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Improving sequential decisions – Efficiently accounting for future learning

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ABSTRACT

In sequential field development planning, past decisions not only directly affect the maximum achievable expected NPV but also influence the future information that can be used to reduce geological uncertainty. To act optimally, when choosing actions, we must also take into account the opportunities to improve the optimal strategy by reducing future uncertainty. In most applications, however, the effect of future information on the optimal decisions is ignored because it would be computationally intractable to update the reservoir model and re-optimize to account for all possible outcomes of future observations. To efficiently make optimal decisions while considering future possibilities for learning through actions, we developed a flexible workflow built on the key-feature-based value of information (VOI) analysis, which is obtained by identifying key reservoir features for optimization problems and key observations for improving future decisions. Instead of considering future information from all remaining actions, we only consider the important information from key actions to reduce the uncertainty with the largest influence on the optimal strategy – that which would be most helpful in improving future decisions. The efficiency of the method results from the focus on the use of key observations to reduce key uncertainty, rather than using all observations to reduce all uncertainties.

In this work, we built supervised-learning algorithms to identify the optimal combination of observations for reducing key uncertainty and simultaneously to estimate the information's reliability. This allows automatic detection of key observations and direct computation of the posterior probability distribution of key uncertainty based on Bayes' rule, avoiding the need for full history matching to re-estimate the uncertainty. Moreover, the entire key observation space is divided into a limited number of disjointed subspaces, such that observations located in the same subspace have almost the same prediction precision for key uncertainty reduction. It is then only necessary to update the reservoir model for each subspace instead of for all distinct sets of observations. Our methods are illustrated by the application of the drilling-order problem in a synthetic field model, for which the drilling sequence of wells is an important contributor to the reservoir's profitability and for which the optimal solution changes significantly with key reservoir features. Results show that using such a simplified VOI analysis based on key actions and key observations can efficiently improve the expected outcome of an optimal strategy with very little performance loss. Although the key actions provide important information for key uncertainty reduction, taking key action rather than the initial optimal decision for the current uncertainty state is not always worthwhile even if the information is obtained without explicit cost. Since there may be an indirect cost of information caused by taking an action that appears to be sub-optimal based on past information, it is necessary to consider both the possibility of key uncertainty reduction and the possibility of high expected NPV to determine whether it is worth taking the action to improve future decisions.

1. Introduction

Almost all published reservoir management or field development optimization studies have as a goal, the generation of a sequence of actions that is optimal for the current level of knowledge. There is an implicit assumption that the sequence that is delivered should be adhered to, whatever the results of the drilling or the control settings. These strategies, would in fact be optimal if there was no opportunity to later make revisions. In reality, one would, of course, modify the drilling schedule or the operation of wells as soon as one obtained new information that revealed a different picture of the reservoir.

How should we account for the possibility of learning from actions

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when optimizing field development for expected net present value (NPV)? To account for future learning requires computation of the value of information (VOI), as it may be advantageous to "pay" for information by making a decision that appears to be sub-optimal for the current assessment of uncertainty in reservoir characterization. If the value of the information obtained by taking an action is greater than the loss of expected NPV, then it is beneficial to take the action.

Unfortunately, while the need to account for the sequential nature of the field development problem is well known, it has generally been ignored in reservoir optimization (Jansen et al., 2005; Sarma et al., 2006; Wang et al., 2009; Chen et al., 2009). The key challenge is that in order to rigorously compute the value of information one must consider all possible values of data that might be obtained from an action then solve a history matching problem with uncertainty assessment for each possible outcome of the data (Barros et al., 2015a, 2016, 2020; Hong et al., 2018). Then optimization must be performed to determine what action should be taken and a value assigned to all possible outcomes. Although several approaches have been proposed to estimate the value of information (Godaet al., 2017; Chen et al., 2017; Eidsvik et al., 2017; He et al., 2018; Barros et al., 2020) for problems in which production flow data must be assimilated, the cost of the combined history matching and optimization is prohibitive for realistic problems. Hence most applications that have considered future learning have had very few decision options (for example drill or not to drill a well) or to problems in which the data assimilation is extremely easy and there are few possible data. Even in those cases however the optimization applications or the data simulation were relatively simple (Cunningham and Begg, 2008; Barros et al., 2015b; Hong et al., 2018).

The problem of robust optimization, taking into account the possibility of uncertainty reduction through the acquisition of data, is closely related to the concept of value of information (Schlaifer, 1959; Grayson, 1960; Bratvold et al., 2009). The application to closed-loop reservoir management (CLRM) is of particular interest. Barros et al. (2016) compute the value of information obtained from an optimal CLRM strategy with traditional production observations. The information is then used to re-estimate uncertainty and re-optimize the controls. They showed that it was possible to compute the value of CLRM in a rigorous, but highly expensive, way, but they did not use the value of information to modify the optimal controls. In an application to the optimization of drilling order, Hanea et al. (2019) also investigated the value of information but, like Barros et al. (2016), did not use the value of information to improve the expected outcome of optimization. Torrado et al. (2017) applied partially observable Monte-Carlo planning algorithm to optimize the drilling schedule considering future uncertainty reduction based on observations through an entire drilling sequence. Their approach is similar to VOI analysis while potentially evaluating only the strategies with high expected values, and the posterior probabilities of uncertainty are estimated by sampling deterministic realizations at given previous observations instead of through a history matching process. Even so, in the case with only two possible observations from each well, many expensive simulations were still needed to compute the optimal solution, since drilling sequences with all distinct sequential observations had to be evaluated and the number of possible combinations was large. A more general application, in which the value of future information was used to optimize bottom-hole pressure controls on wells in a single inverted 5-spot pattern, has been described by Barros et al. (2020). The procedure was shown to increase the expected value of the field although, as in other applications, the computational cost appears to make the method impractical without substantial modification.

In this paper, we consider a realistic problem in which there are many possible decisions at each step, and many possible data, which are determined by the decisions, and but we make the computation manageable by identifying key information that would help in making optimal decisions and key actions that would result in obtaining that information. Through VOI analysis, we aim to obtain a more robust decision considering the opportunities to improve optimal strategy resulting from future uncertainty reduction. Not all decision alternatives, however, may be able to provide information for making better future decisions. Instead of considering the effects of future information from all possible decisions, an efficient and effective way to account for the possibility of future learning is only taking into account the important information from key actions for characterizing key reservoir features for optimization problems. In this way, a standard VOI analysis with extensive form can be simplified with very little performance loss based on key uncertainty with the largest influence on optimal strategy and key observations for improving future decisions. Moreover, the entire key observation space can be divided into a limited number of disjoint subspaces, i.e., observations located in the same subspace have almost the same prediction precision for key uncertainty reduction. In that case, it is only necessary to update the reservoir model for each observation subspace instead of for all distinct sets of observations. Using such a simplified key-feature-based VOI analysis, it is possible to make optimal decisions efficiently considering future learning possibilities. The performance of this approach is illustrated by the application of the drilling-order problem in a synthetic field model. When evaluating the optimal sequence, we neglect the possibility of learning at later times because that information at late time will generally have smaller effect on the optimization of the first few steps in the sequence.

By identifying key uncertainty for the optimization problems, we can identify key actions that would provide the most valuable future information for improving optimal decisions. To efficiently identify key observations, we build supervised-learning algorithms that are able to capture the mapping between observations and key reservoir features to automatically detect the optimal combination of observations and simultaneously evaluate the information's reliability for each observation subspace. This allows the direct computation of posterior probability of key uncertainty using Bayes' rule, avoiding the need for full history matching to re-estimate the uncertainty. Note that here we are dealing with information content of hypothetical data – data that might be obtained after drilling a well. The actual data that is obtained will be different because the rates schedule will be different, and the wells may be controlled by tubing head pressure (THP) instead of bottom-hole pressure (BHP), etc. When the actual data are obtained, it is feasible to perform an actual history match and update the model, because only one set of data needs to be history matched in that case.

Hong et al. (2018) carefully articulated the concept of VOI from the perspective of decision analysis, and demonstrated the value of obtaining saturation information in a 2D waterflooded reservoir for design of a polymer flood. They conclude, however, that VOI analysis plays no role for water, oil, and gas production rate data and well BHP data "because the data have already been or will definitely be gathered." In contrast, our interest is in focused on the value of information that can be obtained from production data, as the actions that we take in the field control the type of information that is obtained and the timing of the acquisition. Although the information obtained from production data may be obtained without explicit cost, it may have a hidden cost if obtaining it requires one to operate a field sub-optimally for the current uncertainty. An obvious case is the running of a pressure shut-in test to obtain an estimate of reservoir pressure or wellbore skin. If the well is already equipped with a downhole gauge, the cost of the information is largely due to deferred production and the information content from the data is not due only to the fact that pressures are recorded, but also to the fact that the control setting has been altered. In our drilling sequence problem, the timing of information acquisition is at the control of the operator and the "cost" is the loss of expected NPV incurred by drilling the wells in a sub-optimal sequence. To determine whether it is worth taking key action earlier in sequence to obtain the information for improving future decisions, we must evaluate the *net* expected value of information with this indirect cost that is associated with changing optimal decisions for current uncertainty state to key action.

This paper is organized as follows. Section 2 introduces the robust decision-making problem under uncertainty and the technologies we use for solving this problem, including key-feature-based VOI analysis for considering future learning possibilities, the supervised-learning algorithm for identifying key observations, the learned heuristic search method for optimizing sequence of discrete actions (Wang and Oliver, 2019) and bias-correction methods for estimating the expected NPV (Wang and Oliver, 2020). Section 3 presents the numerical results of the drilling-order problem in a synthetic model. In this section, we investigate the effects of various geological features on the optimal drilling sequence, the reliability of the key observations identified using supervised learning models with regard to key uncertainty reduction, and VOI analysis performances through key actions with different initial probabilities of key uncertainty. Finally, the conclusions of this study are provided in Section 4.

2. Methodology

2.1. Robust decision making under uncertainty

The general purpose of robust field development optimization is to identify an optimal strategy that maximizes the expectation of an objective function (e.g., expected NPV) in an uncertain reservoir model. Geological uncertainty frequently results in large uncertainty in reservoir performance, but that uncertainty can be reduced using observations obtained from past decisions through history matching or data assimilation. In traditional CLRM, the model is updated and uncertainty is re-estimated based on past observations before making the next decision and the optimal decision for each decision stage is obtained by performing a re-optimization in the currently updated reservoir model. In other words, the optimal decision is typically determined by maximizing the expected NPV over the *current assessment of uncertainty*. The decision we make at the current time will also influence the possibility of obtaining information that might reduce the reservoir uncertainty and improve the optimized strategy. Therefore, the true optimal action for each decision step depends on both the past decisions and the consequences on future uncertainty reduction. In this section, we use the drilling-order problem (i.e., maximize the expected NPV by optimizing the drilling sequence of wells) as an example to demonstrate the optimal decisions obtained with different concerns.

Suppose that we need to optimize the drilling schedule of N_w wells and each drilled well results in observations that can be used to re-estimate uncertainty before choosing the next well to be drilled. Hence, after drilling each new well, the reservoir model is updated based on previously obtained observations, before optimizing the next decision. Fig. 1 shows an example path generated by an ordered sequence of N_w drilling actions with sequential observations from all wells. The set of actions $a_1, a_2, \dots, a_{N_w}$ represents the sequence of N_w wells drilled at time $t_0, t_1, \dots, t_{N_w-1}$. The sequence $o_1, o_2, \dots, o_{N_w}$ denotes the observations obtained from the drilling of each well. We have assumed that these observations are immediately available for updating the reservoir model. The state $s_0, s_1, \dots, s_{N_w}$ denotes the specific environments at each decision step constrained to the past decisions and corresponding observations. Uncertainty at each environment state s_j is re-estimated based on

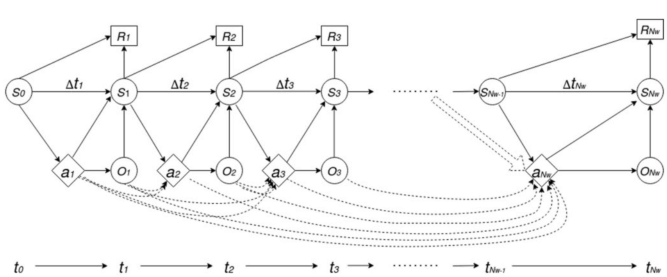


Fig. 1. Example of an observation-based dynamic drilling-sequence planning with N_w wells.

the observations $o_1, o_2, \dots, o_j$ from the previously j drilled wells. The expected NPV is the cumulative reward consisting of the sum of rewards $R_1, R_2, \cdots, R_{N_w}$ over the time periods $\Delta t_1, \Delta t_2, \cdots, \Delta t_{N_w}$. As illustrated in Fig. 1, the previous and the current decisions affect both the possibility of the future choices of actions and the possibility of future observations. Therefore, the robust optimal decision at each decision step should be determined by considering the possibility of achieving high expected NPV, and the opportunity to improve the optimal strategy based on the future uncertainty state, namely, the future learning possibilities.

2.1.1. Optimization ignoring future learning possibilities

In most applications of CLRM optimization, to reduce the complexity of the sequential decision problem under uncertainty, the effect of future uncertainty reduction on the optimal strategy is ignored when making decisions. In other words, the optimal action for each decision step is computed by performing a re-optimization in the updated reservoir model based on the current assessment of uncertainty, without considering the consequence of this decision on the future uncertainty state (e.g., Jansen et al., 2005). In that case, after completing the drilling of j wells, the next best action a^*_{j+1} is drilling the well that leads to the maximum expected NPV of complete drilling sequences over the current uncertainty state u_j based on the observations in history h_j, i.e.,

$$a^*_{j+1} = \underset{a_{j+1}\in A_{j+1}(h_j)}{\arg\max}\ \mathrm{EV}^*\left(h_j, a_{j+1}, u_j\right), \tag{1}$$

where h_j is an observable history consisting of a sequence of selected actions (i.e., j drilled wells) and observation pairs, $h_j = (a_1, o_1, \dots, a_j, o_j)$, where observation o_j obtained from each past action a_j might be a single datum (e.g., types of facies) or a collection of data (e.g., production data of various types over a time interval); $A_{j+1}(h_j)$ is the current action space at a given history h_j, which consists of the $(N_w - j)$ remaining wells; u_j is the current assessment of uncertainty based on the past observations from j drilled wells in history h_j; $\mathrm{EV}^*(h_j, a_{j+1}, u_j)$ is the maximum expected NPV for complete drilling sequences over the uncertainty state u_j constrained to history h_j followed by taking a_{j+1} as the next decision. Note that in this approach any possible future information from the remaining $N_w - j$ actions is not considered, including the observation o_{j+1} from the current decision alternatives, a_{j+1}. Thus, the EV^* in Eq. (1) is evaluated over the uncertainty state u_j instead of u_{j+1}. To compute the optimal decision a^*_{j+1}, learned heuristic search (Wang and Oliver, 2019) is an efficient approach, which allows for optimizing either only the first few decisions or a complete strategy. The key advantage of this approach is that an approximation of the maximum expected value EV^* constrained to the past decisions can be accurately estimated without finding the entire optimal strategy.

2.1.2. Fully structured robust decision making

As discussed in the previous section, when selecting an action that will increase the expected NPV, we should also take into account the possibility of future uncertainty reduction, rather than basing our decision solely on the maximization of expected NPV over current uncertainty. The optimal choice of the next well after sequentially drilling j wells should therefore be based on the expected value over all possible observations from all remaining wells (assuming no explicit cost for collecting information from each drilled well),

$$a^{*^{\mathrm{fs}}}_{j+1} = \underset{a_{j+1}\in A_{j+1}(h_j)}{\arg\max} \sum_{o\in O_{a_{j+1}}} p\left(o \middle| h_j, a_{j+1}\right) Q^*_{N_w-(j+1)}\left(h_{j+1}\right), \tag{2}$$

where $O_{a_{j+1}}$ is the observation space obtained from a_{j+1}; $p(o|h_j, a_{j+1})$ are the marginal probabilities of distinct observations; and $Q^*_{N_w-j}(h_{j+1})$ is the optimal expected value considering all possible future observations constrained to history h_{j+1} including the observation from a_{j+1}. The optimal expected value can be calculated in a backward induction procedure, i.e.,

$$Q^*_{N_w-(j+k)}\left(h_{j+k}\right) = \max_{\substack{a_{j+k+1}\in A_{j+k+1}\left(h_{j+k}\right)}} \sum_{o\in O_{a_{j+k+1}}} p\left(o\middle|h_{j+k},a_{j+k+1}\right)Q^*_{N_w-(j+k+1)}\left(h_{j+k+1}\right), \text{for} \quad k=1,2,\ldots,N_w-(j+1), \tag{3}$$

where $Q^*_0(h_{N_w})$ is the expected NPV over the final uncertainty state updated using all sequential observations from a complete drilling sequence in history h_{N_w}. Using backward induction to solve the optimization problem in Eq. (2) is also known as the standard VOI decision analysis process with extensive form (Neumann and Morgenstern, 1944; Raiffa and Schlaifer, 1961). This approach is a fully structured decision tree that considers all possible combinations of the sequences of remaining actions with distinct sequential observations (Hong et al., 2018).

As a simple illustration, suppose that N_w -2 wells have been drilled sequentially resulting in history h_{N_w-2} and the optimal next well is chosen from the two remaining wells W_a, W_b. Each well can provide two possible distinct observations o_{f_1}, o_{f_2} about the type of facies. Fig. 2 shows a simple example of determining the optimal next well from W_a and W_b through the backward induction procedure. In this case, determining the optimal action considering the future information from the two remaining wells, requires consideration of 8 possible combinations of sequences with distinct observations. Because the number of options is small, the optimal action $a^{*^{fs}}_{j+1}$ based on the expected values over all possible future observations (Eqs. (2) and (3)), is easily determined to drill W_a as the next well. However, as this optimization requires information about the expected NPV from all possible combinations of sequences with observations and about the marginal probabilities of all possible observations from remaining actions, the size of the decision tree is exponential in the number of distinct states related to both the action space and the observation space obtained from each action. Consequently, if there were 8 possible remaining wells while each well provides only two distinct observations, then there would be $8!\times 2^8 \approx 1\times 10^7$ possible combinations of drilling sequences with distinct sequential observations. The use of such a fully structured decision tree will be computationally intractable even before taking into account the cost of updating the reservoir model.

Although it is possible to approximately solve Eq. (2) by formulating the problem as a partially observable Markov decision process (POMDP) (Åström, 1965; Sondik, 1971), the cost of solving a POMDP can be prohibitive for reservoir applications (Torrado et al., 2017), since the evaluations of the expected values require many expensive simulations and the number of the states that need to be evaluated in a POMDP can be large, especially when many various combinations are likely to generate high expected values. Hence, computing the optimal decision $a^{*^{fs}}_{j+1}$ that considers all possible future observations is only applicable to reservoir simulation-based problems with small numbers of distinct actions and distinct observations in practice.

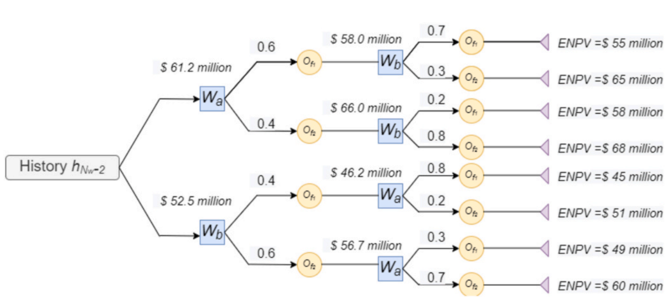


Fig. 2. A fully structured decision tree for determining the order of two remaining wells in consideration of all possible future observations.

2.1.3. Accounting for future learning through the current decision

Instead of using a fully structured decision tree, a more feasible way to obtain an optimal decision that considers the future learning possibilities is to take into account the effects of future observations resulting from only the current decision step,

$$a^{*^{fl}}_{j+1} = \underset{a_{j+1}\in A_{j+1}\left(h_j\right)}{\arg\max} \sum_{o\in O_{a_{j+1}}} p\left(o\middle|h_j,a_{j+1}\right)\mathrm{EV}^*\left(h_j,a_{j+1},u^o_{j+1}\right), \tag{4}$$

where $\mathrm{EV}^*(h_j,a_{j+1},u^o_{j+1})$ is the maximum achievable expected NPV constrained to the previous actions in history h_j and the current decision alternative a_{j+1}. This expectation is evaluated over the uncertainty state u^o_{j+1} updated based on the future possible observation o from a_{j+1}. Note that $\mathrm{EV}^*(h_j,a_{j+1},u^o_{j+1})$ is different from the expected value $Q^*_{N_w-(j+1)}(h_{j+1})$ in Eq. (2) which accounted for future information from all remaining decisions.

In the terminology of VOI, $a^{*^{fl}}_{j+1}$ (Eq. (4)) is the optimal decision based on the expected value with additional information (EVWI) through one decision point (Hong et al., 2018),

$$a^{*^{fl}}_{j+1} = \underset{a_{j+1}\in A_{j+1}\left(h_j\right)}{\arg\max}\ \mathrm{EVWI}_{a_{j+1}}, \mathrm{EVWI}_{a_{j+1}} = \sum_{o\in O_{a_{j+1}}} p\left(o\middle|h_j,a_{j+1}\right)\mathrm{EV}^*\left(h_j,a_{j+1},u^o_{j+1}\right), \tag{5}$$

while a^*_{j+1} (Eq. (1)), that ignores the effects of all possible future observations, is the optimal decision determined by the expected value without additional information (EVWOI),

$$a^*_{j+1} = \underset{a_{j+1}\in A_{j+1}\left(h_j\right)}{\arg\max}\ \mathrm{EVWOI}_{a_{j+1}}, \mathrm{EVWOI}_{a_{j+1}} = \mathrm{EV}^*\left(h_j,a_{j+1},u_j\right). \tag{6}$$

We assume that there is no cost for acquiring information from $a_{j+1}\in A_{j+1}(h_j)$. Because $a^{*^{fl}}_{j+1}$ is obtained considering the possibility of future learning before committing to a decision, $a^{*^{fl}}_{j+1}$ generally is a more robust decision than a^*_{j+1}, which ignores the effect of all future information.

Figs. 3 and 4 show the VOI decision trees from the example of two remaining wells with and without considering the effect of future observations from current decision alternatives. As illustrated in Fig. 4, the optimal choice for the next well that ignores the future learning possibilities is W_b, which is obtained by maximizing the expected NPV over the current assessment of uncertainty (Eq. (6)). When the effect of future possible observations is considered (Fig. 3), the optimal next well is W_a, which has a higher EVWI and is a more robust decision. For a large problem with many decision alternatives, although the optimal decision $a^{*^{fl}}_{j+1}$ that is obtained from a simplified VOI decision tree might not be identical to $a^{*^{fs}}_{j+1}$ that is obtained from a standard VOI analysis with extensive form, the cost of computing $a^{*^{fl}}_{j+1}$ is much lower than that in $a^{*^{fs}}_{j+1}$. In general, the

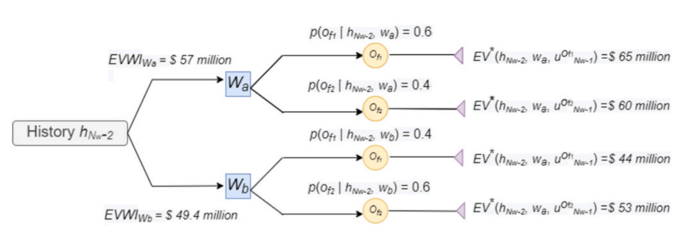


Fig. 3. VOI analysis considering the future information obtained from current decision alternatives.

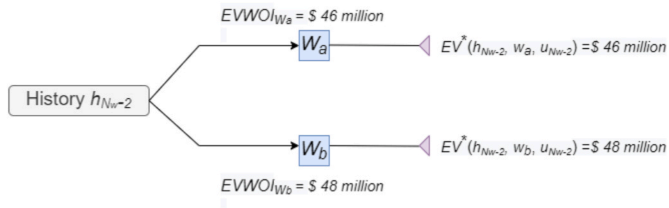


Fig. 4. A simple example of the decision tree ignoring the future learning possibilities.

information obtained from the later decision stages has a smaller impact on improving the optimal strategy. We expect that simplifying the VOI analysis by only considering the information from the current decision step would not incur much performance loss, i.e., $a^{*\mathrm{fl}}_{j+1}$ is expected to be an approximation solution close to the optimal decision $a^{*\mathrm{fs}}_{j+1}$. In this work, we focus on how to efficiently improve the optimal decision a^{*}_{j+1} to $a^{*\mathrm{fl}}_{j+1}$.

Although $a^{*\mathrm{fl}}_{j+1}$ does not require a fully structured decision tree, directly solving Eq. (5) is still prohibitive in most reservoir applications for which the costs of history matching and optimization are large. If there are N_d decision alternatives and N_o distinct observations from each decision, it would be necessary to update the reservoir model and perform the optimization $N_d \times N_o$ times to obtain the optimal decision $a^{*\mathrm{fl}}_{j+1}$. Hence, it is desirable to make the computation manageable and design a more practical way to compute the optimal decision $a^{*\mathrm{fl}}_{j+1}$ in consideration of the possibilities of future learning.

In general, we might expect that gained information will reduce the uncertainty, thereby leading to better future decisions. Some decisions, however, may result in little information or information that is irrelevant to the optimization of the objective. In that case, accounting for the possibility of future uncertainty reduction will only increase the cost of making decisions, while the optimal decision may not be changed (i.e., the optimal strategies for maximizing the expected NPV over uncertainty state u^{o}_{j+1} and u_j are the same). Hence, because the computational cost of considering many possible observations is high, it is more important to account for the future information that is most likely to improve decisions than to consider as much information as possible from remaining actions. In this work, we use a simplified VOI decision analysis to efficiently account for the possibility of future learning when choosing actions, in which only the key information that would have a large influence on the optimal decisions is taken into account.

2.2. Planning for future learning

2.2.1. Learning through key action

Due to limited observations of the reservoir, the properties of the subsurface (e.g., porosity, permeability, fluid contact locations, fault transmissibilities) may be highly uncertain. Uncertainty in some properties may have little effect on the optimal decisions and information on those nonessential properties would not be beneficial for optimally managing the reservoir, even if the uncertainties could be reduced significantly. Hence, when evaluating the desirability of performing an action to learn about the reservoir, we can focus our attention on obtaining information from a few key actions that can be used to reduce key uncertainty that have large impact on optimal decisions. Then, the optimal decision can be made based on the trade-off between the key action that would provide the most important information to reduce key uncertainty and the action that would achieve the maximum expected value over current uncertainties.

Fig. 5 shows a feasible workflow that efficiently accounts for the possibility of future learning of key information through key actions that would be most helpful for improving future decisions. By identifying key uncertainty for the optimization problem, we can identify the key information-gathering action that would provide the most important observations for reducing key uncertainty, potentially leading to better future decisions. The key action could provide a large number of observations from various available information sources. To avoid the cost of formal history matching, we select observations for which the connection to uncertainty reduction in key reservoir features is straightforward. In this work, we build supervised-learning models to identify the optimal combination of observations for key uncertainty reduction and simultaneously evaluate the reliability of information. Then, the probability of key uncertainty with the given observations can be computed directly using Bayes' rule instead of using data assimilation algorithms, which typically require many expensive simulations to obtain estimates of the posterior probability distribution. Hence, the workflow is applicable for reducing key uncertainty for optimization problem without the requirement of an expensive history matching process to update the reservoir model.

Although information obtained from a key action is most likely to improve future decisions, taking a key action to acquire important future observations is not necessarily worthwhile, even if these observations may be obtained without an explicit cost. In some cases, the optimal strategy obtained without acquiring future information (i.e., the optimal strategy for current assessment of uncertainty) may have a higher expected value than that of the optimal strategy considering the possibility of future learning through key action (i.e., the optimal strategy with additional information from key action). The cause of this situation is that there is a hidden cost when taking the key action would lead to a sub-optimal solution, so that value gained by using additional information from key action may not be able to compensate for this hidden cost. To determine whether it is worth taking the key action, we need to assess an implicit *net* expected value of information associated with a change in decisions (i.e., change action obtained from the optimal strategy for the current assessment of uncertainties to key action), which is the difference between the expected value with future information from key action and the expected value without any future information.

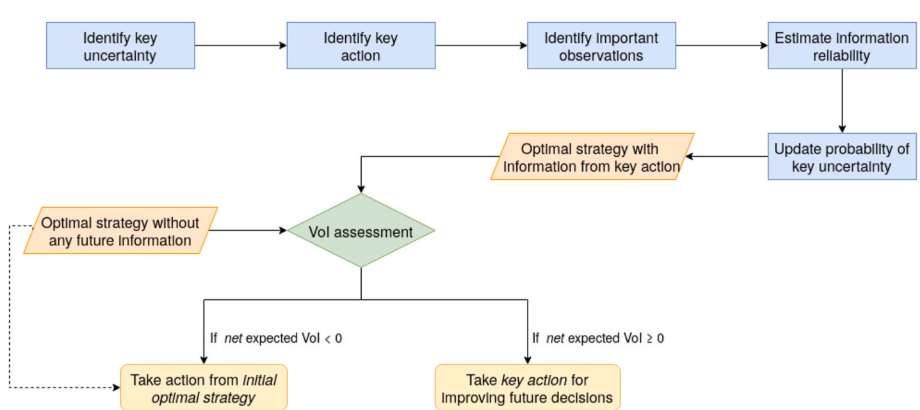


Fig. 5. Decision making while considering future learning through key action and key information.

2.2.2. Value-of-information analysis through key action

Instead of computing the actual $\mathrm{EVWI}_{a_{j+1}}$ for all possible decision alternatives $a_{j+1} \in A_{j+1}(h_j)$ (Eq. (5)) to obtain an optimal decision that considers future learning possibilities, we simplify the VOI decision tree to only two decision alternatives, i.e., a^*_{j+1} and a^{key}_{j+1}, by considering only future information from the key action (Fig. 6). a^*_{j+1} is the decision obtained from the optimal strategy for the current assessment of uncertainties (Eq. (1)), while a^{key}_{j+1} is the key action that would provide important information for improving future decisions. In other words, a^{key}_{j+1} is the decision alternative from the current action space $A_{j+1}(h_j)$ that is expected to result in a high *net* EVOI associated with changing decision a^*_{j+1} to a^{key}_{j+1} defined as

$$\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}} = \mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} - \mathrm{EV}^*\left(h_j, a^*_{j+1}, u_j\right), \tag{7}$$

where $\mathrm{EV}^*(h_j, a^*_{j+1}, u_j)$ is the expected value from the optimal strategy for current uncertainty without any future information (Eq. (1) and Eq. (6)) and $\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}}$ is the expected value with additional information from a^{key}_{j+1}, which could be computed in the following way,

$$\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} = \sum_{o\in O_{a^{\mathrm{key}}_{j+1}}} p\left(o\middle|h_j, a^{\mathrm{key}}_{j+1}\right) \times \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u^o_{j+1}\right), \tag{8}$$

where $p(o\,|\,h_j, a^{\mathrm{key}}_{j+1})$ is the marginal probability of a specific observation; $O_{a^{\mathrm{key}}_{j+1}}$ is set of all possible distinct observations from a^{key}_{j+1}; $\mathrm{EV}^*(h_j, a^{\mathrm{key}}_{j+1}, s^{o_i}_{j+1})$ is the expected value from the optimal strategy for uncertainty state u^o_{j+1} updated with observations from a^{key}_{j+1}.

As shown in Eq. (7), performing the VOI analysis through a^{key}_{j+1} is similar to a simplified VOI analysis with the decision alternative $a_{j+1} \in A_{j+1}(h_j)$ that has a high $\mathrm{EVWI}_{a_{j+1}}$. If a^{key}_{j+1} has the maximum $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$, a^{key}_{j+1} would be identical to the optimal decision $a^{*\mathrm{fl}}_{j+1}$ from Eq. (5). In that case, simplifying the VOI decision tree to a^{key}_{j+1} would not incur performance loss compared to directly solving Eq. (5). The main advantage of using $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ instead of $\mathrm{EVWI}_{a_{j+1}}$ is that a^{key}_{j+1} with a high $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ can be identified without comparing the actual $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ of all possible decisions.

The $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ from changing decision a^*_{j+1} to a^{key}_{j+1} (Eq. (7)) can be rewritten as

$$\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}} = \left(\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} - \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u_j\right)\right) - \left(\mathrm{EV}^*\left(h_j, a^*_{j+1}, u_j\right) - \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u_j\right)\right), \tag{9}$$

where the first group of terms is the standard definition of EVOI for a^{key}_{j+1}, i.e., it is the difference in the expected values with and without additional information from a^{key}_{j+1},

$$\mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}} = \mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} - \mathrm{EVWOI}_{a^{\mathrm{key}}_{j+1}} = \mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} - \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u_j\right). \tag{10}$$

The second group of terms in Eq. (9) is the expected cost of information (ECOI), or the hidden cost, caused by a sub-optimal solution constrained to a^{key}_{j+1} chosen as the next decision using the current assessment of uncertainty,

$$\mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}} = \mathrm{EV}^*\left(h_j, a^*_{j+1}, u_j\right) - \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u_j\right). \tag{11}$$

Thus, $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ actually is an implicit *net* $\mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}}$ accounting for the hidden cost $\mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$,

$$\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}} = \mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}} - \mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}. \tag{12}$$

If a^{key}_{j+1} is identical to a^*_{j+1}, there is no hidden cost, i.e., $\mathrm{ECOI}_{a^{\mathrm{key}}_{j+1}\to a^{\mathrm{key}}_{j+1}} = 0$. In general, however, a^{key}_{j+1} will not be the same as the optimal decision a^*_{j+1} for the current uncertainty state, especially when the decision space is large.

According to Eq. (12), a^{key}_{j+1} is the decision alternative that is expected to result a large $\mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}}$ but a small $\mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$. The standard $\mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}}$ (Eq. (10)) depends on whether a^{key}_{j+1} is able to provide useful information for making better future decisions, i.e., important information for key uncertainty reduction. The hidden cost $\mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ depends on whether the optimal strategies constrained to a^{key}_{j+1} can achieve a high expected NPV over current uncertainty state, which is possible to be evaluated when computing the optimal decision a^*_{j+1} that ignores the effects of future possible observations. Therefore, by considering the possibility of obtaining valuable information for reducing key uncertainty and the possibility of achieving a high expected NPV for current assessment of uncertainty, we can identity the a^{key}_{j+1} that is likely to result in a high $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$; that is the decision alternative $a_{j+1} \in A_{j+1}(h_j)$ with a high $\mathrm{EVWI}_{a_{j+1}}$.

To judge whether it is preferable to take action a^{key}_{j+1} over a^*_{j+1}, we need to assess $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ before committing to a decision, noting that $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ may be negative value to the hidden cost. If $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}} > 0$, the value gained by using the information from a^{key}_{j+1} can compensate for the hidden cost caused by a sub-optimal solution, i.e., $\mathrm{EVOI}_{a^{\mathrm{key}}_{j+1}} > \mathrm{ECOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$. It is then worth taking a^{key}_{j+1} to acquire the information that will help in improving future decisions.

The simplified VOI analysis based on $\mathrm{EVOI}_{a^*_{j+1}\to a^{\mathrm{key}}_{j+1}}$ does not take in account the future learning possibilities through a^*_{j+1} (Fig. 6). In some cases, a^*_{j+1} may also be able to provide important future information for reducing key uncertainty. To obtain a more robust decision, we can take into account the possibilities of future learning through both a^{key}_{j+1} and a^*_{j+1},

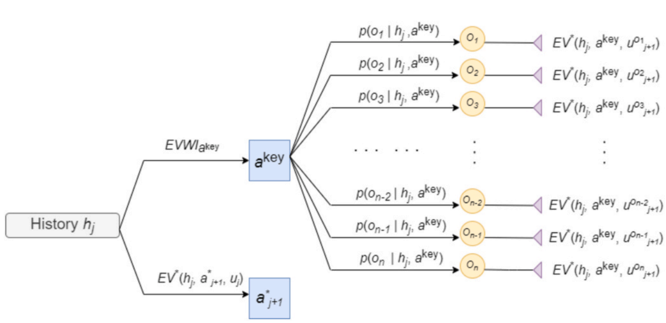


Fig. 6. Simplified VOI decision tree with future information only from key action.

$$\widehat{a}_{j+1}^{*\mathrm{fl}} = \arg\max_{a\in\left[a^*_{j+1},a^{\mathrm{key}}_{j+1}\right]} \mathrm{EVWI}_a = \arg\max_{a\in\left[a^*_{j+1},a^{\mathrm{key}}_{j+1}\right]} \sum_{o\in O_a} p\left(o\middle|h_j,a\right) \times \mathrm{EV}^*\left(h_j,a,u^o_{j+1}\right). \tag{13}$$

However, this approach will increase the computational cost of making a decision since the evaluation of EVWI for each action is based on the maximum expected values, $\mathrm{EV}^*(h_j,\, a,\, u^o_{j+1})$, corresponding to various observations. This requires re-optimization multiple times to obtain all expected values. Consequently, directly solving Eq. (7) or Eq. (13) is likely to be impractical when all distinct observations are accounted for, although the number of decision alternatives in VOI analysis is reduced by identifying a^{key}_{j+1}. To the computation of EVWI manageable, we will present a methodology in the following section for efficiently estimating EVWI by using key observations to reduce key uncertainty, rather than using all observations to reduce all uncertainties.

2.2.3. Key observation selection

In ensemble-based methods, a set of N_e model realizations is used to represent the uncertainties in reservoir properties. To reduce the effects of sampling error, N_e is typically chosen to be on the order of 100. Each model realization is capable of generating a specific set of simulated observations obtained by taking action a^{key}_{j+1}, e.g., production data over a certain period, in which case there would be N_e distinct sets of observations – one set from each ensemble member. If each set of observations is used in the estimation of $\mathrm{EVWI}_{a^{\mathrm{key}}}$, the computational cost will be high since both history matching and optimization have to be performed N_e times to obtain the maximum expected values from all posterior ensembles. Hence, it is generally infeasible to consider all N_e distinct realizations of future outcomes in standard VOI analysis with history matching for making a decision, especially when the observation space or decision space is large. Moreover, when all observations obtained from a^{key}_{j+1} are used to simultaneously re-estimate uncertainty, the largest decrease in uncertainty may be in properties that are irrelevant to current decisions. All data will, of course, be used to update model uncertainty after an action has been taken.

As mentioned previously, the additional value with information is achieved by the reduction in uncertainty of model parameters that will affect the optimal decisions, rather than all reductions in uncertainties in model parameters. Instead of using all observations to reduce all uncertainties, we can approximately compute $\mathrm{EVWI}_{a^{\mathrm{key}}}$ by reducing key uncertainties of the optimization problem based on key observations that are most helpful in exploring key reservoir properties. Such an approximation of $\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}}$ can be used to indicate the importance of a^{key}_{j+1}. Because VOI analysis is performed by ranking the importance of decision alternatives based on the expected values and $\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}}$ deals with the information content of hypothetical data, we expect (without proof) that using the $\mathrm{EWVI}_{a^{\mathrm{key}}_{j+1}}$ computed based on key information would not incur performance loss in VOI analysis and the optimization framework. When the actual observations are obtained from an action that has been executed, an actual history match will be performed with all observations to update various uncertainties in reservoir properties.

Performing the action a^{key}_{j+1} can provide a large number of observations, but the reduction in key uncertainties from some observations may be very small. Accounting for all information in the VOI, including those nonessential observations, will increase the computational effort associated with updating the reservoir model. Hence, we would like to use a reduced set of important observations to update key uncertainties, i.e., key information is defined to be a subset of observations that are most helpful in reducing key uncertainties. Instead of updating key uncertainties for each possible outcome of key observations obtained from an individual realization, we divide the entire observation space R^n_b associated with the best subset, B, into a limited number of disjoint subspaces (e.g., $R^n_b = \Omega^b_1 \cup \Omega^b_2$ and $\Omega^b_1 \cap \Omega^b_2 = \varnothing$). Suppose that observations located in the same subspace have almost the same prediction precision to reduce key uncertainties, then the posterior probability distributions of key uncertainties conditioned on observations in the same subspace would be similar. In that case, there is no need to compute N_e posterior ensembles considering all distinct sets of observations from individual realizations. $\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}}$ could be efficiently evaluated by performing the optimization process only in a few posterior ensembles associated with the observation subspaces Ω^b_k,

$$\mathrm{EVWI}_{a^{\mathrm{key}}_{j+1}} = \sum_{k=1}^{N_{\Omega^b}} p\left(o^b \in \Omega^b_k \middle| h_j, a^{\mathrm{key}}_{j+1}\right) \times \mathrm{EV}^*\left(h_j, a^{\mathrm{key}}_{j+1}, u^{\Omega^b_k}_{j+1}\right), \tag{14}$$

where N_{Ω^b} is the number of observation subspaces, which is much smaller than the ensemble size ($N_{\Omega^b} \le N_e$); $u^{\Omega^b_k}_{j+1}$ is the updated uncertainty state for observed values $o^b \in \Omega^b_k$. Fig. 7 shows an example of VOI analysis considering only the important future observations through key action for key uncertainty reduction, while the entire key observation space is divided into four disjoint subspaces. We refer to such a simplified VOI decision tree as *key-feature-based* VOI analysis. It is performed only through the key action and the key information that have been identified for exploring reservoir features with large influences on the optimal decisions.

To ensure the usefulness of key observations and their subspaces in reducing key uncertainties, the entire key observation space is divided such that each subspace Ω^b_k will have high a probability $P(\Omega^b_k|\Theta^m_k)$ for indicating a specific key uncertainties subregion Θ^m_k, while the probability $P(\Omega^b_k|\Theta^m_i)$ for key uncertainties located in other subregions Θ^m_i is low. Suppose that distribution of key uncertainties is divided into N_{Ω^b} disjoint subregions $\Theta^m = [\Theta^m_1, \Theta^m_2, \ldots, \Theta^m_{N_{\Omega^b}}]$. The best observation space division $\Omega^b = [\Omega^b_1, \Omega^b_2, \ldots, \Omega^b_{N_{\Omega^b}}]$ can then be described as

$$\Omega^b = \arg\max_{\Omega^b} \sum_{k=1}^{N_{\Omega^b}} \left[P\left(\Omega^b_k \middle| \Theta^m_k\right) - \sum^{N_{\Omega^b}} \begin{matrix} i=1 \\ i\neq k \end{matrix} P\left(\Omega^b_k \middle| \Theta^m_i\right) \right]. \tag{15}$$

For key uncertainties with categorical variables, each category can be set as one specific subregion Ω^m_i. For continuous variables, instead of randomly dividing the distribution of key uncertainties into a limited number of subregions, the division of Θ^m can be optimized based on the performance of the corresponding Ω^b,

$$\Theta^m = \arg\max_{\Theta^m} \sum^{N_{\Omega^b}} k = 1 P\left(\Omega^b_k \middle| \Theta^m_k\right), \tag{16}$$

which is a simplification of Eq. (15) since $\sum_{k=1}^{N_{\Omega^b}} P(\Omega^b_k|\Theta^m_i) = 1$ for a specific key uncertainties subregion Θ^m_i.

Based on the prior probability $P(\Theta^m_i)$ of each subregion Θ^m_i and the information's reliability $P(\Omega^b_k|\Theta^m_i)$, the posterior probability $P(\Theta^m_i|\Omega^b_k)$ can be computed using Bayes' rule,

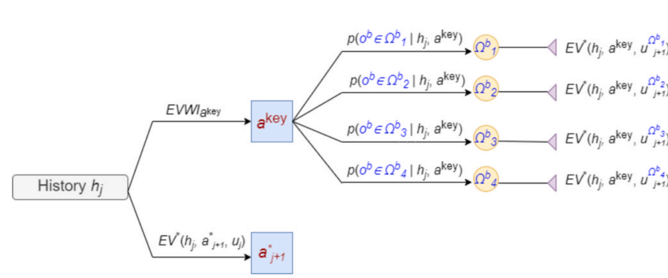


Fig. 7. Key-feature-based VOI analysis obtained by identifying key uncertainties for optimization problems and key observations for reducing key uncertainty.

$$P\left(\Theta_i^m \middle| \Omega_k^b\right) = \frac{P\left(\Omega_k^b \middle| \Theta_i^m\right) \times P\left(\Theta_i^m\right)}{\sum_{i=1}^{N_{\Omega^b}} P\left(\Omega_k^b \middle| \Theta_i^m\right) \times P\left(\Theta_i^m\right)}, \tag{17}$$

where $\sum_{i=1}^{N_{\Omega^b}} P(\Omega_k^b|\Theta_i^m) \times P(\Theta_i^m)$ is the marginal probability of observing $o^b \in \Omega_k^b$ in the prior ensemble.

Approximations applied to solve Eq. (15) would affect the performance of key uncertainty reduction. To ensure the effectiveness of using Ω^b to reduce key uncertainties, an appropriate approach that can effectively identify the observation subspaces Ω^b with high information's reliability $\sum_{k=1}^{N_{\Omega^b}} P(\Omega_k^b|\Theta_k^m)$ is required. In this work, we apply multiple supervised-learning algorithms to identify the optimal observation subset and the corresponding best space division Ω^b. Meanwhile, the reliability of information $P(\Omega_k^b|\Theta_k^m)$ for each subspace can also be estimated when evaluating the learning algorithm's performance with the optimal observation subset. Then, the posterior probability of key uncertainties can be computed using Bayes' theorem (Eq. (17)). Consequently, there is no need to use data assimilation algorithms that update every model parameter for VOI analysis in our workflow. Building supervised-learning models to identify key observations requires a dataset (also called the original dataset) that includes all possible observations and the corresponding distribution of key uncertainties. This dataset can be obtained by collecting relevant information from a number of individual realizations applied with key action.

Fig. 8 shows the process of selecting key observations from the original dataset by using filter and wrapper methods with an inductive learning model that is able to capture the mapping between the inputs (i.e., observations from key action) and the outputs (i.e., key uncertainty). The original dataset may contain hundreds to thousands of observations, which are known as features in learning models. Note that in supervised-learning algorithms, the observations acquired from key action are input variables and are called "features", while the values of key reservoir properties are output variables. A large number of features would make the model more complex and may lead to overfitting due to the curse of dimensionality. To avoid these issues, we first apply filter methods (Lazar et al., 2012) to quickly remove redundant and irrelevant features by ranking the features using some relevance measure, regardless of learning algorithms, obtaining a subset of features. The selected feature subset after filtering is generally not the optimal feature subset for key uncertainty reduction. Thus, a wrapper method (Kohavi and John, 1997) involved in supervised-learning models is used to select the best combination of features that gives the optimal results for learning algorithms, i.e., a feature subset that leads to high prediction accuracy. To avoid overfitting in learning models, we split the original dataset into separate training and test subsets and use resampling methods (e.g., cross-validation) to evaluate learning models' performance with limited data samples. The prediction accuracy (i.e., reliability of information) is estimated from the test error associated with specific learning models.

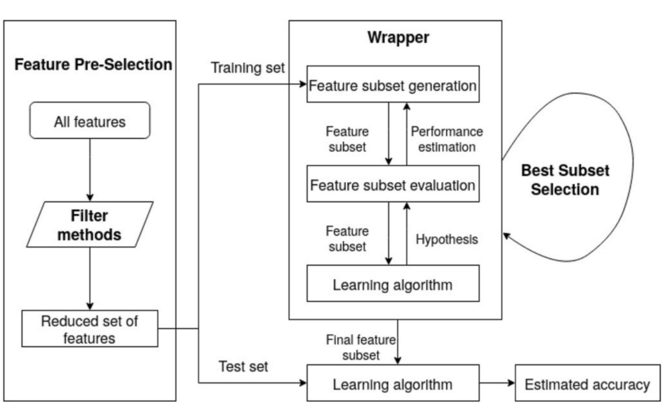


Fig. 8. Key feature selection process based on filter and wrapper methods.

Suppose that N_{sl} individual realizations are applied to generate the dataset for building supervised-learning models. The cost of updating the key uncertainty for all observation subspaces will be N_{sl} simulations. Using multiple supervised-learning algorithms will not increase the simulation cost since the learning models are built using the same samples. If N_{opt} simulations are required for a single robust optimization, it will require $N_{\mathrm{sl}} + N_{\mathrm{opt}} \times (N_\Omega^b + 1)$ simulations to perform the VOI analysis through key action (Eq. (7)) and key information (Eq. (14)), which is much lower than the cost of solving Eq. (4) with an exhaustive history matching and optimization procedure that requires $N_d \times N_e \times (N_{\mathrm{hm}} + N_{\mathrm{opt}})$, where N_d is the number of decision alternatives and N_{hm} is the cost of history matching. Even accounting for future learning possibilities through all N_d decision alternatives, performing VOI analysis through key information still requires many fewer simulations than directly solving Eq. (4). In that case, the cost of VOI analysis through key information is $N'_{\mathrm{sl}} + N_s \times N_\Omega^b \times N_{\mathrm{opt}}$, in which N'_{sl} simulations are performed to investigate the reliability of information from all N_d possible decision alternatives, and N_s is the number of decisions that are identified with reliable information for key uncertainty reduction and small hidden cost caused by sub-optimal solutions, which is generally smaller than N_d.

In this paper, the performance of key-feature-based VOI analysis (Fig. 7) applied with supervised-learning algorithms (Fig. 8) is illustrated by an application of the drilling-order problem in a synthetic model (REEK field). The key uncertainty that has the largest influence on the optimal drilling sequence is whether one fault is completely sealing or not, for which the output variable in learning models is a category. In that case, there is no need to optimize the division of key uncertainty since each subregion Θ_i^m corresponds to a specific category. We first use the Minimum Redundancy Maximum Relevance (mRMR) method (Peng et al., 2005; Estevez et al., 2009; Ramírez-Gallego et al., 2017) to eliminate some less important features, then further reduce the number of features by using the area under the receiver operating characteristic (ROC) curve (Hanley and McNeil, 1982), which measures the classification performance at various thresholds. These two steps are independent of any learning algorithms. To obtain the optimal combination of observations, we investigate four different classification models (i.e., k-Nearest Neighbor, Logistic Regression, Support Vector Machine, and Random Forest). We then use the best-performing learning algorithm to identify the best observation subset and evaluate each subspace's prediction accuracy based on the test dataset.

2.3. Robust optimization of well drilling schedule

In order to perform the optimization efficiently, we require two additional technologies to deal with the search for an optimal sequential solution, and to deal with uncertainty in the reservoir characteristics. Learned heuristic search is an effective and efficient search method for solving the optimization problems with discrete actions (Wang and Oliver, 2019). This approach allows for optimizing only the first few actions by limiting the search depth so that the optimal well for each decision step could be obtained at a reduced cost without finding the entire optimal drilling sequence (Wang and Oliver, 2020). The key point of this method is that an accurate approximation of the maximum achievable expected NPV constrained to previous wells, i.e., $\mathrm{EV}^*(h_j, a_{j+1}, u_j)$ in Eq. (1), can be evaluated by first using a crude heuristic function to estimate the maximized value. The accuracy of the heuristic is then improved by learning the errors of the initial approximate values obtained from previous decision steps. In this way the search direction can be guided toward the optimal solution quickly and effectively. In this paper, we apply two different online-learning techniques (i.e., single-step adjustment and multiple-time-periods learning) to improve the initial imprecise heuristic values, which can be inexpensively obtained by assuming that all remaining wells are drilled simultaneously at the next step and then put on production immediately after completing the drilling of wells.

The second technology is the application of bias-correction methods to the estimate of NPV obtained from the mean reservoir model to efficiently compute a good approximation of the expected NPV over an ensemble of reservoir model realizations (Wang and Oliver, 2020). Although the mean model $\overline{m}$ generally provides a poor estimate of the expected value when the objective function $J(x,m)$ at control x is nonlinear in the uncertain model parameter m, this approximation can be significantly improved by estimating a multiplicative bias correction factor $\alpha(x)$. The estimation only requires information from individual simulations with distinct controls and model realizations, i.e.,

$$E[J(x,m)] \approx \widehat{\alpha}(x) J(x,\overline{m}), \quad \widehat{\alpha}(x) = G(\beta_1, \beta_2, \cdots, \beta_n, x), \tag{18}$$

where $\widehat{\alpha}(x)$ is the estimate of bias correction factor between the ensemble average value and the value obtained from the mean model, i.e., $\widehat{\alpha}(x) \approx \frac{\sum_{j=1}^{N_e} J(x,m_i)}{N_e J(x,\overline{m})}$. G is an estimating function for $\alpha(x)$ based on a set of observations β obtained by applying n randomly sampled controls to individual realizations and the mean reservoir model, where β_j is the partial correction factor at a random control x_j of a random individual realization m_j, i.e., $\beta_j = \frac{J(x_j,m_j)}{J(x_j,\overline{m})}$. When estimating $\alpha(x)$, high weights would be assigned to the observed values of β from similar controls because they are expected to provide more useful information. Using such a bias-correction method, robust optimization requires additional simulations only from the mean model during the optimization process, in which case the robust optimal solution could be obtained at a much lower cost compared to using the ensemble average of simulation results to estimate the expected value.

There are three different ways to estimate $\alpha(x)$: distance-based localization, regularized localization, and optimal weights based on the covariance of correction factors. In this work, we estimate the expected NPV by applying distance-based localization to correct the bias in $J(x,\overline{m})$ since the other two methods require additional information such as the variance of the bias correction factor, which is generally unknown. To efficiently identify similar drilling sequences, a well-position based distance metric is used to measure the similarity of drilling sequences in terms of the bias correction factor.

3. Results and discussion

3.1. Reservoir model

REEK Field is a three-phase black-oil reservoir model with specified locations for eight vertical wells (five producers and three injectors) that need to be drilled sequentially (Fig. 9). It consists of 40× 64× 14 grid cells, of which 34,770 are active. The maximum rates of production and injection wells are 6000 m^3/day and 10,000 m^3/day, respectively. The minimum BHP of the producers is 250 bar, while the BHP of the injectors cannot exceed 320 bar. The porosity field, permeability field, and fault transmissibility multipliers are all uncertain. Recent studies have used this model for optimizing the drilling order of wells, in which the control variables are discrete (Wang and Oliver, 2019, 2020; Hanea et al., 2019; Leeuwenburgh et al., 2016). However, these studies do not consider the possibilities of future learning through action in the optimization process, i.e., the optimal well for each decision step is based on the current assessment of uncertainty (Eq. (1)). To obtain a more robust optimal solution, we will apply a simplified VOI decision analysis based on key reservoir features and key observations (Fig. 7). This method will enable us to make the optimal decision more efficiently while considering the future reduction of key uncertainty in the drilling-order problem.

In the original REEK model, the drilling schedule of wells is an important contributor to the reservoir profitability, i.e., the expected NPV of the optimized drilling sequence could be as much as 25% higher than random drilling schedules. However, the deterministic optimal drilling sequence does not change significantly with the geological uncertainty (Wang and Oliver, 2020). To increase the effect of geological properties on the drilling-order problem, we modified the original REEK Field by extending fault F5 (Fig. 10) so that the reservoir model could be separated into two compartments, and we assume this extended fault F5 would either be non-sealing or sealing in individual model realizations. When fault F5 is completely sealing, the injector WI_1 will be totally isolated from all the other wells, and there is no benefit from drilling WI_1. In that case, the optimal drilling sequence of the remaining wells may change significantly. Table 1 shows the economic parameters used for computing NPV (10 years), the reservoir properties, and control variables in this modified REEK Field. In this work, we have used the same values for economic parameters as in Hanea et al. (2019), which investigated the impact of history matching well data on creating value after re-optimizing the drilling schedule of wells in the REEK model. Although the oil price has consequently been set to a low value, it does not affect the conclusions from our experiment results.

To obtain useful information for improving optimal decisions, we first identify the key uncertainty of the drilling-order problem in this modified REEK Field, i.e., illustrating whether a non-sealing/sealing fault F5 has the largest influence on the optimal drilling sequence. Then, we identify the key action that would provide the most important observations for reducing the key uncertainty. To compute a robust, optimal, and complete solution, we could apply the workflow that accounts for future learning possibilities through key actions (Fig. 5) at each decision step before drilling a new well. Since information obtained at the early stage is most effective for improving the future decisions and thereby increasing the expected NPV after re-estimating the geological uncertainty (Hanea et al., 2019), we simplify the drilling-order problem in our application and neglect the possibilities of future learning at later stages when evaluating the optimal complete drilling sequence, i.e., only

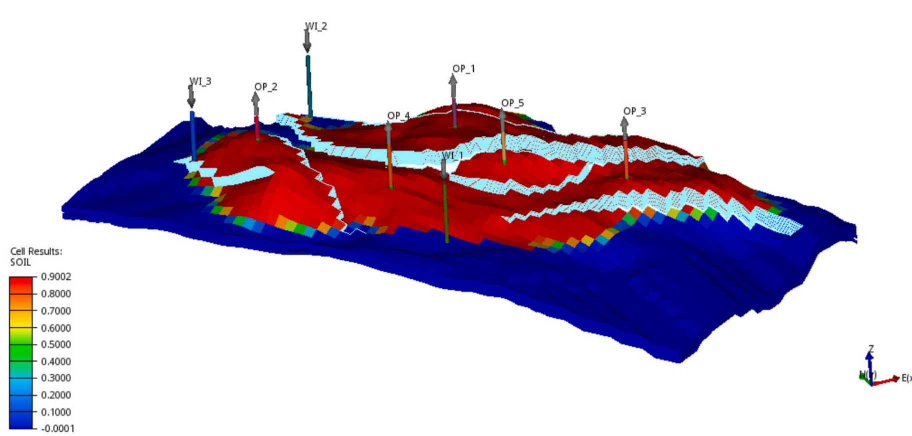


Fig. 9. Well and fault locations in the REEK Field and initial oil saturation in one realization.

identify the key action for the *first decision step* and update the reservoir model based on the *key observations* from that action before drilling the second well. In this paper, we did not reduce the key uncertainty through a history matching process, although that would naturally occur after production data is obtained. Instead, we updated the reservoir model directly using Bayes' rule because the reliability of information collected for the reduction of key uncertainties can be evaluated simultaneously when identifying the best observation subset using a supervised learning model. Because the VOI assessment depends both on the accuracy of information and the prior probabilities, we will study the performances of VOI analysis through key action for the drilling-order problem with different initial probabilities of key uncertainty.

3.2. Key uncertainty for drilling-order problem

To identify the key uncertainty of the drilling-order problem, we performed a simple Monte Carlo experiment in which all variables were perturbed simultaneously, and then optimized for each realization for studying the sensitivity of the optimal solution to different geological features. In this work, we use the Manhattan distance, i.e., the sum of the absolute differences between positions of wells in the drilling sequence, to measure the similarity between optimal drilling sequences (Wang and Oliver, 2020). Fig. 11a compares the distributions of Manhattan distance between deterministic optimal drilling sequences of individual realizations in two cases, one in which fault F5 is always non-sealing (histogram in yellow) and another in which fault F5 alternates between being non-sealing and sealing (histogram in blue). To obtain reliable results, we repeated the experiment 100 times for each case. The optimal drilling sequence clearly varies more significantly when fault F5 is changed from non-sealing to sealing. For the individual realizations with various porosity fields, permeability fields, and fault transmissibility multipliers while fault F5 is always non-sealing, the average Manhattan distance between the deterministic solutions was substantially smaller than when F5 is not always non-sealing, and the optimal well order for some positions in the drilling sequence were almost independent of the geological uncertainty. In this modified REEK model, it seems that whether fault F5 is completely sealing or not has a relatively larger influence than other geological features on the optimal drilling sequence.

In some cases, optimal solutions with a large Manhattan distance might have similar economic values, for example, if only the optimal wells for the later decision stages are changed or their positions are swapped. Hence, we also investigated the variability in NPV obtained in a fixed reference model applied with different deterministic optimal solutions to further illustrate the importance of the sealing properties of fault F5 as a key feature for the drilling-order problem (Fig. 11b). The results show that the NPV changes by less than 2% in most cases when various deterministic drilling sequence solutions for a non-sealing fault F5 are applied to a fixed reference model (histogram in green), even if the deterministic optimized drilling sequence might vary with a Manhattan distance larger than 22 in several cases. In cases with solutions obtained respectively from individual realizations with a non-sealing (i.e., reference model) and a sealing fault F5 (histogram in red), the relative change in NPV could be as much as 10% in the reference model. Thus, in terms of either a change in optimal decisions or a potential improvement in NPV, we observed that the key uncertainty for the drilling-order problem in this modified REEK field is whether F5 is completely sealing or not. To improve future decisions by using additional information, the acquired information should be able to provide useful observations for exploring this key reservoir feature.

3.3. Identifying key action and collecting information

In this paper, to reduce key uncertainties in the drilling-order problem, we use information from production and pressure data that can be obtained from standard oil-field monitoring. In the modified

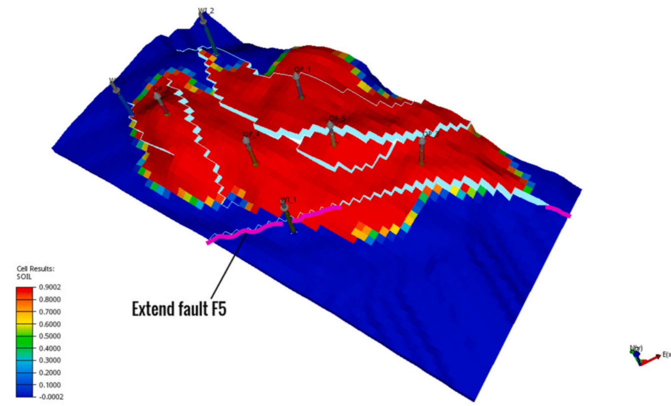
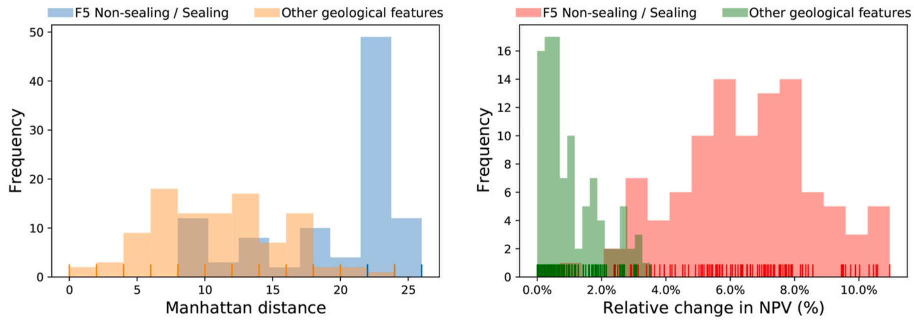


Fig. 10. Modified REEK model with extended fault F5 near Injector WI_1.

REEK Field (Fig. 10), Producers OP_3, OP_4, OP_5 and WI_1 are located near fault F5. We expect that, compared to the other wells, observations from these wells may be potentially more useful for predicting whether fault F5 is completely sealing or not. In this case, although OP_5 is close to fault F5, an examination of the information from OP_5 showed that it was less reliable as a source of information than what could be obtained from OP_3 or OP_4. When WI_1 or OP_5 is drilled as the first well, the hidden cost of information caused by sub-optimal solutions (i.e., it can be estimated when computing the optimal solution over the current uncertainty state) is larger than that from OP_3 and OP_4. Therefore, based on the possibility of obtaining valuable information for key uncertainty reduction and the possibility of achieving high expected NPV, only OP_3 and OP_4 are considered as possible key actions for the first decision step.

Fig. 12 shows the oil production rate and BHP in the first 6 months (i.e., the assumed drilling period for each well) obtained from 100 individual realizations with a non-sealing and sealing fault F5, where OP_3 and OP_4 are drilled as the first well, respectively. In almost all cases, production is first constrained to a maximum rate of 6000 m^3/day and then decreased to hold the producer at a minimum BHP of 250 bar. When fault F5 is sealing, the production rate (red curves in Fig. 12a and c) decreases more rapidly than when the fault F5 is non-sealing while maintaining the pressure at 250 bar, and the pressure (magenta curves in Fig. 12b and d) drops faster while maintaining a production rate of 6000 m^3/day. It seems that both OP_3 and OP_4 can potentially provide useful information for reducing the uncertainty about whether fault F5 is non-sealing or sealing, which would influence the rates of decline in both production and pressure.

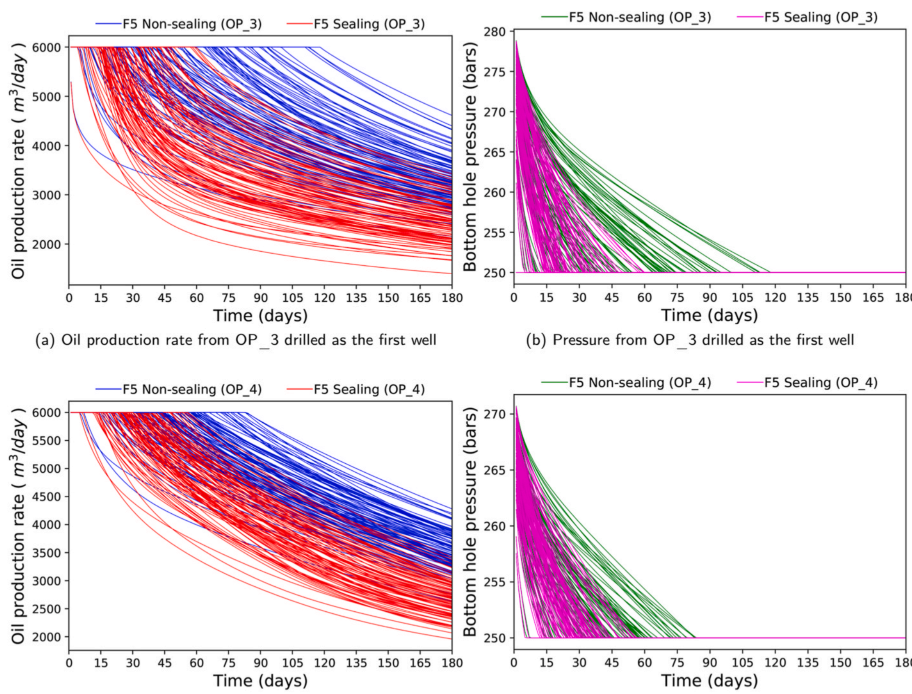
Fig. 13 shows the derivative of production rate when the producer is held at the minimum BHP of 250 bar and the pressure derivative when the producer is controlled by the maximum rate 6000 m^3/day. Note that in Figs. 13a and c the derivative of the production rate is shown with time starting from the first day when BHP = 250 bar, while the x-axis in Figs. 13b and d for the pressure derivative displays the time starting from the first day of production. Here we use the normalized logarithmic derivative of pressure to compute the pressure derivative, i.e., $\frac{t\frac{dp}{dt}}{-b_0}$, where $-b_0$ is the initial value of logarithmic pressure derivative at the beginning of production. Compared to the results obtained from the production rate and pressure (Fig. 12), it seems that the separation between model realizations with a non-sealing and sealing fault F5 is better when using the derivative information, especially with regard to observations of the pressure derivative when OP_3 is drilled as the first well (Fig. 13b). Note that computation of the normalized pressure derivative requires evaluation of 4 pressure values, so it may not be surprising that it is more informative for identifying potential barriers than a pressure measurement. Although we may intuitively find a good observation (e.g., pressure derivative at day 10) for predicting whether F5 is non-sealing or sealing in this example, that might not be the case in other



(a) Distribution of Manhattan distance between the optimal drilling sequences from individual realizations

(b) Distribution of the relative changed in NPV in the reference model with different solutions

Fig. 11. Variability in the optimal drilling sequences and variability in the NPV in the reference model with different optimized drilling sequences.



(a) Oil production rate from OP_3 drilled as the first well

(b) Pressure from OP_3 drilled as the first well

(c) Oil production rate from OP_4 drilled as the first well

(d) Pressure from OP_4 drilled as the first well

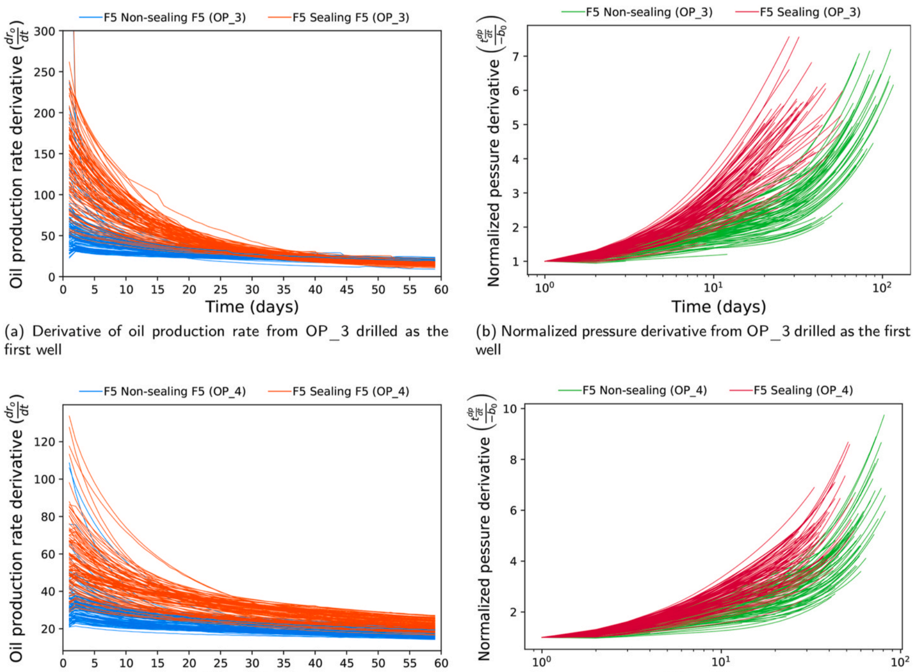
Fig. 12. Oil production rate and pressure in first 6 months obtained from individual realizations with non-sealing or sealing fault F5 when OP_3 and OP_4 are drilled as the first well respectively.

situations. The information obtained from a key action could contain hundreds or thousands of observations. Manual identification of the important observations from such a large dataset is laborious and time-consuming. Moreover, to obtain highly reliable information for key uncertainty reduction, we usually need to combine multiple observations. Therefore, it is necessary to apply a practical method for the automatic defection of key observations.

In this work, we build supervise-learning models that are able to capture the mapping between the inputs (observations) and the outputs (non-sealing/sealing fault F5) to select the best observation subset with high prediction accuracy. Because the best subset might contain observations from different sources, we consider all information related to production rate, pressure, and their derivatives in the process of identifying key observations. In our application, it takes only a few minutes to identify the optimal combination of observations from the original information dataset with more than 700 observations using supervised-learning algorithms.

3.4. Selecting the best observation subset

In the six-month period after drilling the first well a large amount of production and pressure data are recorded. Some of the data are apparently unaffected by whether fault F5 is non-sealing or sealing (e.g., BHP of OP_3 is 250 bar in the last 2 months production before drilling the next well). To identify the important observations from such a large original dataset, we first use the mRMR feature selection method to remove irrelevant and redundant observations and obtain a small observation subset that may provide useful information. This is followed by a ROC curve analysis to further reduce the size of the subset, leaving only the observations with relatively good classification performance. In this way, the dimension of the observation dataset can be reduced quickly without incurring the loss of important information. Finally, a supervised-learning model can be applied with a small observation subset (i.e., few input variables) containing most of the useful information needed to efficiently identify the optimal combination of observations with highly reliable information for key uncertainty reduction. Although we did not consider the effect of observation error in the following analysis, the only change required in methodology



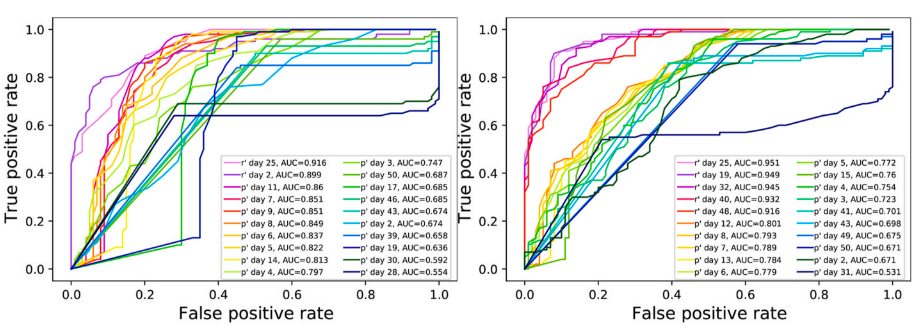
(a) Derivative of oil production rate from OP_3 drilled as the first well

(b) Normalized pressure derivative from OP_3 drilled as the first well

(c) Derivative of oil production rate from OP_4 drilled as the first well

(d) Normalized pressure derivative from OP_4 drilled as the first well

Fig. 13. Derivative of production rate and pressure derivative obtained from individual realizations with non-sealing or sealing fault F5 when OP_3 and OP_4 are drilled as the first well respectively.



(a) ROC curves of the best 20 observations from OP_3

(b) ROC curves of the best 20 observations from OP_4

Fig. 14. Comparison of the receiver operating characteristic (ROC) plots for the 20 top-ranked observations when OP_3 and OP_4 are drilled as the first well respectively.

would be to add random noise to the modeled observations.

Fig. 14a and b shows the ROC curves of 20 best observations obtained from the production/pressure data when OP_3 and OP_4 are drilled as the first well, respectively. In these figures the true positive rate (TPR, y-axis) is plotted against the false positive rate (FPR, x-axis) at various thresholds. The TPR represents the proportion of positive samples (i.e., individual realizations with a non-sealing fault F5) that are correctly identified, while the FPR is the proportion of negatives samples (i.e., individual realizations with a sealing fault F5) that are incorrectly identified as positive cases. The classification performance of a single observation is quantified by using the area under the ROC curve (AUC), which measures the entire two-dimensional area underneath the ROC curve from (0,0) to (1,1). A large AUC score indicates that the single observation has a good ability to distinguish between different classes. All of the 20 best observations are from r' or p', and this shows that information obtained from pressure and rate derivatives (Fig. 13) provides more useful observations for predicting whether or not fault F5 is sealing than directly using production rates or pressure (Fig. 12). Of the 20 top-ranked observations, most are associated with the pressure derivative, however, observations from the production rate derivative have higher AUC scores. In this case, it seems that p' provides more observations with useful information, while observations from r' provide better predictive performance in distinguishing between realizations with a non-sealing or sealing fault F5. If we use only the pressure data for reducing key uncertainty, the drilling of OP_3 as the first well would yield more important observations than the drilling of OP_4; seven observations of p' from OP_3 have AUC scores larger than 0.8, while only one observation of p' with an AUC ¿ 0.8 emerges from OP_4. If only considering the information from rate data, drilling OP_4 as the first well would provide more key observations with higher AUC scores than those from OP_3.

Fig. 15 shows the distributions of observed values obtained from individual realizations at the best single observation r'_b and p'_b (largest AUC score) identified from the derivatives of production rate and pressure, respectively. The red vertical lines represent the best cutting point that maximizes the difference between the TPR and FPR. The probabilities of the individual realizations with observed values located in each region determined by the optimal threshold are summarized in

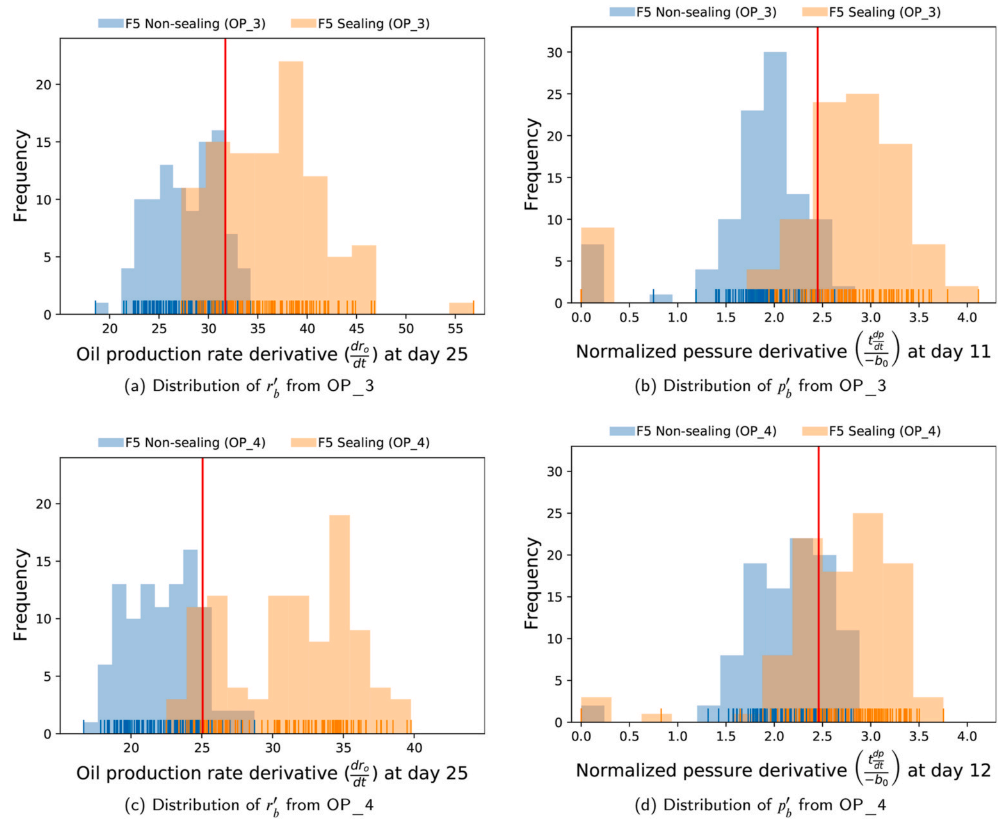


Fig. 15. Distributions of the observed values of r'_b and p'_b obtained from individual model realizations with non-sealing or sealing fault F5.

Table 1
Economic parameters for NPV and reservoir properties in modified REEK model.

Field	REEK model
Start time	December 1, 1999
Time period of NPV (years)	10
Discount factor	0.08
Produced-oil price (USD/m^3)	60
Water-separation cost (USD/m^3)	5
Water-injection cost (USD/m^3)	1
Drilling cost of each well (USD)	1 million
Number of grid blocks	40 × 64 × 14 (34,770 active cells)
Number of faults	6 (fault F5 is extended)
Number of wells (all vertical wells)	8 (5 producers and 3 injectors)
Drilling period of wells (months)	6
Maximum production rate (m^3/day)	6000
Minimum BHP of producers (bars)	250
Maximum injection rate (m^3/day)	10000
Maximum BHP of injectors (bars)	320
Number of geological realizations	100
Permeability (md)	0 to 3500 (average 733)
Porosity	0 to 0.45 (average 0.159)
Fault transmissibility multiplier of other faults	0 to 1 (average 0.105)
Fault transmissibility multiplier of fault F5	0 (sealing) or ¿ 0 (non-sealing)

Table 2
Probabilities of the individual realizations with the observed values located in each region determined by the optimal threshold of the best single observation r'_b or.p'_b

Best single observation	Information from OP_3	Information from OP_4
$P(r'_b < \delta_{r'_b} \vert \mathrm{F5}_{\mathrm{non-sealing}})$	0.89	0.90
$P(r'_b \geq \delta_{r'_b} \vert \mathrm{F5}_{\mathrm{sealing}})$	0.80	0.87
$P(p'_b < \delta_{p'_b} \vert \mathrm{F5}_{\mathrm{non-sealing}})$	0.94	0.76
$P(p'_b \geq \delta_{p'_b} \vert \mathrm{F5}_{\mathrm{sealing}})$	0.74	0.72

Table 2. When OP_4 is drilled as the first well, the prediction accuracy reaches as high as 90% based on r'_b, but the accuracy rate based on p' is less than 75%. When OP_3 is drilled as the first well, r'_b also provides information with a higher reliability than that of p'_b, but using p'_b to identify the models with a non-sealing fault F5 would be more effective, i.e., 94 of 100 individual realizations with a non-sealing F5 have $p'_b < \delta_{p'_b}$. In this example, although it is possible to reduce key uncertainty using only a single observation, this might not always be the case in other problems. In most applications, it is generally necessary to use multiple observations to obtain reliable information and thereby reduce key uncertainty. A quick, simple way is the direct use of a combination of the observations with high AUC scores. However, when the classification performances of these observations are similar, the prediction accuracy will most likely not improve.

By using supervised-learning algorithms, we can efficiently identify the optimal observation subset with high prediction accuracy. We should expect that a good observation subset consists of observations with useful information (e.g., AUC ¿ 0.5). Based on the 20 top-ranked observations identified through the ROC curve analysis, we apply supervised-learning algorithms to identify the best combination of observations for predicting whether F5 is non-sealing or sealing. To avoid overfitting of the learning models, the original observation dataset obtained from 200 samples (100 individual realizations with non-sealing and sealing fault F5 respectively) is split into a training set (80%) and a test set (20%). The learning model is built based on the training set while the performance of the model is evaluated in the test set. To acquire a more statistically reliable estimate of performance, we use k-fold cross-validation resampling method to evaluate the learning model on the limited dataset.

Fig. 16 shows the performances of four classification models applied to predict whether or not fault F5 is sealing using optimized observation subsets with different size based on the information from OP_3. The performance score on the y-axis represents the prediction accuracy

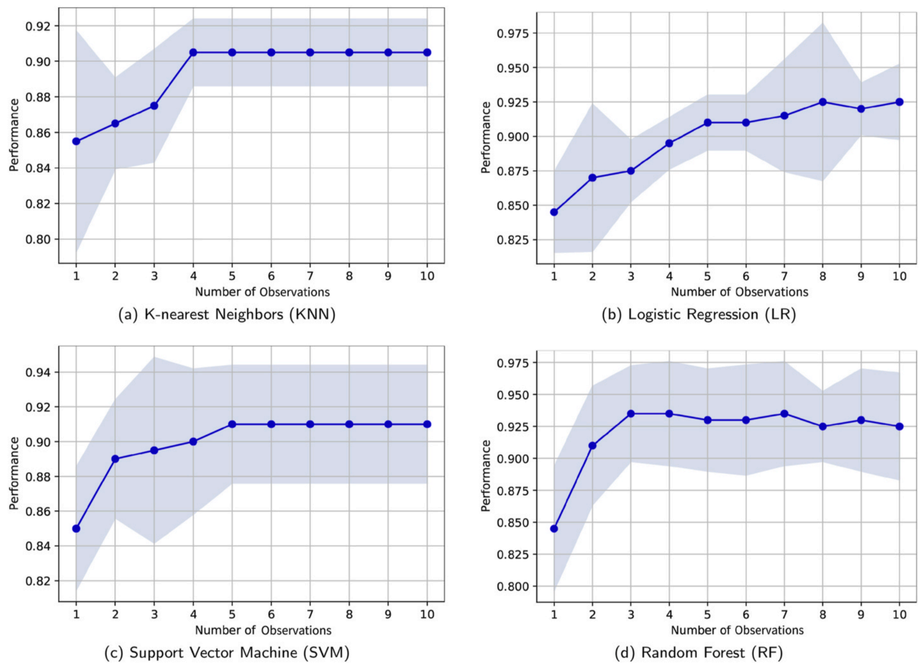


Fig. 16. Performances of four supervised-learning models for predicting whether F5 is sealing or not using the optimal observation subset based on information from OP_3.

measured in the test set. The light blue area indicates the standard deviation of prediction accuracy estimated through 5-fold cross validation. Results show that the Random Forest model (Fig. 16d), which is an ensemble machine learning algorithm based on bootstrap aggregation (bagging), performed better than the other three supervised-learning algorithms (i.e., k-Nearest Neighbors, Logistic Regression, Support Vector Machine). Compared with the case involving only one single observation, prediction accuracy improves using an optimized combination of two or three observations. However, when using more than three observations, the performance of the learning model does not improve with the addition of new observations; rather, as the number of input variables increases, the model becomes more complex, making it more prone to overfitting the training set. In this case, we can use only three key observations to predict whether or not F5 is sealing with high accuracy. When OP_4 is drilled as the first well, we observe similar properties in the process of identifying the optimal observations subset: the Random Forest algorithm extracts an observation subset with relatively higher prediction accuracy than the subsets produced by the other algorithms and an optimal number of approximately three observations.

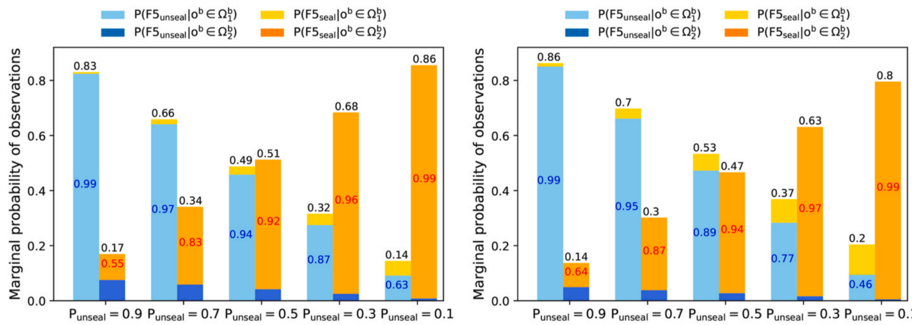
Table 3 shows the optimal observation subset with three key observations identified using the Random Forest model, as well as the probabilities of individual realizations with observed values located in the two best disjoint subspaces Ω_1^b, Ω_2^b for predicting whether fault F5 is non-sealing and sealing, respectively. We note that, although the best single observation r'_b and p'_b have good classification performances (Fig. 15), the optimal combination of observations does not necessarily include r'_b or p'_b. The optimal observation subset from OP_3 has one observation of pressure derivative $p^{'}$ obtained at day 19, and this is only the 18th-best single observation with AUC = 0.636 (Fig. 14a). However, after additional two observations from $r^{'}$ are combined, the accuracy rate for identifying models with non-sealing and sealing fault could reach 92% and 94%, respectively. When OP_4 is drilled as the first well, the optimal observation subset consists of three observations from $r^{'}$, whereas the best single observation r'_b at day 25 with AUC = 0.951 is not in the optimal subset. However, the prediction accuracy is increased to 95% when fault F5 is non-sealing. The above results show that drilling either OP_3 or OP_4 at the first decision step will result in obtaining highly reliable information for reducing key uncertainty in the drilling-order problem (i.e., whether F5 is completely sealing or not). By using a supervised-learning algorithm, we efficiently identified key observations from both OP_3 and OP_4 as well as the best division of space for prediction purposes, and we also simultaneously estimated the reliability of information (i.e., prediction accuracy) in each subspace (Table 3). This allows the direct computation of the posterior probability of key uncertainty using Bayes' theorem, thereby avoiding the need for history matching to re-estimate uncertainty.

Table 3
Best observation subset and the reliability of the information in each observation subspace obtained from Random Forest model.

$N_f^b = 3$	Information from OP_3	Information from OP_4
Optimal observation subset	$o^b = (r'_{\text{day2}}, r'_{\text{day25}}, p'_{\text{day19}})$	$o^b = (r'_{\text{day19}}, r'_{\text{day32}}, r'_{\text{day48}})$
$P(o^b \in \Omega_1^b \mid \text{F5}_{\text{non-sealing}})$	0.916	0.945
$P(o^b \in \Omega_2^b \mid \text{F5}_{\text{sealing}})$	0.941	0.878

3.5. Assessing value of information through key action

By identifying the key reservoir feature that has the largest influence on the optimal drilling sequence, we obtained two possible key actions at the first step (i.e., OP_3 and OP_4) that are more likely to provide useful information for reducing the key uncertainty in the drilling-order problem: whether fault F5 is sealing or not. Key observations obtained from both OP_3 and OP_4 are demonstrated to have high predictive accuracy for indicating if fault F5 is sealing or non-sealing. Although reducing key uncertainty would potentially lead to better future decisions, taking a key action to acquire the useful information for key uncertainty reduction is not always worthwhile, since there may be a high hidden cost of obtaining information caused by the sub-optimality of the solution which uses the key action. To judge whether taking the key action increases the maximum expected NPV in current uncertainty state, we must evaluate the EVOI associated with this hidden cost (Eq. (7)). To compute the expected value of information, we need to obtain the expected NPV from the initial optimal solution obtained utilizing the



(a) Key uncertainty reduction based on information from OP_3 (b) Key uncertainty reduction based on information from OP_4

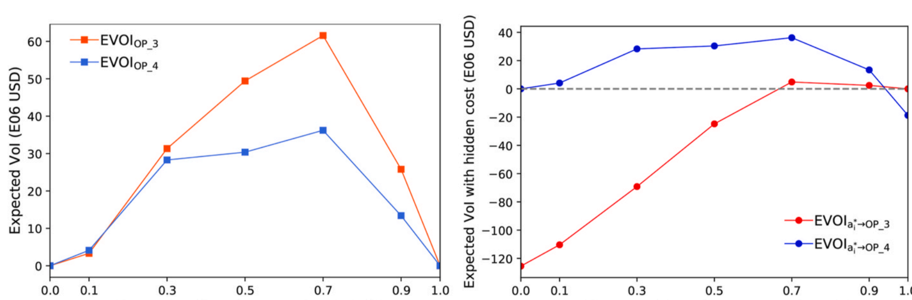
Fig. 17. Posterior probabilities and marginal probabilities of observations obtained from different prior probability distributions of key uncertainty.

prior probability of key uncertainty and the optimal solutions (i.e., constrained to the selected key action) in the posterior probability, and the marginal probabilities of observations. All of these values are related to the prior probability. Hence, instead of evaluating the EVOI only in one specific prior probability, we investigate the performances of the VOI analysis through key action with different prior probabilities of key uncertainty.

Fig. 17 shows effects of using the best observation subset o^b from OP_3 or OP_4 to evaluate the posterior probabilities of a non-sealing and sealing fault F5 with different initial probabilities, i.e., $P_{\text{unseal}} = (0.9, 0.7, 0.5, 0.3, 0.1)$. The black values on the tops of bars represent the marginal probabilities of observations located in the best two disjoint subspaces Ω_1^b and Ω_2^b, i.e., $P(o^b \in \Omega_1^b)$ and $P(o^b \in \Omega_2^b)$, irrespective of the reservoir features. Since the prediction accuracy for observations located in the same subspace is almost the same, we only need to compute the posterior probabilities at $o^b \in \Omega_1^b$ and $o^b \in \Omega_2^b$ (marked with different colors in the bars) considering all possible observations associated with o^b. Note that the values in blue indicate the posterior probabilities $P(\text{F5}_{\text{unseal}}|o^b \in \Omega_1^b)$ while the values in red represent $P(\text{F5}_{\text{seal}}|o^b \in \Omega_2^b)$. Overall, using key observations from both OP_3 and OP_4 can significantly reduce the uncertainty about whether fault F5 is completely sealing or not, but the posterior probabilities are strongly influenced by the prior probabilities. The posterior probabilities with observations $o^b \in \Omega_1^b$ and $o^b \in \Omega_2^b$ change considerably especially when $0.3 < P_{\text{F5}_{\text{unseal}}} < 0.7$. When $P_{\text{F5}_{\text{unseal}}} = 0.5$, the posterior probabilities $P(\text{F5}_{\text{unseal}}|o^b \in \Omega_1^b)$ and $P(\text{F5}_{\text{seal}}|o^b \in \Omega_2^b)$ based on the key observations obtained from OP_3 could reach 0.94 and 0.92, respectively. When $P_{\text{F5}_{\text{unseal}}}$ ¿ 0.9 or ¡ 0.1, $o^b \in \Omega_1^b$ and $o^b \in \Omega_2^b$ are almost perfect observations for indicating a non-sealing and sealing fault F5, respectively. At a fixed prior probability, both marginal probabilities of observations and posterior probability distributions change slightly when key observations respectively from OP_3 and OP_4 are used, although the reliability of information is different (Table 3). In this case, drilling either OP_3 or OP_4 as the first well can result in obtaining information with similar effectiveness in terms of the reduction of key uncertainty for the drilling-order problem, i.e., whether F5 is sealing or not.

The purpose of collecting information for key uncertainty reduction is to obtain a better optimal drilling order solution with higher expected value. To study whether key observations identified from OP_3 and OP_4 are useful in increasing expected profitability, we computed the EVOI (Eq. (10)) to evaluate their potential for increasing the expected NPV from improved optimal solutions. Fig. 18a compares the $\text{EVOI}_{\text{OP}_3}$ and $\text{EVOI}_{\text{OP}_4}$ obtained from the difference between the expected values of optimal drilling sequences with and without use of additional information (i.e., key observations) to reduce the uncertainty about whether F5 is sealing or not. Results show that both $\text{EVOI}_{\text{OP}_3}$ and $\text{EVOI}_{\text{OP}_4}$ are positive when $0 < P_{\text{F5}_{\text{unseal}}} < 1$. This indicates that the optimal drilling sequence of the remaining wells is improved after reducing the key uncertainty in the drilling-order problem. Also, the key observations from either OP_3 or OP_4 are always helpful in making better future decisions with different prior probabilities. When the initial probability of a non-sealing fault F5 is near 0.7, both $\text{EVOI}_{\text{OP}_3}$ and $\text{EVOI}_{\text{OP}_4}$ reach their maximum values. Although the effects of using information from OP_3 and OP_4 to reduce key uncertainty are similar (Fig. 17), $\text{EVOI}_{\text{OP}_3}$ is always higher than $\text{EVOI}_{\text{OP}_4}$, i.e., more *additional* value can be created with the information from OP_3. If we only consider the EVOI for choosing a key action, it seems that OP_3 should be preferred to OP_4. However, there might be a large hidden cost of information caused by a sub-optimal solution when OP_3 or OP_4 is drilled as the first well. Hence, we should evaluate the net EVOI with this hidden cost to determine whether it is worth drilling OP_3 or OP_4 first to obtain information for improving future decisions, rather than taking the optimal decision for achieving the maximum expected NPV over the current uncertainty state.

Fig. 18b shows the net EVOI including the hidden cost of information when OP_3 or OP_4 is drilled as key action for first decision step. Note that there is no need to compute the EVOI (Eq. (10)) and the cost of information ECOI (Eq. (11)) separately for obtaining the net EVOI through a key action. This net EVOI can be computed based on the



(a) EVOI based on key observations from OP_3 or OP_4 (b) EVOI with hidden cost caused by sub-optimal solutions

Fig. 18. Comparison of standard EVOI and EVOI with the hidden cost of obtaining information from OP_3 or OP_4.

$\mathrm{EVOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$ obtained from changing the initial optimal decision a_i^* for current uncertainty state to key action a_{j+1}^{key} (Eq. (7)). Although $\mathrm{EVOI}_{\mathrm{OP}_3} > \mathrm{EVOI}_{\mathrm{OP}_4}$, the hidden cost of obtaining information from OP_3 is much larger than the cost from OP_4 especially when there is a high probability that fault F5 is sealing. When the initial probability of a non-sealing fault F5 is less than 0.7, the $\mathrm{EVOI}_{a_i^* \to \mathrm{OP}_3}$ is negative, which means that the additional value created by using the information from OP_3 to improve the optimal drilling sequence of the remaining wells is not enough to compensate for the hidden cost of obtaining information using a sub-optimal drilling sequence when OP_3 drilled as the first well, i.e., $\mathrm{EVOI}_{\mathrm{OP}_3} < \mathrm{ECOI}_{a_i^* \to \mathrm{OP}_3}$. In that case, there is no benefit to drilling OP_3 first even if it can provide highly reliable information for reducing key uncertainty for the drilling-order problem, and OP_3 is preferred to a_i^* only when $P_{\mathrm{F5}_{\mathrm{unseal}}} \geq 0.7$. However, $\mathrm{EVOI}_{a_i^* \to \mathrm{OP}_4}$ is always positive and larger than $\mathrm{EVOI}_{a_i^* \to \mathrm{OP}_3}$ unless the prior probability of non-sealing fault F5 is extremely high. After consideration of the hidden cost of information, when $P_{\mathrm{F5}_{\mathrm{unseal}}} < 0.94$, OP_4 will be a better choice of key action than OP_3 while OP_3 will be preferred to OP_4 if $P_{\mathrm{F5}_{\mathrm{unseal}}} \geq 0.94$. In this case, performing the key-feature based VOI analysis (Fig. 7), which considers only the future learning possibility through OP_3 or OP_4 that has a higher $\mathrm{EVOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$, leads to the same optimal decision with consideration of the future information from all possible decision alternatives, which illustrates that by taking into account future information from key action, we are able to make optimal decisions which account for the possibilities of further learning without sacrificing the quality of solution.

Instead of computing all $\mathrm{EVOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$ to identify the preferred a_{j+1}^{key}, we can study the initial optimal drilling sequence to quickly obtain a good a_{j+1}^{key} with a potentially higher $\mathrm{EVOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$. Since the hidden cost of information is caused by a sub-optimal solution and the wells that have important contributions to increase the expected NPV are generally preferable for drilling at an early stage, we expect that the cost $\mathrm{ECOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$ from a_{j+1}^{key} that is drilled at a later stage along the initial optimal complete drilling sequence will be potentially larger than the hidden cost of obtaining information from a_{j+1}^{key} drilled at an early stage. When there are several possible a_{j+1}^{key} with similar reliability of information, we can choose the one that is supposed to be drilled earlier for maximizing the expected NPV in the current uncertainty state to avoid a high hidden cost of information. In this work, we used learned heuristic search with mean model bias-correction methods to efficiently obtain robust optimal drilling sequence under uncertainty. During the search process, the maximum expected NPV constrained to different selected wells is estimated without finding the actual optimal solution. Hence, we could also obtain an approximation of $\mathrm{ECOI}_{a_i^* \to a_{j+1}^{\mathrm{key}}}$ when computing the initial optimal decision a^* and without incurring additional costs.

For the drilling-order problem, we observe that the optimal drilling sequence always starts with OP_4 when $P_{\mathrm{F5}_{\mathrm{unseal}}} < 0.94$, and then changes to OP_3 drilled as the first well when $P_{\mathrm{F5}_{\mathrm{unseal}}} \geq 0.94$, which indicates that one of the possible key actions (e.g., OP_3 or OP_4) has no hidden cost of information caused by a sub-optimal solution since it is identical to the initial optimal decision a^*, i.e., $\mathrm{ECOI}_{a^* \to a^*} = 0$. When a^* is also able to provide useful information for key uncertainty reduction, we can consider the future information from both a^* and a_{j+1}^{key} that is identified from the other decision alternatives to make a more robust decision in consideration of future learning possibilities (Eq. (13)). In this example, the optimal decision obtained after taking into account the future information from both OP_3 and OP_4 is still the initial optimal decision a^* since taking a^* is able to both generate highly reliable information for key uncertainty reduction (i.e., valuable information for improving future decisions) and maximize the expected NPV for the current assessment of uncertainty (i.e., no hidden cost of information), although this might not always be the case in other problems. Note that here, we only investigated the future learning possibilities at the first decision step. After drilling a new well, the reservoir model will be updated through history matching based on the actual data. The key-feature-based VOI analysis (Fig. 7) could then be performed again to determine the next optimal well. At the second and later decision stages, however, information from the remaining actions will generally have a smaller potential for improving the optimal strategy, and there may be no clear key uncertainties for the optimization problem. In that case, there would be no need to consider the effect of future information when making the optimal decision, and one could simply use a standard robust optimization method (Wang and Oliver, 2020).

4. Conclusion

In this paper, we proposed a flexible workflow built on a key-feature-based value of information analysis to make optimal decisions efficiently while accounting for the possibilities for future learning through actions. Taking into account the effects of future information before committing to a decision allows improvement of the optimal strategy. However, it is infeasible and unnecessary to account for all possible future observations from remaining actions (i.e., a standard VOI analysis with extensive form). In our approach, the VOI analysis is only performed on a small number of key actions that will provide key information for reducing the most important uncertainties in the optimization problem. i.e., information that will be for making better future decisions. Then, the optimal decision is made based on the trade-off between the key actions and the initial optimal decision obtained without considering any future information. The simplified VOI analysis based only on key actions and key information might not result in the same optimal solution as the complete VOI analysis, but it offers a practical way to obtain a near-optimal decision that accounts for the possibilities of future learning (i.e., the opportunities to improve optimal strategy resulting from future uncertainty reduction). The key actions can be identified by considering the possibility of obtaining valuable information for reducing key uncertainties and the possibility of achieving high expected NPVs for the current uncertainty state, so that there is no need to compare the actual expected values of all possible decisions. The focus on the use of key information to reduce key uncertainties avoids the need for full history matching to re-estimate all uncertainties in the optimization. Instead of considering all distinct sets of observations obtained from all ensemble members when updating the reservoir model, we divide the entire key observation space into a limited number of disjointed subspaces, i.e., each subspace will have high information reliability for indicating a specific key uncertainties subregion, and observations located in the same subspace have similar prediction precision for key uncertainty reduction. Consequently, we only have to re-estimate key uncertainties for each observation subspace and perform the optimization process in a few posterior ensembles for computing the expected value with information. The following conclusions can be drawn from the present study:

- Although many uncertainties arise in reservoir characterization, some of them have little influence on the optimal decisions, even if they might be reduced significantly by assimilation of acquired observations. By identifying key uncertainties for the optimization problems, we can identify key actions that would provide the most useful information for improving future decisions.
- When all observations are used to simultaneously re-estimate uncertainty, the largest decrease in uncertainty may be in properties that are irrelevant to current decisions, and the reduction in key uncertainties from some observations might be very small. However, the computational penalty of including those nonessential observations in updating the reservoir model can be large. Thus, instead of only reducing the decision space by identifying key actions, we also

need to identify the most important observations for reducing key uncertainties to make the computation manageable.

- Performing a key action to acquire information for reducing key uncertainty is not necessarily worthwhile, even if there is no explicit cost in obtaining the information and future decisions could be improved. When taking key action to obtain information leads to a sub-optimal solution, there is a hidden cost in obtaining the information. Instead of using the additional value that could be created with information to judge whether it is worth taking action, the criteria should be the *net* expected value of information, including the hidden cost associated with changing the optimal decision for the current uncertainty state to the key action.
- The initial action in the optimal sequence of actions based on current information might, in some cases, be a possible key action. To obtain a robust decision, it may be necessary to consider the possibility of future learning through both the initial optimal decision and alternative key actions identified from the remaining decision alternatives.
- The expected value of information attributed to key actions will depend on the prior probability distribution of key uncertainties. Hence, changing the prior probabilities will not only affect the standard (naïve) computation of the optimal solution, but will also affect the optimal decision obtained from VOI analysis.

Although the methodology is illustrated by the application of drilling-order problems, it can be extended to general sequential decision-making problems under uncertainty while considering the effect of future information. The key point is to effectively identify key actions and key observations that are associated with the key reservoir features for optimization problems. In our example application, *key uncertainties* are identified by studying the sensitivity of deterministic optimization solutions to different individual uncertainties. *Key actions* are identified by evaluating the reliability of information for reducing key uncertainties and the hidden cost of obtaining information from key actions. For large problems, it may be necessary to explore more generalized and efficient approaches for identifying key uncertainties and key actions. To efficiently identify *key observations*, we built supervised-learning algorithms that can automatically detect the optimal combination of observations as well as the best division of space for reducing key uncertainty. At the same time, we estimate the prediction accuracy (i.e., the information's reliability) for observations located in each subspace. This allows directly computing the posterior probability of key uncertainty based on Bayes' theorem, avoiding the necessity of expensive data assimilation algorithms to update the entire reservoir model. Using learning algorithms to identify the important observations is applicable for optimization problems with multiple key uncertainties that are continuous or categorical variables. For continuous variables, the distribution of key uncertainties could be divided into a set of optimized subregions based on the performance of the observation subspaces in reducing key uncertainties. Our simplified VOI analysis considers the future information only resulting from the current decision step. If the key information can only be obtained by taking at least two actions (i.e., individual decision alternatives are shown to be unreliable as a source of information for reducing key uncertainty), we could extend the VOI analysis by considering the possibility of future learning through the following two decision steps, i.e., a combination of information from two actions. The simultaneous consideration of information from two actions would increase the complexity of VOI analysis, computational cost of expected value with information, and the hidden cost of obtaining key information caused by sub-optimal solution, which will be constrained to more past decisions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Nomenclature

α	bias correction factor
$\overline{m}$	expected value of model parameter
β	partial correction factor
$\mathrm{ECOI}_{a^*\rightarrow a^{\mathrm{key}}}$	hidden cost of information caused by sub-optimal solutions constrained to decision a^{key}
EVOI	expected value of information
$\mathrm{EVOI}_{a^*\rightarrow a^{\mathrm{key}}}$	expected value of information changing decision a^* to a^{key}
EVWI	expected value with information
EVWOI	expected value without information
Ω^b	key observation subspace
Θ^m	subregion of key uncertainty
A	decision space
a	decision alternative
a^*	optimal decision over the current assessment of uncertainty
$a^{*^{\mathrm{fl}}}$	optimal decision considering future information from current decision stage
$a^{*^{\mathrm{fs}}}$	optimal decision considering future information from all remaining decision stages
a^{key}	decision alternative providing important information for key uncertainty reduction
EV^*	maximum expected value over current uncertainty state
h	history of past decisions and observations
J	objective function
m	model parameter
N_{Ω^b}	Number of observation subspaces
N_e	Ensemble size

O observation space
o observation obtained from specific decision
o^b best observation subset for key uncertainty reduction
$P(\Omega^b|\Theta^m)$ probability of observing $o^b \in \Omega^b$ at key uncertainty subregion Θ^m
$P(\Theta^m|\Omega^b)$ posterior probability of key uncertainty subregion Θ^m with observation $o^b \in \Omega^b$
$P(\Theta^m)$ prior probability of key uncertainty subregion Θ^m
$P(o|h, a)$ probability of observing o from decision a following history h
Q^* maximum expected value over all possible future observations from all remaining actions
u uncertainty state
x control variable

Subscripts
i key uncertainty subregion index
j decision stage index/model realization index (depending on context)
k key observation subspace index

Superscripts
b best observation subset
m model parameter
o observation

Authors contribution

Lingya Wang - conducted the experiments, analyzed the data and wrote the paper. Dean S. Oliver - conceived and designed the study. All authors contributed to manuscript revisions and approved the final version of the manuscript.

References

Åström, K., 1965. Optimal control of Markov processes with incomplete state information. J. Math. Anal. Appl. 10, 174–205. https://doi.org/10.1016/0022-247X(65)90154-X.
Barros, E.G.D., Van den Hof, P.M.J., Jansen, J.D., 2016. Value of information in closed-loop reservoir management. Comput. Geosci. 20, 737–749. https://doi.org/10.1007/s10596-015-9509-4.
Barros, E.G.D., Van den Hof, P.M.J., Jansen, J.D., 2020. Informed production optimization in hydrocarbon reservoirs. Optim. Eng. 21, 25–48. https://doi.org/10.1007/s11081-019-09432-7.
Barros, E.G.D., Jansen, J.D., Van den Hof, P.M.J., 2015a. Value of information in parameter identification and optimization of hydrocarbon reservoirs. IFAC-PapersOnLine 48, 229–235. https://doi.org/10.1016/j.ifacol.2015.08.036.
Barros, E.G.D., Leeuwenburgh, O., Van den Hof, P.M.J., Jansen, J.D., 2015b. Value of multiple production measurements and water front tracking in closed-loop reservoir management, in: SPE Reservoir Characterisation and Simulation Conference and Exhibition, Society of Petroleum Engineers, Abu Dhabi, UAE. doi:10.2118/175608-MS.
Bratvold, R.B., Bickel, J.E., Lohne, H.P., 2009. Value of information in the oil and gas industry: past, present, and future. SPE Reservoir Eval. Eng. 12, 630–638. https://doi.org/10.2118/110378-PA.
Chen, B., He, J., Wen, X.H., Chen, W., Reynolds, A.C., 2017. Uncertainty quantification and value of information assessment using proxies and Markov chain Monte Carlo method for a pilot project. J. Petrol. Sci. Eng. 157, 328–339. https://doi.org/10.1016/j.petrol.2017.07.039.
Chen, Y., Oliver, D.S., Zhang, D., 2009. Efficient ensemble-based closed-loop production optimization. SPE J. 14, 634–645. https://doi.org/10.2118/112873-PA.
Cunningham, P., Begg, S., 2008. Using the value of information to determine optimal well order in a sequential drilling program. The American Association of Petroleum Geologists 92, 1393–1402. https://doi.org/10.1306/06040808071.
Eidsvik, J., Dutta, G., Mukerji, T., Bhattacharjya, D., 2017. Simulation-regression approximations for value of information analysis of geophysical data. Math. Geosci. 49, 467–491. https://doi.org/10.1007/s11004-017-9679-9.
Estevez, P.A., Tesmer, M., Perez, C.A., Zurada, J.M., 2009. Normalized mutual information feature selection. IEEE Trans. Neural Network. 20, 189–201. https://doi.org/10.1109/TNN.2008.2005601.
Goda, T., Ohno, K., Sato, K., 2017. Value of information in optimizing reservoir development under geological uncertainty. J. Jpn. Petrol. Inst. 60, 41–52. https://doi.org/10.1627/jpi.60.41.
Grayson, C.J., 1960. Decisions under Uncertainty: Drilling Decisions by Oil and Gas Operators. Harvard University Press, Boston, Massachusetts.
Hanea, R., Casanova, P., Hustoft, L., Bratvold, R., Nair, R., Hewson, C.W., Leeuwenburgh, O., Fonseca, R.M., 2019. Drill and learn: a decision-making work flow to quantify value of learning. SPE Reservoir Eval. Eng. 22, 1131–1143. https://doi.org/10.2118/182719-PA.
Hanley, J., McNeil, B., 1982. The meaning and use of the area under a receiver operating characteristic (roc) curve. Radiology 143, 29—36. https://doi.org/10.1148/radiology.143.1.7063747.
He, J., Sarma, P., Bhark, E., Tanaka, S., Chen, B., Wen, X.H., Kamath, J., 2018. Quantifying expected uncertainty reduction and value of information using ensemble-variance analysis. SPE J. 23, 428–448. https://doi.org/10.2118/182609-PA.
Hong, A., Bratvold, R., Thomas, P., Hanea, R., 2018. Value-of-information for model parameter updating through history matching. J. Petrol. Sci. Eng. 165, 253–268. https://doi.org/10.1016/j.petrol.2018.02.004.
Jansen, J.D., Brouwer, D.R., Nævdal, G., Van Kruijsdijk, C.P.J.W., 2005. Closed-loop reservoir management. First Break 23, 43–48. https://doi.org/10.3997/1365-2397.2005002.
Kohavi, R., John, G.H., 1997. Wrapper for feature subset selection. Artif. Intell. 97, 273–324. https://doi.org/10.1016/S0004-3702(97)00043-X.
Lazar, C., Taminau, J., Meganck, S., Steenhoff, D., Coletta, A., Molter, C., De Schaetzen, V., Duque, R., Bersini, H., Nowé, A., 2012. A survey on filter techniques for feature selection in gene expression microarray analysis. IEEE ACM Trans. Comput. Biol. Bioinf 9, 1106–1119. https://doi.org/10.1109/TCBB.2012.33.
Leeuwenburgh, O., Chitu, A.G., Nair, R., Egberts, P.J.P., Ghazaryan, L., Feng, T., Hustoft, L., 2016. Ensemble-based methods for well drilling sequence and time optimization under uncertainty, in: ECMOR XV-15th European Conference on the Mathematics of Oil Recovery. doi:10.3997/2214-4609.201601871.
Neumann, J.V., Morgenstern, O., 1944. Theory of Games and Economic Behavior. Princeton University Press, Princeton, NJ, USA.
Peng, H., Long, F., Ding, C., 2005. Feature selection based on mutual information: criteria of max-dependency, max-relevance, and min-redundancy. IEEE Trans. Pattern Anal. Mach. Intell. 27, 1226—1238. https://doi.org/10.1109/tpami.2005.159.
Raiffa, H., Schlaifer, R., 1961. Applied Statistical Decision Theory. Harward University Press, Boston, USA.
Ramírez-Gallego, S., Lastra, I., Martínez-Rego, D., Bolón-Canedo, V., Benítez, J.M., Herrera, F., Alonso-Betanzos, A., 2017. Fast-mrmr: fast minimum redundancy maximum relevance algorithm for high-dimensional big data. Int. J. Intell. Syst. 32, 134–152. https://doi.org/10.1002/int.21833.
Sarma, P., Durlofsky, L.J., Aziz, K., Chen, W.H., 2006. Efficient real-time reservoir management using adjoint-based optimal control and model updating. Comput. Geosci. 10, 3–36. https://doi.org/10.1007/s10596-005-9009-z.
Schlaifer, R., 1959. Probability and Statistics for Business Decisions : an Introduction to Managerial Economics under Uncertainty. McGraw-Hill Book Company, New York.
Sondik, E.J., 1971. The Optimal Control of Partially Observable Markov Processes. Ph.D. thesis. Stanford University. Stanford, CA, USA.
Torrado, R.R., Rios, J., Tesauro, G., 2017. Optimal sequential drilling for hydrocarbon field development planning, in: Proceedings of the Twenty-Ninth AAAI Conference on Innovative Applications (IAAI-17), pp. 4734–4739. URL: https://aaai.org/ocs/index.php/IAAI/IAAI17/paper/view/14577.
Wang, C., Li, G., Reynolds, A.C., 2009. Production optimization in closed-loop reservoir management. SPE J. 14, 506–523. https://doi.org/10.2118/109805-PA.
Wang, L., Oliver, D.S., 2019. Efficient optimization of well drilling sequence with learned heuristics. SPE J. 24, 2111–2134. https://doi.org/10.2118/195640-PA.
Wang, L., Oliver, D.S., 2020. Fast robust optimization using bias correction applied to the mean model. Comput. Geosci. 1573 https://doi.org/10.1007/s10596-020-10017-y.