TECHNISCHE UNIVERSITÄT DARMSTADT

Fachbereich Elektrotechnik und Informationstechnik
Bioinspired Communication Systems

Bayesian Inference of Information Transfer in Graph-Based Continuous-Time Multi-Agent Systems

Master-Thesis
Elektro- und Informationstechnik

Eingereicht von

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am
07.07.2020

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Abstract

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2.1 Continuous Time Bayesian Networks

The entire communication is defined in the framework of continuous-time Bayesian network (CTBN).

2.1.1 Time-homogenous continuous-time Markov Processes

The messages that are emitted by the cells are modelled as independent time-homogeneous continuous-time Markov processes (CTMP). These processes are defined by transition intensity matrices Q_X and Q_Y, whose intensities does not depend on time. In this matrix, the intensity q_i represents the instantaneous probability of leaving state i and $q_{i,j}$ represents the instantaneous probability of switching from state i to j.

Infinitesimal transition probabilities in terms of the entries of transition matrices q_{ij} can be written as [1]:

$$p_{i,j}(h) = \delta_{ij} + q_{i,j}h + o(h) \tag{2.1}$$

where $p_{i,j}(t) \equiv Pr(X^{(t+s)} = j \mid X^{(s)} = i)$ are Markov transition functions and o(.) is a function decaying to zero faster than its argument.

The *forward* or *master equation* is then derived as follows:

$$p_j(t) = \sum_{\forall i} p_{i,j}(h)p_i(t-h) \tag{2.2}$$

$$
\begin{aligned}
\lim_{h\to 0} p_j(t) &= \lim_{h\to 0} \sum_{\forall i} \left[\delta_{ij} + q_{i,j}h + o(h)\right] p_i(t-h) \\
&= \lim_{h\to 0} p_j(t-h) + \lim_{h\to 0} h \sum_{\forall i} q_{i,j} p_i(t-h)
\end{aligned}
\tag{2.3}
$$

$$
\lim_{h\to 0} \frac{p_j(t) - p_j(t-h)}{h} = \lim_{h\to 0} \sum_{\forall i} q_{i,j} p_i(t-h) \tag{2.4}
$$

$$
\begin{aligned}
\frac{d}{dt} p_j(t) &= \sum_{\forall i} q_{i,j} p_i(t) \\
&= \sum_{\forall i \neq j} \left[q_{i,j} p_i(t) - q_{j,i} p_j(t)\right]
\end{aligned}
\tag{2.5}
$$

Eq.2.5 can be written in matrix form:

$$
\frac{d}{dt} p = p\mathbf{Q} \tag{2.6}
$$

The solution to ODE, the time-dependent probability distribution $p(t)$ is,

$$
p(t) = p(0) \exp(t\mathbf{Q}) \tag{2.7}
$$

with initial distribution $p(0)$.

The amount of time staying in a state i is exponentially distributed with parameter q_i. The probability density function f for staying in the state i:

$$
f(t) = q_i \exp\left(-q_i t\right), t \geq 0 \tag{2.8}
$$

2.1.1.1 The Likelihood Function

To write down the likelihood of a trajectory sampled from a homogenous CTMC X, first let us consider one transition $d =< i, j, t >$, where transition happens after time t from state i to j. The likelihood of this transition is:

$$
L_X(d \mid \mathbf{Q}) = \left(q_i \exp\left(-q_i t\right)\right) \left(\frac{q_{i,j}}{q_i}\right) \tag{2.9}
$$

We define sufficient statistics over a dataset D as $T[i]$, total amount of time spent in state i, $M[i,j]$ total number of transitions from state i to j, we can write down the likelihood of a trajectory $X^{[0,T]}$,

$$
\begin{aligned}
L_X(\mathbf{Q} : D) &= \prod_{d \in D} L(d \mid \mathbf{Q}) \\
&= \left(\prod_{\forall i} q_i^{M[i]} \exp\left(-q_i T[i]\right)\right) \left(\prod_{\forall i} \prod_{\forall j \neq i} \left(\frac{q_{i,j}}{q_i}\right)^{M[i,j]}\right)
\end{aligned}
\tag{2.10}
$$

with $M[i] = \sum_{\forall j} M[i,j]$.

2.1.2 Time-inhomogeneous continuous-time Markov Processes

In an conventional CTBN, while every node is a Markov process itself, the leaf nodes are *conditional* Markov processes, a type of inhomogeneous Markov process, where the intensities change over time, but not as a function of time rather as a function of parent states. [2]

For inhomogeneous Markov processes, Eq.2.8 becomes:

$$f(t) = q_i(t) \exp\left(-\int_0^t q_i(u)du\right) \tag{2.11}$$

2.1.2.1 The Likelihood Function

Let X be an inhomogeneous Markov process, and $X^{[0,T]}$ is a trajectory sampled from this process. We define m number of transitions, with $0 = t_0 < t_1 < ... < t_m$ are the times where transition occurred, and $x_0, x_1, ..., x_m$ are the observed states. The likelihood of trajectory $X^{[0,T]}$ is as follows:

$$L(\mathbf{Q}_X : X^{[0,T]}) = \prod_{k=1}^{m} \left[q_{x_{k-1}}(t_k) \exp\left(-\int_{t_{k-1}}^{t_k} q_{x_{k-1}}(u)du\right) \frac{q_{x_{k-1},x_k}(t_k)}{q_{x_{k-1}}(t_k)} \right] \tag{2.12}$$

We can formulate our graphical model as $S = [X, Y, Z]$, where $Par(Z) = X, Y$ and $Par(X) = Par(Y) = \emptyset$. We can then write the conditional probability:

$$\begin{aligned} P\left(S^{(t+h)} = s' | S^{(t)} = s\right) =& P\left(X^{(t+h)} = x' | X^{(t)} = x, Par(X)^{(t)} = y, z\right) \\ & P\left(Y^{(t+h)} = y' | Y^{(t)} = y\right) P\left(Z^{(t+h)} = z' |^{(t)} = z\right) \end{aligned} \tag{2.13}$$

2.2 Partially observable Markov Decision Processes

In our problem, cell Z does not have a direct access to its parents' states, rather it observes a summary of them. This is different than observing the states through noisy binary channels, and presents a POMDP problem. Even though it is assumed that the policy of Z is optimal given the true states of parents, the problem of inferring the observational model remains unsolved and to be addressed.

In a conventional POMDP, given the *transition function*, $T(s, a, s')$ and *observation function*,

$O(s', a, o)$, the belief state update is computed as follows [3] [1]:

$$
\begin{aligned}
b'\left(s'\right) &= \Pr\left(s'|o, a, b\right) = \Pr\left(s'|o, b\right) \\
&= \frac{\Pr\left(o|s', b\right)\Pr\left(s'|b\right)}{\Pr(o|b)} \\
&= \frac{\Pr\left(o|s'\right)\sum_{s\in\mathcal{S}}\Pr\left(s'|b, s\right)\Pr(s|b)}{\Pr(o|b)} \\
&= \frac{O\left(s', o\right)\sum_{s\in\mathcal{S}}T\left(s, s'\right)b(s)}{\Pr(o|b)}
\end{aligned}
\tag{2.14}
$$

In Eq.2.14, the transition function is time-independent. However, in our setting, the parent nodes X and Y are modelled as CTMCs with time-dependent transition matrices. To derive *continuous-time belief state*, $b(t)$, first we need to get the joint transition matrix of X and Y, two independent processes. This operation called *amalgamation* is described in detail by Nodelman *et al* [2]. Let us denote this matrix by Q_{XY}. Now we can derive the belief state as follows:

$$
b_{s'}(t) = \frac{\Pr\left(o|s'\right)\sum_{s\in\mathcal{S}}\Pr\left(S(t) = s'|S(t-h) = s\right)b_s(t-h)}{\Pr(o|b)}
\tag{2.15}
$$

$$
\begin{aligned}
\lim_{h\to 0} b_{s'}(t) &= C\lim_{h\to 0}\left[\sum_{s\in\mathcal{S}} p_{s,s'}(t)b_s(t-h)\right] \\
&= C\lim_{h\to 0}\left[\sum_{s\in\mathcal{S}}\left[\delta_{ss'} + q_{s,s'}h + o(h)\right]b_s(t-h)\right] \\
&= C\lim_{h\to 0}\left[b_{s'}(t-h) + \sum_{s\in\mathcal{S}} q_{s,s'}hb_s(t-h)\right]
\end{aligned}
\tag{2.16}
$$

Due to the scaling factor $C = \frac{\Pr(o|s')}{\Pr(o|b)}$, cannot derive $\frac{d}{dt}b = \mathbf{Q}b$, .
However, we can use the conditional distribution of Markov processes as the transition function $T(s'|s)$ to derive the continuous-time belief state. The conditional distribution over the value of X, a homogenous Markov process, at time t given its value at an earlier time s can be written as

$$
\Pr\left(X(t) \mid X(s)\right) = \exp(\mathbf{Q}(t-s))
\tag{2.17}
$$

Then the continuous-time belief state becomes:

$$
b_{s'}(t) = \frac{\Pr\left(o|s'\right)\sum_{s\in\mathcal{S}}\left[\exp(\mathbf{Q}(h))\right]_{s,s'}b_s(t-h)}{\Pr(o|b)}
\tag{2.18}
$$

[1] Since for now, we assume there is no affect of Z's action on the observation or transition function, a is omitted from the equation.

3 Methodology

3.1 Data Generation - Simulation

3.2 Inference of Deterministic Observation Model

Our dataset contains a number of trajectories from all the nodes involved in the communication. $\mathbf{D} = \{D_1, ..., D_N\}$. Every trajectory comprises of state transitions in time interval $[0, T]$, and the times of these transitions.

4 Experimental Results and Evaluation

4.1 Bayesian Nonparametrics

5 Conclusion

5.1 Discussion

5.2 Future Work

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