

CMSC 460 - HW6

Gudjon Einar Magnusson

November 17, 2016

1

1.a Singular Matrices

The following matrices are singular because the span of Ax contains the zero vector when x is not zero. That tells me that zero is an eigenvalue of A .

$$\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} / 4$$

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} / 4$$

1.b Complex λ

The following matrices have complex eigenvalues because x and Ax never become parallel.

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} / 4$$

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} / 4$$

1.c Double λ

The following matrices have repeated eigenvalues.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 6 & 4 \\ -1 & 2 \end{bmatrix} / 4$$

2

2.a

For my image matrix I of size $540 \times 960 \times 3$, or 540×2880 after reshaping

$$[V, S, U] = \text{svd}(I, 0)$$

Gives me a large non-square sigma matrix, which is not what I want.

$$[V, S, U] = \text{svd}(I', 0)$$

Gives me a 540×540 sigma matrix.

2.b

Figure 2 shows how my image approximated by matrices of different ranks. The original image is of rank 540 but the difference is almost undetectable down to rank 100. At rank 20 the image still readable but the quality drops quickly at ranks below that. That makes sense if we look at the log plot of the singular values in figure 1. The first few singular values drop very quickly but then levels off somewhere around the 20th value and starts dropping slower.

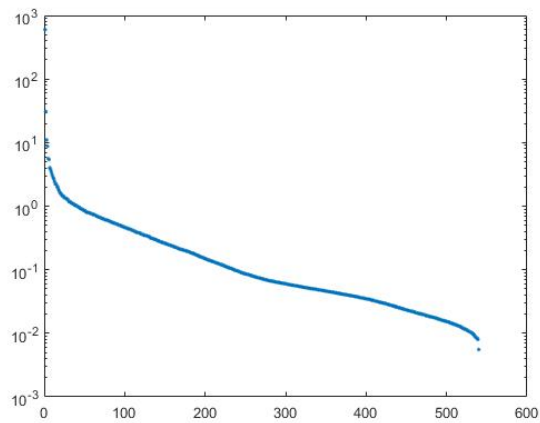
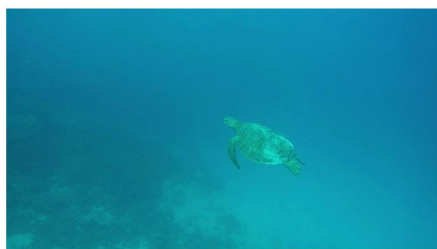
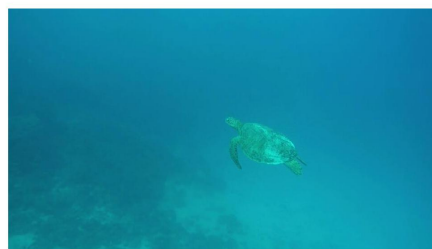


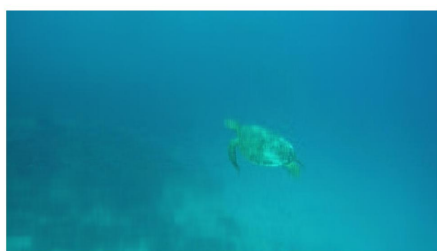
Figure 1



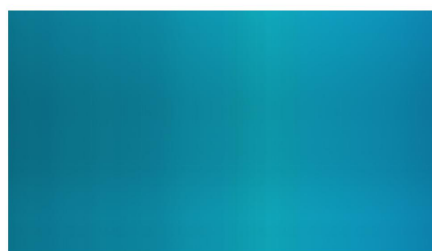
(a) Original image, rank 540



(b) Rank 100 approximation



(c) Rank 20 approximation



(d) Rank 1 approximation

Figure 2: $Rank_k$ approximation of an image