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Comparison of Markov Theory and Monte Carlo Simulations for Analysis of Marine Operations Related to Installation of an Offshore Wind Turbine

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ABSTRACT

This paper contains a comparison study of two different methods for estimating weather downtime and duration of marine operations related to the installation of bottom-fixed offshore wind turbines. The first method is based on numerical Monte Carlo simulations. The second method is based on an approach using Markov theory as published in (Anastasiou and Tsekos, 1996), and it differs from the Monte Carlo technique in that it establishes analytically, not through simulation, the probability distribution of the duration of the marine operations.

Having established the probability distribution of individual activities in the operations scenario, the probability distribution of the duration of an entire marine operation is determined with the aid of the Probabilistic Network and Evaluation Technique (PNET), see (Ang et al., 1975). The combination of Markov theory and PNET provides a useful means for predicting the duration of a marine operation. Both activities which do and do not require a weather window are included in the analysis, as well as the possibility to execute activities as a combination of parallel and sequential operations. A comparison of the computational time efforts using the analytical approach and Monte Carlo simulation are presented in in this paper, where Markov theory based approach is several times faster than Monte Carlo simulations.

The paper includes an illustrative example related to the installation process of bottom-fixed offshore wind turbines.. The sea state generator used in the example is based on observed weather data for the North Sea. The paper concludes with a summary of advantages and disadvantages of the two approaches based on Monte Carlo simulations and Markov Theory/PNET.

Keywords : marine operations, project duration, weather window, Markov process.

1. INTRODUCTION

Marine operations involve many uncertain variables such as a weather/surface conditions and require a use of risk analysis, which is applied to project planning and scheduling. To perform this risk analysis experts use networks to represent an occurrence of activities involved in the project. Such a network consists of activities and links. Each activity represents a task in the project, where links are used to indicate relationships between the tasks. A sequence of activities is called a path. Failure to complete the project on time occurs when one or more paths take longer time to complete than expected. A probabilistic approach incorporating the correlation between the network paths was presented in (Ang et al., 1975).

Several probabilistic methods use risk analysis for construction/marine operations: 1) Program Evaluation Review Technique (PERT); 2) Probabilistic Network Evaluation Technique (PNET); 3) Narrow Reliability Bounds (NRB); 4) Monte Carlo Simulations (MCS); 5) Simplified Monte Carlo Simulations (SMCS). PERT and MCS are most widely used in practice. The detailed description of the use of these methods in construction industry can be found in (Diaz et al., 1993).

Monte Carlo simulation is a powerful tool which allows the determination of the probability distribution (PD) of the activity duration and can be adapted to any complex operational scenario. Long series of environmental data are simulated within an appropriate time interval where critical conditions are taken into account. PD of the activity duration is defined after the execution of a proper number of simulation runs. The environmental record needs to be sufficiently large especially for long activities. The correlation between individual Monte Carlo runs is low. Hence, Monte Carlo simulations can be very expensive in computational time.

A Markov chain is a set of states of a system for which there is a defined probability of a transition from any state to any other or to itself. This can be used to describe a project in which transitions can occur in three ways: normal transitions to the next sequential activity; transitions to earlier activities which need rectification; transitions back to the start of the current activity when it needs to be repeated. The behaviour of a Markov chain is described by a transition matrix. This matrix specifies the transition probabilities between states and the probability of future states can be calculated by multiplying the transition matrices together.

The use of PNET algorithm introduced by (Ang et al., 1975) is presented in this paper. PNET is based on different modes of failure that a network can have. Failure means completion of a project in a time longer than the target duration. Each path in the network can become a mode of failure. Thus, the completion of the project can be delayed by one or more paths in the network.

In PNET each activity must have an expected time and a standard deviation. The paths are ranked in the order of the longest duration. In the case when two paths have the same duration, the path with a higher standard deviation is assigned as a critical path. A correlation coefficient between two paths is defined in the Appendix. A set of critical paths may exist for large projects; then the paths are divided by groups. The path with a maximum mean duration is a representative path of each group. The probability distribution of the duration of the entire project is established as a product of the probability distributions of the representative paths.

Both activities which do and do not require a weather window can be included in a marine project as presented in this paper. The first group of activities may be interrupted during its execution due to unacceptable weather conditions and may be recommenced when fair conditions apply. The second group of activities may not be interrupted during its execution. A variability in the performance as a function of the environmental conditions which affects the execution of the activity is considered by introducing efficiency states with respect to critical parameters which affect the activity execution, for example wave height or wind speed. Each efficiency state is associated with an efficiency factor which is considered by technical requirements related to the activity execution.

A comparison of the computational time efforts using the analytical approach and Monte Carlo simulation are presented in this paper, where Markov theory based approach is seventeen times faster than Monte Carlo simulations. This paper includes an illustrative example related to the installation process of bottom-fixed offshore wind turbines within a realistic marine operations project. Some selected activities of the project are shown in Figure 1 (pile upending, driving and transfer), where the reader can see that some operations strongly depend on sea surface conditions. The sea state generator used in the example is based on observed weather data for the North Sea. The paper concludes with a summary of advantages and disadvantages of the two approaches based on Monte Carlo simulations and Markov Theory/PNET algorithm. In addition, MATLAB code to evaluate the project duration probability is presented in the Appendix. Two different correlation coefficients are used to demonstrate a difference in PNET algorithm based probability distributions of the project duration, where more conservative predictions occur when both paths are critical. The code presented in the Appendix can easily be adapted to other project networks. The proposed method allows to spot potential bottlenecks in offshore marine operations. It is also possible to add a combination of different weather conditions such as wave, wind, ice, etc. into the project evaluation algorithm. The methodology presented in this paper enables the generation of risk and cost estimations.

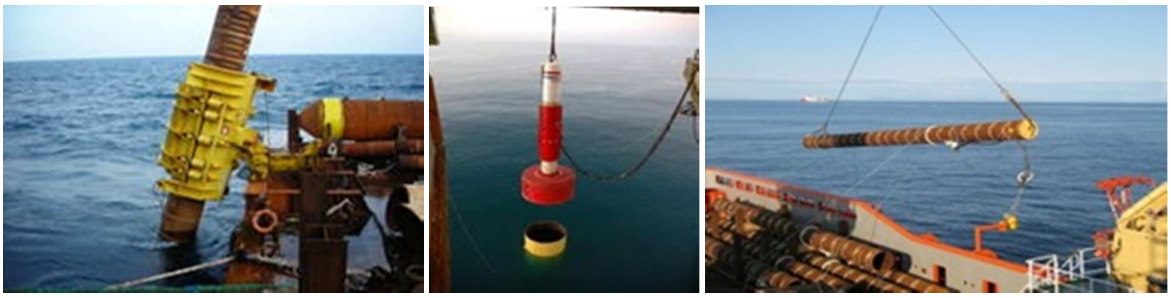

FIGURE 1. Some Examples of Marine Operations for Bottom Fixed Offshore Wind Turbines.

2. CASE STUDY

In this section the statistical parameters and probability distribution of the activity durations derived from the Markov process based model with observed values and Monte Carlo simulations are presented. The observed data correspond to wave height in North Sea for a period of 4 years where the sampling interval is 1 hour. The recorded data is partitioned into four seasons to avoid seasonal variability.

In order to examine the effectiveness of activity durations in the project the efficiency states and factors are defined according to marine operation requirements provided by Sway, where activities are united into four groups. The first group has the efficiency upper limit of one metre and the activity should be stopped when the wave height is greater than the upper limit. The other three groups have the limits of 1.5, 2 and 2.5 metres, respectively. The efficiency states, factors and ranges for different activity groups are presented in Table 1. The transition matrices of winter and summer efficiency states for each group are found as described in (Anastasiou and Tsekos, 1996). The matrices are shown below. In this paper five efficiency states are defined.

$$\mathbf{P_{1W}} = \begin{bmatrix} 0.738 & 0.2513 & 0.0053 & 0 & 0.0053 \\ 0.0238 & 0.8402 & 0.1295 & 0.0025 & 0.004 \\ 0 & 0.107 & 0.7675 & 0.1158 & 0.0092 \\ 0 & 0 & 0.139 & 0.7092 & 0.1518 \\ 0.0001 & 0.0012 & 0.0022 & 0.0509 & 0.9455 \end{bmatrix} \mathbf{P_{2S}} = \begin{bmatrix} 0.7586 & 0.2328 & 0 & 0 & 0.0086 \\ 0.0089 & 0.8889 & 0.1003 & 0.0006 & 0.0013 \\ 0 & 0.0684 & 0.8524 & 0.074 & 0.0051 \\ 0 & 0 & 0.1234 & 0.7893 & 0.0869 \\ 0 & 0.0012 & 0.0055 & 0.1031 & 0.8902 \end{bmatrix}$$

$$\mathbf{P_{2W}} = \begin{bmatrix} 0.8270 & 0.1689 & 0.001 & 0 & 0.0031 \\ 0.0446 & 0.8685 & 0.0809 & 0 & 0.0056 \\ 0 & 0.0791 & 0.8468 & 0.0566 & 0.0175 \\ 0 & 0 & 0.2088 & 0.6087 & 0.1825 \\ 0.0008 & 0.0053 & 0.0159 & 0.0502 & 0.9279 \end{bmatrix} \mathbf{P_{2S}} = \begin{bmatrix} 0.8577 & 0.1398 & 0 & 0 & 0.0025 \\ 0.0257 & 0.9186 & 0.0529 & 0 & 0.0028 \\ 0 & 0.0858 & 0.8822 & 0.027 & 0.0048 \\ 0 & 0 & 0.2182 & 0.674 & 0.1078 \\ 0.0028 & 0.0211 & 0.0214 & 0.101 & 0.8537 \end{bmatrix}$$

$$\mathbf{P_{3W}} = \begin{bmatrix} 0.8747 & 0.1213 & 0 & 0 & 0.0041 \\ 0.0571 & 0.8670 & 0.0663 & 0.0002 & 0.0092 \\ 0 & 0.1149 & 0.7996 & 0.0678 & 0.0177 \\ 0 & 0.0008 & 0.1441 & 0.7492 & 0.1059 \\ 0.0034 & 0.0159 & 0.0159 & 0.0565 & 0.9082 \end{bmatrix} \mathbf{P_{3S}} = \begin{bmatrix} 0.9011 & 0.0974 & 0.0003 & 0.0003 & 0 \\ 0.0422 & 0.9212 & 0.0333 & 0.0002 & 0.003 \\ 0 & 0.1385 & 0.8204 & 0.0381 & 0.003 \\ 0 & 0 & 0.1751 & 0.7419 & 0.083 \\ 0.0095 & 0.0694 & 0.0315 & 0.0986 & 0.791 \end{bmatrix}$$

$$\mathbf{P_{4W}} = \begin{bmatrix} 0.9135 & 0.0825 & 0 & 0.0003 & 0.0037 \\ 0.0580 & 0.8832 & 0.0445 & 0.0002 & 0.014 \\ 0.0005 & 0.1059 & 0.8102 & 0.0691 & 0.0143 \\ 0 & 0 & 0.1853 & 0.7031 & 0.1116 \\ 0.0059 & 0.0327 & 0.0164 & 0.0481 & 0.8969 \end{bmatrix} \mathbf{P_{4S}} = \begin{bmatrix} 0.935 & 0.0625 & 0.0002 & 0 & 0.0024 \\ 0.06 & 0.9169 & 0.0199 & 0 & 0.0031 \\ 0 & 0.1383 & 0.831 & 0.0285 & 0.0022 \\ 0 & 0 & 0.1445 & 0.7457 & 0.1098 \\ 0.0545 & 0.099 & 0.0248 & 0.0891 & 0.7327 \end{bmatrix}$$

The index shows a group number, indexes "S" and "W" meaning winter or summer period. Due to the limited space in this paper the marine project is analysed for the winter season only.

TABLE 1. Efficiency states, ranges and factors.

States	State 1	State 2	State 3	State 4	State 5
Efficiency Factors	100%	80%	60%	40%	0%
Group 1 (m)	[0 0.25)	[0.25 0.5)	[0.5 0.75)	[0.75 1]	>1
Group 2 (m)	[0 0.375)	[0.375 0.75)	[0.75 1.25)	[1.25 1.5]	>1.5
Group 3 (m)	[0 0.5)	[0.5 1)	[1 1.5)	[1.5 2]	> 2
Group 4 (m)	[0 0.625)	[0.625 1.25)	[1.25 1.875)	[1.875 2.5]	>2.5

The marine project example related to the installation process of bottom-fixed offshore wind turbines provided by Sway (Norway) is shown in Figure 2, where numbers in parentheses define the uninterrupted activity durations in hours assuming 100% efficiency. The activity details are presented in Table 2, where α and β are numbers of time intervals required to restart and suspend an activity, λ is a minimum number of intervals required between two successive passages. The superscript "*" indicates an activity where the weather window is needed. Such an activity must be completed within the weather window. It cannot be suspended and restarted like the other type of activity.

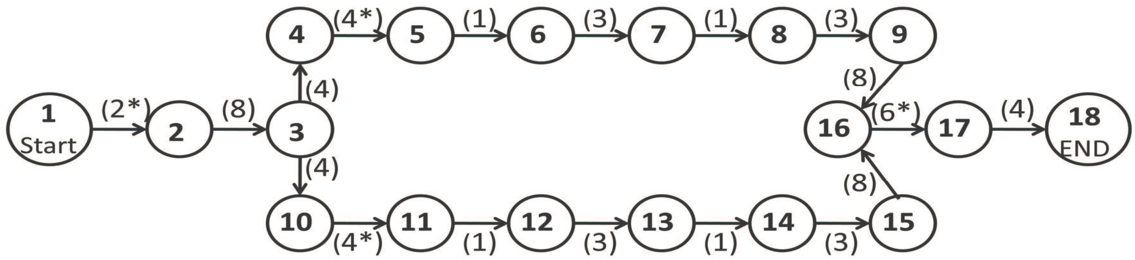

FIGURE 2. Network of marine project.

TABLE 2. Operational characteristics of the marine project.

Operation	Activity	Duration, h	α, h	β, h	λ, h	Trans. Matrix
Transit *	1-2	2	0	0	2	$\mathbf{P_{3w}}$
Jack Up	2-3	8	3	2	6	$\mathbf{P_{2w}}$
Desk Operations	3-4 ; 3-10	4	1	1	3	$\mathbf{P_{2w}}$
Pile Transfer *	4-5 ; 10-11	4	0	0	4	$\mathbf{P_{1w}}$
Pile Upending	5-6; 7-8; 11-12; 13-14	1	0	0	1	$\mathbf{P_{3w}}$
Pile Driving	6-7; 8-9; 12-13; 14-15	3	1	1	2	$\mathbf{P_{4w}}$
Measurements	9-16; 15-16	8	2	1	4	$\mathbf{P_{3w}}$
Sea fastening *	16-17	6	0	0	6	$\mathbf{P_{2w}}$
Jack Down	17-18	4	2	1	3	$\mathbf{P_{2w}}$

The probability distribution function of the activity duration is associated with a mean and variance of the return intervals (Anastasiou and Tsekos, 1996), hence the differences between the Markov Theory and Monte Carlo simulations based curves occur. The comparison of the probability distributions based on both methods is presented in Figure 3. This figure illustrates the probability distribution of four operations, where two operations (pile transfer and sea fastening) require a weather window and the other two operations (pile upending and jack down) do not require a weather window and hence can be suspended and restarted. The dashed lines indicate the activities required a weather window and solid lines indicate the activities which do not require a weather window. The numbers of the activities refer to Table 2. The distributions derived from the Monte Carlo simulations have been defined from 1000 simulations runs. A comparison of the statistical parameters of all activities obtained from the Markov Theory and Monte Carlo simulations are shown in Table 3 where the difference between the parameters depend on simplifying assumptions made for the Markov model by (Anastasiou and Tsekos, 1996). The difference between the kurtosis values is greater than the difference between expected values. The mean and variance of the duration of each activity have been defined according to the procedures presented in (Anastasiou and Tsekos, 1996). The skewness and kurtosis of each activity have been found numerically from the curves. MATLAB code of these procedures is presented in Appendix A and B.

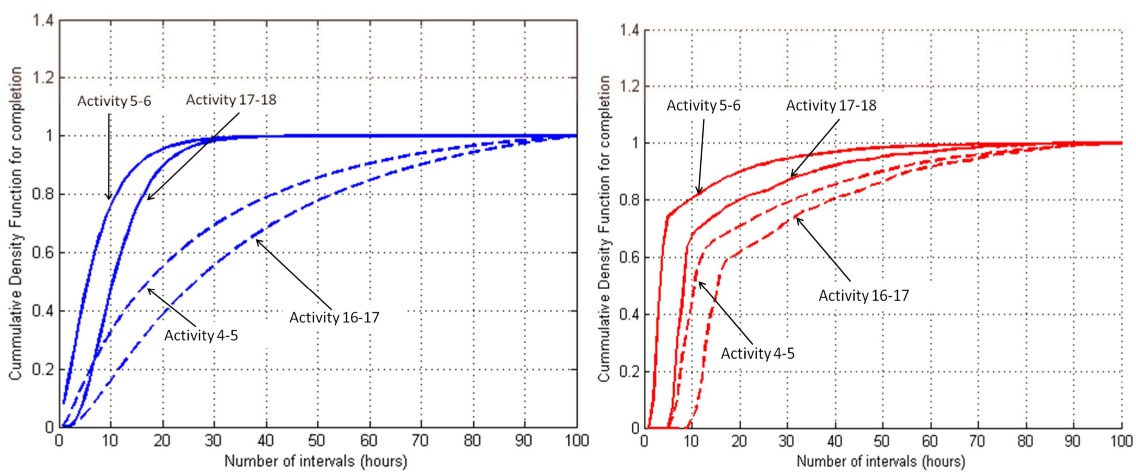

FIGURE 3. Probability distributions based on Markov Theory (left) and Monte Carlo simulations (right).

TABLE 3. Statistical parameters of the activity durations according to the Markov model and Monte Carlo simulations.

Activity	Exp. Value Markov/M. Carlo	Variance Markov/M. Carlo	Skewness Markov/M. Carlo	Kurtosis Markov/ M.Carlo
1-2	7.60/11.82	74.02/90.91	1.59/2.8	2.95/8.35
2-3	21.67/24.98	144.36/231.09	0.52/1.55	0.77/1.33
3-4	13.42/19.06	147.64/225.22	0.89/1.95	0.16/3.0
4-5	14.18/25.34	189.52/196.09	0.51/1.75	0.48/2.02
5-6	4.56/10.18	50.56/130.31	1.75/3.81	2.95/16.52
6-7	5.95/11.50	34.08/112.67	1.66/4.2	2.87/12.95
9-16	19.75/21.62	80.47/188.40	0.82/2.67	0.14/7.17
16-17	15.18/32.39	75.17/187.82	0.08/1.9	0.68/4.4
17-18	19.36/20.48	137.28/230.33	0.86/1.62	0.24/1.66

The probability distribution of the entire project duration is defined from the probability distribution of a set of individual activities presented in Table 2. In this paper a combination of Markov processes and PNET methodology is used to evaluate the project duration. Probabilistic Network and Evaluation Technique is described in (Ang et al., 1975). The project shown in Figure 2 consists of two paths, where the mean duration and variance of each path are determined as the sum of mean durations and variances of the individual activities. The critical path is defined as a path with maximum mean duration. The second path is correlated in relation to the empirical transitional correlation index ρ_0.

In PNET method a correlation between any two paths is based on the duration of their common activities. In (Ang et al., 1975) and (Ökmen et al., 2008) the transitional correlation index is defined as ρ_0=0.5, which is appropriate for ordinary construction projects. However, a higher degree of correlation is required for marine projects where weather conditions are more critical. In this paper two correlation indices are analysed and illustrated in Figure 4, where a higher index yields two critical paths (blue dotted line) and results have more conservative predictions. The Monte Carlo simulation results are also shown in Figure 3 by dashed red line.

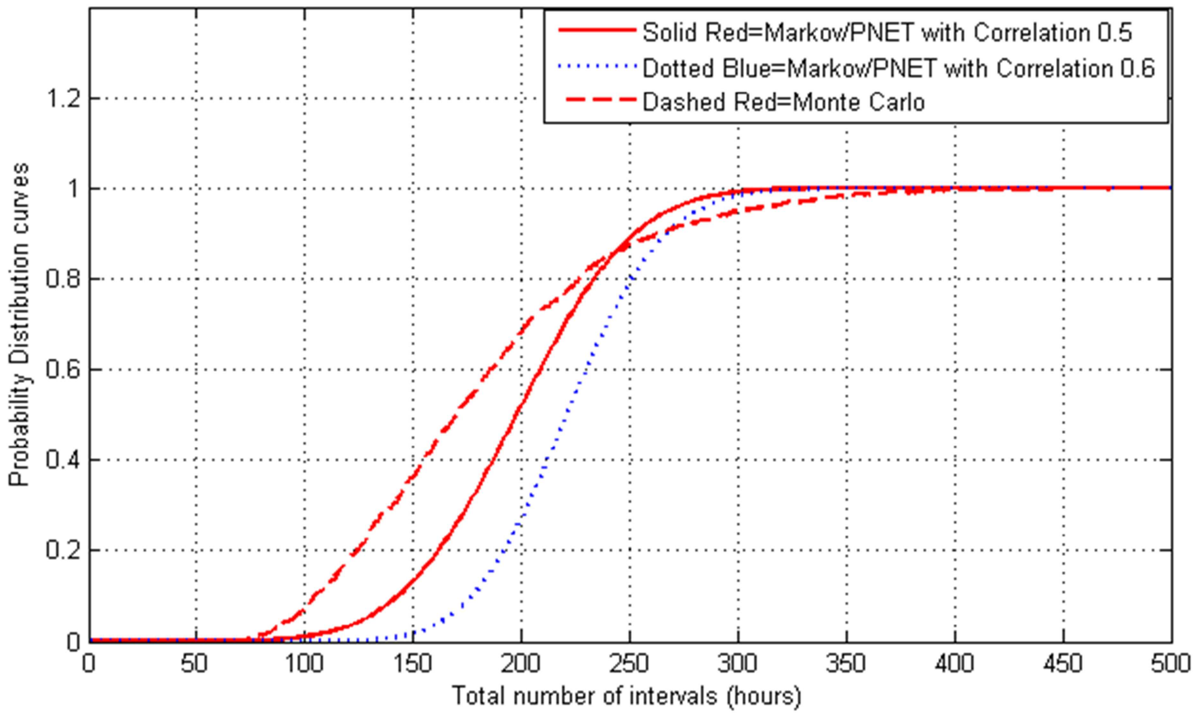

FIGURE 4. Comparison between the probability distribution of the project duration obtained from Monte Carlo simulations (dashed line) and Markov Theory (solid line).

3. CONCLUSIONS

The project network completion-time probability evaluation methodology presented in this paper is based on a combination of Markov theory and PNET algorithm. The Markov processes may be effectively used to evaluate both the activities which require a weather window and the activities where a weather window is not required. A use of Markov Theory in project scheduling has some benefits, such as an ability to apply constraints (different environmental conditions) and flexible efficiency levels which may affect the activity duration.

The values of the statistical parameters derived from the Markov Theory are compared to the parameters derived from Monte Carlo simulations. The parameters related to the first order moments have excellent agreement but higher moment orders are significantly different, especially kurtosis. It probably depends on simplifying assumptions made for the Markov theory as well as limited records which do not enable an accurate derivation of return intervals of the operable states.

Both probability distribution curves of the activity durations related to Markov model and Monte Carlo simulations have a reasonable agreement for the activities which do not require a weather window or require small uninterrupted duration. Hence, long uninterrupted activities should be divided to shorter activities to improve simulation accuracy. In most cases MCS results in more conservative values of probability distribution than PNET method because MCS considers every path and activity in the network as a potential cause of failure of completion of the project in a given time. A computational time using PNET algorithm with Markov chain is 17 times faster than those required by MCS where MCS has 10,000 simulation runs. In comparison, PNET method is only 2 times faster when MSC has 1,000 runs, but in this case the MCS results are less accurate.

The authors of this paper found that the correlation coefficient (ρ_0) determines whether the result is liberal or conservative. In regard to choosing a coefficient value, there are several considerations that a scheduler should keep in mind. In (Ang et al., 1975) the proposed ρ_0=0.5 which suits many construction networks, however, a higher coefficient value is more appropriate for marine operation networks. If most of the paths in the network are closely correlated (share a large number of activities), the coefficient value between 0.6 and 0.8 gives more conservative results closer to MCS. Until more research is done in a field of correlation coefficient analysis, the authors suggest to solve a problem using two different values of correlation even than the networks are large. The objective of this paper did not include a parametric study of correlation due to a limited project network size, however a sensitivity analysis of value of correlation coefficient is one of the further directions of the research project.

A combination of Markov processes and PNET provides a convenient and effective tool for the marine projects evaluation under uncertainty. A main contribution of the paper is the MATLAB code presented in the Appendix A and B. The Markov theory presented by (Anastasiou and Tsekos, 1996) is not easily implemented (both for activities which require and do not require a weather window). The authors believe that by making the MATLAB code available, the Markov approach to evaluating marine operations will become more accessible for other researchers and practitioners.

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REFERENCES

Anastasiou, K. and Tsekos, C. 1996. Operability analysis of marine project based on Markov theory. Applied Ocean Research, Vol. 18 pp. 329-352.

Ang, A.H.S., Abdenour, J. and Chaker, A.A. 1975. Analysis of Activity Networks under Uncertainty. Journal of the Engineering Mechanics Division, Vol. 101, No. 4, pp. 373-387.

Diaz, C.F. and Hadipriono, F.C. 1993. Nondeterministic Networking Methodds. Journal of Construction Engineering and Management. Vol. 119, No. 1, pp. 40-56.

Ökmen, Ö. and Öztas, A. 2008. Construction Project Network Evaluation with Correlated Schedule Risk Analysis Model. Journal of Construction Engineering and Management. Vol. 134, No. 1, pp. 49-63.

APPENDIX A – MATLAB CODE FOR MARKOV THEORY

In this Appendix a MATLAB based code is presented. The code is used to calculate a whole project duration based on a combination of Markov Theory and PNET algorithm. The function `chap3monte_markov` which generates a mean and standard deviation for a weather window is not presented here due to the limited space. These parameters are found simulatiouslly from an observed data.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%   MAIN MODULE  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all
r = [1 0.8 0.6 0.4 0]; % here is a vector of Efficiency Factors
%%%%%%%%%%%%%%%%%%%%%%%%%%%% Activities%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   Activity 1-2 WITH a weather window
[ P ] = get_P( 3 ); % Function to get Transmissibility Matrix from observed %data according to
the efficiency states. Activity 1-2 has matrix P_{3W}
alpha=0; beta=0; lambda=2; T=2; % number of intervals to restart and suspend
% operation, number of minimum number of operatable intervals and operation duration.
NDays=40; % number of time intervals used to define an activity duration
[ A12,Avv12,ExpVal12,n12,ft12,Omega ] = markov_probability_w_weather(P,r,NDays,T);
[ set12 ] = markov_statistics(A12,ExpVal12,n12,ft12);
% Two function above are used to calculate statistical parameters,
% probability distribution help variables related to an activity there a
% weather window is required. All other activities with a weather window
% are found in a similar manner,where alpha, beta, lambda and duration may % vary.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Activity 2-3 WITHOUT a weather window
[ P ] = get_P( 2 ); alpha=3; beta=2; lambda=6; T=8; NDays=60;
E=markovfunc(P,r,alpha,beta,lambda,T);
% A function above generates the parameters associated with Markov processes
[ A23,Avv23,ExpVal23,n23,ft23 ] = markov_probability_wo_weather(E,NDays,T);
[ set23 ] = markov_statistics(A23,ExpVal23,n23,ft23)
% Twelve other activities without a weather window are found in a similar %manner
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Two existing paths T1 and T2:
% 1->2->3->4->5->6->7->8->9->16->17->18
% 1-> 2->3->10->11->12->13->14->15->16->17->18
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Get mu and sigma^2 (paths) eq. 61-62 p 346 - (Anastasiou and Tsekos, 1996)
MuT1T2=set12(1)+set23(1)+set34(1)+set45(1)+set56(1)+set67(1)+set56(1)+set67(1)+set916(1)+set16
17(1)+set1718(1);
SigT1T2=set12(2)+set23(2)+set34(2)+set45(2)+set56(2)+set67(2)+set56(2)+set67(2)+set916(2)+set1
617(2)+set1718(2);
STDT1T2=set12(3)+set23(3)+set34(3)+set45(3)+set56(3)+set67(3)+set56(3)+set67(3)+set916(3)+set1
617(3)+set1718(3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Correlation between the paths T1 and T2
Cor12=(set12(2)+set23(2)+set1617(2)+set1718(2))/(SigT1T2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% gamma1 and gamma2 - eq.67 p.346 (Anastasiou and Tsekos, 1996)
gamma11=(set12(4)*set12(3)^3+set23(4)*set23(3)^3+set34(4)*set34(3)^3+set45(4)*set45(3)^3+set56
(4)*set56(3)^3+set67(4)*set67(3)^3+set56(4)*set56(3)^3+set67(4)*set67(3)^3+set916(4)*set916(3)
^3+set1617(4)*set1617(3)^3+set1718(4)*set1718(3)^3)/(SigT1T2)^(3/2);
gamma21=(set12(5)*set12(3)^4+set23(5)*set23(3)^4+set34(5)*set34(3)^4+set45(5)*set45(3)^4+set56
(5)*set56(3)^4+set67(5)*set67(3)^4+set56(5)*set56(3)^4+set67(5)*set67(3)^4+set916(5)*set916(3)
^4+set1617(5)*set1617(3)^4+set1718(5)*set1718(3)^4)/(SigT1T2)^(2);
% Norma=(x-MuT2)/(sqrt(SigT2))
%%%%%%%%%%%%%%%%%%%%%% Project duration%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Case 1: Critical Path 1 : (cumsum(PDF12/A12)+SecPart12)-red
% Case 2: Critical paths 1 and 2 : P23
NDays=500; n=1:1:NDays;
for i=1:length(n)
    Norma12(i)=(n(i)-MuT1T2)/(sqrt(SigT1T2));
    PDF12(i)=(1/sqrt(2*pi*SigT1T2))*exp(-(Norma12(i))^2/2);
    Brek12(i)=(gamma11*(1-Norma12(i)^2)/(6)+gamma21*(3*Norma12(i)-
Norma12(i)^3)/(24)+gamma11^2*(-15*Norma12(i)+10*Norma12(i)^3-Norma12(i)^5)/(72));
    SecPart12(i)=PDF12(i)*Brek12(i);
end
A12=sum(PDF12); TTT=cumsum(PDF12/A12);
for i=1:length(n)
    P23(i)= (TTT(i)+SecPart12(i))*(TTT(i)+SecPart12(i));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of Main Module %%%%%%%%%%%%%%%%%%%

function E=markovfunc(P,r,alpha,beta,lambda,T)
```

```
N=size(P);
% Partition of Transition Probabilities
G=P(1:N(1)-1,1:N(2)-1); V=P(1:N(1)-1,N(2)); Q=P(N(1),1:N(2)-1); pNN=P(N(1),N(2));
H,D]=eig(G); % Eigenvalues and Eigenvectors
d=diag(D); % These are the eigenvalues of G, lambda_s
Erho  = pNN + Q*H*diag((2-d)./(1-d).^2)*inv(H)*V;
Erho2 = pNN + Q*H*diag((d.*d-3*d+4)./(1-d).^3)*inv(H)*V;
N=length(P); R=   inv(H)*diag(r(1:N-1))*H; Rhat=inv(H)*diag(r(1:N-1).^2)*H;
for(s=1:N-1)
   for(t=1:N-1)
      if(s==t)
C(s,t)=R(s,s)*d(s)^(lambda-1)/(1-d(s))*(lambda-alpha-beta-1+1/(1-d(s)));
Chat(s,t)=Rhat(s,s)*d(s)^(lambda-1)/(1-d(s))*(lambda-alpha-beta-1+1/(1-  d(s)));
F(s,t)=R(s,s)*(d(s)^(lambda-1))/(1-d(s)) * ( (lambda^2*(1-d(s))^2 + 2*lambda*(1-d(s)) + d(s) +
1)/((1-d(s))^2) - (alpha+beta+1)*(lambda+1/(1-d(s))));
      else
C(s,t)=R(s,t)*d(s)^(-1)*d(t)^(-1)/(1-d(s)/d(t))*((d(s)/d(t))^(alpha+1)*d(t)^(lambda+1)/(1-
d(t))-(d(s)/d(t))^(-beta)*d(s)^(lambda+1)/(1-d(s)));
Chat(s,t)=Rhat(s,t)*d(s)^(-1)*d(t)^(-1)/(1-
d(s)/d(t))*((d(s)/d(t))^(alpha+1)*d(t)^(lambda+1)/(1-d(t))-(d(s)/d(t))^(-
beta)*d(s)^(lambda+1)/(1-d(s)));
F(s,t)=R(s,t)*(d(s)^(-1)*d(t)^(-1))/(1-d(s)/d(t)) * (
(d(s)/d(t))^(alpha+1)*(d(t)^(lambda+1))/(1-d(t))*(lambda+1/(1-d(t))) - (d(s)/d(t))^(-
beta)*(d(s)^(lambda+1))/(1-d(s))*(lambda+1/(1-d(s))) );
      end;
       for(u=1:N-1)
          if((s==t) && (t==u))
mu(s,t,u)=R(s,s)*d(s)^(lambda-1)/(1-d(s))*( (lambda*(1-d(s))*(lambda*(1-
d(s))+2)+d(s)+1)/(2*(1-d(s))^2) - (alpha+beta+1.5)*(lambda+1/(1-d(s))) + 0.5*((alpha+beta-
1)^2+5*(alpha+beta-1)+6));
          elseif(t==u)
mu(s,t,u)=R(t,t)*d(s)^(-1)*d(t)^(-1)/(1-d(s)/d(t))*( (d(s)/d(t))^(alpha+1)*(lambda+1/(1-d(t))-
alpha-beta-1-1/(1-d(s)/d(t)))*d(t)^(lambda+1)/(1-d(t)) + (d(s)/d(t))^(-beta)/(1-d(s)/d(t)) *
(d(s)^(lambda+1))/(1-d(s)) );
          elseif(s==t)
mu(s,t,u)=R(t,u)*1/(1-d(t)/d(u))*( (d(u)^(-2))/(1-d(t)/d(u)) * (
(d(t)/d(u))^(alpha+1)*(d(u)^(lambda+1))/(1-d(u)) - (d(t)/d(u))^(-beta-1)*(d(t)^(lambda+1))/(1-
d(t)) ) - (d(t)/d(u))^(-beta)*d(u)^(-1)*(d(t)^lambda)/(1-d(t)) * (lambda-alpha-beta-2+1/(1-
d(t))));
          elseif(s==u)
mu(s,t,u)=R(t,u)*1/(1-d(t)/d(u))*( -(d(u)^(-2))/(1-d(u)/d(t)) * ( (d(u)/d(t))^(alpha+beta+1) *
(d(t)^(lambda+1))/(1-d(t)) - (d(u)/d(t))^(-1) * d(u)^(lambda+1)/(1-d(u))) +
(d(t)*d(u)^(lambda-2))/(1-d(u))*(lambda-alpha-beta-2+1/(1-d(u))));
          else
mu(s,t,u)=R(t,u)*(d(s)^(-1))/(1-d(t)/d(u)) * ( (d(t)*d(u)^(-2))/(1-d(s)/d(u)) *
((d(s)^(alpha+1))*d(u)^(lambda-alpha)/(1-d(u)) - d(s)^(lambda-beta)*d(u)^(beta+1)/(1-d(s))) -
(d(u)^(beta-1))/(1-d(s)/d(t)) * (d(t)^(lambda-alpha-beta)*d(s)^(alpha+1)/(1-d(t)) -
(d(t)*d(s)^(lambda-beta))/(1-d(s)) ));
          end; end; end; end;
for(s=1:N-1)
    for(u=1:N-1)
        THETA(s,u)=0;
        for(t=1:N-1)
            THETA(s,u)=THETA(s,u)+R(s,t)*mu(s,t,u);
        end; end; end;
EWv = Q*H*C*inv(H)*V; EWv2 = Q*H*(Chat+2*THETA)*inv(H)*V; EWvrho = Q*H*F*inv(H)*V;
M = EWv/Erho; EZv2 = EWv2 - 2*M*EWvrho + M^2*Erho2; B = EZv2/Erho;
E=[Erho Erho2 EWv EWv2 EWvrho EZv2 M B];
%%%%%%%%%%%%%%%%%%%%%% End of Function %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [ A,Avv,ExpVal,x,fs,Omega  ] = markov_probability_w_weather(P,r,NDays,T)
Nrho=NDays;            % Maximum number of intervals (days) in the calculations
r=[r(1) r(2) r(3) r(4)]; N=length(P);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 3a in procedure described in Section 3.5 in (Anastasiou and Tsekos, 1996) %
% Transition Probabilities P2 excluding non-operable state N
for(i=1:N-1)
    P2(i,:)=P(i,1:N-1)/(1-P(i,N));
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 3c in procedure described in Section 3.5 in (Anastasiou and Tsekos, 1996)
alpha=0; beta=0; lambda=1; PI=P2^40; PI=PI(1,:)/sum(PI(1,:)); M=sum(r*PI');
Omega=round(T/M); sigmak=0; [window_mean, window_std]=chap3monte_markov(T,Omega,P);
for(ktemp=1:N-1)
    [P3,r3]=shufflestates(P2,r,ktemp); E=markovfunc(P3,r3,alpha,beta,lambda,T);
    sigmak=sigmak+E(6); end;
```

```
sigmak=sigmak/(N-1); %Average
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 3e in procedure described in Section 3.5 in (Anastasiou and Tsekos, 1996)
% Here we need the probability of the length of the weather window: Prob(nrho=n)
x=1:Nrho; %Days
Probnrho=normpdf(x,window_mean,window_std); Probnrho=Probnrho/sum(Probnrho);
i=find(x==Omega); ProbYv=sum(Probnrho(1:i+1))+sum(Probnrho(i+2:round(Omega/min(r)+1)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 3d in procedure described in Section 3.5 in (Anastasiou and Tsekos, 1996)
mu_nrho=0; sigma2_nrho=0;
for(n=1:Omega+1)
    mu_nrho=mu_nrho+n*Probnrho(n); %Eq.(43), part 1
    sigma2_nrho=sigma2_nrho+n^2*Probnrho(n); %Eq.(44), part 1
end;
for(n=Omega+2:round(Omega/min(r)+1))
    EYv=M*(n-1); Eq.(34) in paper
    VarYv=mean(PI)*sigmak*(n-1); % Eq.(38)
    mu_nrho=mu_nrho+n*Probnrho(n)*normcdf(n,Omega-EYv,sqrt(VarYv)); %Eq.(43), part 2
    sigma2_nrho=sigma2_nrho+n^2*Probnrho(n)*normcdf(n,Omega-EYv,sqrt(VarYv)); %Eq.(44)
end;
mu_nrho=mu_nrho/ProbYv; %Eq.(43)
sigma2_nrho=sigma2_nrho/ProbYv-mu_nrho^2; %Eq.(44)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 3g in procedure described in Section 3.5 (Anastasiou and Tsekos, 1996)
fp=zeros(Nrho+1,1); fp(1)=1-ProbYv; %Eq.(50)
for(rho=1:Nrho)
    for(k=1:10) %10 should be enough, increase if necessary
       fp(rho+1)=fp(rho+1)+normpdf(rho,k*mu_nrho,sqrt(k*sigma2_nrho))*ProbYv^k; %Eq.(49)
    end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEPS 4a and 4b in procedure described in Section 3.5 (Anastasiou and Tsekos, 1996)
for(n=1:Nrho) %Intervals
    EDn=M*n; VarDn=mean(PI)*sigmak*n; %Eq.(53) %Eq.(52)
    fDn_n=normpdf(T,EDn,sqrt(VarDn)); SfDn=0;
    for(l=T:round(T/min(r)))
        SfDn=SfDn+normpdf(T,M*l,sqrt(mean(PI)*sigmak*l));
    end;
    fn(n)=fDn_n/sum(SfDn); %Eq.(55)
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STEP 5 in procedure described in Section 3.5 in (Anastasiou and Tsekos, 1996)
for(s=1:Nrho)
    fs(s)=0;
    for(rho=1:s)
        fs(s)=fs(s)+fp(rho)*fn(1+s-rho);
    end;
    Aver(s)=fs(s)*s;
end;
fp=fp/sum(fp); fn=fn/sum(fn); A=sum(fs); Avv=sum(Aver); ExpVal=Avv/A;
End
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [ A,Avv,ExpVal,n,ft ] = markov_probability_wo_weather(E,NDays,T)
n=0:0.1:NDays;
for i=2:length(n)
ft(i)=1/sqrt(2*pi*E(8)*n(i))*exp(-((T-n(i)*E(7))^2)/(2*E(8)*n(i))); Aver(i)=ft(i)*n(i);
end
A=sum(ft); Avv=sum(Aver); ExpVal=Avv/A; end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [ set ] = markov_statistics(A,ExpVal,n,ft)
for i=2:length(n)
 AverSq(i)=(n(i)-ExpVal)^2*ft(i)/A;
end;
Variance=sum(AverSq); STD=sqrt(Variance);
for i=2:length(n)
Skr(i)=((n(i)-ExpVal)^3*ft(i))/(A*STD^3); Kur(i)=((n(i)-ExpVal)^4*ft(i))/(A*STD^4);
end;
Skewness=sum(Skr);Kurtosis=sum(Kur)-3; set(1)=ExpVal; set(2)=Variance; set(3)=STD;
set(4)=Skewness; set(5)=Kurtosis; end
```

APPENDIX B – MATLAB CODE FOR MONTE CARLO SIMULATIONS

This section contains the base functions monte2.m and monte3.m for the Monte Carlo Simulations

```
% This function returns the number of days required to complete an activity, given the wanted
% score (T), the number of restart days (alpha), number of suspend days (beta) and number
% of effective days (lambda). No weather window required
function i=monte2(T,alpha,beta,lambda,P)
r = [1.0 0.8 0.6 0.4 0.0];  % Efficiency factors:
NN = length(r);             % Index of non-operational state
state=1+round(rand(1)*4);   % Initial state
score=0;                    % We simulate number of days until score = T
bSuspend=false;             % We are in suspend mode
bRestart=true;              % We are in restart mode
bOperating=false;           % We are in operational mode
NNindex=0;                  % Days between non-operational states
RestartDays=0;              % Count how many days we have been in restart mode
SuspendDays=0;              % Count how many days we have been in suspend mode
i=0;                        % Hr number
while(score<T)
    i=i+1;
    if(bRestart)
        RestartDays=RestartDays+1;
        if(RestartDays>alpha)
            bRestart=false; bOperating=true; RestartDays=0; end;
    elseif(bSuspend)
        SuspendDays=SuspendDays+1;
        if(SuspendDays>beta)
            bSuspend=false; SuspendDays=0;
        end;
    elseif(i-NNindex>lambda)
        bOperating=true;
    end;
    if(bOperating)
        score=score+r(state);
    end;
    state=newstate(state,P);
    if(state==NN)
        bOperating=false;bRestart=false;RestartDays=0;bSuspend=false;SuspendDays=0;NNindex=i;
    end; end;

% This function returns the number of days i until a weather window occurs. Omega = Required
% score of the weather window. T = Required score for the marine operation
function i=monte3(T,Omega,P)
r = [1.0 0.8 0.6 0.4 0.0];  % Efficiency factors:
NN = length(r);             % Index of non-operational state
state=1+round(rand(1)*4);   % Initial state
score=0;                    % We simulate number of days until score = T
i=0;                        % Number of days until weather window complete
windowstart=0;              % Index for when the window started
% Step 1: Wait for the weather window
while(score<Omega)
    i=i+1; state=newstate(state,P);
    if(state==NN)
        windowstart=i; score=0;
    else
        score=score+r(state);
    end;
end;
i=windowstart;
% Step 2: Perform the activity. Transition Probabilities P2 excluding non-operable state N
N=length(P);
for(j=1:N-1)
    P2(j,:)=P(j,1:N-1)/(1-P(j,N));
end;
score=0; state=1+round(rand(1)*3);   % Initial state
while(score<T)
    i=i+1; state=newstate(state,P2); score=score+r(state);
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function xout=newstate(x,P)
Prob=cumsum(P(x,:)); y=rand(1); xout=1;
for(i=1:length(Prob)-1)
    if(y>Prob(i))    xout=i+1;    end;end;
```