GRAPHIC ERA HILL UNIVERSITY
Department of Mathematics
TMA-316 : Discrete Mathematical Structures and Combinatorics
(Assignment No: 4)

Last Date of Submission : 20-Nov-2023

1. Which of the followings are statements :-

   a) The earth is round.　　b) $2 + 3 = 5$.　　　　c) Do you speak English ?

   d) $3 - x = 5$.　　　　　e) Take two aspirins.　　f) The sun will come out tomorrow.

   g) Study logic.　　　　　h) $x^2 + x + 1 = 0$.　　　i) If stock price fall, I will lose money.

2. Give the negations of the following statements :-

   a) $p : 2 + 3 > 1$,　　　　b) q : It is cold,　　　c) $2 + 7 \leq 11$,　　　d) 2 is even integer.

3. Find the truth tables of the followings :-

   a) $\sim p \wedge q$,　　　　　　b) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$,　　　c) $(p \wedge (q \wedge r)) \vee \sim ((p \vee q) \wedge (r \vee s))$,

   d) $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r))$,　　　e) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$.

4. Consider the followings :-
   $$p : \text{Today is Tuesday}, \qquad q : \text{It is raining}, \qquad r : \text{It is cold}.$$
   Write in simple sentences the meaning of the followings :-

   a) $q \Rightarrow r$,　　b) $\sim q \Rightarrow (r \wedge p)$,　　c) $\sim p \Rightarrow (q \vee r)$,　　d) $(p \vee q) \Leftrightarrow r$,　　e) $(p \wedge \sim q) \Rightarrow \sim r$.

5. Show that the following propositions are tautologies :-

   a) $p \vee \sim p$,　　　　b) $\sim (p \vee q) \vee [(\sim p) \wedge q] \vee p$,　　c) $(p \wedge q) \Rightarrow p$,

   d) $\sim p \Rightarrow (p \Rightarrow q)$,　　e) $[p \wedge (p \Rightarrow q)] \Rightarrow q$,　　　f) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$,

   g) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$.

6. Check whether the following propositions are logically equivalent or not :-

   a) $\sim (p \wedge q)$ and $\sim p \vee \sim q$,　　b) $p \wedge (\sim q \vee q)$ and p,　　　　　c) $p \vee (q \wedge \sim q)$ and p,

   d) $p \wedge q$ and $q \wedge p$,　　　　f) $\sim (p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$,　　e) $p \vee (p \wedge q)$ and q,

   g) $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$,　　　　h) $(p \vee q) \Rightarrow r \equiv (p \Rightarrow r) \wedge (q \Rightarrow r)$.

7. Write an equivalent formula for $p \wedge (q \Leftrightarrow r) \vee (r \Leftrightarrow p)$ which does not involve biconditional.

8. Establish the equivalence $p \Rightarrow q \vee r \equiv p \wedge \sim p \Rightarrow r \equiv p \wedge \sim r \Rightarrow q$.
   Hence rewrite the following sentence in two different ways :-
   $$\text{If } n \text{ is prime then } n \text{ is odd or } n \text{ is 2.}$$

9. Obtain the disjunctive normal form of the followings :-

   a) $\sim (p \Rightarrow (q \wedge r))$,　　　　b) $\sim (p \vee q) \Leftrightarrow p \wedge q$,　　　c) $\sim (\sim (p \Leftrightarrow q) \wedge r)$,

   d) $(\sim p \Rightarrow r) \wedge (p \Leftrightarrow q)$,　　　e) $(p \wedge \sim (q \wedge r)) \vee (p \Rightarrow q)$.

10. Obtain the conjunctive normal form of the followings :-

    a) $\sim ((p \vee \sim q) \wedge \sim r)$,　　　　b) $\sim (p \vee q) \Leftrightarrow p \wedge q$,　　　c) $(\sim p \Rightarrow r) \wedge (q \Leftrightarrow p)$,

    d) $(p \wedge q) \vee (\sim p \wedge q) \vee (q \wedge r)$,　　e) $(q \vee (p \wedge r)) \wedge \sim ((p \vee r) \wedge q)$.

11. Using truth table and without using truth table, find the principal disjunctive normal forms of the followings :-
    a) $p \vee q$,   b) $p \Leftrightarrow q$,   c) $\sim (p \wedge q)$,   d) $(q \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$.

12. Using truth table and without using truth table, find the principal conjunctive normal forms of the followings :-
    a) $q \wedge (p \vee \sim q)$,   b) $(p \wedge q) \vee (\sim q \wedge r)$,   c) $\sim (p \wedge q)$,   d) $(q \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$.

13. Show that $p \Rightarrow (s \vee t)$ is a valid conclusion from the premises $p \Rightarrow (q \vee r), q \Rightarrow s$ and $r \Rightarrow t$.

14. Show that $t \Rightarrow s$ is a valid conclusion from the premises $(p \wedge q) \vee (r \Rightarrow s), t \Rightarrow r, \sim (p \wedge q)$.

15. Show that $p \Rightarrow \sim r$ is a valid conclusion from the premises $p \Rightarrow q, r \Rightarrow \sim q$.

16. Show that t is a valid conclusion from the premises $\sim p \wedge q, r \Rightarrow p, \sim r \Rightarrow s$ and $s \Rightarrow t$.

17. Chek the validity of the following arguments :-
    a) "If Shalini has completed MCA or MBA, then she is assured of a good job. If Shalini is assured a good job, she is happy. Shalini is not happy. So Shalini has not completed MBA".
    b) "If I study, I will pass in examination. If I do not go to cinema, then I will study. But I failed in examination. Therefore, I went to cinema".
    c) "If today is Sunday, then yesterday was Saturday. Yesterday was Saturday. Therefore, today is Sunday".
    d) "If I drive to work then I will arrive in time. I do not drive to work. Therefore, I will not arrive in time".
    e) "If I try hard and I have a talent, then I will become a mathematician. If I become mathematician, then I will be happy. Therefore, If I will not be happy, then I did not try hard or I do not have talent".

18. Write short note (define with suitable examples) on the followings:-
    a) Propositions,   b) Compound propositions,   c) Logical Equivalence,   c) Tautology,
    d) Contradiction   d) Fallacy argument,   e) Derived Connectives (NAND, NOR & XOR).

19. Using the rules of inference, determine whether the following inference patterns are valid or not :-
$$\sim t \Rightarrow \sim r, \ \sim s, \ t \Rightarrow w; \text{ i.e. } \frac{r \vee s}{w}.$$

20. Determine whether the conclusion C is valid in the following premises :-
$$H_1 : P \Rightarrow (Q \Rightarrow R); \ \ H_2 : P \wedge Q; \ \ C : R.$$

21. If p : *Today is Monday*;
    q : *The grass is wet*;
    r : *The dish ran away with the spoon.*

    Write an English sentence that corresponds to each of the following :-

    a) $\sim r \wedge q$,   b) $\sim q \vee r$,   c) $\sim (p \vee q)$, $p \vee \sim r$.

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