QUANTITATIVE MANAGEMENT MODELING HW 3

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R Markdown

Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?
1. Formulate and solve this transportation problem using R
2. Formulate the dual of this transportation problem
3. Make an economic interpretation of the dual

Objective Function
$MinimizeTC = 622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$
These are subject to the following constraints
$x_{11} + x_{12} + x_{13} >= 100$
$x_{21} + x_{22} + x_{23} >= 120$
these are the supply constraints
$x_{11} + x_{21} >= 80$
$x_{12} + x_{22} >= 60$
$x_{13} + x_{23} >= 70$
these are the demand constraints

These are all subject to non-negativity where x_{ij}>=0 where i=1,2 and j=1,2,3
Here is what the table looks like:

```
library(Matrix)
library("lpSolve")
display <- matrix(c(22,14,30,600,100,
                 16,20,24,625,120,
                 80,60,70,"-","210/220"),ncol=5,nrow=3,byrow=TRUE)
colnames(display) <- c("Warehouse1","Warehouse2","Warehouse3","Prod Cost","Prod Capacity")
rownames(display) <- c("PlantA","PlantB","Monthly Demand")
display <- as.table(display)
display
```

```
##                Warehouse1 Warehouse2 Warehouse3 Prod Cost Prod Capacity
## PlantA         22         14         30         600       100
## PlantB         16         20         24         625       120
## Monthly Demand 80         60         70         -         210/220
```

Being the capacity is equal to 220 and Demand is equal to 210 we need to add a “dummy” row where a Warehouse4 would be. It will contain 0 and 0 for each of the plants and the dummy will add to the total up to 220. The table would then look like this:

```
display1 <- matrix(c(622,614,630,0,100,
                 641,645,649,0,120,
                 80,60,70,10,220),ncol=5,nrow=3,byrow=TRUE)
colnames(display1) <- c("Warehouse1","Warehouse2","Warehouse3","Dummy","Production Capacity")
rownames(display1) <- c("PlantA","PlantB","Monthly Demand")
display1 <- as.table(display1)
display1
```

```
##                Warehouse1 Warehouse2 Warehouse3 Dummy Production Capacity
## PlantA                622        614        630     0                 100
## PlantB                641        645        649     0                 120
## Monthly Demand         80         60         70    10                 220
```

This table now satisfies the need for a balanced problem. Now we are ready to solve within R. First we want to make the costs matrix:

```
costs <- matrix(c(622,614,630,0,
                  641,645,649,0),nrow=2, byrow = TRUE)
```

Next we will identify the Production Capacity in the row of the matrix:

```
row.rhs <- c(100,120)
row.signs <- rep("<=", 2)
```

Then we will identify the Monthly Demand with double variable of 10 at the end. Above we added the 0,0 in at the end of each of the columns:

```
col.rhs <- c(80,60,70,10)
col.signs <- rep(">=", 4)
```

Now we are ready to run LP Transport command:

```
lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

Here is the solution matrix:

```
lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

This gives us the following that $Z = 132,790$dollars. This gives us the following results for each of the variables: $60x_{12}$ which is Warehouse 2 from Plant A.

$40x_{13}$ which is Warehouse 3 from Plant A.
$80x_{21}$ which is Warehouse 1 from Plant B.
$30x_{23}$ which is Warehouse 3 from Plant B.
and because "10" shows up in the 4th variable $10x_{24}$ it is a "throw-away variable"
This would complete the answer for question 1.

We know that number of variables in primal is equal to the number of constants in dual. The first question is the primal of the LP. Since we took the minimization in the primal we will maximize in the dual. Let's use the variables u and v for the dual problem

```
display2 <- matrix(c(622,614,630,100,"u_1",
                 641,645,649,120,"u_2",
                 80,60,70,220,"-",
                 "v_1","v_2","v_3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(display2) <- c("W1","W2","W3","Prod Cap","Supply (Dual)")
rownames(display2) <- c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
display2 <- as.table(display2)
display2
```

```
##                W1  W2  W3  Prod Cap Supply (Dual)
## PlantA         622 614 630 100      u_1
## PlantB         641 645 649 120      u_2
## Monthly Demand 80  60  70  220      -
## Demand (Dual)  v_1 v_2 v_3 -        -
```

From here we are going to create our objective function based on the constraints from the primal. Then use the objective function from the primal to find the constants of the dual.
Maximize $Z = 100u_1 + 120u_2 + 80v_1 + 60v_2 + 70v_3$
this objective function is subject to the following constraints.
$u_1 + v_1 <= 622$
$u_1 + v_2 <= 614$
$u_1 + v_3 <= 630$
$u_2 + v_1 <= 641$
$u_2 + v_2 <= 645$
$u_2 + v_3 <= 649$
These constants are taken from the transposed matrix of the Primal of Linear Programming function. An easy way to check yourself is to transpose the f.con into the matrix and match to the constants above in the Primal. These are unrestricted where u_k, v_l where u=1,2 and v=1,2,3

```
#Constants of the primal are now the objective function variables.
f.obj <- c(100,120,80,60,70)
#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1),nrow=6, byrow = TRUE)

#these change because we are MAX the dual not min
f.dir <- c("<=",
"<=",
"<=",
"<=",
"<=",
"<=")

f.rhs <- c(622,614,630,641,645,649)

lp ("max", f.obj, f.con, f.dir, f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp ("max", f.obj, f.con, f.dir, f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

So Z=139,120 dollars and variables are: $u_1 = 614$ which represents Plant A
$u_2 = 633$ which represents Plant B
$v_1 = 8$ which represents Warehouse 1
$v_3 = 16$ which represents Warehouse 3

OBSERVATION
The minimal Z=132790 (Primal) and the maximum Z=139120(Dual). What are we trying to max/min in this problem. We found that we should not be shipping from Plant(A/B) to all three Warehouses. We should be shipping from:

$60x_{12}$ which is 60 Units from Plant A to Warehouse 2.
$40x_{13}$ which is 40 Units from Plant A to Warehouse 3.
$80x_{21}$ which is 60 Units from Plant B to Warehouse 1.
$30x_{23}$ which is 60 Units from Plant B to Warehouse 3.
Now we want to Max the profits from each distribution in respect to capacity.

Now I have been working very hard to try and get the third question correct from the problem.
From the notes taken on Tuesday we would have the following:
$u_1^0 - v_1^0 <= 622$
then we subtract v_1^0 to the other side to get$u_1^0 <= 622 - v_1^0$
To compute that value it would be $614<=(-8+622) which is true. We would continue to evaluate these equations:
$u_1 <= 622 - v_1$===614<=622-8=614 = TRUE
$u_1 <= 614 - v_2$===614<=614-0=614 = TRUE
$u_1 <= 630 - v_3$===614<=630-16=614 = TRUE
$u_2 <= 641 - v_1$===633<=641-8=633 = TRUE
$u_2 <= 645 - v_2$===633<=645-0=645 = NOT TRUE

$u_2 <= 649 - v_3$===633<=649-16=633= TRUE
Also learning from the Duality-and-Sensitivity.pdf we can test for the shadow price by updating each of the column. We change the 100 to 101 and 120 to 121 in our LP Transport. You can see the work below in R.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=", 2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=", 4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=", 2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=", 4)
lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```
lp.transport(costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```
lp.transport(costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

Since we are taking the min of this specific function seeing the number go down by 19 means the shadow price is 19, that was found from the primal and adding 1 to each of the Plants. However with Plant B does not have a shadow price. We also found that the dual variable v_2 where Marginal Revenue (MR) <= Marginal Cost (MC). Recalling the equation which was $u_2 <= 645 - v_2$===633<=645-0=645 = NOT TRUE which was found by using $u_1^0 - v_1^0 <= 622$ also that

```
lp ("max", f.obj, f.con, f.dir, f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

v_2 was = to 0.
CONCLUSION:
from the primal:
$60x_{12}$ which is 60 Units from Plant A to Warehouse 2.
$40x_{13}$ which is 40 Units from Plant A to Warehouse 3.
$80x_{21}$ which is 60 Units from Plant B to Warehouse 1.
$30x_{23}$ which is 60 Units from Plant B to Warehouse 3.
from the dual
We want the MR=MC. Five of the six MR<=MC. The only equation that does not satisfy this requirement is Plant B to Warehouse 2. We can see that from the primal that we will not be shipping any AED device there.