

VERIFICATION OF THE DYNAMICS OF THE 5-DOF GANTRY-TAU PARALLEL KINEMATIC MACHINE

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ABSTRACT

In this paper the 5-DOF dynamics of a full-scale parallel kinematic machine (PKM) is verified. The dynamics of PKMs are more difficult and computationally expensive than the dynamics for serial machines. To our knowledge, this is the first time that such a dynamics verification has been published. The experimental results show that the dynamics model fits the experiments with errors in the range of only 1 to 25 percent. The structures of most PKMs allow these machines to be used in applications requiring high speeds and accelerations. An accurate dynamics model is a pre-requisite when designing controllers for such applications.

KEY WORDS

Robotics, parallel, modeling, identification, Tau.

1 Introduction

In this paper the verification of the rigid-body dynamics model of the 5-degree-of-freedom (DOF) Gantry-Tau parallel kinematic machine (PKM) is presented. The rigid-body dynamics analysis is based on a relatively new general approach for dynamics modelling of PKMs presented in [1]. To our knowledge, this paper is the first where the methods in [1] have been experimentally verified on full-scale 5-DOF PKM. Inverse dynamics modelling and verification of PKMs is not a new topic, see for example [2]. The new aspects presented in this paper are as follows: 1) a 5-DOF dynamics model of the Gantry-Tau is presented for the first time, 2) to our knowledge, the approach taken in [1] is experimentally verified on a full-scale 5-DOF PKM for the first time. The approach in [1] is attractive, because it is based on familiar Jacobian matrices. There are surprisingly few published articles describing experimental dynamics verification of PKMs, both 3-DOF and 5-DOF. One possible reason for the lack of published work, is the relatively large effort required to build a mechanical prototype including a real-time control and measurement system. Most prototypes are built to verify workspace and the kinematic motions. Rigid-body dynamics verification requires more robust prototypes, ideally backlash-free and rigid, such that they are able to withstand large accelerations without exciting elasticity effects. An example of experimental verifica-

tion is [2], which is based on the Hamiltonian formulation but the effects of friction were ignored. The experiment in [2] was the first ever for the Delta robot, performed on one of the prototypes at EPFL, Lausanne.

Identification and verification of dynamics models for serial robots is a much more established research area, see for example [3] and the references in that paper. In [3] the mean square model accuracy of a serial industrial manipulator was approximately $90N^2m^2$ for a motor excitation signal of approximately $200Nm$. This is equivalent to a root-mean-square error of approximately 5%. The results in [3] are comparable or slightly higher than the results obtained in this paper for the Gantry-Tau, where the final identified model errors are approximately 1%. As in [3] multi-sines were used for the identification and model verification.

The kinematic description of the 5-DOF Gantry-Tau PKM was presented in [4] and a picture of the prototype is shown in Fig. 1. Due to lack of space, the kinematic description of the machine is not repeated in this paper. The

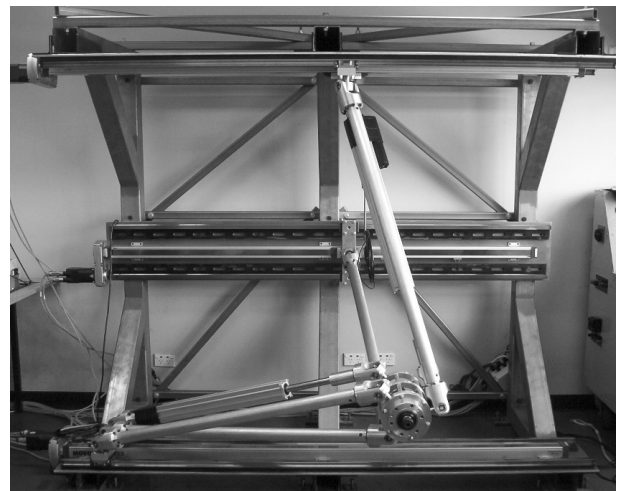


Figure 1. The Gantry-Tau 5-DOF parallel kinematic machine.

5-DOF Gantry-Tau has three linear actuators fixed at the base and six links supporting the tool. Two of the six links are telescopic actuators, resulting in a total of 5-DOF. The

5-DOF consist of three Cartesian positions and two Cartesian rotations.

2 Inverse Dynamics Model

In [1] a general approach to the inverse dynamics of parallel robots was presented. A summary of this approach applied to the 5-DOF Gantry-Tau is given below. The inverse dynamics model of a parallel structure is given by the following general form

$$\Gamma = \mathbf{J}_r^T \left[\mathbf{F}_p + \sum_{k=1}^m \left(\frac{\delta \mathbf{X}_k}{\delta \mathbf{X}} \right)^T \mathbf{J}_k'^T \mathbf{H}_k \right] \quad (1)$$

where Γ is a 5×1 vector of actuator forces, \mathbf{J}_r is the 6×5 direct robot Jacobian, \mathbf{F}_p is a vector containing the 6-DOF platform forces, \mathbf{X}_k is a vector containing 3-DOF Cartesian coordinates of platform point k , \mathbf{X} is a vector containing the 6-DOF Cartesian coordinates of the TCP, \mathbf{J}_k' is the link variables to platform point Jacobian of link i . For a fixed length link the dimensions of this Jacobian is 3×3 and for a telescopic link the dimension of this Jacobian is 4×3 . \mathbf{H}_k is the inverse dynamics model of link i , which has dimensions 3×1 for a fixed length link and dimensions 4×1 for a telescopic link. Both \mathbf{J}_k' and \mathbf{H}_k are calculated with the platform disconnected from the links. Fig. 2 shows the four

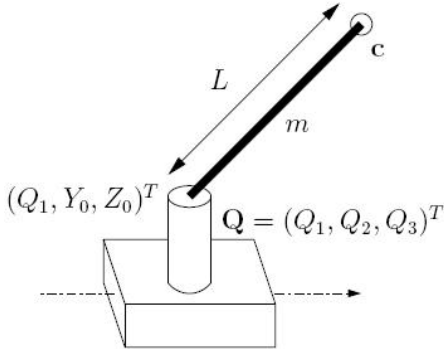


Figure 2. Variables for a telescopic link

actuator variables for a telescopic link. The first variable, Q_1 , is a pure translation caused by the linear base actuator. The next two variables, Q_2 and Q_3 , describe the unactuated joint rotations and the final variable L is the variable telescopic link length. The link also has dynamic parameters such as mass, centre of mass and inertias. A fixed length link has the same parameters, except that the parameter L is fixed. More details about the dynamics model in general can be found in [1] and for the Gantry-Tau in particular in [5].

3 Experiment Results

The trajectories chosen for the base actuators are multi-sines of the following form

$$q_i(t) = \sum_{k=1}^N (\sin(w_{k,i}t + \varphi_i) + \cos(w_{k,i}t + \varphi_i)) \quad (2)$$

where $i = 1, 2, 3$. The frequencies $w_{k,i}$ and phases φ_i are chosen to, if possible, avoid periodic behavior in the TCP trajectory, i.e. getting more information of the system for the estimation and validation sets later on.

The first experiment kept the length of the telescopic actuators fixed. The results with the trajectories in eq. (2) are shown in Figure 3. For this experiment, only a 3-DOF inverse dynamics model was used, ignoring the rotations of the tool platform. It can be seen that the model predicts the output relatively well and that the model has the most problem with the second actuator. The RMS values, defined by

$$\text{RMS} = \left(\frac{1}{N} \sum_{k=1}^N |y[k] - \hat{y}[k]|^2 \right)^{1/2} / (\max |y[k]|) \quad (3)$$

where y are the measured values and \hat{y} are values predicted by the model, are given in Table 1. The RMS values can be interpreted as the prediction error of the model. The RMS values include a division by the largest absolute value, to make the values a percentage estimate rather than a force estimate in Newtons. The experimental results for the 3-

Table 1. The RMS values for the simulation of the 3-DOF Gantry-Tau model for the actuator positions given in (2).

	Act 1	Act 2	Act 3
3-DOF	16.10%	17.87%	12.14%
5-DOF I	9.53%	24.76%	20.22%
5-DOF II	15.88%	16.98%	18.39%
3-DOF Id	1.39%	1.15%	1.09%

DOF motion in Table 1 show that the model errors vary between 12 and 18 percent. The physical parameters of the model, such as lengths, masses and centre of masses were measured when the machine was disassembled. The most difficult parameters to estimate manually were the inertia parameters of the links and the moving platform.

Note that the inverse dynamics in eq. (1) only describe the dynamics of the links and the moving platform, and not the actuator dynamics. The actuator dynamics contain for example forces caused by friction and elasticity effects that can be as significant as the forces in eq. (1). The actuator dynamics of the Gantry-Tau were studied extensively in [6]. In this paper the effects from the actuator dynamics have been removed by running the trajectories in eq. (2) twice; first with the PKM structure connected and second with the PKM structure disconnected from the base actuators. The forces from the PKM structure can then be isolated by subtracting the actuator forces measured in the second step from the forces measured in the first step.

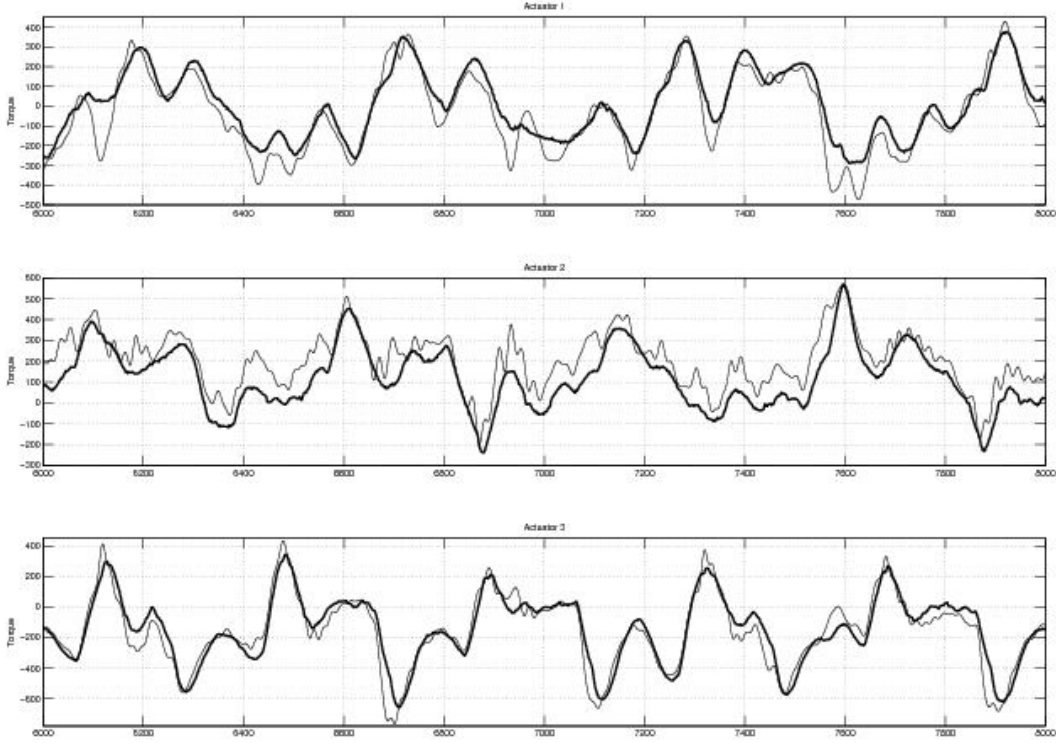


Figure 3. Experimental results when the actuator trajectories in eq. (2) are applied. The thick line shows the measurement of the system when the Tau structure is connected and the thin line shows the sum of the disconnected actuator torques and the model output.

Fig. 5 shows the same experimental result as in Fig. 4, except that the complete 5-DOF inverse dynamics model was used. The lengths of the telescopic actuators were kept fixed as in the 3-DOF experiment. Note that with the current experimental setup it was not possible to measure the forces in the telescopic links. Hence, the model fit was only calculated for the three base actuators. The results are given in Table 1. The model fit is better compared to the 3-DOF model for actuator 1, but worse for actuators 2 and 3. The model fits vary between 9 and 25 percent.

Fig. 6 shows the experimental results for the most complex motion and model. Multi-sines were applied to all the actuators, including the telescopic links. Even if the telescopic forces were not measured, the telescopic link lengths were measured. The position, velocity and acceleration of the link lengths influence the forces in the base actuators through the 5-DOF inverse dynamics model. The results are given in Table 1. The model fits vary between 15 and 19 percent.

4 Identification

The general dynamics formula in eq. (1) applied to the Gantry-Tau can be written, if assuming diagonal inertia ma-

trices, as a linear regression

$$\Gamma = \mathbf{J}_r^T \begin{pmatrix} \tilde{\mathbf{F}}_p & \tilde{\mathbf{F}}_s & \tilde{\mathbf{F}}_l & \tilde{\mathbf{F}}_t \end{pmatrix} \begin{pmatrix} \theta_p \\ \theta_s \\ \theta_l \\ \theta_t \end{pmatrix} \triangleq \varphi^T \theta \quad (4)$$

where the subscripts stand for the platform, the short stiff link, the long stiff links, and the telescopic links, which all have different dynamic properties, i.e.

$$\theta_p = \begin{pmatrix} m_p \\ m_p c_x \\ m_p c_z \\ I_{p_{yy}} \end{pmatrix}, \quad \theta_k = \begin{pmatrix} m_k \\ I_{k_{xx}} \\ I_{k_{yy}} \\ I_{k_{zz}} \end{pmatrix}, \quad k = s, l, t \quad (5)$$

where m is the mass of the object, c is the coordinates for the centre of mass of the platform and I is the moment of inertia of the object.

A batched model for the identification is used, i.e. for a data set of N samples the following equation applies

$$\underbrace{\begin{pmatrix} \Gamma(t_1) \\ \Gamma(t_2) \\ \vdots \\ \Gamma(t_N) \end{pmatrix}}_{Y_N} = \underbrace{\begin{pmatrix} \varphi^T(t_1) \\ \varphi^T(t_2) \\ \vdots \\ \varphi^T(t_N) \end{pmatrix}}_{\Phi_N} \theta \quad (6)$$

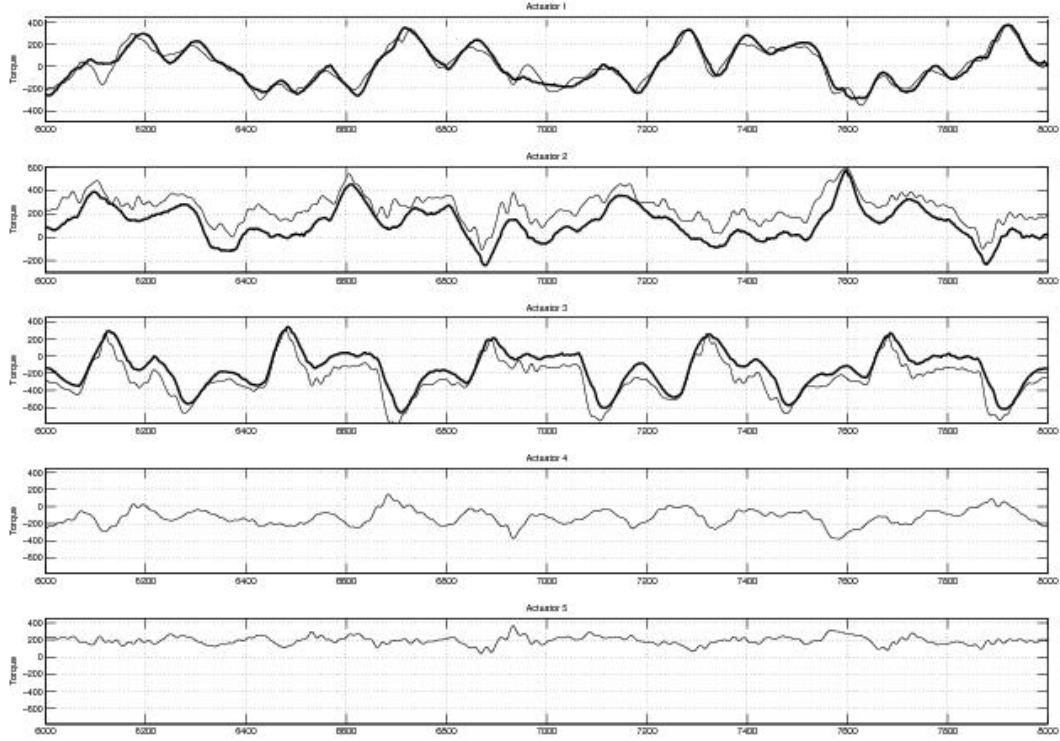


Figure 4. *Experimental results when the actuator trajectories in (2) are applied. The thick line shows the measurement of the system when the Tau structure is connected and the thin line shows the sum of the disconnected actuator torques and the model output.*

The solution to a least squares (LS) criterion is then given by

$$\hat{\theta}_{ls} = \Phi_N^\dagger Y_N \quad (7)$$

where Φ_N^\dagger is the pseudo inverse

$$\Phi_N^\dagger = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T \quad (8)$$

The actuator trajectories used for identification are in the form specified in eq. (2). By making use of the 5-DOF inverse dynamics model different fixed telescopic link lengths can be enabled for each data sets. This yields different platform orientations for each set and therefore more information about the system. Cross validation was also used, i.e. different data sets for estimation and validation.

The estimated LS parameters are given in Table 2 and the nominal values used for the experiments in Section 3 are presented in Table 3. In Figure 6 the model validation results with the LS parameters are given and the last row of Table 1 presents the RMS values defined in (3).

As can be seen in both Figure 6 and Table 1 the estimated model performs quite well and predicts the output within 1%-2% of the maximal torque.

Unfortunately, the estimated parameters, given in Table 2, are not physical since some of the moment of inertias are negative and the masses differ significantly from the measured ones, given in Table 3. This is thought to be the result of the chosen trajectories. The movements of the

actuators are too small to be able to excite all the dynamical parameters in the system. Especially the rotations of the links and the platform, which yield information about the moments of inertias, are difficult to make large because of the large masses in the system. This can also be seen when studying the standard deviations of the parameters in Table 2 where all the moment of inertias have large values, which indicate that they were not properly excited during the experiments. When driving the chosen actuator trajectories, one could feel vibrations in the frame of the manipulator, which indicate that the acceleration of the actuators could not have been chosen any larger without exciting unmodelled elasticity effects.

5 Discussion and Conclusions

In this paper a 5-DOF inverse dynamics model for the Gantry-Tau parallel kinematic machine has been presented. The model errors were in the range of only 10 to 25 percent for a complex 5-axis motion. These model fits are acceptable, since all the physical parameters were estimated manually before the machine was assembled. In conclusion, the inverse dynamics model presented in [1] is a good description of the Gantry-Tau dynamics. The model errors were reduced to about only 1 percent when the least-squares estimation was applied. Such a model fit is exceptionally good, however, some of the estimated inertias have large

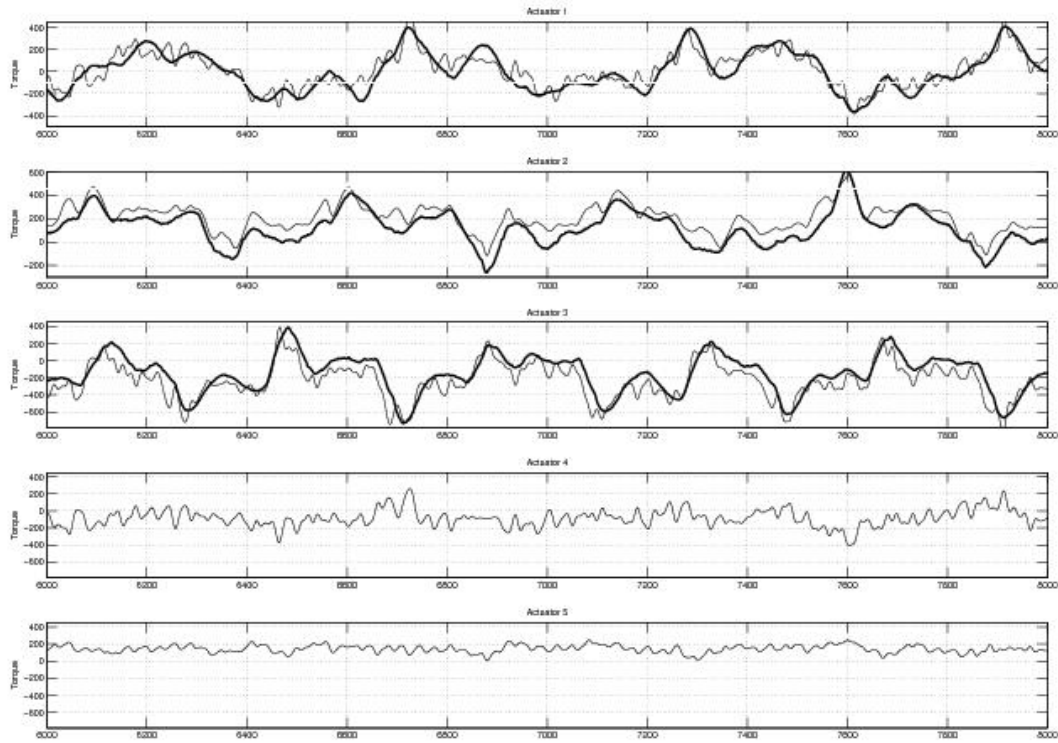


Figure 5. Experimental results when the actuator trajectories in (2) are applied for the 5-DOF model. The thick line shows the measurement of the system when the Tau structure is connected and the thin line shows the sum of the disconnected actuator torques and the model output.

variances and non-physical mean values.

A future extension of the work presented in this paper is to check for which actuator trajectories the system parameters are identifiable. One approach would be to calculate optimal excitation trajectories as described for example in [7]. It should be noted that such an optimisation would be more difficult and computationally time consuming for PKMs compared to serial machines.

Another future extension of the work would be to measure the forces in the telescopic actuators and to calculate the model errors for the telescopic forces also. The authors have no reasons to believe that the model fits for the telescopic forces will be much different compared to the base actuators.

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Table 2. The estimated parameters for the linear regression (4) with the parameter standard deviation within parentheses.

m_p	$m_p c_x$	$m_p c_z$	$I_{p_{yy}}$
6.0072 (± 0.7440)	0.9469 (± 0.1864)	-0.9141 (± 0.2977)	8.9927 (± 0.5617)
m_s	$I_{s_{xx}}$	$I_{s_{yy}}$	$I_{s_{zz}}$
9.6144 (± 0.6499)	-43.1484 (± 9.2828)	-3.4402 (± 1.9457)	-1.0233 (± 0.7477)
m_l	$I_{l_{xx}}$	$I_{l_{yy}}$	$I_{l_{zz}}$
2.4416 (± 0.7821)	1.4515 (± 0.8624)	-3.4395 (± 0.9741)	4.0691 (± 0.7117)
m_t	$I_{t_{xx}}$	$I_{t_{yy}}$	$I_{t_{zz}}$
8.2764 (± 0.8560)	-13.8664 (± 1.3054)	8.3159 (± 1.9866)	-1.1505 (± 1.0506)

Table 3. The nominal parameter values used for the simulation in Section 3.

m_p	$m_p c_x$	$m_p c_z$	$I_{p_{yy}}$	m_s	$I_{s_{xx}}$	$I_{s_{yy}}$	$I_{s_{zz}}$
11.790	0	0	0.0614	3.970	0.3931	0.3931	0.0020
m_l	$I_{l_{xx}}$	$I_{l_{yy}}$	$I_{l_{zz}}$	m_t	$I_{t_{xx}}$	$I_{t_{yy}}$	$I_{t_{zz}}$
4.220	0.5495	0.5495	0.0021	10.370	1.3503	1.3503	0.0187

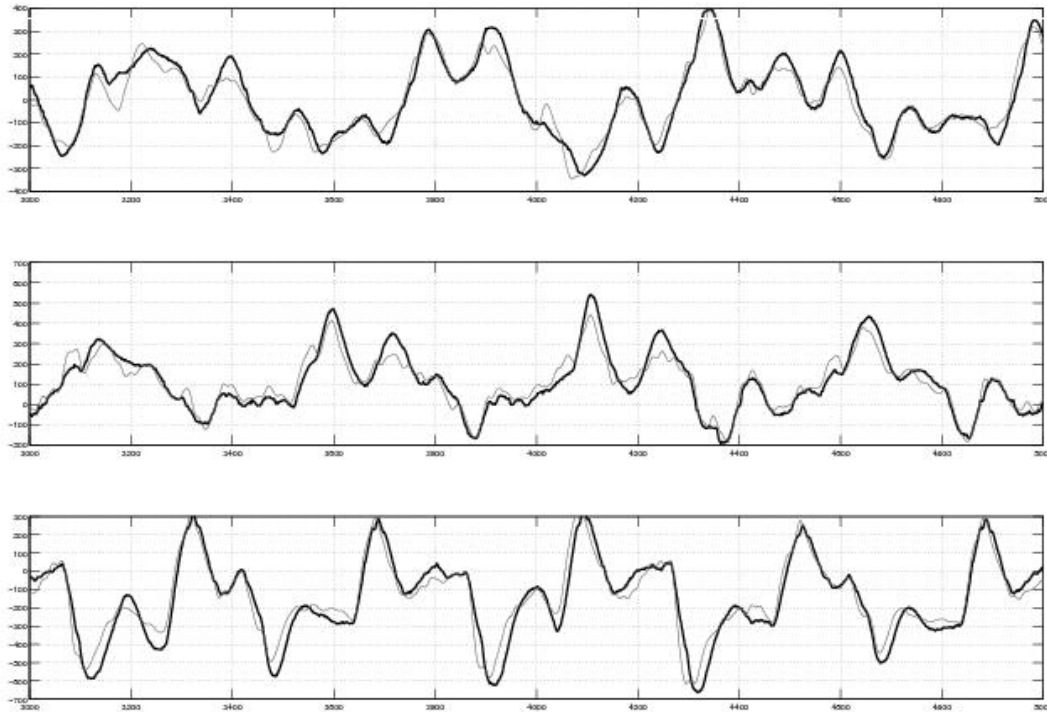


Figure 6. Experimental results with the least square parameters when the actuator trajectories (2) are applied. The thick line shows the measurement of the system when the Tau structure is connected and the thin line shows the sum of the disconnected actuator torques and the model output.