Optimal TCSC Placement Based on Line Flow Equations via Mixed-Integer Linear Programming

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ABSTRACT

Research effort has been given to find the optimal location and initial settings of Thyristor-Controlled Series Capacitor (TCSC) using Mixed-Integer Linear Programming (MILP). As a useful tool for combinatorial optimisation over integer and continuous variables, mixed-integer programming approach can provides robust performance as well as high computation efficiency when solving complex optimal problems. Artificial Intelligence (AI) techniques, as the mainstream resolvent of system planning, are intractable for large scale systems when dealing with mixed integer problems. Previous work with MILP on power system planning issues employed dc load flow model ignoring reactive power balance, power loss and tapped transformer ratios. In this paper, a novel approximation planning methodology is proposed for preliminary system design. The objective of this planning strategy is to improve system loadability in the network by choosing optimal locations and settings of TCSC. Simulation results are presented for IEEE 9- and 14-bus systems.

1. INTRODUCTION

With the introduction of competitive electricity markets, power industries worldwide are undergoing profound changes with deregulation and reconstruction. As countries enjoy the benefits of economic effects as lower customer rates and higher efficiency, the separation of generation system from the transmission system gives rise to the situation where transmission has no direct role on deciding generation patterns, both in short and long terms. The main consequence of this has marked a rise in the level of venture and uncertainty associated with the increase of unscheduled power exchange due to the competition among utilities and types of contracts concluded between vendors and vendees [1].

In response to this challenge, transmission systems management must be flexible to react to diverse generation and load patterns. Since the investment of new transmission equipments is restricted for both political and environmental issues, transmission lines become stressed and operate close to thermal ratings.

Studies have been investigated to how to improve system capacity and different types of stabilities without transmission and generation expansions. These investigations suggest that it can be achieved by applying Flexible Alternating Current Transmission System (FACTS) devices for their power flow controllability. FACTS devices may provide strategic benefits through better utilization of existing transmission assets, increased reliability and security, and enabling environmental benefits [2]. For series capacitors offer certain advantages in load flow control over shunt counterparts [3], in this paper, benefits of installing thyristor-controlled series capacitor (TCSC) is investigated to improve system performance. The main task is to determine the optimal numbers, locations and initial parameter settings of TCSC.

There are numbers of approaches proposed in literature for optimising allocation and parameter settings of FACTS devices from different viewpoints [1], [4], [5], [6], [7]. Some previous work is reported to design metaheuristic approaches. In [1], evolutionary approaches are used for multi-type FACTS devices placement to enhance the system loadability, however, the number of FACTS devices is predetermined and without economic investment considerations. In [4], an improved genetic algorithm is proposed for thyristor-controlled phase shifter transformer (TCPST) planning to maximise social surplus and system capacity, but the computational time is not provided.

In addition to Artificial Intelligence (AI) methodologies, sensitivity analysis [5], extended voltage phasors approach (EVPA) [6] has been studied to enhance loadability and voltage profile respectively. However, both strategies just give approximate numbers and settings of FACTS devices, and do not employ investment considerations

In [7], to identify the size and location of thyristor-controlled phase shifter transformer (TCPST), mixed-integer linear programming (MILP) is employed with respect to dc power flow model. The paper proposes a two steps planning strategy in application for large-scale systems to optimise the loadability and investment.

In this paper, a new strategy for TCSC planning is proposed using Mixed-Integer Linear Programming (MILP) method. This approach is based on the most recent advances in branch and bound algorithms [8], through which remarkable gains in computational speed can be achieved. The main challenge of the problem formulation is to accommodate nonlinear power flow constraints into a set of linear constraints. Binary variables will be associated with TCSC installation locations. A decision of installation of TCSC will be made based on the outcome of these binary variables.

The software CPLEX which contains these advances is used for TCSC planning. The proposed model is tested and verified with the IEEE 9- and 14-bus systems. It will be shown that the approach could generate optimal solution with impressive computational speed.

The work is organized as follows. In section 2 the ideal TCSC planning model, Mixed-Integer Programming and line-flow based equations are presented, as well as the proposed planning model. Section 3 shows the results and discussion of case studies. Since the model is approximate with respect to operation constraints, the proposed optimisation approach can be employed as preliminary planning design followed by detailed studies.

2. MIP BASED TCSC PLANNING MODEL

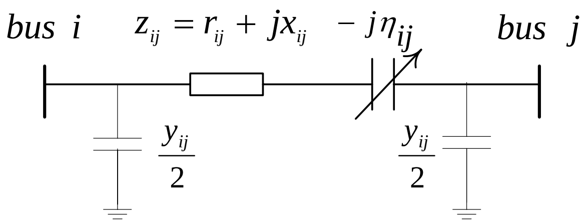

Figure 1: Model of TCSC installed in transmission line [i, j].

The ideal TCSC model is to be represented as a variant capacitive reactance with resistance ignored, which is suitable for long term planning. Figure 1 shows a branch [i, j] with original impedance z_{ij}, and an installed ideal TCSC modelled by variant reactance $-j\eta_{ij}$. A decision variable δ is introduced, which flags if a TCSC is to be installed or not on branch [i, j]. In the proceeding text, δ will always be bundled with η as the compensation capacity of each line.

2.1. MIP BRIEF REVIEW

A Mixed-Integer Programming (MIP) problem is one where some of the decision variables are restricted to have only integer values at the optimal solution. MIP includes two types of problems, as mixed-integer linear programming (MILP) problems and mixed-integer quadratic programming (MIQP) problems. The mathematical form of a generalized MIP [9] problem is:

$$
\begin{aligned}
&\min \quad f(x) = \frac{1}{2} x^T Q x + g^T x \\
&S.t. \\
&\qquad x_l \le x \le x_u \\
&\qquad b_l \le Ax \le b_u \\
&\qquad x_i \ \text{integer}, \quad \mathrm{i} \in \mathrm{I}
\end{aligned}
$$

Where $x, g, x_l, x_u \in R^n, Q \in R^{m\times n}, b_l, b_u \in R^m$, and I is a set of integer variables indices. If $Q == 0$, then it is a MILP problem, otherwise a MIQP problem. Note that the Q matrix must be positive definite or semi-definite in resolving MIQP problems. The use of integer variables greatly expands the scope of useful optimisation problems. Several studies have been carried out to explore the application of mixed-integer linear programming (MILP) in power system planning and expansion and obtained fast and robust behaviours [7], [10], [11]. In this paper, LP method will be investigated as the approaches to exploit TCSC planning problem with consideration of system loadability.

2.2. MATHEMATICAL FORMULATION

The selection of linearised power flow model is dominant while employing mixed-integer programming based algorithms in solving power system issues. The dc load flow is popularly used as a simplified model to carry out system planning [7], [10]. The well-known limitations of this model are the ignorance of reactive power balance, tap dependence in the transformer reactance, as well as absence of active and reactive power loss. In this section, a new linearised load flow model is described and employed for TCSC planning. Before the proposition of it, a brief review for line flow based equations [13] is provided.

2.2.1. LINE FLOW BASED EQUATIONS

The line flow based equations are described below:

$$A \cdot p + A' \cdot l = P_{GL} \tag{1}$$

$$A_1 \cdot q - H_1 \cdot V_{pq}^2 + A_1' \cdot m = Q_{GL} \tag{2}$$

$$
\begin{aligned}
&2R \cdot p + 2X \cdot q - \\
&\qquad (\Lambda A_{1+}^T + A_{1-}^T) V_{pq}^2 + k = \Lambda A_c^T V_{V\_gen}^2
\end{aligned} \tag{3}
$$

Where

A	Full bus incidence matrix
A'	Modified bus incidence matrix, with all '-1' in A are set to zero
A_1	Bus incidence matrix associated to PQ nodes.
H_1	Diagonal matrix associated to reactive power compensation devices of PQ nodes
A_1'	Modified bus incidence matrix, with all '-1' in A_1 are set to zero.
R	Diagonal matrix of line resistance
X	Diagonal matrix of line reactance
Λ	Diagonal matrix associated to tapped transformers
A_{1+}	Modified bus incidence matrix, all '-1' in A_1 are set to zero.
A_{1-}	Modified bus incidence matrix, with all '+1' in are set to zero
A_c	Bus incidence matrix, associated to PV nodes.
p	Vector of active power at line receiving end

q	Vector of reactive power at line receiving end
V_{pq}^2	Vector of PQ bus voltage magnitude square
$V_{V\_gen}^2$	Vector of generator bus voltage magnitude square
l	Vector of active power loss at each line
m	Vector of reactive power loss at each line
k	Vector of a composite variable
P_{GL}	Vector of bus active power injection
Q_{GL}	Vector of bus reactive power injection

The first two equations were derived by using graph theory with respect to power equilibriums at each node. The third equation was yielded from branch voltage drop equation.

2.2.2. TCSC PLANNING MODEL

The linearised planning model is proposed based on above equations with the introduction of loadability factor ξ, which is associated with active and reactive load increase. The model is described as:

$$A\cdot p + A^{'}\cdot l - P_g + P_L \cdot \xi = 0 \tag{4}$$

$$A\cdot q - H\cdot V^2 + A^{'}\cdot m - Q_g + Q_L \cdot \xi = 0 \tag{5}$$

$$2R\cdot p + 2X\cdot q + CM\cdot V^2 + k = 0 \tag{6}$$

Where

V^2	Vector of squared bus voltage of the end node of line [i, j]
P_g	Vectors of active power generation
Q_g	Vectors of reactive power generation
P_L	Vectors of active power load
Q_L	Vectors of reactive power load
H	Diagonal matrices associated to reactive power compensation devices at each node
CM	Coefficient matrix of V^2

For TCSC planning, the expression of branch impedance is:

$$X^{'} = X + \eta\delta \tag{7}$$

Where

$X^{'}$	Branch reactance after TCSC installed
η	Compensation settings of TCSC
δ	Vector of binary variables which flags TCSC installations on branches

Then equation (9) is rewritten as:

$$2R\cdot p + 2X\cdot q + CM\cdot V^2 + k + 2\eta q\delta = 0 \tag{8}$$

2.2.2.1. *TCSC Planning Objective Function*

The objective function of TCSC planning can be described as follows:

$$\max f(x) = \xi$$

The set of total variables of problem is:

$$X^T = \left(p, q, V^2, P_g, Q_g, l, m, k, \eta, \delta, \eta\delta, \xi\right)$$

2.2.2.2. *Equality and Inequality Constraints*

Equations (7) and (8) are the equality constraints of the performed optimisation, corresponding to power balance of each node. Due to the quadratic component $q\eta\delta$, equation (11) may be transformed into the following inequality constraints:

$$-Rp - Xq - \frac{1}{2}CM\cdot V^2 - \frac{1}{2}k \le -q_{\max}\eta\delta \tag{9}$$

$$-Rp - Xq - \frac{1}{2}CM\cdot V^2 - \frac{1}{2}k \ge q_{\max}\eta\delta \tag{10}$$

The variant range of variables (in p.u) obeys:

$$p_{\min} \le p \le p_{\max} \tag{11}$$

$$q_{\min} \le q \le q_{\max} \tag{12}$$

$$V_{\min}^2 \le V^2 \le V_{\max}^2 \tag{13}$$

$$P_g^{\min} \le P_g \le P_g^{\max} \tag{14}$$

$$Q_g^{\min} \le Q_g \le Q_g^{\max} \tag{15}$$

$$l_{\min} \le l \le l_{\max} \tag{16}$$

$$m_{\min} \le m \le m_{\max} \tag{17}$$

$$k_{\min} \le k \le k_{\max} \tag{18}$$

$$\xi_{\min} \le \xi \le \xi_{\max} \tag{19}$$

$$\delta \in [0,1],\ \text{integer variables} \tag{20}$$

The variable $\eta\delta$ is the product of continuous and binary variables. It can be expressed as a set of linear constraints [14]:

$$z = \delta \cdot f(x) = \begin{cases} f(x), & \delta == 1 \\ 0, & \delta == 0 \end{cases} \tag{21}$$

is equivalent to:

$$\begin{cases} -M\delta + z \le 0 \\ m\delta - z \le 0 \\ -m\delta + z \le f(x) - m \\ M\delta - z \le -f(x) + M \end{cases} \tag{22}$$

M, m *are the maximum and minimum values of* $f(x)$

During the procedure of rescheduling power in optimisation, certain network elements, especially transmission lines, may exceed their thermal ratings. To prevent this consideration, the following rule is obeyed when the bounds of P and Q are determined.

$$p_{\max}^2 + q_{\max}^2 \le SRate^2 \tag{23}$$

where $SRate$ is the thermal limits of branches.

3. CASE STUDIES

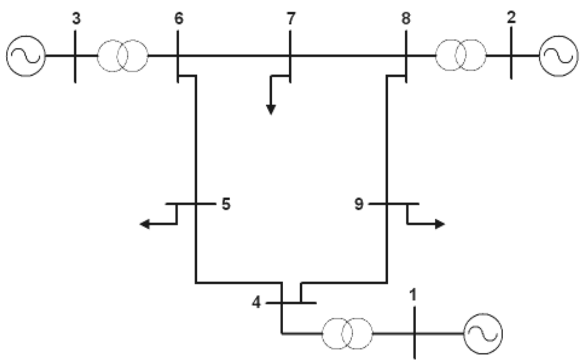

Figure 2: IEEE 9-Bus test system

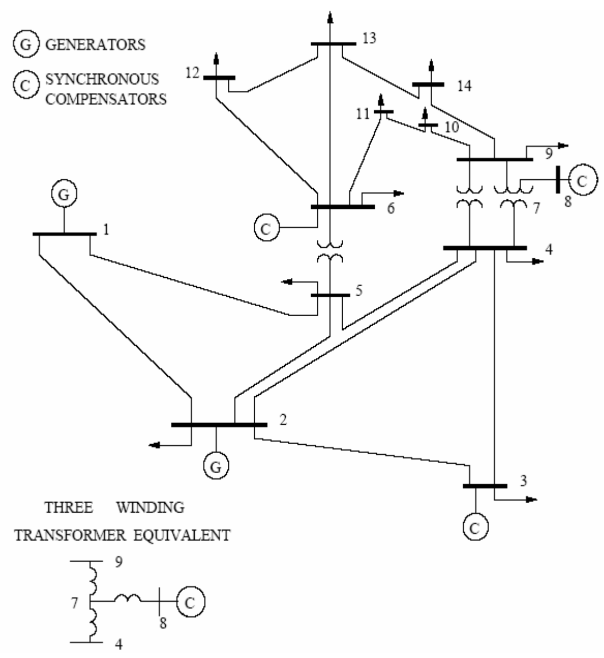

Figure 3: IEEE 14-Bus test system

System loadability can be improved by a few well-located FACTS devices [1], [7], however, it cannot go beyond system topology and operation constraints. In this section, systems loadability is to be maximised by selecting the number, location and percentage compensation of TCSC optimally. This is to quest for the interrelationships between device number and loadability enhancement. Simulations are carried out on IEEE 9-, 14-bus systems with a tolerance gap of 0.01% to explore the characteristics of loadability enhancement. Figure 2-3 show the topology diagrams of the test systems. The specification and mathematic model scales of testing systems is shown in Table I.

The percentage compensation of TCSC is allowed to vary from 30% to 70% in each line. As introduced in section 1, the problem is solved through the CPLEX 8.0 solver [9]. All cases are done on Dell OptiPlex GX520 Desktop.

Systems	9-bus system	14-bus system
Number of branches	9	20
Total active load (MW)	315	259
Max active power generation (MW)	820	772.4
Total reactive load (MVar)	115	73.5
Max reactive power generation (MVar)	900	148
Number of variables	88	185
Equality constraints	19	29
Inequality constraints	257	550

Table I: Specification of IEEE test systems and problem scale

Table II lists the maximum loadability of 2 test systems in two steps. In the first step, the number of devices is not limited, which means the maximum allowed number of installed TCSC is up to the number of branches. This is expected to find the maximum loadability of systems while avoiding getting a suboptimal factor ξ. Since no constraint is imposed on number of TCSC installation, maximising system loadability would yield a large number of TCSC installations. In the second step, the device number will be allowed to start from zero to find out the minimum number of TCSC that are needed to achieve the same gain as obtained in the first step. It will help to explore how loadability is affected by the number of devices. Figures 4-5 show the variation of system loadability versus number of TCSCs installed. It can be seen clearly that the improvement of system loadability tends to level off after installing several well-allocated devices. Despite of the fact that the test systems are small in scale, in order to achieve the maximum loadability we need 6 TCSCs for the 9 bus system and 8 for the 14 bus system respectively. It also can be seen that most of the loadability improvement is achieved by the first small number of well-located devices More specifically, 33.4 % improvement can be achieved by the first 4 TCSCs for the 9 bus system and 23.5%

improvement can be achieved by the first 3 installations for the 14 bus system. This provides very useful information to guide further detailed TCSC planning considering other factors such as budget. It should be noted that there are some room for loadability improvement even without TCSC installed – see the value of loadability factor at 0 TCSC numbers. Table III gives the detailed TCSC locations and parameter settings in the two test systems.

Systems	9-bus system	14-bus system
Maximum loadability	1.3363	1.2446
TCSC number (without limits)	8	15
CPU time (s)	0.2500	0.4850
Optimal TCSC number	6	8

Table II: Comparison of TCSC numbers with max loadability

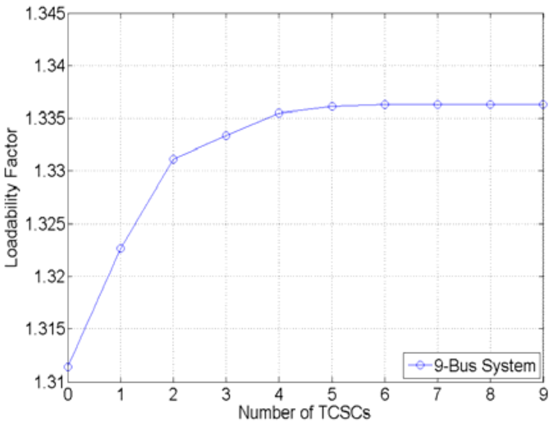

Figure 4: 9-Bus system loadability enhancement versus number of TCSC

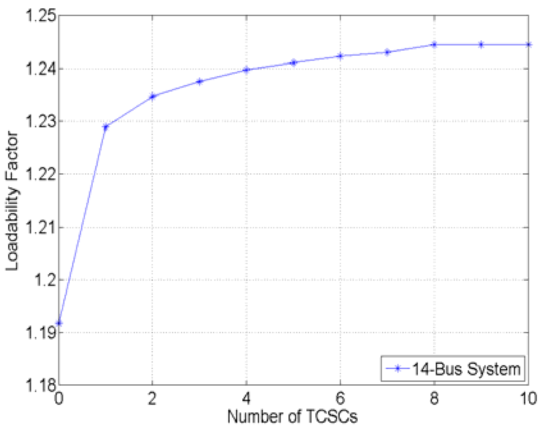

Figure 5: 14-Bus system loadability enhancement versus number of TCSC

4. Conclusion

System loadability is a complex quantity to model and compute, influenced by the network topology, line parameters, operation constraints, as well as generators and loads distributions. This paper adhibites the latest advances in Mixed-Integer Linear Programming (MILP) for TCSC planning issue to improve loadability. A new mathematical model which is built on line flow based equations is developed for TCSC planning with respect to system operation constraints. The proposed methodology is tested on 9- and 14-bus systems. From the IEEE case studies, it shows high efficiency in calculating optimal combinatorial TCSC planning problem as well as excellent robust behaviour. However, due to the approximate essence of the model, it is suitable for preparatory power system design and the result need further amelioration by full ac model involved security and stability considerations.

Systems	TCSCs locations	Reactance (p.u.)
9-Bus system	1-4	-0.0403
	4-5	-0.0566
	5-6	-0.0510
	6-7	-0.0362
	8-9	-0.0992
	9-4	-0.0345
14-Bus system	4-9	-0.3893
	5-6	-0.1764
	6-11	-0.0814
	6-13	-0.0530
	7-8	-0.1233
	9-10	-0.0352
	10-11	-0.0794
	13-14	-0.1420

Table III: Optimal TCSC Allocation of maximum loadability

References

[1] S. Gerbex, R. Cherkaoui, and A. J. Germond, "Optimal location of multi-type FACTS devices in a power system by means of genetic algorithms," Power Systems, IEEE Transactions on, vol. 16, pp. 537-544, 2001.

[2] K. Habur, and D. Oleary, "FACTS-Flexible AC Transmission Systems, for cost effective and reliable transmission of electrical energy," http://www.siemenstd.com/TransSys/pdf/CostEffectiveReliabTrans.pdf

[3] R. M. Mathur and R. K. Varma, Thyristor-Based FACTS Controllers for Electrical Transmission Systems, 2002.

[4] L. Ippolito and P. Siano, "Selection of optimal number and location of thyristor-controlled phase shifters using genetic based algorithms," Generation, Transmission and Distribution, IEE Proceedings, vol. 151, pp. 630-637, 2004.

[5] S. N. Singh and A. K. David, "A new approach for placement of FACTS devices in open power markets," Power Engineering Review, IEEE, vol. 21, pp. 58-60, 2001.

[6] N. K. Sharma, A. Ghosh, and R. K. Varma, "A novel placement strategy for FACTS controllers," Power Delivery, IEEE Transactions on, vol. 18, pp. 982-987, 2003.

[7] F. G. M. Lima, F. D. Galiana, I. Kockar, and J. Munoz, "Phase shifter placement in large-scale systems via mixed integer linear programming," Power Systems, IEEE Transactions on, vol. 18, pp. 1029-1034, 2003.

[8] R. Bixby, M. Fenelon, Z. Gu, E. Rothberg, R. Wunderling, "MIP: Theory and Practice-Closing the Gap," In System Modelling and Optimization: Methods, Theory, and Applications, M. J. D. Powell and S. Scholtes, Eds, Netherlands: Kluwer Academic Publishers, 2000.

[9] Tomlab/CPLEX v8.0 User's Guide, Tomlab Optimization Inc, 2002.

[10] L. Bahiense, G. C. Oliveira, M. Pereira, and S. Granville, "A mixed integer disjunctive model for transmission network expansion," Power Systems, IEEE Transactions on, vol. 16, pp. 560-565, 2001.

[11] P. C. Paiva, H. M. Khodr, J. A. Dominguez-Navarro, J. M. Yusta, and A. J. Urdaneta, "Integral planning of primary-secondary distribution systems using mixed integer linear programming," Power Systems, IEEE Transactions on, vol. 20, pp. 1134-1143, 2005.

[12] J. M. Maciejowski, Predictive control : with constraints. Harlow, England: Prentice Hall/Pearson Education, 2002.

[13] Y. Ping and A. Sekar, "Analysis of radial distribution systems with embedded series FACTS devices using a fast line flow-based algorithm," Power Systems, IEEE Transactions on, vol. 20, pp. 1775-1782, 2005.

[14] D. Mignone, "The really big collection of logic propositions and linear inequalities," Technical report, AUT01-11, Automatic Control Laboratory, ETH Zurich, Switzerland, 2001.

APPENDIX

Bus #	Generation		Load	
	P(MW)	Q(MVar)	P(MW)	Q(MVar)
1	0	0	0	0
2	163.00	0	0	0
3	85.00	0	0	0
4	0	0	0	0
5	0	0	90.00	30.00
6	0	0	0	0
7	0	0	100.00	35.00
8	0	0	0	0
9	0	0	125.00	50.00

Table IV: Bus data of 9-bus system

Systems	branches	Resistance (p.u.)	Reactance (p.u.)
9-Bus system	1 4	0	0.0576
	4 5	0.0170	0.0920
	5 6	0.0390	0.1700
	3 6	0	0.0586
	6 7	0.0119	0.1008
	7 8	0.0085	0.0720
	8 2	0	0.0625
	8 9	0.0320	0.1610
	9 4	0.0100	0.0850

Table V: Branch data of IEEE 9-bus system

Bus #	Generation		Load	
	P(MW)	Q(MVar)	P(MW)	Q(MVar)
1	232.40	-16.90	0	0
2	40.00	42.40	21.70	12.70
3	0	23.40	94.20	19.00
4	0	0	47.80	-3.90
5	0	0	7.60	1.60
6	0	12.20	11.20	7.50
7	0	0	0	0
8	0	17.40	0	0
9	0	0	29.50	16.60
10	0	0	9.00	5.80
11	0	0	3.50	1.80
12	0	0	6.10	1.60
13	0	0	13.50	5.80
14	0	0	14.90	5.00

Table VI: Bus data of 14-bus system

Systems	branches	Resistance (p.u.)	Reactance (p.u.)
14-Bus system	1 2	0.0194	0.0592
	1 5	0.0540	0.2230
	2 3	0.0470	0.1980
	2 4	0.0581	0.1763
	2 5	0.0570	0.1739
	3 4	0.0670	0.1710
	4 5	0.0134	0.0421
	4 7	0	0.2091
	4 9	0	0.5562
	5 6	0	0.2520
	6 11	0.0950	0.1989
	6 12	0.1229	0.2558
	6 13	0.0662	0.1303
	7 8	0	0.1762
	7 9	0	0.1100
	9 10	0.0318	0.0845
	9 14	0.1271	0.2704
	10 11	0.0820	0.1921
	12 13	0.2209	0.1999
	13 14	0.1709	0.3480

Table V: Branch data of IEEE 14-bus system