Solution to Exercise 2: Leaky integrate-and-fire (LIF) neuron

(i) If $V = 0$ at a time zero, it will with a constant input current I reach the threshold $V = \theta$ after at time $t = t^*$ given by

$$\theta = R\,I\left(1 - e^{-t^*/\tau_m}\right) \tag{1}$$

In the present LIF model, the neuron will fire a spike when $V = \theta$, and afterward the potential is immediately reset to $V = 0$. With a constant current input I, the time between each spike will thus according to equation (1) be

$$t^* = -\tau_m \ln\left(1 - \frac{\theta}{RI}\right) = -\tau_m \ln\left(\frac{RI - \theta}{RI}\right) \quad . \tag{2}$$

The firing rate f will be the inverse of the time t^* between two spikes, that is,

$$f = \frac{1}{t^*} = -\frac{1}{\tau_m}\frac{1}{\ln\left(\frac{RI-\theta}{RI}\right)} \quad . \tag{3}$$

This formula looks a bit 'ugly', but we can learn a couple of things about the $f - I$ curve from it prior to plotting:

1. For $RI < \theta$, the argument inside the logarithm becomes a negative number. The logarithm of a negative number is not defined, signalling that the formula is not applicable here. In this situation the current I is so small that the maximum membrane potential $V_{\max} = RI$ is below the threshold. Thus there will be no firing at all and $f = 0$

2. Firing can only occur for $RI > \theta$, so the input current will have to be larger than the threshold current $I_{\text{thresh}} = \theta/R$ for firing to occur.

A sketch of the $f - I$ curve is given in Figure 1. Explanation of the detailed shape requires further analysis (see below), but at least the clear threshold behaviour can be understood from the above argument.

(ii) We can expect the threshold current to be lower as the noise current occasionally will push the membrane potential above threshold, even for input currents that are in themselves too low.

Python exercises:

Example code will be given.

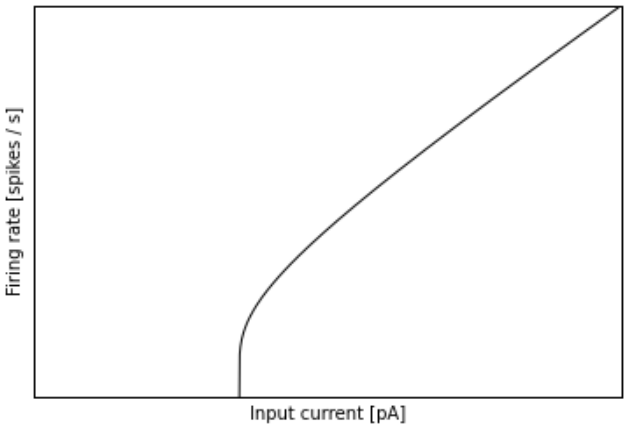

Figure 1: 'Sketch' of $f - I$ curve, cf. problem (i).

(iii) See example of solution in Figure 2.

(iv)–(v) See Figure 3.

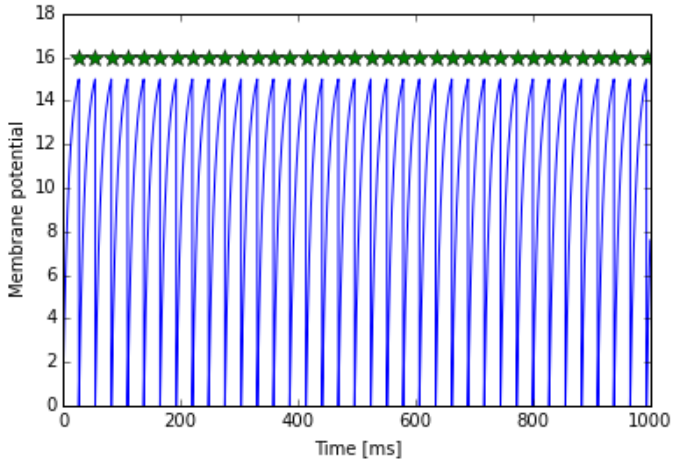

Figure 2: Example of solution in problem (iii). The blue line shows the membrane potential, and the green stars mark the spikes.

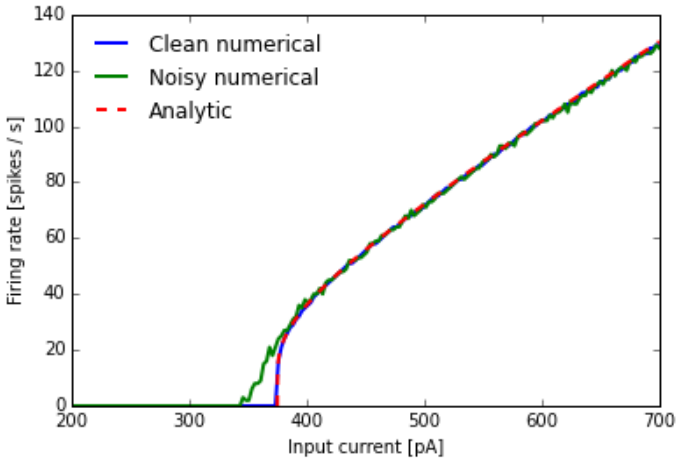

Figure 3: Example of solution of problems (iv) and (v). The red line is from the analytical solution in equation (3), while the blue and green line are numerical solutions from problems (iv) and (v).