Resonance

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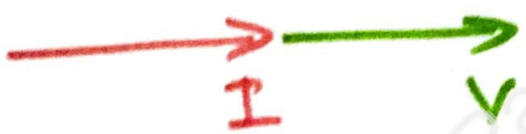
Now because the input voltage (V) and Input current (I) are in phase.
Hence, the phase difference ϕ is ZERO.



} Both are in phase ($\phi = 0$)

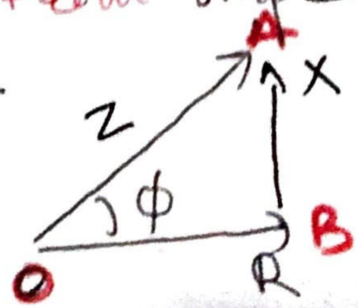
If $\phi = 0°$; the power factor, $\cos\phi = 1$

i.e $\cos(0) = 1$ (unity power factor)

Hence, resonance is the condition at which the given ac circuit posses UNITY power factor.

Now; $\cos\phi = 1$

also ; $\cos\phi = \frac{R}{Z}$ {from Impedance Δ}



OB = R
OA = Z
so; $\cos\phi = \frac{R}{Z}$

at Resonance
$\cos\phi = 1$
$\frac{R}{Z} = 1$
$\boxed{Z = R}$

$\frac{OB}{OA} = \cos\phi$

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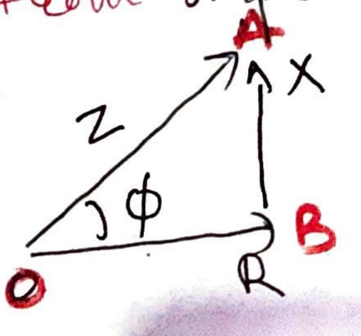

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at Resonance

$\cos\phi = 1$

$\frac{R}{Z} = 1$

$\boxed{Z = R}$

Hence, we can say that, Resonance is the condition at which the impedance (Z) of the given AC circuit is equal to R.

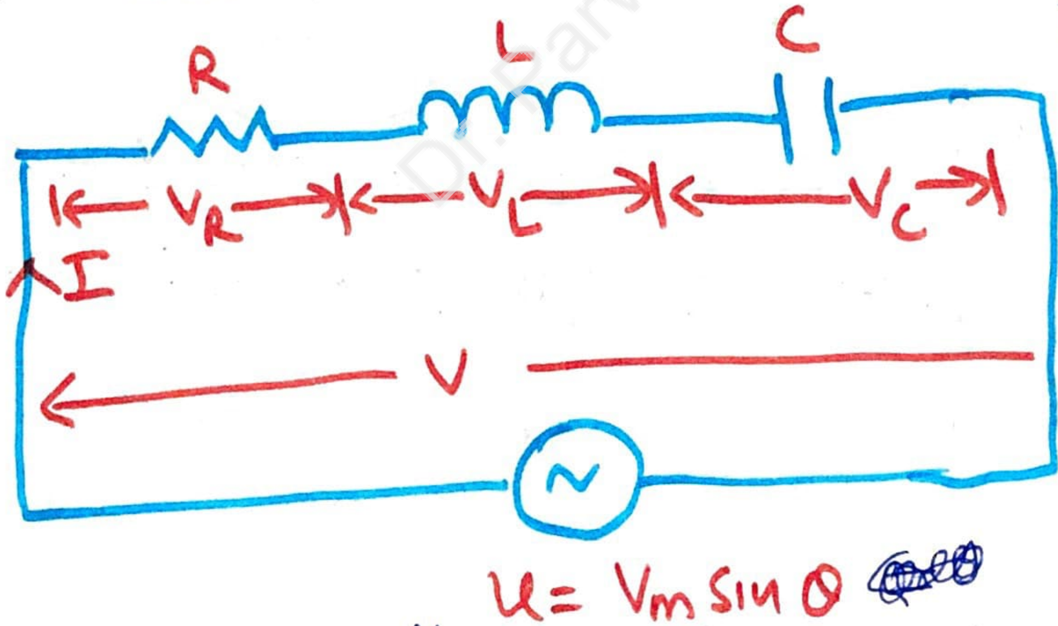
i.e the circuit behaves as a pure Resistive circuit.

i.e $Z = R + jX$; at resonance; $Z = R$

$Y = G + jB$; at resonance, $Y = G$

Resonance in RLC circuit

Let us consider a series RLC circuit as shown below:



{SERIES RESONANT CIRCUITS ARE the Acceptor circuits}

V and I are the RMS values of v and i

v and i are the instantaneous quantities (voltage) and (current)

$V_R \Rightarrow$ RMS voltage across R

$V_L \Rightarrow$ RMS voltage across L

$V_C \Rightarrow$ RMS voltage across C

In a series circuit, the resonance is known as "Series Resonance" or "Voltage Resonance" (3)

The current "I" is the common parameter.

$V_L = I X_L$ $\quad\quad X_L = 2\pi f L$ (Inductive Reactance)

$V_R = IR$

$V_C = I X_C$ $\quad\quad X_C = \frac{1}{2\pi f C}$ (Capacitive Reactance)

Net Reactance in the circuit is : $\boxed{X = X_L - X_C}$

Total Impedance of the circuit; $Z = R + j(X_L - X_C)$

$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

Now; ~~according~~ based on net reactance, we have three cases in this circuit.

1) If $X_L - X_C > 0$ (i.e $X_L > X_C$) : Inductive circuit.

2) If $X_L - X_C < 0$ (i.e $X_L < X_C$) : capacitive circuit.

3) If $X_L - X_C = 0$ (i.e $X_L = X_C$) then

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + (0)^2}$$

$$= \sqrt{R^2}$$

$\boxed{Z = R}$ ⇐ circuit is behaving as pure resistive circuit.

Hence, at $X_L = X_C$; the given RLC series AC circuit behaves as a pure resistive circuit. So, $X_L = X_C$ is also the condition for resonance in series RLC ac circuit.

Hence, we can say that:
In series RLC ac circuit, "resonance is the condition that occurs when the inductive reactance (X_L) is equal to the capacitive reactance (X_C)"

At resonance (in series RLC ac circuit)

$$X_L = X_{L0} \quad \text{and} \quad X_C = X_{C0}$$

Impedance is minimum and is given by

$$\boxed{Z_0 = R}$$

At resonance, the circuit impedance is known as Dynamic Impedance (Z_0)

At resonance,

$$X_{L0} = X_{C0}$$

or

$$I X_{L0} = I X_{C0}$$

$$\boxed{V_{L0} = V_{C0}}$$

where V_{L0} = RMS voltage across Inductor at resonance and, V_{C0} = RMS voltage across capacitor at resonance.

Also, Total voltage, $V = V_R + j(V_{L0} - V_{C0})$

or $|V| = \sqrt{V_R^2 + (V_{L0} - V_{C0})^2}$

$\because V_{L0} = V_{C0}$

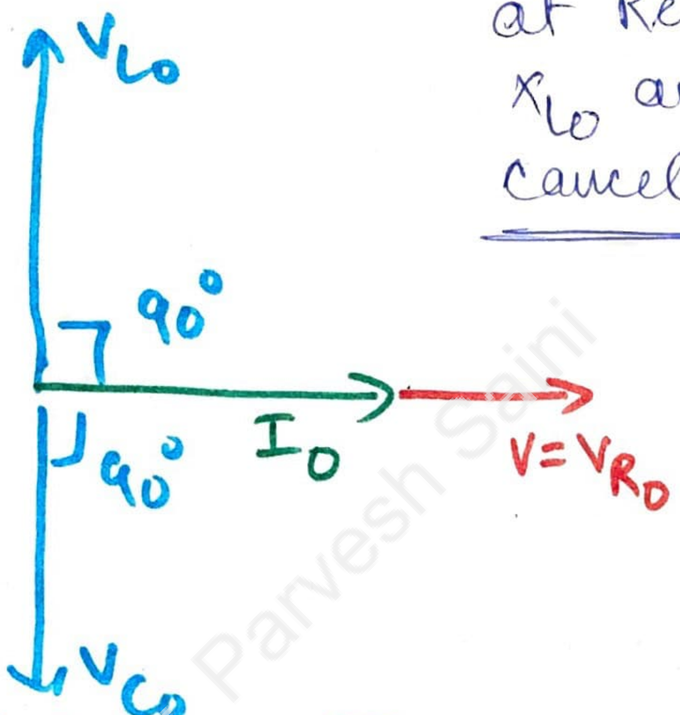
Hence, $V = \sqrt{V_R^2 + 0}$

$$\boxed{V = V_R}$$

At resonance; $\boxed{V_R = I_0 R}$

$I_0 \Rightarrow$ current in the RLC series ac circuit at resonance.

Phasor Diagram at Resonance



at Resonance X_{L0} and X_{C0} cancel each other

Resonant frequency :-

It is the frequency at which Resonance occurs. It is denoted by f_0 (or f_r).

The expression of resonant frequency is described as:

At Resonance,

$$X_{L0} = X_{C0}$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$2\pi f_0 L \times 2\pi f_0 C = 1$$

$$4\pi^2 f_0^2 LC = 1$$

or $$4\pi^2 f_0^2 = \frac{1}{LC}$$

$$f_0^2 = \frac{1}{4\pi^2 LC}$$

$$f_0 = \sqrt{1/4\pi^2 LC}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The Resonant frequency is given by :

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec.}$$

From the above expressions; it is observed that: the series resonance effect may be produced by either of the following two ways:

(a) vary the value of frequency (f) and keep the values of "L" and "C" constant.

(b) By keeping "f" constant and vary "L" or "C" (or both).

Summary of Series resonant AC Circuit

At resonance, a series RLC ac circuit has following characteristics

(i) $\underline{V}$ and I are in same phase (i.e $\phi = 0°$)

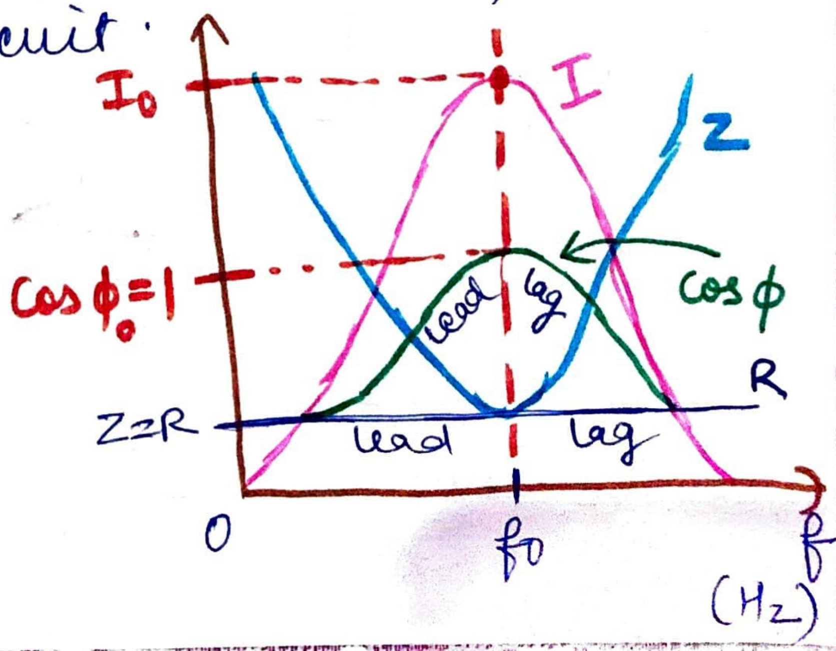
(ii) power factor, $\cos\phi_0 = 1$ (i.e unity power factor)

(iii) Z_0 is minimum ($Z_0 = R$). Hence $I_0 = \frac{V}{Z_0}$ is maximum

(iv) $X_{L0} = X_{C0}$; $V_{L0} = V_{C0}$; and maximum power dissipation in the circuit.

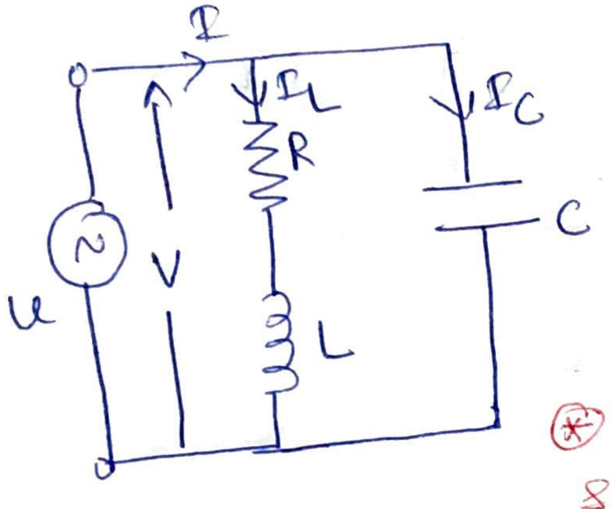
$\Longrightarrow$

The diagram shows the variation of various parameters with frequency and their values at resonance (i.e at $f = f_0$)



Resonance in parallel AC RLC circuit

The most common form of parallel resonant circuit in ~~parallel~~ practical use is shown below:
(Known as Tank circuit)



Tank circuit

In practical tank circuit, at resonance, L and C exchange the energy between each other.

(*) Tank circuit is used to stabilize the electrical frequency of an AC oscillator circuit.

In many ways a parallel resonance circuit is exactly the same as the ~~set~~ series resonance circuit.

In a practical tank circuit as shown above, a coil of Inductance (L) and series resistance (R) is connected in parallel with a capacitor.

At resonance : (a) The resultant current (I) is in phase with the supply voltage (V).

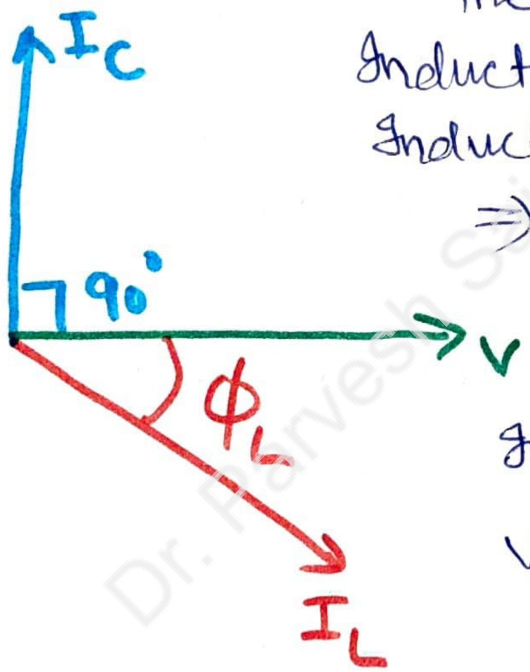
(b) The parallel ac circuit produces parallel resonance (also known as Anti-Resonance) (or Current Resonance)

(c) The current starts circulating between L and C, due to the exchange of energy.

A parallel resonant circuit stores the energy of the circuit as magnetic field of the inductor and as electric field in a capacitor.

This energy is constantly transferred back and forth between the inductor and capacitor at Resonance.

Phasor diagram of the practical tank circuit



The R-L branch is Inductive but not pure Inductor. Hence

$\Rightarrow I_L$ lags V by angle ϕ

In capacitor, I_C leads V by angle $90°$.

RESONANT FREQUENCY OF TANK CIRCUIT:

The expression for resonant frequency in parallel RLC tank circuit can be derived as:→

Admittance of the circuit is

$$Y = \frac{1}{Z} = \frac{1}{R+j\omega L} + j\omega C \quad \text{—(1)}$$

or $Y = j\omega C + \dfrac{R-j\omega L}{R^2+\omega^2 L^2}$

or, $Y = \underbrace{\dfrac{R}{R^2+\omega^2 L^2}}_{\text{Real}} + j\omega\underbrace{\left(C - \dfrac{L}{R^2+\omega^2 L^2}\right)}_{\text{Imaginary}}$ —(2)

Now, at resonance, the imaginary term is taken as ZERO and $w = w_0$. Hence

$$jw\left(C - \frac{L}{R^2 + w_0^2 L^2}\right) = 0$$

or $$C - \frac{L}{R^2 + w_0^2 L^2} = 0$$

or, $$R^2 + w_0^2 L^2 = \frac{L}{C}$$

or, $$w_0^2 L^2 = \frac{L}{C} - R^2$$

or, $$w_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

or $$w_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{--- (3)}$$

Now

$$w_0 = 2\pi f_0.$$

Hence

$$2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{--- (4)}$$

Now, if R is small then $\frac{R^2}{L^2} << \frac{1}{LC}$, hence it is neglected. So,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - 0} = \frac{1}{2\pi\sqrt{LC}}$$

and

$$w_0 = \sqrt{\frac{1}{LC} - 0} = \frac{1}{\sqrt{LC}}$$

Admittance of the given practical tank circuit at resonance is:

$$Y = Y_0, \quad w = w_0.$$

Hence, from eq^n (2) we have

$$Y_0 = \frac{R}{R^2 + w_0^2 L^2} \quad \text{--- (5)}$$

Impedance of given tank circuit at resonance is

$$Z_0 = \frac{1}{Y_0} = \frac{R^2 + w_0^2 L^2}{R}$$

$$Z_0 = R + \frac{w_0^2 L^2}{R}$$

But $w_0^2 L^2 = \frac{L}{C} - R^2$

Hence, $Z_0 = R + \frac{1}{R}\left(\frac{L}{C} - R^2\right)$

$$Z_0 = \cancel{R} + \frac{L}{RC} - \cancel{R} = \frac{L}{RC}$$

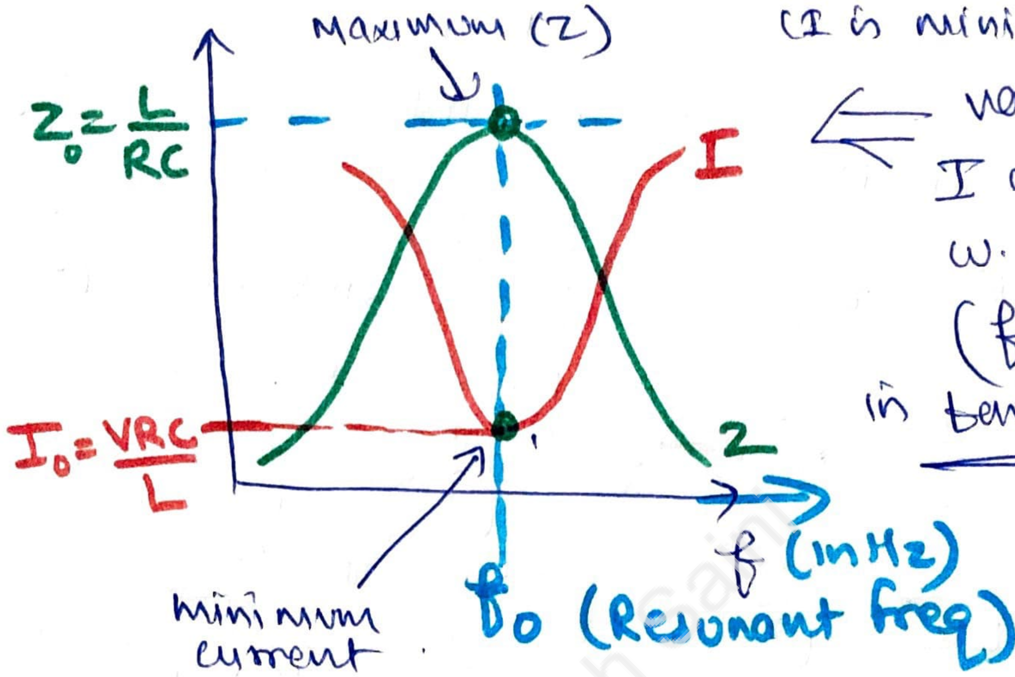
$$\boxed{Z_0 = \frac{L}{RC}}$$

$Z_0 \in$ dynamic Impedance.

The resultant current at resonance is

$$\boxed{I_0 = \frac{V}{Z_0} = \frac{V}{L/RC} = \frac{VRC}{L} \qquad I_0 = \frac{VRC}{L}}$$

In such circuits, at Resonance Impedance (Z) is maximum and current (I is minimum)



⟸ variation of I and Z w.r.t. frequency (f) in tank circuit

Applications of Resonance :

1. The most common use of resonance effect is tuning. for example : when we tune a radio to a particular station, the LC circuits are set at resonance for that particular carrier frequency.
2. Series Resonant circuit provides voltage magnification.
3. Parallel resonant circuit provides current magnification.
4. A parallel resonant circuit can be used as load impedance in output circuits of RF amplifiers because due to high impedance at resonance, the gain of the amplifier is maximum.
5. Both series and parallel resonant circuits are used in Induction heating.
6. These can also be used as filters (to filter out frequency)