Contents

1 Les modèles factoriels (1976-)

1.1 Introduction

1.1.1 v1

Initié par Ross en 1976 dans le cadre de l'APT. L'idée est de supposer que les rendements des actifs financiers s'expliquent par des facteurs.

1. Définition (Les Modèles Factoriel linéaire - forme générale). On suppose que le marché est constitué d'actifs financiers $(S^1, S^2, .., S^d)$. Le modèle suppose l'existence de facteurs $F_1, F_2, .., F_k$ tel que

$$R_{it} = \alpha_i + \sum_{j=1}^{k} b_{ij} f_{jt} + \varepsilon_{it} = \alpha_i + \boldsymbol{b}_i' \mathbf{f_t} + \varepsilon_{it}$$

Où les $b_{i,j}$ sont les sensbilités (factor's exposure, loadings) au facteur F_j et ε_i fournit le risque spécifique. De plus:

$$\begin{aligned} & \mathbb{E}[\mathbf{f_t}] = \mu_f, \quad \text{cov}\left(\mathbf{f}_t\right) = E\left[\left(\mathbf{f}_t - \boldsymbol{\mu}_f\right)\left(\mathbf{f}_t - \boldsymbol{\mu}_f\right)'\right] = \boldsymbol{\Omega}_f \\ \text{(Strong exogeneity)} \quad & \mathbb{E}[\varepsilon_i] = 0 \\ & cov(\varepsilon_{it}, f_{kt}) = 0, \forall k, i, t \\ \text{(Serially uncorrelated)} \quad & cov(\varepsilon_i, \varepsilon_j) = \sigma_i^2; \forall i = j \text{ and } t = s; \\ & \qquad\qquad\quad = 0 \text{ Otherwise} \end{aligned}$$

En utilisant l'hypothèse (1) on obtient que $\mathbb{E}[R^i] = a_i$ ainsi:

$$R_{it} = \mathbb{E}[R^i] + \sum_{j=1}^{k} \boldsymbol{b}_{ij} \mathbf{f}_{jt} + \varepsilon_i; \quad i = 1, .., n$$

- CAPM vs modèle factoriel linéaire. Si on pose que $\forall i$, $\boldsymbol{b_i} = (b_{i1}, .., b_{ik})$ et $\mathbf{f} = (F_1, .., F_k)$. On peut réécrire:

$$\begin{aligned} R_{it} = \mathbb{E}[R_i] + \boldsymbol{b}_i \mathbf{f}' + \varepsilon_i \iff \mathbb{V}[R_{it}] &= Var(\boldsymbol{b}_i \mathbf{f}') + var(\varepsilon_i) \\ &= \underbrace{\boldsymbol{b_i} \Sigma_F \boldsymbol{b_i'}}_{=\text{risk systematique}} + \underbrace{Var(\varepsilon_i)}_{=\text{risk specifique}} \quad ; \quad \Sigma_F = (cov(F_i, F_j))_{j=1,..,d} \end{aligned}$$

Donc les modèles factoriels linéaires sont une généralisation du CAPM/MEDAF avec un nombre plus importants de facteurs qui représentent le risque systématique.

- Modèle factoriel et Portefeuille. On va pouvoir réécrire l'expression du portefeuille constitué d'actifs financiers $(S^1, S^2, .., S^d)$

$$\begin{aligned} R(x) = \sum_{i=1}^{d} R_{it} x_{it} &= \sum_{i=1}^{d} \left(\mathbb{E}\left[R^i\right] + \sum_{j=1}^{K} b_{ij} f_{jt} + \varepsilon_{ij} \right) x_{it} \\ &= \sum_{i=1}^{d} \left(\mathbb{E}\left[R^i\right] x_{it} + \sum_{j=1}^{K} b_{ij} f_{jt} x_{it} + \varepsilon_{ij} x_{it} \right) \\ &= \mathbb{E}\left[R(x)\right] + \sum_{i=1}^{d} \sum_{j=1}^{K} b_{ij} f_{jt} x_{it} + \varepsilon(x) \\ &= \mathbb{E}\left[R(x)\right] + \sum_{j=1}^{K} b_{xj} f_{jt} + \varepsilon(x); \quad b_{xj} = \sum_{i=1}^{d} x_i b_{ij} = x \cdot b_{??} \end{aligned}$$

On peut ensuite calculer la variance du portefeuille:

$$\begin{aligned}\mathbb{V}(R(x)) &= Var(\sum_{j=1}^{K} b_{xj} f_{jt}) + Var(\varepsilon(x)) \\ &= b_x \Sigma_F b_x' + x \Sigma_E x'\end{aligned}$$

Cette décomposition de variance peut ensuite être utilisé comment concurrent à la frontière de Markowitz:

$$\text{(Markowitz)}: \quad \begin{cases} E(R(x)) = R^0 + x\left(\mathbb{E}[R] - R^0\mathbf{1}\right)' \\ \text{Var}(R(x)) = x\Sigma x' \end{cases}$$

Frontière des Fondamentaux:

$$\text{(Fundamentals)}: \quad \begin{cases} E(R(x)) = R^0 + x\left(\mathbb{E}[R] - R^0\mathbf{1}\right)' \\ \text{Var}(R(x)) = b_x \Sigma_F b_x' + x\Sigma_E x' \end{cases}$$

En dehors de l'aspect décomposition le second avantage est calculatoire. En effet si $N = 14,000$ actifs alors la matrice de variance "naive" sera de:

$$[\Sigma]_{N\times N} = cov(R_i, R_j) \Rightarrow N \times N = 890,700$$

Alors que celle utilisant les facteurs si $K = 65$ sera de:

$$[F]_{65\times 65} = cov(f_i, f_j)$$

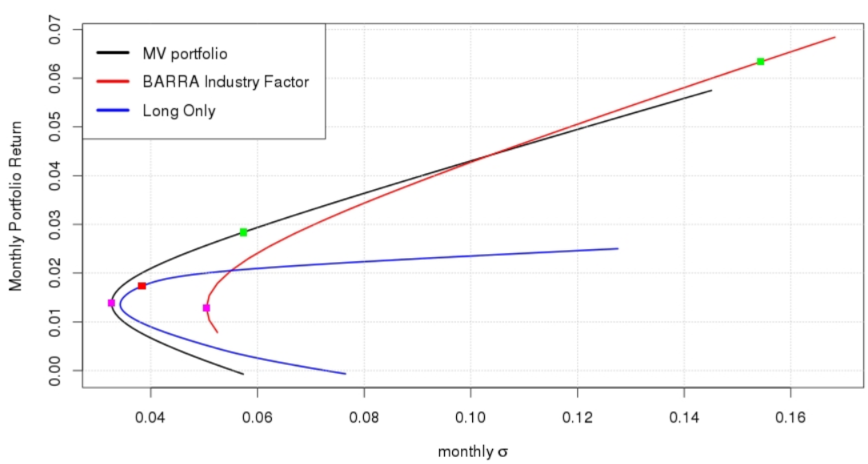


Figure 1: Efficient frontiers for a Mean-variance portfolio vs Barra factor model

2. Définition (Cross-sectional regression). Dans ce cadre on se fixe le temps t avec nos $i = 1, .., N$ actifs avec $\alpha_t = \mathbb{E}[R]$

$$R_t = \alpha_t + B\mathbf{f}_t + \varepsilon_t \iff \begin{array}{c}\text{AAPL}\\ \text{MSFT}\\ \vdots\end{array} \underbrace{\begin{pmatrix} R_{1t} \\ R_{2t} \\ \vdots \end{pmatrix}}_{N\times 1} = \underbrace{\begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \vdots \end{pmatrix}}_{N\times 1} + \begin{array}{c}\\ \text{AAPL}\\ \text{MSFT}\\ \vdots\end{array} \underbrace{\begin{array}{cc} \text{HML} & \text{SMB} \\ \multicolumn{2}{c}{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \end{pmatrix}} \end{array}}_{N\times K} \underbrace{\begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \end{pmatrix}}_{K\times 1} \begin{array}{c}\text{HML}\\ \text{SMB}\\ \vdots\end{array} + \underbrace{\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \end{pmatrix}}_{N\times 1} \begin{array}{c}\text{AAPL}\\ \text{MSFT}\\ \vdots\end{array}$$

Avec R le vecteur des rentabilités, B est la matrice $B = (b_{i,k})_{1\leq i\leq n; 1\leq k\leq K}$, F est le vecteur des facteurs et ε le vecteur des risques idiosyncratiques. Avec $i = 1, .., N$ actifs et le temps de $t = 1, .., T$. Auquel on a la propriété que :

$$E\left[\varepsilon_t \varepsilon_t' \mid \mathbf{f}_t\right] = \mathbf{D} = \operatorname{diag}\left(\sigma_1^2, \ldots, \sigma_N^2\right)$$

On peut lui associer une matrice de variance-covariance:

$$\operatorname{cov}\left(\mathbf{R}_t\right) = \mathbf{\Omega} = \mathbf{B}\mathbf{\Omega}_f \mathbf{B}' + \mathbf{D}$$

3. Définition (Time-series regression). Le modèle multifactoriel peut aussi être réécris sous forme d'une régression en série temporelle pour l'actif $i = 1$ dit "AAPL". Ce qui amène pour $i = 1, .., N$

$$\underset{(T\times 1)}{\mathbf{R}_i} = \underset{(T\times 1)}{\mathbf{1}_T}\underset{(1\times 1)}{\alpha_i} + \underset{(T\times K)}{\mathbf{F}}\underset{(K\times 1)}{\boldsymbol{\beta}_i} + \underset{(T\times 1)}{\boldsymbol{\varepsilon}_i} \iff \underset{T\times 1}{\begin{array}{c} 1990 \\ 1991 \\ \vdots \end{array}\begin{pmatrix} R_{11} \\ R_{12} \\ \vdots \end{pmatrix}} = \underset{T\times 1}{\begin{pmatrix} 1 \\ 1 \\ \vdots \end{pmatrix}} \alpha_{AAPL} + \underset{T\times K}{\begin{array}{cc} & \begin{array}{cc} \text{HML} & \text{SMB} \end{array} \\ \begin{array}{c} 1990 \\ 1991 \\ \vdots \end{array} & \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \end{pmatrix} \end{array}} \underset{K\times 1}{\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \vdots \end{pmatrix}\begin{array}{c} \text{HML} \\ \text{SMB} \\ \vdots \end{array}} + \underset{T\times 1}{\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \end{pmatrix}\begin{array}{c} 1990 \\ 1991 \\ \vdots \end{array}}$$

Avec la propriété de :

$$\text{(serial correlation)} \quad E\left[\varepsilon_i \boldsymbol{\varepsilon}_i'\right] = \sigma_i^2 \mathbf{I}_T$$

4. Définition (Multivariate regression). En collectionant l'ensemble des données de $t = 1, .., T$ du modèle de cross-section permet de:

$$R_t = \alpha_t + B\mathbf{f}_t + \varepsilon_t \iff \underset{N\times 1}{\begin{array}{c} \text{AAPL} \\ \text{MSFT} \\ \vdots \end{array}\begin{pmatrix} R_{1t} \\ R_{2t} \\ \vdots \end{pmatrix}} = \underset{N\times 1}{\begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \vdots \end{pmatrix}} + \underset{N\times K}{\begin{array}{cc} & \begin{array}{cc} \text{HML} & \text{SMB} \end{array} \\ \begin{array}{c} \text{AAPL} \\ \text{MSFT} \\ \vdots \end{array} & \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ \vdots & \vdots \end{pmatrix} \end{array}} \underset{K\times 1}{\begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \end{pmatrix}\begin{array}{c} \text{HML} \\ \text{SMB} \\ \vdots \end{array}} + \underset{N\times 1}{\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \end{pmatrix}\begin{array}{c} \text{AAPL} \\ \text{MSFT} \\ \vdots \end{array}}$$

permet au modèle d'être exprimé sous forme matricielle:

$$\underset{(N\times T)}{\mathbf{R}} = \underset{(N\times 1)}{\boldsymbol{\alpha}} + \underset{(N\times K)}{\mathbf{B}}\underset{(K\times T)}{\mathbf{F}} + \underset{(N\times T)}{\mathbf{E}}$$

5. Nature des facteurs. On peut classer les modèles factoriels selon trois types:

(a) **Les modèles factoriels macroéconomiques.** On se fixe des facteurs observables de l'économie comme des indices boursiers sectoriels, taux de change, d'inflation, taux d'intérêts ... Dans ce type de modèles, les facteurs sont observables, mais les sensibilités b_{ik} doivent être estimées.

- **Exemple.** On dit que si l'inflation augmente de +1% alors on aura une augmentation de +0.2% de la rentabilité moyenne (expected return). Et à l'inverse si le taux d'intérêt augmente de +1% alors on aura une baisse de 0.4% de la rentabilité moyenne.

$$R^i = 0.11 + \underbrace{0.2}_{b_{i1}}\underbrace{(inflation)}_{F_1} - \underbrace{0.4}_{b_{i2}}\underbrace{(\text{interest rates})}_{F_2}$$

(b) **Les modèles factoriels fondamentaux.** Les facteurs proviennent du secteur d'activité d'où les actifs sont issues: taille de la société, taux de dividende, indicateur du secteur d'activité. Eg

- Dans le modèle de type Barra les sensibilités b_{ik} correspondent à des attributs de l'actif et les facteurs sont estimés.
- Dans le modèle de Fama-French, les attributs des actifs sont utilisés pour définir les facteurs, les sensibilités b_{ik} sont alors estimées.

(c) **Les modèles factoriels statistiques.** On utilise des méthodes statistiques tells que l'ACP pour estimer les facteurs et les sensibilités. Ainsi, les facteurs sont traités comme des variables non-observables ou variables latentes. Ce type d'approche peut aboutir à des facteurs qui peuvent être interprétés commme des indices boursiers, maias parfois l'interprétation économique ou financière des facteurs résultant n'est pas évidente.

1.1.2 v2

We are going to look at expected excess return

$$E(R^{ei}) = \beta_{i,f}\lambda_f + \alpha_1$$

The beta comes from time series regression it's a two step model and a cross-sectional model. α is the deviation and λ is the slope relating expected returns to factor risk premium

$$R_t^{ei} = a_i + \beta_{i,f} \cdot f_t + \varepsilon_t^i, \quad t = 1,..,T$$

But this is for a general model what can we put for f ? How do we avoid getting exposed mean variance efficient portfolio ? What does it mean we have "explained expected return by factor model "?

So far we only have one factor model: the consumption factor model where the factor is consumption growth

$$f_t = \Delta C_t$$

Our goal is to find other factors like the CAPM which defines f_t as the excess return on market portfolio R_t^{em} with

$$E(R^{ei}) = \beta_{i,M}\lambda_M$$

Expected excess returns are linear on lambda. Recall that you might have rather seen the notation of

$$E(R^{ei}) = \beta_{im}E(R^{em}) \quad (1)$$

So we have to connect those two. Regarding the market $\beta_M = 1, E(R^{em}) = 1$. That's what the graph shows; So when the factor is a traded excess return, the mean of the factor should equal the factor risk premium

$$\lambda_i = E(f)$$

And a deeper implication is that we look at the time series regression again:

$$R_t^{ei} = \alpha_i + \beta_{iM}R_t^{em} + \varepsilon_t^i$$

Where α_i is the intercept, β_{im} is the slope coefficient and R is the right hand variable.
We can take the unconditional mean

$$E(Rt^{ei}) = \alpha_i + \beta_{iM}(R_t^{em})$$

Here for the cross-sectional relationship β is the right hand variable, E(R) is the slope coefficient and α_i is the error. That's why he chooses λ to emphasize that distinction. (note that technique works only on traded portfolio not eg for the consumption factor model).

Actually when we look at formula 1; we have an extra α_i here so it means that the prediction of our model is 0 and α_i is the error of the cross-sectional regression is that error should be zero.

We will look also at the ICAPM: here the factors go beyond the market return they are "innovation to state variable for investment opportunities outside of income".

But for today we will look at two logics:

- Equilibrium pricing logic: it is like the consumption model $\Delta C_t \leftarrow f_t$ but rather we use theories, we use the representation theorems that we demonstrated
- APT

Overall the history was: CAPM then ICAPM, APT and ΔC the consumption factor model that encompasses all of them.

1.1.3 v3

CAPM simple 2 period - Our objective is to get a discount factor that is linear function of the market return

$$m_{t+1} = a - bR^M_{t+1}$$

because by of one of our theorem once we get:

$$m_{t+1} = a - bf_{t+1} \iff E(R^{ei}) = \beta_{if}\lambda_f$$

So how to get there we need a set of assumptions: we assume a quadratic utility function we don't use it after "the bliss point" so only before c^*:

$$u(c) = -\frac{1}{2}\left(c^* - c_t\right)^2$$
$$m_{t+1} = \beta\frac{c^* - c_{t+1}}{c^* - c_t}$$

the investor Live 2-periods, and then die after $t+1$,no job or outside income; Then we get that tomorrow consumptions is the rate of return on invested wealth times the wealth left over, start with W_t eats c_t; Then next line the discount factor M_{t+1}

$$C_{t+1} = W_{tu} = R^W_{t+1}\left(W_t - c_t\right)$$

$$\begin{aligned} M_{t+1} = \beta\frac{c^* - R^w_n\left(w_t - c_t\right)}{c' - c_t} &= \left[\beta\frac{c'}{c' - c_t}\right]\cdot\left[\beta\frac{w_t - c_t}{c' - c_r}\right]R^w_{t+1} \\ &= a_t \cdot b_t R^w_{t+1} \\ &\iff E_t(R^{e_i}_{t+1}) = \beta_{it}\lambda_{t+1} \end{aligned}$$

We get two constants that varies over time a_t and b_t so we have a conditional CAPM.

Points of assumptions: what do we need the CAPM to work

- We need the returns on the wealth portfolio R^W_{t+1} drives tomorrow consumption C_{t+1}; To make that we said that the guy dies after tomorrow and
- the second thing we need is that m is a linear function of the wealth portfolio, for that we use quadratic utility.

1.2 Arbitrage Pricing theory (1976)

L'apparition du modèle APT ou Modèle d'évaluation par Arbitrage (MEA), développé par Ross (1976), est l'une des premières réponses aux critiques du MEDAF. Ce modèle multifactoriel admet la présence de plus d'un facteur comme variables explicatives du rendement.

- Hypothèses. De plus il néglige toutes les hypothèses du MEDAF en considérant uniquement l'absence d'opportunité d'arbitrage. Si une opportunité d'arbitrage se produit, alors il sera vite exploité par des agents.

- Modèle. Pour un actif:

$$\mathbb{E}[R^i] = r_f + \beta_1 f_1 + \beta_2 f_2 + .. + \beta_n f_n$$

Avec β_n la sensbilité au facteur n et f_n le n^{th} factor price. Unlike the CAPM, the APT does not specify the factors.

- Expériences. Selon la recherche de Stephen Ross et Richard Roll les facteurs les plus importants sont: La variation de l'inflation, les changements en production industrielle, variation des risk premiums et les modification de la shape of the term structure of interest rates.

1.3 Modèles factoriels fondamentaux

1.3.1 Cross-section of Expected Returns (1992)

Paper - Cross-Section of Expected Stock Returns (1992). Our goal is to evaluate the joint roles of market β, size, E/P (Earning/Price), leverage and book-to-market equity in the cross-section of average returns on NYSE..

Their conclusion: are two-folds:

- β does not seem to help explain the cross-section of average stock returns [in the period 1941-1990]
- The combination of size (ME=Market Equity=Market capitalization) and book-to-market equity (Book/ME) absorbs the roles of leverage and E/P in average stock returns. [during their period of 1963-1990 sample]

- Data. They use nonfinancial firms (since financial firms have high leverage). page 431 relire le truc sur les returns of july ...

β estimation: details - In june of each year all stocks are sorted by ME to determine the NYSE decile breakpoints. The breakpoints in FF are calculated using only NYSE stocks. Then all stocks (NYSE, AMEX and NASDAQ listed stocks) are sorted into portfolios based on these breakpoints. Since what we define today as a Big Cap as the precise dollar value of market cap taht delimits the top 70% from the rest evolve across time (the total market cap evolve). Like 1 billion was huge 50 years ago but not now.

1.3.2 Fama-French 3 factors (1996)

1.3.2.1 Introduction

The FF-3 factors was released in the multifactor explanations of asset pricing Anomalies (1996). The aim of the paper is to demonstrate that the anomalies (see below) largely disappear in a 3 factor model.

- Anomalies. Average returns on common stocks are related to firms characteristics like size, E/P, CF/price, BM/E, past sales growth.. Because those patterns are not explained by the CAPM they are called anomalies.

- Motivation behind their factors. Note they call book to market equity BE/ME (BE=Book of common equity)

- Using HML to explain returns is in line with the evidence of Chan and Chen (1991) that there is covariation in returns that is not captured by market return.
- Same for SMB but based on the Huberman and Kandel paper, there is covaraition in the returns on small stocks that is not captured by market return.
- FF (1995) show
  - FF (1995) shows that BE/ME and slopes on HML (h_i) proxy for relative distress. Weak firms with persistently low earnings tend to have high BE/ME and positive slopes on HML $> h_i$;
  - strong firms with persistently high earnings have low BE/ME and negative slopes on HML $< h_i$.

- Model. Recall that for the CAPM

$$E(R_i) - r_f = b_i[E(R_M) - r_f)]$$

Most importantly recollect that the α_i in the CAPM is the part of the model that is not explained by β. So our goal is to expand it to find the hidden anomalies inside it.

As such FF3 defines:

$$\mathrm{E}\left(R_i\right) - R_f = b_i\left[\mathrm{E}\left(R_M\right) - R_f\right] + s_i \mathrm{E}(\mathrm{SMB}) + h_i \mathrm{E}(\mathrm{HML})$$

$$R_i - R_f = \alpha_i + b_i\left(R_M - R_f\right) + s_i \mathrm{SMB} + h_i \mathrm{HML} + \varepsilon_i .$$

factor sensitivities or loadings, b_i, s_i, and h_i, are the slopes in the time-series regression,

Note on data processing. Every june they look at the size and BE/ME of every stocks then the following january they form portfolios they divide the world in 25 bins (portfolios) by market cap & values.

Table 1. Value stocks those with a very low market price relative to counting book value. Growht price is very high market price like Google. In parallel they take all the portfolios to build the HML and SMB.

The question of the table is: is there a difference in average returns between the Value/Growth/Small/Big stocks ? This table demonstrates that the monthly $\mathbb{E}[R]$ increase in between both extremes. It showcases that small cap stocks tends to have higher returns than Big cap stocks. eg it ranges from 36 to 82 basis points And high BE/ME stocks have higher returns than low BE/ME stocks.

Book-to-Market Equity (BE/ME) Quintiles

Growth stocks — Value stocks

Panel A: Summary Statistics

	Size	Means: Low	2	3	4	High	Standard Deviations: Low	2	3	4	High
Small cap	Small	0.31	0.70	0.82	0.95	1.08	7.67	6.74	6.14	5.85	6.14
	2	0.48	0.71	0.91	0.93	1.09	7.13	6.25	5.71	5.23	5.94
	3	0.44	0.68	0.75	0.86	1.05	6.52	5.53	5.11	4.79	5.48
	4	0.51	0.39	0.64	0.80	1.04	5.86	5.28	4.97	4.81	5.67
Big cap	Big	0.37	0.39	0.36	0.58	0.71	4.84	4.61	4.28	4.18	4.89

We observe an increase
of E[R] in both directions

Figure 2: Table of monthly expected returns where we divide our portfolios in those 5x5 =25 portfolios by quantiles

- Analysis.

- The question from Table 1: perhaps those higher average returns $\mathbb{E}[R]$ have higher β's ? Companies that are really in trouble should have higher β and small companies (?). In several papers they demonstrated that this is not the case and here we observe no variation in the b sensibility. So $b_i \not\Rightarrow R^i$
- For SMB we observe an $s_i \nearrow \Rightarrow R^i \nearrow$ from Big cap to Small cap which makes sense since its factor is SMB
- For HML we observe an $h_i \nearrow \Rightarrow R^i \nearrow$ from growth stocks to value stocks
- The intercept should be close to zero they correspond to the errors of the cross-sectional regression measure (How off are we from the CAPM line). The closer the α_i are to zero the better we have encompassing the informations to predict the average returns. (Note there are a 0.45 and 0.20 which they are cognizant)

Book-to-Market Equity (BE/ME) Quintiles

Growth stocks — Value stocks

	Size	Low	2	3	4	High	Low	2	3	4	High
		Panel B: Regressions: $R_i - R_f = a_i + b_i(R_M - R_f) + s_i SMB + h_i HML + e_i$									
		a					t(a)				
Small cap	Small	−0.45	−0.16	−0.05	0.04	0.02	−4.19	−2.04	−0.82	0.69	0.29
	2	−0.07	−0.04	0.09	0.07	0.03	−0.80	−0.59	1.33	1.13	0.51
	3	−0.08	0.04	−0.00	0.06	0.07	−1.07	0.47	−0.06	0.88	0.89
	4	0.14	−0.19	−0.06	0.02	0.06	1.74	−2.43	−0.73	0.27	0.59
Big cap	Big	0.20	−0.04	−0.10	−0.08	−0.14	3.14	−0.52	−1.23	−1.07	−1.17
		b					t(b)				
	Small	1.03	1.01	0.94	0.89	0.94	39.10	50.89	59.93	58.47	57.71
	2	1.10	1.04	0.99	0.97	1.08	52.94	61.14	58.17	62.97	65.58
	3	1.10	1.02	0.98	0.97	1.07	57.08	55.49	53.11	55.96	52.37
	4	1.07	1.07	1.05	1.03	1.18	54.77	54.48	51.79	45.76	46.27
	Big	0.96	1.02	0.98	0.99	1.07	60.25	57.77	47.03	53.25	37.18
		s					t(s)				
	Small	1.47	1.27	1.18	1.17	1.23	39.01	44.48	52.26	53.82	52.65
	2	1.01	0.97	0.88	0.73	0.90	34.10	39.94	36.19	32.92	38.17
	3	0.75	0.63	0.59	0.47	0.64	27.09	24.13	22.37	18.97	22.01
	4	0.36	0.30	0.29	0.22	0.41	12.87	10.64	10.17	6.82	11.26
	Big	−0.16	−0.13	−0.25	−0.16	−0.03	−6.97	−5.12	−8.45	−6.21	−0.77
		h					t(h)				
	Small	−0.27	0.10	0.25	0.37	0.63	−6.28	3.03	9.74	15.16	23.62
	2	−0.49	0.00	0.26	0.46	0.69	−14.66	0.34	9.21	18.14	25.59
	3	−0.39	0.03	0.32	0.49	0.68	−12.56	0.89	10.73	17.45	20.43
	4	−0.44	0.03	0.31	0.54	0.72	−13.98	0.97	9.45	14.70	17.34
	Big	−0.47	0.00	0.20	0.56	0.82	−18.23	0.18	6.04	18.71	17.57

Figure 3: Table of expected returns of the sensitivities of the LR

- Statistical tests. Are t-statistics is a good way of saying this is a good model ? No this model is not about the t-statistics it tells you it is well measures. And for the R^2 we think a big value is good ? But it doesn't tell us if their model is a good model.

- R^2 explains variation [over time] in returns; it tells you this is a good model to explain covariance but not the mean which is what we want.
- α explains variation [across portfolios] in average returns $\Rightarrow$ Focus on the CAPM market line and its α is better than the R^2.

Here we make the same point visually, where the expected return are high is where the h coefficients are high.

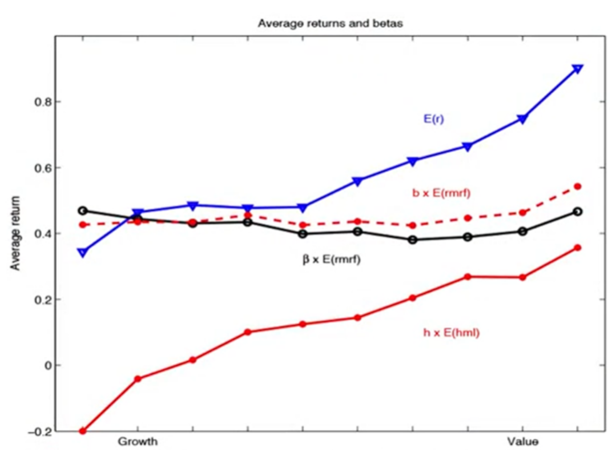


Figure 4: Average returns and β for FF 10 B/M sorted portfolios. Monthly data 1963-2010.

To test a model of average returns you ask are all the α jointly zero ? The $F - test$ of Gibbons et al rejects the hypothesis that the FF3 explains the average returns. ("it destroys the model"). Even though they revolutionize the research the model is still good they set a style for empirical work they removed the pure hypothesis research that was ongoing.

- Applications. The question is now: I can explain size and B/M how is that useful for other things ? It helps to understand other puzzle, similar to the CAPM that is not here to explain the market portfolios but the anomalies.

Once this model is built we can wonder if we should:

- Buy stocks with strong 5 years of sales growth (Google)
- Or disastrous 5 years sales decline (Sears) ?

First look at the average returns we earn twice at much rate of return by investing in Sears.

To do so we use a metric of: The five-year sales rank for June of year t, it is the weighted average of the annual sales growth ranks for the prior five years:

$$5 - \text{Yr SR}(t) = \sum_{j=1}^{5} (6 - j) \times \text{Rank}(t - j)$$

They build the table by splitting stocks into 10 categories every june they look at sales performance
What we observe from the table:

- β should explain when a company goes bad but again we do not observe anything
- The s have a U-shaped it doesnt help our trend; only the smallest stock is large since smallest stocks do wild things.
- h has a strong patterns, $h_i \nearrow \Rightarrow R^i \nearrow$ from Disastrous to Strong sales

The conclusion is that we should actually buy Sears rather than Google. We have found an anomaly which is not related to size and BM $\Rightarrow$ We have shown that slow sales behave like value stocks (they are not exactly value stocks). When the value stocks returns go down all this bad sales companies their stocks goes down.

Strong sales (Google)					Deciles					Disastrous sales (Sears)		
	1	2	3	4	5	6	7	8	9	10	GRS	$p(GRS)$
5-Yr SR	**High**									**Low**		
Mean	0.47	0.63	0.70	0.68	0.67	0.74	0.70	0.78	0.89	1.03		
Std. Dev.	6.39	5.66	5.46	5.15	5.22	5.10	5.00	5.10	5.25	6.13		
t (Mean)	1.42	2.14	2.45	2.52	2.46	2.78	2.68	2.91	3.23	3.21		
Ave. ME	937	1233	1075	1182	1265	1186	1075	884	744	434		
5-Yr SR	**High**									**Low**		
a	−0.21	−0.06	−0.03	−0.01	−0.04	−0.02	−0.04	0.00	0.04	0.07		
b	1.16	1.10	1.09	1.03	1.03	1.03	1.00	0.99	0.99	1.02		
s	0.72	0.56	0.52	0.49	0.52	0.51	0.50	0.57	0.67	0.95		
h	−0.09	0.09	0.21	0.20	0.24	0.33	0.33	0.36	0.47	0.50		
$t(a)$	−2.60	−0.97	−0.49	−0.20	−0.61	−0.25	−0.66	0.07	0.47	0.60	0.87	0.563
$t(b)$	59.01	70.59	67.65	65.34	56.68	68.89	62.49	54.12	50.08	34.54		
$t(s)$	25.69	25.11	22.59	21.65	20.15	23.64	21.89	21.65	23.65	22.34		
$t(h)$	−2.88	3.55	8.05	7.98	8.07	13.63	12.80	12.13	14.78	10.32		
R^2	0.95	0.96	0.95	0.95	0.93	0.95	0.94	0.93	0.92	0.87		

Figure 5: Table of expected returns of the sensitivities of the LR

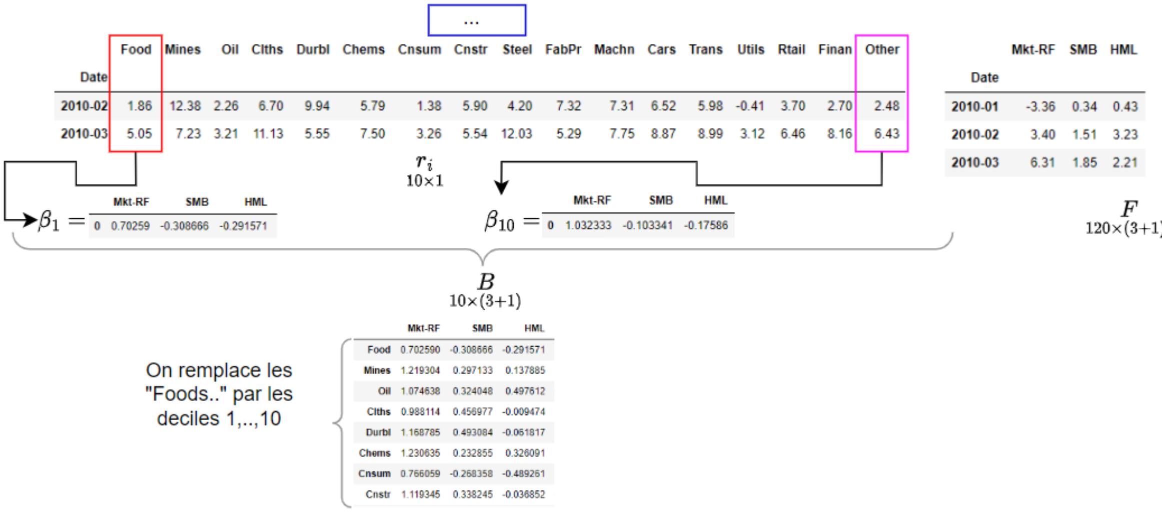


Figure 6: Visual "idea" of the regression process, the B matrix give you the slope coefficients (Fake data)

- Table 6. Momentum and reversal. Do stocks show momentum, reversal or random walk ? It means suppose we form a portfolios of all stocks that have gone up, up until day t, will they continue to go up and those that went down continue to go down? (on average) Or they reverse, those coming up will go down and those that come down will go up (on average). Note that he added a distribution since they are means. Or are they random walk ? Let's look at the table 6

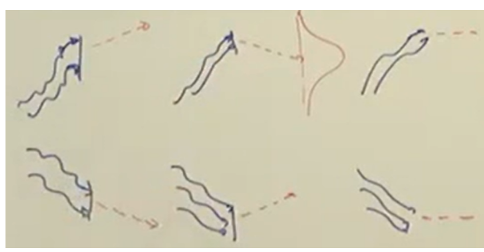


Figure 7: Momentum, Reversal and Random Walk

They say it depends on the horizon when we look at table 6, they show us a stock formed on different portfolio formation period. That's how long you look back to see winers and loosers. What they saw is: for momentum if the period is one year then you see a momentum; if -5years until -1 year leaving out the momentum they see reversal.

In the table: look at 12-2 we see

- Next video. They say that with their model they have found three portfolios that have parsimonious description of returns and average returns, and so they can absorb most of the anomalies of the CAPM. Aka at least it is an APT the R^2 is huge and $\alpha \approx 0$ it satisfies the APT this is the definition of "parsimonious description". Rest of the explanation is messy

1.3.2.2 The process of constructing the factors

(i) Retrieving Fundamental and Market Data. The first step is to extract the databases

Figure 8: Datasets from Compustat And Refinitiv

(ii) Merge the database. By merging the databasis we can construct the first steps of the HML and SMB factors. Susbequently by performing a *"weighted" groupby* we group all the stocks by size and value relative to their market cap (the weights).

$$\text{HML} = \frac{1}{2}(\text{small value} + \text{big value}) + \frac{1}{2}(\text{small growth} + \text{big growth})$$

And:

$$\text{SMB} = \frac{1}{3}(\text{Small value} + \text{Small neutral} + \text{Small growth}) - \frac{1}{3}\text{Big value} + \text{Big neutral} + \text{Big growth}$$

And for *mkt* this is the weigthed average (by market cap weight) of the returns whatever their categories.

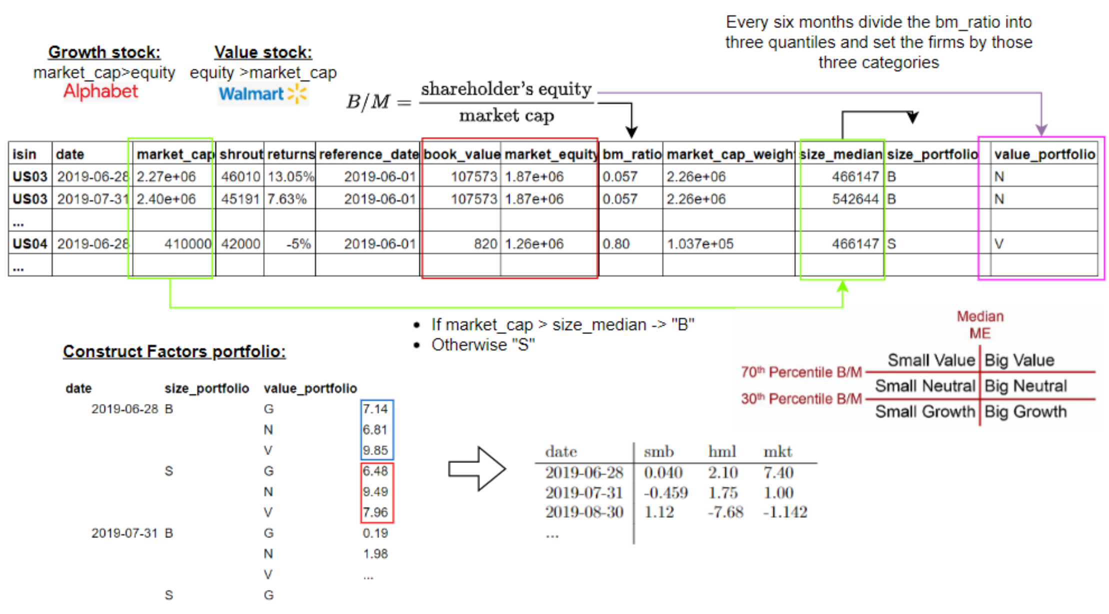


Figure 9: Merging the Databases

1.3.3 Draft

Notes. they realized that if we look historically, two types of portfolios have an $\alpha > 0$ that is the return is better than the CAPM. so they wanted to improve their models by saying that small caps stocks tends to outperform large cap stsocks so there is a size effect. Also companies with a large BTM value stock ¿ growth stocks. So we have two effects that they understood historically. Now we have three factors to study systematic risk.

2:40 - SMB they say what if we puchrase stocks with small caps and we are short with large cap stocks.

HML: long and short

Momentum: He invented the momentum cause we can observe that companies with low returns often remains at a low return and those with high stays with high; so we are long on the highest 30% return and short on the 30% lowest.

Fama-French three factor model (1992) Au cours des années 80 des travaux empiriques sont venus infirmer le modèle de Black. L'un des plus notoires est celui de Fama et French (1992). La conclusion de leur travail est que le bêta ne suffit pas à expliquer les rentabilités des actifs, d'autres éléments interviennent: les effetse de taille (capitalisation des titres), le ratio book to market, le ratio dette/actif ...

$$R_i - R_0 = \alpha + \beta_1(R_M - R_0) + \beta_2 SMB + \beta_3 HML$$

Les trois facteurs sont small minus big (SMB), high minus low (HML) et le β du marché. Le HML represénte le spread in returns between companies with high Book to market and those with low book-to-market ratio.

Database.

- Step 1. We use the CRSP (Center of Research in Security prices) database to get monthly data of securities with their closing price (altprc), shares outstanding (shrout), exchange code aka NYSE/NASDAQ or AMEX (exchd), their industry code which relies on the Standard Industrial Classification codes (siccd), share code aka common stock or preferred stock (shrcd) and finally we have to deal with unlisted stocks, their delisting code (dlstcd) and delisting returns (dlret). We will adjust the returns of those unlisted with ret_adj.

- Step 2. We use the Compustat DB (CCM). It contains thousands of annuals & quarterly income statement, balance sheet ... for active and inactive companies. This will be used to calculate the *book to market* ratio

Les facteurs:

(a) Market Risk.

(b) SMB. We sort all of our company by *Market Cap*. We split in two groups using quantiles (0.5) to obtain two groups {small S et Big B}. Overall the SMB is just the difference of average returns of the big cap companies vs small cap:

$$\begin{aligned} SMB = & \frac{1}{3}(\text{Small value} + \text{Small neutral} + \text{Small Growth}) \\ & - \frac{1}{3}(\text{Big Value} + \text{Big Neutral} + \text{Big Growth}) \end{aligned}$$

(c) HML.

- Book to market ratio (Valeur comptable par action). La Book Value (=Valeur Comptable = Capitaux propres). On a le ratio book to market

$$\text{B/M} = \frac{\text{Common Shareholder Equity}}{\text{Market cap}}$$

On pose que

$$\text{Shareholder Equity} = \text{Total Assets} - \text{Total Liabilities}$$
$$\iff \text{Capitaux propres} = \text{Total Actifs} - \text{Total des dettes}$$

Et pour rappel:

$$\text{Market Cap} = \text{Nb d'actions} \times \text{Closing price}$$

- Lorsque *Book* > Market cap (Value stock): l'entreprise a une valeur comptable plus importante que sa valeur marchande. Le cours de l'action perform à un niveau inférieur, l'entreprise est donc sous-évaluée. En achetant ce genre d'action de potentiels gains sont à prévoir si le cours de l'action revient à son cours normal
- Lorsque *Book* < Market cap (Growth Stock): L'entreprise a une valeur marchande plus importante que celle de ses comptes. Aujourd'hui l'immense majorité des entreprises sont dans ce cas
- Si *Book* = Market cap (Neutral stock)

- HML. En utilisant notre *book to market* ratio on va diviser notre DB en trois groupes $growth \in [0, 30]$; $neutral \in [30, 70]$ et $high \in [70, 100]$. Puis on aura plus qu'à faire la moyenne des rentabilités de ces deux groupes.

$$HML = \frac{1}{2}(\text{Small Value} + \text{Big Value}) + \frac{1}{2}(\text{Small Growth} + \text{Big Growth})$$

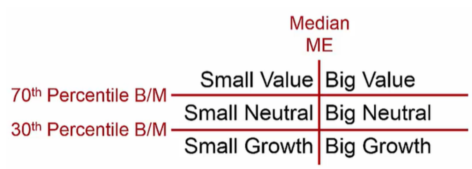


Figure 10: The CAPM only assumes one source of systematic risk: Market Risk

1.3.4 Carhart 4 factors model (1997)

1.3.4.1 Analysis

Motivation. The aim of this paper is to tell if managers have skills, that is, they are capable of finding underpriced stocks and make α ? Or can we replicate their portfolios using indexes so I don't have to pay the manager ?

The Carhart four-factor model (1997) The UML factor was firstly studied by Jagadeesh and Titman (1993) and next by Carhart (1997)
On Persistence in Mutual Fund Performance by Carhart (1997)

$$R_i - R_0 = \alpha + \beta_1(R_M - R_0) + \beta_2 SMB + \beta_3 HML + \beta_4 UMD$$

Table 3.
- Practicalities. Mutual funds are sorted on 01-01-X from 1963 to 1993 into decile portfolios based on their one year cumulative return. The author take into account all the Mutual funds even the dead ones.

- Comments. Why is it okay to use momentum ? is that a state variable for investment opportunities ? well no this is because we ask a different question than before, in our last segment we wanted asset pricing that could generate any risk premium. Here we only ask do the return of the managers are just based on a mechanic strategy (following the momentum) ? Or do I have to pay the manager for his skills ?

β

We observe good funds did 0.68% of the next year.

Good portfolio

He sorted the portfolios into 10 portfolios and 3 sub portfolios according to their performance on the previous years. Then he is going to look at their performance on the next year.

Bad portfolio

Bad portfolio are really bad with 0.22% gap with 9

Portfolio	Monthly Excess Return	Std Dev	CAPM Alpha	CAPM VWRF	CAPM Adj R-sq	4-Factor Alpha	4-Factor RMRF	4-Factor SMB	4-Factor HML	4-Factor PR1YR	4-Factor Adj R-Sq
1A	0.75%	5.45%	0.27%	1.08	0.777	−0.11%	0.91	0.72	−0.07	0.33	0.891
			(2.06)	(35.94)		(−1.11)	(37.67)	(19.95)	(−1.65)	(11.53)	
1B	0.67%	4.94%	0.22%	1.00	0.809	−0.10%	0.86	0.59	−0.05	0.27	0.898
			(2.00)	(39.68)		(−1.08)	(40.66)	(18.47)	(−1.38)	(10.63)	
1C	0.63%	4.95%	0.17%	1.02	0.843	−0.15%	0.89	0.56	−0.05	0.27	0.927
			(1.70)	(44.65)		(−1.92)	(49.76)	(20.86)	(−1.61)	(12.69)	
1 (high)	0.68%	5.04%	0.22%	1.03	0.834	−0.12%	0.88	0.62	−0.05	0.29	0.933
			(2.10)	(43.11)		(−1.60)	(50.54)	(23.67)	(−1.86)	(13.88)	
2	0.59%	4.72%	0.14%	1.01	0.897	−0.10%	0.89	0.46	−0.05	0.20	0.955
			(1.75)	(57.00)		(−1.78)	(66.47)	(22.95)	(−2.25)	(12.43)	
3	0.43%	4.56%	−0.01%	0.99	0.931	−0.18%	0.90	0.34	−0.07	0.16	0.963
			(−0.08)	(70.96)		(−3.65)	(76.80)	(18.99)	(−3.69)	(11.52)	
4	0.45%	4.41%	0.02%	0.97	0.952	−0.12%	0.90	0.27	−0.05	0.11	0.971
			(0.33)	(85.70)		(−2.81)	(90.03)	(18.18)	(−3.12)	(9.40)	
5	0.38%	4.35%	−0.05%	0.96	0.960	−0.14%	0.90	0.22	−0.05	0.07	0.970
			(−1.10)	(93.93)		(−3.31)	(89.65)	(14.42)	(−3.27)	(6.18)	
6	0.40%	4.36%	−0.02%	0.96	0.958	−0.12%	0.90	0.22	−0.04	0.08	0.968
			(−0.46)	(91.94)		(−2.82)	(86.16)	(14.02)	(−2.37)	(6.01)	
7	0.36%	4.30%	−0.06%	0.95	0.959	−0.14%	0.90	0.21	−0.03	0.04	0.967
			(−1.39)	(92.90)		(−3.09)	(85.73)	(13.17)	(−1.62)	(2.89)	
8	0.34%	4.48%	−0.10%	0.98	0.951	−0.13%	0.93	0.20	−0.06	0.01	0.958
			(−1.86)	(85.14)		(−2.52)	(75.44)	(10.74)	(−3.16)	(0.84)	
9	0.23%	4.60%	−0.21%	1.00	0.926	−0.20%	0.93	0.22	−0.10	−0.02	0.938
			(−3.24)	(67.91)		(−3.11)	(60.44)	(9.69)	(−3.80)	(−1.17)	
10 (low)	0.01%	4.90%	−0.45%	1.02	0.851	−0.40%	0.93	0.32	−0.08	−0.09	0.887
			(−4.58)	(46.09)		(−4.33)	(42.23)	(9.69)	(−2.23)	(−3.50)	
10A	0.25%	4.78%	−0.19%	1.00	0.864	−0.19%	0.91	0.33	−0.11	−0.02	0.891
			(−2.05)	(48.48)		(−2.16)	(42.99)	(10.27)	(−3.20)	(−0.76)	
10B	0.02%	4.92%	−0.42%	1.00	0.817	−0.37%	0.91	0.32	−0.09	−0.09	0.848
			(−3.84)	(40.67)		(−3.45)	(35.52)	(8.24)	(−2.16)	(−2.99)	
10C	−0.25%	5.44%	−0.74%	1.05	0.736	−0.64%	0.98	0.32	−0.04	−0.17	0.782
			(−5.06)	(32.16)		(−4.49)	(28.82)	(6.29)	(−0.73)	(−4.09)	
1-10 spread	0.67%	2.71%	0.67%	0.01	−0.002	0.29%	−0.05	0.30	0.03	0.38	0.231
			(4.68)	(0.39)		(2.13)	(−1.52)	(6.30)	(0.53)	(10.07)	
1A-10C spread	1.01%	3.87%	1.00%	0.02	−0.002	0.53%	−0.07	0.40	−0.02	0.50	0.197
			(4.90)	(0.42)		(2.72)	(−1.61)	(5.73)	(0.32)	(8.98)	
9-10 spread	0.22%	1.22%	0.23%	−0.02	0.004	0.20%	−0.01	−0.10	−0.01	0.07	0.118
			(3.64)	(−1.60)		(3.13)	(−0.40)	(−4.30)	(−0.60)	(3.87)	

Then they run CAPM to check if the best one are good because they take lot of risks. CAPM is a failure 1.03, .. is beta

Then 4 factor model, we observe roughly the same SMB; more interesting is the momentum factor, loadings on the momentum match the average return, leaving us alpha pretty much flat

Figure 11: An illustration of the WML's construction

- Summary of the facts. fact 1 - one year return persist and the best group keep doing a bit better; fact 2 - you can explain this return persistence with the 4 factor model and especially the loading of the mometum factor; fact 3 - how long do the return persist, during the formation year we observe a wide gap between winners and loosers but as time passes the return difference reduces.

Plot. As time passes the returns between bad and good portfolio are mixed up

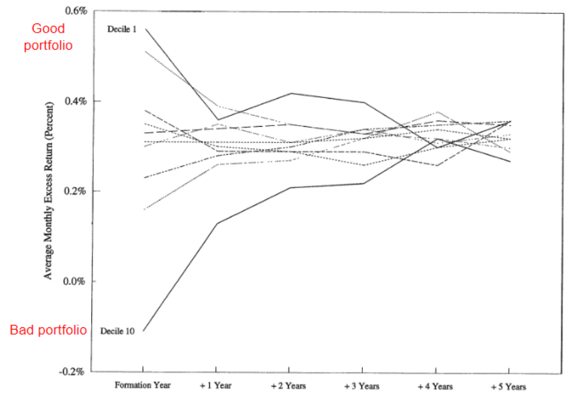


Figure 12: Up to five years performance

Fact 4: maybe we observe momentum funds, ???

Picture: the winning funds are funds that happens to hold stocks that went up in the previous year, those stocks are likely to go up again, a little bit, the next year with a lot of risk. In average they go up. Alternatively some stocks keep going down.

Table 5, do fees and turnover help investors or hurt investors ? When a fund charges higher fees have fancier offices and sell a lot of stocks is that good for you the investors ? In fact simple logic the answer should be yes. The fund manager will tell you higher fees mean more research and more money for you and me.

Independent Variables (Coefficients × 100)	Estimate	*t*-statistic
Expense ratio (t)	−1.54	(−5.99)
Turnover (t) (Mturn)	−0.95	(−2.36)
ln TNA (t-1)	−0.05	(−0.66)
Maximum Load (t-1)	−0.11	(−3.55)
Buy turnover (t)	−0.43	(−1.16)
Sell turnover (t)	−1.26	(−3.00)

Figure 13: Table

Table F cross regressions, LHS returns to investors RHS expense ratio turn over... if the fund charges 1% more in fees do the investors get 0.5% more or nothign more ? No he get -1.5% more fees is less return. Similarly turnover they buy new stocks it turns out you loose about 0.95%.

No carhart. What happens since then ? we've applied this techniques to hedge fund, soverign fund... along the way the nb of factors has exploted now we look at put ptions, carry trade... here we have a graph performance attribution of the equity market neutral index. Red is the hedge fund and blue is one year return n the stock market. Up until 2008, the equity market neutral hedge fund seems okay, they makes money, but at 2008 it was collapsed.

1.3.4.2 Construction the Monthly Momentum Factor

Pour le construire on suit la méthodologie de NEFIN au lieu de UMD ils l'appellent WML:

- Every month t, we (ascending) sort the eligible stocks into 3 quantiles (portfolios) according to their cumulative returns between month $t-12$ and $t-2$.
- Then we compute the equal-weighted returns of the first portfolio ("Losers") and the third portfolio ("Winners").

- The WML Factor is the return of the "Winners" portfolio minus the return of the "Losers" portfolio. ou mean(returns, tercile=3) - mean(returns, tercile=1)

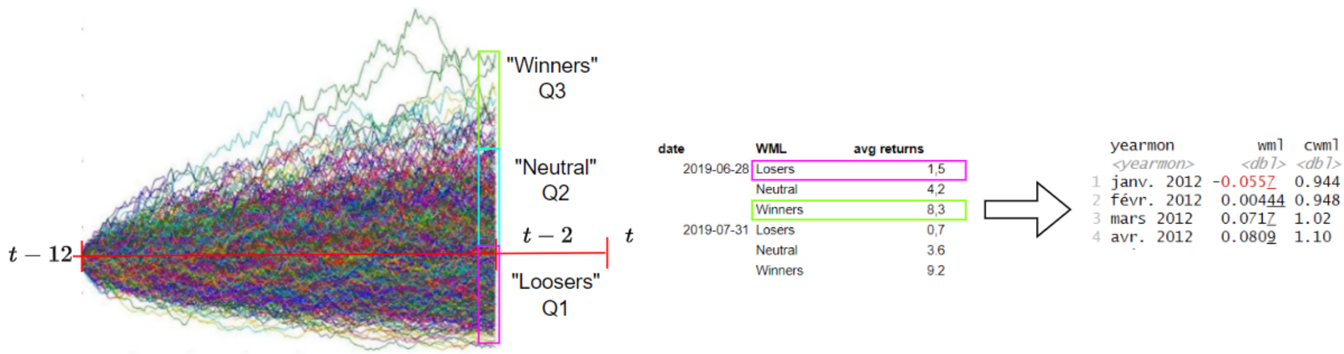


Figure 14: An illustration of the WML's construction

1.3.5 FF Luck versus Skill in the Cross-Section of Mutual Fund Returns (2010)

They took a completely different approach, so far we search for skills by looking at some characteristics that could let us see the good funds going forward. One year return is noisy, 5 years is noisy. FF attacks this qst good vs bad funds without having to find the good funds.

We can detect they are good directly.

THey start with the point of equilibrium accounting; it is important to recollect that active investment must also be a zero sum game. The average investors must hold the market portfolio doing anything else is a zero sum game. The average alpha for the market portfolio it is zero.

But who agrees to be the looser ?

So FF run a simulation they ask, suppose all funds have zero alpha, of course by luck there will be some good and bad ones. If all funds have zero true alpha how many should we see of positive alpha or negative ?

They run a regression of return and four factors. They build the t statistic it remains the influence of very short time funds that could be lucky.

So what should be the distribution of t statistics if there is no alpha at all ? We know our statistics 5% of the funds should have an alpha t-statistics greater than 1.64, and 2.5% ¿ 1.96. If there is some skills we should see more good funds not due to chance They didn't do that since if the errors are correlated across the funds or error not normal ? what they did is a bootstrap a simulation, In graph form, we find the distribution of alpha do to luck.

Table. 95 and 1.68, it means 5% of the funds should have a t stat greater than 1.68 if there were no skills at all. In the world we have [1.54, 1.71] depresing higher tail; for the 5% of the funds should have less than 1.71, we have 1.74 for 20% and the median is -0.62. So there are more bad fund due to chance.

Then for 1 billion they look even worst,

Here graphical version of the same thing of the table, the blue ones are from fama and thick blue is smoothed; we can see that the average is shifted to the left, and we were hping to see to the right.

Other way of doing it

1.3.6 Berk and Green: Five Myths of Active Portfolio Management (2005)

The world we have seen so far is puzzling FF paper summarizes 45 years of academic research that without a itch have found bad performance for mutual funds, hedge funds ... the alpha are negative with the fees. WIth carhart we have fund the performance of the best fund lasts only a year and comes up from momentum in the underlying stocks. Yet investors not only choose high fees active management but they chase past returns.

Paper from juldy ... Funds that did very well last year gets a lot more money last year; and for older/larger fund it is the same. It doens't make sense since we have seen that being good last year doesns't say much about performance in the future.

Furthemore manager are paid hugely in a competitive market even though academia found bad performance, index funds were invented in the 1970. Saying this is irationality doesn't make much sense, what don't we try to explain those phenomena in a normal competitive market ? Nobody knows previousy but Berk have found.

We boil it down with an example, the manager can only run 10$m of investment. Any additional money will put in the index. 1% fee is the normal for mutual funds.

Berks lists the 5 hypothesis that underly our previous discussion (i) alpha doesn't measure skill, it is alpha times assets under management is skills (iii)

Some of the problems FF and Berks are a discussion, FF says we should see alpha before fees and no alpha after fees, berk replied that alpha get swamped by indexing, let's measure alpha times assets there is still alpha. FF says berks mean net alpha should be zero on avg, berk said if you only take factors like value.

1.3.7 Asness et al - Value and momentum everywhere (2013)

This paper wants to establish a link between value and momentum. THey show that this is negatively correlated. Recall we said value and profitability are negatively correlated. Since they are the overall portfolio risk decreases (if we invest in 50/50). In this paper they show that value and momentum are negatively correlated. So they propose a portfolio with 50% value stocks and 50% momentum. They also do a PCA they consider 8 asset classes, they figure out that momentum grows positively on PC1 and value negatively on PC1. They explains that the link is fund liquidity that connects both momentum and value stocks. But also some drawbacks in this paper later.

2nd page: they argue that you can separate market liquidity from fund liquity, and the second one is the driver in their story. They also argue as the market becomes less liquid values generate higher returns which doesn't make sense but they argue it. Also, that the negative correlation might be of practical relevance for portfolio management since it reduces the portfolio risk. At the end of the page they come up with an explanation for why this is so ?

section 1. Next we investigate 8 asset classses, they use only big stocks which are more liquid by df and anomalies are more pronounced in small stocks. Strategy on small stocks there is a risk you cannot trade them because of lack of liquidity.

section 1.2 they describe the next asset class: global equity indices, they use 18 indices from dvp markets . Section A.3 they use currencies A.4 govt bonds, A.5 commodities future. For him one of the problem is that they don't justify why those datasets.

Straightforward of how they build a value portfolio the use lagged six month book value and divide by Market cap. Then they sort from low to high B/M; And for momentum they cumulate the returns from t-12 to t-2. Then they are short with the stocks that have the lowest cumulative return and long the winner stock with the highest cumulative return. Then we increase by +1 and we repeat the process. we rebalance at the beginning of each month. But for commodities we cannot build a B/M so instead, they have y-axis Price of commodities (currencies..), we look 5 years in the past, and we are at t=0. let's $P_{t-5} = 50$ and $P_t = 20$, the difference is $-30/P_0 = -60\%$ then they take its negative 60%. Then they sort from low value to high, then they sell low values and buy those that have long values. It is a proxy of the book value. In the example that he draw we buy it, since asset that decrease a lot are expected to move back to their true value. this is the idea behind value strategies. 26:42 he re-explains with drawing

More stuff on momentum: short-term reversal effect, where the winners of last month tends to underperform those stocks that have performed very poorly. He calls it the "return cycle".

They have 48 test portfolios. section D we skip.

First good table is table 1.

1.3.8 Novy Marx - Gross profitability (2013)

Theoretically "Profitability" can be defined as the earnings E_t. But what Novy Marx would like to do is to find a better proxy for it that yields more informations:

$$M_t = \sum_{\tau=0}^{\infty} \frac{\mathbf{E}_t\left[E_{t+\tau} - dB_{t+\tau}\right]}{(1+r)^{\tau}}$$

Table 1. We aim at empirically demonstrating that Gross profitability vs Earnings and FCF is the best proxy for the "profitability" factor. The experiments are realized with Fama McBeth.

Controlling either Gross Profitability, Earnings or FCF we observe that the GP is dramatically more significant (t-value of 5.49). And if we pair GP with E_t/FCF we keep observing the significant effect of GP. Conclusion: Gross profitability captures the effects of both Earnings and FCF, as such this is the best proxy for profitability.

	Slope coefficients ($\times 10^2$) and [test-statistics] from regressions of the form $r_{tj} = \beta' \mathbf{x}_{tj} + \epsilon_{tj}$						
Independent variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Straight profitability variables							
Gross profitability	0.75			0.69	0.62		0.61
	[5.49]			[5.22]	[4.63]		[4.59]
Earnings		0.22		0.08		−0.02	−0.07
		[0.84]		[0.31]		[−0.06]	[−0.27]
Free cash flow			0.27		0.20	0.39	0.33
			[2.28]		[1.64]	[3.17]	[2.67]
log(B/M)	0.35	0.30	0.26	0.34	0.30	0.27	0.31
	[5.98]	[4.97]	[4.59]	[5.54]	[5.17]	[4.48]	[5.05]
log(ME)	−0.09	−0.12	−0.13	−0.11	−0.11	−0.13	−0.11
	[−2.29]	[−3.24]	[−3.20]	[−2.78]	[−2.80]	[−3.34]	[−2.92]
$r_{1,0}$	−5.57	−5.49	−5.52	−5.64	−5.66	−5.56	−5.70
	[−13.8]	[−13.7]	[−13.7]	[−14.1]	[−14.1]	[−13.9]	[−14.3]
$r_{12,2}$	0.76	0.78	0.78	0.74	0.74	0.76	0.73
	[3.87]	[4.02]	[4.02]	[3.80]	[3.80]	[3.93]	[3.74]

Figure 15: Fama McBeth Regression.

- Justification. First off we define gross profitability as the ratio:

$$\text{Gross profitability} = \frac{\text{gross profit}}{assets} = \frac{\text{sales - COGS}}{assets}$$

The advantage of this metric is that it is not driven by accruals or R&D expenditures.

- Critisim of Fama McBeth. There are two problems: (a) first this method do not take into account the weights. As a consequence low and large market caps have the same explanatory power whereas it shouldn't. Since low cap have more anomalous behaviors and they will bias the model. Additionally there is a risk of multicolinearity in the model.

Table 2. He proves that gross profitability has the opposite effect as the B/M. Indeed we again observe that the returns increase from Low to High profitability or B/M. Additionally the HML is supposed to divide Growth vs value stocks and we observe the opposing effects between the two. (High-Low is negative for profitability).

	Portfolio	r^e	α	MKT	SMB	HML
			Alphas and three-factor loadings			
	Panel A: Portfolios sorted on gross profits-to-assets					
Low profitability	Low	0.31	−0.18	0.94	0.04	0.15
		[1.65]	[−2.54]	[57.7]	[1.57]	[5.87]
	2	0.41	−0.11	1.03	−0.07	0.20
		[2.08]	[−1.65]	[67.5]	[−3.13]	[8.51]
	3	0.52	0.02	1.02	−0.00	0.12
		[2.60]	[0.27]	[69.9]	[−0.21]	[5.42]
	4	0.41	0.05	1.01	0.04	−0.24
		[1.94]	[0.83]	[70.6]	[1.90]	[−11.2]
High profitability	High	0.62	0.34	0.92	−0.04	−0.29
		[3.12]	[5.01]	[58.3]	[−2.03]	[−12.3]
	High–low	0.31	0.52	−0.03	−0.08	−0.44
		[2.49]	[4.49]	[−0.99]	[−2.15]	[−10.8]

Book to market Panel (same as FF3)

Panel B: Portfolios sorted on book-to-market						
Low	0.39	0.13	0.98	−0.09	−0.39	Growth stocks
	[1.88]	[2.90]	[90.1]	[−5.62]	[−23.9]	
2	0.45	−0.02	0.99	0.05	0.04	
	[2.33]	[−0.29]	[78.1]	[2.61]	[2.23]	
3	0.56	0.03	0.96	0.04	0.22	
	[2.99]	[0.53]	[63.5]	[2.09]	[9.71]	
4	0.67	−0.00	0.96	0.10	0.53	
	[3.58]	[−0.03]	[74.8]	[5.66]	[27.1]	
High	0.80	0.07	1.01	0.25	0.51	Value stocks
	[3.88]	[1.04]	[60.7]	[10.7]	[20.5]	
High–low	0.41	−0.06	0.03	0.34	0.91	
	[2.95]	[−0.71]	[1.44]	[12.0]	[30.0]	

Figure 16: Linear regression where portfolios are sorted by either Profitability or B/M.

- Hedge portfolio. As such if I create a portfolio of half value stocks and half profitable stocks. And both are uncorrelated (value vs growth stocks) then the overall risk (variance) of the portfolio will decrease:

$$R_p = x_1 R^1 + x_2 R^2 \Rightarrow \mathbb{V}[R_p] = x_1^2 \sigma_1 + x_2^2 \sigma_2 + 2 \cdot x_1^2 x_2^2 cov(R^1, R^2)$$

Additionally we have empirically proven that high profitability stocks behave like growth stocks and high b/m as value stocks. Additionally Lakonishok et al. 1994 has shown that growth stocks are typically overpriced and value stocks underpriced. As a consequence the strategy consists in buying value stocks and selling growth stocks to exploite misvaluations in the cross-section.

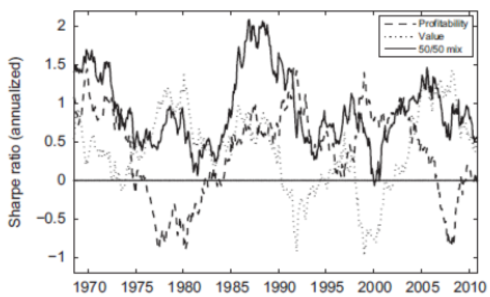


Figure 17: Performance over time of our mixed portfolio from June 1963 to December 2010.

Robustness test.

Variable	IB/A	FCF/A	B/M	ME	$r_{12,2}$	$r_{1,0}$
Gross profitability (GP/A)	0.45	0.31	−0.18	−0.03	0.09	0.02
	[58.7]	[17.9]	[−17.2]	[−2.51]	[7.00]	[1.87]
Earnings (IB/A)		0.60	−0.25	0.36	0.23	0.07
		[16.7]	[−8.89]	[30.4]	[14.4]	[6.27]
Free cash flow (FCF/A)			−0.03	0.20	0.17	0.07
			[−1.33]	[10.7]	[10.6]	[7.12]
Book-to-market (B/M)				−0.26	−0.09	0.02
				[−13.1]	[−4.98]	[1.37]
Market equity (ME)					0.26	0.13
					[11.3]	[9.20]
Prior year's performance ($r_{12,2}$)						0.08
						[5.15]

Figure 18: Correlation table.

1.3.9 Fama-French 5 factors (2015)

1.3.9.1 Analysis

Theoretical analysis.

$$M_t = \sum_{\tau=1}^{\infty} \mathbb{E}\left(E_{t+\tau} - dB_{t+\tau}\right)/(1+r)^{\tau} \Rightarrow \frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathbb{E}\left(E_{t+\tau} - dB_{t+\tau}\right)/(1+r)^{\tau}}{B_t}.$$

- Profitability. Which can be proxied by the E_t if the LHS is fixed then if $E_t \nearrow \Rightarrow r \nearrow$.

- Investment. It can be proxied by dB_t the variation in book values over two years. Again if the LHS is fixed then to equal things out we get $dB_t \nearrow \Rightarrow r \searrow$ and conversely $dB_t \searrow \Rightarrow r \nearrow$.

- B/M. If $B_t >> M_t \iff B/M \nearrow$ as such on the RHS we need a small figure on the denominator so $r \nearrow$. And conversely $B/M \searrow \Rightarrow B/M \nearrow$.

Table 1. In blue sky we observe the exact same pattern for B/M, E_t and dB_t as theoretically as we look at the average expected returns of assets. Juste note that in red the patterns loose their meanings.

		Low	2	3	4	High
	Panel A: Size-B/M portfolios					
microcaps	Small	0.26	0.81	0.85	1.01	1.15
	2	0.48	0.72	0.94	0.94	1.02
	3	0.50	0.78	0.79	0.88	1.07
	4	0.60	0.57	0.71	0.85	0.86
megacaps	Big	0.46	0.51	0.48	0.56	0.62
	Panel B: Size-OP portfolios					
microcaps	Small	0.56	0.94	0.90	0.95	0.88
	2	0.59	0.78	0.84	0.81	0.98
	3	0.53	0.77	0.72	0.78	0.94
	4	0.57	0.65	0.63	0.70	0.82
megacaps	Big	0.39	0.33	0.43	0.47	0.57
	Panel C: Size-Inv portfolios					
microcaps	Small	1.01	0.98	0.99	0.89	0.35
	2	0.92	0.91	0.92	0.90	0.48
	3	0.90	0.93	0.81	0.82	0.50
	4	0.79	0.72	0.71	0.75	0.54
megacaps	Big	0.71	0.52	0.49	0.48	0.42

Effect is increasing for the two factors

Weird behaviors for high investment or high profit or low B/M

This table is polluted since they don't take into account the inter effect of the factors. Addtionally FF has shown that the three variables are correlated. Solution is

Figure 19: Table 1.

- HML is redundant. Looking at the GRS we observe that adding HML do not add any explanatory power in any settings. In our methodology we add one factor after the other:

	2 × 3 Factors			
	GRS	$A\lvert a_i\rvert$	$\frac{A\lvert a_i\rvert}{A\lvert \bar{r}_i\rvert}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$
Panel A: 25 Size-B/M portfolios				
HML	3.62	0.102	0.54	0.38
HML RMW	3.13	0.095	0.50	0.24
HML CMA	3.52	0.101	0.53	0.39
RMW CMA	2.84	0.100	0.53	0.22
HML RMW CMA	2.84	0.094	0.50	0.23
Panel B: 25 Size-OP portfolios				
HML	2.31	0.108	0.68	0.51
RMW	1.71	0.067	0.42	0.12
HML RMW	1.64	0.062	0.39	0.16
HML CMA	3.02	0.137	0.86	0.90
RMW CMA	1.87	0.075	0.47	0.12
HML RMW CMA	1.87	0.073	0.46	0.12
Panel C: 25 Size-Inv portfolios				
HML	4.56	0.112	0.64	0.57
CMA	4.03	0.105	0.60	0.47
HML RMW	4.40	0.106	0.61	0.57
HML CMA	4.00	0.099	0.57	0.43
RMW CMA	3.33	0.085	0.49	0.29
HML RMW CMA	3.32	0.085	0.49	0.29

Mkt + SMB + X

The GRS test says that all their model are incomplete descriptions of expected returns. BUT we observe a decrease in GRS when we add more factors which means they are still useful.

Figure 20: Table 1.

Next up we can build a linear regression where $Y = \{HML\}$. On observe que un CMA $\nearrow\Rightarrow$ entreprise conservatrice sur l'investissement ie que ici on aurait un value stock qui investis pas trop d'après le HML ce qui est OK. Par contre ça dit aussi qu'elle serait très robuste ces entreprises alors que les value stocks sont censé être moins profitable. Since recollect that

$$CMA = \frac{1}{2}(\text{Small conservative} + \text{Big conservative}) - \frac{1}{2}(\text{Small aggressive} + \text{Big aggressive})$$

Dependent variable	*Int*	$R_M - R_F$	*SMB*	*HML*	*RMW*	*CMA*	R^2
2 × 3 Factors							
$R_M - R_F$							
Coef	0.82		0.25	0.03	−0.40	−0.91	0.24
t-Statistic	4.94		4.44	0.38	−4.84	−7.83	
SMB							
Coef	0.39	0.13		0.05	−0.48	−0.17	0.17
t-Statistic	3.23	4.44		0.81	−8.43	−1.92	
HML							
Coef	−0.04	0.01	0.02		0.23	1.04	0.51
t-Statistic	−0.47	0.38	0.81		5.36	23.03	
RMW							
Coef	0.43	−0.09	−0.22	0.20		−0.44	0.21
t-Statistic	5.45	−4.84	−8.43	5.36		−7.84	
CMA							
Coef	0.28	−0.10	−0.04	0.45	−0.21		0.57
t-Statistic	5.03	−7.83	−1.92	23.03	−7.84		

Figure 21: Table 1.

1.3.9.2 Factors Construction

Fama-French five factor model (2015)

$$R_i - R_0 = \alpha + \beta_1(R_M - R_0) + \beta_2 SMB + \beta_3 HML \beta_4 RMW + \beta_5 CMA$$

(a) RMW. the profitability factor aka robust minus weak

$$RMW = \frac{1}{2}(\text{Small robust} + \text{Big Robust}) - \frac{1}{2}(\text{Small weak} + \text{Big weak})$$

- Gross profit. Gross profit will appear on a company's income statement. A company with high gross profit has the opportunity to make wise decisions on how they allocate capital (re-invest, reduce debt, shareholders). A company with low gross profit has a lower probability of being successful

$$\text{Profitability} = \text{revenues - cost of good sales (COS)}$$

Le COS (=Coût/prix de revient) est la sommes des coûts supportés par la production et la distribution d'un bien ou d'un service. C'est une somme de deux coûts:

- En charges directes (eg achats des matières premières, le temps passé pour chaque client aka taux horaire)
- En charges indirectes qui regroupent tous les autres frais: locations, assurances ...

- Profitability (Gross profit to assets).

$$\text{Profitability} = \frac{\text{firm gross profit}}{\text{assets}} = \frac{\text{revenues - cost of good sales (COS)}}{\text{assets}}$$

This ratio measures how productively assets are being used in the company A double digits figure is taken to be indicative of a productive, efficient company that may have a competitive advantage.

(b) CMA. the investment factor

- Investment. The investment ratio is used to rank aggressiveness vs conservativeness of firms:

$$INV = \frac{\text{total assets}_t}{\text{total assets}_{t-1}}$$

This ratio is calculated at the end of each fiscal years.

$$CMA = \frac{1}{2}(\text{Small conservative} + \text{Big conservative}) - \frac{1}{2}(\text{Small aggressive} + \text{Big aggressive})$$

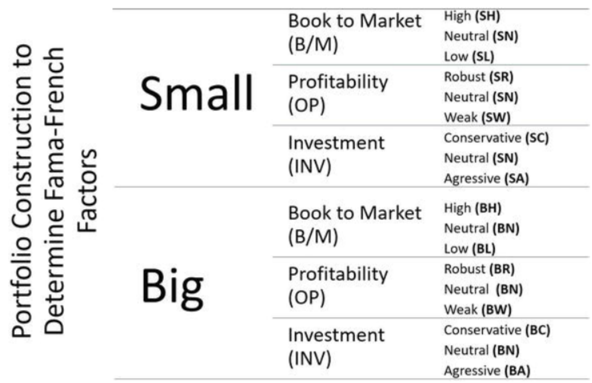


Figure 22: An illustration of the WML's construction

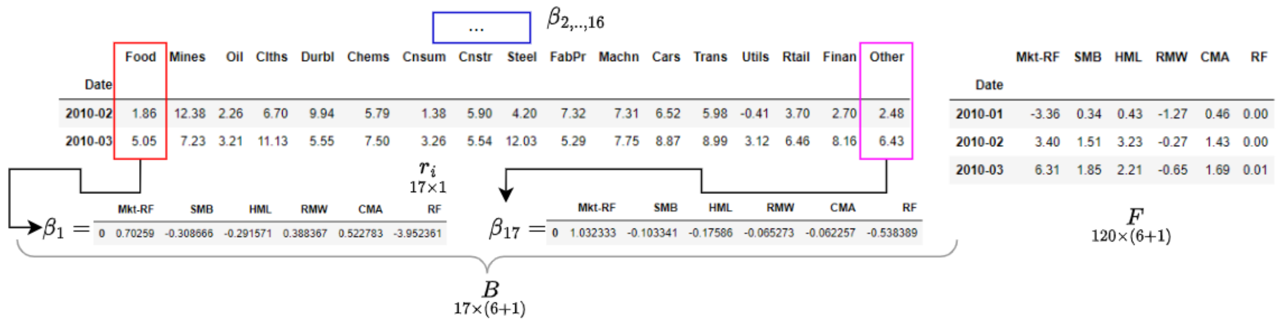
1.3.9.3 Fama McBeth regression for FF5

Input. We have $N = 17$ portfolios, $K = 5$ risk factors and $T = 120$ periods of data.

Algorithm (Fama McBeth). It is a two stage model:

- 1st stage. We perform $N = 17$ time-series regression

$$\underset{T\times 1}{R_i} = \underset{T\times(K+1)}{F}\underset{(K+1)\times 1}{\beta_i} + \underset{T\times 1}{\varepsilon}, \quad i = 1, .., 17$$



Date	Food	Mines	Oil	Clths	Durbl	Chems	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans	Utils	Rtail	Finan	Other
2010-02	1.86	12.38	2.26	6.70	9.94	5.79	1.38	5.90	4.20	7.32	7.31	6.52	5.98	-0.41	3.70	2.70	2.48
2010-03	5.05	7.23	3.21	11.13	5.55	7.50	3.26	5.54	12.03	5.29	7.75	8.87	8.99	3.12	6.46	8.16	6.43

	Mkt-RF	SMB	HML	RMW	CMA	RF
0	0.70259	-0.308666	-0.291571	0.388367	0.522783	-3.952361

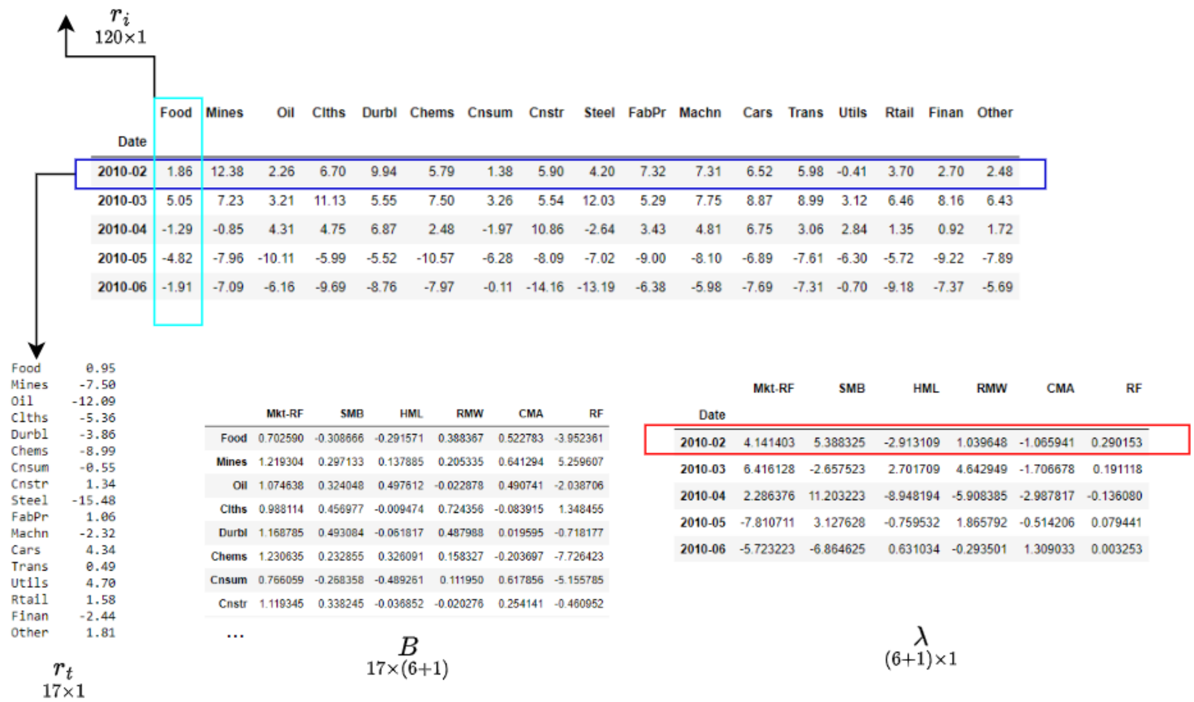
	Mkt-RF	SMB	HML	RMW	CMA	RF
0	1.032333	-0.103341	-0.17586	-0.065273	-0.062257	-0.538389

Date	Mkt-RF	SMB	HML	RMW	CMA	RF
2010-01	-3.36	0.34	0.43	-1.27	0.46	0.00
2010-02	3.40	1.51	3.23	-0.27	1.43	0.00
2010-03	6.31	1.85	2.21	-0.65	1.69	0.01

Figure 23: First stage

- 2nd stage. We perform $T = 120$ cross-sectional, one for each time period:

$$\underset{N\times 1}{R_t} = \underset{N\times(K+1)}{B}\underset{(K+1)\times 1}{\mathbf{f}_t}, \quad t = 1, .., 120$$



Date	Food	Mines	Oil	Clths	Durbl	Chems	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans	Utils	Rtail	Finan	Other
2010-02	1.86	12.38	2.26	6.70	9.94	5.79	1.38	5.90	4.20	7.32	7.31	6.52	5.98	-0.41	3.70	2.70	2.48
2010-03	5.05	7.23	3.21	11.13	5.55	7.50	3.26	5.54	12.03	5.29	7.75	8.87	8.99	3.12	6.46	8.16	6.43
2010-04	-1.29	-0.85	4.31	4.75	6.87	2.48	-1.97	10.86	-2.64	3.43	4.81	6.75	3.06	2.84	1.35	0.92	1.72
2010-05	-4.82	-7.96	-10.11	-5.99	-5.52	-10.57	-6.28	-8.09	-7.02	-9.00	-8.10	-6.89	-7.61	-6.30	-5.72	-9.22	-7.89
2010-06	-1.91	-7.09	-6.16	-9.69	-8.76	-7.97	-0.11	-14.16	-13.19	-6.38	-5.98	-7.69	-7.31	-0.70	-9.18	-7.37	-5.69

Food	0.95
Mines	-7.50
Oil	-12.09
Clths	-5.36
Durbl	-3.86
Chems	-8.99
Cnsum	-0.55
Cnstr	1.34
Steel	-15.48
FabPr	1.06
Machn	-2.32
Cars	4.34
Trans	0.49
Utils	4.70
Rtail	1.58
Finan	-2.44
Other	1.81

	Mkt-RF	SMB	HML	RMW	CMA	RF
Food	0.702590	-0.308666	-0.291571	0.388367	0.522783	-3.952361
Mines	1.219304	0.297133	0.137885	0.205335	0.641294	5.259607
Oil	1.074638	0.324048	0.497612	-0.022878	0.490741	-2.038706
Clths	0.988114	0.456977	-0.009474	0.724356	-0.083915	1.348455
Durbl	1.168785	0.493084	-0.061817	0.487988	0.019595	-0.718177
Chems	1.230635	0.232855	0.326091	0.158327	-0.203697	-7.726423
Cnsum	0.766059	-0.268358	-0.489261	0.111950	0.617856	-5.155785
Cnstr	1.119345	0.338245	-0.036852	-0.020276	0.254141	-0.460952

Date	Mkt-RF	SMB	HML	RMW	CMA	RF
2010-02	4.141403	5.388325	-2.913109	1.039648	-1.065941	0.290153
2010-03	6.416128	-2.657523	2.701709	4.642949	-1.706678	0.191118
2010-04	2.286376	11.203223	-8.948194	-5.908385	-2.987817	-0.136080
2010-05	-7.810711	3.127628	-0.759532	1.865792	-0.514206	0.079441
2010-06	-5.723223	-6.864625	0.631034	-0.293501	1.309033	0.003253

Figure 24: Second stage

1.3.10 Barra Factor Model (1998)

1.3.10.1 Barra-type "Industry" Factor Model - USEX

On va considérer un modèle de Barra avec des facteurs du type "Industrie" avec K industries exclusives. Les sensibilités β_{ik} pour chaque actif sont invariant au temps et sont de la forme:

$$\begin{aligned}\beta_{ik} &= 1 \text{ if asset } i \text{ is in industry } k\\ &= 0, \text{ otherwise}\end{aligned}$$

On pourra donc interpréter f_{kt} comme la réalisation du facteur pour la $k^{\text{ème}}$ industrie au temps t. On peut noter que le facteur β est une variable dummy qui indique si l'actif appartient à une seule industrie. Ainsi les facteurs f_{kt} devront être estimés. Le modèle des facteurs d'industrie avec K industries peut être résumé par:

$$\begin{aligned}R_{it} &= \beta_{i1} f_{1t} + \cdots + \beta_{iK} f_{Kt} + \varepsilon_{it}, i = 1, \ldots, N; t = 1, \ldots, T\\ \operatorname{var}(\varepsilon_{it}) &= \sigma_i^2, i = 1, \ldots, N\\ \operatorname{cov}(\varepsilon_{it}, f_{jt}) &= 0, j = 1, \ldots, K; i = 1, \ldots, N\\ \operatorname{cov}(f_{it}, f_{jt}) &= \sigma_{ij}^f, i, j = 1, \ldots, K\end{aligned}$$

Où $\beta_{ik} = 1$ si l'actif i est dans l'industrie k $(k = 1, .., K)$ et zéro sinon. On suppose qu'il y'a N_k firmes dans la kth industrie de sorte que $\sum_{k=1}^{K} N_k = N$. Eg. si l'actif i et appartient à l'industrie 1 et 4 respectivement on aura:

$$R_{it} = \beta_{i1} f_{1t} + \varepsilon_{it}; \quad R_{jt} = \beta_{j4} f_{4t} + \varepsilon_{jt}$$

- Forme matricielle (Industry).

$$\mathbf{R}_t = \underset{N\times K}{\mathbf{B}} \underset{K\times 1}{\mathbf{f}_t} + \varepsilon_t = \begin{pmatrix} | & | & \ldots & | \\ \beta_1 & \beta_2 & \ldots & \beta_K \\ | & | & \ldots & | \end{pmatrix} \begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{Kt} \end{pmatrix}$$

De plus, puisque les industries sont mutuellement exclusives il en suit que:

$$\boldsymbol{\beta}_j' \boldsymbol{\beta}_k = N_k \text{ for } j = k, 0 \text{ otherwise} \iff B^T B = diag(N_1, .., N_k)$$

Where N_k is the count of assets in industry k.

Eg:

$$B = \begin{array}{c c} & \begin{array}{cc} \text{Tech} & \text{Industry} \end{array} \\ \begin{array}{c} \text{AAPL} \\ \text{MSFT} \\ \text{EDF} \end{array} & \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \end{array} \Rightarrow \beta_1' \beta_1 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2 = N_1$$

Barra Factor (OLS - biased). An unbiased but inefficient estimate of the factor realizations $\mathbf{f}_t$ can be obtained by OLS giving

$$\widehat{\mathbf{f}}_{t,\text{OLS}} = (\mathbf{B}'\mathbf{B})^{-1} \mathbf{B}'\mathbf{R}_t$$

or

$$\begin{pmatrix} \widehat{f}_{1t,\text{OLS}} \\ \vdots \\ \widehat{f}_{Kt,\text{OLS}} \end{pmatrix} = \begin{pmatrix} \frac{1}{N_1} \sum_{i=1}^{N_1} R_{it}^1 \\ \vdots \\ \frac{1}{N_K} \sum_{i=1}^{N_K} R_{it}^K \end{pmatrix}$$

using (15.10) where R_{it}^{k} denotes the return on asset i if it is in industry k. Here, the estimated factor realizations $\widehat{f}_{kt}$ have nice interpretations. They represent an equally weighted average return in time period t on the industry k assets. Of course, this is expected given the nature of the binary industry factor beta values.

- Estimator of Factor Covariance matrix. Given the time series of factor realizations, the covariance matrix of the industry factors may be computed as the time series sample covariance

$$\widehat{\boldsymbol{\Omega}}_{\mathrm{OLS}}^{F} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\widehat{\mathbf{f}}_{t,\mathrm{OLS}} - \overline{\mathbf{f}}_{\mathrm{OLS}}\right)\left(\widehat{\mathbf{f}}_{t,\mathrm{OLS}} - \overline{\mathbf{f}}_{\mathrm{OLS}}\right)'$$

$$\overline{\mathbf{f}}_{\mathrm{OLS}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{f}}_{t,\mathrm{OLS}}$$

- Estimation of Residual variances. The residual variances, $\operatorname{var}(\varepsilon_{it}) = \sigma_i^2$, can be estimated from the time series of residuals from the T cross-section regressions given in (15.9) as follows. Let $\widehat{\boldsymbol{\varepsilon}}_{t,\ \mathrm{OLS}}\ , t = 1,\ldots,T$, denote the $(N \times 1)$ vector of OLS residuals from (15.9), and let $\widehat{\varepsilon}_{it,\ \mathrm{OLS}}$ denote the i^{th} row of $\widehat{\varepsilon}_{t,\ \mathrm{OLS.}}$. Then σ_i^2 may be estimated using

$$\widehat{\sigma}_{i,\mathrm{OLS}}^{2} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\widehat{\varepsilon}_{it,\mathrm{OLS}} - \bar{\varepsilon}_{i,\mathrm{OLS}}\right)^2, i = 1,\ldots,N$$

$$\bar{\varepsilon}_{i,\mathrm{OLS}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\varepsilon}_{it,\mathrm{OLS}}$$

- Overall Covariance matrix. The covariance matrix of the N assets is then estimated by

$$\widehat{\boldsymbol{\Omega}}_{\mathrm{OLS}} = \mathbf{B}\widehat{\boldsymbol{\Omega}}_{\mathrm{OLS}}^{F}\mathbf{B}' + \widehat{\mathbf{D}}_{\mathrm{OLS}}$$

where $\widehat{\mathbf{D}}_{\mathrm{OLS}}$ is a diagonal matrix with $\widehat{\sigma}_{i,\ \mathrm{OLS}}^{2}$ along the diagonal.

biblio:

- S plus documentation
- A blog on 100% industry model
- Slides of the S plus doc

1.3.10.2 Barra-type "Industry + Market" Factor Model - USEX

- Forme matricielle (Industry + Market). The market plus industry sector model has the form

$$\begin{aligned}\mathbf{R}_t &= \begin{pmatrix} \mathbf{1} & \vdots & \mathbf{B}_i \end{pmatrix}\mathbf{f}_{mi,t} + \boldsymbol{\varepsilon}_t, \quad t = 1,2,\cdots,T \\ &= \mathbf{B}_{mi}\mathbf{f}_{mi,t} + \boldsymbol{\varepsilon}_t\end{aligned}$$

where $\mathbf{B}_{mi}$ is an $N \times (K+1)$ matrix with $\mathbf{B}_i$ an $N \times K$ matrix of $1's$ and $0's$ representing K industry sectors, with each stock belonging to one and only one sector over the given time interval, and

$$\mathbf{f}_{mi,t} = (f_{0,t}, f_{1,t}, , f_{2,t}, \cdots, f_{K,t})'$$

It follow that the sum of the K column vectors of $\mathbf{B}_i$ is a vector of $1's$, and $\mathbf{B}_{mi}$ is rank deficient with rank K instead of $K+1$. Consequently the use of least squares to fit the above model for each time period does not lead to a unique solution.

bibliography:

- An implementation of industry with intercept
- Github model with

1.3.10.3 Barra-type "Size" with Style Factors - USEX

If attribute k represents "size", for example, then $\beta_{k,i}$ is typically a z-score type variable constructed by sorting all stocks by size (e.g. market capitalization) and then standardizing the sorted data

$$\beta_{k,i} = \frac{\text{market cap}_i - \text{mean (market cap)}}{\text{sd(market cap)}}$$

Then $\beta_{k,i} > 2$ indicates a very large firm; $\beta_{k,i} < -2$ indicates a very small firm.

Bibliography:

- here

1.3.10.4 Barra-type "Industry + Market + Style + Country" Factors - USE4

The GEM2 factor structure consists of a World factor, 55 country factors, 34 industry factors and 8 styles:

$$R_t =$$

Since every stock belongs to a country and industry, the factor structure above contains two exact collinearities. In order to impose a unique regression solution, we impose two constraints:

- The cap-weighted country factor returns sums to zero
- The cap-weighted industry factor returns sum to zero.

bibliography:

- Github model with

1.3.10.5 Barra-type "Industry + Style + Country" Factors - GEM2/USE4

bibliography:

- Lulu Wang model + explanations for covariance updates..
- Github model with
- Characteristics of factor portfolios Mencheron 2010; the author quotes:
  - An introduction to classical econometric theory by Ruud
  - Does industrial structure explain the benefits of international diversification ? Heston et al. 1994

1.3.10.6 To do job

to do expliuqer les différentes types de BARRA, la y'a BIM (Barra Integrated Model), Barra US Equity Factor Model ....

1.4 Risk decomposition

bibliograpby:

- slides
- blog on the topic

1.5 Informations complémentaires

According to Cochrane (p. 435, 2005) the difference btw two types of papers are:

- Time series: How average returns change over time
- Cross section: How average returns change across different stock or portfolios

If we study cross section of stock returns, we want to answer the question of why stock A earns higher/lower returns than stock B. Hence the name cross-section: at one point in time, we check the cross section of many stocks. For that you don't need a full time-series but rather one point in time. Eg. the CAPM, Fama-French .. are cross-section models.

biblio other models here

1.6 Fama McBeth

Step 1. Let's denote the excess returns of stock i at time t as $y_{t,i}$ within $[Y]_{T,1}$; and x_{jt} denotes the risk factor j at time t (eg HML, size..). Let's also assume we have a sample of $i = 1, .., N$ assets (stocks). Then we run N times OLS regressions of the Equations:

$$y_{t,i} = \alpha_i + \beta_{i,1}x_{i,1} + \beta_{i,2}x_{i,2} + .. + \beta_{i,K}x_{i,K} + \varepsilon_{i,j}$$

Where $\varepsilon_{i,j} \sim IID(0, \sigma^2)$. Furthermore, given each asset its parameter dimension is obviously $j = 1, .., K$ and the time dimension is $t = 1, .., T$.

$\Rightarrow$ each Equation gives us the estimated parameter vector $(\beta_{i,1}, .., \beta_{i,K})$. Then we repeat for $i = 1, .., N$ stocks so we get 500 vectors.

$$X_i = \begin{pmatrix} 1 & R_1^{Mkt} & R_1^{SMB} & R_1^{HML} \\ | & | & | & | \\ 1 & R_T^{Mkt} & R_T^{SMB} & R_T^{HML} \end{pmatrix}_{T\times K} \Rightarrow \beta_i = (X'X)^{-1}X'y_i = \begin{pmatrix} \alpha_1 & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix}_{4,1}$$

$\Rightarrow$ The whole system of equations gives us the matrix $\mathbf{B}$ that contains the estimated $\beta's$ vectors across all Equations. Moreover, we add a vector of ones since we don't care about the intercept:

$$B = \begin{pmatrix} 1 & \beta_{1,1} & \dots & \beta_{1,K} \\ | & | & | & | \\ 1 & \beta_{N,1} & \dots & \beta_{N,K} \end{pmatrix}_{N\times(K+1)}$$

Step 2. Calculate the Y vector that contains the sample averages of every single asset returns:

$$Y = \begin{pmatrix} \bar{y_1} \\ \vdots \\ \bar{y_N} \end{pmatrix}_{N,1} \qquad \bar{y_i} = \frac{1}{T}\sum_{t=1}^{T} y_{it}, i = 1, .., N$$

Step 3. Now cross-section regression, let denote

$$[\hat{\lambda}]_{k\times1} = (B'B)^{-1}B'Y = \begin{pmatrix} \hat{\lambda}_0 \\ \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_K \end{pmatrix}$$

- The risk premium's. The $\hat{\lambda}_1$ that's the risk premium associated with the first risk factor etc. Lambda states that: How are the variations in B related to the average returns ? If the lambda are statistically significant then we can say that the risk premium matter.

- The "intercept". Now what $\hat{\lambda}_0$ means ? It measures the average return that is unexplained by the variations in the B. It is the average "mispricing" so to speak. Recall previously we tested the α jointly then we conclude that the FF3 model is not able to price our model correctly. Here the $\hat{\lambda}_0$ gives us an estimate the "mispricing". So if it is statisticall different than 0 then the B exposure shows that it cannot price the cross-section of expected return correctly.

step 4. t-statistics. We use the expression of the lambda

$$[\hat{\lambda}]_{k\times1} = (B'B)^{-1}B' \begin{pmatrix} y_{1,1} \\ y_{1,2} \\ \vdots \\ y_{1,N} \end{pmatrix}_{t=1} = \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \vdots \\ \lambda_k \end{pmatrix}_{t=1}$$

We can then repeat for $t = 1, .., T$ times. It let us build the matrix:

$$\Lambda = \begin{pmatrix} \lambda_{0,1} & \lambda_{0,2} & \cdots & \lambda_{0,T} \\ \lambda_{1,1} & \lambda_{1,2} & \cdots & \lambda_{1,T} \\ \cdots & & & \vdots \\ \lambda_{K,1} & \lambda_{K,2} & \cdots & \lambda_{K,T} \end{pmatrix}_{(K+1)\times T}$$

Step 5. Now with the Λ matrix we have the evolution of the lambda across time. Now for example if we want the t-statistic for λ_1 we take the sample average across time since it should the same as above:

$$\bar{\lambda_1} = \frac{1}{T}\sum_{t=1}^{T}\lambda_{1,t} \Rightarrow \frac{\bar{\lambda_1}}{\sqrt{\frac{Var(\hat{\lambda_1}}{T}}} = t - stat$$

Step 6.
- For λ_0. We can build the hypothesis that $H_0 : \lambda_0 = 0$ or $H_a : \lambda_0 \neq 0$

- If the t-stat of $\hat{\lambda}_0$ is > 1.96 then we would conclude that the model cannot explain the cross-section of expected returns
- Alternatively we would conclude that it can explain the price of the cross-section of expected return correctly

- The λ_i. Again we test $H_0 : \lambda_i = 0$ or $H_a : \lambda_i \neq 0$:

- If the t-stat of $\hat{\lambda}_0$ is > 1.96 then we would conclude that eg the size factor in the cross-section explains significantly the expected returns.

Extra step - Extra. some people use rolling window regression they use the first 60 months and run the Fama McBeth and get the beta matrix and estimate the lambdas; Then they run the window forward 1 month ... So they get T-60 estimates for lambda.