

Note: see figures on last page.

1a) Calculate the (x,y,z) value of the 3<sup>rd</sup> order (cubic) Hermit Spline when  $s = 0.8$  given the following points and tangents (expressed as derivatives vs  $s$ ).

$$p_1 = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix} \quad p_2 = \begin{bmatrix} 3.5 \\ 9 \\ 5 \end{bmatrix} \quad \frac{dp_1}{ds} = \begin{bmatrix} 1.2 \\ 1 \\ 2 \end{bmatrix} \quad \frac{dp_2}{ds} = \begin{bmatrix} 1 \\ 2 \\ 0.5 \end{bmatrix}$$

**NOTE:** this can be accomplished by treating the different dimensions of the problem separately in terms of  $s$ , and then combining them in a vector (the calculations for each dimension do not affect each other). In other words, first calculate the  $x$  value using only the  $x$  projection of the points (2 and 3.5) and tangents (1.2 and 1), then do the same for the  $y$  and  $z$  projections. Figure 1 may provide intuition.

1b) Calculate the parametric form of the hermit spine.

**Note:** again this is accomplished by handling each dimension separately to find:

$$x = g(s), \quad y = f(s), \quad \text{and} \quad z = h(s)$$

2) Assume the manipulator arm in figure 2. You are provided the following transformations between the various coordinate frames:

$${}^L T_A, {}^A T_B, {}^B T_C, {}^C T_D, {}^D T_E, {}^E T_F, {}^W T_F$$

Given a point  ${}^W p$  expressed in the  $W$  coordinate frame, determine its value in the  $L$  coordinate frame.

3) There is a Dubins vehicle depicted in Figure 3 (it is the dark point), it is located at (0,0) and heading 45 degrees off of the x axis. The vehicle will trace a circle of radius 1 when turning at the maximum turning rate.

3a) Draw the (shortest) Dubins path to the configuration located at point  $\mathbf{a} = (2,0)$  and with heading 120 degrees off the x-axis. **NOTE:** just draw it, you do not need to calculate anything.

3a) Draw the (shortest) Dubins path to the configuration located at point  $\mathbf{b} = (4,0)$  and with heading 80 degrees off the x-axis. **NOTE:** just draw it, you do not need to calculate anything.

4a) Assume you are given an empty FIFO-Queue Q and nodes A,B,C,D,E,F,G. Draw the order of the nodes within Q after the following sequence of operations (make sure to label which end is the front and which is the back).

Q.INSERT(A), Q.INSERT(C), Q.POP(), Q.INSERT(B), Q.INSERT(G), Q.POP(), Q.INSERT(F), Q.TOP(),  
Q.INSERT(D), Q.INSERT(E), Q.POP(), Q.TOP()

4b) Now Assume you are instead given an empty Stack (FILO-Queue) Q. Draw the order of the nodes within Q after the same sequence of operations as in (4a).

5) Assume you are given the graph in Figure 4, and assume the order of the neighbor lists are as appears below.

**Neighbor list orders:** F->[D,G] means that node F has edges to D and to G, and calling F.NextNeighbor will return D the first time, G the second time, and then empty set.

A->[B]

B->[D]

C->[A]

D->[C,E,F,H]

E->[D]

F->[D,G]

G->[F,H]

H->[D,G]

5a) Assume start=F and that Breadth-First Search is run until the entire graph is explored (this can happen if the goal is not connected to the graph), what is the order that nodes are discovered? Draw the final search tree (i.e., the tree of parent pointers).

5b) Assume start=F and that Depth-First Search is run until the entire graph is explored, what is the order that nodes are discovered? Draw the final search tree (i.e., the tree of parent pointers).

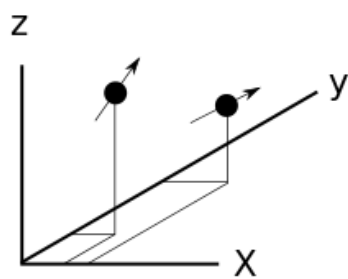


Figure 1

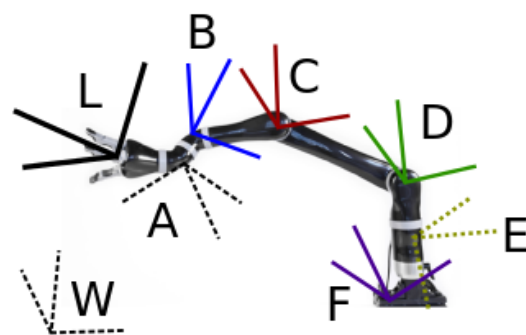


Figure 2

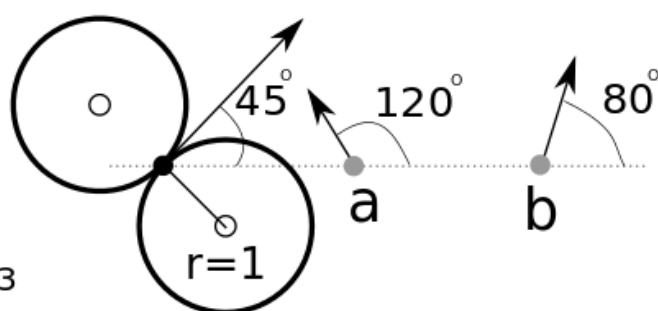


Figure 3

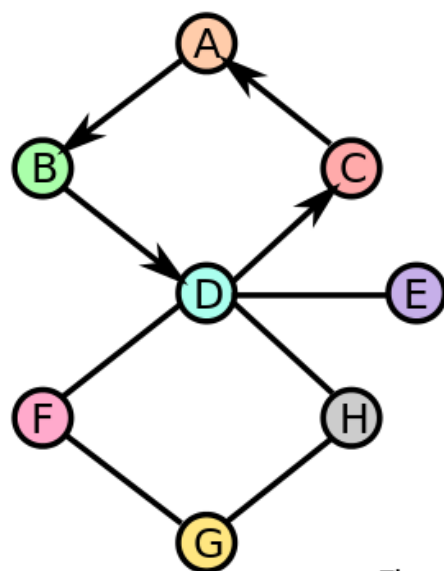


Figure 4