CMSC 460 - HW4

Gudjon Einar Magnusson

October 22, 2016

1

1.a

$p(x) = 816x^3 - 3835x^2 + 6000x - 3125 = 0$, $x = \frac{25}{17}, \frac{25}{16}, \frac{5}{3}$

1.b

Figure 1 shows a graph of the polynomial $p(x)$ on the interval $[1.43, 1.71]$ with the zeros marked with dots.

1.c

Newtons method finds a zero in 11 iterations. It finds the zero at $x = \frac{25}{17}$.

Below are values of x_i where i is the iteration.

```
X_0:  1.500000000000000
X_1:  1.416666666666667
X_2:  1.450677267373387
X_3:  1.466352192549958
X_4:  1.470328933587513
X_5:  1.470587167764815
X_6:  1.470588235275914
X_7:  1.470588235294096
X_8:  1.470588235293973
X_9:  1.470588235294035
X_10: 1.470588235294158
X_11: 1.470588235294158
```

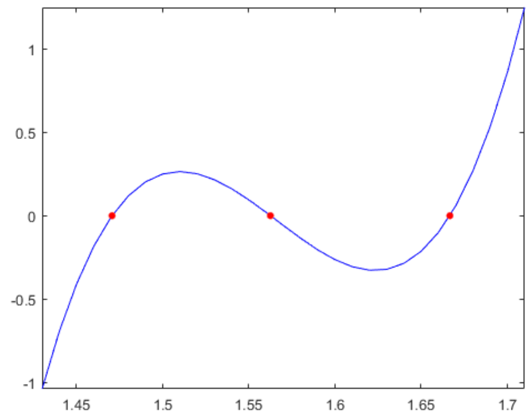


Figure 1: The 3 roots of the polynomial $p(x)$

1.d

The secant method finds a zero in 12 iterations. It finds the zero at $x = \frac{5}{3}$.

Below are values of x_i where i is the iteration.

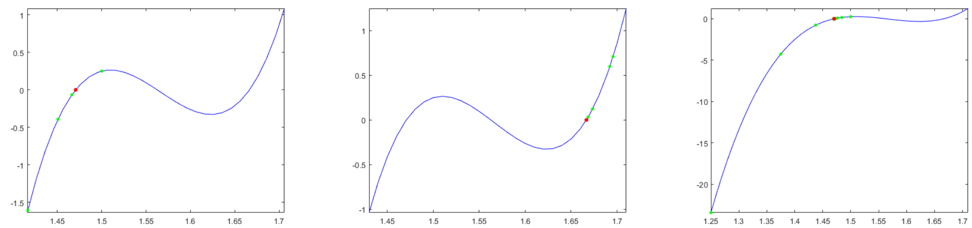
```
X_0:  1.000000000000000
X_1:  2.000000000000000
X_2:  1.695652173913044
X_3:  1.692189044157721
X_4:  1.673392092119985
X_5:  1.668509164558399
X_6:  1.666832623252804
X_7:  1.666671061078255
X_8:  1.666666677366379
X_9:  1.666666666667267
X_10: 1.666666666666721
X_11: 1.666666666666585
X_12: 1.666666666666653
X_13: 1.666666666666653
```

1.e

The bisection method finds a zero in 52 iterations. It finds the zero at $x = \frac{25}{17}$.

Below are the upper and lower bound of the search interval in each iteration.

```
step 0:  a = 1.000000000000000, b = 2.000000000000000
step 1:  a = 1.000000000000000, b = 1.500000000000000
step 2:  a = 1.250000000000000, b = 1.500000000000000
step 3:  a = 1.375000000000000, b = 1.500000000000000
step 4:  a = 1.437500000000000, b = 1.500000000000000
step 5:  a = 1.468750000000000, b = 1.500000000000000
step 6:  a = 1.468750000000000, b = 1.484375000000000
step 7:  a = 1.468750000000000, b = 1.476562500000000
step 8:  a = 1.468750000000000, b = 1.472656250000000
step 9:  a = 1.468750000000000, b = 1.470703125000000
step 10: a = 1.469726562500000, b = 1.470703125000000
step 11: a = 1.470214843750000, b = 1.470703125000000
step 12: a = 1.470458984375000, b = 1.470703125000000
step 13: a = 1.470581054687500, b = 1.470703125000000
step 14: a = 1.470581054687500, b = 1.470642089843750
step 15: a = 1.470581054687500, b = 1.470611572265625
step 16: a = 1.470581054687500, b = 1.470596313476563
step 17: a = 1.470581054687500, b = 1.470588684082031
step 18: a = 1.470584869384766, b = 1.470588684082031
step 19: a = 1.470586776733398, b = 1.470588684082031
step 20: a = 1.470587730407715, b = 1.470588684082031
step 21: a = 1.470588207244873, b = 1.470588684082031
step 22: a = 1.470588207244873, b = 1.470588445663452
step 23: a = 1.470588207244873, b = 1.470588326454163
step 24: a = 1.470588207244873, b = 1.470588266849518
step 25: a = 1.470588207244873, b = 1.470588237047195
step 26: a = 1.470588222146034, b = 1.470588237047195
step 27: a = 1.470588229596615, b = 1.470588237047195
step 28: a = 1.470588233321905, b = 1.470588237047195
step 29: a = 1.470588235184550, b = 1.470588237047195
step 30: a = 1.470588235184550, b = 1.470588236115873
step 31: a = 1.470588235184550, b = 1.470588235650212
step 32: a = 1.470588235184550, b = 1.470588235417381
step 33: a = 1.470588235184550, b = 1.470588235300966
step 34: a = 1.470588235242758, b = 1.470588235300966
step 35: a = 1.470588235271862, b = 1.470588235300966
step 36: a = 1.470588235286414, b = 1.470588235300966
step 37: a = 1.470588235293690, b = 1.470588235300966
step 38: a = 1.470588235293690, b = 1.470588235297328
step 39: a = 1.470588235293690, b = 1.470588235295509
step 40: a = 1.470588235293690, b = 1.470588235294599
step 41: a = 1.470588235294144, b = 1.470588235294599
step 42: a = 1.470588235294144, b = 1.470588235294372
step 43: a = 1.470588235294144, b = 1.470588235294258
step 44: a = 1.470588235294144, b = 1.470588235294201
step 45: a = 1.470588235294144, b = 1.470588235294173
step 46: a = 1.470588235294144, b = 1.470588235294159
step 47: a = 1.470588235294152, b = 1.470588235294159
step 48: a = 1.470588235294155, b = 1.470588235294159
step 49: a = 1.470588235294155, b = 1.470588235294157
step 50: a = 1.470588235294156, b = 1.470588235294157
step 51: a = 1.470588235294156, b = 1.470588235294156
step 52: a = 1.470588235294156, b = 1.470588235294156
```



(a) Newton method (b) Secant method (c) Bisection method

Figure 2: Points evaluated by each of the zero finding methods

1.f

fzerotx returns a value of 1.666666666666669, corresponding to the zero at $x = \frac{5}{3}$.

fzerotx uses an improved version of the secant algorithm. It approaches 0 in a similar way as secant, following the slope from $x = 2$ towards $x = 1$. Thats why it it finds the zero closest to $x = 2$ at $x = \frac{5}{3}$.

fzerotx gives a slightly less accurate answer than my implementation of secant because it uses a threshold that is approximately 2 times machine epsilon.

2

To find where $x = tan(x)$ I defined the function $f(x) = x - tan(x)$ and found where that is equal to zero. *tan* is π periodic and continuous on the interval $[c\pi - \frac{\pi}{2}, c\pi + \frac{\pi}{2}]$ where c is some whole number. Since *tan* is only positive on the upper half of the interval, thats where I searched for zeros. I used *fzero* on the interval $[c\pi, c\pi + \frac{\pi}{2} - \epsilon]$ for $c = 1, 2, \ldots, 10$. ϵ is a small value I subtracted from the upper bound to avoid falling on the wrong side of the singularity and getting a value with the wrong sign.

Below are the 10 values of x that satisfy the condition $x = tan(x)$. These values accurate to within about 10^{-11}.

```
x = 4.493409457909064
x = 7.725251836937708
x = 10.904121659428899
x = 14.066193912831473
x = 17.220755271930770
x = 20.371302959287561
x = 23.519452498689006
x = 26.666054258812672
x = 29.811598790892958
x = 32.956389039822476
```

3

Since every thing is given except the depth, I can rewrite the formula to get the temperature as a function of depth. $T(x) = 35\,\mathrm{erf}(\frac{x}{2\sqrt{\alpha t}}) - 15$. We want to know at what depth is the temperature equal to zero. To find zero I used *fzerotx* on the function T and the interval $[0, 5]$. I picked 5 arbitrarily based on my intuition, I don't think even the coldest parts of Siberia freeze down to 5 meters.

According to *fzerotx*, $T(x) = 0$ where $x = 0.946280996267692$. About 95cm.