Basic Electrical Engineering (TEE 101)

Lecture 17: Numerical Practice

Content

This lecture covers numerical on:

- **AC Quantities**
- **Average Value**
- **R.M.S. or Effective Value**

Example 1. *An alternating current i is given by ;* ***i = 141·4 sin 314 t***

Find:

(i) the maximum value

(ii) frequency

(iii) time period and

(iv) the instantaneous value when t is 3 ms.

Solution. Comparing the given equation of alternating current with the standard form $\boldsymbol{i = I_m \sin\omega t}$, we have,

(*i*) Maximum value, I_m = **141·4 A**

(*ii*) Frequency, $f = \omega/2\pi = 314/2\pi$ = **50 Hz**

(*iii*) Time period, $T = 1/f = 1/50$ = **0·02 s**

(*iv*) $i = 141{\cdot}4 \sin 314\, t$

When $t = 3$ m s $= 3 \times 10{-}3$ s,

$i = 141{\cdot}4 \sin 314 \times 3 \times 10{-}3$ = **114· 35 A**

Example 2. *An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for the instantaneous value.*

Solution. *Max. value of current, Im = 120 A ; Frequency, f = 60 Hz*

(i) The instantaneous value of current is given by ;

$i = I_m \sin \omega t = I_m \sin 2\pi f t = 120 \sin 2\pi \times 60 \times t$

$\therefore$ ***i = 120 sin 120 π t***

Example 3. *An alternating current is given by ; i = 10 sin 942 t. Determine the time taken from t = 0 for the current to reach a value of + 6 A for a first time.*

Solution. Figure 1 shows the waveform of the given alternating current. Let the current become +6A for the first time after *t* second. Then,

$6 = 10 \sin 942\, t$

Or, $\sin 942\, t = 6/10 = 0{\cdot}6$

$\therefore$ $942\, t = \sin^{-1} 0{\cdot}6 = 0{\cdot}643$ rad

or $t = 0.643/942$

$= 0{\cdot}68 \times 10^{-3}$ s = **0·68 ms**

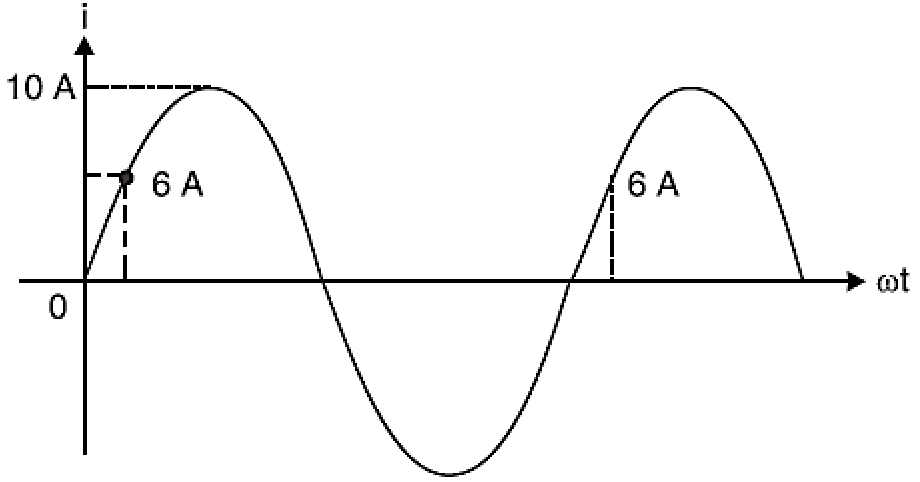

Figure 1

Example 4. *Determine the average value of a sinusoidal current given by ;*
$i = I_m \sin\theta$

Solution. The equation of an alternating current varying sinusoidally is given by ; $\boldsymbol{i = I_m \sin\theta}$

Consider an elementary strip of thickness $d\theta$ in the first half-cycle of current wave as shown in Figure 2. Let i be the mid-ordinate of this strip.

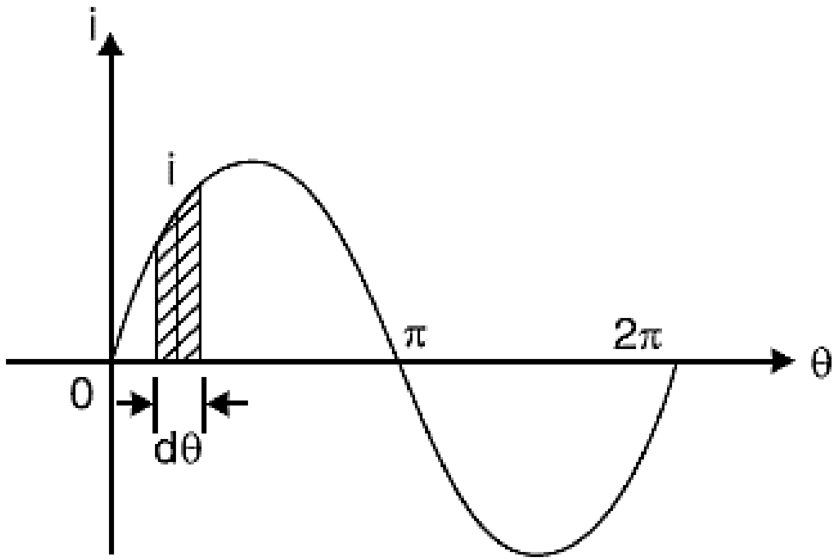

Figure 2

$$\text{Area of strip} = i\,d\theta$$

$$\text{Area of half-cycle} = \int_0^{\pi} i\,d\theta$$

$$= \int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= I_m[-\cos\theta]_0^{\pi} = 2I_m$$

$$\therefore \quad \text{Average value, } I_{av} = \frac{\text{Area of half-cycle}}{\text{Base length of half-cycle}} = \frac{2I_m}{\pi}$$

or $\quad \boldsymbol{I_{av} = 0{\cdot}637\, I_m}$

Hence, the half-cycle average value of a.c. is 0·637 times the peak value of a.c.

For positive half-cycle, $I_{av} = +\,0{\cdot}637\ I_m$

For negative half-cycle, $I_{av} = -\,0{\cdot}637\ I_m$

- *Clearly, average value of a.c. over a complete cycle is zero.*
- *Similarly, it can be proved that for alternating voltage varying sinusoidally,* ***Vav = 0·637 Vm.***

Example 5. *Determine the R.M.S value of a sinusoidal current given by ;*
$i = I_m \sin \theta$

Solution. The equation of an alternating current varying sinusoidally is given by ; $\boldsymbol{i = I_m \sin \theta}$

Consider an elementary strip of thickness $d\theta$ in the first half-cycle of current wave as shown in Figure 2. Let i be the mid-ordinate of this strip.

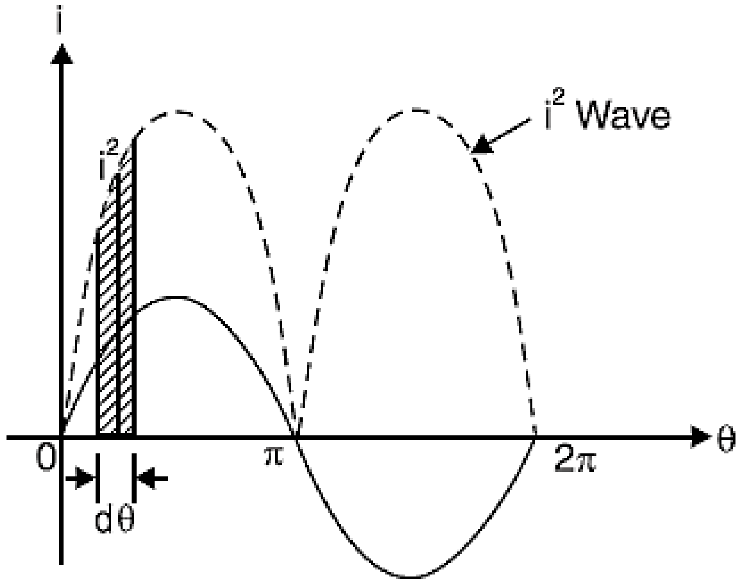

Figure 3

Area of strip $= i^2 d\theta$

Area of half-cycle of the squared wave

$$= \int_0^{\pi} i^2 d\theta$$

$$= \int_0^{\pi} I_m^2 \sin^2 \theta d\theta$$

$$= I_m^2 \int_0^{\pi} \sin^2 \theta d\theta = \frac{{}^*\pi I_m^2}{2}$$

$$I_{r.m.s.} = \sqrt{\frac{\text{Area of half-cycle squared wave}}{\text{Half-cycle base}}}$$

$$= \sqrt{\frac{\pi I_m^2/2}{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$I_{r.m.s.} = 0{\cdot}707\, I_m$$

- Similarly, it can be proved that for alternating voltage varying sinusoidally, ***Vr.m.s. = 0·707 Vm.***

$$* \quad \int_0^{\pi} \sin^2 \theta d\theta = \int_0^{\pi} \frac{1-\cos 2\theta}{2} = \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{\pi}{2}$$

Example 6. *Find the average value and r.m.s of ac voltage whose waveform is shown in Figure 4.*

Solution. (i) Average Value: One cycle of waveform extends from t = 0 to t = 0·6 s, so that the time period **T = 0·6s**.

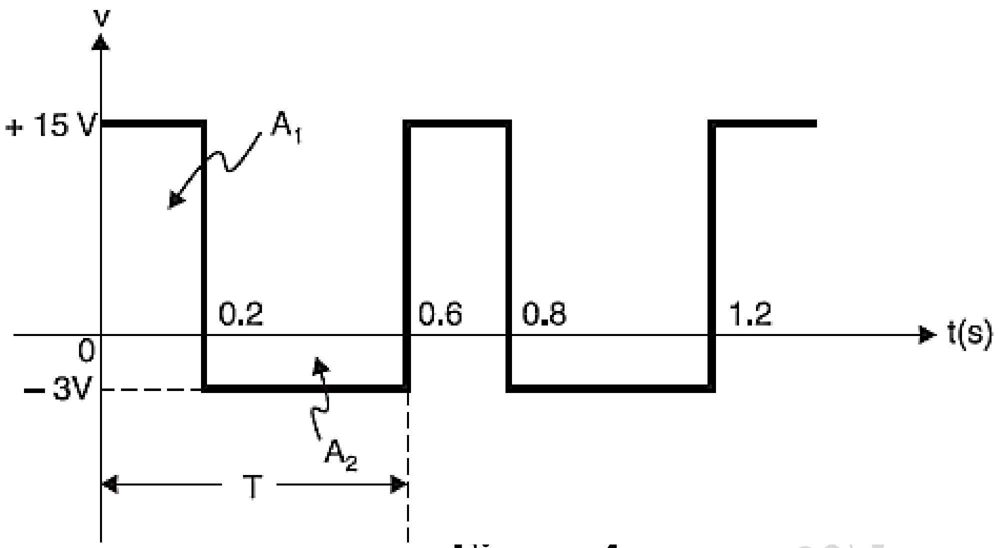

Figure 4

$$V_{avg} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{0.6}\left[\int_0^{0.2} v_1 dt + \int_{0.2}^{0.6} v_2 dt\right]$$

$$v_1 = 15V \ from\ t = 0\ to\ t = 0.2$$

$$v_2 = -3V \ from\ t = 0.2\ to\ t = 0.6$$

$$V_{avg} = \frac{1}{0.6}\left[\int_0^{0.2} 15dt + \int_{0.2}^{0.6}(-3)dt\right]$$

$$V_{avg} = \left[\frac{15}{0.6}\int_0^{0.2} dt - \frac{3}{0.6}\int_{0.2}^{0.6} dt\right]$$

$$V_{avg} = \left[\frac{15}{0.6}|t|_0^{0.2} - \frac{3}{0.6}|t|_{0.2}^{0.6}\right] = \left[25(0.2-0) - 5(0.6-0.2)\right]$$

$$V_{avg} = \left[25\times0.2 - 5\times0.4\right] = 5 - 2 = 3V$$

Solution. (ii) r.m.s Value:

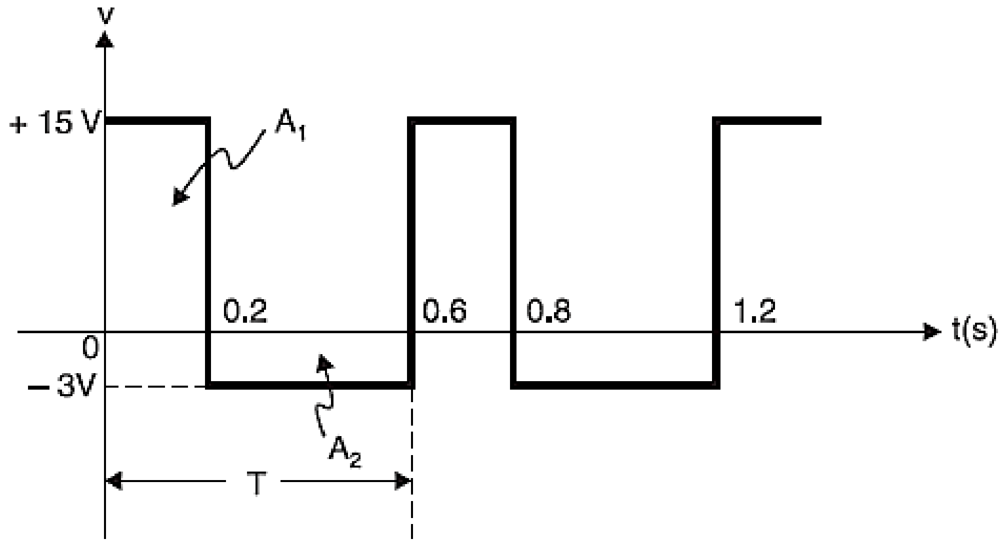

Figure 4

$$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = \sqrt{\frac{1}{0.6}\left[\int_0^{0.2} v_1^2 dt + \int_{0.2}^{0.6} v_2^2 dt\right]}$$

$$v_1 = 15V \; from \; t = 0 \; to \; t = 0.2$$

$$v_2 = -3V \; from \; t = 0.2 \; to \; t = 0.6$$

$$V_{r.m.s} = \sqrt{\frac{1}{0.6}\left[\int_0^{0.2} (+15)^2 dt + \int_{0.2}^{0.6} (-3)^2 dt\right]}$$

$$V_{r.m.s} = \sqrt{\left[\frac{225}{0.6}\int_0^{0.2} dt + \frac{9}{0.6}\int_{0.2}^{0.6} dt\right]}$$

$$V_{r.m.s} = \sqrt{\frac{225}{0.6}\left|t\right|_0^{0.2} + \frac{9}{0.6}\left|t\right|_{0.2}^{0.6}}$$

$$V_{r.m.s} = \sqrt{\frac{225}{0.6}(0.2-0) + \frac{9}{0.6}(0.6-0.2)}$$

$$V_{r.m.s} = \sqrt{\frac{225}{0.6}\times 0.2 + \frac{9}{0.6}\times 0.4} = \sqrt{75+6} = 9V$$

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Example 7. *Find the*

(i) *average value and r.m.s value, for halfwave rectified alternating current and*

(ii) *average value for full-wave rectified alternating current.*

Solution. *(i)* Average value of half-wave rectified a.c.

Figure 5 shows half-wave rectified a.c. in which one half-cycle is suppressed *i.e.* current flows for half the time during complete cycle. ***(Time period, T = 2π)***

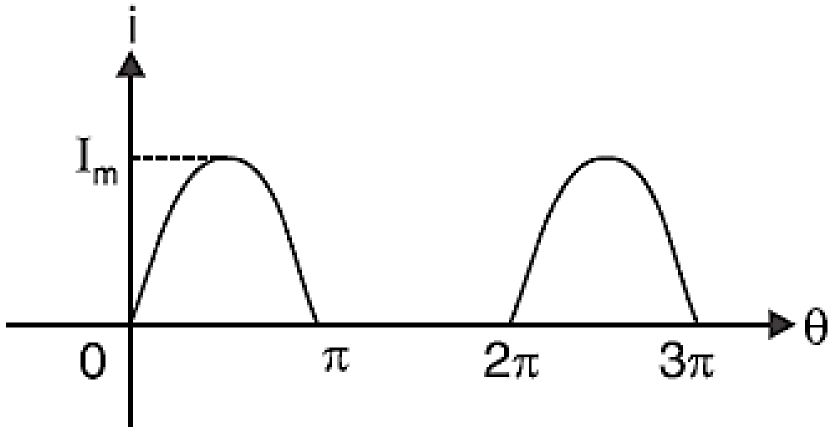

Figure 5

$$I_{av} = \frac{1}{T}\int_0^T i\,dt$$

$$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} i_1 dt + \int_{\pi}^{2\pi} i_2 dt\right]$$

$$i_1 = I_m \sin(\theta)\ from\ \theta = 0\ to\ \theta = \pi$$

$$i_2 = 0\ from\ \theta = \pi\ to\ \theta = 2\pi$$

$$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin(\theta)d\theta + \int_{\pi}^{2\pi} 0.d\theta\right]$$

$$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin(\theta)d\theta\right]$$

$$I_{av} = \frac{I_m}{2\pi}\left[\int_0^{\pi} \sin(\theta)d\theta\right]$$

$$I_{av} = \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$I_{av} = -\frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$I_{av} = -\frac{I_m}{2\pi}\left[-1-1\right]$$

$$I_{av} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi}\ \text{(answer)}$$

Solution. r.m.s value of half-wave rectified a.c.

The r.m.s value is expressed as:

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

(Time period, T = 2π)

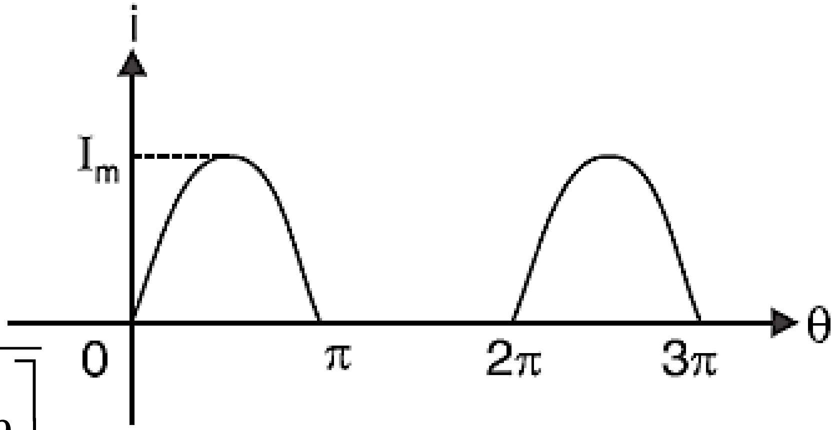

Figure 5

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} i_1^2 dt + \int_{\pi}^{2\pi} i_2^2 dt\right]}$$

$$i_1 = I_m \sin(\theta)\, from\ \theta = 0\ to\ \theta = \pi$$

$$i_2 = 0\ from\ \theta = \pi\, to\ \theta = 2\pi$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2(\theta) d\theta + \int_{\pi}^{2\pi} 0.d\theta\right]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{2\pi}\left[\int_0^{\pi} \sin^2(\theta) d\theta\right]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{2\pi}\left[\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta\right]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{4\pi}\left[\int_0^{\pi} (1-\cos 2\theta) d\theta\right]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{4\pi}\left[(\pi - 0) - \left(\frac{\sin 2\pi - \sin 0}{2}\right)\right]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{4\pi}[\pi]}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \sqrt{\frac{I_m^2}{4}}$$

$$\mathrm{I}_{\mathrm{r.m.s}} = \frac{\mathrm{I}_\mathrm{m}}{2}(\text{answer})$$

Solution. ***(ii)* Average value of full-wave rectified a.c.**

Figure 6 shows full-wave rectified a.c. in which both half-cycles appear in the output *i.e*. current flows in the same direction for both half-cycles.
Since the wave is symmetrical, half-cycle may be considered for various computations.***(Time period, T = π)***

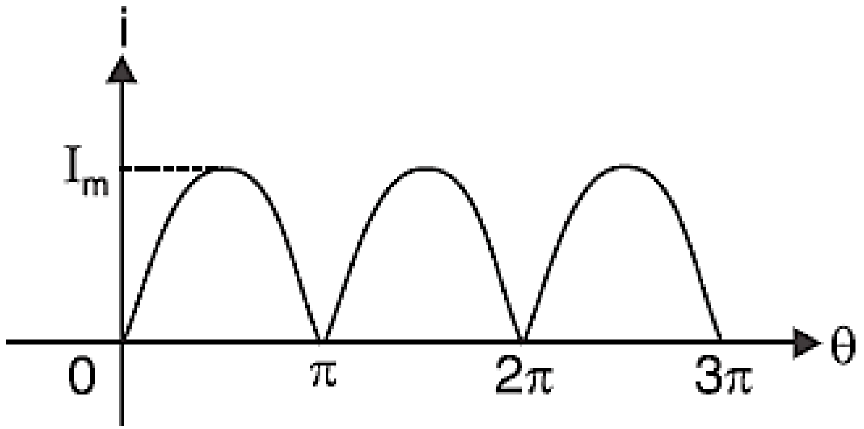

Figure 6

$$\mathrm{I_{av}} = \frac{1}{T}\int_0^T i\,dt$$

$$\mathrm{I_{av}} = \frac{1}{\pi}\left[\int_0^{\pi} i\,dt\right]$$

$$i = I_m \sin(\theta)\ from\ \theta = 0\ to\ \theta = \pi$$

$$\mathrm{I_{av}} = \frac{1}{\pi}\left[\int_0^{\pi} I_m \sin(\theta)d\theta\right]$$

$$\mathrm{I_{av}} = \frac{I_m}{\pi}\left[\int_0^{\pi} \sin(\theta)d\theta\right]$$

$$\mathrm{I_{av}} = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$\mathrm{I_{av}} = -\frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$\mathrm{I_{av}} = -\frac{I_m}{\pi}[-1-1]$$

$$\mathrm{I_{av}} = \frac{2\mathrm{I_m}}{\pi}\ (\text{answer})$$

Thank You