Basic Electrical Engineering (TEE 101)

Lecture 23: Power in Resistor, Inductor and Capacitor

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Content

This lecture covers:

- Power in a resistor
- Power in an inductor
- Power in a capacitor

Power in a Resistor

Power. In any circuit, electric power consumed at any instant is the product of voltage and current at that instant

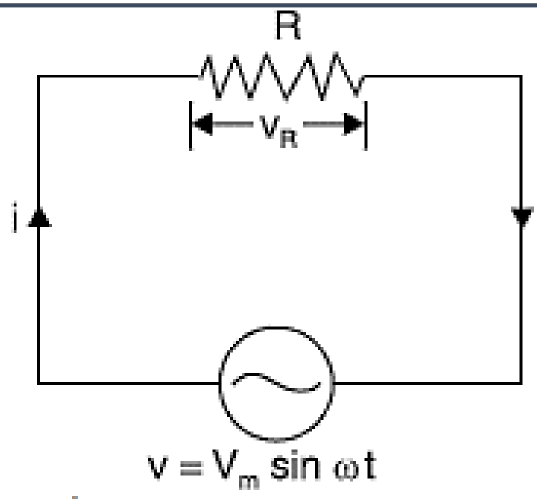

Figure 1: Circuit Diagram

Consider a circuit containing a pure resistance of $R\ \Omega$ connected across an alternating voltage source. Let the alternating voltage be given by the equation :

$$v = V_m \sin \omega t$$

The current through the resistance is given by:

$$i = I_m \sin \omega t$$

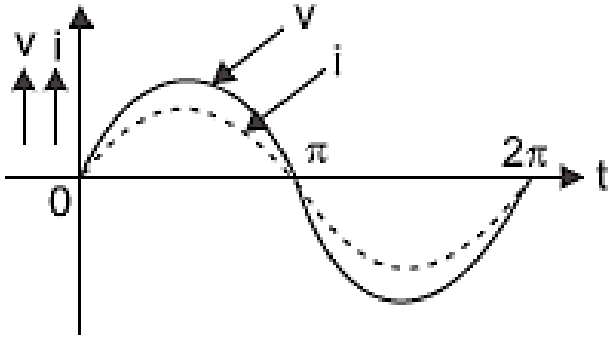

Figure 3: Waveform

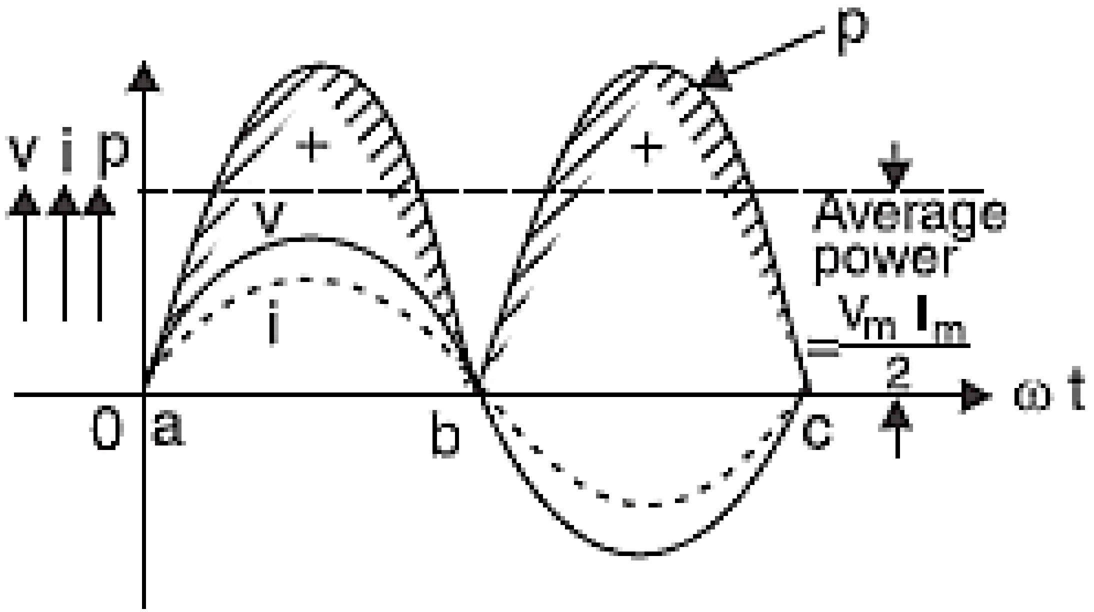

Figure 2: power curve for a pure resistive circuit

- Fig. 2 shows the power curve for a pure resistive circuit.
- Points on the power curve are obtained from the product of the corresponding instantaneous values of voltage and current.
- It is clear that power is always positive except at points *a*, *b* and *c* at which it drops to zero for a moment.
- This means that the voltage source is constantly delivering power to the circuit which is consumed by the circuit.

Mathematical proof of the power waveform (figure 2)

The power consumed by the resistance is

$$P_{av} = \frac{1}{T}\int_0^T p\,dt$$

$$p = v i = V_m I_m \sin^2 \omega t$$

$$P_{av} = \frac{1}{\pi}\int_0^\pi V_m I_m \sin^2 \omega t \, d(\omega t) \quad \text{— (1)}$$

$$P_{av} = \frac{1}{\pi}(V_m I_m)\int_0^\pi \sin^2 \omega t \, d(\omega t)$$

$$\sin^2 \omega t \rightarrow \left[\frac{1-\cos 2\omega t}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{\pi}\int_0^\pi \frac{1-\cos 2\omega t}{2} d(\omega t) \quad \text{— (2)}$$

$$P_{av} = \frac{V_m I_m}{2\pi}\left| \omega t - \frac{\sin 2\omega t}{2}\right|_0^\pi$$

$$P_{av} = \frac{V_m I_m}{2\pi}\left[(\pi - 0) - \underbrace{\frac{\sin 2\pi - \sin 0}{2}}_{=0}\right]$$

$$P_{av} = \frac{V_m I_m}{2\pi}\left[\pi - 0\right]$$

$$P_{av} = \frac{V_m I_m}{2\cancel{\pi}} \times \cancel{\pi} = \frac{V_m I_m}{2} \quad \text{— (3)}$$

In terms of RMS

$$\frac{V_m I_m}{2} = \left(\frac{V_m}{\sqrt{2}}\right) \times \left(\frac{I_m}{\sqrt{2}}\right) = V I$$

where V and I are the RMS values of the instantaneous v and i respectively

i.e $\boxed{P = V I}$ ⇐ Power consumed by a Resistance

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Power in an Inductor

Consider an alternating voltage applied to a pure inductance of *L* henry as shown in Fig. 4. Let the equation of the applied alternating voltage be :

$$v = V_m \sin \omega t$$

The current through the inductance is given by:

$$i = I_m \sin (\omega t - \pi/2)$$

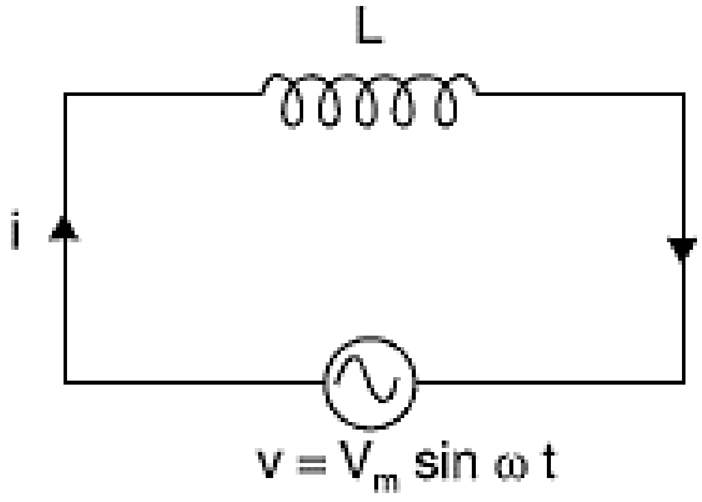

Figure 4: Circuit Diagram

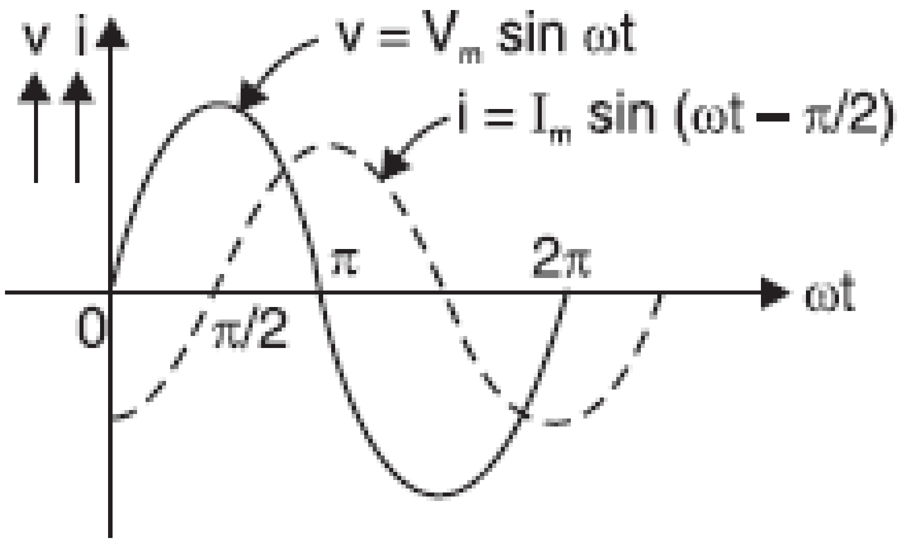

Figure 5: Waveform

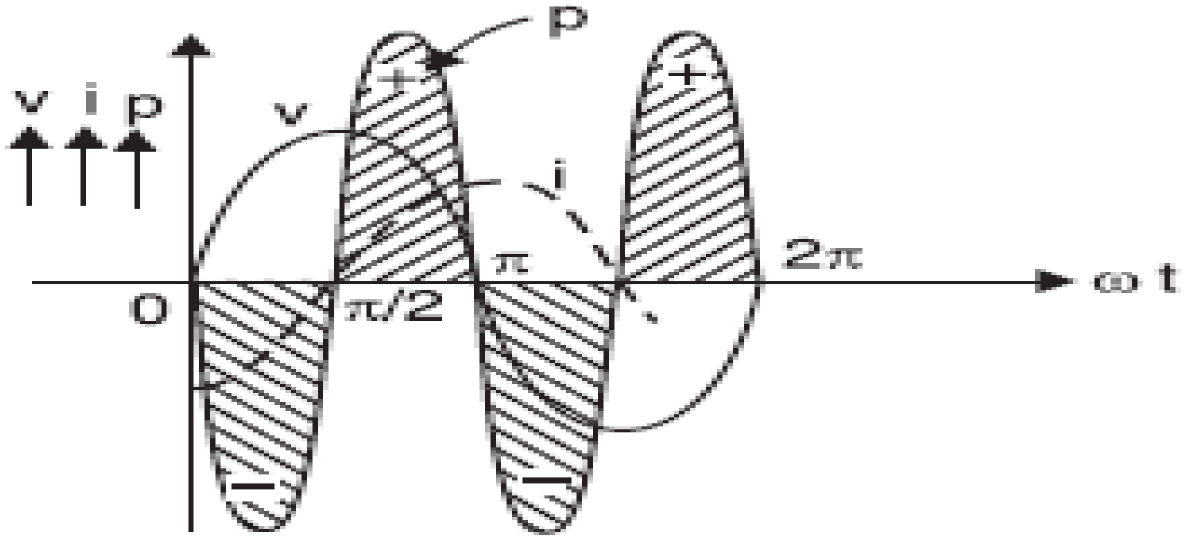

Figure 6: power curve for a pure inductive circuit

- Fig. 3 shows the power curve for a pure inductive circuit.
- During the first 90º of the cycle, the voltage is positive and the current is negative.
- Therefore, the power supplied is negative.
- This means the power is flowing from the coil to the source.
- During the next 90º of the cycle, both voltage and current are positive and the power supplied is positive.
- Therefore, power flows from the source to the coil.

Mathematical proof of the power waveform (figure 6) for an Inductor

The instantaneous power is given by

$$p = vi = V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$$

$$= V_m I_m [\sin\omega t \times \sin(\omega t - \pi/2)]$$

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$p = vi = V_m I_m \left[-\frac{\sin 2\omega t}{2}\right]$$

or $$p = -\frac{V_m I_m}{2} \sin 2\omega t \quad \text{——— (1)}$$

The power consumed by an inductor is

$$P_{av} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{\pi}\int_0^{\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t)$$

$$P_{av} = \frac{1}{\pi}\left[-\frac{V_m I_m}{2}\right]\int_0^{\pi} \sin 2\omega t \, d(\omega t)$$

$$P_{av} = -\frac{V_m I_m}{2\pi}\int_0^{\pi} \sin 2\omega t \, d(\omega t) \quad \text{——— (2)}$$

$$P_{av} = -\frac{V_m I_m}{2\pi}\left| -\frac{\cos 2\omega t}{2} \right|_0^{\pi}$$

$$= \frac{V_m I_m}{4\pi}\left[\cos 2\pi - \cos 0\right]$$

$$= \frac{V_m I_m}{4\pi}\left[1 - 1\right]$$

$$= \frac{V_m I_m}{4\pi} \times 0 = 0$$

$$\boxed{P_{av} = 0} \Leftarrow$$ The power consumed by an inductor is ZERO.

i.e an inductor does not consume power (or dissipate power)

Then what it does with the power??

An inductor stores the power.

Hence power absorbed in pure inductance is zero.

Power in a Capacitor

Consider an alternating voltage applied to a capacitor of capacitance *C* farad as shown in Fig. 7. Let the equation of the applied alternating voltage be :

$$\boldsymbol{v = V_m \sin \omega t}$$

The current through the capacitor is given by:

$$\boldsymbol{i = I_m \sin (\omega t + \pi/2)}$$

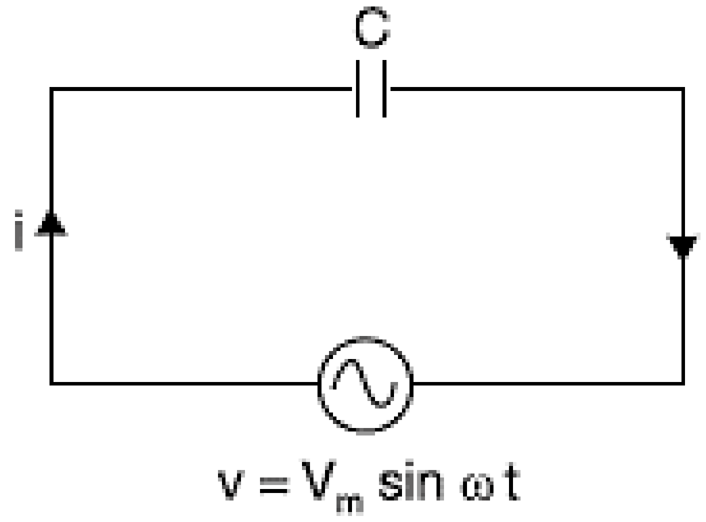

Figure 7: Circuit Diagram

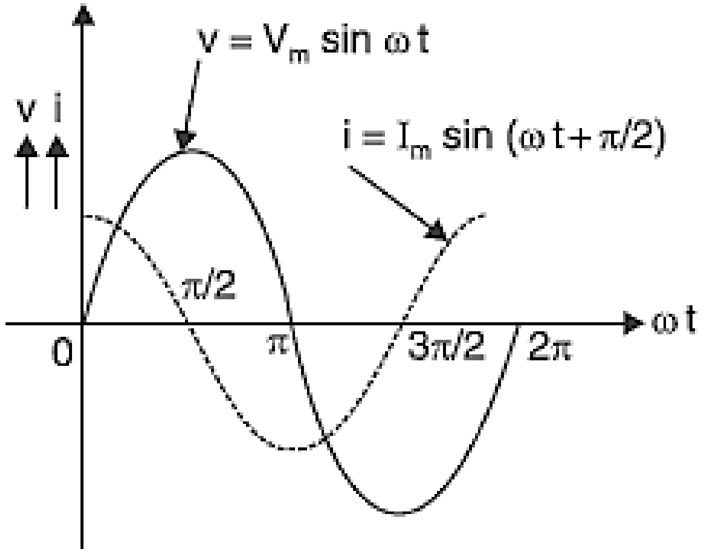

Figure 8: Waveform

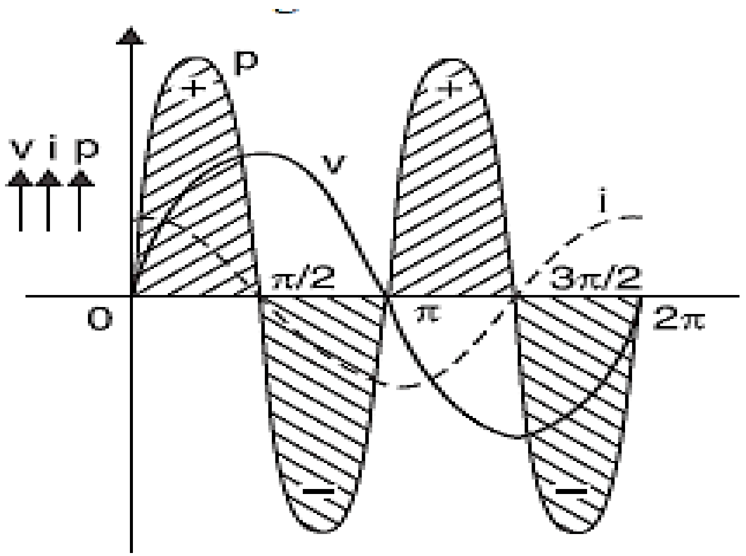

Figure 9: power curve for a pure capacitive circuit

Instantaneous power is given by ;

$$p = v\,i = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2) = V_m I_m \sin \omega t \cos \omega t$$

$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

$$\text{Average power, } P = \text{Average of } p \text{ over one cycle}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence power absorbed in a pure capacitance is zero.

- Fig. 9 shows the power curve for a pure capacitive circuit.
- The power curve is similar to that for a pure inductor because now current leads the voltage by 90º.
- It is clear that positive power is equal to the negative power over one cycle.
- Hence net power absorbed in a pure capacitor is zero.

Thank You