GRAPHIC ERA HILL UNIVERSITY

Department of Mathematics

TMA-316 : Discrete Mathematical Structures and Combinatorics

(Assignment No: 1)

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1. Define a set and give examples to illustrate the difference between a collection and a set. What are the different ways to specify a set ?

2. Define Countable and Uncountable Sets.

3. Show that the set Z of all integers is countable. Also Show that every infinite set has a denumerable (countable infinity) subset.

4. Prove that sets of real numbers and sets of complex numbers are uncountable.

5. Let A be a finite set. Identify, whether the followings are true or not ?
   a) $A \subseteq P(A)$ b) $A \in P(A)$ c) $\phi \subseteq P(A)$ d) $\phi \in P(A)$ e) $\phi \subseteq A$ f) $\phi \in A$.

6. For a set A, followings are true or not ?
   a) $\phi \in 2^A$ b) $\phi \subseteq 2^A$ c) $A \in 2^A$ d) $A \subseteq 2^A$ e) $A \in 2^A$ f) $|A| \leq |2^A|$.

7. Determine $|P(P(P(P(\phi))))|$.

8. Uf $A = \{\text{Fine, Yang}\}$ and $B = \{\text{president, vice-president, secretary, treasurer}\}$, write:-
   a) $A \times B$, b) $B \times A$, c) $A \times A$, d) $A \times \phi$, e) $\phi \times A$.

9. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{5, 6, 7\}$, $A_3 = \{8, 9, 10\}$, $A_4 = \{4, 8, 10\}$, $A_5 = \{4, 5, 7, 9\}$, $A_6 = \{1, 2, 3, 6, 8, 10\}$. Which of the following are partitions of A ?
   a) $\{A_1, A_2, A_5\}$, b) $\{A_1, A_3, A_5\}$, c) $\{A_3, A_6\}$, d) $\{A_2, A_3, A_4\}$, e) $\{A_2, A_4\}$.

10. Let $A = \{a, b, c, d, ..., z\}$. Give the partition $\mathcal{P}$ of A such that $|\mathcal{P}| = 4$ and one element of $\mathcal{P}$ contains only the letters needed to spell your first name.

11. Let $A = \{a, b, c, d, ..., z\}$. Give the partition $\mathcal{P}$ of A such that $|\mathcal{P}| = 3$ and each element of $\mathcal{P}$ contains at-least five elements.

12. The number of partitions of a set with n elements into k subsets satisfies the recurrence relation $S(n, k) = S(n-1, k-1) + k.S(n-1, k)$ with initial conditions $S(n, 1) = S(n, n) = 1$. Find the number of partitions of a set with three elements into two subsets, i.e. $S(3, 2)$. Hence, verify your result on a set $A = \{1, 2, 3, 4\}$.

13. Which of the followings are not true ?
    a) $A - B = A \cap B^c$ b) $A - (A - B) = A \cap B$ c) $A - B = A \cap B^c$ d) $A \cap B = B$
    e) $B \cup (A \cap B) = B$ f) $(A \cap B^c) \cup (A \cap B) = A$ g) $A - (A \cap B) = A - B$ h) $B^c \subset A^c$
    i) $B - A = \phi$ j) if $A \subset \phi$, then $A = \phi$ h) $A - (A \cap B) = B$ i) $A \cup B = A$.

14. Which of the followings are not true ?
    a) $(A - B) - C = A - (C - B)$ b) $(A - B) - C = (A - C) - B)$
    c) $(A \cup B) - (B \cup C) = A - (A \cup C) - (A - B)$ d) $(A - B) - C = A - (B \cup C)$

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15. Find the cardinality of a set of integers, defined as $X = \{n | 1 \leq n \leq 123, n \text{ is divisible by 2 or 3}\}$.

16. Find the cardinality of a set of integers, defined as $X = \{n | 1 \leq n \leq 123, n \text{ is divisible by 2 and 3}\}$.

17. Find the cardinality of the power set of the set $\{0, 1, 2, ..., 10\}$.

18. Write down the following sets:
    a) $A = \{x : x^2 = 4\}$, b) $B = \{x : x^2 = 9, x - 3 = 5\}$,
    c) $C = \{x : x^2 + 1 = 0, x \text{ is a complex number}\}$, d) $D = \{x : x^2 + 1 = 0, x \text{ is a real number}\}$.

19. Which of the following sets are equal ? Also examine their nature (*i.e.* null, singleton etc.)
    a) $A = \{x : x^2 = 11, x \text{ is an even integer}\}$, b) $B = \{0\}$, c) $C = \{x : x + 6 = 6\}$,
    d) $D = \{x : x^2 = 7, 3x = 5\}$, e) $E = \{x : x \in N, 5 < x < 14\}$.

20. Determine the power sets of the following sets:
    a) $\{a\}$, b) $\{\{a\}\}$, c) $\{\phi, \{\phi\}\}$.

21. Let $A = \{\phi, b\}$, construct the following sets:
    a) $A - \phi$, b) $\{\phi\} - A$, c) $A \cup P(A)$, d) $\phi - A$.

22. Show that union of two sets is: (where symbol U stands for universal set)
    a) commutative, b) Associative, c) Idempotent, d) $A \cup \phi = A$, e) $U \cup A = U$.

23. Show that intersection of two sets is:
    a) commutative, b) Associative, c) Idempotent, d) $A \cap U = A$, e) $A \cap \phi = \phi$.

24. Simplify the expression $[\{(A \cup B) \cap C\}^c \cup B^c]^c$, where superscript c represents the compliment.

25. Show that union is distributive over intersection
    $$\text{i.e.} \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

26. Show that intersection is distributive over union
    $$\text{i.e.} \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

27. Let $A_n = \{x : x \text{ is divisible by } n\}$, where $n \in N$. Find $A_3 \cap A_5$ and $A_3 \cup A_5$.

28. Prove the followings:
    a) $A - B = A \Leftrightarrow A \cap B = \phi$, b) $A - B = \phi \Leftrightarrow A \subseteq B$,
    c) $A - (A - B) = A \cap B$, d) $(A - C) \cap (B - C) = (A \cap B) - C$,
    e) $A - (A \cap B) = A - B$, f) $A \cap (B - C) = (A \cap B) - (A \cap C)$,
    g) $A \oplus B$ or $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

29. Prove that $P(A \cap B)$ is equal to $P(A) \cap P(B)$.

30. In a class of 25 students, 12 have taken economics, 8 have taken economics but not political science. Find the number of students who have taken economics and political science and those who have taken political science but not economics.

31. Among the first 500 positive integers,
    a) Determine the integers which are neither divisible by 5, 7 nor 9.
    b) Determine the integers which are divisible by 5 but not by 7 and 9.

32. In a group 200 people, each of whom is at-least accountant or management consultant or sales-manager, it was found that 80 are accountants, 110 are management consultants, 130 are sales-managers, 25 are accountants as-well-as sales-managers, 70 are management consultants as-well-as sales managers, 10 are accountants, management consultants as-well-as sales managers. Find the number of those people who are accountant as-well-as management consultants but not sales-managers.

33. Let $A = \{1, 2, 4, a, b, c\}$, identify each of the following as true or false :-
    a) $2 \in A$, b) $3 \in A$, c) $c \notin A$, d) $\phi \in A$, e) $\{\} \notin A$, f) $A \in A$.

34. Let $A = \{x : x$ is a real number and $x < 6\}$, identify each of the following as true or false :-
    a) $3 \in A$, b) $6 \in A$, c) $5 \notin A$, d) $8 \notin A$, e) $-8 \in A$, f) $3.4 \notin A$.

35. In each part, give the set by listing its elements :-
    a) The set of all positive integers that are less than 10, b) $A = \{x : x \in \mathbb{Z}$ is and $x^2 < 12\}$,
    c) AARDVARK, d) BOOK, e) MISSISSIPPI.

36. Let $A = \{1, \{2, 3\}, 4\}$. Identify each the following as true or false : -
    a) $3 \in A$, b) $\{1, 4\} \subseteq A$, c) $\{2, 3\} \subseteq A$, d) $\{2, 3\} \in A$, e) $\{4\} \in A$, f) $\{1, 2, 3\} \subseteq A$.

37. Let $A = \{1\}, B = \{1, a, 2, b, c\}, C = \{b, c\}, D = \{a, b\}$ and $E = \{1, a, 2, b, c, d\}$. Each of the following part, replace the symbol $\diamond$ with either $\subseteq$ or $\nsubseteq$ to give a trure statement: -
    a) $A \diamond B$, b) $\phi \diamond A$, c) $B \diamond C$, d) $C \diamond E$, e) $D \diamond C$, f) $B \diamond E$.

38. If $P(B) = \{\{\}, \{m\}, \{n\}, \{m, n\}\}$, then find B.

39. If $A = \{3, 7, 2\}$, find :- a) $P(A)$, b) $|A|$, c) $|P(A)|$.

40. Find all the partition of $X = \{a, b, c, d\}$.

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End of the assignment..