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ABSTRACT

Our goal is to develop insights about the maximum squared Sharpe ratio for model factors as a metric for ranking asset pricing models. We consider nested and non-nested models. The nested models are the capital asset pricing model, the three-factor model of Fama and French (1993), the five-factor extension in Fama and French (2015), and a six-factor model that adds a momentum factor. The non-nested models examine three issues about factor choice in the six-factor model: (1) cash profitability versus operating profitability as the variable used to construct profitability factors, (2) long-short spread factors versus excess return factors, and (3) factors that use small or big stocks versus factors that use both.

JEL classification: G12

Keywords: Asset pricing tests, factor model, Sharpe ratio, max squared Sharpe ratio

1. Introduction

Harvey, Liu, and Zhu (2015) catalogue 316 anomalies proposed as potential factors in asset-pricing models and they note that there are others that do not make their list. Given the plethora of factors that might be included in a model, choosing among competing models is an open challenge.

Previous work takes two approaches. The left-hand-side (LHS) approach judges competing models on the intercepts (unexplained average returns) they leave in time-series regressions to explain excess returns on sets of LHS portfolios. See, for example, Fama and French (FF, 1993, 2012, 2015, 2016, 2017), Hou, Xue, and Zhang (2015, 2016), and Harvey and Liu (2016). A limitation of this approach is that inferences can vary across sets of LHS portfolios.

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An alternative, right-hand-side (RHS) approach uses spanning regressions to judge whether individual factors contribute to the explanation of average returns provided by a model. Each candidate factor is regressed on the model's other factors. If the intercept in a spanning regression is nonzero, that factor adds to the model's explanation of average returns in that sample period. [Fama (1998) provides an early proof. FF (1996, 2015, 2016, 2017) provide examples.] The *GRS* statistic of Gibbons, Ross, and Shanken (GRS, 1989) produces a test of whether multiple factors add to a base model's explanation of expected returns. We shall see that the RHS approach is useful for choosing among nested models, where the question is whether factors should be added. Non-nested models in which competing models have distinct factors are not suited for this approach.

We examine a performance metric proposed by Barillas and Shanken (BS, 2016) that also focuses on RHS factors of competing models and is potentially useful for choosing among nested and non-nested models. BS assume the factors of competing models are among the LHS returns each model is asked to explain. Formally, suppose R is the target set of nonfactor LHS excess returns, f_i is the factors of model i, and f_{Ai} is the union of the factors of model i's competitors. In the BS approach, the set of LHS returns for model i, Π_i, combines R and f_{Ai}, with linearly dependent components deleted.

Another BS assumption is that competing models should be judged on what we call the maximum (max) squared Sharpe ratio for the intercepts from time-series regressions of LHS returns on a model's factors. Define a_i as the vector of intercepts from regressions of Π_i on f_i and Σ_i as the residual covariance matrix. The max squared Sharpe ratio for the intercepts is

$$Sh^2(a_i) = a_i'\Sigma_i^{-1}a_i, \tag{1}$$

and the winner among competing models is the one that produces the smallest $Sh^2(a_i)$.

GRS (1989) show that $a_i'\Sigma_i^{-1}a_i$ is the difference between the max squared Sharpe ratio one can construct from f_i and Π_i together and the max for f_i alone,

$$Sh^2(a_i) = Sh^2(\Pi_i, f_i) - Sh^2(f_i). \tag{2}$$

Since Π_i includes the factors of all model i's competitors, the union of Π_i and f_i, which we call Π, does not depend on i. Eq. (2) then simplifies to

$$Sh^2(a_i) = Sh^2(\Pi) - Sh^2(f_i). \quad (3)$$

If the goal is to minimize $Sh^2(a)$ (and we ignore measurement error), the best model is the one whose factors have the highest $Sh^2(f)$. The BS argument that leads to $Sh^2(f)$ as the metric for judging asset pricing models implies that if Π spans the factors of competing models, inferences do not depend on what is in R, the nonfactor assets in Π. Since the goal of asset pricing models is to capture expected returns on all assets, our preference is to assume R includes all assets and to interpret the best model as the one whose $Sh^2(f)$ is closest to that produced by all assets.

If the inputs in $Sh^2(f)$ are population parameters, ordering models on $Sh^2(f)$ is clean. A problem arises, however, when the inputs are sample estimates. Speaking loosely, factors whose average returns are high relative to expected returns get too much weight in the estimated tangency portfolio, and vice versa. Sampling errors in the factor covariance matrix also affect the optimization. As a result, the estimate of the denominator of $Sh^2(f)$ is likely to be low relative to its population value. The end result is that estimates of $Sh^2(f)$ are upward biased. The bias is likely larger for models with more factors because more sampling errors are used in $Sh^2(f)$ estimates. The bias is also larger in smaller samples since parameter estimates have more sampling error.

The bias in estimates of $Sh^2(f)$ does not undermine our comparisons of nested models, which use spanning regressions to make inferences about $Sh^2(f)$. The t-statistic on the intercept from the spanning regression for one additional factor and the *GRS* test on the intercepts from spanning regressions for multiple additional factors account for sampling error. The bias is a problem when we compare non-nested models. Our solution is bootstrap simulations of in-sample (IS) and out-of-sample (OS) $Sh^2(f)$ estimates for competing models. The tests use US stock returns for July 1963-June 2016. We split the 636 months into 318 adjacent pairs: months (1, 2), (3, 4) … (635, 636). Each simulation run draws (with replacement) a random sample of 318 pairs and randomly assigns a month from each pair to the IS sample (using that month repeatedly if the pair is drawn more than once). We use the in-sample months to compute the run's values of IS $Sh^2(f)$ for all models. Each model's IS $Sh^2(f)$ identifies weights for factors in its IS tangency portfolio for a simulation run. We combine these weights with the unused months of the chosen pairs to compute the run's out-of-sample estimate of the Sharpe ratio. IS $Sh^2(f)$ are subject to the upward bias described above, but since monthly returns are close to serially uncorrelated, OS Sharpe ratios are free of the bias. A benefit of the paired observation approach to the simulations is that the effects of parameter nonstationarity are similar in sample and out of sample. Since the bias in IS $Sh^2(f)$ induced by sampling error can differ across models, OS and IS squared Sharpe ratios can rank models differently.

Bias-free OS estimates of $Sh^2(f)$ come at a cost. IS and OS samples are half the size (318 months) of the full sample (636 months), so the parameters in IS and OS $Sh^2(f)$ estimates have more sampling error than the parameters in full-sample (FS) simulations that estimate $Sh^2(f)$ from random samples (with replacement) of 636 months. We see later that because they use parameters with less sampling error, bootstrap distributions of FS $Sh^2(f)$ are less disperse than IS and OS distributions, and FS estimates are less upward biased than IS estimates. We examine FS, IS, and OS estimates.

We use $Sh^2(f)$ as the metric in two limited tasks. The first is comparing nested versions of the five-factor model of FF (2015) augmented (by popular demand) with a momentum factor. Motivated by the dividend discount model, FF (2015) add profitability and investment factors to the three-factor model of FF (1993). Here we add a momentum factor to the five-factor model. The time-series regression for the resulting six-factor model is

$$R_{it} - F_t = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_{Ot} + c_i CMA_t + m_i UMD_t + e_{it}. \tag{4}$$

In this equation, R_{it} is the month t return on asset i, F_t is the one-month US Treasury bill rate observed at the beginning of t, Mkt_t is the return on the value-weight (VW) portfolio of NYSE, AMEX, and Nasdaq stocks in excess of F_t, SMB_t (small minus big) and HML_t (high minus low book-to-market equity) are the size and value factors of the FF (1993) three-factor model, RMW_{Ot} (robust minus weak) is a profitability factor, CMA_t (conservative minus aggressive) is an investment factor, and UMD_t (up minus down) is a momentum factor. All factors are described in detail later.

The nested models are the capital asset pricing model (CAPM) in which (dropping the time subscript) Mkt is the only explanatory variable, the FF (1993) three-factor model that adds SMB and

HML, the FF (2015) five-factor extension that includes *RMW* and *CMA*, and the six-factor model. The results for these nested models are not surprising (the six-factor model wins), but they illustrate that tests on the intercepts from spanning regressions are equivalent to ordering models on $Sh^2(f)$.

Our second and more challenging task is to illustrate how $Sh^2(f)$ can be used to choose among non-nested models. IS, OS, and FS simulations are the tools. We study three issues about the factors in the six-factor model suggested by previous work. The first concerns the profitability factor RMW_O in Eq. (4). It is constructed by sorting stocks on the accruals-based operating profitability (*OP*) measure suggested by Novy-Marx (2013), and it is the profitability factor in the five-factor model of FF (2015, 2016, 2017). Ball, Gerakos, Linnainmaa, and Nikolaev (2016) argue that cash profitability (*CP*), that is, profitability unaffected by accruals, produces a factor that better captures average returns in sorts on accruals. We test whether replacing RMW_O by a cash profitability factor, RMW_C, increases $Sh^2(f)$.

The second issue about factor choice centers on *HML*, *CMA*, *RMW*, and *UMD*. Each is an average of spread portfolio returns for small and big stocks. For example, *HML* is the average of HML_S and HML_B, where the spread portfolio HML_S is the difference between the returns on high and low book-to-market equity portfolios of small stocks and HML_B is the return difference for high and low book-to-market portfolios of big stocks. The small stock components of *HML*, *CMA*, *RMW*, and *UMD* have larger average returns than the big stock components, and most patterns in average returns uncovered in previous research are stronger for small stocks. It is thus possible that factors that equal weight small and big stock components underestimate the premiums in small stock average returns and overestimate the premiums for big stocks. As an alternative, we consider models that use *Mkt*, *SMB*, and just the small or big components of *HML*, *CMA*, *RMW*, and *UMD*.

In the six-factor model of Eq. (4), *Mkt* is an excess return (the market return in excess of the risk-free rate), but the other factors are spread portfolio returns. For example, *HML* is the difference between returns on portfolios of high and low book-to-market stocks (*H* and *L*). If one uses Merton's (1973) intertemporal capital asset pricing model to motivate multifactor models, the natural explanatory returns are *Mkt* and either excess returns on the *K* relevant state variable mimicking portfolios or [in the terminology of Fama (1996)] excess returns on K multifactor minimum variance portfolios. The third factor choice issue is whether spread factors produce higher $Sh^2(f)$ than factors that are excess returns on the long or short ends of spread factors.

Our version of the argument that leads to $Sh^2(f)$ as a metric for judging asset pricing models assumes the goal is to find the model that minimizes the max squared Sharpe ratio of the intercepts for all assets. Minimizing $Sh^2(a)$ is not the only reasonable objective. For example, the weighting scheme in $Sh^2(a) = a'\Sigma^{-1}a$ typically has extreme long and short positions that may not capture the relative importance of assets in applications. We see later that $Sh^2(f)$ also uses extreme positions in factors. Thus, we also examine how $Sh^2(f)$ lines up with common performance metrics that equal weight functions of intercepts for interesting sets of LHS assets (the LHS approach). When interpreting the results, we keep in mind the warning of Roll and Ross (1994) and Kandel and Stambaugh (1995) that unless a model captures expected returns on all assets, different sets of LHS portfolios can order models differently.

Harvey, Liu, and Zhu (2016) emphasize that undisciplined search for the best model in a large set of potential factors can create an overwhelming multiple comparisons problem that preempts statistical inference. If we are to order models in a reliable way, the number of models considered must be limited. The obvious path is to use theory to limit the set of competing models. In the ideal case, theory provides fully specified models that lead to precise statements about the relation between an asset's measurable characteristics and its expected return. The CAPM of Sharpe (1964) and Lintner (1965) is a prime example, and its fully specified predictions about risk and expected return explain its lasting attraction.

The empirical failures of the CAPM and its fully specified competitor, the consumption-based capital asset pricing model (CCAPM) of Lucas (1978) and Breeden (1979), lead to factor models motivated by theory that does not produce fully specified models but just suggests variables and derived factors likely to be important in describing expected returns. An early example is Ball (1978), who argues that average returns are related to price ratios like book-to-market equity (*BE*/*ME*) because of the discount rate effect. Stocks with high expected returns have low prices relative to future expected cash flows. If current fundamentals are reasonable proxies for expected cash flows, low prices relative to fundamentals should be related to higher expected returns. This argument is the motivation for *HML*, the *BE*/*ME* value factor of the three-factor model of FF (1993), and (with a stretch) it can motivate the size factor, *SMB*, which is based on market capitalization (price times shares outstanding), not the size of assets or book equity.

Ball's (1978) discount rate effect is essentially an appeal to the dividend discount model, which FF (2015) use to motivate the addition of profitability and investment factors, RMW_O and *CMA*, to the FF (1993) three-factor model. The dividend discount model is an umbrella: regardless of the process generating prices, there is a discount rate, which we call the long-run expected return, that links a stock's price to its expected dividends. Since the dividend discount model says nothing about what determines expected returns, it offers no clues about whether the FF (2015) five-factor model collapses to three-factor or CAPM pricing. The model is useful for our purposes, however, because the factors it spawns are limited to those linked to expected future cash flows.

Theory, even umbrella theory such as the dividend discount model or the production-based model of Cochrane (1991) invoked by Hou, Xue, and Zhang (2015), helps limit the range of competing models. Robustness of results is another limiting consideration. Factors that seem important often lose explanatory power out-of-sample (McLean and Pontiff, 2016; Hou, Xue, and Zhang, 2016; Harvey, Liu, and Zhu, 2015). In contrast, most if not all the factors of the FF (1993, 2015) three-factor and five-factor models, initially studied in US data beginning in 1963, survive tests on an earlier US sample (Davis, Fama and French, 2000; Wahal, 2017) and on international data (FF, 2012, 2016).

Thorny issues arise for factors that have no theoretical motivation but are robust in out-of-sample tests. Without a model that identifies the forces responsible for a meaningful pattern in observed returns, it is hard to assess the likelihood the pattern will persist. One could draw a line in the sand and exclude such factors, even when they enhance model performance. The models in previous versions of this paper, for example, exclude momentum factors. We include momentum factors (somewhat reluctantly) now to satisfy insistent popular demand. We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside: the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging that produces a long list of factors with little hope of sifting through them in a statistically reliable way.

To limit data mining that would cloud statistical inference, we consider a limited set of models that (except for inclusion of momentum factors) are nested in the dividend discount model, and a limited set of factor construction issues suggested by previous research. We do not consider all factor construction issues [for example, FF (2015)], and we do not consider all additional theory-motivated factors that might help capture average returns. This is reasonable discipline, in line with previous work. For example, Hou, Xue, and Zhang (2015, 2016) use the LHS approach and a wide range of LHS portfolios to choose among five models. Harvey, Liu, and Zhu (2015) consider a long list of factors, but they do not attempt to extract the best overall model. Indeed, their work is a cautionary tale on the multiple comparisons problem in an undisciplined search for the best model in a long list of potential factors.

Our story unfolds as follows. Section 2 presents an expression for a factor's marginal contribution to $Sh^2(f)$, which later helps us estimate factor contributions to different models. Summary statistics for factors are in Section 3. Section 4 considers choice among nested models. The heavy lifting is done by spanning regressions and *GRS* tests of whether adding factors to a model increases $Sh^2(f)$. The rest of the paper addresses choice among competing versions of the six-factor model. Section 5 shows that in the models we study, cash profitability (*CP*) factors produce higher $Sh^2(f)$ than operating profitability (*OP*) factors. Section 6 uses $Sh^2(f)$ to rank models that use *CP* factors. Section 7 studies marginal contributions of factors to $Sh^2(f)$ and factor weights in tangency portfolios implied by $Sh^2(f)$. Section 8 examines how $Sh^2(f)$ compares with other measures of model performance. Section 9 summarizes and concludes with general comments on discipline for the sake of inference when choosing among factor models.

2. Marginal contributions to $Sh^2(f)$

The GRS (1989) result in Eq. (2) provides a simple way to measure a factor's marginal contribution to $Sh^2(f)$, the max squared Sharpe ratio for a model's factors. Let a_i be the intercept in the spanning regression of factor i on the model's other factors and let σ_i be the standard deviation of the regression residuals. If we interpret factor *i* as the single LHS return to be explained and the other factors as *f*, then Eqs. (1) and (2) imply that the increase in the max squared Sharpe ratio for a model's factors when *i* is added to the model is

$$a_i^2/\sigma_i^2 = Sh^2(f, i) - Sh^2(f). \tag{5}$$

In this equation, $Sh^2(f, i)$ is the max squared Sharpe ratio for the expanded model that includes *f* and *i*, and $Sh^2(f)$ is the max squared Sharpe ratio for *f*. Eq. (5) says a factor's marginal contribution to a model's max squared Sharpe ratio is small if the factor's expected return is explained well by other factors (a_i is close to zero), its variation not explained by other factors (σ_i) is large, or both.

3. The candidate factors

Our sample is NYSE, AMEX, and Nasdaq stocks with Center for Research in Security Prices (CRSP) share codes 10 or 11 and the Compustat data required for a sort. A stock can be in one set of portfolios even if it does not have the accounting data necessary for other sorts. Definitions of the sort variables are in the Appendix.

Our non-nested models are 12 versions of the six-factor model. The excess market return, *Mkt*, is in every model, but models differ on how other factors are defined. *HML*, the value factor in Eq. (4), is from annual (end of June) independent sorts of stocks into two size groups and three book-to-market equity (*BE/ME*) groups. The accounting variables for these and other sorts at the end of June of year *t* are for the fiscal year ending in the previous calendar year, and market cap *ME* in *BE/ME* is for the end of December of *t*-1. The breakpoints for all sorts use NYSE stocks. The size break is the NYSE median *ME* at the end of June, and the *BE/ME* breakpoints are the 30th and 70th percentiles of *BE/ME* for NYSE stocks. The intersections of the sorts produce six portfolios, L_S, N_S, H_S, L_B, N_B, and H_B, where *L*, *N*, and *H* indicate growth, neutral, and value (low 30%, middle 40%, and high 30% of *BE/ME*) and subscripts *S* and *B* indicate small and big. We compute monthly VW portfolio returns from July of year *t* to June of *t*+1. We construct value minus growth spread factors for small and big stocks, $HML_S = H_S\text{-}L_S$ and $HML_B = H_B\text{-}L_B$, and *HML*, the combined spread factor, is the average of the small and big spread factors.

The investment factor, *CMA* (conservative minus aggressive), is constructed like *HML* except the second sort at the end of June of year *t* is on the rate of growth of total assets (low to high) for the fiscal year ending in the previous calendar year. The profitability factor in Eq. (4), RMW_O (robust minus weak operating profitability), is also formed like *HML* except the second sort is on operating profitability (net of interest expense and scaled by book equity) for the fiscal year ending in the previous calendar year. The cash profitability factor RMW_C mimics RMW_O except profitability is cash profits (operating profits minus the effect of accruals) divided by book equity. *CMA*, RMW_O, and RMW_C are averages of small stock and big stock spread factors: CMA_S and CMA_B, RMW_{OS} and RMW_{OB}, and RMW_{CS} and RMW_{CB}.

In previous drafts, we follow Ball, Gerakos, Linnainmaa, and Nikolaev (2016) and measure operating and cash profitability before research and development (R&D) expense. This is equivalent to treating R&D as an infinitely lived asset (which means it should be added to investment). Here we follow the Financial Accounting Standards Board and measure profitability net of R&D. This change (not the addition of six months to the sample) accounts for the main changes in results in this draft.

The 2x3 sorts for *HML*, RMW_O, RMW_C, and *CMA* produce four size factors: SMB_{BM}, SMB_{OP}, SMB_{CP}, and SMB_{Inv}. For example, SMB_{BM} is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns from the *ME-BE/ME* sorts. We combine the four size factors into two factors. SMB_O is the average of SMB_{BM}, SMB_{OP}, and SMB_{Inv}, and SMB_C is the average of SMB_{BM}, SMB_{CP}, and SMB_{Inv}. Since SMB_{OP} and SMB_{CP} use all stocks with the required accounting data, differences between SMB_O and SMB_C are trivial. For example, average SMB_O is 0.26% per month (t = 2.11), average SMB_C is 0.27% (t = 2.15), and the correlation between them exceeds 0.999. Because the size factors are so similar and the models that use cash profitability factors beat models that use operating profitability factors, we use SMB_C or its excess return factor, S_C-*F*, in the tests, and they are labeled *SMB* and *S-F*.

The momentum factor, *UMD*, is defined like *HML*, except it is updated monthly instead of annually, and the sort for portfolios formed at the end of month *t*-1 is based on *Mom*, the average return from *t*-12 to *t*-2. In contrast, *SMB*, *HML*, RMW_O, RMW_C, and *CMA* are updated annually using data that, except for size, are at least six months old. Like *HML*, RMW_O, RMW_C, and *CMA*, *UMD* is the average of a small stock spread factor, UMD_S, and a big stock spread factor, UMD_B.

Each spread factor is parent to two factors that are excess returns on its long and short ends. The long and short excess return factors are denoted by the first and third letters of the parent spread factor's name. For example, *H-F* and *L-F* are excess returns on the long and short ends of *HML*, H_S-*F* and L_S-*F* are excess returns on the long and short ends of the small stock spread factor HML_S, and the long and short ends of the big stock spread factor HML_B produce H_B-*F* and L_B-*F*. In general, the subscript *S* or *B* on a factor indicates that it includes only small or big stocks. Absence of subscript *S* or *B* means a factor is an equal-weight combination of small and big components.

Table 1 shows summary statistics for the factors. All spread factors that combine small and big stocks have strong average returns (premiums) during the July 1963 to June 2016 period. The market premium is 0.50% per month (t = 2.84). The value premium (average *HML* return), the investment premium (average *CMA* return), and the two profitability premiums (average RMW_O and RMW_C returns) are 0.24% to 0.36% per month and 2.75 to 4.71 standard errors above zero. The correlation (not shown) between the two profitability factors is low, 0.66. This suggests that accruals are important in the variation in RMW_O. The average *UMD* return is large, 0.69% per month (t = 4.09).

Insert Table 1 near here.

The average small spreads in *HML*, RMW_O, RMW_C, *CMA*, and *UMD* are between 0.31% and 0.92% per month, the average big spreads are between 0.17% and 0.46%, and except for the profitability factors, a factor's average small spread exceeds its average big spread by more than two standard errors.

Average monthly returns for long excess return factors that combine small and big stocks range from 0.74% (R_O-*F*, t = 3.79) to 0.96% (*U-F*, t = 4.56). These high average returns are not surprising because they all in effect include the average excess market return, which is strong during the sample period, and they get the benefit of the high average return for small stocks, whose 50% weight in combined factors is much greater than their weight in the market portfolio. In contrast, average returns for most short excess return factors are below the average excess market return.

4. Nested models

Presenting summary statistics for factors is a ritual in papers that test asset pricing models. The message from Eq. (4), however, is that if models are judged on the max squared Sharpe ratio produced by their factors, $Sh^2(f)$, the relevant average return for measuring the marginal contribution of a factor to a model is the intercept in the spanning regression of the factor on the model's other factors, tempered by the variance of the spanning regression residuals. We use spanning regression intercepts and *GRS* tests on the intercepts in a simple task – choosing among nested models, specifically, the FF (1993) three-factor model versus the CAPM, the FF (2015) five-factor model versus the three-factor model, and the five-factor model versus a six-factor model that adds *UMD*.

The *GRS* test on the intercepts from the spanning regressions of *SMB* and *HML* on *Mkt* (Table 2) rejects the hypothesis that the intercepts are jointly zero with a *p*-value that is zero to at least three decimals. This result implies that adding *SMB* and *HML* to *Mkt* produces reliably higher $Sh^2(f)$ than *Mkt* alone: the CAPM loses to the three-factor model. The intercept in the spanning regression for *HML* is 0.43% per month (t = 4.01). The intercept in the regression for *SMB* is less impressive, 0.17 (t = 1.46).

Insert Table 2 near here.

Spanning regressions of the profitability and investment factors on *Mkt*, *SMB*, and *HML* are center stage in the choice between three-factor and five-factor models. Table 2 shows results for two sets of regressions. One pairs *CMA* with RMW_O, the operating profitability version of the profitability factor, and the other uses the cash profitability version, RMW_C. *GRS* tests on the intercepts from both sets reject (*p*-value = 0.000) the three-factor model in favor of the five-factor model. The intercepts in the profitability factor regressions are strong, 0.34% per month (t = 4.01) for RMW_O and 0.48% (t = 7.88) for RMW_C. The intercept in the investment factor regression is also strong, 0.20% (t = 3.53). The *GRS* tests imply that adding profitability and investment factors to the three-factor model improves $Sh^2(f)$.

The intercepts and their *t*-statistics in the spanning regression of *UMD* on the factors of the five-factor model suffice to show that the momentum factor adds to five-factor $Sh^2(f)$. The *UMD* regression intercept and *t*-statistic are larger when the profitability factor is RMW_O (0.73% per month, t = 4.34), but the intercept is also far from zero (0.61%, t = 3.55) when the profitability factor is RMW_C.

We turn now to a more challenging task, using $Sh^2(f)$ to choose among six-factor models. The Appendix shows results for five-factor models that drop momentum factors.

5. Operating or cash profitability?

Do cash profitability (*CP*) factors deliver larger $Sh^2(f)$ than operating profitability (*OP*) factors? Table 3 addresses this question for two sets of six-factor models that are identical except one set uses *OP* and the other uses *CP* factors. The list of models is driven by the other questions we address: Do factors that combine small and big stocks produce larger $Sh^2(f)$ than factors limited to small or big stocks? Do spread factors produce larger $Sh^2(f)$ than excess return factors? These questions are directed at value, profitability, investment, and momentum factors. The market portfolio is the centerpiece of asset pricing models and *Mkt* is in all our models. *SMB* is the size factor in spread factor models, and *S-F* (excess return on the small stock end of *SMB*) is the size factor in excess return models. Models in which the other factors include only big stocks perform poorly, and we do not show results for them.

Insert Table 3 near here.

Panel A of Table 3 shows Actual $Sh^2(f)$ and means and medians of $Sh^2(f)$ from 100,000 full-sample, in-sample, and out-of-sample bootstrap simulation runs. Again, OS estimates apply the weights for factors in the tangency portfolio implied by IS $Sh^2(f)$ to the factor returns of the matched sample of adjacent months. Actual, FS, and IS estimates of $Sh^2(f)$ are upward biased because they maximize in part on sampling error. OS estimates are free of this bias. In each trial, the FS, IS, or OS bootstrap months are the same for all models.

Some results in Table 3 are predictable. For example, since an IS simulation run has half as many observations as an FS run, the bias caused by sampling error in the maximization is bigger in IS simulations. As a result, means and medians of $Sh^2(f)$ are higher in IS than in FS simulations. More sampling error in IS $Sh^2(f)$ also means more dispersion. Skipping the details, the 5th percentile of IS $Sh^2(f)$

is always below the 5th percentile of FS estimates for the same model, and the 95th percentile of IS $Sh^2(f)$ is (further) above the 95th percentile of FS estimates. The bias in FS and IS estimates is most apparent in the large drops in means and medians of $Sh^2(f)$ from FS and IS simulations to OS simulations.

The means of FS, IS, and OS estimates of $Sh^2(f)$ are higher than the medians. The implied right skewness arises because sampling variation in the variance in the denominator of $Sh^2(f)$ has asymmetric effects on $Sh^2(f)$. Skewness is stronger in IS than in FS estimates (the means of $Sh^2(f)$ are further above the medians) because smaller samples imply more sampling variation in the denominator variance.

Despite differences in the levels of Actual, FS, IS, and OS $Sh^2(f)$ in Panel A of Table 3, Panel B shows that average spreads between $Sh^2(f)$ for matched *CP* and *OP* models are similar to median spreads, FS and IS average and median spreads are similar to sample Actual spreads, and average and median spreads between $Sh^2(f)$ for *CP* and *OP* models are only slightly smaller in OS simulations.

Most important, Panel B of Table 3 shows that on all summary metrics, six-factor models that use *CP* factors produce higher $Sh^2(f)$ than matching models that use *OP* factors. The differences are typically large and statistically reliable. For example, replacing RMW_O with RMW_C in the spread factor model of Eq. (4) increases average OS $Sh^2(f)$ by almost 50%, from 0.108 to 0.159. Models that use *CP* factors produce higher $Sh^2(f)$ than matching models that use *OP* factors in more than 95% of FS simulation runs. With the smaller sample size in IS and OS simulations, models that use *CP* factors still win on $Sh^2(f)$ in more than 82% of simulation runs. Skipping the details, we can report that with the power of 100,000 replications, average differences between $Sh^2(f)$ for *CP* and *OP* models are more than 160 standard errors from zero in the three sets of simulations.

Appendix Table A1 confirms that five-factor (no momentum) models with *CP* factors have higher $Sh^2(f)$ than models that use *OP* factors. We continue to show results for the RMW_O spread factor model of Eq. (4) for perspective, but we drop other *OP* models from remaining tests.

6. Ordering six-factor models on $Sh^2(f)$

The model that includes *Mkt*, *SMB*, and small stock spread factors (HML_S, *RMWcs*, CMA_S, and UMD_S) has the highest sample Actual $Sh^2(f)$, 0.226 (Panel A of Table 3). A surprising second ($Sh^2(f)$ =

0.210) is the model that uses *Mkt*, *S-F*, and excess returns L_S-*F*, W_{CS}-*F*, A_S-*F*, and D_S-*F* on the short ends of HML_S, RMW_{CS}, CMA_S, and UMD_S. The standard model that combines *Mkt*, *SMB*, and spread factors *HML*, *RMWc*, *CMA*, and *UMD* that include small and big stocks places third ($Sh^2(f)$ = 0.190). The order of models on Actual $Sh^2(f)$ is the same as the order on mean and median $Sh^2(f)$ in the FS and IS simulations. Mean and median squared Sharpe ratios are lower in the OS simulations, but the declines are similar for all models, and the order of models is unchanged.

Are the values of $Sh^2(f)$ for the three top models statistically distinguishable from those for other models or from one another? Table 4 summarizes 100,000 FS, IS, and OS bootstrap samples that address this question. The table shows average and median $Sh^2(f_{Column})$-$Sh^2(f_{Row})$, the difference between $Sh^2(f)$ for a column model and a row model, and the percent of simulation runs in which the difference is negative (the column model loses). The column models are the top three on $Sh^2(f)$ in Panel A of Table 3.

Insert Table 4 near here.

For each model, there are systematic differences across the averages of FS, IS, and OS $Sh^2(f)$ and across the medians (Panel A of Table 3). In Table 4, however, average (and median) $Sh^2(f_{Column})$-$Sh^2(f_{Row})$ are similar across FS, IS, and OS simulations. This suggests that upward bias in $Sh^2(f)$ in FS and IS simulations largely cancels in differences between $Sh^2(f)$ for different models. Given that bias is not important, we can lean on the larger FS simulation samples (636 months versus 318 in IS and OS simulations) for more precise inferences about the reliability of $Sh^2(f)$ margins for winning models.

Table 4 confirms that the first-place model that combines *Mkt*, *SMB*, and small stock spread factors is better on $Sh^2(f)$ than other models in a preponderance of simulation runs. It loses to the second-place model (*Mkt*, *S-F*, and excess returns on the short ends of small stock spread factors) in only 17.5% of FS simulation runs. The third-place model (*Mkt*, *SMB*, and spread factors that combine small and big stocks) beats it in only 8.7% of FS simulation runs. Other models lower on Actual $Sh^2(f)$ beat the winning model in less than 3.2% of FS simulation runs. The smaller samples of OS simulations produce more disperse $Sh^2(f_{Column})$-$Sh^2(f_{Row})$, but the top model on $Sh^2(f)$ beats the second- and third-place models in 74.1% and 80.0% of simulation runs, and it beats other models in at least 84.6% of simulation runs.

Appendix Tables A1 and A2 repeat Tables 3 and 4 for five-factor models. The results are different. No clear five-factor winner emerges. The top three five-factor models (which, except for the absence of momentum factors, are the same as the top three six-factor models in Table 3) are statistically indistinguishable on $Sh^2(f)$, but they are reliably better than other models. Thus, if momentum factors are dropped, the standard model that uses spread factors that combine small and big stocks performs as well on $Sh^2(f)$ as any of the other models we consider.

7. Deconstructing $Sh^2(f)$

For the top three models on $Sh^2(f)$ in Table 3, Table 5 reports spanning regressions that explain each of the six factors in a model with the other five. The table also shows $Sh^2(f)$ and marginal contributions of factors to $Sh^2(f)$. From Eq. (5), the marginal contributions are a_i^2/σ_i^2, the square of a factor's spanning regression intercept over the regression's residual variance.

Insert Table 5 near here.

7.1. Spread factor models

The *t*-statistic for the intercept in a factor's spanning regression measures the statistical reliability of the factor's marginal contribution to $Sh^2(f)$. For the two models that use spread factors (combined or small), the spanning regressions for *Mkt*, *SMB*, and RMW_C or RMW_{CS}, *CMA* or CMA_S, and *UMD* or UMD_S produce only one intercept (t = 2.87 for *CMA*) less than 3.5 standard errors from zero.

As in FF (2015), in the models that use spread factors, only the value factors (*HML* and HML_S) do not contribute much to $Sh^2(f)$. The *t*-statistics for the intercepts in the *HML* and HML_S regressions are 0.90 and -0.59 (Table 5), and the marginal contributions of *HML* and HML_S to $Sh^2(f)$ are trivial. The *HML* and HML_S regressions say that large average *HML* and HML_S returns (0.35, t = 3.15, and 0.51, t = 4.00, in Table 1) are absorbed by strong positive slopes on *CMA* and (for HML_S) $RMWC_S$. In contrast, in the *Mkt* and *SMB* regressions, negative slopes on profitability and (for *Mkt*) investment factors lead to intercepts that are about twice average *Mkt* and *SMB* returns (Table 1). These results show that average returns can be misleading for judging the importance of factors in multifactor models.

Sometimes the multivariate regression slopes in Table 5 mimic univariate characteristics and sometimes they do not. For example, small stocks tend to be less profitable firms (FF, 1996), which is in line with negative slopes on RMW_C or RMW_{CS} in spanning regressions for *SMB*. Value stocks tend to be associated with low investment (FF, 1996), which is in line with positive *CMA* slopes in *HML* and HML_S regressions. But value stocks also tend to be less profitable, which is not in line with positive *RMW* and $RMWC_S$ slopes in *HML* and HML_S regressions.

Table 5 also shows that marginal contributions of factors to $Sh^2(f)$ depend on residual variances in spanning regressions as well as on regression intercepts. For example, in the two regressions that use spread factors, the intercepts for RMW_C and RMW_{CS} are less than half those for *Mkt*, but the residual variances in the regressions for the profitability factors are also less than half those in the *Mkt* regressions. As a result, the marginal contributions of RMW_C and RMW_{CS} to $Sh^2(f)$ are similar to those of *Mkt*. The intercepts for *SMB* are similar to those for RMW_C or RMW_{CS}, but residual variances are higher in the *SMB* regressions, and the marginal contributions of *SMB* to $Sh^2(f)$ are less than half those of RMW_C or RMC_S. The spanning regressions to explain *UMD* and UMD_S also illustrate the role of residual variances in marginal contributions to $Sh^2(f)$. The intercepts, 0.61 and 0.96 (Table 5), are close to the large average returns for *UMD* and UMD_S, 0.69 and 0.92 (Table 1), but the regression R^2 are only 0.10 and 0.05, far below R^2 in the spanning regressions for other factors. As a result, the residual variances for *UMD* and UMD_S are high and their marginal contributions $Sh^2(f)$ for the spread models are only 0.023 and 0.054.

In sum, in the model that uses spread factors that include small and big stocks, *Mkt* and *RMW* are by far the biggest marginal contributors to $Sh^2(f)$ (0.080 and 0.085), followed by *SMB* (0.034), *UMD* (0.023), *CMA* (0.015), and *HML* (0.002). Marginal contributions to $Sh^2(f)$ are more bunched in the model that uses small stock spread factors, 0.070 and 0.066 for *Mkt* and RMW_S, followed by 0.054 for UMD_S, 0.042 for CMA_S, and 0.031 for *SMB*, with HML_S a distant last, 0.001. The larger marginal contributions of CMA_S and UMD_S, relative to those of *CMA* and *UMD*, probably trace to investment and momentum premiums that are stronger for small stocks, which is less true for the profitability premium (Table 1).

7.2. Short excess return factors

Spanning regression results in Table 5 are different for the model that combines *Mkt* and *S-F* with L_S-*F*, W_S-*F*, A_S-*F*, and D_S-*F*, which are excess returns on the short (low average return) ends of the small stock spread factors, HML_S, RMW_{CS}, CMA_S, and UMD_S. A surprising result is that *Mkt*, a top marginal contributor to $Sh^2(f)$ in spread factor models, is redundant in the model that uses excess returns on the short ends of small stock spread factors (intercept = -0.03, t = -0.41, marginal contribution to $Sh^2(f)$ = 0.000). L_S-*F*, the excess return on the short end of the small stock value factor HML_S, is also largely redundant (intercept = 0.07, t = 1.81, marginal contribution to $Sh^2(f)$ = 0.006). In contrast, *S-F*, the excess return on the long end of *SMB*, makes by far the biggest marginal contribution to $Sh^2(f)$, 0.179. If *S-F* is dropped, $Sh^2(f)$ falls from 0.210 to a puny 0.031. The marginal contributions of the profitability, investment, and momentum factors, W_S-*F*, A_S-*F*, and D_S-*F*, to $Sh^2(f)$ are moderate, 0.048, 0.031, and 0.034, but we see next that negative spanning regression intercepts for these factors mean their main role in the tangency portfolio that produces $Sh^2(f)$ is to feed short sale receipts to *S-F*.

The spanning regressions in Table 5 say that value factors add little to six-factor descriptions of average returns for July 1963–June 2016. Table A3 confirms this result for five-factor (no momentum) models. Thus, for these models and this period, value factors can be dropped in the interest of parsimony. If one contemplates this route in applications for a different sample period, it is worth confirming that redundancy holds since spanning results can be sensitive to period (FF, 2017).

7.3. Weights for factors in $Sh^2(f)$

The marginal contribution of a factor to a model is the increase in $Sh^2(f)$ when that factor is added to the model's other factors. Marginal contributions are not a breakdown of six-factor $Sh^2(f)$. Direct perspective on how all factors of a six-factor model combine to produce $Sh^2(f)$ is in Table 6, which shows factor weights that deliver the monthly excess return on the tangency portfolio T_f implied by $Sh^2(f)$, for the models of Table 4. The weights are percentages of \$1 invested in the tangency portfolio. The weights are scaled to make the sum of long and short positions 100%, which implies no net investment in the risk-free asset: we are at the tangency portfolio T_f and not a point along the line from *F* through T_f. For spread factor models, this means the weight for the market portfolio is 100% since net investment in each (long-short) spread factor is zero. For models that use excess return factors, the sum of the weights for all factors is 100%. Table 6 also shows leverage in T_f, that is, dollars sold short per dollar invested. For a spread factor model, leverage is the sum of the absolute values of percentage weights for the model's spread factors divided by 100. For a model that uses excess return factors, leverage is the sum of the absolute values of any negative percentage weights for model factors divided by 100.

Insert Table 6 near here.

Tangency portfolios sell short liberally. The spread factor model that combines small and big stocks uses the least leverage, \$5.5 per \$1 net investment. The two models that include excess returns on the long ends of spread factors employ the most leverage, \$16.3 and \$17.8 per \$1 invested.

Weights in tangency portfolios need not line up neatly with marginal contributions to $Sh^2(f)$. In the cash profitability model that uses spread factors constructed with small and big stocks, for example, the marginal contribution of *CMA* to $Sh^2(f)$, 0.015 in Table 5, is low relative to that of *Mkt*, 0.080, but the weight for *CMA* in $Sh^2(f)$, 116.7%, is slightly higher than that of *Mkt*, 100%. For this model, the marginal contribution of the momentum factor *UMD* to $Sh^2(f)$ is 0.023, versus 0.015 for *CMA*, but *CMA* gets more than twice the weight of *UMD* (116.7% versus 49.3%) in the tangency portfolio that produces $Sh^2(f)$. In short, even if adding a factor to a model does not produce a large increase in $Sh^2(f)$, the rebalancing that occurs can give the new factor heavy weight in the tangency portfolio.

Marginal contributions and tangency portfolio weights line up better in the model that uses *Mkt*, *S-F*, and excess returns on the short ends of small stock spread factors (L_S-F, W_{CS}-F, A_S-F, D_S-F). In this model, *S-F*, the excess return on the small stock portfolio, is the biggest marginal contributor to $Sh^2(f)$, 0.179 in Table 5, followed by W_{CS}-F (0.048), D_S-F (0.034), and A_S-F (0.031). Negative intercepts in the regressions for W_{CS}-F, A_S-F, and D_S-F in Table 5 foreshadow their role in $Sh^2(f)$ in Table 6, which is to provide short sale dollars to *S-F* and, to a lesser extent L_S-F, the excess return on the short end of HML_S.

Extreme allocations to *S-F* (726.7%), W_{CS}-F (-324.4%), and A_S-F (-313.4%) in the model that uses *Mkt*, *S-F*, and excess returns on the short ends of small stock spread factors suggest that this model focuses on the low average returns of small stocks of firms that invest a lot despite low profitability. Such stocks are a pervasive problem for the five-factor model in FF (2015, 2016, 2017). Skipping the details, the model that includes *Mkt*, *S-F*, L_S-*F*, W_{CS}-*F*, A_S-*F*, and D_S-*F* produces less extreme intercepts for these portfolios than its two main competitors, but more extreme intercepts for other portfolios.

8. Comparing $Sh^2(f)$ to other measures of model performance

Leaning on Barillas and Shanken (2016), we evaluate asset-pricing models on $Sh^2(f)$, the max squared Sharpe ratios produced by their factors. The motivation is that the model with the largest $Sh^2(f)$ minimizes $Sh^2(a) = a'\Sigma^{-1}a$, the max squared Sharpe ratio for the intercepts from regressions of all asset returns on a model's factors. In the spirit of GRS (1989), $a'\Sigma^{-1}a$ is an attractive metric for tests of asset-pricing models, but the often extreme weights for pricing errors implied by Σ^{-1} may not capture the importance of different assets in applications. We next examine whether models that deliver high $Sh^2(f)$ look good on other metrics that equal weight functions of intercepts from time-series regressions of LHS returns on model factors, what we call the LHS approach in contrast to the RHS approach of BS.

8.1. Left-hand-side portfolios and measures of performance

The LHS portfolios we use and the performance metrics we compare to $Sh^2(f)$ are those in FF (2015, 2016). The initial tests of the five-factor model in FF (2015) use six sets of LHS portfolios: three 5x5 quintile sorts on *ME* and *BE/ME*, *OP*, or *Inv* and three 2x4x4 sorts on *ME* and pairs of *BE/ME*, *OP*, and *Inv*. The 2x4x4 sorts add little to the 5x5 sorts, so to limit the number of LHS portfolios, we drop them. Since we are also interested in cash profitability and momentum, we add portfolios from 5x5 sorts on *ME* and *CP*, and *ME* and *Mom* to the LHS portfolios. The portfolios are formed with NYSE quintile breakpoints and the sorts of each pair are independent. We form *ME-Mom* portfolios monthly, but the others are formed at the end of June each year.

The LHS portfolios described above are from finer versions of the sorts that produce the RHS factors. For a more challenging test, FF (2016) ask the five-factor model to describe average returns for anomalies known to cause problems for the FF (1993) three-factor model. The anomalies are the flat relation between market β and average return that has long plagued tests of the CAPM (Black, Jensen, and Scholes, 1972; Fama and MacBeth, 1973), the high average returns after share repurchases and the low returns after share issues (Ikenberry, Lakonishok, and Vermaelen, 1995; Loughran and Ritter, 1995), the low average returns of stocks with high return volatility, measured using daily returns or daily residuals from the three-factor model (Ang, Hodrick, Xing, and Zhang, 2006), and the low average returns of stocks of firms with large accounting accruals (Sloan, 1996).

Anomaly patterns in average returns are stronger for small stocks, and our first pass sort for the anomaly portfolios assigns stocks to NYSE quintiles on *ME*. The second sort, on an anomaly variable, also assigns stocks to NYSE quintiles. The exception is net share issues, for which we form seven groups, including net repurchases, zero net issues, and quintiles of positive net issues. The *ME*- β, *ME*-accruals, and *ME*-net issuance portfolios are formed at the end of each June, using independent sorts for *ME* and the anomaly variable. The two sets of volatility portfolios are formed monthly and, because large stocks with highly volatile returns are rare, we condition the quintile breakpoints for variance and residual variance on *ME* quintile.

We use the LHS portfolios described above to examine how $Sh^2(f)$ lines up with the *GRS* statistic and three measures of model performance in FF (2015, 2016, 2017). Using *A* to indicate an average value, the simplest of the three is $A|a_i|$, the average absolute intercept for a set of portfolios. To estimate the cross-section dispersion in average returns missed by a model, FF (2016) first define $\bar{r}_i$ as the difference between the average return on LHS portfolio *i* and the average VW market return. The average squared intercept over the average squared value of $\bar{r}_i$, $Aa_i^2/A\bar{r}_i^2$, is the unexplained dispersion of LHS average returns relative to their total dispersion. FF (2016) also report estimates of the proportion of unexplained dispersion in average returns due to sampling error, $As^2(a_i)/Aa_i^2$, where $As^2(a_i)$ is the average of the squared sample standard errors of the intercepts and Aa_i^2 is the average squared intercept. A low value of $Aa_i^2/A\bar{r}_i^2$ is good news for a model. It says intercept dispersion is low relative to the dispersion of LHS average returns. In contrast, a low value of $As^2(a_i)/Aa_i^2$ is bad news. It says dispersion of the true intercepts is more important than sampling error in the dispersion of the estimated intercepts. We call $A|a_i|$, $Aa_i^2/A\bar{r}_i^2$, and $As^2(a_i)/Aa_i^2$ equal weight metrics because the averages in the statistics weight

LHS portfolios equally. Finally, we also show $Sh^2(a)$, the max squared Sharpe ratio for the intercepts from regressions of LHS returns on a model's factors.

8.2. Relations between $Sh^2(f)$ and other measures of performance

Table 7 reports $Sh^2(f)$ and the other performance metrics for the seven models of Table 4. Panel A summarizes time-series regressions for monthly excess returns on the all the LHS portfolios described above. Panel A thus tests competing models on 260 LHS portfolios that cover a broad set of known patterns in average returns. Panel B shows results for the 125 portfolios from the 5x5 sorts on *ME* and *BE/ME*, *OP*, *CP*, *Inv*, or *Mom*. Panel B is a family tournament in which LHS portfolios are more finely sorted offspring of RHS factors. Panel C, which shows results for the 135 anomaly portfolios, tests competing models on prominent nonfamily patterns in average returns. In each panel, models are sorted on $Sh^2(f)$ to facilitate comparisons of $Sh^2(f)$ with other performance metrics.

Insert Table 7 near here

We expect that $Sh^2(f)$ is negatively related to *GRS* and $Sh^2(a)$. $Sh^2(a)$ is the difference between the max squared Sharpe ratio produced by combining the LHS portfolios, ω, and the factors of a model and the max squared Sharpe ratio for the factors,

$$Sh^2(a) = Sh^2(\omega, f) - Sh^2(f). \tag{6}$$

If ω includes all assets Π, $Sh^2(\omega, f) = Sh^2(\omega)$, so all variation in $Sh^2(a)$ across models is due to $Sh^2(f)$, and $Sh^2(a)$ and $Sh^2(f)$ are perfectly negatively related. The relation need not be perfect for a subset of LHS assets because there is variation across models in $Sh^2(\omega, f)$ not linearly related to $Sh^2(f)$.

Similarly, we can write the *GRS* statistic as

$$GRS = \gamma\left(\frac{1+Sh^2(\omega,f)}{1+Sh^2(f)} - 1\right). \tag{7}$$

The constant γ is a function of the number of observations (636), factors (six), and LHS portfolios (260, 125, and 135 in Panels A to C of Table 7). It varies across the panels of Table 7, but not across models in a panel. As with $Sh^2(a)$, cross-model variation in $Sh^2(f)$ and $Sh^2(\omega, f)$ drives variation in *GRS*. The ratio in Eq. (7) creates complications that do not affect Eq. (6), but the main driver for a less than perfect negative relation between *GRS* and $Sh^2(f)$ is again variation in $Sh^2(\omega, f)$ not linearly related to $Sh^2(f)$.

The negative relations between $Sh^2(f)$ and *GRS* are near perfect. In Panels A and B of Table 7 sorting models from higher to lower $Sh^2(f)$ produces monotone increasing *GRS*. In Panel C, the monotone increase in *GRS* is violated by only one model. The relation between $Sh^2(f)$ and $Sh^2(a)$ is less consistent. The model that uses small stock spread factors produces both the highest (best) $Sh^2(f)$ and the lowest (best) $Sh^2(a)$ in Panels A and B. In Panels A and B, the models in the top three on $Sh^2(f)$ are also best on $Sh^2(a)$. In Panel C (anomalies), however, the model that uses excess returns on the short ends of spread factors is second to last on $Sh^2(f)$ but best (lowest) on $Sh^2(a)$. The model that produces the highest $Sh^2(f)$ must produce the lowest $Sh^2(a)$ in tests that include all possible LHS assets. The results for LHS anomaly portfolios show that it need not produce the lowest $Sh^2(a)$ for subsets of assets.

The LHS assets in Panel A of Table 7 are all those in Panels B and C. Thus, in Panel A, $Sh^2(a)$ exploits all cross-section variation (including sampling error) in the regression intercepts in Panels B and C and a richer residual covariance matrix. As a result, for each model, $Sh^2(a)$ in Panel A is more than twice those in Panels B and C. But inclusion of all assets makes the maximization bias in $Sh^2(a)$ most extreme in Panel A. This sampling error problem is accounted for in *GRS*, and model-by-model *GRS* statistics in Panel A are larger than those in Panel B but smaller than those in Panel C.

Eqs. (6) and (7) say that $Sh^2(a)$, *GRS*, and $Sh^2(f)$ are all members of the *GRS* family, so strong correlations of $Sh^2(f)$ with $Sh^2(a)$ and *GRS* are not surprising. There are no direct links, however, between $Sh^2(f)$ and the three EW measures of model performance in Table 7, and our main interest in this section is whether $Sh^2(f)$ and the EW metrics rank models similarly. The most positive result is that the winner on $Sh^2(f)$, the model that combines *Mkt* and *SMB* with the small stock spread factors (HML_S, RMW_{CS}, CMA_S, and UMD_S), also wins on the three EW metrics. It produces the highest $As^2(a_i)/Aa_i^2$ and the lowest $A|a_i|$ and $Aa_i^2/A\bar{r}_i^2$ in the three panels of Table 7.

Otherwise, the links between $Sh^2(f)$ and the EW metrics are tenuous. For example, the model that combines *Mkt*, *S-F*, and excess returns on the short ends of small stock spread factors (L_S-F, W_S-F, A_S-F, and D_S-F) has the second highest $Sh^2(f)$, but on the three EW metrics it is no better than the two models lowest on $Sh^2(f)$. Judged on $Sh^2(f)$, models that use cash profitability factors dominate the same models that use operating profitability factors. Table 7 reiterates, for example, that substituting RMW_C for RMW_O in the model that uses spread factors that combine small and big stocks raises $Sh^2(f)$ from 0.135 (last place) to 0.190 (third place). The spread factor model that uses RMW_C is also better on $Sh^2(a)$ for the three sets of portfolios in Table 7. But the two models are close on the three EW metrics in Table 7. Thus, on EW metrics, the *CP* spread factor does not have a clear advantage over the *OP* factor for a wide range of LHS portfolios. The way intercepts are weighted is clearly important in model rankings.

To check the time consistency of model performance, Appendix Table A6 repeats the tests of Table 7 on the two equal subperiods, July 1963-December 1989 and January 1990-June 2016. In the first subperiod, the model that uses *Mkt*, *S-F*, and excess returns on the short ends of small stock spread factors (L_S-F, W_S-F, A_S-F, and D_S-F) shines. It wins on $Sh^2(f)$ and the three EW performance metrics. In the second subperiod, this model is third on $Sh^2(f)$ but toward the bottom on the EW metrics. Its relatively strong performance for the full sample period thus owes much to the first half. In contrast, the model that combines *Mkt*, *S-F*, and the small stock spread factors HML_S, RMW_{CS}, CMA_S, and UMD_S is best on $Sh^2(f)$ and all but one EW metric in the second half of the sample period and second best on all metrics in the first half. The winning performance of this model in the Table 7 tests for the full sample period is thus due to strong performance in both halves.

9. Conclusions

If the goal is to minimize the max squared Sharpe ratio for the intercepts for all assets, models should be ranked on the max squared Sharpe ratio for model factors, $Sh^2(f)$. Among the six-factor models we consider, the winner on $Sh^2(f)$ combines *Mkt* and *SMB* with the small stock spread factors, HML_S, RMW_{CS}, CMA_S, and UMD_S. The simulations in Table 4 say this model is reliably better on $Sh^2(f)$ than the other models we consider. This model also performs best on the three EW metrics applied to a wide range of LHS portfolios in Table 7, and it is the most consistent strong performer in the split period tests of Table A6.

We are not convinced, however, that we have enough evidence to propose a switch to small stock spread factors in applications. The base model that combines small and big stocks in its spread factors *HML*, RMW_C, *CMA*, and *UMD* performs well in all tests and if momentum factors are dropped, it performs as well on all metrics as any of the models we consider. Our guess is that the small and combined spread factor models produce similar results in applications. As a check, it would be interesting to examine whether the two models produce similar inferences in tests of mutual fund performance like those in FF (2010). Likewise, judged on $Sh^2(f)$, cash profitability factors dominate operating profitability factors (Table 3), but on other metrics (Table 7) there is not much to choose between the two. Since robustness of inferences is the prime consideration both in tests of asset pricing models and in applications, it would also be instructive to add the base model Eq. (4) that uses the operating profitability factor RMW_O to the mutual fund tests.

Factor models are a response to the empirical failures of the CAPM of Sharpe (1964) and Lintner (1965) and the CCAPM of Lucas (1978) and Breeden (1979). The attraction of the CAPM and CCAPM is that they specify the relevant measure of risk and the relation between expected returns and risk, which provides discipline for empirical tests. In contrast, factor models are motivated by observed patterns in average returns. For example, the FF (1993) three-factor model is motivated by the size and value patterns in average returns. Through time, many patterns in average returns are discovered and become potential candidates for inclusion in factor models. A danger is that, in the absence of discipline from theory, factor models degenerate into long lists of factors that come close to spanning the ex post mean-variance-efficient (MVE) tangency portfolio of a particular period, in other words, empty data dredging exercises.

What discipline can we add to limit data dredging? We suggest that model comparisons in any paper should be limited by theory, even an umbrella theory like the dividend discount model, and by evidence on model robustness out-of-sample (different time periods and markets). For example, FF (2015, 2016) invoke the dividend discount model to motivate the five-factor model. Here, in our tests of nested models, we, in addition, invoke history to limit comparisons to the three-factor model versus the CAPM, the five-factor model versus the three-factor model, and the six-factor model versus the five-factor model.

Despite the absence of theoretical justification, we (somewhat reluctantly) add a momentum factor to the five-factor model. Thus, we limit both the total number of factors and the number of competing models. The spanning-test rejections of the CAPM, three-factor, and five-factor models are so strong that, if we invoke Bonferroni's inequality to take account of the number of comparisons (three, or five if we count alternative profitability factors), all inferences survive.

Suppose instead we start with the six-factor model Eq. (4) and ask which subset of the six best describes average returns. The six factors support 63 combinations, and a factor cannot be ruled globally redundant without testing all models that include it. For example, *SMB* adds little to the FF (1993) three-factor model in Table 2, but it makes a powerful comeback in the five-factor and six-factor models of Table A3 and Table 5. Even with just six factors, examining all 63 combinations leads to a serious multiple comparisons problem. If we get ambitious and ask which subset of the 316 factors identified by Harvey, Liu, and Zhu (2015) best describe average returns (they rightfully do not address this question), multiple comparisons issues make meaningful statistical inference impossible, at least with the testing frameworks currently available.

In general, if inference is to have content, the list of models considered in a study must be relatively short, generically like the limited set of nested and non-nested alternatives considered here. Moreover, if factor modeling is not to degenerate into meaningless dredging for the ex post MVE portfolio, the number of factors in models must also be limited. Establishing ground rules, however, awaits more experience.

Finally, statistical inference is always clouded by multiple comparisons issues. In asset pricing, we typically use data scoured by many before us, and the questions addressed are conditioned by previous work, typically on much the same data. In the absence of fresh data, inferences about reliability should reflect the union of all earlier tests (reported and unreported) – an impossible goal. Our point is that, if we limit the models considered in a study, we have a shot at ordering them in a statistically meaningful way, even though the overall level of *p*-values is unavoidably clouded.

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Appendix

The data are from the Center for Research in Security Prices and Compustat, supplemented with hand-collected book equity data, as in Davis, Fama, and French (2000). We form most portfolios at the end of June in each year *t*, but we sort firms on the momentum, *Mom*, and variance measures, *Var* and *RVar*, every month. We use only stocks with CRSP share codes of 10 or 11. Stocks are included in every sort for which they have the necessary data. Thus, a stock can be in an *ME-Inv* portfolio for year *t* and not an *ME-BE/ME* portfolio. Stocks of firms with non-positive book equity are excluded from *BE/ME* sorts. The sort variables are defined as follows.

ME—Market cap, price times shares outstanding.

BE/ME—Ratio of book value of equity to market value of equity. Book equity in the sort for June of year *t* is total assets for the last fiscal year-end in calendar year *t*-1, minus liabilities, plus balance sheet deferred taxes and investment tax credit if available, minus preferred stock liquidating value if available, or redemption value if available, or carrying value, adjusted for net share issuance from the fiscal year-end to the end of December of *t*-1. Market equity (market cap) is price times shares outstanding at the end of December of *t*-1, from CRSP.

OP—Operating profitability. *OP* in the sort for June of year *t* is measured with accounting data for the fiscal year ending in year *t*-1 and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Unlike in earlier versions of the paper, research and development expenses reduce operating profitability.

CP—Cash profitability. In the sort for June of year *t*, *CP* is *OP* minus accruals for the fiscal year ending in *t*-1. To compute cash profitability, we follow Ball, Gerakos, Linnainmaa, and Nikolaev (2016) and define accruals as the change in accounts receivable from *t*-2 to *t*-1, plus the change in prepaid expenses, minus the change in accounts payable, inventory, deferred revenue, and accrued expenses.

Inv—Investment. In the sort for June of year *t*, *Inv* is the change in total assets from the fiscal year ending in *t*-2 to the fiscal year ending in *t*-1 divided by total assets at *t*-2.

NS—Net stock issuance, the implied growth in split-adjusted shares outstanding from the end of June in year *t*-1 to the end of June in *t*. *NS* is zero if CRSP's shares outstanding does not change over the 12 months. Otherwise, we compute *NS* by comparing the total growth in *ME* from June *t*-1 to June *t*, *ME*(*t*)/*ME*(*t*-1), with the growth implied by compounding the monthly without-dividend stock returns over the same period, $\Pi(1+RetX_i)$,

$$NS = \frac{ME(t)/ME(t-1)}{\prod(1+RetX_i)} - 1.$$

Ac/B—Accruals, the change in operating working capital per split-adjusted share from *t*-2 to *t*-1 divided by book equity per split-adjusted share at *t*-1. Operating working capital is current assets minus cash and short-term investments minus current liabilities plus debt in current liabilities. We use operating working capital per split-adjusted share to adjust for the effect of changes in the scale of the firm caused by share issuances and repurchases.

β—Market β. Measured at the end of each June, it is the sum of the slopes from the regression of monthly returns on the current and first lag of monthly market returns. The regression uses the preceding 60 months (24 minimum) of returns.

Var—Variance of daily total returns. Each stock's *Var* is estimated monthly using 60 days (20 minimum) of lagged returns.

RVar—Variance of daily residuals from the Fama and French (1993) three-factor model. Each stock's *RVar* is estimated monthly using 60 days (20 minimum) of lagged returns.

Mom—Momentum. For portfolios formed at the end of month *t*-1, *Mom* is the average monthly return from *t*-12 to *t*-2.

Table 1

Summary statistics for monthly factor returns, July 1963-June 2016

Mkt is the difference between the value-weight (VW) market return, *M*, and the one-month US Treasury bill rate, *F*. At the end of each June, NYSE, AMEX, and Nasdaq stocks are allocated to two size groups (Small and Big) using the NYSE median market cap (*ME*) as breakpoint. Stocks are allocated independently to three *BE/ME* groups (Low to High), using NYSE 30th and 70th percentile breakpoints. The intersections of the two sorts produce six VW *ME-BE/ME* portfolios. In the sort for June of year *t*, *BE* is book equity at the end of the fiscal year ending in year *t*-1 and *ME* is market cap at the end of December of year *t*-1, adjusted for changes in shares outstanding between the measurement of *BE* and the end of December. The intersections of the *ME-BE*/*ME* sorts produce six portfolios, L_S, N_S, H_S, L_B, N_B, and H_B, where *L*, *N*, and *H* indicate low 30%, middle 40%, and high 30% of *BE/ME* and subscripts *S* and *B* indicate small and big. We compute monthly VW returns for each portfolio from July of year *t* to June of *t*+1. We construct spread factors for small and big stocks, $HML_S = H_S\text{-}L_S$ and $HML_B = H_B\text{-}L_B$, and *HML*, the combined spread factor, is the average of the small and big spread factors. The ends of the three spread factors provide three long (*H-F*, $H_S\text{-}F$, and $H_B\text{-}F$) and three short (*L-F*, $L_S\text{-}F$, and $L_B\text{-}F$) excess return factors. The investment factor, *CMA*, and the profitability factors, RMW_O and RMW_C, are constructed like *HML*. For *CMA*, the second sort at the end of June of year *t* is on *Inv*, the rate of growth of total assets (low to high) for the fiscal year ending in the previous calendar year. The second sort for RMW_O is on operating profitability (net of interest expense and scaled by book equity), again for the fiscal year ending in the previous calendar year. The cash profitability factor, RMW_C, mimics RMW_O, except the profitability sort is on cash profits (operating profits minus the effect of accruals) divided by book equity. The momentum factor, *UMD*, is defined like *HML*, except it is updated monthly instead of annually, and the sort for portfolios formed at the end of month *t*-1 is on average return from *t*-12 to *t*-2. (The sort variables are defined in the Appendix.) We decompose each of the combined spread factors, *CMA*, RMW_O, RMW_C, and *UMD* into small stock and big stock spread factors, three long excess return factors, and three short excess return factors. The long and short excess return factors are denoted by the first and third letters of the spread factor names. The 2x3 sorts used to construct *HML*, RMW_O, RMW_C, and *CMA* produce four size factors, SMB_{BM}, SMB_{OP}, SMB_{CP}, and SMB_{Inv}. For example, SMB_{BM} is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns from the *ME-BE/ME* sorts. *SMB* is the average of SMB_{BM}, SMB_{CP}, and SMB_{Inv}. The table shows averages and standard deviations of monthly factor returns and *t*-statistics for the average returns. Panel A shows summary statistics for *Mkt*, *SMB*, and the excess return factors *S-F* and *B-F* constructed from the long and shorts ends of *SMB*. Panel B shows summary statistics for value (*Value*), operating profitability ($Prof_O$), cash profitability ($Prof_C$), investment (*Inv*), and momentum (*Mom*) factors, grouped as they are in the models.

Panel A: Summary statistics for market and size factors

	Average return				*t*-statistic			
Factors	*Mkt*	*SMB*	*S-F*	*B-F*	*Mkt*	*SMB*	*S-F*	*B-F*
Market and size factors	0.50	0.26	0.78	0.52	2.84	2.17	3.41	3.02

Table 1 (continued)

Panel B: Summary statistics for value, operating profitability, cash profitability, investment, and momentum factors

	Average return					*t*-statistic				
Factors	*Value*	$Prof_O$	$Prof_C$	*Inv*	*Mom*	*Value*	$Prof_O$	$Prof_C$	*Inv*	*Mom*
HML, RMW, CMA, UMD	0.35	0.24	0.36	0.31	0.69	3.15	2.75	4.71	3.88	4.09
HML_S, RMW_S, CMA_S, UMD_S	0.51	0.31	0.45	0.41	0.92	4.00	2.96	4.61	5.20	5.47
HML_B, RMW_B, CMA_B, UMD_B	0.19	0.17	0.27	0.21	0.46	1.57	1.76	2.91	1.96	2.47
H-F, R-F, C-F, U-F	0.84	0.74	0.79	0.79	0.96	4.21	3.79	4.29	4.05	4.56
H_S-F, R_S-F, C_S-F, U_S-F	1.00	0.90	0.97	0.93	1.17	4.51	3.83	4.45	3.87	4.76
H_B-F, R_B-F, C_B-F, U_B-F	0.68	0.58	0.62	0.65	0.76	3.52	3.34	3.65	3.78	3.95
L-F, W-F, A-F, D-F	0.49	0.49	0.44	0.48	0.27	2.24	2.21	1.92	2.16	1.10
L_S-F, W_S-F, A_S-F, D_S-F	0.49	0.59	0.52	0.52	0.25	1.80	2.25	1.96	2.02	0.88
L_B-F, W_B-F, A_B-F, D_B-F	0.48	0.40	0.36	0.44	0.30	2.65	1.97	1.69	2.16	1.29
$HML_{S\text{-}B}$, $RMW_{S\text{-}B}$, $CMA_{S\text{-}B}$, $UMD_{S\text{-}B}$	0.32	0.14	0.18	0.20	0.46	2.80	1.35	1.65	2.01	4.14
$H_{S\text{-}B}$, $R_{S\text{-}B}$, $C_{S\text{-}B}$, $U_{S\text{-}B}$	0.32	0.32	0.35	0.28	0.41	2.76	2.43	2.78	1.88	3.27
$L_{S\text{-}B}$, $W_{S\text{-}B}$, $A_{S\text{-}B}$, $D_{S\text{-}B}$	0.00	0.18	0.16	0.08	-0.05	0.03	1.31	1.13	0.62	-0.35

Table 2
Spanning tests for nested models: July 1963-June 2016

This table tests whether the excess market return, *Mkt*, spans the size spread factor, *SMB*, and the value spread factor, *HML;* whether *Mkt*, *SMB*, and *HML* span the investment spread factor, *CMA*, and the profitability spread factors, RMW_O or RMW_C; and whether *Mkt*, *SMB*, *HML*, *CMA*, and RMW_O or RMW_C span the momentum spread factor, *UMD*. The tests center on the intercepts from spanning regressions of additional factors on base factors: *SMB* and *HML* regressed on *Mkt; CMA* and RMW_O or RMW_C on *Mkt*, *SMB*, and *HML;* and *UMD* on *Mkt*, *SMB*, *HML*, *CMA*, and RMW_O or RMW_C. Panel A shows regression coefficients and *t*-statistics for the coefficients. Panel B shows *GRS* statistics from Gibbons, Ross, and Shanken (1989) and their *p*-values testing whether the additional factors jointly improve the maximum squared Sharpe ratio, $Sh^2(f)$, produced by the factors of the base model, either the capital asset pricing model (CAPM) or the three-factor model. The *t*-statistics for the regression intercepts provide tests for individual additional factors.

Panel A: Spanning regressions

	Coefficient							*t*(Coefficient)								
LHS	*Int*	*Mkt*	*SMB*	*HML*	RMW_O	RMW_C	*CMA*	*Int*	*Mkt*	*SMB*	*HML*	RMW_O	RMW_C	*CMA*	R^2	*s*(*e*)
SMB	0.17	0.18						1.46	6.89						0.07	2.93
HML	0.43	-0.17						4.01	-6.89						0.07	2.71
RMW_O	0.34	-0.07	-0.23	0.01				4.01	-3.70	-8.16	0.25				0.14	2.07
RMW_C	0.48	-0.14	-0.26	0.06				7.88	-10.12	-13.05	2.70				0.39	1.49
CMA	0.20	-0.10	0.01	0.46				3.53	-7.57	0.35	22.34				0.52	1.39
UMD	0.73	-0.13	0.08	-0.54	0.25		0.41	4.34	-3.07	1.37	-6.74	3.12		3.47	0.09	4.05
UMD	0.61	-0.09	0.14	-0.53		0.46	0.34	3.55	-1.97	2.30	-6.72		4.29	2.91	0.10	4.02

Panel B: Multi-factor tests

Model	LHS returns	*GRS*	*p*-value
CAPM	*SMB*, *HML*	9.20	0.000
Three-factor model	RMW_O, *CMA*	17.99	0.000
Three-factor model	RMW_C, *CMA*	37.46	0.000

Table 3

Comparison of six-factor models that include an operating profitability (*OP*) or a cash profitability (*CP*) factor: July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, momentum, and profitability spread factors (*HML*, *CMA*, *UMD*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, UMD_S, and RMW_{OS} or RMW_{CS}, and HML_B, CMA_B, UMD_B, and RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F*, and HML_S is parent to H_S-*F* and L_S-*F*. A subscript *S* indicates that a factor is constucted from small stocks. Absence of a subscript means a factor uses small and big stocks. Each model has two versions, one with an operating profitabilility factor (indicated with a subscript *O*) and one with a cash profitability factor (subscript *C*). Each line of Panel A shows the factors in a model; the sample Actual maximum squared Sharpe ratio, $Sh^2(f)$, for the model's factors; and average and median $Sh^2(f)$ from 100,000 full-sample (FS), in-sample (IS), and out-of-sample (OS) simulation runs. FS simulations estimate $Sh^2(f)$ from random samples (with replacement) of 636 months from the 636 months of July 1963-June 2016. IS and OS simulations split the 636 sample months into 318 adjacent pairs: months (1, 2), (3, 4), … (635, 636). A simulation run draws a random sample with replacement of 318 pairs. The IS simulation run chooses a month randomly from each pair in the run. We calculate IS $Sh^2(f)$ for all models on that sample of months. A model's IS $Sh^2(f)$ identifies weights for factors in the IS tangency portfolio for the factors. These weights and the unused months of the simulation pairs produce an OS estimate of the Sharpe ratio for the IS tangency portfolio. For each model, Panel B shows Actual $Sh^2(f_C)$-$Sh^2(f_O)$, the difference between $Sh^2(f)$ when the model uses a *CP* or *OP* factor; the means and medians of $Sh^2(f_C)$-$Sh^2(f_O)$ from 100,000 FS, IS, and OS simulation runs; and the percent of simulation runs in which the model that uses the *OP* factor beats the model that uses the *CP* factor.

Panel A: Levels of $Sh^2(f)$

		Full-sample		In-sample		Out-of-sample	
Model	Actual	Average	Median	Average	Median	Average	Median
Six-Factor Operating Profitability							
Mkt, *SMB*, *HML*, RMW_O, *CMA*, *UMD*	0.135	0.152	0.149	0.177	0.169	0.108	0.102
Mkt, *SMB*, HML_S, RMW_{OS}, CMA_S, UMD_S	0.199	0.217	0.213	0.244	0.236	0.169	0.162
Mkt, *S-F*, *H-F*, R_O-*F*, *C-F*, *U-F*	0.134	0.148	0.146	0.172	0.166	0.106	0.100
Mkt, *S-F*, H_S-*F*, R_{OS}-*F*, C_S-*F*, U_S-*F*	0.167	0.182	0.180	0.206	0.200	0.138	0.132
Mkt, *S-F*, *L-F*, W_O-*F*, *A-F*, *D-F*	0.111	0.127	0.124	0.152	0.144	0.085	0.079
Mkt, *S-F*, L_S-*F*, W_{OS}-*F*, A_S-*F*, D_S-*F*	0.174	0.192	0.189	0.220	0.212	0.144	0.137
Six-Factor Cash Profitability							
Mkt, *SMB*, *HML*, RMW_C, *CMA*, *UMD*	0.190	0.208	0.205	0.236	0.228	0.159	0.152
Mkt, *SMB*, HML_S, RMW_{CS}, CMA_S, UMD_S	0.226	0.244	0.241	0.274	0.265	0.194	0.186
Mkt, *S-F*, *H-F*, R_C-*F*, *C-F*, *U-F*	0.177	0.193	0.191	0.218	0.212	0.147	0.141
Mkt, *S-F*, H_S-*F*, R_{CS}-*F*, C_S-*F*, U_S-*F*	0.188	0.204	0.201	0.229	0.221	0.159	0.153
Mkt, *S-F*, *L-F*, W_C-*F*, *A-F*, *D-F*	0.160	0.176	0.173	0.204	0.196	0.128	0.121
Mkt, *S-F*, L_S-*F*, W_{CS}-*F*, A_S-*F*, D_S-*F*	0.210	0.229	0.226	0.259	0.250	0.177	0.170

Table 3 (continued)

Panel B: Differences between $Sh^2(f)$ for cash and operating profitability factors

		Full-sample			In-sample			Out-of-sample		
Model	Actual	Average	Median	% < 0	Average	Median	% < 0	Average	Median	% < 0
Cash - Operating Profitability										
Mkt, SMB, HML, RMW, CMA, UMD	0.055	0.056	0.054	0.0	0.059	0.055	1.1	0.050	0.047	2.2
Mkt, SMB, HML_S, RMW_S, CMA_S, UMD_S	0.027	0.028	0.027	1.0	0.030	0.027	8.2	0.025	0.023	10.0
Mkt, S-F, H-F, R-F, C-F, U-F	0.043	0.045	0.043	0.0	0.047	0.044	1.1	0.041	0.039	2.0
Mkt, S-F, H_S-F, R_S-F, C_S-F, U_S-F	0.021	0.022	0.021	4.5	0.023	0.020	17.3	0.022	0.020	15.4
Mkt, S-F, L-F, W-F, A-F, D-F	0.048	0.049	0.047	0.2	0.052	0.047	4.1	0.043	0.040	5.0
Mkt, S-F, L_S-F, W_S-F, A_S-F, D_S-F	0.036	0.037	0.036	0.0	0.039	0.035	1.8	0.033	0.031	5.4

Table 4

Distributions of differences between $Sh^2(f)$, column model minus row model: July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, momentum, and profitability spread factors (*HML*, *CMA*, *UMD*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, UMD_S, and RMW_{OS} or RMW_{CS}, and HML_B, CMA_B, UMD_B, and RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F*, and HML_S is parent to H_S-*F* and L_S-*F*. A subscript *S* indicates that a factor is constucted from small stocks. Absence of a subscript means a factor is constructed from small and big stocks. The table shows average and median differences between $Sh^2(f)$ for a column model and a row model from 100,000 full-sample, in-sample, and out-of-sample simulation runs, and the percent of simulation runs in which the row model has higher $Sh^2(f)$ than the column model (% < 0). The column models are the three that produce the highest sample actual $Sh^2(f)$ in Panel A of Table 3.

Table 4 (continued)

Model	$Mkt, SMB, HML, RMW_C, CMA, UMD$			$Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S$			$Mkt, S\text{-}F, L_S\text{-}F, W_{CS}\text{-}F, A_S\text{-}F, D_S\text{-}F$		
	Average	Median	% < 0	Average	Median	% < 0	Average	Median	% < 0
Full sample									
$Mkt, SMB, HML, RMW_C, CMA, UMD$				0.037	0.036	8.7	0.021	0.021	21.0
$Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S$	-0.037	-0.036	91.3				-0.015	-0.015	82.5
$Mkt, S\text{-}F, L_S\text{-}F, W_{CS}\text{-}F, A_S\text{-}F, D_S\text{-}F$	-0.021	-0.021	79.0	0.015	0.015	17.5			
$Mkt, SMB, HML, RMW_O, CMA, UMD$	0.056	0.054	0.0	0.093	0.091	0.0	0.077	0.076	0.0
$Mkt, S\text{-}F, H\text{-}F, R_C\text{-}F, C\text{-}F, U\text{-}F$	0.015	0.014	24.6	0.051	0.050	3.1	0.036	0.035	12.3
$Mkt, S\text{-}F, H_S\text{-}F, R_{CS}\text{-}F, C_S\text{-}F, U_S\text{-}F$	0.004	0.003	45.7	0.040	0.040	2.8	0.025	0.025	21.4
$Mkt, S\text{-}F, L\text{-}F, W_C\text{-}F, A\text{-}F, D\text{-}F$	0.032	0.031	0.6	0.068	0.068	1.4	0.053	0.052	3.1
In sample									
$Mkt, SMB, HML, RMW_C, CMA, UMD$				0.038	0.036	21.5	0.023	0.022	31.8
$Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S$	-0.038	-0.036	78.5				-0.015	-0.014	69.6
$Mkt, S\text{-}F, L_S\text{-}F, W_{CS}\text{-}F, A_S\text{-}F, D_S\text{-}F$	-0.023	-0.022	68.2	0.015	0.014	30.4			
$Mkt, SMB, HML, RMW_O, CMA, UMD$	0.059	0.055	1.1	0.097	0.092	0.9	0.082	0.078	1.7
$Mkt, S\text{-}F, H\text{-}F, R_C\text{-}F, C\text{-}F, U\text{-}F$	0.018	0.016	31.5	0.056	0.051	13.7	0.041	0.038	23.7
$Mkt, S\text{-}F, H_S\text{-}F, R_{CS}\text{-}F, C_S\text{-}F, U_S\text{-}F$	0.007	0.006	45.5	0.045	0.043	11.8	0.030	0.029	29.7
$Mkt, S\text{-}F, L\text{-}F, W_C\text{-}F, A\text{-}F, D\text{-}F$	0.032	0.031	7.4	0.070	0.068	10.1	0.055	0.054	13.8
Out of sample									
$Mkt, SMB, HML, RMW_C, CMA, UMD$				0.035	0.033	20.0	0.018	0.017	33.5
$Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S$	-0.035	-0.033	80.0				-0.017	-0.016	74.1
$Mkt, S\text{-}F, L_S\text{-}F, W_{CS}\text{-}F, A_S\text{-}F, D_S\text{-}F$	-0.018	-0.017	66.5	0.017	0.016	25.9			
$Mkt, SMB, HML, RMW_O, CMA, UMD$	0.050	0.047	2.2	0.085	0.081	1.1	0.068	0.064	3.1
$Mkt, S\text{-}F, H\text{-}F, R_C\text{-}F, C\text{-}F, U\text{-}F$	0.012	0.010	35.5	0.047	0.043	14.5	0.030	0.027	27.7
$Mkt, S\text{-}F, H_S\text{-}F, R_{CS}\text{-}F, C_S\text{-}F, U_S\text{-}F$	-0.001	-0.001	51.0	0.034	0.033	15.4	0.017	0.016	36.5
$Mkt, S\text{-}F, L\text{-}F, W_C\text{-}F, A\text{-}F, D\text{-}F$	0.031	0.029	7.6	0.066	0.062	7.8	0.049	0.046	12.5

Table 5
Spanning regressions and marginal contributions to $Sh^2(f)$ for three models that produce highest $Sh^2(f)$ in Table 4: July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, momentum, and cash profitability spread factors (*HML*, *CMA*, *UMD*, and RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, UMD_S, RMW_{CS} and HML_B, CMA_B, UMD_B, RMW_{CB}). L_S-F, W_{CS}-F, A_S-F, and D_S-F are excess returns on the short ends of the small stock spread factors, HML_S, CMA_S, RMW_{CS}, and UMD_S. For the three models that produce the highest $Sh^2(f)$ in Table 4, the table shows intercepts *a*, *t*-statistics for the intercepts *t*(*a*), slopes, R^2, and residual standard errors *s*(*e*) from spanning regressions of each of the factors of a model on the model's other five factors. The table also shows $Sh^2(f)$ and each factor's marginal contribution to a model's $Sh^2(f)$, that is, $a^2/s^2(e)$.

Panel A: Spanning regressions for combined spread factor model

LHS	*a*	*Mkt*	*SMB*	*HML*	RMW_C	*CMA*	*UMD*	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	1.04		0.06	0.04	-0.87	-0.70	-0.07	6.77	0.31	3.70	0.190	0.080
SMB	0.48	0.03		0.04	-0.83	0.03	0.06	4.35	0.27	2.59	0.190	0.034
HML	0.08	0.01	0.03		0.16	0.94	-0.13	0.90	0.51	1.95	0.190	0.002
RMW_C	0.43	-0.14	-0.27	0.09		-0.02	0.06	7.00	0.41	1.47	0.190	0.085
CMA	0.17	-0.10	0.01	0.47	-0.02		0.04	2.87	0.53	1.38	0.190	0.015
UMD	0.61	-0.09	0.14	-0.53	0.46	0.34		3.55	0.10	4.02	0.190	0.023

Panel B: Spanning regressions for small stock spread factor model

LHS	*a*	*Mkt*	*SMB*	HML_S	RMW_{CS}	CMA_S	UMD_S	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	1.02		0.18	-0.16	-0.36	-0.51	-0.13	6.19	0.23	3.88	0.226	0.070
SMB	0.45	0.08		0.03	-0.60	0.09	-0.01	4.03	0.27	2.60	0.226	0.031
HML_S	-0.05	-0.04	0.01		0.69	0.84	-0.09	-0.59	0.64	1.91	0.226	0.001
RMW_{CS}	0.42	-0.07	-0.24	0.51		-0.42	0.04	6.03	0.56	1.64	0.226	0.066
CMA_S	0.30	-0.07	0.03	0.48	-0.32		0.05	4.73	0.47	1.45	0.226	0.042
UMD_S	0.96	-0.14	-0.03	-0.42	0.22	0.37		5.41	0.05	4.12	0.226	0.054

Panel C: Spanning regressions for small stock short factor model

LHS	*a*	*Mkt*	*S-F*	L_S-F	W_{CS}-F	A_S-F	D_S-F	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	-0.03		0.71	0.19	-0.66	0.42	0.03	-0.41	0.81	1.93	0.210	0.000
S-F	0.31	0.10		-0.27	0.53	0.52	0.04	10.50	0.98	0.73	0.210	0.179
L_S-F	0.07	0.04	-0.44		0.55	0.88	-0.05	1.81	0.98	0.93	0.210	0.006
W_{CS}-F	-0.19	-0.13	0.71	0.45		-0.02	0.02	-5.11	0.98	0.85	0.210	0.048
A_S-F	-0.12	0.06	0.48	0.50	-0.01		0.04	-4.06	0.99	0.70	0.210	0.031
D_S-F	-0.52	0.07	0.65	-0.41	0.21	0.60		-4.30	0.85	2.79	0.210	0.034

Table 6
Weights (in percent) and leverage in $Sh^2(f)$ tangency portfolios for the models of Table 4: July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, momentum, and profitability spread factors (*HML*, *CMA*, *UMD*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, UMD_S, and RMW_{OS} or RMW_{CS}, and HML_B, CMA_B, UMD_B, and RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F*, and HML_S is parent to H_S-*F* and L_S-*F*. A subscript *S* indicates that a factor is constucted from small stocks. Absence of subscript means a factor uses small and big stocks. Each line of the table shows the *Mkt*, *Size*, *Value*, *Prof*, *Inv*, and *Mom* factors in a model; $Sh^2(f)$, the maximum squared Sharpe ratio for the model's factors; the factor weights for the tangency portfolio that produces $Sh^2(f)$; and tangency portfolio leverage, expressed as a proportion of $1 invested in the tangency portfolio.

		Weight in tangency portfolio						
Model	$Sh^2(f)$	*Mkt*	*Size*	*Value*	*Prof*	*Inv*	*Mom*	Leverage
Mkt, SMB, HML, RMW_C, *CMA, UMD*	0.190	100.0	93.7	26.1	259.6	116.7	49.3	5.5
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	0.226	100.0	99.2	-20.0	230.5	207.8	82.9	6.4
Mkt, S-F, L_S-*F*, W_{CS}-*F*, A_S-*F*, D_S-*F*	0.210	-11.7	726.7	106.2	-324.4	-313.4	-83.5	7.3
Mkt, SMB, HML, RMW_O, *CMA, UMD*	0.135	100.0	69.7	38.9	152.5	197.6	77.4	5.4
Mkt, S-F, H-F, R_C-*F*, *C-F, U-F*	0.177	-955.3	-673.5	66.7	857.7	485.5	318.8	16.3
Mkt, S-F, H_S-*F*, R_{CS}-*F*, C_S-*F*, U_S-*F*	0.188	-25.6	-1756.6	273.7	699.0	438.3	471.1	17.8
Mkt, S-F, L-F, W_C-*F*, *A-F, D-F*	0.160	323.7	369.1	-61.1	-361.0	-118.0	-52.6	5.9

Table 7

Summary statistics for regression intercepts, July 1963-June 2016

The 125 left-hand-side (LHS) portfolios in Panel B are from five 5x5 quintile sorts on *ME* and, independently, on *BE/ME*, *OP*, *CP*, *Inv*, or *Mom*. (See Appendix for definitions of all variables.) Like the 2x3 portfolios used to construct the factors, the *ME-Mom* portfolios are formed monthly and the other portfolios in Panel B are formed at the end of June. Eighty-five of the anomaly portfolios in Panel C are from independent, end-of-June, 5x5 sorts on *ME* and market β, 5x5 sorts on *ME* and accruals, and 5x7 sorts on *ME* and net issuance (repurchases, zero net share issues, and quintiles of positive net share issues). The remaining 50 anomaly portfolios are from monthly 5x5 sorts on *ME* and either *Var*, the variance of daily returns, or *RVar*, the variance of daily residuals from the Fama and French (1993) three-factor model. The breakpoints for *Var* and *RVar* quintiles are conditional on *ME* quintile. The 260 LHS portfolios in Panel A are all the LHS portfolios in Panels B (125) and C (135). The table shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989) and its *p*-value, *p*(*GRS*); $A|a_i|$, the average absolute intercept; $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$, which is the difference between the average return on LHS portfolio *i* and the average return on the value-weight market; $As^2(a_i)/Aa_i^2$, the average of the squared sample standard errors of the intercepts over the average squared intercept; the average of the regression R^2, AR^2; $Sh^2(f)$, the maximum squared Sharpe ratio for the model's factors; and $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios.

Table 7 (continued)

Model	*GRS*	*p*(*GRS*)	*A*|*a*|	$Aa_i^2/A\bar{r}_i^2$	$As^2(a_i)/Aa_i^2$	AR^2	$Sh^2(f)$	$Sh^2(a)$
Panel A: All portfolios in Panels B and C								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	2.26	0.000	0.087	0.14	0.42	0.91	0.226	1.926
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	2.33	0.000	0.103	0.18	0.32	0.91	0.210	1.958
Mkt, SMB, HML, RMW_C, CMA, UMD	2.35	0.000	0.095	0.18	0.33	0.91	0.190	1.943
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	2.39	0.000	0.111	0.28	0.22	0.91	0.188	1.975
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	2.40	0.000	0.106	0.25	0.25	0.90	0.177	1.961
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	2.47	0.000	0.094	0.19	0.31	0.91	0.160	1.989
Mkt, SMB, HML, RMW_O, CMA, UMD	2.54	0.000	0.094	0.19	0.30	0.91	0.135	2.010
Panel B: 5x5 sorts on ME and BE/ME, OP, CP, Inv, and Mom								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	1.99	0.000	0.072	0.09	0.58	0.92	0.226	0.599
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	2.07	0.000	0.091	0.14	0.35	0.92	0.210	0.613
Mkt, SMB, HML, RMW_C, CMA, UMD	2.17	0.000	0.082	0.13	0.38	0.92	0.190	0.631
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	2.22	0.000	0.084	0.14	0.37	0.92	0.188	0.647
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	2.24	0.000	0.081	0.14	0.36	0.92	0.177	0.645
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	2.31	0.000	0.086	0.15	0.33	0.92	0.160	0.657
Mkt, SMB, HML, RMW_O, CMA, UMD	2.43	0.000	0.079	0.12	0.37	0.92	0.135	0.675
Panel C: 5x5 sorts on ME and accruals, β, variance of daily returns and residuals, and 5x7 sorts on ME and net share issuance								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	2.62	0.000	0.100	0.19	0.36	0.90	0.226	0.867
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	2.63	0.000	0.114	0.22	0.30	0.90	0.210	0.858
Mkt, SMB, HML, RMW_C, CMA, UMD	2.66	0.000	0.106	0.23	0.31	0.89	0.190	0.853
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	2.80	0.000	0.136	0.41	0.17	0.89	0.188	0.899
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	2.82	0.000	0.130	0.35	0.20	0.89	0.177	0.895
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	2.71	0.000	0.102	0.23	0.30	0.89	0.160	0.849
Mkt, SMB, HML, RMW_O, CMA, UMD	2.88	0.000	0.108	0.25	0.26	0.90	0.135	0.881

Table A1

Comparison of five-factor (no momentum) models that include operating profitability or cash profitability factors, July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, and profitability spread factors (*HML*, *CMA*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, RMW_{OS} or RMW_{CS} and HML_B, CMA_B, RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F* and HML_S is parent to H_S-*F* and L_S-*F*. A subcript S indicates that a factor is constucted from small stocks. Absence of a subcript means a factor uses small and big stocks. Each model has two versions, one with an operating profitability factor (indicated with a subcript *O*) and one with a cash profitability factor (subcript *C*). Each line of Panel A shows the factors in a model; the sample Actual max squared Sharpe ratio, $Sh^2(f)$, for the model's factors; and average and median $Sh^2(f)$ from 100,000 full-sample (FS), in-sample (IS) and out-of-sample (OS) simulation runs. FS simulations estimate $Sh^2(f)$ from random samples (with replacement) of 636 months from the 636 months of July 1963-June 2016. IS and OS simulations split the 636 sample months into 318 adjacent pairs: months (1, 2), (3, 4), … (635, 636). A simulation run draws with replacement a random sample of 318 pairs. The IS simulation run chooses a month randomly from each pair in the run. We calculate IS $Sh^2(f)$ for all models on that sample of months. A model's IS $Sh^2(f)$ identifies weights for factors in the IS tangency portfolio for the factors. These weights and the unused months of the simulation pairs produce an OS estimate of the Sharpe ratio for the IS tangency portfolio. For each model, Panel B of the table shows Actual $Sh^2(f_C)$-$Sh^2(f_O)$, the difference between $Sh^2(f)$ when the model uses a *CP* or *OP* factor; the means and medians of $Sh^2(f_C)$-$Sh^2(f_O)$ from 100,000 FS, IS, and OS simulation runs; and the percent of the simulation runs in which the model that uses the *OP* factor beats the model that uses the *CP* factor.

Panel A: Levels of $Sh^2(f)$

		Full-sample		In-sample		Out-of-sample	
Model	Actual	Average	Median	Average	Median	Average	Median
Operating Profitability							
Mkt, SMB, HML, RMW_O, *CMA*	0.103	0.114	0.112	0.132	0.127	0.083	0.078
Mkt, SMB, HML_S, RMW_{OS}, CMA_S	0.142	0.153	0.151	0.171	0.166	0.120	0.116
Mkt, S-F, H-F, R_O-*F*, *C-F*	0.085	0.097	0.095	0.114	0.110	0.066	0.061
Mkt, S-F, H_S-*F*, R_{OS}-*F*, C_S-*F*	0.090	0.102	0.100	0.119	0.114	0.071	0.066
Mkt, S-F, L-F, W_O-*F*, *A-F*	0.090	0.101	0.099	0.119	0.113	0.070	0.065
Mkt, S-F, L_S-*F*, W_{OS}-*F*, A_S-*F*	0.140	0.152	0.150	0.169	0.164	0.119	0.114
Cash Profitability							
Mkt, SMB, HML, RMW_C, *CMA*	0.167	0.180	0.178	0.201	0.195	0.143	0.137
Mkt, SMB, HML_S, RMW_{CS}, CMA_S	0.172	0.185	0.183	0.204	0.199	0.149	0.144
Mkt, S-F, H-F, R_C-*F*, *C-F*	0.134	0.147	0.144	0.166	0.161	0.112	0.107
Mkt, S-F, H_S-*F*, R_{CS}-*F*, C_S-*F*	0.114	0.126	0.124	0.144	0.138	0.094	0.089
Mkt, S-F, L-F, W_C-*F*, *A-F*	0.146	0.158	0.156	0.179	0.173	0.122	0.117
Mkt, S-F, L_S-*F*, W_{CS}-*F*, A_S-*F*	0.176	0.189	0.187	0.209	0.203	0.153	0.148

Table A1 (continued)

Panel B: Differences between $Sh^2(f)$ for cash and operating profitability factors

		Full-sample			In-sample			Out-of-sample		
Model	Actual	Average	Median	% < 0	Average	Median	% < 0	Average	Median	% < 0
Cash - Operating Profitability										
Mkt, SMB, HML, RMW, CMA	0.064	0.066	0.064	0.0	0.069	0.065	0.5	0.061	0.057	0.9
Mkt, SMB, HML_S, RMW_S, CMA_S	0.030	0.032	0.031	0.4	0.034	0.031	6.3	0.029	0.027	7.0
Mkt, S-F, H-F, R-F, C-F	0.048	0.050	0.049	0.0	0.052	0.049	0.7	0.046	0.043	0.8
Mkt, S-F, H_S*-F*, R_S*-F*, C_S*-F*	0.023	0.024	0.023	1.8	0.025	0.023	13.8	0.023	0.021	10.5
Mkt, S-F, L-F, W-F, A-F	0.056	0.057	0.055	0.1	0.060	0.055	2.8	0.052	0.048	3.2
Mkt, S-F, L_S*-F*, W_S*-F*, A_S*-F*	0.036	0.037	0.036	0.0	0.040	0.036	1.5	0.034	0.032	5.0

Table A2

Distributions of differences between $Sh^2(f)$, column model minus row model (without momentum factors): July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, and profitability spread factors (*HML*, *CMA*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, and RMW_{OS} or RMW_{CS}, and HML_B, CMA_B, and RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F*, and HML_S is parent to H_S-*F* and L_S-*F*. A subscript *S* indicates that a factor is constucted from small stocks. Absence of a subscript means a factor is constructed from small and big stocks. The table shows average and median differences between $Sh^2(f)$ for a column model and a row model from 100,000 full-sample, in-sample, and out-of-sample simulation runs, and the percent of simulation runs in which the row model has higher $Sh^2(f)$ than the column model (% < 0). The column models are the three that produce the highest sample actual $Sh^2(f)$ in Panel A of Table A1.

Table A2 (continued)

Model	*Mkt, SMB, HML,* RMW_C*, CMA*			*Mkt, SMB,* HML_S*,* RMW_{CS}*,* CMA_S			*Mkt, S-F,* L_S*-F,* W_{CS}*-F,* A_S*-F*		
	Average	Median	% < 0	Average	Median	% < 0	Average	Median	% < 0
Full-sample									
Mkt, SMB, HML, RMW_C*, CMA*				0.005	0.005	41.9	0.009	0.009	35.8
Mkt, SMB, HML_S*,* RMW_{CS}*,* CMA_S	-0.005	-0.005	58.1				0.004	0.004	38.4
Mkt, S-F, L_S*-F,* W_{CS}*-F,* A_S*-F*	-0.009	-0.009	64.2	-0.004	-0.004	61.6			
Mkt, SMB, HML, RMW_O*, CMA*	0.066	0.064	0.0	0.071	0.070	0.0	0.075	0.074	0.0
Mkt, S-F, H-F, R_C*-F, C-F*	0.034	0.033	2.9	0.038	0.038	6.0	0.043	0.043	7.0
Mkt, S-F, H_S*-F,* R_{CS}*-F,* C_S*-F*	0.054	0.053	2.1	0.059	0.058	0.0	0.063	0.062	0.4
Mkt, S-F, L-F, W_C*-F, A-F*	0.022	0.021	3.8	0.027	0.027	16.7	0.031	0.031	11.7
In-sample									
Mkt, SMB, HML, RMW_C*, CMA*				0.003	0.004	46.8	0.007	0.008	42.8
Mkt, SMB, HML_S*,* RMW_{CS}*,* CMA_S	-0.003	-0.004	53.2				0.005	0.005	42.6
Mkt, S-F, L_S*-F,* W_{CS}*-F,* A_S*-F*	-0.007	-0.008	57.2	-0.005	-0.005	57.4			
Mkt, SMB, HML, RMW_O*, CMA*	0.069	0.065	0.5	0.072	0.070	3.2	0.077	0.074	2.1
Mkt, S-F, H-F, R_C*-F, C-F*	0.035	0.033	13.0	0.038	0.037	19.1	0.042	0.042	19.3
Mkt, S-F, H_S*-F,* R_{CS}*-F,* C_S*-F*	0.058	0.055	10.6	0.061	0.058	0.3	0.065	0.063	5.4
Mkt, S-F, L-F, W_C*-F, A-F*	0.022	0.021	15.4	0.025	0.026	29.8	0.030	0.030	25.5
Out-of-sample									
Mkt, SMB, HML, RMW_C*, CMA*				0.006	0.006	43.6	0.009	0.010	40.0
Mkt, SMB, HML_S*,* RMW_{CS}*,* CMA_S	-0.006	-0.006	56.4				0.004	0.004	43.3
Mkt, S-F, L_S*-F,* W_{CS}*-F,* A_S*-F*	-0.009	-0.010	60.0	-0.004	-0.004	56.7			
Mkt, SMB, HML, RMW_O*, CMA*	0.061	0.057	0.9	0.066	0.064	2.7	0.070	0.067	2.1
Mkt, S-F, H-F, R_C*-F, C-F*	0.031	0.030	11.3	0.037	0.036	16.4	0.041	0.039	17.0
Mkt, S-F, H_S*-F,* R_{CS}*-F,* C_S*-F*	0.049	0.046	11.1	0.055	0.053	0.5	0.058	0.056	5.2
Mkt, S-F, L-F, W_C*-F, A-F*	0.021	0.020	16.0	0.027	0.026	26.1	0.030	0.030	21.7

Table A3
Spanning regressions and marginal contributions to $Sh^2(f)$ for the three without-momentum models that produce the highest $Sh^2(f)$ in Table A1: July 1963-June 2016

Mkt is *M-F*, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, and cash profitability spread factors (*HML*, *CMA*, and RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, RMW_{CS} and HML_B, CMA_B, RMW_{CB}). L_S-*F*, W_{CS}-*F*, and A_S-*F* are excess returns on the short ends of the small stock spread factors, HML_S, CMA_S, and RMW_{CS}. For the three models that produce the highest $Sh^2(f)$ in Table A1, the table shows intercepts *a*, *t*-statistics for the intercepts *t*(*a*), slopes, R^2, and residual standard errors *s*(*e*) from spanning regressions of each of the factors of a model on the model's other four. The table also shows $Sh^2(f)$ and each factor's marginal contribution to a model's $Sh^2(f)$, that is, $a^2/s^2(e)$.

Panel A: Spanning regressions for combined spread factor model

	a	*Mkt*	*SMB*	*HML*	RMW_C	*CMA*	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	1.01		0.05	0.08	-0.90	-0.72	6.56	0.30	3.70	0.167	0.074
SMB	0.52	0.02		0.01	-0.81	0.05	4.76	0.27	2.60	0.167	0.040
HML	-0.00	0.02	0.01		0.11	0.96	-0.01	0.48	2.02	0.167	0.000
RMW_C	0.48	-0.15	-0.27	0.06		0.00	7.88	0.40	1.49	0.167	0.104
CMA	0.20	-0.10	0.01	0.46	0.00		3.34	0.52	1.39	0.167	0.020

Panel B: Spanning regressions for small stock spread factor model

	a	*Mkt*	*SMB*	HML_S	RMW_{CS}	CMA_S	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	0.92		0.18	-0.11	-0.40	-0.57	5.61	0.22	3.91	0.172	0.055
SMB	0.44	0.08		0.03	-0.61	0.09	4.01	0.27	2.59	0.172	0.029
HML_S	-0.14	-0.03	0.02		0.70	0.84	-1.68	0.63	1.94	0.172	0.005
RMW_{CS}	0.46	-0.07	-0.24	0.50		-0.41	6.73	0.55	1.65	0.172	0.078
CMA_S	0.34	-0.08	0.03	0.47	-0.32		5.64	0.46	1.46	0.172	0.056

Panel C: Spanning regressions for small stock short factor model

	a	*Mkt*	*S-F*	L_S-*F*	W_{CS}-*F*	A_S-*F*	*t*(*a*)	R^2	*s*(*e*)	$Sh^2(f)$	$a^2/s^2(e)$
Mkt	-0.05		0.74	0.18	-0.66	0.44	-0.63	0.81	1.93	0.176	0.001
S-F	0.29	0.11		-0.30	0.55	0.56	9.93	0.98	0.74	0.176	0.158
L_S-*F*	0.10	0.04	-0.48		0.55	0.87	2.45	0.98	0.94	0.176	0.011
W_{CS}-*F*	-0.20	-0.13	0.72	0.45		-0.01	-5.50	0.98	0.85	0.176	0.053
A_S-*F*	-0.15	0.06	0.51	0.50	-0.01		-4.86	0.99	0.71	0.176	0.042

Table A4

Weights (in percent) and leverage in $Sh^2(f)$ tangency portfolios for the five-factor models of Table A2 (no momentum): July 1963-June 2016

Mkt is M-F, the monthly excess return on the value-weight market portfolio. The size spread factor, *SMB*, is the difference between small stock and big stock portfolio returns (*S* and *B*). The value, investment, momentum, and profitability spread factors (*HML*, *CMA*, *UMD*, and RMW_O or RMW_C) are averages of small stock and big stock spread factors (HML_S, CMA_S, RMW_{OS} or RMW_{CS}, and HML_B, CMA_B, RMW_{OB} or RMW_{CB}). Each of the combined and small stock spread factors is parent to two excess return factors constructed from its long and short ends and identified by the first and last letters of the spread factor's name. For example, *HML* is parent to *H-F* and *L-F*, and HML_S is parent to H_S-*F* and L_S-*F*. A subcript *S* indicates that a factor is constucted from small stocks. Absence of a subcript means a factor uses small and big stocks. Each line of the table shows the *Mkt*, *Size*, *Value*, *Prof*, and *Inv* factors in a model; $Sh^2(f)$, the maximum squared Sharpe ratio for the model's factors; the factor weights in the tangency portfolio that produces $Sh^2(f)$; and tangency portfolio leverage, expressed as a proportion of $1 invested in the tangency portfolio.

		Weight in tangency portfolio					
Model	$Sh^2(f)$	*Mkt*	*Size*	*Value*	*Prof*	*Inv*	Leverage
Mkt, *SMB*, *HML*, RMW_C, *CMA*	0.167	100.0	105.1	-0.2	294.7	139.1	5.4
Mkt, *SMB*, HML_S, RMW_{CS}, CMA_S	0.172	100.0	109.1	-61.7	281.9	269.9	7.2
Mkt, *S-F*, L_S-*F*, W_{CS}-*F*, A_S-*F*	0.176	-19.4	744.2	155.1	-378.1	-401.8	8.0
Mkt, *SMB*, *HML*, RMW_O, *CMA*	0.103	100.0	84.2	-3.4	190.7	254.5	5.3
Mkt, *S-F*, *H-F*, R_C-*F*, *C-F*	0.134	-783.6	-479.7	-26.8	906.1	484.0	12.9
Mkt, *S-F*, H_S-*F*, R_{CS}-*F*, C_S-*F*	0.114	-4.6	-1178.2	98.1	678.6	506.2	11.8
Mkt, *S-F*, *L-F*, W_C-*F*, *A-F*	0.146	312.1	368.0	-23.8	-405.0	-151.3	5.8

Table A5

Summary statistics for five-factor (no momentum) regression intercepts, July 1963-June 2016

The one hundred left-hand-side (LHS) portfolios in Panel B are from four 5x5 quintile sorts on *ME* and *BE/ME*, *OP*, *CP*, or *Inv*. (See Appendix for definitions of all variables.) The 135 anomaly portfolios in Panel C are from 5x5 sorts on *ME* and market β, 5x5 sorts on *ME* and accruals, 5x7 sorts on *ME* and net issuance (repurchases, zero net share issues, and quintiles of positive net share issues), 5x5 sorts on *ME* and *Var*, the variance of daily returns, and 5x5 sorts on *ME* and *RVar*, the variance of daily residuals from the Fama and French (1993) three-factor model. The *ME-Var* and *ME-RVar* sorts are monthly and the *Var* and *RVar* quintiles are conditional on *ME* quintile. All other portfolios are formed annually at the end of June and the sorts are independent. The 235 LHS portfolios in Panel A are all the LHS portfolios in Panels B (100) and C (135). The table shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989) and its *p*-value, *p*(*GRS*); $A|a_i|$, the average absolute intercept; $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$, which is the difference between the average return on LHS portfolio i and the average return on the value-weight market; $As^2(a_i)/Aa_i^2$, the average of the squared sample standard errors of the intercepts over the average squared intercept; the average of the regression R^2, AR^2; $Sh^2(f)$, the maximum squared Sharpe ratio for the model's factors; and $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios.

Table A5 (continued)

Model	GRS	$p(GRS)$	$A\|a\|$	$Aa_i^2/A\bar{r}_i^2$	$As^2(a_i)/Aa_i^2$	AR^2	$Sh^2(f)$	$Sh^2(a)$
Panel A: All portfolios in Panels B and C								
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$	2.39	0.000	0.107	0.23	0.27	0.91	0.176	1.657
Mkt, SMB, HML_S, RMW_{CS}, CMA_S	2.41	0.000	0.098	0.24	0.27	0.90	0.172	1.659
Mkt, SMB, HML, RMW_C, CMA	2.42	0.000	0.099	0.23	0.28	0.90	0.167	1.659
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$	2.50	0.000	0.095	0.22	0.29	0.90	0.146	1.687
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$	2.54	0.000	0.113	0.31	0.20	0.90	0.134	1.692
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$	2.65	0.000	0.121	0.37	0.16	0.90	0.114	1.736
Mkt, SMB, HML, RMW_O, CMA	2.67	0.000	0.102	0.27	0.22	0.91	0.103	1.735
Panel B: 5x5 sorts on ME and BE/ME, OP, CP, and Inv								
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$	1.97	0.000	0.091	0.17	0.32	0.92	0.176	0.432
Mkt, SMB, HML_S, RMW_{CS}, CMA_S	2.00	0.000	0.077	0.13	0.44	0.92	0.172	0.438
Mkt, SMB, HML, RMW_C, CMA	2.06	0.000	0.080	0.14	0.39	0.92	0.167	0.449
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$	2.17	0.000	0.082	0.14	0.41	0.92	0.146	0.463
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$	2.21	0.000	0.080	0.15	0.36	0.92	0.134	0.467
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$	2.37	0.000	0.090	0.17	0.31	0.92	0.114	0.493
Mkt, SMB, HML, RMW_O, CMA	2.41	0.000	0.078	0.14	0.35	0.92	0.103	0.496
Panel C: 5x5 sorts on ME and accruals, β, variances of daily returns and residuals, and 5x7 sorts on ME and net share issuance								
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$	2.76	0.000	0.119	0.27	0.24	0.90	0.176	0.875
Mkt, SMB, HML_S, RMW_{CS}, CMA_S	2.82	0.000	0.114	0.31	0.22	0.89	0.172	0.893
Mkt, SMB, HML, RMW_C, CMA	2.77	0.000	0.113	0.29	0.24	0.89	0.167	0.873
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$	2.79	0.000	0.105	0.28	0.25	0.89	0.146	0.863
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$	3.00	0.000	0.137	0.42	0.16	0.89	0.134	0.918
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$	3.11	0.000	0.144	0.50	0.13	0.89	0.114	0.934
Mkt, SMB, HML, RMW_O, CMA	3.06	0.000	0.121	0.35	0.19	0.89	0.103	0.910

Table A6

Summary statistics for regression intercepts, July 1963-December 1989 and January 1990-June 2016, 318 observations each

Panels A-C and D-F show results for July 1963-December 1989 and January 1990-June 2016, respectively. The 125 left-hand-side (LHS) portfolios in Panels B and E are from five 5x5 quintile sorts on *ME* and, independently, on *BE/ME*, *OP*, *CP*, *Inv*, or *Mom*. (See Appendix for definitions of all variables.) The *ME-Mom* portfolios are formed monthly and the other portfolios in Panels B and E are formed at the end of June. Eighty-five of the anomaly portfolios in Panels C and F are from independent, end-of-June, 5x5 sorts on *ME* and market β, 5x5 sorts on *ME* and accruals, and 5x7 sorts on *ME* and net issuance (repurchases, zero net share issues, and quintiles of positive net share issues). The remaining 50 anomaly portfolios are from monthly 5x5 sorts on *ME* and either *Var*, the variance of daily returns, or *RVar*, the variance of daily residuals from the Fama and French (1993) three-factor model. The breakpoints for *Var* and *RVar* quintiles are conditional on *ME* quintile. The 260 LHS portfolios in Panels A and D are all the LHS portfolios in Panels B and C or E and F. The table shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989) and its *p*-value, *p*(*GRS*); $A|a_i|$, the average absolute intercept; $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$, which is the difference between the average return on LHS portfolio i and the average return on the value-weight market; $As^2(a_i)/Aa_i^2$, the average of the squared sample standard errors of the intercepts over the average squared intercept; the average of the regression R^2, AR^2; $Sh^2(f)$, the maximum squared Sharpe ratio for the model's factors; and $Sh^2(a)$, the max squared Sharpe ratio for the intercepts for a set of LHS portfolios. Models are sorted on $Sh^2(f)$ for July 1963-June 2016.

Table A6 (continued)

Model	GRS	$p(GRS)$	$A\|a\|$	$Aa_i^2/A\bar{r}_i^2$	$As^2(a_i)/Aa_i^2$	AR^2	$Sh^2(f)$	$Sh^2(a)$
Panel A: LHS portfolios: All 5x5 and 5x7 returns including momentum, July 1963-December 1989								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	2.23	0.000	0.123	0.21	0.33	0.93	0.390	15.382
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	2.26	0.000	0.108	0.17	0.41	0.93	0.425	16.006
Mkt, SMB, HML, RMW_C, CMA, UMD	2.63	0.000	0.160	0.37	0.18	0.93	0.299	16.957
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	2.61	0.000	0.134	0.31	0.22	0.93	0.298	16.828
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	2.42	0.000	0.171	0.48	0.14	0.93	0.321	15.893
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	2.97	0.000	0.149	0.37	0.16	0.93	0.217	17.961
Mkt, SMB, HML, RMW_O, CMA, UMD	2.75	0.000	0.146	0.33	0.19	0.93	0.264	17.284
Panel B: LHS portfolios: 5x5 sorts on ME and BE/ME, OP, CP, Inv, and Mom, July 1963-December 1989								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	2.08	0.000	0.097	0.12	0.52	0.94	0.390	1.921
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	1.96	0.000	0.090	0.10	0.62	0.94	0.425	1.858
Mkt, SMB, HML, RMW_C, CMA, UMD	2.32	0.000	0.134	0.25	0.22	0.94	0.299	1.999
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	2.36	0.000	0.097	0.16	0.38	0.94	0.298	2.037
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	2.28	0.000	0.128	0.26	0.23	0.94	0.321	2.003
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	2.58	0.000	0.126	0.26	0.20	0.94	0.217	2.087
Mkt, SMB, HML, RMW_O, CMA, UMD	2.43	0.000	0.115	0.20	0.28	0.94	0.264	2.042
Panel C: LHS portfolios: 5x5 sorts on ME and accruals, β, Var, and RVar, and 5x7 sorts on ME and net share issuance, July 1963-December 1989								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	3.71	0.000	0.147	0.30	0.26	0.93	0.390	3.904
Mkt, $S\text{-}F$, $L_S\text{-}F$, $W_{CS}\text{-}F$, $A_S\text{-}F$, $D_S\text{-}F$	3.65	0.000	0.125	0.23	0.33	0.93	0.425	3.944
Mkt, SMB, HML, RMW_C, CMA, UMD	3.98	0.000	0.184	0.48	0.15	0.93	0.299	3.920
Mkt, $S\text{-}F$, $H_S\text{-}F$, $R_{CS}\text{-}F$, $C_S\text{-}F$, $U_S\text{-}F$	4.00	0.000	0.168	0.45	0.16	0.92	0.298	3.932
Mkt, $S\text{-}F$, $H\text{-}F$, $R_C\text{-}F$, $C\text{-}F$, $U\text{-}F$	4.01	0.000	0.211	0.68	0.11	0.92	0.321	4.011
Mkt, $S\text{-}F$, $L\text{-}F$, $W_C\text{-}F$, $A\text{-}F$, $D\text{-}F$	4.26	0.000	0.170	0.47	0.14	0.93	0.217	3.927
Mkt, SMB, HML, RMW_O, CMA, UMD	4.12	0.000	0.175	0.46	0.16	0.92	0.264	3.947

Table A6 (continued)

Model	GRS	$p(GRS)$	$A\|a\|$	$Aa_i^2/A\bar{r}_i^2$	$As^2(a_i)/Aa_i^2$	AR^2	$Sh^2(f)$	$Sh^2(a)$
Panel D: LHS portfolios: All 5x5 and 5x7 returns including momentum, January 1990-June 2016								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	1.70	0.011	0.107	0.30	0.60	0.89	0.183	10.018
Mkt, S-F, L_S-F, W_{CS}-F, A_S-F, D_S-F	1.70	0.011	0.129	0.45	0.39	0.90	0.169	9.887
Mkt, SMB, HML, RMW_C, CMA, UMD	1.74	0.008	0.112	0.34	0.55	0.89	0.176	10.173
Mkt, S-F, H_S-F, R_{CS}-F, C_S-F, U_S-F	1.75	0.008	0.131	0.47	0.41	0.89	0.147	9.975
Mkt, S-F, H-F, R_C-F, C-F, U-F	1.73	0.009	0.111	0.36	0.55	0.89	0.162	9.972
Mkt, S-F, L-F, W_C-F, A-F, D-F	1.82	0.005	0.115	0.36	0.52	0.89	0.155	10.421
Mkt, SMB, HML, RMW_O, CMA, UMD	1.82	0.005	0.109	0.32	0.54	0.89	0.138	10.310
Panel B: LHS portfolios: 5x5 sorts on ME and BE/ME, OP, CP, Inv, and Mom, January 1990-June 2016								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	1.48	0.007	0.092	0.21	0.72	0.91	0.183	1.166
Mkt, S-F, L_S-F, W_{CS}-F, A_S-F, D_S-F	1.58	0.002	0.113	0.34	0.42	0.91	0.169	1.230
Mkt, SMB, HML, RMW_C, CMA, UMD	1.46	0.009	0.089	0.22	0.69	0.91	0.176	1.142
Mkt, S-F, H_S-F, R_{CS}-F, C_S-F, U_S-F	1.72	0.000	0.114	0.31	0.53	0.90	0.147	1.313
Mkt, S-F, H-F, R_C-F, C-F, U-F	1.58	0.002	0.087	0.21	0.76	0.91	0.162	1.218
Mkt, S-F, L-F, W_C-F, A-F, D-F	1.54	0.004	0.096	0.25	0.60	0.91	0.155	1.179
Mkt, SMB, HML, RMW_O, CMA, UMD	1.56	0.003	0.087	0.21	0.68	0.91	0.138	1.179
Panel C: LHS portfolios: 5x5 sorts on ME and accruals, β, Var, and RVar, and 5x7 sorts on ME and net share issuance, January 1990-June 2016								
Mkt, SMB, HML_S, RMW_{CS}, CMA_S, UMD_S	1.96	0.000	0.122	0.39	0.54	0.88	0.183	1.756
Mkt, S-F, L_S-F, W_{CS}-F A_S-F, D_S-F	1.97	0.000	0.143	0.54	0.37	0.88	0.169	1.748
Mkt, SMB, HML, RMW_C, CMA, UMD	2.01	0.000	0.134	0.45	0.48	0.87	0.176	1.788
Mkt, S-F, H_S-F, R_{CS}-F, C_S-F, U_S-F	2.01	0.000	0.147	0.62	0.35	0.87	0.147	1.749
Mkt, S-F, H-F, R_C-F, C-F, U-F	1.98	0.000	0.134	0.49	0.46	0.87	0.162	1.744
Mkt, S-F, L-F, W_C-F A-F, D-F	2.01	0.000	0.133	0.45	0.48	0.87	0.155	1.758
Mkt, SMB, HML, RMW_O, CMA, UMD	1.99	0.000	0.130	0.43	0.47	0.88	0.138	1.713