d) Find the children of c and e.

Sol. The children of c ~~and~~ are d and e are g and h.

e) Find the descendants of the vertices a and c.

Sol The descendants of a are b, c, d, e, f, g, h. The descendants of c are d, e, f, g, h.

Ex 2. Every nontrivial tree T has at least two vertices of degree 1.

Sol. Let $n$ = the no. of vertices of T $(n \geq 2)$ and

$m$ = the no. of vertices of degree 1.

Let $v_1, v_2, \dots v_m$ denote the $m$ vertices of degree 1. Then each of the remaining $n-m$ vertices $v_{m+1}, v_{m+2}, \dots v_n$ has at least degree 2.

Thus, $\deg(v_i) = 1$ for $i = 1, 2, \dots m$

$\geq 2$ for $i = m+1, m+2, \dots n$.

$$\sum_{i=1}^{n} \deg(v_i) = \sum_{i=1}^{m} \deg(v_i) + \sum_{i=m+1}^{n} \deg(v_i)$$

$$= m + \sum_{i=m+1}^{n} \deg(v_i)$$

$$\geq m + 2(n-m) = 2n - m$$

Again $\sum_{i=1}^{n} \deg(v_i) = 2e = 2(n-1) = 2n-2$

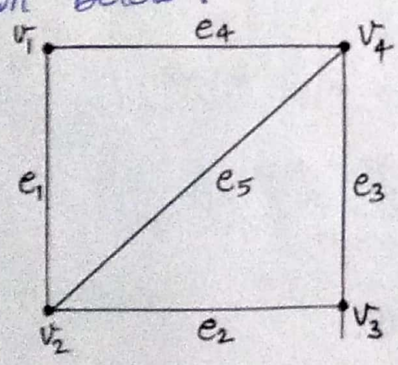
Hence $2n-2 \geq 2n-m \Rightarrow m \geq 2$

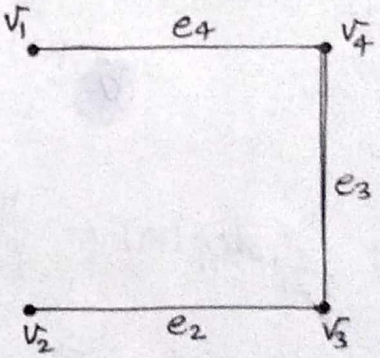
This proves that T contains at least two vertices of degree 1.

## Fundamental Circuits :

let T be a spanning tree of a graph G. Then the edges of G that are in T are called branches of G. The edge of G that is not in T is called a chord of G with respect to T. A circuit formed by adding a chord e to a spanning tree T of a graph is called a Fundamental circuit of G with respect to spanning tree T relative to chord e. The cut set containing exactly on branch of T is called fundamental cut set of G to w.r.t. T.

Consider the spanning tree T of the graph G as shown below:

G: 


T: 


The branches of G are $e_2$, $e_3$ and $e_4$.

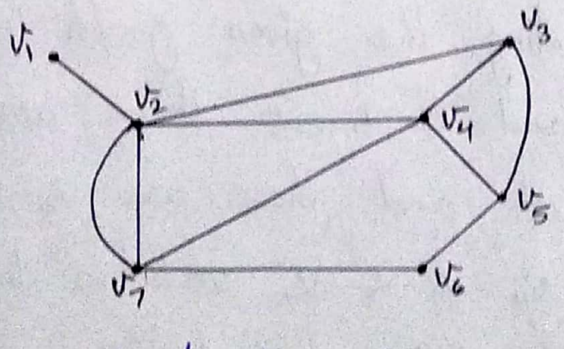
The chords of G are $e_1$ and $e_5$.

If the chord $e_1$ is added to the spanning tree, then one circuit $v_1 - v_4 - v_3 - v_2 - v_1$ is formed and is known as fundamental circuit. If the chord $e_5$ is added, then the circuit $v_2 - v_3 - v_4 - v_2$ is another fundamental circuit.

NOTES

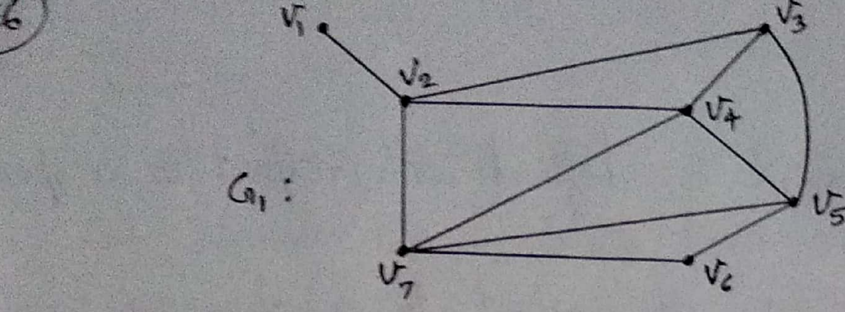
1. Fundamental circuit is defined with respect to a spanning tree.
2. Fundamental circuit with respect to a spanning tree in a graph is not unique.
3. A given circuit may be fundamental circuit with respect to one spanning tree but not so with respect to other spanning tree in the same graph.
4. If $G$ is a connected simple graph with $n$ vertices and $e$ edges, it has $r = e-(n-1)$ chords with respect to any spanning tree $T$, so it has $r$ fundamental circuits w.r.t. $T$.
5. For every branch there is corresponding cutset since removal of any branch from a spanning tree breaks the spanning tree into two trees.

Example 3. Find fundamental circuits for the graphs shown below:




Sol. First we find the spanning tree. For this first delete all loops and parallel edges. Deleting one parallel edge $(v_2, v_7)$ we get the graph $G_1$.

56

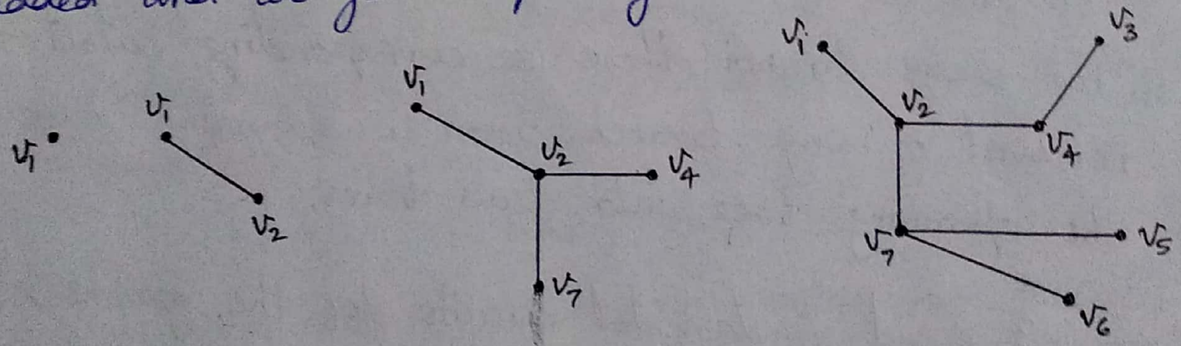



(i) Choose the vertex $v_1$ to be the root.

(ii) Add edge incident with all vertices to $v_1$, so that edge $(v_1, v_2)$ is added.

(iii) Add edges from this vertex $v_2$ to adjacent vertices not already in the tree. Hence $(v_2, v_4)$ and $(v_2, v_7)$ are added.

(iv) Add edges from $v_4$ and $v_7$ to adjacent vertices not already in tree. Hence $(v_7, v_6)$, $(v_7, v_5)$ and $(v_4, v_3)$ are added and we get a spanning tree.




After converting the given graph in simple graph, we have $e = 11$ and $n = 7$. So there are $r = 11 - 7 + 1 = 5$ fundamental circuits and these are $v_2 - v_3 - v_4 - v_2$ relative to edge $(v_2, v_3)$, $v_4 - v_7 - v_2 - v_4$ relative to edge $(v_4, v_7)$, $v_4 - v_5 - v_7 - v_2 - v_4$ relative to edge $(v_4, v_5)$, $v_5 - v_6 - v_7 - v_5$ relative to edge $(v_5, v_6)$ and $v_3 - v_4 - v_2 - v_7 - v_5 - v_3$ relative to edge $(v_3, v_5)$.

## Binary Tree :

A binary tree is a rooted tree in which each vertex has atmost two children. Each child in a binary tree is designated either a **left child** or a **right child** (not both), and an internal vertex has at most one left and one right child. A **full binary** is a tree in which each internal vertex has exactly two children.

Given an internal vertex $v$ of a binary tree $T$, the **left subtree** of $v$ is the binary tree whose root is the left child of $v$, whose vertices consist of the left child of $v$ and all its descendants, and whose edges consist of all those edges of $T$ that connect the vertices of the left subtree together. The **right subtree** of $v$ is defined analogously.

Fig 1 (a) is a binary tree and fig 1. (b) is a full binary tree since each of its internal vertices has two children.

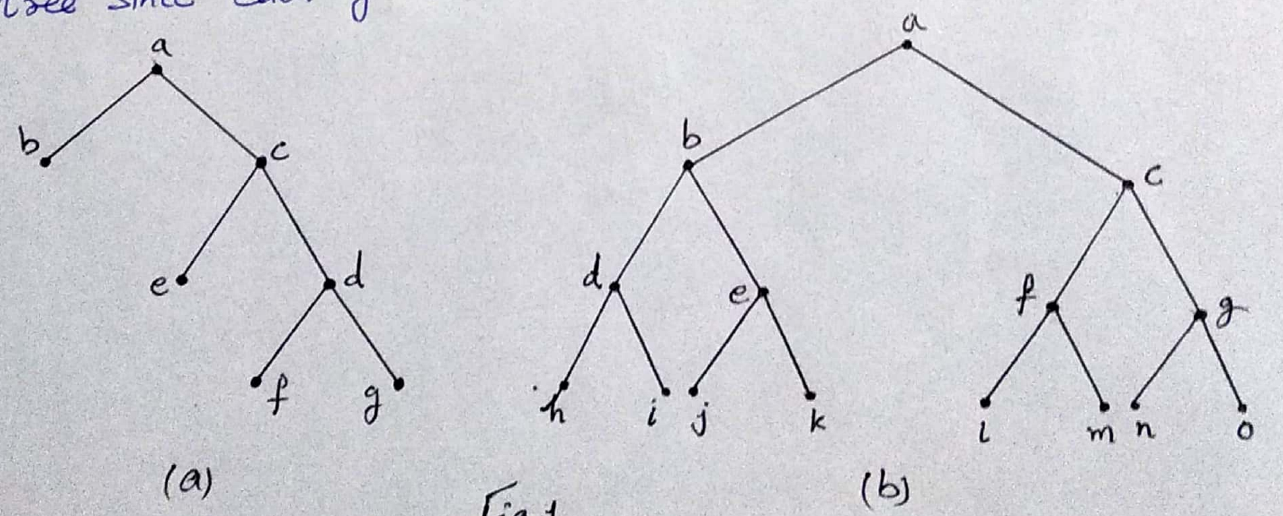



Fig 1.

A tree in which there is exactly one vertex of degree two and each of the other vertices is of degree one or three is called a binary tree. The vertex of degree two is called root of the tree.

Ex.4 What are the left and right children of b shown in Fig.2? What are the left and right subtrees of a?

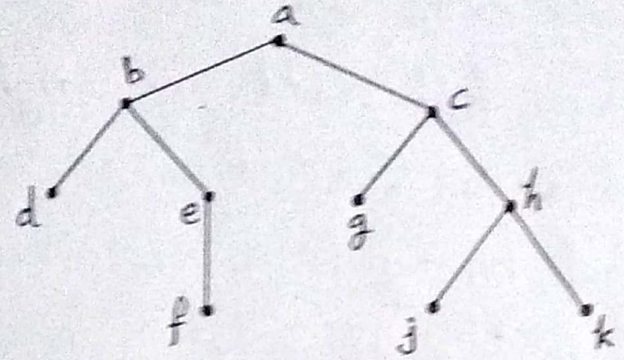



Fig 2

Sol. The left child of b is d and the right child is e. The left subtree of the vertex a consists of the vertices b, d, e and f and the right subtree of a consists of the vertices c, g, h, j and k whose figures are shown in Fig3 (a) and (b) respectively.

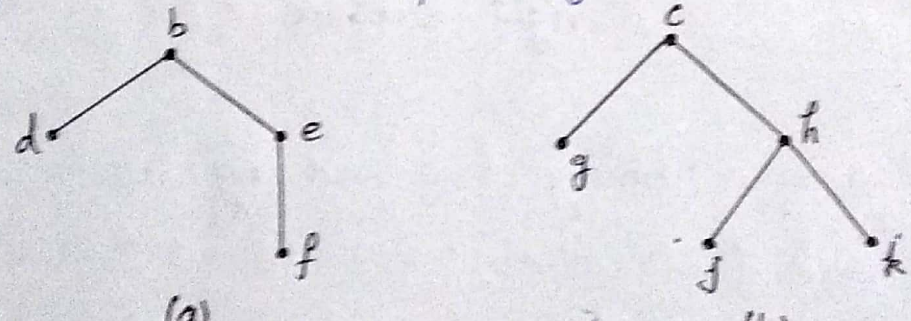



Fig3.