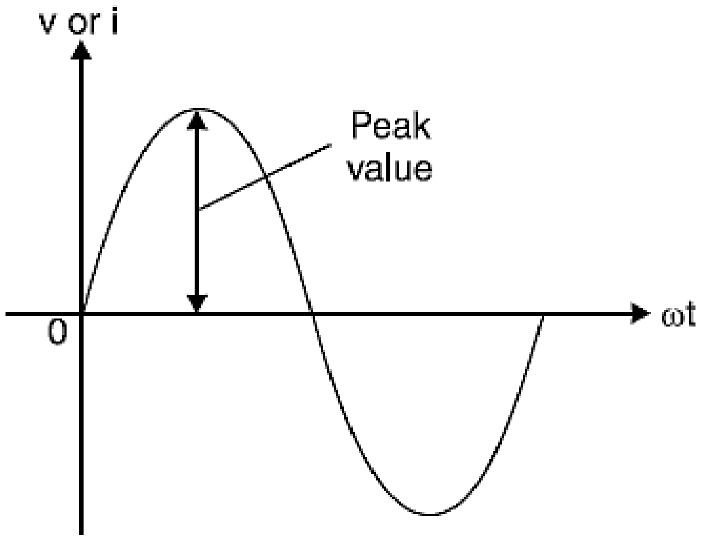
It is the maximum value attained by an alternating quantity.

The peak or maximum value of an alternating voltage or current is represented by *Vm* or *Im*.

The knowledge of peak value is important in case of testing materials.

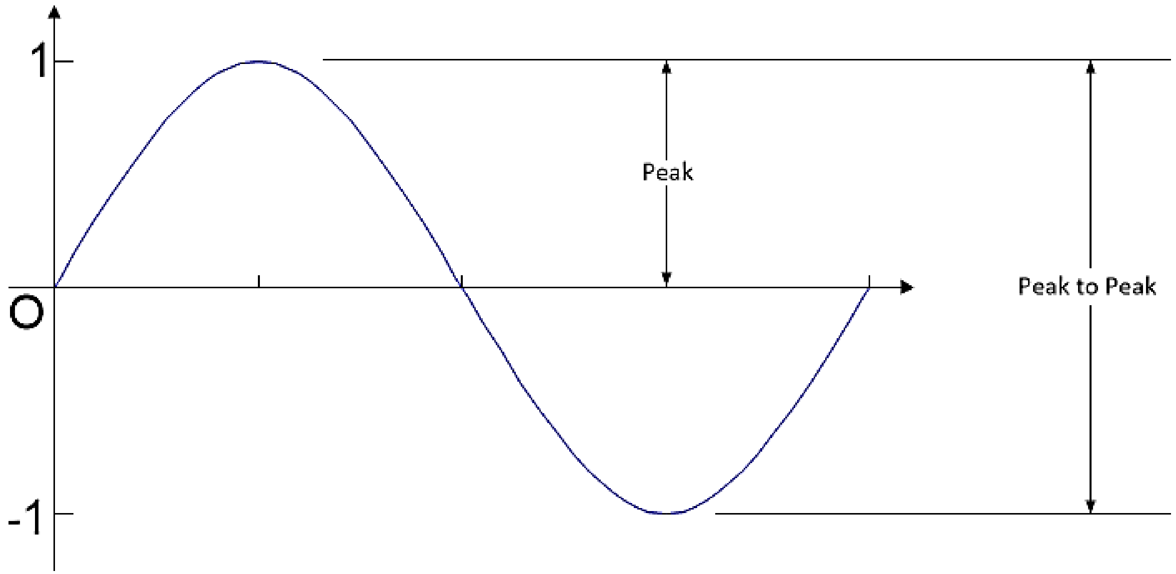
However, peak value is not used to specify the magnitude of alternating voltage or current.

Instead, we generally use r.m.s. values to specify alternating voltages and currents.



Peak to Peak Value

It is the difference between the positive peak and negative peak of an alternating signal over a complete cycle.



Average Value of an alternating quantity

The average value of a waveform is the average of all its values over a period of time.

The average value can be computed by adding all the magnitude of all instantaneous values over one half cycle and dividing the sum with the number of instants.

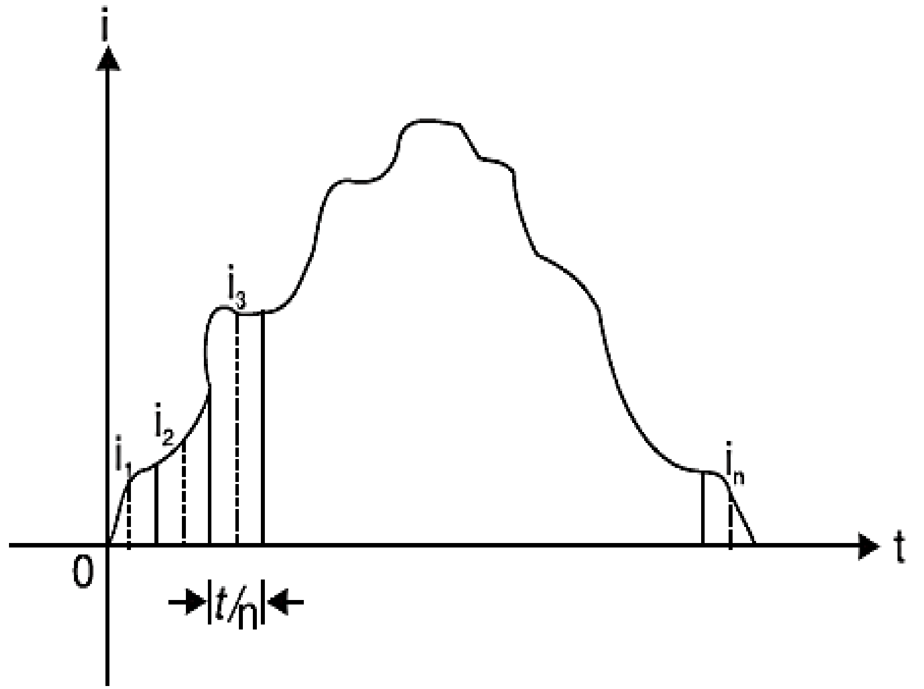
The average value of a symmetric alternating signal (such as sinusoidal signal) over its entire cycle is ZERO.

Because the average value obtained for one half cycle is a positive and for another half cycle is a negative.

Average value can be determined by the following mathematical relationship:

$$\text{Average Value} = \frac{\text{Total (net) area under the curve}}{\text{Base Length}}$$

This can be understood through figure as shown here.



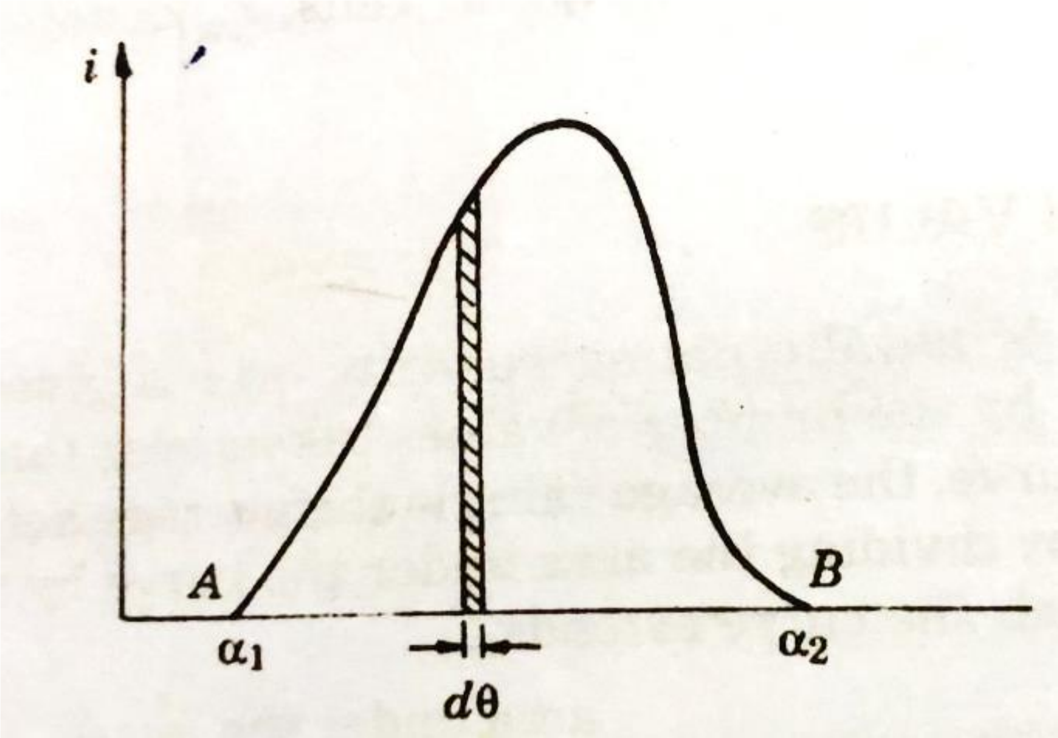
$$\textit{Average Value} = \frac{i_1 + i_2 + i_3 + \ldots i_n}{n}$$

Where, i_1, i_2, i_3 …… i_n are the mid-ordinate values of alternating current

Average Value of alternating quantity through Method of Integration

- The average value of an alternating signal can also be calculated through the "Method of Integration".
- This is explained below:
- In this method, the curve is integrated for infinite number of mid-ordinates.
- To understand, let us consider a strip of width ***dθ*** (as shown by shaded area in the curve)
- The area of the strip dA = i(dθ)
- The total area under the curve can be obtained by integrating the area of strip dA over the range α_1 to α_2 (i.e. from $\theta = \alpha_1$ to $\theta = \alpha_2$)
- Hence the base length of the curve is

$$AB = \alpha_2 - \alpha_1$$



Total area under the curve is $$A = \int_{\alpha_1}^{\alpha_2} dA = \int_{\alpha_1}^{\alpha_2} i d\theta$$

- The average value of the given curve is the average height of the curve.

$$\text{Hence, the average height of the curve} = \frac{\text{Total area under the cuve}}{\text{base length of the curve}} = \frac{\int_{\alpha_1}^{\alpha_2} i d\theta}{\alpha_2 - \alpha_1}$$

$$= \frac{1}{(\alpha_2 - \alpha_1)} \int_{\alpha_1}^{\alpha_2} i d\theta$$

Let, $\alpha_2 - \alpha_1$ is the time period (T) of the alternating signal (current and/or voltage). Then the average value of alternating current "i" or "v" over the period "T" is given as

$$I_{avg} = \frac{1}{T} \int_0^T i dt$$

$$V_{avg} = \frac{1}{T} \int_0^T v dt$$

Where, "i" and "v" are the instantaneous values of the alternating current and voltage respectively

Root Mean Square (R.M.S) Value

The R. M. S value is also known as "Effective Value" (because it is this value which tells the energy transfer capability of a.c. source.)

The average value cannot be used to specify a sinusoidal voltage or current.

It is because its value over one-cycle is zero and cannot be used for power calculations.

Therefore, we must search for a more suitable criterion to measure the effectiveness of an alternating current (or voltage).

The obvious choice would be to measure it in terms of direct current that would do work (or produce heat) at the same average rate under similar conditions.

This equivalent direct current is called the root-mean-square (r.m.s.) or effective value of alternating current.

The **effective or r.m.s. value** *of an alternating current is that steady current* (d.c.) *which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.*

For example, when we say that the r.m.s. or effective value of an alternating current is 5A, it means that the alternating current will do work (or produce heat) at the same rate as 5A direct current under similar conditions.

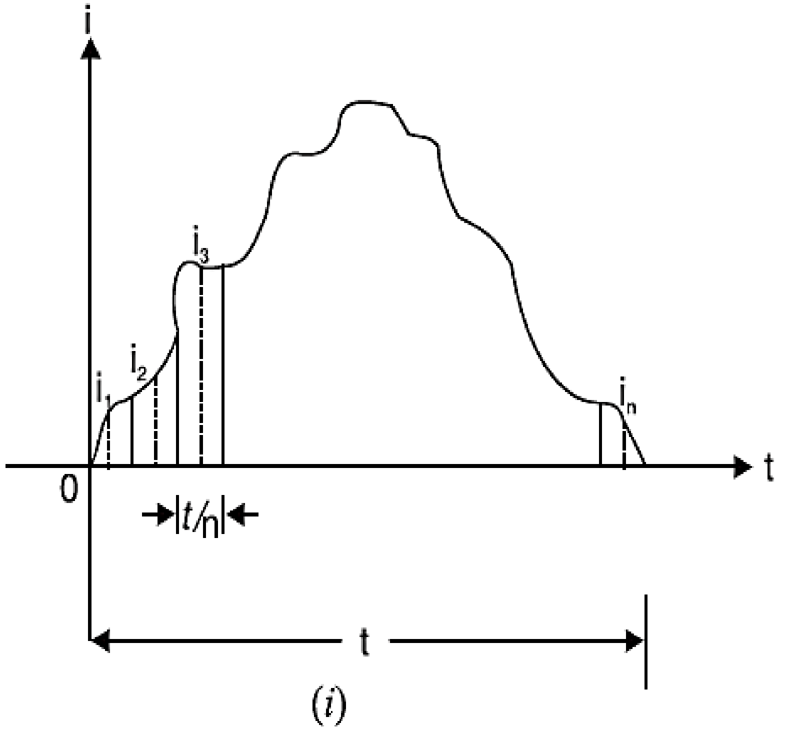
Calculation of R. M. S value of an alternating quantity

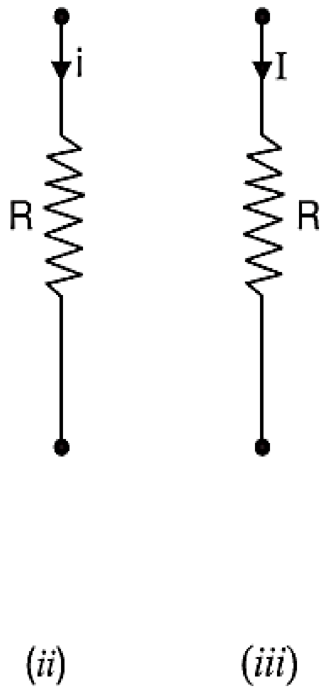
The r.m.s. or effective value of alternating current (or voltage) can be determined as follows.

Consider the half-cycle of a non-sinusoidal alternating current i , flowing through a resistance $R\Omega$ for t seconds.

Divide the time t in n equal intervals of time, each of duration t/n second.

Let the mid-ordinates be i_1, i_2, i_3,.....i_n.





Each current i_1, i_2, i_3,.....i_n will produce heating effect when passed through the resistance R

Suppose the heating effect produced by current i in R is the same as produced by some direct current I flowing through the resistance R for the same time t. Then direct current I is the r.m.s. or effective value of alternating current i.

The heating effect of various components of alternating current will be

$$i_1^2 R\frac{t}{n},\ i_2^2 R\frac{t}{n},\ i_3^2 R\frac{t}{n}\ldots\ldots\ldots\ldots i_n^2 R\frac{t}{n}$$

Total heat produced by alternating current *i*

$$\left(i_1^2 R\frac{t}{n}+\ i_2^2 R\frac{t}{n}+\ i_3^2 R\frac{t}{n}+\ldots\ldots\ldots\ldots+i_n^2 R\frac{t}{n}\right)$$

$$\left(i_1^2+\ i_2^2+\ i_3^2+\ldots\ldots\ldots\ldots+i_n^2\right)R\frac{t}{n}$$ Joules

Heat produced by equivalent direct current
I* = *I*² *R t joules

Since heat produced in both cases is the same. Hence,

$$I^2 Rt=\left(i_1^2+\ i_2^2+\ i_3^2+\ldots\ldots\ldots\ldots+i_n^2\right)R\frac{t}{n}$$

or

$$I^2=\frac{\left(i_1^2+\ i_2^2+\ i_3^2+\ldots\ldots\ldots\ldots+i_n^2\right)}{n}$$

or

$$I=\sqrt{\frac{\left(i_1^2+\ i_2^2+\ i_3^2+\ldots\ldots\ldots\ldots+i_n^2\right)}{n}}$$

i.e. *I = Square* ***root*** *of the* ***mean*** *of the* ***squares*** *of the current*
I = ***root-mean-square (r.m.s.)*** *value*

Similarly, The r.m.s. or effective value of an alternating voltage can similarly be expressed as :

$$V\text{r.m.s.}=\sqrt{\frac{\left(v_1^2+\ v_2^2+\ v_3^2+\ldots\ldots\ldots\ldots+v_n^2\right)}{n}}$$

Where, v_1, v_2, v_3 …… v_n are the mid-ordinate values of alternating voltage

Calculation of R. M. S value of an alternating quantity using Method of Integration

Let us consider that an alternating current, 'i' is flowing in a resistor 'R' for duration 'dt'
The heat produced by this current 'i' in resistor 'R' in time 'dt' is i^2Rdt.
The total heat produced in time period 'T'

$$H_{a.c.} = \int_0^T i^2 R dt$$

Now, the heat produced by the equivalent direct current 'I' in the same resistor 'R' in time 'T' is given by:

$$H_{d.c.} = I^2 RT$$

Now, as per the statement of R.M.S value of alternating quantity, $H_{a.c.}$ *is equal to* $H_{d.c.}$ *Hence,*

$$I^2 RT = \int_0^T i^2 R dt \quad \text{OR} \quad I^2 T = \int_0^T i^2 dt$$

Or,

$$I^2 = \frac{1}{T}\int_0^T i^2 dt$$

$$I_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

Similarly, The r.m.s. or effective value of an alternating voltage can similarly be expressed as :

$$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

Where, "i" and "v" are the instantaneous values of the alternating current and voltage respectively

Thank You