# Basic Electrical Engineering (TEE 101)

## Lecture 30: Numerical Practice AC - 3

By: Dr. Parvesh Saini

# Content

**This lecture covers Numerical on:**

- AC circuits (Series)
- AC circuits (Parallel)
- Power and Power Factor
- Resonance in AC circuit

# AC circuits

**Example 1.** *A pure inductive coil allows a current of 10 A to flow from a 230 V, 50 Hz supply. Find*

*(i) Inductive reactance*
*(ii) Inductance of the coil*
*(iii) Power absorbed.*
*(iv) Write down the equations for voltage and current.*

**Solution.** **(*i*)** Circuit current, $I = V/X_L$ $\qquad (V_L = V)$

$\therefore$ Inductive reactance, $X_L = V/I = 230/10 = \mathbf{23\ \Omega}$

**(*ii*)** Now, $X_L = 2\pi f L \quad \therefore \quad L = \dfrac{X_L}{2\pi f} = \dfrac{23}{2\pi \times 50} = \mathbf{0.073\ H}$

**(*iii*)** Power absorbed = **Zero**

$V_m = 230 \times \sqrt{2} = 325.27\text{ V}\ ;\ I_m = 10 \times \sqrt{2} = 14.14\text{ A}\ ;\quad \omega = 2\pi \times 50 = 314\text{ rad/s}$

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are :

$$\mathbf{v = 325.27 \sin 314\, t \quad ; \quad i = 14.14 \sin (314\, t - \pi/2)}$$

**Example 2.** *The current through an 80 mH inductor is 0·1 sin (440 t − 25o) A. Write the mathematical expression for the voltage across it.*

**Solution.** Inductive reactance is

$$X_L = 2\pi f L = 400 \times 80 \times 10^{-3} = 32\ \Omega$$

$$V_m = I_m X_L = 0{\cdot}1 \times 32 = 3{\cdot}2\ \text{V}$$

Since the voltage leads the current by 90º, we must add 90º to the phase angle of voltage.

$\therefore$ $$v = V_m \sin(400\,t - 25^\circ + 90^\circ)$$

or $$\mathbf{v = 3{\cdot}2 \sin(400\,t + 65^\circ)\ V}$$

**Example 3.** *A 318 μF capacitor is connected across a 230 V, 50 Hz system.*

*Determine*

*(i) The capacitive reactance*

*(ii) r.m.s. value of current, and*

*(iii) Equations for voltage and current.*

**Solution.** (*i*) Capacitive reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{10^6}{2\pi \times 50 \times 318} = \mathbf{10\Omega}$

(*ii*) R.M.S. value of current, $I = V/X_C = 230/10 = \mathbf{23\ A}$

(*iii*) $V_m = 230 \times \sqrt{2} = 325{\cdot}27$ volts ; $I_m = \sqrt{2} \times 23 = 32{\cdot}53$ A ; $\omega = 2\pi \times 50 = 314$ rad/s

$\therefore$ Equations for voltage and current are :

$$\mathbf{v = 325{\cdot}27 \sin 314\,t \quad ; \quad i = 32{\cdot}53 \sin(314\,t + \pi/2)}$$

# AC circuits (Series)

**Example 4.** *A coil having a resistance of 7 Ω and an inductance of 31·8 mH is connected to 230 V, 50 Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed and (v) voltage drop across resistor and inductor.*

**Solution.** (*i*) Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Coil impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$

∴ Circuit current, $I = V/Z = 230/12.2 =$ **18·85 A**

(*ii*) $\tan\phi = X_L/R = 10/7$

∴ Phase angle, $\phi = \tan^{-1}(10/7) =$ **55° lag**

(*iii*) Power factor $= \cos\phi = \cos 55° =$ **0.573 lag**

(*iv*) Power consumed, $P = VI\cos\phi = 230 \times 18.85 \times 0.573 =$ **2484·24 W**

(*v*) Voltage drop across $R = IR = 18.85 \times 7 =$ **131.95V**

Voltage drop across $L = IX_L = 18.85 \times 10 =$ **188.5 V**

**Example 5.** *An inductor coil is connected to a supply of 250 V at 50 Hz and takes a current of 5 A. The coil dissipates 750 W. Calculate:*

***(i) Power factor***

***(ii) Resistance of coil, and***

***(iii) Inductance of coil.***

**Solution.** (*i*) Power consumed, $P = VI \cos\phi$

$$\therefore \quad \text{Power factor, } \cos\phi = \frac{P}{VI} = \frac{750}{250 \times 5} = \mathbf{0.6\ lag}$$

(*ii*) Impedance of coil, $Z = V/I = 250/5 = 50\ \Omega$

Resistance of coil, $R = Z\cos\phi = 50 \times 0.6 = \mathbf{30\ \Omega}$

$$(iii) \quad \text{Reactance of coil, } X_L = \sqrt{Z^2 - R^2} = \sqrt{(50)^2 - (30)^2} = 40\ \Omega$$

$$\therefore \quad \text{Inductance of coil, } L = \frac{X_L}{2\pi f} = \frac{40}{2\pi \times 50} = \mathbf{0.127\ H}$$

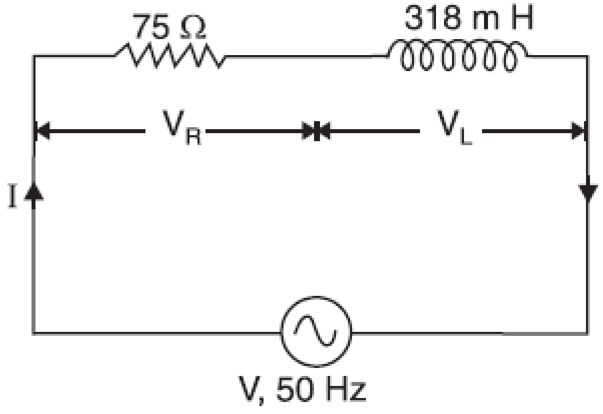
**Example 6.** *A pure inductance of 318 mH is connected in series with a pure resistance of 75 Ω. The circuit is supplied from 50 Hz source and the voltage across 75 Ω resistor is found to be 150 V. Calculate the supply voltage and the phase angle.*

**Solution.** The circuit diagram and the phasor diagram are shown in Fig. (i).

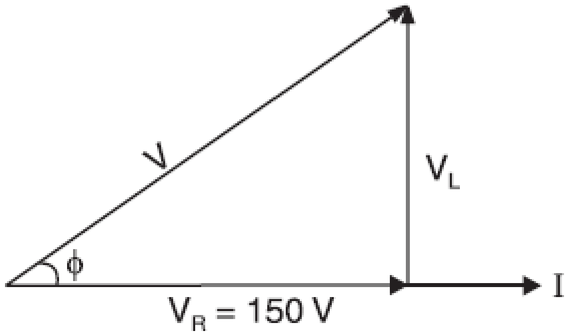
Circuit current, $I = V_R/R = 150/75 = 2$ A

Reactance of coil, $X_L = 2p\, f\, L = 2\pi \times 50 \times 318 \times 10^{-3} = 100\ \Omega$

Voltage across $L$, $V_L = IX_L = 2 \times 100 = 200$ V




(*i*)




(*ii*)

Referring to the phasor diagram of the circuit in Fig. (*ii*),

$$\text{Supply voltage, } V = \sqrt{V_R^2 + V_L^2} = \sqrt{150^2 + 200^2} = \mathbf{250\ V}$$

Now, $\tan\phi = X_L/R = 100/75 = 1.33$

$$\therefore \quad \text{Phase angle, } \phi = \tan^{-1} 1.33 = \mathbf{53.06^\circ\ lag}$$

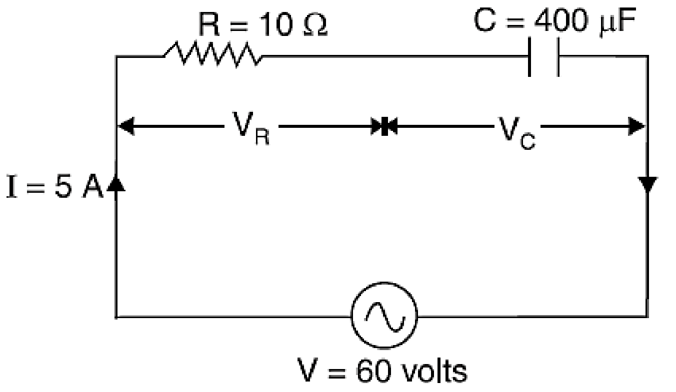
**Example 7.** *A 10 W resistor and 400 μF capacitor are connected in series to a 60-V sinusoidal supply. The circuit current is 5 A.*

*Calculate:*

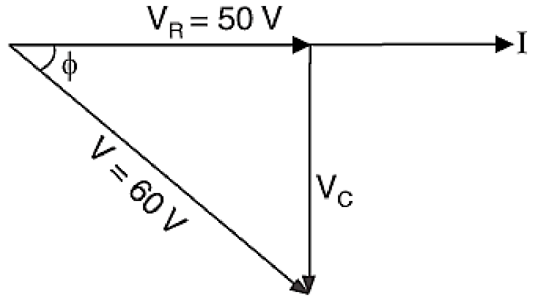
i. *The supply frequency, and*

ii. *Phase angle between the current and voltage.*

**Solution.** Fig. (*i*) shows the circuit diagram whereas Fig. (*ii*) shows phasor diagram.




(*i*)




(*ii*)

$$\text{Voltage across } R,\ V_R = IR = 5 \times 10 = 50 \text{ V}$$

$$\text{Voltage across } C,\ V_C = \sqrt{V^2 - V_R{}^2} = \sqrt{60^2 - 50^2} = 33.17 \text{ V}$$

$$\text{Reactance of capacitor, } X_C = V_C / I = 33{\cdot}17/5 = 6{\cdot}634\ \Omega$$

$$\therefore \quad \text{Supply frequency, } f = \frac{1}{2\pi C X_C} = \frac{10^6}{2\pi \times 400 \times 6 \cdot 634} = \mathbf{60\ Hz}$$

$$\tan\phi = V_C / V_R = 33.17/50 = 0.6634$$

$$\therefore \quad \text{Phase angle, } \phi = \tan^{-1} 0.6634 = \mathbf{33.6^\circ\ lead}$$

**Example 8.** *A two-element series circuit consumes 700 W and has a p.f. of 0·707 leading. If the applied voltage is v = 141.1 sin (314 t + 30°), find the circuit constants.*

**Solution.**

Since the circuit p.f. is leading, one circuit element must be a capacitor. Further, power consumed in the circuit is 700 W. This suggests that other circuit element is a resistor. Therefore, it is *RC* series circuit.

R.M.S. value of applied voltage, $V = V_m/\sqrt{2} = 141.1/\sqrt{2} = 100$ volts

Power consumed, $P = VI\cos\phi$ or $700 = 100 \times I \times 0.707 \quad \therefore \quad I = 10\text{A}$

Also, $P = I^2R \quad \therefore \quad R = P/I^2 = 700/(10)^2 = 7\ \Omega$

Now, $Z = V/I = 100/10 = 10\Omega \quad \therefore \quad X_C = \sqrt{Z^2 - R^2} = \sqrt{(10)^2 - (7)^2} = 7\Omega$

$\therefore$ Capacitance, $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi \times 50 \times 7} = 450 \times 10^{-6}\ \text{F} = \mathbf{450\ \mu F}$

**Example 9.** *A circuit when connected to 200 V, 50 Hz mains takes a current of 10 A, leading the voltage by one-twelfth of time period. Calculate (i) resistance (ii) capacitive reactance and (iii) capacitance of the circuit.*

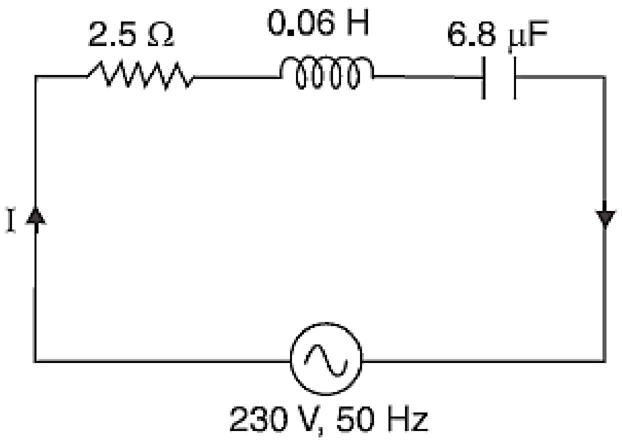
**Solution.** One time period corresponds to a phase difference of 2ϕ radians or 360°. Hence one-twelfth of time period corresponds to a phase difference of $\phi = 360°/12 = 30°$. This means that current leads the voltage by 30°.

**(*i*)** Circuit impedance, $Z = V/I = 200/10 =$ **20** Ω
Circuit resistance, $R = Z\cos\phi = 20\cos 30° =$ **17.32** Ω
**(*ii*)** Capacitive reactance, $X_C = Z\sin\phi = 20\sin 30° =$ **10** Ω

(*iii*) Capacitance, $C = \dfrac{1}{2\pi f\ X_C} = \dfrac{1}{2\pi \times 50 \times 10} = \mathbf{318 \times 10^{-6}\ F}$

**Example 10.** *A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6·8 μF capacitor.*

**Calculate** *(i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed.*




(*i*)

**Solution.** Fig. (*i*) shows the conditions of the problem.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.06 = 18.85\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\ \Omega$$

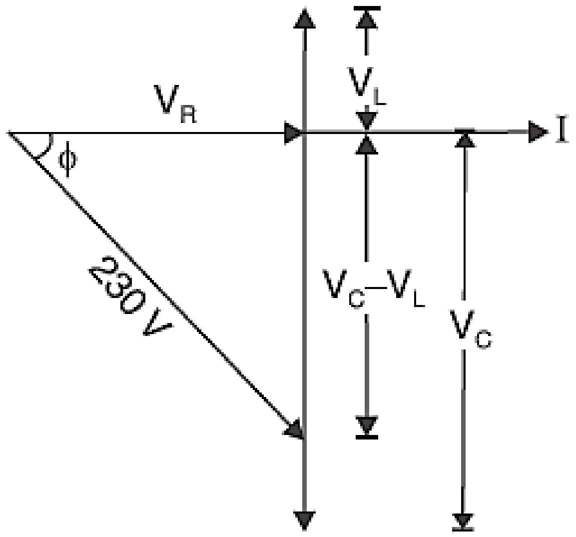
(*i*) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.85 - 468)^2} = \mathbf{449.2\ \Omega}$

(*ii*) Circuit current, $I = V/Z = 230/449.2 = \mathbf{0.512\ A}$

(*iii*) $\tan\phi = \frac{X_L - X_C}{R} = \frac{18.85 - 468}{2.5} = -179.66$

∴ Phase angle, $\phi = \tan^{-1} - 179.66 = -89.7° = \mathbf{89.7°}$ ***lead***

The negative sign with ϕ shows that current is leading the voltage [ See the phasor diagram in Fig. (*ii*)].




(*ii*)

(*iv*) Power factor, $\cos\phi = \frac{R}{Z} = \frac{2.5}{449.2} = \mathbf{0.00557}$ ***lead***

(*v*) Power consumed, $P = VI\cos\phi = 230 \times 0.512 \times 0.00557 = \mathbf{0.656\ W}$

By: Dr. Parvesh Saini

**Example 11.** *A coil of p.f. 0.8 is connected in series with a 110 μF capacitor. The supply frequency is 50 Hz. The potential difference (p.d.) across the coil is found to be equal to the p.d. across the capacitor. Calculate the resistance and inductance of the coil.*

**Solution.** Fig. (i) shows the conditions of the problem.

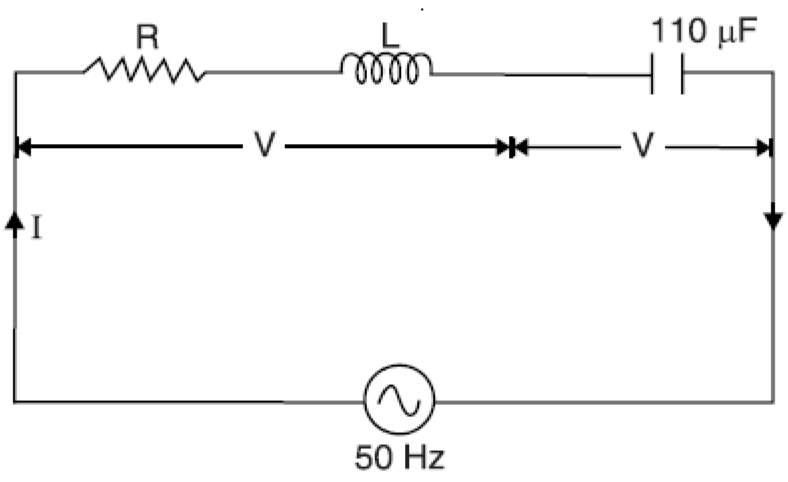



Fig. (i)

$$\text{Reactance of capacitor, } X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi\times 50\times 110} = 29\ \Omega$$

$$\text{Now,} \quad I Z_{coil} = IX_C \quad \therefore \quad Z_{coil} = X_C = 29\ \Omega$$

$$\text{For the coil, } \cos\phi = R/Z_{coil} \quad \therefore \quad R = Z_{coil}\cos\phi = 29\times 0.8 = \mathbf{23.2\ \Omega}$$

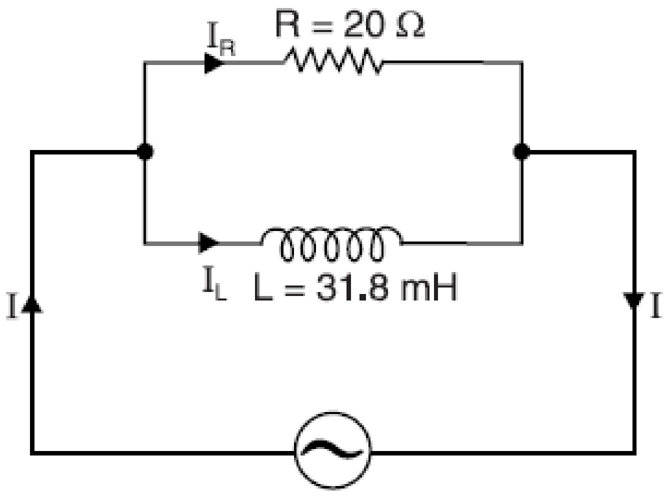
$$\text{Reactance of coil, } {}^*X_L = Z_{coil}\sin\phi = 29\times 0.6 = 17.4\ \Omega$$

$$\therefore \quad \text{Inductance of coil, } L = \frac{X_L}{2\pi f} = \frac{17.4}{2\pi\times 50} = \mathbf{0.055\ H}$$

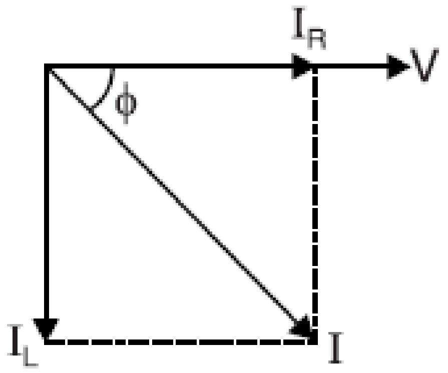
# AC circuits (Parallel)

**Example 12.** *A resistance of 20 Ω and a coil of inductance 31·8 mH and negligible resistance are connected in parallel across 230 V, 50 Hz supply.*
*Find (i) the line current (ii) power factor and power consumed by the circuit.*

**Solution.** Fig. (*i*) shows the circuit diagram while Fig. (*ii*) shows the phasor diagram of the circuit.



(*i*)



(*ii*)

$$I_R = V/R = 230/20 = 11.5 \text{ A in phase with } V$$
$$X_L = 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$$
$$I_L = V/X_L = 230/10 = 23 \text{ A lagging } V \text{ by } 90°$$

The line current $I$ is the phasor sum of $I_R$ and $I_L$.

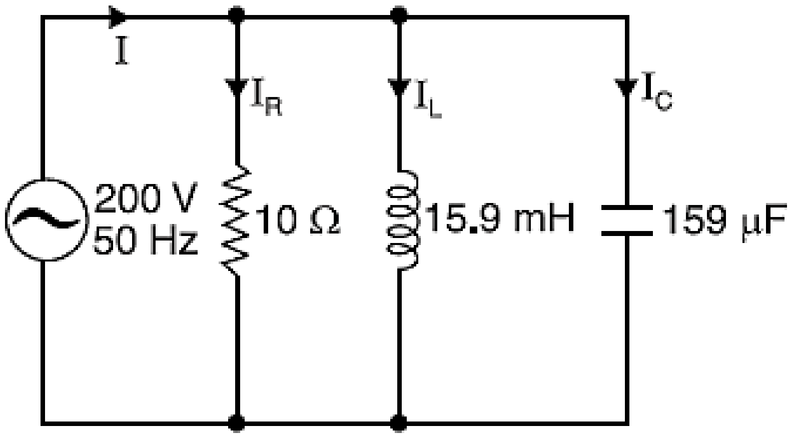
(*i*) Line current, $I = \sqrt{I_R^2 + I_L^2} = \sqrt{(11.5)^2 + (23)^2} =$ **25.71 A**

(*ii*) Power factor, $\cos\phi = I_R/I = 11.5/25.71 =$ **0.447 lag**

Power consumed, $P = VI\cos\phi = 230 \times 25.71 \times 0.447 =$ **2643 watts**

**Example 13.** *A 10 Ω resistor, a 15.9 mH inductor and 159 μF capacitor are connected in parallel to a 200 V, 50 Hz source.*
*Calculate the supply current and power factor*.

**Solution.** Figures below shows the circuit diagram and the phasor diagram of the circuit



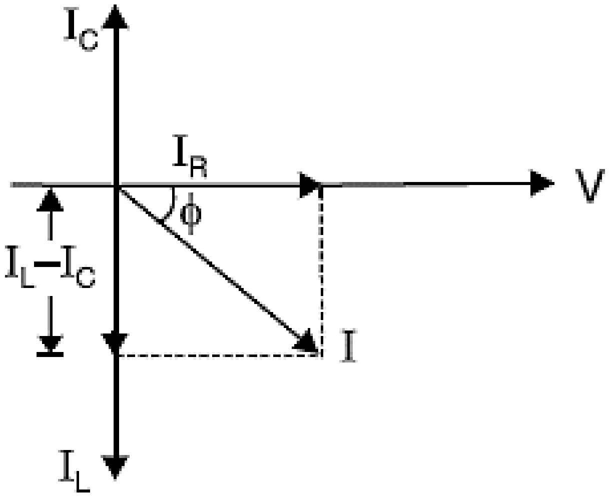

$$X_L = 2\pi f L = 2\pi \times 50 \times 15.9 \times 10^{-3} = 5\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 159} = 20\ \Omega$$

$$I_R = V/R = 200/10 = 20\ \text{A} \quad \text{...in phase with } V$$

$$I_L = V/X_L = 200/5 = 40\ \text{A} \quad \text{...lags } V \text{ by } 90°$$

$$I_C = V/X_C = 200/20 = 10\ \text{A} \quad \text{...leads } V \text{ by } 90°$$




Note that $I_L$ and $I_C$ are 180° out of phase with each other. The supply current $I$ is the phasor sum of $I_R$ and $(I_L - I_C)$.

$$\text{Supply current, } I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{20^2 + (40-10)^2} = \mathbf{36\ A}$$

$$\text{Circuit p.f.} = \cos\phi = I_R/I = 20/36 = \mathbf{0.56}\ \boldsymbol{lag}$$

# Resonance in AC circuits

**Example 14.** *A coil of resistance 100 Ω and inductance 100 μH is connected in series with a 100 pF capacitor. The circuit is connected to a 10 V variable frequency source. Calculate (i) the resonant frequency (ii) current at resonance (iii) voltage across L and C at resonance*

**Solution.** (*i*) Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{10^{-4}\times10^{-10}}}$ $= \mathbf{1.59 \times 10^6\ Hz}$

(*ii*) Current at resonance, $I_r = V/R = 10/100 = \mathbf{0.1\ A}$

(*iii*) At resonance, $X_L = 2\pi f_r L = 2\pi \times 1.59 \times 10^6 \times 10^{-4} = 1000\ \Omega$

At resonance, $V_L = I_r X_L = 0.1 \times 1000 = \mathbf{100\ V}$

At resonance, $V_C = I_r \times X_C = 0.1 \times 1000 = \mathbf{100\ V}$

**Example 15.** *A series RLC circuit has R = 5 Ω, L = 0.2 H and C = 50 μF. The applied voltage is 200 V. Find the resonant frequency*

**Solution.** $R = 5\ \Omega$ ; $L = 0.2$ H ; $C = 50 \times 10^{-6}$ F ; $V = 200$ volts

Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.2\times50\times10^{-6}}}$ $= \mathbf{50.33\ Hz}$

# Thank You