13.21 IMPORTANT FORMULAE

(1) $L^{-1}\left(\frac{1}{s}\right) = 1$

(2) $L^{-1}\frac{1}{s^n} = \frac{t^{n-1}}{(n-1)!}$

(3) $L^{-1}\frac{1}{s-a} = e^{at}$

(4) $L^{-1}\frac{s}{s^2-a^2} = \cosh at$

(5) $L^{-1}\frac{1}{s^2-a^2} = \frac{1}{a}\sinh at$

(6) $L^{-1}\frac{1}{s^2+a^2} = \frac{1}{a}\sin at$

(7) $L^{-1}\frac{s}{s^2+a^2} = \cos at$

(8) $L^{-1}F(s-a) = e^{at}f(t)$

(9) $L^{-1}\frac{1}{(s-a)^2+b^2} = \frac{1}{b}e^{at}\sin bt$

(10) $L^{-1}\frac{s-a}{(s-a)^2+b^2} = e^{at}\cos bt$

(11) $L^{-1}\frac{1}{(s-a)^2-b^2} = \frac{1}{b}e^{at}\sinh bt$

(12) $L^{-1}\frac{s-a}{(s-a)^2-b^2} = e^{at}\cosh bt$

(13) $L^{-1}\frac{1}{(s^2+a^2)^2} = \frac{1}{2a^3}(\sin at - at\cos at)$

(14) $L^{-1}\frac{s}{(s^2+a^2)^2} = \frac{1}{2a}t\sin at$

(15) $L^{-1}\frac{s^2-a^2}{(s^2+a^2)^2} = t\cos at$

(16) $L^{-1}(1) = \delta(t)$

(17) $L^{-1}\frac{s^2}{(s^2+a^2)^2} = \frac{1}{2a}[\sin at + at\cos at]$

MULTIPLICATION by *s*

$$L^{-1}[s\,F(s)] = \frac{d}{dt}f(t) + f(0)\,\delta(t)$$

Division by *s* (multiplication by $\frac{1}{s}$)

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,dt = \int_0^t f(t)\,dt$$

FIRST SHIFTING PROPERTY

$L^{-1}F(s) = f(t)$, then $L^{-1}F(s+a) = e^{-at}L^{-1}[F(s)]$

SECOND SHIFTING PROPERTY

$$L^{-1}\left[e^{-as}F(s)\right] = f(t-a)\,U(t-a)$$

INVERSE LAPLACE TRANSFORMS OF DERIVATIVES

$$L^{-1}\left[\frac{d}{ds}F(s)\right] = -t\,L^{-1}[F(s)] = -t\,f(t) \quad \text{or} \quad L^{-1}[F(s)] = -\frac{1}{t}L^{-1}\left[\frac{d}{ds}F(s)\right]$$

.27 INVERSE LAPLACE TRANSFORM OF INTEGRALS

$$\mathrm{L}^{-1}\left[\int_s^{\infty} F(s)\,ds\right] = \frac{f(t)}{t} = \frac{1}{t}\mathrm{L}^{-1}[F(s)] \qquad \text{or} \qquad \mathrm{L}^{-1}[F(s)] = t\,\mathrm{L}^{-1}\left[\int_s^{\infty} F(s)\,ds\right].$$