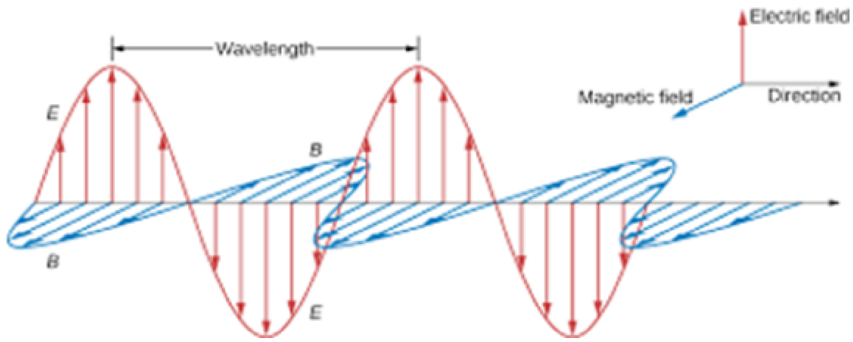
By Dr. Vishal Chauhan

Polarisation

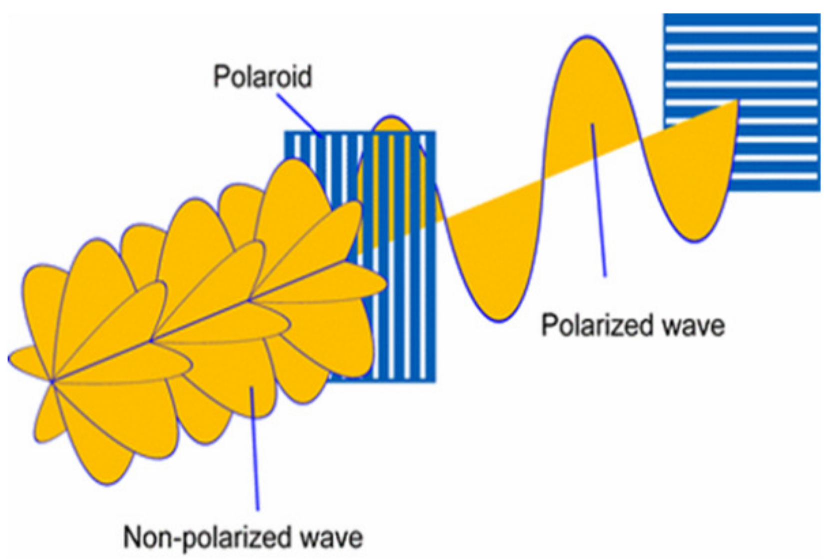
Propagation of light waves:

Initially, Huygens proposed that light waves are longitudinal waves, but later it was found out that light waves are transverse in nature which means particles move perpendicular to the direction of wave.



Natural sunlight and almost every other form of artificial illumination transmits light waves whose electric field vectors vibrate in all perpendicular planes with respect to the direction of propagation. When the electric field vectors are restricted to a single plane by filtration, then the light is said to be polarized with respect to the direction of propagation and all waves vibrate in the same plane.

Polarization is a phenomenon peculiar to transverse waves. Longitudinal waves such as sound cannot be polarized. Light and other electromagnetic waves are transverse waves made up of mutually perpendicular, fluctuating electric and magnetic fields.



Unpolarized light:

Light is called unpolarized if the electric field fluctuates symmetrically in all directions in a plane perpendicular to the direction of propagation of light. Many common light sources such as sunlight, halogen lighinting, LED spotlights, and incandescent bulbs produce unpolarized light.

Polarized light:

If the direction of the electric field of light is well defined, it is called polarized light. The most common source of polarized light is a laser.

Double Refraction

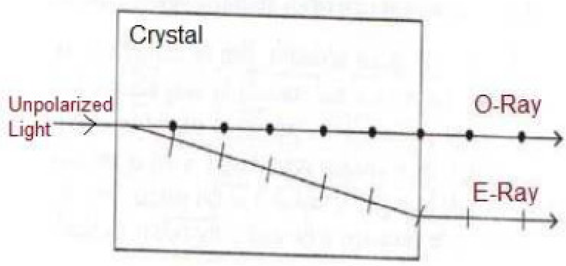
Light passing through calcite crystal is split into two rays. This process, first reported by Erasmus Bartholinus in 1669, is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular.

Ordinary and Extraordinary rays

For normal incidence (a 0° angle of incidence), Snell's law predicts that the angle of refraction will be 0°. In the case of double refraction of a normally incident ray of light, at least one of the two rays must violate Snell's Law as we know it. For calcite, one of the two rays does indeed obey Snell's Law; this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray).

- All transparent crystals except those in the cubic system have the property of double refraction. For most crystals the image separation is not large enough to be visible. However, we will observe other optical properties that result from the double refraction.
- For hexagonal and tetragonal crystals, there will be one O-ray and one E-ray.
- For orthorhombic, monoclinic, and triclinic crystals, there will be two E-rays. In general, the refractive indices for non-cubic crystals depend on vibration direction.
- Non-cubic crystals are, therefore, said to be optically anisotropic.

  In most cases the refractive indices for the two rays produced by double refraction are not the same. One of the two rays will have a higher refractive index (and a lower velocity); this ray is called the slow ray. The other ray is then the fast ray.



(Double Refraction phenomenon)

If a beam of light incident normally on a double refracting crystal then the o-ray passes through the crystal without any deviation. While e-ray deviates inspite of normal incidence.

The velocities of o-ray and e-ray are different in all the directions except in one particular direction known as optic axis. In this optic-axis, both o-ray and e-ray travel with same velocities.

The o-ray has same velocity in all direction while e-ray is changed if the refractive index is changed.

Both o-ray and e-ray are linearly polarized.

If the refractive index of o-ray is μ_o, then

$$\mu_o = \frac{c}{v_o}$$

c = speed of light in vacuum
v_o = velocity of o-ray in crystal

$$\boxed{v_o = \frac{c}{\mu_o}}$$

If the refractive index of e-ray is μ_e then

$$\mu_e = \frac{c}{v_e}$$

c = speed of light in vacuum
v_e = velocity of e-ray in crystal

$$\boxed{v_e = \frac{c}{\mu_e}}$$

Uniaxial Crystal

- This crystal is described by one optical axis and two principal refractive indices.
- Examples of uniaxial Crystals are calcite, KDP, quartz, rutile etc.
- When light beam passes through such crystal, it splits into o-ray and e-ray (We know that o-ray passes through it without any deviation where as e-ray deviates at air to crystal interface).
- There are two forms of uniaxial crystals viz. positive uniaxial crystal and negative uniaxial crystal.

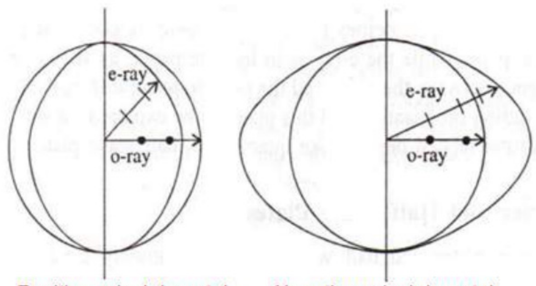
Isotropic medium

Many transparent solids are optically isotropic, meaning that the index of refraction is equal in all directions throughout the crystalline lattice. Examples of isotropic solids are glass, table salt etc.

Anisotropic medium

Anisotropic medium refers to the medium in which the properties are different in all the directions. These have different and inconsistent chemical bonding. These are used for polarisers, wedges. These have many refractive indexes.

Positive Uniaxial crystal vs Negative Uniaxial crystal



Positive uniaxial crystal   Negative uniaxial crystal

- When light is propagated through uniaxial crystal the wavefronts (i.e. Huygen's wavesurfaces) due to o-ray and e-ray are based on relative velocities of o-ray and e-ray following which two cases may arise.

- Case-1: If Vo > Ve (i.e. $\mu_o < \mu_e$) in all the directions except along optical axis. In this case, spherical wavefront due to o-ray would be outside of elliptical wavefront due to e-ray. In this condition, two wavefronts from o-ray and e-ray touch only at two diametrically opposite points on the optical axis. Such crystals are known as positive uniaxial crystals. Examples of such crystals are quartz, rutile etc.

- Case-2: If Vo < Ve (i.e. $\mu_o > \mu_e$), the elliptical wavefront due to e-ray is outside of the spherical wavefront due to o-ray. Such crystals are known as negative uniaxial crystals. Examples of such crystals are calcite, KDP etc.

Retardation plates

The crystal which retard the velocity of either e-ray or o-ray and produce a path difference between e-ray and o-ray are known as retardation plates.

Types of retardation plates

1. **Half wave plates:** The crystal which produce a path difference of $\left(\frac{\lambda}{2}\right)$ between e-ray and o-ray.

2. **Quarter wave plates:** The crystal which produce a path difference of $\left(\lambda/4\right)$ between e-ray and o-ray.

Production and detection of plane, circular, and eliptically polarized light

Superposition of linearly polarized vibrations

suppose the equation of e-ray is $x = a\sin(\omega t + \delta)$ ——— (1)

and the equation of o-ray is $y = b\sin\omega t$ ——— (2)

$\delta \rightarrow$ the path difference between e-ray and o-ray.

From eq^n (1) $\frac{x}{a} = \sin(\omega t + \delta)$ $\quad \sin(A+B) = \sin A.\cos B + \cos A.\sin B$

$$\frac{x}{a} = \sin\omega t.\cos\delta + \cos\omega t.\sin\delta \text{ ——— (3)}$$

From eq^n (2) $\frac{y}{b} = \sin\omega t$ ——— (4)

$$\cos\omega t = \sqrt{1-\sin^2\omega t} \qquad \because \cos^2\theta = 1-\sin^2\theta, \quad \cos\theta = \sqrt{1-\sin^2}$$

$$\cos\omega t = \sqrt{1-\frac{y^2}{b^2}} \text{ ——— (5)}$$

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Put from 4 & 5 in 3

$$\frac{x}{a} = \sin\omega t \cos\delta + \cos\omega t \cdot \sin\delta$$

$$\frac{x}{a} = \frac{y}{b}\cdot\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\cdot\sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\cdot\cos\delta = \sqrt{1-\frac{y^2}{b^2}}\cdot\sin\delta$$

square on both side

$$\left[\frac{x}{a} - \frac{y}{b}\cdot\cos\delta\right]^2 = \left[\sqrt{1-\frac{y^2}{b^2}}\right]^2\cdot\sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - 2\frac{xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\cdot\sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}(\sin^2\delta + \cos^2\delta) - \frac{2xy}{ab}\cos\delta - \sin^2\delta = 0$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta - \sin^2\delta = 0}$$

Resultant equation after superposition

This is also known as the equation of oblique ellipse or the general equation.

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Case-I : if $\delta = 0, 2\pi, 4\pi \ldots 2n\pi$

then $\cos\delta = 1$, $\sin\delta = 0$

$$\because \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\frac{x}{a} = \pm\frac{y}{b}$$

$$\boxed{y = \pm\frac{b}{a}x}$$

Equation of straight line

This means that if there is a phase difference between e-ray and o-ray is $2n\pi$, then emergent light from crystal is a plane polarized light.

Case-II : if $\delta = \pi, 3\pi, 5\pi \ldots (2n+1)\pi$

$\cos\delta = -1$, $\sin\delta = 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

$$\boxed{y = \pm\frac{b}{a}x}$$

Case-III : if $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots (2n+1)\frac{\pi}{2}$

$\cos\delta = 0$, $\sin\delta = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 0 = 1 \Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Hence, emergent light is an elliptically polarized light

if $a = b$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1, \quad \boxed{x^2 + y^2 = a^2}$$

if the amplitude of e-ray and o-ray are equal, the emergent light is circullarly poloarized.

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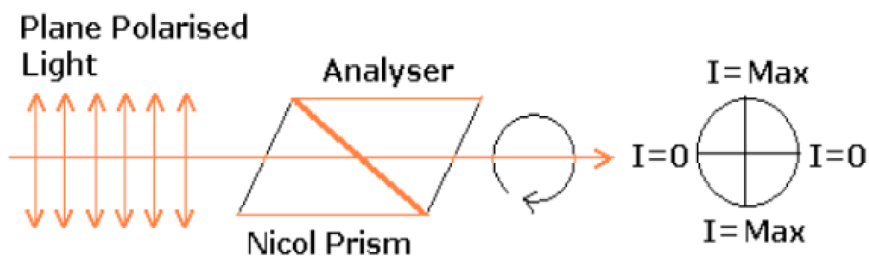
Detection of plane, circular and polarized light

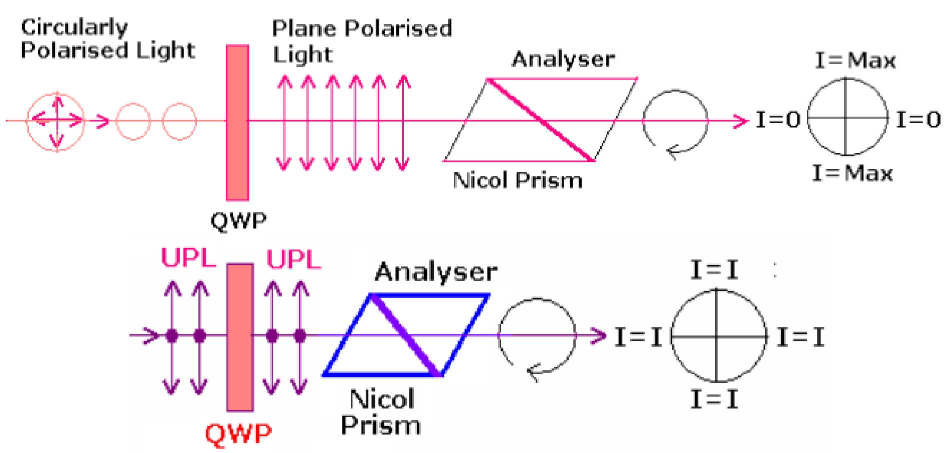
1. Plane polarized light

The given beam of light is passed through a nicol prism (analyser) which is slowly rotated. If the intensity of emerging light varies between maximum and zero twice in every rotation, the given beam of light is plane polarised.



2. Circularly polarized light

If the intensity of the emergent light varies between maximum and zero, the given beam of light is circularly polarised. If the intensity still remains unchanged, the given beam of light is ordinary unpolariosed light.



3. Elliptically Polarized light :

If the intensity of the emergent light varies between maximum and minimum, the given beam of light is either elliptically polarised or a mixture of plane polarised light and ordinary light (partially polarised light).