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Identification of Coupled Elastic Dynamics Using Inverse Eigenvalue Theory

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Abstract

In this paper we present a new method for identification of robot manipulators with coupled inertia dynamics. The main contributions of the work are a) The parameters of a mechanical mass-spring-coupled masses equivalent of any order with coupled mass-terms are identified given only partial state information. b) The method is fully automatic with no operator input and can be applied in the field to update the dynamic model parameters. The approach is particularly useful when the robot operators mount flexible tooling or equipment on the robot arms. c) In the paper we combine an important result from the vibration literature, with the solution of inverse eigenvalue problems. The main advantage of the method compared to other identification methods is the fact that the mechanical structure is exploited and that only motor encoder position and torque are required to identify the springs, masses and coupled masses of an Nth order system.

1. Introduction

Research on inverse eigenvalue problems in mechanics began in the Soviet Union with the work of Gantmakher and Krein, [2]. The work has been translated and made accessible by Gladwell, [3]. Only recently have the powerful results in inverse eigenvalue theory been applied to the online parameter identification of flexible robot mechanics, see for example [1]. Traditionally, before modern signal processing and acquisition electronics were available, inverse eigenvalue experiments were performed first by physically exciting the mechanics by an external impulse response, for example by a hammer. The resonance modes of the unconstrained system would be observed. By unconstrained we mean that the mechanical system is not locked to any external reference points and the position variables are allowed to drift. Second, one position variable would be physically locked (for example by a clamp) to a fixed reference point and the resonance modes of this new system would be observed from a new impulse response. These two sets of resonance modes together with the sum of all masses provided enough information to identify all masses and springs in an Nth order serial-type mechanical system and the complete method is described in [3].

Recently (1996), Ram and Elhay [7] provided an important practical result by proving that the second set of resonance modes (obtained from the constrained system) could always be obtained from the anti-resonance modes of the unconstrained system. The main contribution of our work in [1] was to combine the works in [3] and [7] and demonstrate the usefulness of inverse eigenvalue theory to the online identification of a serial-type Nth order mechanical structure. The need to physically lock the system during experiments was removed. From the results in [1] the complete parameter identification could now be performed from a single frequency response experiment on the unconstrained system.

In the work presented here, we further extend our previous work [1, 5, 6] to a system containing coupled inertia terms and hence removing the restriction of a serial-type mechanical structure. We assume a parallel structure of M serial-type chains, were the final mass in each chain is coupled through an inertia term to all other final masses in all other chains. In each chain there will be N masses and $N-1$ springs, giving a total of MN unknown mass parameters, $M(N-1)$ spring parameters and $\frac{M!}{2!(M-2)!}$ coupled inertia parameters, where $M! = 1 \cdot 2 \cdots M$. Such elasticity systems frequently appear in multiple arm robots with non-coupled transmission elasticities and coupled elasticity modes between the arms.

System identification in general is a very active research field. However, the majority of the results appearing in the literature provide black-box type results. For example, generic type models, such as ARMAX with a subjectively chosen model order, are often assumed and the algorithms provide parameter estimates of such generic models. One advantage of flexible mechanics is the fact that the model structure and order (ie. the dominating elasticity modes) can be found quite accurately by performing physical static load analysis on the mechanics (for example by applying a constant load to one endpoint and measuring forces and deflections by strain gauges and linear transducers). In the work presented here, we exploit the form of the mechanical structure to a large degree and the main advantage is the fact that the identified parameters will have meaningful physical interpretations.

2. Elasticity Model with Coupled Inertia

The elasticity model with coupled inertia is presented in Figure 1. In the Figure we have defined $L_1 = N(M-1)$ and

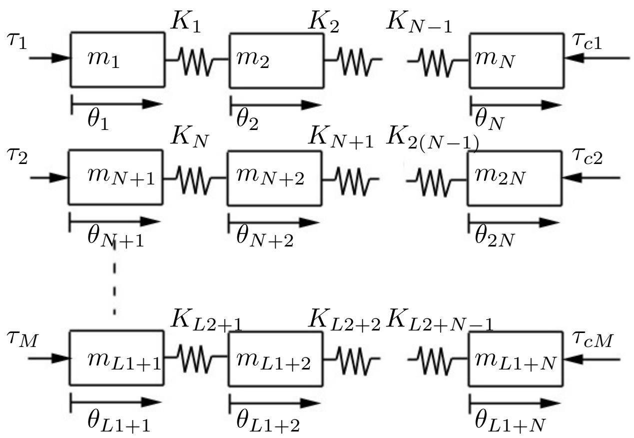

Figure 1: *Illustration of mechanical system with inertia couplings.*

$L_2 = (M-1)(N-1)$, where N is the number of masses in one horizontal branch and M is the number of horizontal parallel branches. The coupling terms are defined as follows.

$$\tau_{ci} = -\sum_{j \in J_i} \left(m_{Ni,Nj} \ddot{\theta}_{Nj} \right) \tag{1}$$

where the set $J_i = \{1, \cdots, M\} \backslash \{i\}$. In other words, all the final masses in a given horizontal branch are coupled through inertia to all the final masses in all the other branches. One example of such a mechanical system is a flexible robot manipulator where only the arm states are coupled through inertia terms, see for example [6], and the motor state variables have no inertia couplings.

In this paper we solve the identification problem under the following assumptions. 1) The torques $\tau_1, \cdots, \tau_M$ are the control variables, 2) The states $\theta_1, \theta_{N+1}, \cdots, \theta_{L1+1}$ can be measured (states on which control variables act, typically motors), 3) No other state variables can be measured, 4) All masses m_i, coupled masses $m_{i,j}$ and springs K_i are unknown.

The assumptions above are realistic for a flexible robot manipulator with M motors and N flexible modes per motor. The N flexible modes per motor are caused by, for example, transmission elasticities or elastic arm modes. There are no direct measurements except for the motor measurements.

3. Inverse Problem

In this section we describe the identification solution. First, the resonance modes and the behaviour at low and high frequencies are estimated, as described in section 3.1. Under certain conditions, the low and high frequency behaviour provide estimates for the motor mass and the sum of masses in each branch. Second, the coupled frequency behaviour gives, under certain conditions, estimates of the coupled inertia terms $m_{Ni,Nj}$ appearing in eq. (1). Third, given the parameters from section 3.1 and 3.2, inverse eigenvalue theory finds a unique solution for the remaining unknown parameters as described in sections 3.3 and 3.4.

3.1. Resonance and Low/High Frequency Estimation

By performing a direct frequency response analysis on each branch in Figure 1, we get an estimate of the transfer function amplitude from motor torque to motor acceleration, ie. $\left|\frac{\theta(\jmath\omega)\cdot\omega^2}{\tau(\jmath\omega)}\right|$, where $\jmath$ is the complex unit. The frequency responses will be generated by sinusoidal excitation of the motor position states, and the acceleration response is given by the position response and the sinusoidal excitation frequency. A typical example is shown in Figure 2 where the frequency response was generated by a system with three branches and three masses in each branch. The frequency response provides the two following equations.

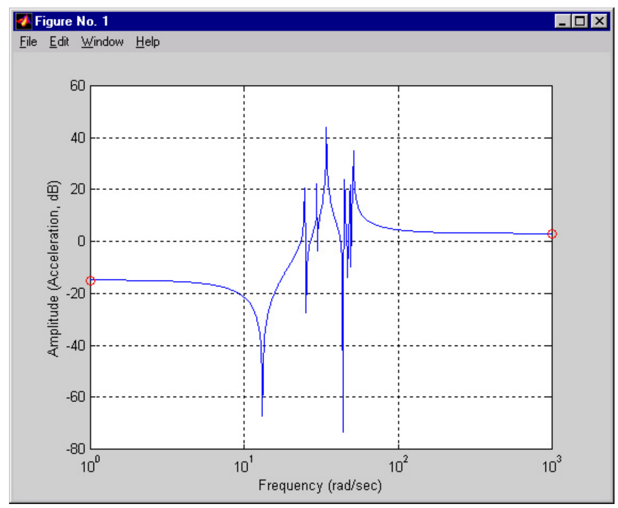

Figure 2: *Frequency amplitude diagram from τ_1 to $\ddot{\theta}_1$. The amplitude to acceleration is generated by multiplying ω^2 to the position amplitude θ_1 for each frequency ω.*

$$A_{iL} = \left(\sum_{j=N(i-1)+1}^{Ni} m_j \right)^{-1} \tag{2}$$

$$A_{iH} = \frac{1}{m_{N(i-1)+1}} \tag{3}$$

where A_{iL} and A_{iH} are the low and high frequency amplitudes, respectively. In Figure 2 A_{1L} and A_{1H} are circled, and we have $A_{1L} = -15.1(dB) = 0.176$ and $A_{1H} = 3.1(dB) = 1.430$. Note that the equation (2) is only correct under the following condition.

$$\tau_{j \in J_i} = K_R(\theta_{0j} - \theta_{N(j-1)+1}) \tag{4}$$

When generating a frequency response for one branch, then all other branches must have a position controller activated, where K_R is the controller gain and θ_{0j} is the position reference. For unconstrained systems, eq. (2) does not hold, since the coupled inertia terms will enter the equation. Since $\tau_1, \cdots, \tau_M$ are the control variables, however, the condition (4) is easy to satisfy.

For the inverse eigenvalue problem in section 3.3, the resonance and anti-resonance frequencies for each branch is required. The resonance frequencies will be the same for all branches, while the anti-resonance frequencies will differ. For the example in Figure 2 we have the eigenvalues

$$\lambda = \left[0, 24.63^2, 29.59^2, 34.16^2, 44.92^2, 48.60^2, 51.28^2\right]^T \quad (5)$$

$$\lambda_0 = \left[13.11^2, 25.29^2, 29.75^2, 43.62^2, 46.88^2, 49.33^2\right]^T \quad (6)$$

where λ are the squared resonance frequencies and λ_0 are the squared anti-resonance frequencies. Note that two eigenvalues at zero are removed for each parallel chain. The extra two eigenvalues for each chain correspond to the other parallel chains and carry no extra information. The remaining eigenvalue at zero is caused by a double-integrator from τ to θ for the chain. The λ_0 vector is generated for all of the horizontal mechanical branches in Figure 1. The eigenvalues are read from the unconstrained frequency response, since the position controller in eq. (4) will change the resonance frequencies.

3.2. Coupled Frequency Response

Let $i, j \in [1, N+1, \cdots, (M-1)N+1]$ and $i \neq j$ be the indices for θ_i and τ_j. The coupled frequency response $\left|\frac{\theta_i(j\omega)\omega^2}{\tau_j(j\omega)}\right|$ will then provide an estimate for the coupled inertia term $m_{Ni,Nj}$. Let A_C be the low frequency gain of the cross-coupled frequency response. We then have

$$m_{Ni,Nj} = \frac{1}{2A_C}\left(-1 \pm \sqrt{1 + 4A_C^2 \frac{1}{A_{iL}A_{jL}}}\right) \quad (7)$$

where A_{iL} is given by eq. (2). Note the $\pm$ sign. The correct solution can be found from the phase response $\angle\left(\frac{\ddot{\theta}_i(j0)}{\tau_j(j0)}\right)$. The sign S of $m_{Ni,Nj}$ is estimated as follows.

$$\begin{aligned} S &= sign\angle\left(\frac{\ddot{\theta}_i(j0)}{\tau_j(j0)}\right) \ \text{if} \ \frac{1}{A_{iL}A_{jL}} < m^2_{Ni,Nj} \\ S &= -sign\angle\left(\frac{\ddot{\theta}_i(j0)}{\tau_j(j0)}\right) \ \text{otherwise} \end{aligned} \quad (8)$$

Note also that equation 7 is only correct given the following condition.

$$\tau_{k\in J_{i,j}} = K_R(\theta_{0k} - \theta_{N(k-1)+1}) \quad (9)$$

where $J_{i,j} = \{1, \cdots, M\}\backslash\{i, j\}$. In other words, a position controller must be activated on all branches which contain neither θ_i nor τ_j.

Figure 3 shows an example of a cross-coupled frequency response. Note that the eigenvalues are different to Figure 2, because of the position controllers in eq. (9). The low frequency amplitude in this case is equal to $A_C = -38.6(dB) = 0.0117$.

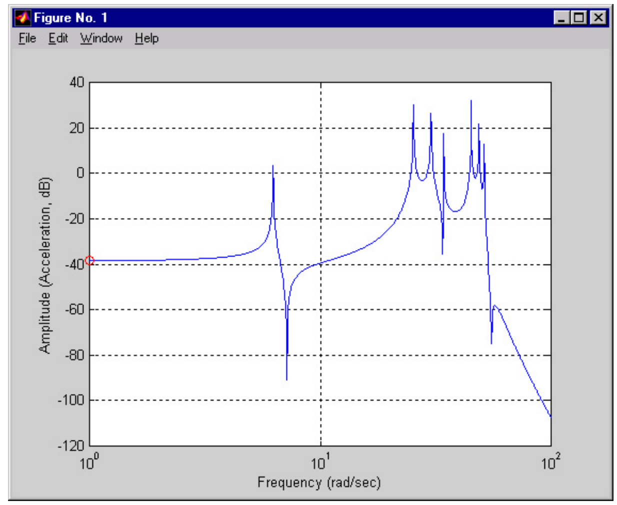

Figure 3: *Frequency amplitude diagram from τ_2 to $\ddot{\theta}_1$. The amplitude to acceleration is generated by multiplying ω^2 to the position amplitude θ_1 for each frequency ω.*

3.3. Inverse Eigenvalue Problem for Mass-Spring System

Let the system in Figure 1 be described by the following equation.

$$\mathbf{M}\ddot{\mathbf{\Theta}} + \mathbf{K}\mathbf{\Theta} = \tau \quad (10)$$

where $\mathbf{\Theta} = [\theta_1, \theta_2, \cdots, \theta_{MN}]$ and $\tau \in \Re^{MN}$ where all terms are zero except for the motor torques $\tau_m = [\tau_1, \tau_2, \cdots, \tau_M]$. $\mathbf{M}$ and $\mathbf{K}$ are the $MN \times MN$ mass and spring matrices respectively. Let the square root of $\mathbf{M}$ be defined as follows.

$$\mathbf{L} = \mathbf{M}^{1/2} = \mathbf{V}\mathbf{D}^{1/2}\mathbf{V}^T \quad (11)$$

where $\mathbf{V}$ is the eigenvector matrix and $\mathbf{D}$ is the diagonal eigenvalue matrix. The square root $\mathbf{D}^{1/2}$ is a diagonal matrix with the square root of the eigenvalues (eigenfrequencies) on the diagonal. For the eigenvalue formulation, we introduce the state variable shift $\mathbf{u} = \mathbf{L}^T\mathbf{\Theta}$, $\ddot{\mathbf{\Theta}} = -\lambda\mathbf{\Theta}$ and we set $\tau = 0$. We then have

$$\mathbf{L}^{-1}\mathbf{K}\mathbf{L}^{-T}\mathbf{u} = \mathbf{B}\mathbf{u} = \lambda\mathbf{I}\mathbf{u} \quad (12)$$

where $\mathbf{I}$ is the identity matrix. The first eigenvector u_1 of $\mathbf{B}$ can be computed from the eigenvalue vectors λ_0 and λ, see [3]. The components of the first eigenvector $\mathbf{u}_N$ are given by

$$[\mathbf{u}_1^{(i)}]^2 = \Pi_{j=1}^{N-1}(\lambda_j^0 - \lambda_i)/\Pi_{j=1}^{N}{}'(\lambda_j - \lambda_i) \quad (13)$$

where $'$ indicates that the term $j = i$ is omitted. Equation (13) is found from solving the eigenvalue problem with one mass locked, see [3], pages 66-67. Note that the eigenvalue vectors λ_0 and λ are now generated by a coupled inertia system for each parallel chain, while in [3] only one serial-type structure was assumed. By using the fact that the eigenvectors are orthogonal, we can now compute B_{11}, B_{12} and B_{22}

as follows.

$$\begin{aligned} B_{11} &= \mathbf{u}_1^T \mathbf{D} \mathbf{u}_1 & (14) \\ \mathbf{d}_2 &= B_{11}\mathbf{u}_1 - \mathbf{D}\mathbf{u}_1 \\ B_{12} &= \sqrt{\mathbf{d}_2^T \mathbf{d}_2} & (15) \\ \mathbf{u}_2 &= \frac{1}{B_{12}} \mathbf{d}_2 \\ B_{22} &= \mathbf{u}_2^T \mathbf{D} \mathbf{u}_2 & (16) \end{aligned}$$

Eqs. (13) to (16) above define the first step of the so-called Lanczos algorithm, see [4].

3.4. Spring and Mass Coefficients

For the system in Figure 1, the $\mathbf{B}$ matrix has a special form which we will exploit. Regardless of the dimensions M and N, we have the following

$$\begin{aligned} B_{11} &= \frac{K_1}{m_1} & (17) \\ B_{22} &= \frac{K_1 + K_2}{m_2} & (18) \\ \frac{B_{12}^2}{B_{11}^2} &= \frac{m_1}{m_2} & (19) \end{aligned}$$

Hence, by knowing B_{11}, B_{12}, B_{22} and m_1, we can solve for K_1, K_2 and m_2. We can always rearrange the branches such that any motor state $i \in [1, N+1, \cdots, (M-1)N+1]$ appears first. Hence, by rearranging the branches, recomputing $\mathbf{B}$ and by knowing the first mass in each branch, we can in general solve for

$$\begin{aligned} K_i \qquad & i \in [1, N, \cdots, (M-1)(N-1)+1] \bigcup \\ & [2, N+1, \cdots, (M-1)(N-1)+2] & (20) \\ m_i \qquad & i \in [2, N+2, \cdots, (M-1)N+2] & (21) \end{aligned}$$

If there are no more than three masses in each branch, then we are done. The final mass is given by

$$m_N = \frac{1}{A_{iL}} - \sum_{i=1}^{N-1} m_i \qquad (22)$$

If there are more than three masses in each branch, we can remove the first two masses in each branch, recompute the vectors λ_0 and λ from the transfer functions, and repeat the procedure described above. The transfer functions can be found from the old transfer functions and knowledge of K_1 and m_1 in each branch, as given by the equations below.

$$\begin{aligned} \tau_2(\jmath\omega) &= \tau_1(\jmath\omega) + m_1\omega^2\theta_1(\jmath\omega) \\ \theta_2(\jmath\omega) &= \theta_1(\jmath\omega) - \frac{1}{K_1}\tau_1(\jmath\omega) \end{aligned}$$

where τ_2 is the left-hand force acting on θ_2 through the spring K_1. Pairs of masses and springs are identified iteratively in this fashion, until only the final mass is left and computed by eq. (22).

In practice, friction distorts the low frequency amplitude gain, while measurement noise can distort the high-frequency gains. The friction effects can be eliminated or at least significantly reduced, see for example [1] and [6]. If the high-frequency noise becomes a problem, an alternative iterative solution will be used. The motor masses are in many cases specified in data-sheets by the manufacturers. Hence, the dependency on equation (3) can be eliminated. Instead of removing two masses in each iterative step as described above, only one mass will be removed. The new first mass in each branch is now already identified from the previous iteration, and hence the high-frequency gain information will never be required.

To our knowledge the results in this section are new. The main difference to previous results on serial-type structures, is the fact that the complete $\mathbf{B}$ matrix in eq. (12) can not be iteratively identified by the Lanczos algorithm because of the introduced coupled inertia terms. Instead, we solve the coupled inertia terms as described in eq. (7). The remaining parameters are solved iteratively by removing two (or one in presence of significant measurement noise) masses in each parallel chain, and performing the first step in the Lanczos algorithm until all parameters are identified. The Lanczos algorithm can be used as long as there are no coupled inertia terms for the current first mass in each chain.

4. Case Study

We now study a particular system with three branches and three masses in each branch. The $\mathbf{M}$ and $\mathbf{K}$ matrices are given as follows.

$$\mathbf{M} = \begin{bmatrix} m_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & m_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & m_3 & \cdot & \cdot & m_{36} & \cdot & \cdot & m_{39} \\ \cdot & \cdot & \cdot & m_4 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & m_5 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & m_{36} & \cdot & \cdot & m_6 & \cdot & \cdot & m_{69} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & m_7 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & m_8 & \cdot \\ \cdot & \cdot & m_{39} & \cdot & \cdot & m_{69} & \cdot & \cdot & m_9 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} K_1 & -K_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -K_1 & K_{12} & -K2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -K_2 & K_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & K_3 & -K_3 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -K_3 & K_{34} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -K_4 & K_4 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_5 & -K_5 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -K_5 & K_{56} & -K_6 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -K_6 & K_6 \end{bmatrix}$$

where $\cdot$ are zero-elements and $K_{ij} = K_i + K_j$. Note that every third mass-element has two coupled inertia terms.

The frequency responses for each branch are shown in Figures 2 and 4, while the cross coupled responses are shown in Figures 3 and 5. For the three branches we then have the following λ_0 vectors.

$$\lambda_{01} = \left[13.11^2, 25.29^2, 29.75^2, 43.62^2, 46.88^2, 49.33^2\right]^T$$

$$\lambda_{02} = \left[14.93^2, 27.05^2, 34.15^2, 43.13^2, 47.94^2, 51.27^2\right]^T$$

$$\lambda_{03} = \left[15.63^2, 27.55^2, 33.55^2, 43.38^2, 48.12^2, 51.17^2\right]^T$$

$$\lambda = \left[0, 24.63^2, 29.59^2, 34.16^2, 44.92^2, 48.60^2, 51.28^2\right]^T$$

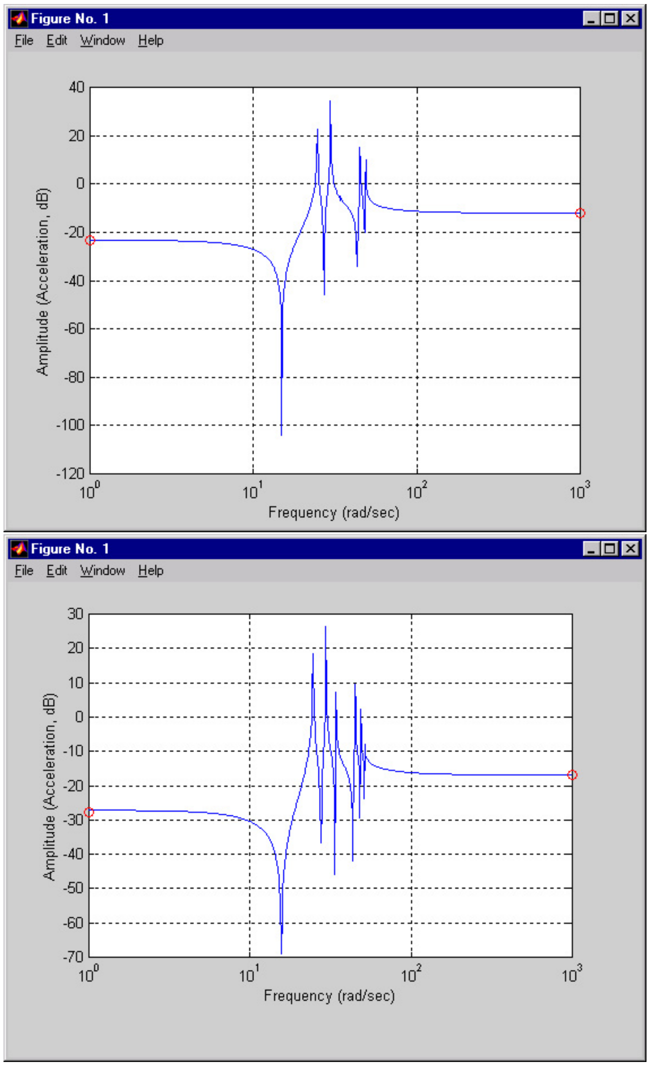

Figure 4: *Frequency amplitude diagram from τ_2 to $\ddot{\theta}_4$ (top) and τ_3 to $\ddot{\theta}_7$ (bottom).*

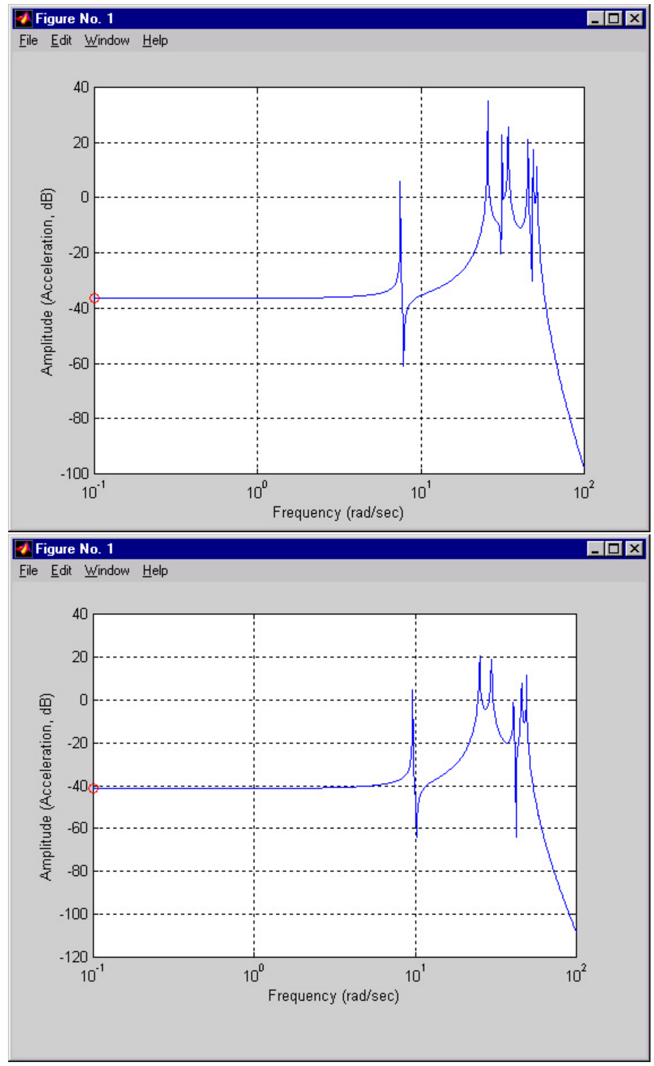

Figure 5: *Frequency amplitude diagram from τ_3 to $\ddot{\theta}_1$ (top) and τ_3 to $\ddot{\theta}_4$ (bottom).*

where λ_{0i} is the anti-resonance vector of the ith parallel chain. The gain values from section 3.1 are given as follows.

$$\begin{aligned} A_{iH} &= [1.431, 0.250, 0.143]^T \\ A_{iL} &= [0.175, 0.067, 0.042]^T \\ A_C &= [0.012, 0.015, 0.009]^T \end{aligned}$$

The elements from section 3.3 are now as follows. For the first branch we have

$$\begin{aligned} \mathbf{u_1} &= [0.3569, 0.1808, 0.1292, 0.7077, 0.2070, 0.2104, 0.4850]^T \\ B_{11} &= 1428.6 \\ \mathbf{d_2} &= [509.89, 148.67, 71.43, 185.14, -122.06, -196.46, -582.52]^T \\ B_{12} &= 845.15 \\ \mathbf{u_2} &= [0.6033, 0.1759, 0.0845, 0.2191, -0.1444, -0.2325, -0.6892]^T \\ B_{22} &= 1500 \end{aligned}$$

We can now solve for all parameters belonging to the first branch.

$$\begin{aligned} m_1 &= \frac{1}{A_{1H}} = 0.7 \\ m_{36} &= 1.0 \quad \text{from eq. (7)} \\ m_{39} &= 2.0 \quad \text{from eq. (7)} \\ K_1 &= B_{11} m_1 = 1000 \\ m_2 &= \frac{m_1 B_{11}^2}{B_{12}^2} = 2.0 \\ K_2 &= B_{22} m_2 - K_1 = 2000 \\ m_3 &= \frac{1}{A_{1L}} - m_1 - m_2 = 3.0 \end{aligned}$$

Similarly, for the second branch we have.

$$\begin{aligned} \mathbf{u_1} &= [0.5248, 0.5016, 0.5723, 0.0368, 0.3019, 0.2289, 0.0218]^T \\ B_{11} &= 750.0 \\ \mathbf{d_2} &= [393.60, 71.97, -71.83, -15.36, -382.91, -369.07, -40.90]^T \\ B_{12} &= 670.82 \\ \mathbf{u_2} &= [0.5867, 0.1073, -0.1071, -0.0229, -0.5708, -0.5502, -0.0610]^T \\ B_{22} &= 1400 \end{aligned}$$

And we get the following parameters.

$$\begin{aligned} m_4 &= \frac{1}{A_{2H}} = 4.0 \\ m_{69} &= 3.0 \quad \text{from eq. (7)} \\ K_3 &= B_{11} m_4 = 3000 \\ m_5 &= \frac{m_4 B_{11}^2}{B_{12}^2} = 5.0 \\ K_4 &= B_{22} m_5 - K_3 = 4000 \\ m_6 &= \frac{1}{A_{2L}} - m_4 - m_5 = 6.0 \end{aligned}$$

For the third branch we have.

$$\begin{aligned} \mathbf{u}_1 &= [0.5538, 0.5238, 0.4761, 0.2561, 0.2881, 0.1890, 0.0894]^T \\ B_{11} &= 714.29 \\ \mathbf{d}_2 &= [395.56, 56.45, -76.76, -115.94, -375.61, -311.47, -171.31]^T \\ B_{12} &= 668.15 \\ \mathbf{u}_2 &= [0.5920, 0.0845, -0.1149, -0.1735, -0.5622, -0.4662, -0.2564]^T \\ B_{22} &= 1375 \end{aligned}$$

Finally, we get the following parameters.

$$\begin{aligned} m_7 &= \frac{1}{A_{3H}} = 7.0 \\ K_5 &= B_{11} m_7 = 5000 \\ m_8 &= \frac{m_7 B_{11}^2}{B_{12}^2} = 8.0 \\ K_6 &= B_{22} m_8 - K_5 = 6000 \\ m_9 &= \frac{1}{A_{3L}} - m_7 - m_8 = 8.9 \end{aligned}$$

All the masses $m_1, \cdots, m_9$, all the springs $K_1, \cdots, K_6$ and all the inertia terms m_{36}, m_{39}, m_{69} have been successfully identified given only low and high-frequency information from sinusoidal frequency responses plus all resonance and anti-resonance frequencies.

5. Conclusions

In this paper we have presented a new method for solving the inverse eigenvalue problem for a parallel mechanical structure consisting of M parallel chains with N masses in each chain and coupled inertia terms between all final masses in each chain. All MN mass parameters, $M(N-1)$ spring parameters and $\frac{M!}{2!(M-2)!}$ coupled inertia terms are identified from sinusoidal frequency responses generated by online measurements.

Because of the coupled inertia terms, the standard Lanczos algorithm can not be applied directly. Instead, we perform an iterative partial identification and iteratively reduce the order of the system. In this fashion, we sequentially identify all masses and springs in all parallel chains. The coupled inertia terms are identified from the low-frequency behaviour of the cross-excited frequency responses. One important restriction is the fact that a P-type controller is required on the first masses in all chains which are not direct part of the cross-coupled frequency excitation in order to identify the cross-coupled terms. The P-type controllers do not impose a loss of generality, since the torques for all motor states can be actively controlled.

In our previous work, [1], we also demonstrated the identification of damper elements which are located in parallel to all the spring elements. The inverse eigenvalue theory does not provide a solution for systems containing damper elements, and a non-linear optimisation method was applied in [1]. Such a method works when the damper elements are small and do not significantly affect the values of the resonance and anti-resonance frequencies. For the mechanical structure with coupled inertia terms presented here, the same optimisation procedure as described in [1] can be applied here.

In the paper we have combined an important result from the vibration literature with the solution of inverse eigenvalue problems. No physical mechanical setups requiring clamps or external impulse responses are required to identify a mechanical system with inertia couplings. To our knowledge, this approach to the identification of flexible mechanical dynamics is new. The method has significant advantages compared to other identification algorithms. The main advantage is the fact that only motor encoder position and motor torque are required to identify the springs, masses and coupled inertias of an Nth order system. Another advantage compared to black-box-type identification, is the fact that the form of the mechanical structure is exploited to a large degree and that physical meaningful identification results are generated.

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