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2018

MCM/ICM

Summary Sheet

Globally, the world currently consume over 11 billion tonnes of fossil fuels every year. The biggest portion of it is being used to power vehicles. If we keep the consumption rate at this point, expert predicts that we will run out of fossil fuels in 2052. This give us only 34 more years left until the fossil fuels extinct.

Tesla is a leading company in electric car manufacturer who has concern about the environmental problem. Since the first time this company stands, Tesla intends to accelerate the advent of sustainable transport. Moreover, Tesla is not only an electric car manufacturer but also a company that puts their customer comfort into its priority. They built public charging stations around the world so cars can always keep on running.

The purpose of this paper is **to influence wide range of countries to develop a national plan to migrate personal transportation from all gasoline to all electric cars**. To build a solid justification, we use modified S.I.R. model to determine how long it will take for a country to migrate its population into using all electric vehicle. Then, we use the modified graph method to determine the optimal number, placement, and distribution of charging stations within the country. After running the simulation, the result give us:

(1) a small optimum number for the charging stations and

(2) an effective distribution of the charging station so the owner will never have a chance running out of battery.

Our model works best under the set of situations when a country have small population growth, average density, and small amount of inhabitable area. Both of our models converge to an optimal solution subjects to our purposes. It is, however, sensitive to some parameters: rate of advertisement and rate of interaction between the susceptible and infected.

We apply our model in the United States condition. If all of the people there migrating, the country will need to have 27,783 charging units. Currently, the U.S. already have 11,875 charging units so the migrating process will take time around 18 years. Furthermore, we apply our model in Uruguay, a country that is still using all gasoline car and hasn't build any supporting facility for the electric car. In there, we need 806 charging station and Uruguay needs 30 years in migrating all of its citizen to all electric car. Our model has proven that if Uruguay doesn't start migrating from all gasoline to all electric cars today, there will be times when they don't have any power to fuel its vehicles. Therefore, it is important to develop a national plan so a country can make the preparation before the world runs out of fossil fuels.

Contents

Electric Vehicle: The New Way to Spark the Future

1. Introduction

In this paper, our team is trying to influence wide range of countries to develop a national plan to migrate personal transportation from all gasoline to all electric cars. We are trying to do this because our earth is running out of fossil fuels in 34 years. Tesla is one of the company that specialize in energy storage and solar panel manufacturing. They took part in the search of renewable energy for cars. And came up with the idea of electric-fueled car. Every purchased car is equipped with Tesla home charging. To facilitate its user to charge the car outside their home, Tesla has provided supercharging and destination charging station spread out across nation. Now we will build a model to estimate the optimal number, placement, and distribution of the charging stations and determine how long it will take for one country to migrating from all gasoline to all electric vehicles.

2. Tesla's Growth in The U.S.A.

Key points: Tesla schedule for the United States switch to all-electric vehicle - Infrastructure preparation for all-electric vehicle era in the United States.

In order to create a grand design for all-electric vehicle era in the United States, Tesla needs to prepare infrastructures to support drivers from all around the country. One of the most essential facility is the charging place availability. Of course Tesla always wants to give the maximum comfort to its driver. However, it is known that building a new charging station in a massive amount can be very costly for the company. Therefore, we are trying to give the best estimation for the lowest amount of charging station needed that can meet the future needs.

2.1 General Assumption

To estimate the future needs, we make several assumptions towards the situations. Those are :

1. The shape of each state is a circle with the size of the actual land area. Each neighbouring state has a straight line that connect one to another
2. Every family in the US consist of 4 persons and only have one Tesla car. They also have a level two High Power Wall Charging (HPWC) installed in their home.
3. No one lived in the connecting line between states This is to avoid the confusing of double counting the population.
4. There is no emigration and migration
5. Destination charging is placed inside the state whereas the supercharger is placed along the connecting line
6. Every charging station has one power outlet

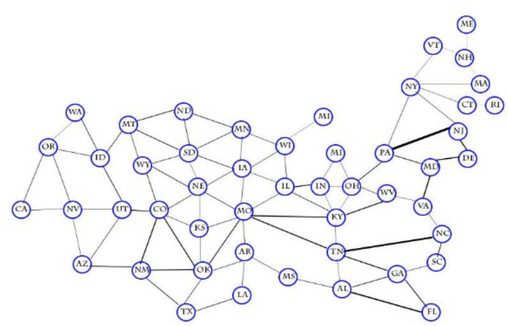

Figure 1

Different with the gas station installation that needs an underground fuel tanks for the fuel storage, Tesla destination charger doesn't need that. Thus, installing two HPWC in one area and installing two HPWC in two different areas will cost the same.

2.2 Urban, Suburban, and Rural Area

We use 3 different area classifications to describe the situation of the area based on the density:

1. Urban Areas
   Urban areas are the most dense area compare to suburban and rural. This is the area where offices, big stores, schools, and other big daily live supporting facilities are located. There are more than 50.000 people that lived in this area.
2. Suburban Areas
   It is usually a residential area that is slightly off the urban area's grid. Here, there are medium size facilities that can support the daily live, such as hospitals and grocery stores. There are around 10.000-50.000 people that lived in this area.
3. Rural Areas
   This area have the lowest amount of population that live in a widely spread area. There are less than 10.000 people that lived in this area.

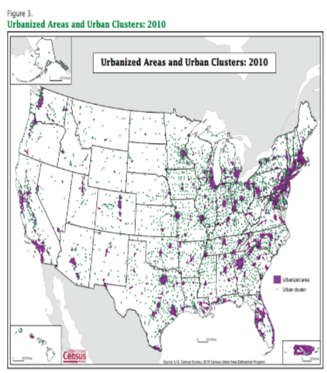

Figure 2:
https://www2.census.gov/

Normally, these three different areas are spread in each state. But then we will end up with too many different areas to be considered. To simplify things, we decide that each state can only be categorized as either rural, suburban, or urban. Based on the 2010 U.S. urbanized area map, we mark a state as an urban state if there are many dense purple dot in the map, sub urban state if the purple dots are less dense but only concentrated in one area, and rural if it's neither urban or suburban. Hence, we got 17 urban states, 15 sub urban states, and 16 rural states.

2.3 Tesla Today

Along with Tesla rapid growth, it must be balanced with the car supporting facilities such as service centers and charging stations. As a growing company, currently Tesla is building two types of public charger in various places: destination charging and supercharging. Destination Charger is a more reachable place to charge your tesla car within the city. The chargers are spread and installed at restaurants and hotels to charge once Tesla owners arrive at their destination. Currently, there are 2,971 destination charger around the U.S. These destination charger are located 53.7% in urban areas, 36.7% in suburban areas, and 9.7% in the rural area. On the other hand, supercharger is prepared for drivers who intend to traveling far. Imagine a rest area beside the toll road, Tesla will develop the super charger station there and it will charge the car while the driver grab a quick bite to eat.

2.4 Tesla Tomorrow: Welcoming All The U.S. Citizens!

Now it is the time to answer the big question: what if all of the U.S. population wants to switch to all-electrical personal passenger vehicles? How many charging station that will be needed? Tesla highly recommend each of the owner to install home charging so the car is always in full charge condition before the driver start their day. So we assume that majority of the owner do so and only small amount of people forget to charge during the night. Therefore, people who use the public charging stations are either forget to charge the car or travelling far.

To determine the amount of charging stations, we have to determine the amount of destination charger and supercharger.

Destination Charger

Each destination charger covers the area with the unique radius depends on the density of the state. Therefore, now we have smaller circle inside our circular state. The amount of the small circle equal with the amount of destination chargers in each state. Density really play a big role in determining the small circle radius. The density value is obtained by dividing the amount of population in one state by the state land area. Using the assumption that one family consist of 4 people, we can divide the density by 4 to determine how many car in one area. However, we don't need one charging station for every car out there. Hence, we take the square root of the density divided by 4. So we have:

$$r = \sqrt{density/4}$$

Logically, if there are more people in one area, and everyone only use electric car, we will definitely need more charging station. But take a closer look at this model, if there is a slightly higher density, we will get a bigger small radius that leads us with less charging station in one state. This is contradiction with the logic. Therefore, we need a correction factor. This give us a better model:

$$r = \sqrt{density/4} * \left(\frac{b}{a}\right)^2$$

$$b = \begin{cases} 1, & urban \\ 2, & suburban \\ 3, & rural \end{cases} \qquad a = \begin{cases} 5, & if\ Area < 10000 \\ 0, & elsewhere \end{cases}$$

After a rigid calculation, the total amount of destination charging needed in the U.S. are 27,447 units. The detail of distribution in each state can be seen in Appendix A.

Supercharger

Based on our model, every state is connected with a straight line to the neighbouring state(s). Based on Tesla's website, every fully charged tesla car has travel range about 220 miles, assuming that there is no traffic. Of course we don't want to run out of battery in the middle of the road. Assuming that the driver will charge the car whenever the car is half empty, we design the supercharger to be 100 miles apart along the connecting line. The length of the connecting line between two states is obtained by measuring the distance from the center of each state. The data is in appendix A. From there, we conclude that 336 supercharging will be the optimum number for the supercharger.

From the results above, we deduce that we need 27,783 charging units, consist of 27,447 destination charging and 336 supercharging. At the first time Tesla was build, its goal is to accelerate the advent of sustainable transport by bringing compelling mass market electric cars to market as soon as possible. The mission is still carried until the present day. Tesla is making its best effort for a complete switch to all-electric in the U.S. But does Tesla's schedule on track ? Or is it behind the schedule?

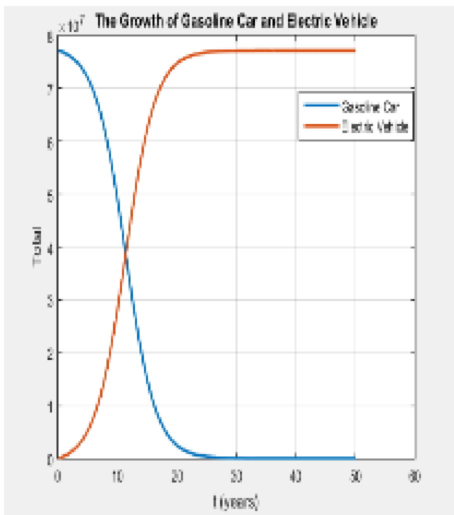

Figure 3

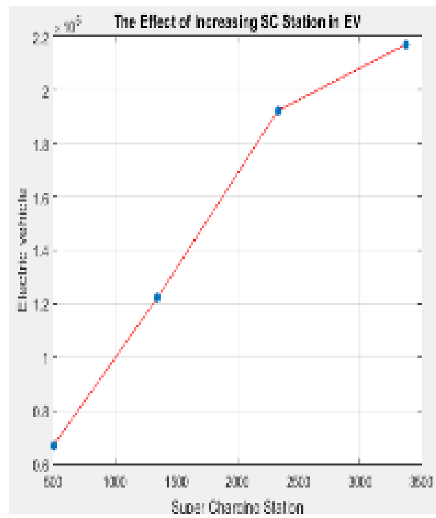

Figure 4

Using the modified S-I-R method that we will explain later in chapter 2, the U.S. will need 24 years to switch all of its citizen in to using all electric cars. In the other hand, we are able to obtain the growth rate of the electric car using the data from 2012-2017, which is 53,757 cars/charging station each year. Therefore, if we assume that the growth rate is constant towards the next 24 years, we can say that Tesla's plan is still on track.

3. Uruguay : Tesla's New Promising Land

Key points : Reason to invest, optimal number, placement, and distribution of charging stations in Uruguay

3.1 Why Uruguay?

Uruguay is a safe developed country located in Latin America. Many foreign investors from abroad loved to invest here because they can enjoy the same incentives as the locals, with no discrimination from the tax point of view, or restrictions for the transfer of profits abroad. Besides that, Uruguay is one of the country who concern on renewable energy. Based on the REN 21 report, it said that this country is the fifth place of the world ranking for level of investment in renewable energies as a percentage of GDP. If we look closely, we can find the similarity between Tesla's mission and the urge of Uruguay in using more renewable energy everyday. Hence, Uruguay is the new promising land for Tesla's company expansion.

3.2 Preparing Uruguay

The growth of the electric car has to be balanced with the growth of the charging station. In determining the amount of charger needed in Uruguay, we use the same model that is being used in the U.S.A., also with the same process and assumptions. In Uruguay, there are 19 departments and we assume that each departments shape is a circle. To categorize each departement, we use the global rural-urban map by http://sedac.ciesin.columbia.edu. The department considered as urban if there are dark red spots within the department, sub urban department if there are orange spots, and rural if it's neither urban or suburban. Hence, we got 3 urban departments, 7 sub urban departments, and 9 rural states.

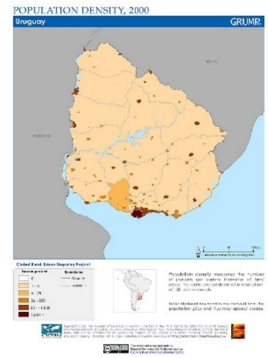

Figure 5
sedac.ciesin.columbia.edu

Applying the previous model with the data that we have in Uruguay, our best estimation is there will be 752 destination charging and 54 supercharging along the connecting line. So, the total amount of the charging station is 806 charging stations.

3.3 The Symbiosis Mutualism

Key points : Tesla bargaining point - charger distribution - evolution timeline - Uruguay budget

Environment is a key factor in maintaining the living quality. As a country who put a big concern in there, shifting into all-electric vehicles is definitely something that Uruguay should consider do. Based on the *www.enelgreenpower.com*, the biggest domestic emissions in 2013 was coming from the transportation sector. This means, by eliminating this factor, Uruguay has eliminate the biggest domestic emissions contributor. Moreover, according the International Energy Agency (IEA), the growth of the renewable energy sector will grow by 43% in the next five years. It is equal to half of what is currently produced by coal, which need 80 years in the making. Renewable energy in the future is certainly a promising source of energy.

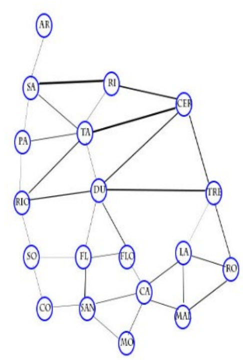

Figure 6

Uruguay has the most developed IT sector in Latin America according to the ICT (Information Communication and Technology) development index (IDI) published by the international Telecommunication Union (ITU) in 2016. This fact is intending that people in Uruguay have a very high exposure to the current technology and understand it. Currently, Tesla is a leading company in electric car manufacturer that offers a very advanced technology. Therefore, if a country is a digital savvy, most likely they will accept and adapt the new technology fast.

Until today, there hasn't been any all-electric vehicle country in the world. Being the first one certainly will benefit so much things for the country itself, such as:

1. Denotes that this country has a very adaptive behaviour towards technologies

2. When a country is adaptive, investor will see that country as a considerate place to invest their money
3. The more investor invest within the country will give a higher transaction rate that can increase the speed of economic growth.
4. Also, it denotes that this country is persistence towards achieving its goal.

In addition, according to Tesla supercharger cost estimator, assumes supercharger cost \$0.2 per kilowatt hour while gasoline cost assume at \$2.73 per gallon. By Tesla calculation, for 200 miles distance driven by car, gasoline will cost about \$26. In the other hand, supercharger will only costs about \$12. That way, the daily cost for using the electric car is way cheaper that using the gasoline car.

Uruguay can take home all of the benefits above. All they have to do is making a right investment and partnership with the company in electric-vehicle field. Tesla is known as the leading company in electric-vehicle manufacturer that not only care about selling its product but also the sustainability of it. In the U.S., Tesla provided over thousands of charging destination, 11,875 to be exact, and it's all on Tesla! The purpose is merely to increase the convenience of its users. Based on statistics on the figure 4, the growth of supercharging station has proved in increasing the amount of electric vehicle. Therefore, if Uruguay willing to invest in the charger, there is no doubt that it will boost the usage growth of the electric car.

The higher the growth of the electric car, the faster it is for Uruguay to achieve its dream to become all-electric vehicle country. We do realize that investing in all those chargers at once can cost this country a tremendous amount of money and also the growth of the car won't happened in one night. Therefore, using the past information that the growth of charger has the linear relations with the growth of the car, it is very reasonable for Uruguay if they want to place those chargers gradually.

The subject in this calculation is the people. When we are facing the large group of population, we can use deterministic mathematics. In this model, each family in the population is assigned to one compartment, each compartment represent a certain stage from an epidemic system.

3.4 The Modifed S.I.R Model

In 1927, W.O. Kermack and A.G McKendrick created a model that have an assumption that there is no change in the population and there are only 3 compartments, Susceptible, Infected, and Recovered.

- Susceptible ($S(t)$): Compartment that is being used to show the amount of individual that hasn't using the electric car and susceptible for that in the future.
- Infected ($I(t)$): Compartment that is being used to show the amount of family that is using the electric car already, this group can influence the susceptible group switching in to the electric car.
- Recovered ($R(t)$): Compartment that is being used to show the amount of people who were using the electric car but not anymore. A Tesla car has the life expectancy about 34 years.

Of course we assume that the owner will take a good care of the car and there is no probabilistic event such as a falling asteroid into this world or the car get stolen. Also, we assume that the duration of the transition from all gasoline to all-electric can happen under Tesla's car expectancy life. Therefore, we eliminate this recovered category.

There are several additional assumptions in this mathematical model, which are :

1. The population of Uruguay is constant, N.
2. The rate is constant.
3. All of the population are homogen.

Here are the parameters that is being used in the model:

Means	Parameter	Units
Advertisement rate	α	1/year
Recruitment rate	μ	1/year
Interaction rate between S and I	β	1/year
Avocation rate	φ	1/year

Table 1

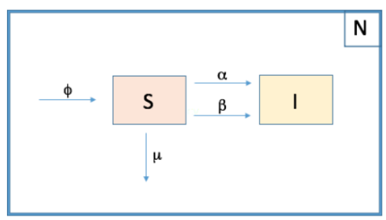

Figure 7

There are factors that affects the population in each compartment.

1. Susceptible

a. There are an increase population that caused by the new family that become susceptible because they are living in the infected environment, for the example they have a neighbour that recently purchase an electric car. The rate is φ.

b. There are interactions between the susceptible and the infected that decrease the amount of population in susceptible compartment. For the example, the family wants to buy the electric but doesn't have the money yet. The rate is β*S*I.

c. There are advertisement factors that decreasing the susceptible population and move them into the infected population. The rate is α*S

d. There are factors that decrease the population because of the avocation rate. The decrease rate is μ*S

Thus, the population rate of the susceptible population each time can be written in the differential equation below:

$$\frac{dS}{dt} = \emptyset - \mu * S - \alpha * S - \beta * S * I$$

2. Infected

a. There are interaction between the susceptible and the infected which result in the increasing number of the infected. The rate is β*S*I.
b. There are advertisement factors which result in the increasing number of the infected. The rate is β*S*I.

$$\frac{dI}{dt} = \alpha * S + \beta * S * I$$

3.5 Uruguay's Future

We used the model above to estimate the amount of time needed to switch from all gasoline into all electric car. Assuming that the relationship between the amount of cars and chargers are linear, we can use this prediction as well to inform the Uruguay's government the checkpoint of how many chargers should they have invested each referral time.

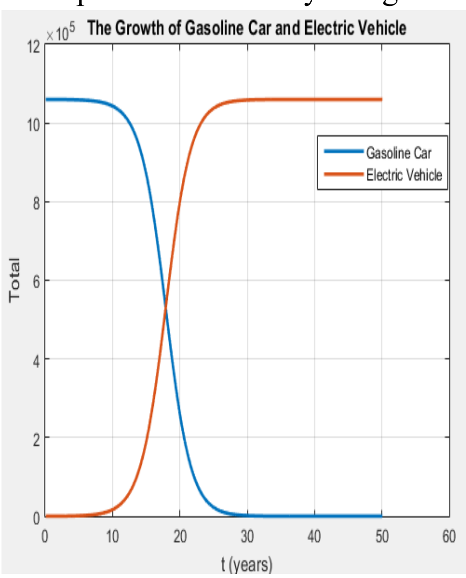

Figure 8

30% of population are using the electric car

According to our model, Uruguay will need 196 months or about 16 to 17 years to switch 30% of all its population to using all electric car. Therefore, based on the previous calculation, Uruguay will already have to implemented 30% of 752 destination charging and 30 % of 54 supercharging, which yield us 242 charging stations.

50% of population are using electric car

Uruguay will need 215 months or about 17 to 18 years.Thus, by this time Uruguay has to already implemented 403 charging stations. In other words, Uruguay will have to implement another 161 charging stations within a year.

100% or all of the population are using electric car

Finally, we are hoping that in in 360 months or 30 years, everyone in Uruguay will only be using all electric car vehicle. Hence, in 30 years from now Uruguay have to implemented 752 destination charging and 54 supercharging, which is another 403 charging stations within 12 years.

Based on the data that we have, Uruguay annual income is $17.8 billion per year. Assuming that each supercharging will cost about $15,000 per unit and destination charging will cost about $2200 per installation, the gradual charger plan that we propose seems reasonable to be implanted in Uruguay. We really think that this investment is one of the greatest option to enhance Uruguay's growth in the future.

Task 3 Sentivity Analysis and Model Strength&Weakness
Key factors : country geographies, growth rate, density, and wealth distribution effect on the models, adjusted models

We had succeeded in building a model that can estimate the optimum number for charging stations needed in order to support the growth of the electric car. However, our model is sensitive to several parameters. Those are rate of advertisement (α) and rate of interaction (β)

Alpha sensitivity

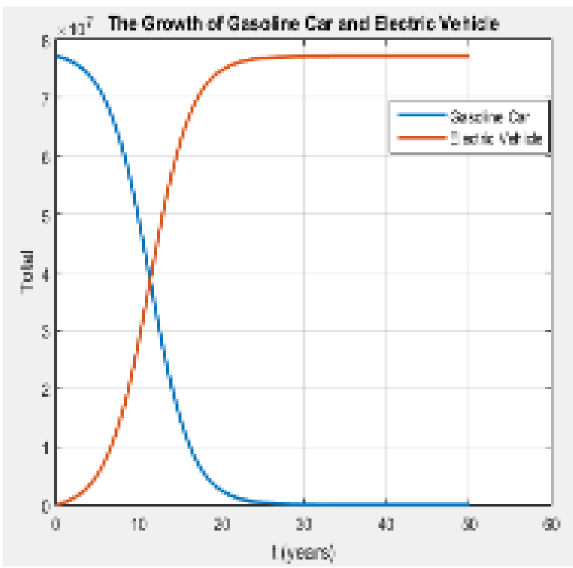

Figure 9

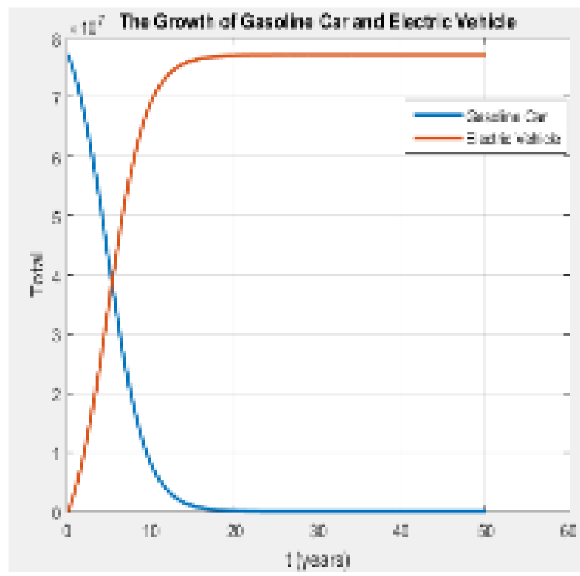

Figure 10

For the first graph, parameters that is being used have values
$\alpha = 0.005$ $\beta = 0.000000005$ $\mu = 0.0000001$ $\emptyset = 0.0000001$.

Whereas, for the second graph, parameters that is being used have values
$\alpha = 0.05$ $\beta = 0.000000005$ $\mu = 0.0000001$ $\emptyset = 0.0000001$.
Note that when the α is changed unsignificantly, it will resul the different grap. Hence, from the graph we can conclude that the bigger the α, the faster the electric vehicle growth.

Beta sensitivity

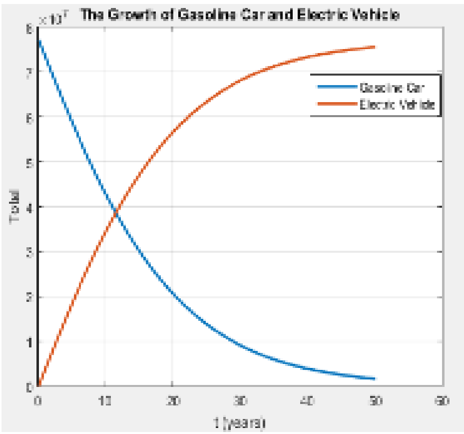

Figure 11

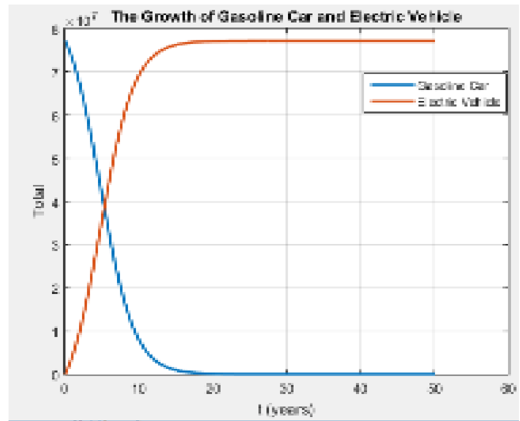

Figure 12

For the first graph, parameters that is being used have values
$\alpha = 0.05$ $\beta = 0.0000000005$ $\mu = 0.0000001$ $\emptyset = 0.0000001$.

Whereas, for the second graph, parameters that is being used have values
$\alpha = 0.05$ $\beta = 0.000000005$ $\mu = 0.0000001$ $\emptyset = 0.0000001$.
Noe that when β is changed unsignificantly, it will result a diffent graph. Hence, from the graph we can conclude that the bigger the β, the fastest the growth of electric vehicle.

Our model strength is the model is simple and easy to be used. Also, it applies in general. Moreover, we can generate a reasonable prediction number. On the other hand, there are several weakness in our model. Therefore, based on these problems we create a classification system that can be a guideline for them in choosing the most suitable model. The 3 specific classifications are :

- Country's growth rate
  If the growth rate of the country is significant, we have to add growth rate in to our model.
- Density
  For a density higher than 100 people/miles2, our model unable to give the most desireable result. We suggest a little justification for the case, that is instead of taking the square root of density, take the higher order root of density. If a state has a larger area but small habitable area, use the habitable area instead to obtain the density of the state.
- The wealth distribution
  In the previous model, we assume that every family can afford one Tesla car. In contrast, this assumption is not very applicable in a country with low Gross Domestic Product (GDP).

4. Will Tesla Lose Against Time?

Key point: Tesla's S.W.O.T. Analysis,

Based on our analysis, assuming that there is no growth in population, we predict that there will be 80,317,500 all electric vehicles in the U.S.A. But this prediction is made without worrying about other growth of sustainable transportation such as car-shares, ride-shares, self-driving cars, and flying cars. Before determining weather the growth of other sustainable transportation will affect the amount of electric car in the future or not, let's take a look at Tesla's S.W.O.T. analysis.

Strength	**Weakness**
• Sustainable Innovation • Fast growing network • Unique product in the market • Strong brand	• High cost vehicle • Time consuming delivery • High production and operating cost
Opportunities	**Threat**
• Growing demand of sustainable product • More low price models for middle class • Global sales expansion	• Legal and regulatory troubles • High competition • Fluctuation in material price

From the analysis, we justify that Tesla has several bargaining position such as its sustainable innovation and fast growing network. However, let's highlight the weakness and the threat of Tesla's car. Currently, Tesla's car price is still considered a high price car. Thus, if there is a mass transportation that can fulfill the need of the society, there is a high chance that instead of purchasing a car, they will try to catch the public transportation. Take Singapore situation for an example. Singapore known as one of the country that have the best public transportation system. Therefore, more and more people are not thinking about having their own private car. This fact will definitely affects the prediction of the amount of electric car in the future.

The Promising New Sustainable Renewable Energy for The Future

Climate change, high pollution and global crisis on fossil fuels, we know that those are domino effects from the careless behaviour of all humankind. Moreover, statistics shows that if we keep using the fossil fuels over 11 billion tonnes every year, the world will run out of it in 34 years.

In order to avoid the global panic in the future, countries should start thinking about the renewable energy alternatives that can replace the use of fossil fuels in the long run. Currently, the most feasible alternatives out there is electric car. Besides using renewable energy to power the car, an electric car produce zero gas emission. Therefore, we can also gradually eliminate the population issue that we currently have.

Tesla is a leading company in electric car manufacturer that has concern about the environmental problem. Tesla is also a company that puts their customer comfort into its priority. They built public charging stations around the world so the cars can always keep on running. Charging station, has a linear relationship with the growth of the amount of car. Hence, if we want to accelerate the electric car's growth, we need to accelerate the growth of charging station provision.

We have created a model that can determine how long it will take for a country to migrate its population into using all electric vehicle with the modified S.I.R. model. In addition, this model can be used for making a prediction to determine the appropriate time puting the gas vehicle ban date. We also create a model to determine the optimal number, placement, and distribution of charging stations within the country with the modified graph model. After running the simulation, the result give us :

(1) a small optimum number for the charging stations and

(2) an effective distribution of the charging station so the owner will never have a chance running out of battery.

Our model works best under the set of situations when a country have small population growth, average density, and small amount of inhabitable area.

We apply our model in the United States condition. If all of the people there migrating, the country will need to have 27,783 charging units. Currently, the U.S. already have 11,875 charging units so the migrating process will take time around 18 years. Furthermore, we apply our model in Uruguay, a country that is still using all fossil fuel car and hasn't build any supporting facility for the electric car. In there, we need 806 charging station and Uruguay needs 30 years in migrating all of its citizen to all electric car. Our model has proven that if Uruguay doesn't start migrating from all gasoline to all electric cars today, there will be times when they don't have any power to fuel its vehicles.

We really don't want this possibility to come true, to your country. Therefore, we are calling you, all the leaders from all around the world, to develop a national plan to migrate personal transportation towards all electric vehicle, as soon as possible.

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APPENDIX A

Below is the data for the United States population, area, density, and amount of charging stations in each state.

State	Population (person)	Area (miles²)	Density (person/miles²)	Radius	Weighted	Circle
WA	7200000	66000	101.2	5	1	830
OR	4000000	96000	39.9	6	2	765
CA	39200000	155000	239.1	7	1	825
NV	2900000	109000	24.6	7	3	626
ID	1600000	82000	19	6	3	610
UT	3000000	82000	33.6	5	2	776
AZ	6900000	113000	56.3	3	1	2555
MT	1000000	145000	6.8	3	3	3016
WY	500000	97000	5.8	3	3	2365
CO	5500000	103000	48.5	3	1	2703
NM	2000000	121000	17	4	2	2265
ND	700000	68000	9.7	4	3	991
SD	800000	75000	10.7	4	3	991
NE	1900000	76000	23.8	7	3	451
KS	2900000	81000	34.9	8	3	328
OK	3900000	68000	54.7	11	3	175
TX	27800000	26000	96.3	4	1	343
MN	5500000	79000	66.6	8	2	377
IA	3100000	55000	54.5	7	2	321
MO	6000000	68000	87.1	9	2	248
AR	2900000	52000	56	7	2	295
LA	4600000	93000	104.9	10	2	282
WI	5700000	54000	105	10	2	163
IL	12800000	55000	231.1	7	1	303
MS	6000000	46000	63.2	7	2	231
MI	9900000	56000	174.8	6	1	407
IN	6600000	35000	181	6	1	246

KY	4400000	39000	109.9	15	3	50
TN	6600000	41000	153.9	12	2	84
AL	4600000	50000	94.4	9	2	168
OH	11600000	40000	282.3	8	1	180
WV	1800000	24000	77.1	13	3	44
GA	10300000	58000	168.4	6	1	438
VA	12700000	44000	283.9	8	1	197
MD	6370000	9800	650	5	2	119
VA	8400000	39000	202.6	7	1	245
NC	10100000	98000	196.1	7	1	636
SC	4900000	30000	153.9	12	2	62
FL	20600000	53000	350.6	9	1	192
NY	19700000	47000	411.2	10	1	145
NJ	8900000	7000	1195.5	3	1	186
DE	900000	2000	460.8	6	3	15
VT	600000	9000	67.9	2	3	468
CT	3500000	4000	738.1	2	1	172
ME	1300000	30000	43.1	9	3	98
NH	1300000	8000	147	3	3	192
MA	6800000	7000	839.4	2	1	265
RI	1000000	1000	1018.1	10	3	3

APPENDIX B

Below is the data for Uruguay population, area, density, and amount of charging stations in each department.

Department	Population	Area	Density	Weighted	Circle	Radius
ARTIGAS	73378	4605.423697	15.93295315	3	40	40
SALTO	124878	5468.361488	22.83645664	3	33	33
PAYSANDU	113124	5375.310925	21.04510819	3	36	36
RIONEGRO	54765	3583.798018	15.28127415	3	33	33
SURYANO	82595	3478.006092	23.74780199	2	46	46
COLONIA	123203	2357.538321	52.25917173	2	14	14
FLORES	25050	1986.108275	12.61260543	2	50	50
SANJOSE	108309	1927.420783	56.19374916	2	10	10
RIVERA	103493	3617.774987	28.60680954	3	17	17
TACUAREMBO	90053	5960.641435	15.10793779	3	55	55
DURAZNO	57088	4495.38465	12.69924699	3	50	50
FLORIDA	67048	4022.023697	16.67021506	2	76	76
CANELONNES	520187	1751.358307	297.0191752	1	7	7
MONTE VIDEO	1319108	204.6340174	6446.181415	1	0	0
CERRO LARGO	84698	5269.518999	16.07319378	2	104	104
TREINTA Y TRES	48134	3735.922174	12.88410137	3	41	41
LAVALLEJA	58815	3867.196827	15.20869059	2	80	80
MALDONADO	164300	1850.586501	88.78266427	1	26	26
ROCHA	68088	4073.761354	16.71379202	3	34	34