Engineering Physics
Course Code : TPH 201

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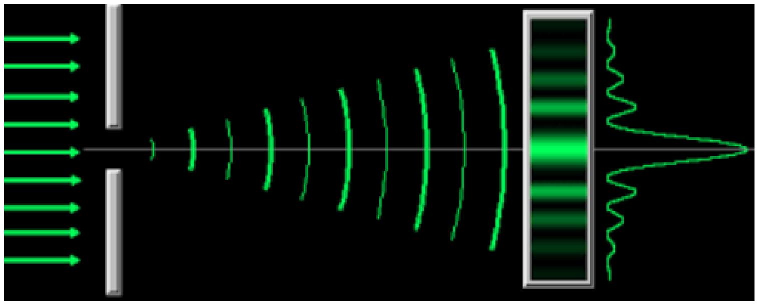
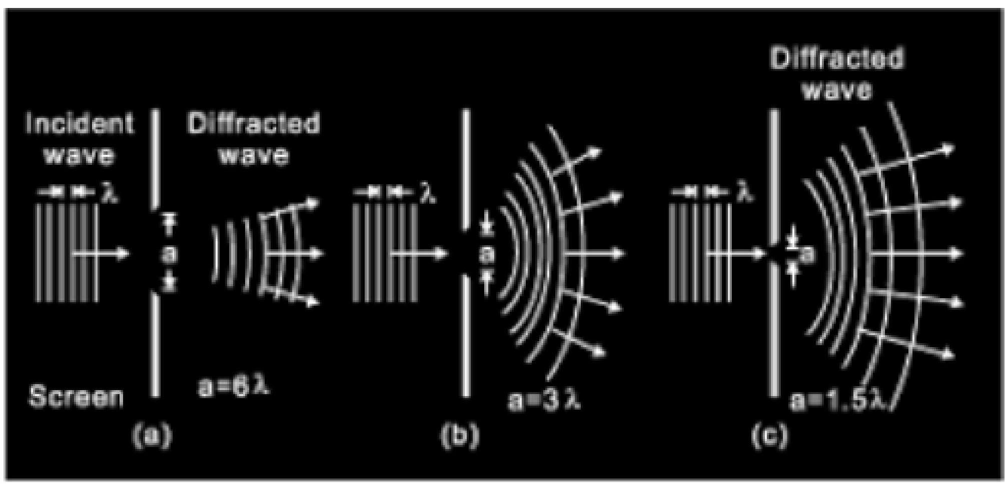
Diffraction of Light



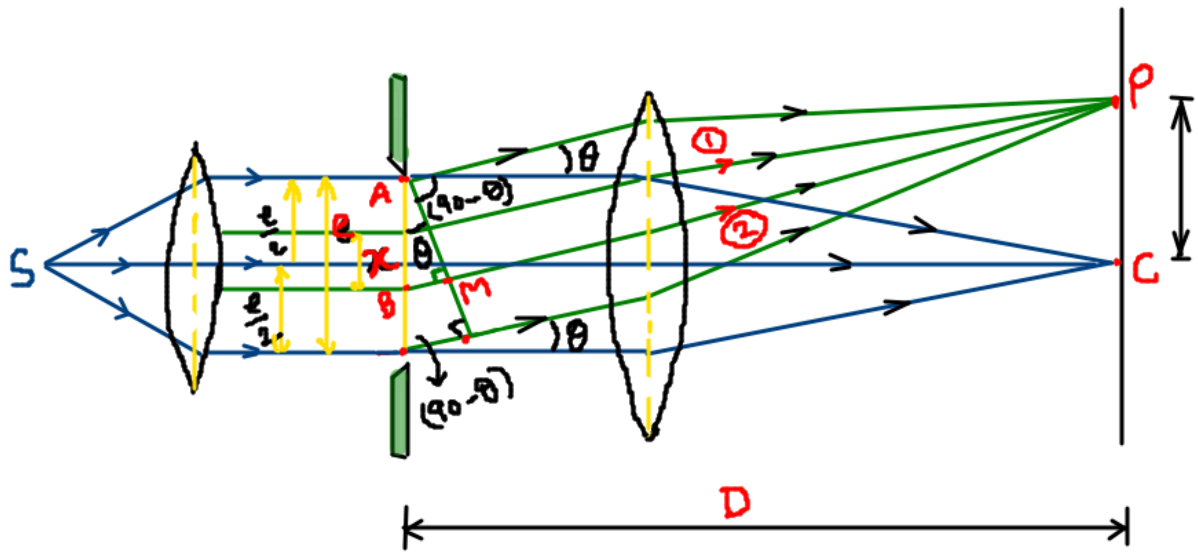
The phenomenon of bending of light round the corners of an obstacle and their spreading into the geometrical shadow (of an object) is called diffraction and the distribution of light intensity resulting in dark and bright fringes is called a diffraction pattern.



Types of Diffraction

1. **Fresnel Diffraction:** (Finite distance b/w source/screen and diffracting element, No requirement of lenses, Incident wavefront is spherical or cylindrical)
2. **Fraunhofer Diffracton:** (Infinite distance b/w source/screen and diffracting element, Lenses required, Incident wavefront is plane)

Fraunhofer Diffraction due to single slit



$AB = x$, $BM =$ $\boxed{\text{Path Diff. between rays (1) and (2)}}$

$BM = AB\sin\theta = x\sin\theta$ ——— (1)

Phase Diff. $(\phi) = \frac{2\pi}{\lambda}(\text{Path diff.})$

$$\boxed{\phi = \frac{2\pi}{\lambda}(x\sin\theta)} \text{ ——— (2)}$$

Let Equation for incident wave before diffraction is $\Rightarrow y = A\cos\omega t$

And after diffraction it becomes $\Rightarrow dy = A\,dx \cdot \cos(\omega t + \phi)$ ——— (3)

$\because y = 2\int_0^{e/2} dy$ Put value of dy from equation (3)

$$y = 2\int_0^{e/2} A\,dx\cos(\omega t + \phi)$$

$\because \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$= 2A\int_0^{e/2} (\cos\omega t \cdot \cos\phi - \sin\omega t \cdot \sin\phi)\,dx$$

$$= 2A\int_0^{e/2} \cos\omega t \cdot \cos\phi \, dx - 2A\int_0^{e/2} \sin\omega t \cdot \sin\phi \, dx$$

Put the value of ϕ from equation (1)

$$= 2A\int_0^{e/2} \cos\omega t \cdot \cos\left(\frac{2\pi x \sin\theta}{\lambda}\right) dx - 2A\int_0^{e/2} \sin\omega t \cdot \sin\left(\frac{2\pi x \sin\theta}{\lambda}\right) dx$$

$$= 2A\cos\omega t \int_0^{e/2} \cos\left(\frac{2\pi x \sin\theta}{\lambda}\right) dx - 2A\sin\omega t \int_0^{e/2} \sin\left(\frac{2\pi x \sin\theta}{\lambda}\right) dx$$

$\int \cos = \sin$ $\quad \because \int \sin = -\cos$

$$= 2A\cos\omega t \left[\frac{\sin\left(\frac{2\pi x \sin\theta}{\lambda}\right)}{\frac{2\pi \sin\theta}{\lambda}}\right]_0^{e/2} + 2A\sin\omega t \left[\frac{\cos\left(\frac{2\pi x \sin\theta}{\lambda}\right)}{\frac{2\pi \sin\theta}{\lambda}}\right]_0^{e/2}$$

$$y = 2A\cos\omega t \left[\frac{\frac{\sin 2\pi\left(\frac{e}{2}\right)\sin\theta}{\lambda}}{\frac{2\pi\sin\theta}{\lambda}} - \underbrace{\frac{\sin(0^\circ)}{\frac{2\pi\sin\theta}{\lambda}}}_{=0}\right] + 2A\sin\omega t\left[\frac{\frac{\cos 2\pi\left(\frac{e}{2}\right)\sin\theta}{\lambda}}{2\pi\sin\theta/\lambda} - \frac{\cos(0^\circ)}{\frac{2\pi\sin\theta}{\lambda}}\right]$$

$\simeq 1 - \simeq 1$

$$y = \cancel{2}A\cos\omega t \left[\frac{\frac{\sin \cancel{2}\pi e \sin\theta}{\cancel{2}\lambda}}{\frac{\cancel{2}\pi\sin\theta}{\lambda}}\right]$$

$$= A\cos\omega t \left[\frac{\frac{\sin \pi e \sin\theta}{\lambda}}{\frac{\pi\sin\theta}{\lambda}}\right] \qquad \text{Let } \boxed{Ae = A_0} \text{ —— (4)}$$

Now, multiply and divide by 'e'

$$= Ae\cos\omega t \left[\frac{\frac{\sin \pi e \sin\theta}{\lambda}}{\frac{\pi e \sin\theta}{\lambda}}\right] \qquad \text{Let } \boxed{\frac{\pi e \sin\theta}{\lambda} = \alpha} \text{ —— (5)}$$

$$y = A_0 \cos\omega t \cdot \frac{\sin\alpha}{\alpha}$$

$$y = \left(A_0 \cdot \frac{\sin\alpha}{\alpha}\right)\cos\omega t$$

take $\boxed{A_0 \cdot \frac{\sin\alpha}{\alpha} = \text{Amplitude of resultant wave}}$

Now the resultant intensity

$$\boxed{I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2}$$

Distribution of resultant Intensity (I)

(i) Central or Principal Maxima :→

$$\because \quad I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)$$

for $I_{p.max}$, α should be min.

or $\alpha = 0$

$$\because \quad I = \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1 \quad (\text{by L' Hospital rule})$$

$$\therefore \boxed{I_{p.max} = A_0^2}$$

$\because \alpha = \frac{\pi e \sin\theta}{\lambda}$

for $\alpha = 0$, πe will not be 0

$\therefore \quad \sin\theta = 0$

$\sin(0^\circ) = 0$

Therefore $\boxed{\theta = 0^\circ}$ for P. Max.

(ii) Principal Minima : $\because I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

In this case $\alpha \neq 0$, but $\sin\alpha$ should be min.

$\therefore \sin\alpha = 0$, $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots \pm n\pi$

$$\because \alpha = \frac{\pi e \sin\theta}{\lambda} \quad \& \quad \alpha = \pm n\pi$$

$$\therefore \frac{\cancel{\pi} e \sin\theta}{\lambda} = \pm n \cancel{\pi}$$

$$\boxed{e\sin\theta = \pm n\lambda}$$

It is the condition for principal minima

(iii) Secondary Maxima : $\because I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

$$\because \boxed{\frac{dI}{d\alpha} = 0} \Rightarrow \frac{d}{d\alpha}\left[A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$A_0^2 = \text{Const.}$

$$\therefore \Rightarrow A_0^2 \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$\Rightarrow A_0^2 \cdot \frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cdot\cos\alpha - \sin\alpha(1)}{\alpha^2}\right) = 0$$

We can take,

$$\Rightarrow \alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \frac{\sin\alpha}{\cos\alpha}, \quad \boxed{\alpha = \tan\alpha}, \quad \left.\begin{array}{l} y = \alpha \\ y = \tan\alpha \end{array}\right\}$$

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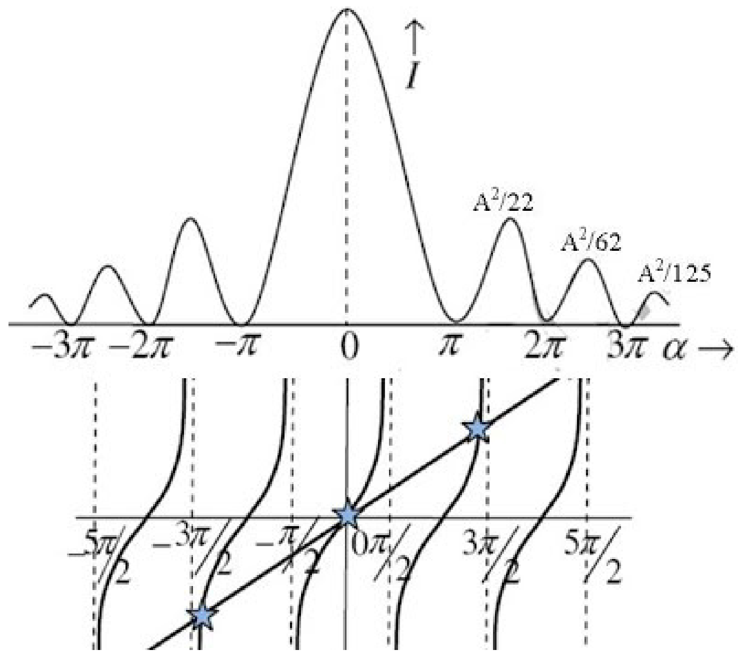
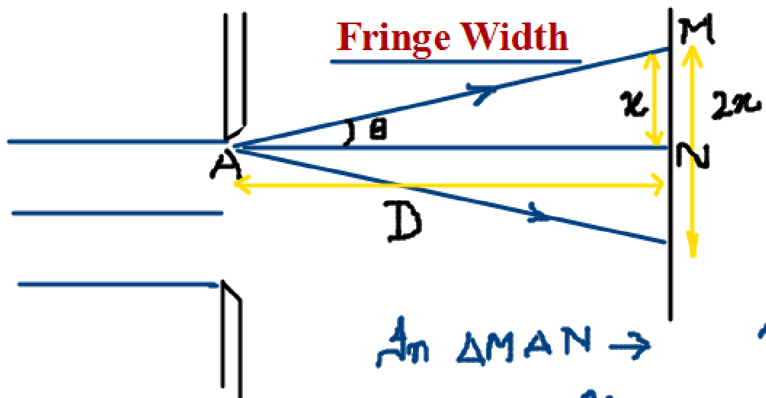




Let Lens is very close to slit

In $\Delta MAN \rightarrow$ $\quad AM \cong AN$

$$\sin\theta = \frac{x}{D} \quad \text{——(1)}$$

& $\because$ $e\sin\theta = n\lambda$

or $\sin\theta = \frac{n\lambda}{e}$, for first order $n=1$

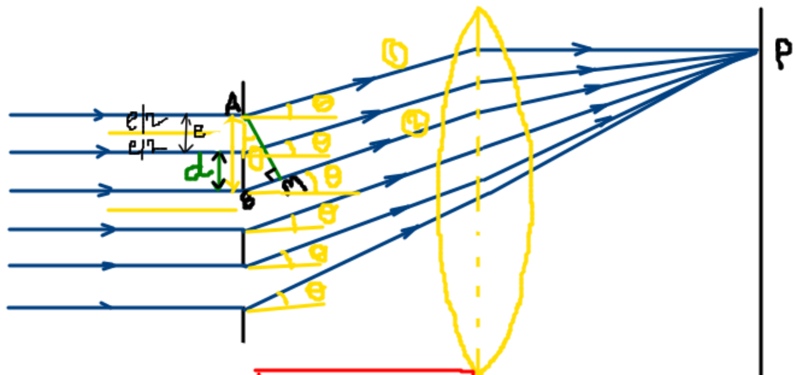
$$\sin\theta = \frac{\lambda}{e} \quad \text{——(2)}$$

$(1)=(2) \Rightarrow \frac{\lambda}{e} = \frac{x}{D} \Rightarrow x = \frac{\lambda D}{e}$, D = focal length $= f$

fringe width $= 2x$

$$\therefore \boxed{\omega = \frac{2\lambda f}{e}}$$

Diffraction due to N slits (Diffraction Grating)



$\boxed{AB = e + d}$ (Grating Element)

Let the equation of incident ray of light is $y = A\cos\omega t$

$\because$ we know that amplitude of diffracted wave is $\dfrac{A_0 \sin\alpha}{\alpha}$

where $\left(\alpha = \dfrac{\pi e \sin\theta}{\lambda}\right)$

Here Path Diff. $(BM) = AB\sin\theta$

$$\boxed{\text{Path diff.} = (e+d)\sin\theta}$$

$$\text{Phase diff.} = \frac{2\pi}{\lambda}(\text{Path diff.})$$

$$= \frac{2\pi}{\lambda}(BM)$$

$$\boxed{\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta}$$

(i) Equation of first diffracted ray (wave): $y_1 = \dfrac{A_0 \sin\alpha}{\alpha}\cdot\cos\omega t$

(ii) Equation of second diffracted ray: $y_2 = \left(\dfrac{A_0 \sin\alpha}{\alpha}\right)\left[\cos(\omega t + \delta)\right]$

(iii) Equation of third diffracted ray: $y_3 = \left(\dfrac{A_0 \sin\alpha}{\alpha}\right)\left[\cos(\omega t + 2\delta)\right]$

(iv) Equation of Nth diffracted ray: $y_N = \left(\dfrac{A_0 \sin\alpha}{\alpha}\right)\left[\cos\{\omega t + (N-1)\delta\}\right]$

(v) Applying superposition principle at P:

$$y = y_1 + y_2 + y_3 \text{----} y_N$$

$$\boxed{y = \frac{A_0 \sin\alpha}{\alpha}\cdot\cos\omega t + \left(\frac{A_0 \sin\alpha}{\alpha}\right)\left[\cos(\omega t+\delta)\right] + \left(\frac{A_0 \sin\alpha}{\alpha}\right)\left[\cos\{\omega t + (N-1)\delta\}\right]}$$

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Intensity of resultant wave = (Max. value of y)^2

$$I = (y_{max})^2$$

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}\left(\frac{1-\cos N\delta}{1-\cos\delta}\right)$$

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}\left(\frac{\cancel{2}\sin^2 N\delta/2}{\cancel{2}\sin^2 \delta/2}\right)$$

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}\left(\frac{\sin^2 N\delta/2}{\sin^2 \delta/2}\right)$$

Let $\delta/2 = \beta$,

$$\boxed{I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)}$$

This is the expression for Intensity when diffracton due to N slits.

(i) Intensity when diffraction due to single slit: $N = 1$

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}\left(\frac{\cancel{\sin^2\beta}}{\cancel{\sin^2\beta}}\right)$$

$$\boxed{I = \frac{A_0^2 \sin^2\alpha}{\alpha^2}}$$

(ii) Intensity when diffraction due to double slit: $N = 2$

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 2\beta}{\sin^2\beta}$$

$$= \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot \frac{(2\sin\beta \cdot \cos\beta)^2}{\sin^2\beta}$$

$$= \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot \frac{4\,\cancel{\sin^2\beta} \cdot \cos^2\beta}{\cancel{\sin^2\beta}}$$

$$\boxed{I = \frac{4A_0^2 \sin^2\alpha}{\alpha^2} \cdot \cos^2\beta}$$