## 13.18 FORMULAE OF LAPLACE TRANSFORM

| S.No. | $f(t)$ | $F(s)$ |
|---|---|---|
| **1.** | $e^{at}$ | $\frac{1}{s-a}$ |
| **2.** | $t^n$ | $\frac{\lceil n+1}{s^{n+1}}$ or $\frac{n!}{s^{n+1}}$ |
| **3.** | $\sin at$ | $\frac{a}{s^2+a^2}$ |
| **4.** | $\cos at$ | $\frac{s}{s^2+a^2}$ |
| **5.** | $\sinh at$ | $\frac{a}{s^2-a^2}$ |
| **6.** | $\cosh at$ | $\frac{s}{s^2-a^2}$ |
| **7.** | $U(t-a)$ | $\frac{e^{-as}}{s}$ |
| **8.** | $\delta(t-a)$ | $e^{-as}$ |
| **9.** | $e^{bt}\sin at$ | $\frac{a}{(s-b)^2+a^2}$ |
| **10.** | $e^{bt}\cos at$ | $\frac{s-b}{(s-b)^2+a^2}$ |
| **11.** | $\frac{t}{2a}\sin at$ | $\frac{s}{(s^2+a^2)^2}$ |
| **12.** | $t\cos at$ | $\frac{s^2-a^2}{(s^2+a^2)^2}$ |
| **13.** | $\frac{1}{2a^3}(\sin at - at\cos at)$ | $\frac{1}{(s^2+a^2)^2}$ |
| **14.** | $\frac{1}{2a}(\sin at + at\cos at)$ | $\frac{s^2}{(s^2+a^2)^2}$ |

## 13.19 PROPERTIES OF LAPLACE TRANSFORM

| S.No. | Property | $f(t)$ | $F(s)$ |
|---|---|---|---|
| 1. | Scaling | $f(at)$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ a $>0$ |
| 2. | Derivative | $\frac{df(t)}{dt}$ | $sF(s)-f(0)$ $s>0$ |
| | | $\frac{d^2f(t)}{dt^2}$ | $s^2F(s)-sf(0)-f'(0)$ $s>0$ |
| | | $\frac{d^3f(t)}{dt^3}$ | $s^3F(s)-s^2f(0)-sf'(0)-f''(0)$ |
| 3. | Integral | $\int_0^t f(t)\,dt$ | $\frac{1}{s}F(s)$ $s>0$ |
| 4. | Initial Value | $\lim_{t\to 0} f(t)$ | $\lim_{s\to\infty} sF(s)$ |
| 5. | Final Value | $\lim_{t\to\infty} f(t)$ | $\lim_{s\to 0} sF(s)$ |
| 6. | First shifting | $e^{-at}f(t)$ | $F(s+a)$ |
| 7. | Second shifting | $f(t)\,U(t-a)$ | $e^{-as}\,\mathrm{L}f(t+a)$ |
| 8. | Multiplication by $t$ | $tf(t)$ | $-\frac{d}{ds}F(s)$ |
| | | $t^nf(t)$ | $(-1)^n\frac{d^n}{ds^n}F(s)$ |
| 9. | Division by $t$ | $\frac{1}{t}f(t)$ | $\int_s^\infty F(s)\,ds$ |
| 10. | Periodic function | $f(t)$ | $\frac{\int_0^T e^{-st}f(t)}{1-e^{-st}}$ *ii*RçQDÇÔçÿ) $f(t+T)=f(t)$ |
| 11. | Convolution | $f(t)*g(t)$ | $F(s)\,G(s)$ |