Project 1 - Notes

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DuBois and Glyde VMC paper

Results

No interaction

Only $\phi_0(r)$ occupied ($n_0 = 1$). Excellent agreement between $\phi_0(r)$ and HO ground-state function for all r. (Good check of method for noninteracting bosons.)

Interaction

Energy per particle, E/N, in units of $\hbar\omega_\perp$ shows a steep inclination. Energies from VMC and Gross-Pitaevskii differ at $N \geq 10^4$. Likely due to excitation of bosons above the condensate, which VMC takes into account. This is very likely not relevant for our values of N.

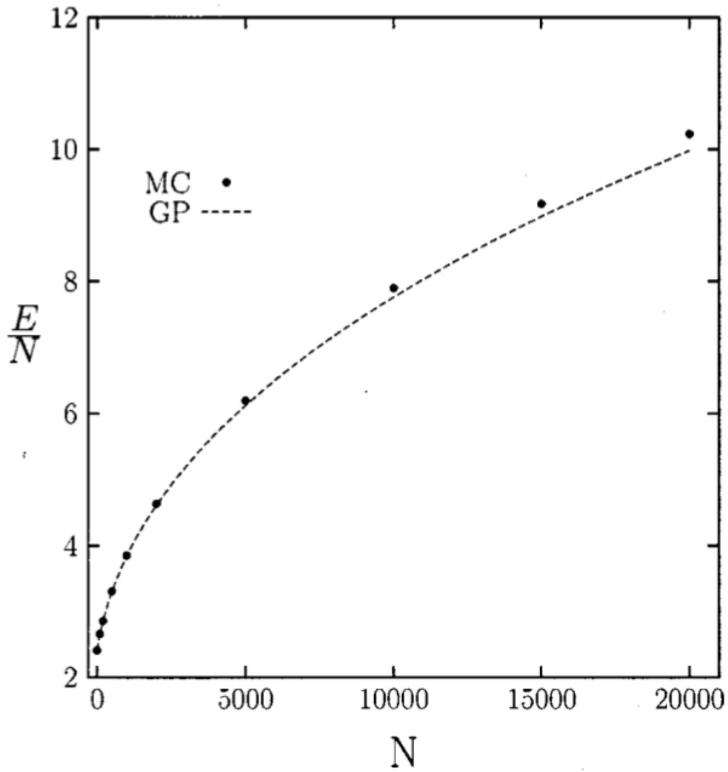

FIG. 2. Energy per particle, in units of $\hbar\omega_\perp$, for hard-sphere bosons in an anisotropic trap as a function of the number of particles N in the trap. Solid circles are the present VMC values for hard spheres with diameter corresponding to the scattering length of Rb, $a_{\mathrm{Rb}}=4.33\times10^{-3}a_{ho}$, where a_{ho} is the trap length in the perpendicular direction. The error bars lie within the solid dots. The dashed line is the value obtained using the Gross-Pitaevskii equation (GP) for the same system [17].

Figure 1: E/N vs N from DuBois paper.

Morten et al paper

Table 1: Energies in units og $\hbar\omega_\perp$ for VMC calculations presentet in table 1. Scattering length $a = 35a_{Rb} = 0.15155a_\perp$, $\lambda = \sqrt{8}$, $N = 500$

E/N	E_{kin}/N	E_{HO}/N
11.12109(14)	4.21520(24)	6.90590(19)

Some potentially relevant equations relating the values in the table above:

$$E_{kin} = \frac{\hbar^2}{2m} \int \mathrm{d}\vec{r} |\nabla \Psi(\vec{r})|^2$$

$$E_{HO} = \frac{m}{2} \int \mathrm{d}\vec{r} (\omega_\perp^2 (x^2 + y^2) + \omega_z^2 z^2) |\nabla \Psi(\vec{r})|^2$$

$$E_1 = \frac{2\pi\hbar^2 a}{m} \int \mathrm{d}\vec{r} |\Psi(\vec{r})|^4$$

$$E_2 = \frac{2\pi\hbar^2 a}{m} \frac{128}{15} (\frac{a^3}{\pi})^{1/4} \int \mathrm{d}\vec{r} |\Psi(\vec{r})|^5$$

The Virial theorem is used to esablish a relation between the different contributions to the energy:

$$2E_{kin} - 2E_{HO} + 3E_1 + \frac{9}{2} E_2 = 0$$

which serves as a proof of the numerical accuracy of the solution of the GP equations (Gross-Pitaevskii).