

Singularity-Free Reconfiguration of the 5-DOF Gantry-Tau Parallel Kinematic Machine

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Abstract: *This paper presents a systematic approach based on Mixed Integer Linear Programming for finding a near optimal singularity-free reconfiguration path of the 5-DOF Gantry-Tau parallel kinematic machine. The results in the paper demonstrate that singularity-free reconfiguration (change of assembly mode) of the machine is possible, which significantly increases the usable workspace. The method has been applied to a full-scale prototype and the singularity-free path has been verified both in simulations and with physical experiments using real-time control of the prototype.*

1 Introduction

In this paper a new approach for automatically reconfiguring the Gantry-Tau 5-Degrees-of-Freedom (DOF) parallel kinematic machine (PKM) is presented. The reconfiguration is possible to perform without passing any type 2 singularities and the reconfiguration significantly increases the usable workspace of the robot since two assembly modes of the robot can be reached seamlessly. The concept for calculating the reconfiguration movements is based on a discretisation of the 5-DOF workspace and Mixed Integer Linear Programming (MILP) is used to find the optimal reconfiguration path. The optimisation criterion is to keep the sum of the condition numbers of the manipulator statics matrix as small as possible with a limited number of optimisation steps. The paper presents simulations and experimental results on automatic reconfiguration of a full-scale prototype located at the University of Agder, Norway. During the reconfiguration, as well as during normal operation, only axial forces (and no torsional moments or bending) are transmitted in the 6-links of the Gantry Tau 5-DOF PKM structure.

Majou, Wenger and Chablat (2001) define the difference between working modes and assembly modes for PKMs. Working modes are associated with the different solutions of the inverse kinematics of the PKM, while assembly modes are associated with the different solutions of the forward kinematics. Moreover, a change of a working mode is associated with a serial singularity, ie. a change of joint position has no effect on tool position. A change of assembly mode is (often) associated with parallel singularities, ie. the tool cannot resist any effort (gains one or more DOFs) and in turn, becomes uncontrollable. This paper demonstrates that a change of assembly mode does not necessarily mean that a parallel singularity needs to be passed. An optimisation approach for the 5-DOF Gantry-Tau which changes both assembly and working modes while avoiding parallel singularities is presented. Serial singularities are not avoided, since they can be driven straight through and represent no danger for physical damage of the machine.

In Budde, Last and Hesselbach (2005) an approach for reconfiguring the Triglade robot was presented. This approach drives the three linear actuators of the robot from one configuration (or assembly mode) to the other and in this way a larger total workspace of the robot can be achieved. However, the approach requires passing an internal parallel singularity where the tool platform acquires an additional degree of freedom. The described approach is based on force control of one of the actuators and using gravity to pull the tool platform through the singularity. This approach has several disadvantages. First, the joints and the links of the Triglade robot must be able to withstand higher torsional moments. Hence, the links and joints will be unnecessarily over-dimensioned. Second, the tool platform may start to rotate at the singularity and there is no guarantee that gravity will pull the platform through. In fact, this platform rotation must actively

be avoided, as the Triglide robot can only be reconfigured if all platform rotations are zero. Moreover, if the platform rotation occurs, gravity may cause large internal forces which can damage the machine.

Chablat and Wenger (1998) and Chablat and Wenger (2004) presented results for a 3-RRR planar parallel manipulator. The authors showed that reconfiguration without passing any singularities was possible. A method for finding singularity-free trajectories based on Octree models and path-connectivity analysis was presented. Unfortunately, the 3-RRR manipulator analysed in the paper has a small workspace to installation space ratio and only 3-DOF with 2-DOF for positioning which limits its use as a general purpose manipulator.

In Zein, Wenger and Chablat (2006) the authors presented an analysis based on cusp points to help determine if singularity-free reconfiguration is possible. Cusp points are special coordinates on singularity curves where triple direct kinematic solutions meet in the joint-space for parallel manipulators. It is proven that change of assembly mode is possible through cusp points. The authors presented a systematic approach for finding the cusp points and the method was exemplified for a 3-RPR manipulator.

The results in this paper are interesting for the following reasons: a singularity-free reconfiguration path has been found for a PKM with 5-DOF and a very large workspace to installation space ratio. The authors believe that the 5-DOF Gantry-Tau is one of the most promising PKM structures to date for industrial applications. The Gantry Tau PKM could even have a higher industrial impact than the successful Delta robot concept, Clavel (1991).

2 The Gantry-Tau Structure

The inverse kinematics (IK) of the Gantry-Tau has two solutions for each of the three base actuators for a fixed TCP position and orientation. Hence, there are in total eight possible solutions for the IK. In addition, as illustrated in Fig. 1, the forward kinematics has two solutions for each set of actuator positions. Each of these solutions is typically called working mode in the literature, as in the past the only way to change the mode has been to dismount and re-assemble the PKM or to run the PKM through a parallel singularity as in Budde, Last and Hesselbach (2005). Figure 1 illustrates the two most important assembly modes of the Gantry-Tau, where the arms are mounted either all left or all right with respect to the three base actuators. The two assembly modes in Figure 1 are important because the robot can only reach both of the workspace extremes in these two modes. The kinematics of both the 3-DOF and 5-DOF versions of the Gantry-Tau have been presented before and will not be repeated in this paper. Interested readers can consult the following papers: Murray, Hovland and Brogårdh (2006), Hovland, Choux, Murray, Tyapin and Brogårdh (2008).

The approach taken in this paper for reconfiguration is based on the statics matrix \mathbf{H} of the manipulator. The derivation of this

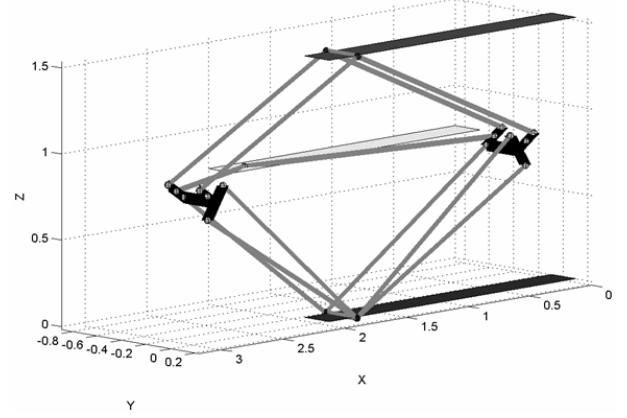


Figure 1: *The two most important assembly modes for Gantry-Tau, all left and all right.*

matrix is given as follows.

$$\begin{aligned} \mathbf{X} &= [X \ Y \ Z]^T & \theta &= [\alpha \ \beta \ \gamma]^T \\ \mathbf{F} &= [F_x \ F_y \ F_z]^T & \mathbf{M} &= [M_x \ M_y \ M_z]^T \\ \mathbf{L} &= [l_1 \ l_2 \ l_3 \ l_4 \ l_5 \ l_6]^T & \mathbf{F}_a &= [F_1 \ F_2 \ F_3 \ F_4 \ F_5 \ F_6]^T \end{aligned}$$

where X, Y, Z are the Cartesian TCP coordinates, α, β, γ are the Cartesian TCP orientation angles, l_i are the link lengths and F_i are link forces where $i = 1, \dots, 6$. F_x, F_y and F_z are the external Cartesian forces acting on the TCP and M_x, M_y and M_z are the external Cartesian torques acting on the TCP. The relationship between the TCP forces and the link forces are:

$$\mathbf{F} = \sum_{i=1}^6 F_i \mathbf{u}_i \quad \mathbf{M} = \sum_{i=1}^6 F_i \mathbf{A}_i \times \mathbf{u}_i \quad (1)$$

where \mathbf{u}_i is a unit vector in the direction of link i and \mathbf{A}_i is a vector pointing from the TCP to the end-point of link i on the platform. The two equations above can be rewritten using the 6×6 statics matrix \mathbf{H} .

$$\begin{bmatrix} \mathbf{F} \\ \mathbf{M} \end{bmatrix} = \mathbf{H} \mathbf{F}_a \quad (2)$$

3 Reconfiguration Using 5-DOF Kinematics

In this section an approach based on Mixed Integer Linear Programming (MILP) optimisation is used to determine a near optimal path for automatically reconfiguring the Gantry-Tau PKM while avoiding Type-II singularities.

Figure 2 shows the workspace in the YZ-plane of the Gantry-Tau. This workspace is discretised into smaller spherical regions and each region is associated with a boolean variable named $\delta_{i,j}$. Figure 3 shows the entire state vector \mathbf{x} for the optimisation problem where the optimised path is discretised into N steps. In addition to the region variables $\delta_{i,j}$, the state vector \mathbf{x} contains the following variables for each step i : κ_i (condition number of the PKM statics matrix \mathbf{H} (see eq.(2))), γ_i (boolean help variable to stop optimisation when the goal region j_N has been reached), $P_{i,x}, P_{i,y}, P_{i,z}, R_{i,x}, R_{i,z}$ (5-DOF TCP position and orientation)

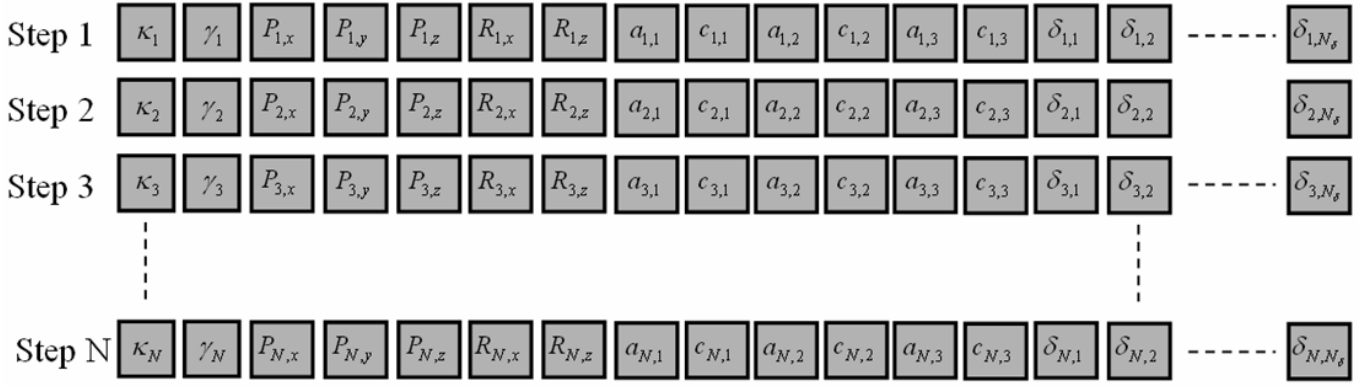


Figure 3: Illustration of state vector \mathbf{x} .

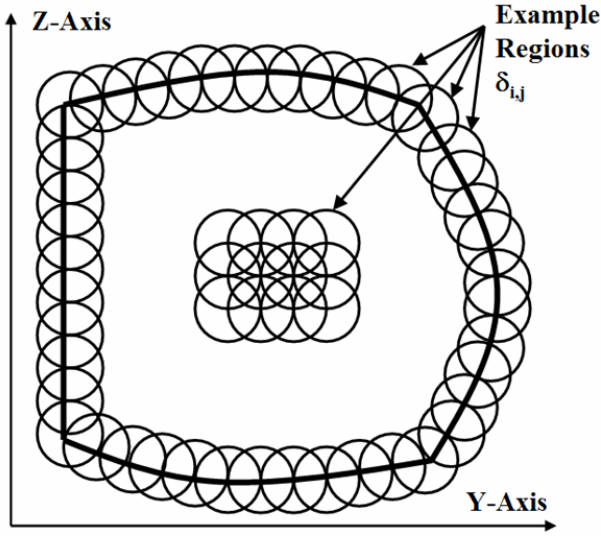


Figure 2: Example of spherical regions $\delta_{i,j}$.

and $a_{i,j}$, $c_{i,j}$ (boolean working mode and working change mode variables for the main actuators $j = 1, 2, 3$).

The optimisation program is given by the following objective function and linear constraints.

$$\min \mathbf{g}^T \mathbf{x} \quad (3)$$

$$\text{subject to} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \quad (4)$$

Some of the variables in the vector \mathbf{x} are continuous while others are boolean. Hence, the optimisation problem defined by eqs. (3-4) is referred to as a MILP, see for example Schrijver (1986). The state vector \mathbf{x} is formulated as illustrated in Figure 3. The variables $\delta_{i,j}$ exemplified in Figure 2 are boolean region variables, while the Cartesian positions at step i , $P_{x,i}$, $P_{y,i}$, $P_{z,i}$, are examples of continuous variables.

The optimisation problem is defined in this case as a minimisation of the sum of the condition numbers of the PKM matrix \mathbf{H} during the reconfiguration path, ie.

$$\min \left(\sum_{i=1}^N \kappa_i \right) \quad (5)$$

Hence, the objective function vector \mathbf{g} equals

$$g(k) = 1 \text{ if } k \in \{1, 13 + N_d + 1, \dots, (13 + N_d)(N - 1) + 1\} \quad (6)$$

$$g(k) = 0 \text{ otherwise} \quad (7)$$

The dimension of the vector \mathbf{g} equals $(13 + N_d)N \times 1$, where N_d is the total number of regions $\delta_{i,j}$ for one step i and N the number of optimisation steps. The constant 13 is required because there are 13 variables in each row of Fig. 3 before the region booleans $\delta_{i,j}$. With this definition of \mathbf{g} , eqs. (3), (6-7) equal eq. (5). The indexes in eq. (6) equal the first column in Figure 3.

Let j_0 be the initial region and j_N be the final goal region. Then, the following constraints are required.

$$\delta_{1,j_0} = 1 \quad (8)$$

$$\delta_{1,j} = 0 \quad j \neq j_0 \quad (9)$$

$$\delta_{N,j_N} = 1 \quad (10)$$

$$\delta_{N,j} = 0 \quad j \neq j_N \quad (11)$$

where N is the final step number. For each step number i only one state $\delta_{i,j}$ can be set to 1, ie.

$$\sum_{j=1}^{N_d} \delta_{i,j} = 1 \quad (12)$$

In order to prevent that the Cartesian distance between two step numbers becomes too large, a new type of constraint based on the TCP coordinates is introduced as follows.

$$-\Delta X \leq P_{x,i} - P_{x,i-1} \leq \Delta X \quad (13)$$

$$-\Delta Y \leq P_{y,i} - P_{y,i-1} \leq \Delta Y \quad (14)$$

$$-\Delta Z \leq P_{z,i} - P_{z,i-1} \leq \Delta Z \quad (15)$$

$$-\Delta R_x \leq R_{x,i} - R_{x,i-1} \leq \Delta R_x \quad (16)$$

$$-\Delta R_z \leq R_{z,i} - R_{z,i-1} \leq \Delta R_z \quad (17)$$

The constraints in eqs. (13-17) prevent that the TCP position changes more than the threshold distances ΔX , ΔY , ΔZ between two step numbers. The threshold distances are chosen such

that the maximum travelled distance equals the diameter of the region spheres illustrated in Figure 2. ΔR_x and ΔR_z are the maximum TCP orientation change values, for examples 5 degrees, between two steps.

To set the TCP positions and orientations for each step number i , the following mixed-integer types of constraints are required.

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } P_{x,i} = X(j) \quad (18)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } P_{y,i} = Y(j) \quad (19)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } P_{z,i} = Z(j) \quad (20)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } R_{x,i} = R_x(j) \quad (21)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } R_{z,i} = R_z(j) \quad (22)$$

For example, the logical constraint in eq. (18) can be implemented by two linear constraints as follows.

$$-m\delta_{i,j} + P_{x,i} \leq X(j) - m \quad (23)$$

$$M\delta_{i,j} - P_{x,i} \leq -X(j) + M \quad (24)$$

where m, M are the minimum and maximum values of $X(j)$ respectively, see for example Bemporad and Morari (1999) for examples of such logical constraints.

The following logical constraints ensure that the summation of the condition numbers in eq. (5) stops once the final goal region j_N has been reached.

$$\text{IF } \delta_{i,j} == 1 \quad \text{THEN } \kappa_i = \gamma_i \kappa(j) \quad (25)$$

$$\begin{aligned} \text{IF } \delta_{i,j_N} == 1 \quad \text{THEN } \gamma_i &= 0 \\ &\text{ELSE } \gamma_i = 1 \end{aligned} \quad (26)$$

The IF-THEN-ELSE type of logical constraint in eq. (26) can be implemented by the following four linear constraints.

$$0.99\delta_{i,j_N} + \gamma_i \leq 1 \quad (27)$$

$$-1.01\delta_{i,j_N} - \gamma_i \leq -1 \quad (28)$$

$$1.01\delta_{i,j_N} + \gamma_i \leq 1.01 \quad (29)$$

$$-0.99\delta_{i,j_N} - \gamma_i \leq -0.99 \quad (30)$$

The constants 0.99 and 1.01 are chosen as the lower and upper limit of the value 1 in eq. (26). The general logical rules for an IF-THEN-ELSE statement are shown in eq. (31-35), where m_1, m_2 and M_1, M_2 are the lower and upper limits on the functions f_1 and f_2 , respectively.

$$\text{IF } \delta == 1 \text{ THEN } z = f_1 \text{ ELSE } z = f_2 \quad (31)$$

$$(m_2 - M_1)\delta + z \leq f_2 \quad (32)$$

$$(m_1 - M_2)\delta - z \leq -f_2 \quad (33)$$

$$(M_2 - m_1)\delta + z \leq f_1 + (M_2 - m_1) \quad (34)$$

$$(M_1 - m_2)\delta - z \leq -f_1 + (M_1 - m_2) \quad (35)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } a_{i,k} = A_k(j) \quad (36)$$

$$\text{IF } \delta_{i,j} == 1 \text{ THEN } c_{i,k} = C_k(j) \quad (37)$$

$$\text{IF } c_{i-1,k} == 0 \text{ THEN } a_{i,k} = a_{i-1,k} \quad (38)$$

The first two logical constraints in eqs. (36-37) set the current working mode and the working mode change variables equal to the corresponding values for the active region $\delta_{i,j}$. The third logical constraint in eq. (38) sets the current working mode for actuator k ($k = 1, 2, 3$) equal to the working mode for the previous step, if the change variable equals zero. Hence, the working mode for actuator k is only allowed to change at step i if the change variable $c_{i-1,k}$ is equal to 1. In order to formulate the entire MILP optimisation problem, an offline pre-processing step is required which associates the following variables with a region j : $X(j), Y(j), Z(j), R_x(j), R_z(j), \kappa(j), A_k(j), C_k(j)$. $C_k(j) = 1$ represents regions in the workspace for two different working modes where the change of actuator k value approaches zero. $A_k(j) \in [0, 1]$ represents the working mode (left or right) for actuator k for region j .

4 Experiments

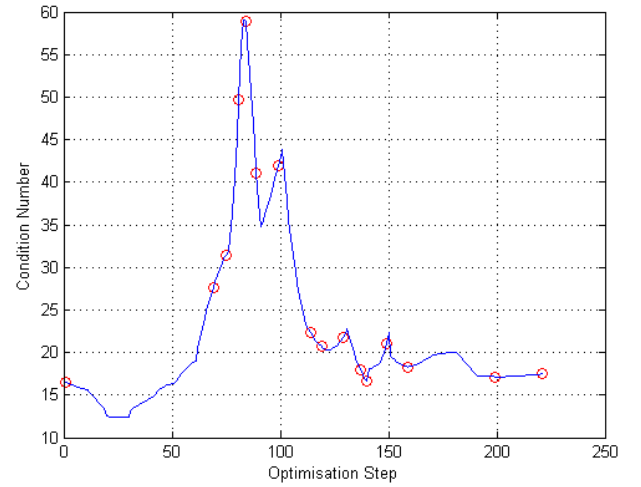


Figure 6: Condition number of matrix H along the optimised re-configuration path illustrated in Figures 4 and 5.

Figure 4 shows an example of an optimised path when the actuator lengths are 2.1m, the support frame has the depth 0.75m and the height 1.5m. The arm lengths are 1.09m for arms 1 and 2, and 1.25m for arm 3 which correspond to the 5-DOF Gantry-Tau machine located at the University of Agder. Arm 1 consists of the single link, arm 2 consists of the upper link pair and arm 3 consists of the lower link triangle.

Figure 5 shows the optimised path in the YZ-plane of the workspace. The actual path of the TCP is illustrated by the solid red line, while the singularities are marked by the lighter coloured lines. For example step 1 has a singularity-free workspace, while step 2 has a singular area when the TCP has Y-values below 0.15m and Z-values above 0.6m. Note that the X values of the TCP have no influence on the condition number of the statics matrix H for the Gantry-Tau.

Figure 6 shows the actual condition number plotted versus the optimisation step number i for the path in Figure 5. At singularities, the condition number of the statics matrix H approaches infinity, while Figure 5 shows that the path is well clear of singular points and the maximum condition number is 59. The max-

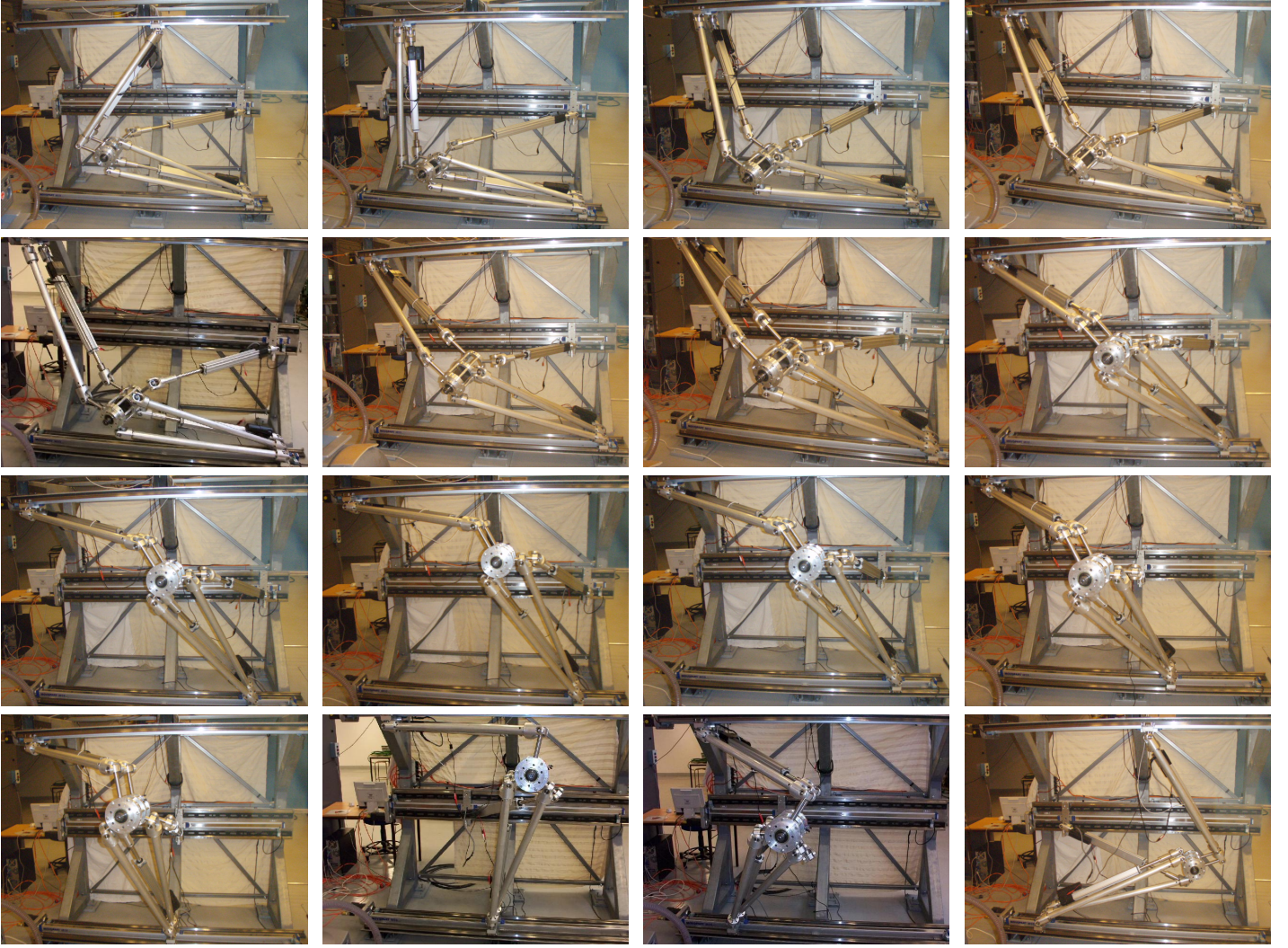


Figure 4: *Example of Reconfiguration Sequence on 5-DOF Gantry-Tau prototype at UiA (sequence moves from left to right and top to bottom).*

imum condition number value takes place at the 5th picture in Figures 4 and 5. The circled dots in Figure 6 correspond to the 16 pictures in Figures 4 and 5. The MILP optimisation problem was solved using ILOG CPLEX version 11 with a total of $N=23$ optimisation steps and $N_d = 45,936$ region booleans $\delta_{i,j}$ per step. The total number of variables for CPLEX to solve equals $(13 + N_d) * N = 1,056,827$ and the solution time is about 15 minutes on a standard computer. Figure 6 illustrates an interpolated path with 220 steps. The 16 pictures in Figures 4 and 5 illustrate 16 selected steps from the total interpolated path of 220 steps.

5 Conclusions

This paper has demonstrated that type 2 singularity-free reconfiguration of the 5-DOF Gantry-Tau parallel kinematic machine is possible. If passing Type-II singularities a manipulated platform will experience unwanted and uncontrollable rotations. The experiments demonstrated in Fig. 4 were conducted without experiencing any unwanted rotations of the manipulated platform.

The reconfiguration sequence is automated and the machine can follow pre-programmed paths to reconfigure in both directions (from left to right and vice versa). Because of the relatively short base actuators used for the prototype (2.1m), the possibility to automatically reconfigure the machine nearly doubles the usable workspace. It should also be noted that a singularity-free reconfiguration can not be found for the 3-DOF version of the Gantry-Tau. The reconfiguration is only possible by tilting the manipulated platform when approaching Type-II singularities.

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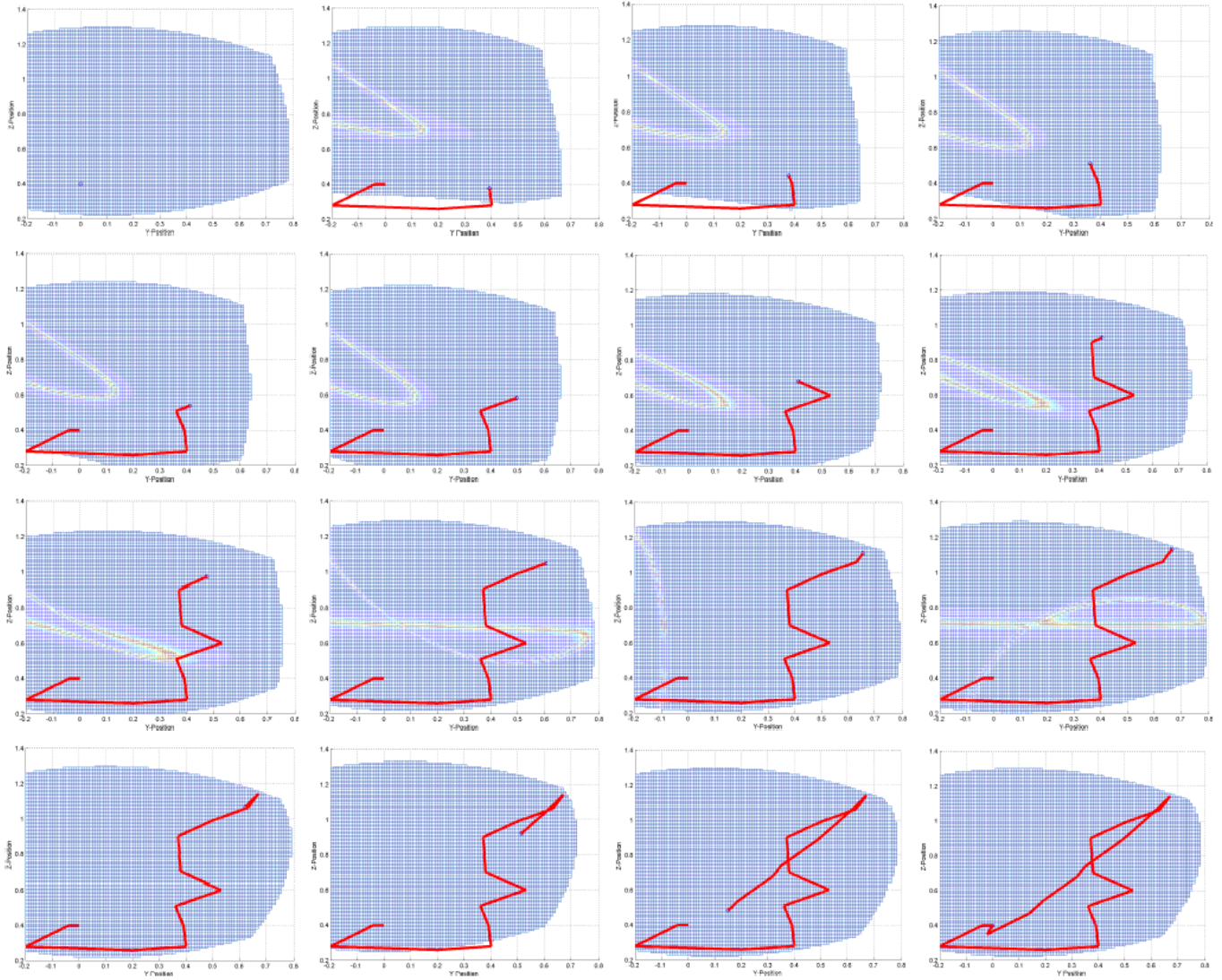


Figure 5: Optimised path of TCP (red line) in the YZ-plane with corresponding condition number maps (light blue loops). The sequence moves from left to right and top to bottom and corresponds to Figure 4.

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