Multi-compartment SEIR model with age classes

In the equations, we have used the following notation: n is an index running over all the n_c countries or compartments. The total poulations of countries n and m are N_n and N_m. We have normalized the variables per country by the total population number N_n (which is why we need the factor N_m/N_n in the interaction term).

We model the interaction between countries using the elements of $\mathbf{R}^\mathrm{C} \in \Re^{n_\mathrm{c}\times n_\mathrm{c}}$ (the diagonal must always be 1). The effective reproductive number per country is a scalar function of time, $R_t(n)$, and is a parameter function that we estimate. We model the relative differences in infections between age groups using the coefficients in $R^\mathrm{A}_{i,j}(n)$ which can differ between countries (the model default is to set all elements $R^\mathrm{A}_{i,j}(n) = 1$ assuming equal transmission rates among age groups). In the equations below, we assume the same $R^\mathrm{A}_{i,j}(n)$ when interacting with other countries as within a country (the only sound alternative would be to set $R^\mathrm{A}_{i,j}(n) = 1$ when $m \neq n$, given the number of coefficients we would otherwise need to specify).

The fractions of fatally ill, severely ill, and mild desease, p_f, p_s, and p_m, can differ between countries. Currently, we have used the same hospitalization fraction of fatally ill p_h for all countries.

In the case with only one country $n = m = 1$, we have $N_m/N_n = 1$, and $R^\mathrm{C}(n,m) = 1$. Thus, the equations reduce to the standard SEIR model. The use of a multi-compartment model only changes the nonlinear interaction term. Besides the interaction term, each country evolve independently of each other.

$$\frac{\partial \mathbf{S}_i(n)}{\partial t} = -\sum_{m=1}^{n_\mathrm{c}} \frac{N_m}{N_n} R^\mathrm{C}_{nm} R_t(n) \left(\sum_{j=1}^{n_\mathrm{a}} \frac{R^\mathrm{A}_{ij}(n)\mathbf{I}_j(m)}{\tau_\mathrm{inf}} \right) \mathbf{S}_i(n) \tag{1}$$

$$\frac{\partial \mathbf{E}_i(n)}{\partial t} = \sum_{m=1}^{n_\mathrm{c}} \frac{N_m}{N_n} R^\mathrm{C}_{nm} R_t(n) \left(\sum_{j=1}^{n_\mathrm{a}} \frac{R^\mathrm{A}_{ij}(n)\mathbf{I}_j(m)}{\tau_\mathrm{inf}} \right) \mathbf{S}_i(n) - \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i(n) \tag{2}$$

$$\frac{\partial \mathbf{I}_i(n)}{\partial t} = \frac{1}{\tau_\mathrm{inc}} \mathbf{E}_i(n) - \frac{1}{\tau_\mathrm{inf}} \mathbf{I}_i(n) \tag{3}$$

$$\frac{\partial \mathbf{Q}_\mathrm{m}(n)}{\partial t} = \sum_{i=1}^{n_\mathrm{a}} \frac{p^i_\mathrm{m}(n)}{\tau_\mathrm{inf}} \mathbf{I}_i(n) - \frac{1}{\tau_\mathrm{recm}} \mathbf{Q}_\mathrm{m}(n) \tag{4}$$

$$\frac{\partial \mathbf{Q}_\mathrm{s}(n)}{\partial t} = \sum_{i=1}^{n_\mathrm{a}} \frac{p^i_\mathrm{s}(n)}{\tau_\mathrm{inf}} \mathbf{I}_i(n) - \frac{1}{\tau_\mathrm{hosp}} \mathbf{Q}_\mathrm{s}(n) \tag{5}$$

$$\frac{\partial \mathbf{Q}_\mathrm{f}(n)}{\partial t} = \sum_{i=1}^{n_\mathrm{a}} \frac{p^i_\mathrm{f}(n)}{\tau_\mathrm{inf}} \mathbf{I}_i(n) - \frac{1}{\tau_\mathrm{hosp}} \mathbf{Q}_\mathrm{f}(n) \tag{6}$$

$$\frac{\partial \mathbf{H}_\mathrm{s}(n)}{\partial t} = \frac{1}{\tau_\mathrm{hosp}} \mathbf{Q}_\mathrm{s}(n) - \frac{1}{\tau_\mathrm{recs}} \mathbf{H}_\mathrm{s} \tag{7}$$

$$\frac{\partial \mathbf{H}_\mathrm{f}(n)}{\partial t} = \frac{p_\mathrm{h}}{\tau_\mathrm{hosp}} \mathbf{Q}_\mathrm{f}(n) - \frac{1}{\tau_\mathrm{death}} \mathbf{H}_\mathrm{f}(n) \tag{8}$$

$$\frac{\partial \mathbf{C}_\mathrm{f}(n)}{\partial t} = \frac{(1-p_\mathrm{h})}{\tau_\mathrm{hosp}} \mathbf{Q}_\mathrm{f}(n) - \frac{1}{\tau_\mathrm{death}} \mathbf{C}_\mathrm{f}(n) \tag{9}$$

$$\frac{\partial \mathbf{R}_\mathrm{m}(n)}{\partial t} = \frac{1}{\tau_\mathrm{recm}} \mathbf{Q}_\mathrm{m}(n) \tag{10}$$

$$\frac{\partial \mathbf{R}_\mathrm{s}(n)}{\partial t} = \frac{1}{\tau_\mathrm{recs}} \mathbf{H}_\mathrm{s}(n) \tag{11}$$

$$\frac{\partial \mathbf{D}(n)}{\partial t} = \frac{1}{\tau_\mathrm{death}} \mathbf{H}_\mathrm{f} + \frac{1}{\tau_\mathrm{death}} \mathbf{C}_\mathrm{f}(n) \tag{12}$$