Computer Vision

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1 Introduction

The difference between computer vision and image processing is the fact that computer vision is the process of extracting information from images, while image processing aims at improving the quality of images. Quite often image processing helps computer vision. The informations we want to extract from the images could be counting, object orientation, object classification, measurements... Computer vision is challenging since we lose depth from the images (3D information becomes 2D), scale and illumination varies, and there's object occlusion (object hiding other objects).

Computer vision started with hand-crafted decision rules that only required few images as example, and evolved to machine learning where the algorithm learns a decision rule, but the training requires hundreds to thousands of images. The big paradigm shift happened with deep learning (e2e learning) which learned both the image representation and the decision rules, but required thousands to millions of words. Deep learning has been enabled by better networks, better hardware and more data.

2 Fundamentals of Image Processing and Computer Vision

2.1 Images

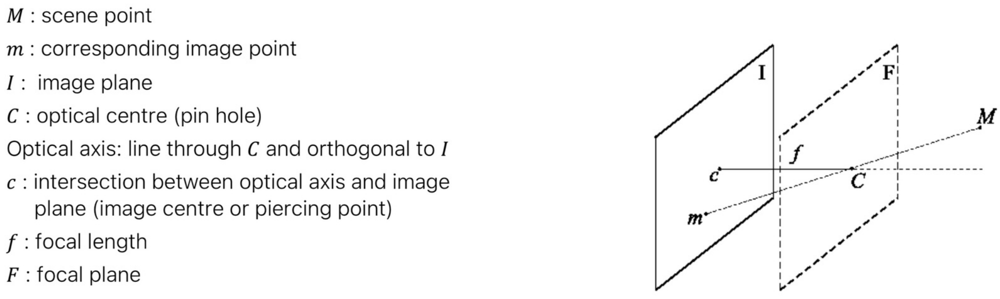
An imaging device gathers the light reflected by 3D objects to create a 2D representation of the scene.

2.1.1 Pinhole camera model

The "pinhole camera" is the simplest camera model we can define. Light goes through the very small pinhole (to not have saturation) and hits the image plane. Geometrically, the image is achieved by drawing straight rays from scene points through the hold up to the image plane.

This simple geometrical model turns out to be a good approximation of the geometry of image formation. However, useful images can hardly be captured by means of a pinhole camera.

The geometric model of image formation in a pinhole camera is known as **perspective projection**.



M : scene point
m : corresponding image point
I : image plane
C : optical centre (pin hole)
Optical axis: line through C and orthogonal to I
c : intersection between optical axis and image plane (image centre or piercing point)
f : focal length
F : focal plane

Figure 1: perspective projection

Which, by writing the poitns as vectors and the plane's coordinates becomes:

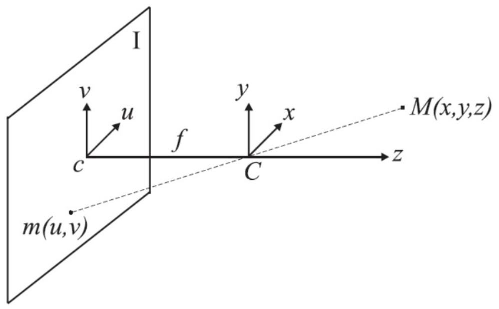


Figure 2: geometric model of image formation

Given the reference frame in 2

- u is the horizontal axis in the image plane.
- v is the vertical axis in the image plane.
- X and Y are the respective axis in the 3D reference system. It's called the **camera reference system** because it is "attached" to the camera.

For the perspective model these axis must be parallel

The equations to map scene points into their corresponding image points are defined as:

$$\frac{u}{x} = -\frac{f}{z} \rightarrow u = -x\frac{f}{z} \qquad \frac{v}{y} = -\frac{f}{z} \rightarrow v = -y\frac{f}{z}$$

The minus sign means the axis gets inverted (as we can see in the visualization, and it's what happens in the brain). We can get rid of the sign, since the image plane can be though of as lying in front rather than behind the optical centre.

Image coordinates are a scaled version of scene coordinates (function of depth), this means that if the distance becomes bigger the object gets smaller. When z increases, since it's at the denominator in both the equations the terms gets smaller (object gets smaller). **The larger the focal length the bigger the object is in the image (and viceversa)**.

As we previously said, the image formation process deals with mapping a 3D space onto a 2D space, and so to a loss of information. A given scene point is mapped into an image point, but an image point is mapped onto a 3D line. For an image point we can only state that its corresponding scene point lays on al ine but cannot disambiguate a specific 3D point along such a line.

2.1.2 Stereo images

The solution is to use multiple images (at least two) to create stereo vision. The human visual system is a stereo vision system. Given correspondences, 3D information can be recovered easily by triangulation. We can use two cameras, or two cameras and an infrared sensor to project guides to align the images.

For standard stereo geometry there are some assumptions we have to make:

- The cameras have parallel (x, y, z) axes.
- The image planes of both cameras are coplanar and aligned.
- Both cameras have identical focal lengths.

Based on this, the transformation between the two reference frames is just a translation, usually horizontal. For stereo vision is also really important to **sense two images at the very same moment**.

The cameras are displaced at a given quantity b called baseline. The **disparity** is the difference between the horizontal coordinates in the left and right images.

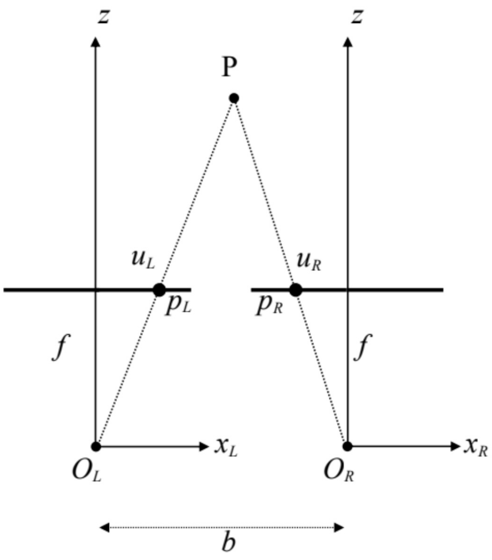


Figure 3: standard stereo geometry

The fundamental relationship in stereo vision is $z = b \cdot \frac{f}{d}$. It's used to calculate the depth of a point in a scene from a pair of stereo images.

In standard stereo geometry since we are given just two 2D images there is no info about the correspondence between two points in the two images. We can recall that the camera have parallel axes, and so we know that we can search for the correspondence alogn the horizontal lines. This task is called stereo matching.

2.1.3 Stereo correspondence

Given a point in one image find it in the other image which is the projection of the same 3D point. Such image points are called corresponding points. **Points farther away have a smaller disparity, while close poitns have a larger display**

The image of a 3D line segment of length L lying in a plane parallel to the image plane at distance z from the optical centre will exhibit a length given by:

$$l = L\frac{f}{z}$$

This relationship is more complicated for an arbitrarily oriented 3D segment, as its position and orientation need to be accounted for as well. For a **given position and orientation, length always shrinks alongside distance**.

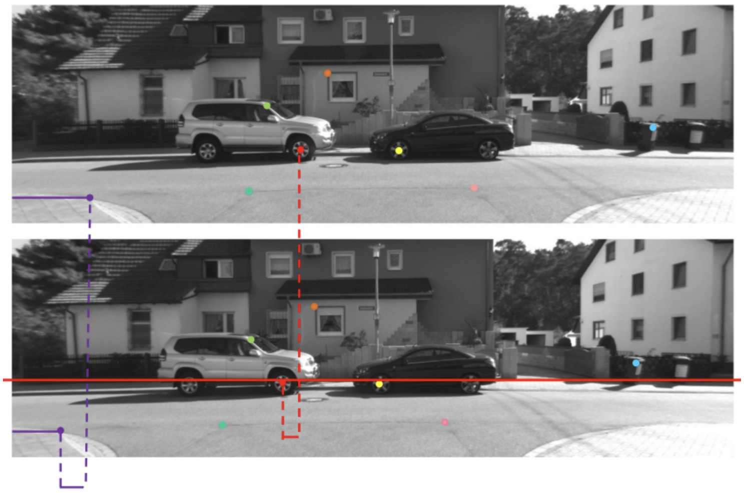


Figure 4: Corresponding points look similar in the two images

Perspective projection maps 3D lines into image lines. **Parallelism between 3D lines is not preserved** (except for lines parallel to the image plane). This is the reason why if we look at a really long road into the distance we have the perception that the road becomes thinner, and the lines of the road intersect in the distance. The images of parallel 3D lines intersect at a point, called **vanishing point**, which isn't necessarily within the image.

If the lines are parallel to the image plane they meet at infinity.

2.1.4 Epipolar geometry

What if the two cameras are no longer aligned? Do we need to search through the whole image? We can project the line related to point P_L in the right plane and search across that line. The issue is that this projection can be computed only if the transformation between the two cameras is known (the relative mapping between the two cameras).

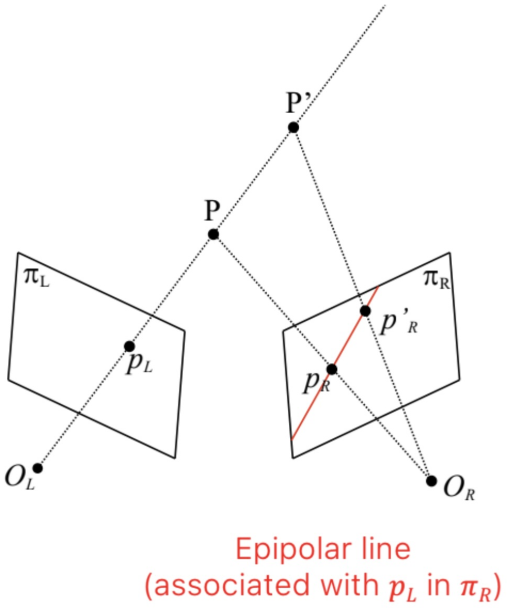


Figure 5: Epipolar line

It is almost impossible to build a stereo rig which is perfectly aligned horizontally. Searching through oblique epipolar lines is awkward, and computationally is less efficient. What people do in practice is to convert epipolar geometry to standard geometry with rectification/warping. We warp the images as if they were acquired through a standard geometry, then we can compute and applyto both images a transformation known as rectification.

2.1.5 Depth of Field (DOF)

A scene point is on focus when all its light rays, gathered by the camera, hit the image plane at the same point. In a pinhole device this happens to all scene points because of the very small size of the hole, so that the camera features an infinite Depth of Field (DOF).

The drawback is that such a small aperture allows gathering a very limited amount of light. The image is really sharp, but has a really low light. If a point is projected onto a circle instead of a point (bigger pinhole) the image is not sharp (not on focus). If we cannot gather through aperture we have to integrate through time, by using a longer exposure time.

2.1.6 Lenses

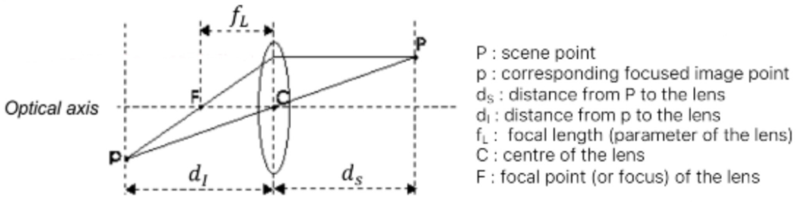
Lenses concentrate light, so we use them to gather more light from a scene point and focus it on a single image point. This enables much smaller exposure times. This way Depth Of Field is no longer infinite, and only points across a limited range of distances can be simultaneously on focus in a given image.

We will consider the approximate model known as thin lens equation, which is useful to graphically determine the position of a focused image point:

- Rays parallel to the optical axis are deflected to pass through F.
- Rays through C are undeflected.

If the image is on focus, the image formation process obeys to the perspective projection model:

- The center of the lens is the optical center.
- The distance v acts as the effective focal length of the projection.



f_L
P
F
C
Optical axis
p
d_I
d_s
P : scene point
p : corresponding focused image point
d_S : distance from P to the lens
d_I : distance from p to the lens
f_L : focal length (parameter of the lens)
C : centre of the lens
F : focal point (or focus) of the lens

$$\frac{1}{d_S} + \frac{1}{d_I} = \frac{1}{f_L}$$

Figure 6: Scheme of a lens

Choosing the distance of the image plane determines the distance at which scene points appear on focus in the image. Scene points in front and behind the focusing plane will result out-of-focus, thereby appearing in the image as circles, known as circles of confusion, or **blur circles**, rather than points.

The advantage of lenses is to have a small exposure time for capturing moving objects but we pay in terms of Depth Of Field.

2.1.7 Diaphragm

In theory, when imaging a scene through a thin lens, only the points at a certain distance can be on focus, all the others appear blurred into circles. However, as long as the circles are smaller than the size of the photosensing elements (a single pixel), the image will still look on-focus.

The range of distances across which the image appears on focus, due to blur circles being small enough, determines the Depth Of Field) of the imaging apparatus. Cameras often deploy an adjustable diaphragm (iris) to control the amount of light gathered through the effective aperture of the lens.

- Reduce aperture → less light → smaller blur circle
- More aperture → more light → larger blur circle.
- Close the diaphragm → increase depth of field → not enough light → increase exposure time → moving object → motion blur → still scenes.

2.1.8 Focusing mechanism (manually changing depth of field)

To focus on objects at diverse distances we need a amechanism that allows the lens (or the lens subsystem) to translate along the optical axis with respect to the fixed position of the image plane.

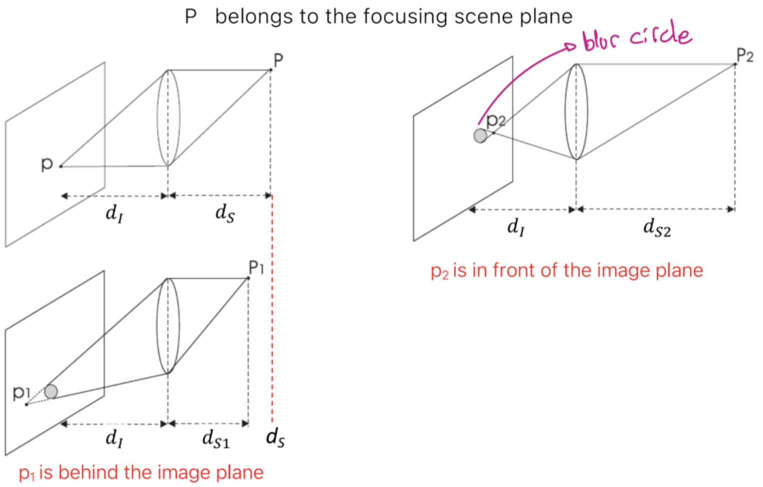

P belongs to the focusing scene plane
blur circle
P
p
d_I
d_S
P1
p1
d_I
d_{S1}
d_S
p1 is behind the image plane
P2
p2
d_I
d_{S2}
p2 is in front of the image plane

Figure 7: Lens focusing at various distances

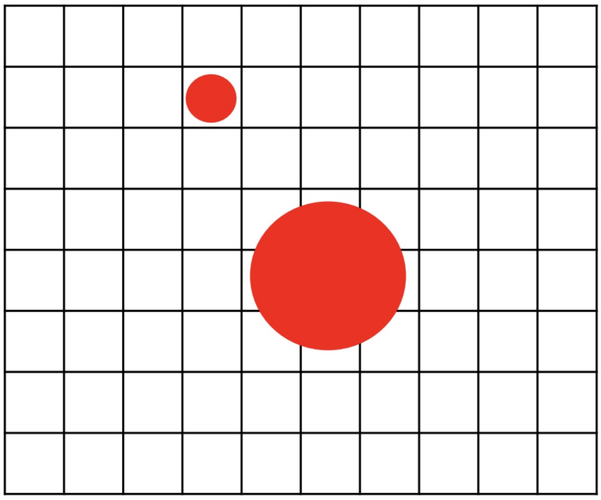


Figure 8: Blurring at pixel level

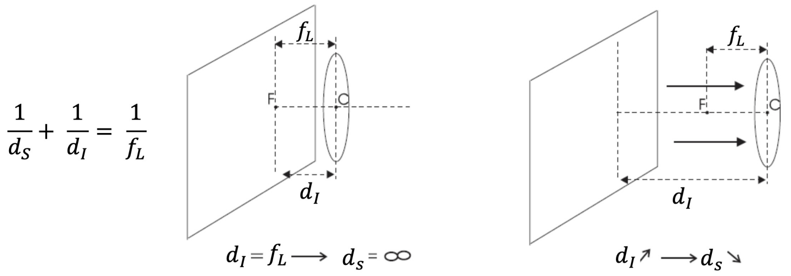
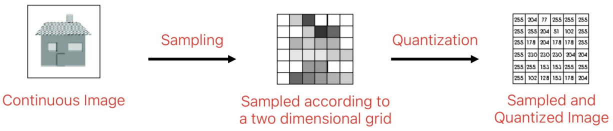

$\frac{1}{d_S} + \frac{1}{d_I} = \frac{1}{f_L}$
f_L
F
C
d_I
$d_I = f_L \longrightarrow d_s = \infty$
f_L
F
C
d_I
$d_I \nearrow \longrightarrow d_s \searrow$

Figure 9: Focusing mechanism

At one end position ($d_I = f_L$) the camera is focused at infinity (objects at inifity are on sharp focus). The focusing mechanism allows the lens to be translated farther away from the image plane up to a certain maximum value, which determines the minimum focusing distance.

2.1.9 Image digitization

How do we convert a continuous image to a discrete one which can be represented on a computer? The process can be divided in two steps: sampling and quanitzation.


Sampling
Quantization
Continuous Image
Sampled according to a two dimensional grid
Sampled and Quantized Image

255	204	77	255	255	255
255	255	204	51	102	255
255	178	204	178	178	255
255	230	230	230	204	204
255	255	153	153	255	255
255	102	128	153	178	204

Figure 10: The two steps of the image digitization process

- **Sampling**: the planar continuous image is sampled along both the horizontal and vertical directions to pick up a matrix of $n \times m$ samples known as pixels.
- **Quantization**: the continuous range of values associated with pixels is quantized into $l = 2^m$ discrete levels known as gray-levels. m is the number of bits used to represent a pixel, with the memory occupancy (in bits) of a gray-scale image given by $B = N \times M \times m$. Coloured digital images are typically represented within computers using 3 bytes per pixels. Both more pixels and more bits per pixel result in a higher quality image.

The more bits we spend for its representation, the higher the quality of the digital image (becomes a closer approximation to the ideal continuous image). This applies both to sampling and quantization.

2.1.10 Camera sensors

The sensor is a matrix of photodetectors. During exposure time, each detector converts the incident light into a proportional electic charge. The companion circuitry reads-out the charge to generate the output signal, which can be either digital or analog. For digital cameras the sensor includes the necessary ADC circuitry.

Today, the two main sensor technologies are:

- Charge Coupled Devices (CCD), where the sensor and circuit are separated.
- CMOS (Complementary Metal Oxide Semiconductor), where everything is on the same circuit.

CCD/CMOS sensors can't sense colors, so we place an array of optical filters in front of the photodetectors, to render each pixel sensitive to a specific range of wavelengths.

2.1.11 SNR

The intensity measured at a pixel under perfectly static conditions varies due to the presence of random noise. The main noise sources are:

- **Photon Shot Noise**: the number of photons collected during exposure time is not constant.
- **Electronic Circuitry Noise**: generated by the electronics.
- **Quantization Noise**: related to the ADC conversion.
- **Dark Current Noise**: random charge observed due to thermal excitement.

SNR can be expressed both in decibels and bits.

2.1.12 Dynamic Range (DR)

If the sensed amount of light is too small, the "true" signal cannot be distinguished from noise. Given $E_{\min}$: the minimum detectable irradiation, and $E_{\max}$, the saturation irradiation. The Dynamic Range (DR) of a sensor is defined as $DR = \frac{E_{\max}}{E_{\min}}$, and like the SNR, it is often specified in decibels or bits.

Like SNR, the higher the DR the better it is. A higher DR corresponds to the ability of the sensor to simultaneously capture in one image both the dark and bright structures of the scene.

2.2 Image Filtering

2.2.1 Noise and image filters

In computer vision we have to deal with noise. Noise is always different and more noticeable in uniform regions of the image. The simplest thing to reduce noise is to output the average of the pixel color over time, to get **almost** the ideal noiseless value.

$$O(p) = \frac{1}{T}\sum_{t=1}^{T} I_k(p) = \frac{1}{T}\sum_{t=1}^{T}(\tilde{I}(p) + n_t(p))$$

This technique works well on still images.

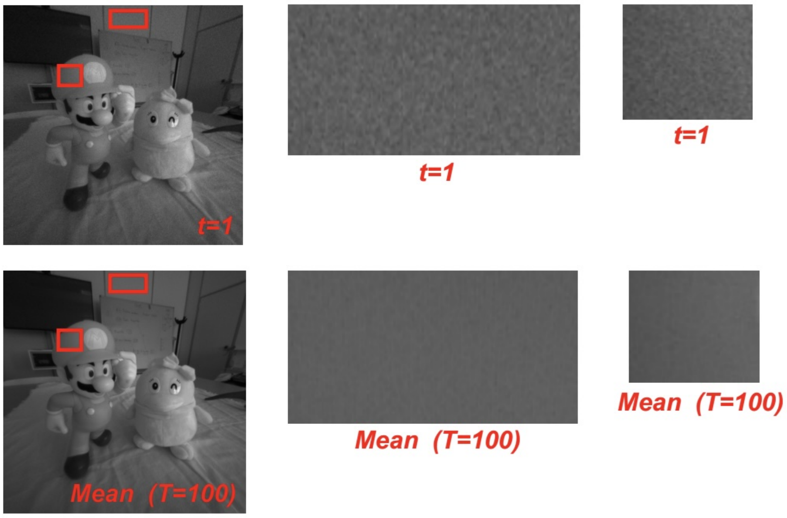


Figure 11: Simple denoising

If we are given a simple image, we may compute a mean across neighbouring pixels, like a spatial rather than temporal mean. The size of the square of the neighbouring pixels is a tradeoff.

This is a very basic denoising filter.

Image filters are image processing operators that compute the new intensity (colour) of a pixel, p, based on the intensities (colours) of those belonging to a neighbourhood (support) of p.

2.2.2 Convolution

An important sub-class of filters is given by **Linear** and **Translation-Equivariant (LTE)** operators.

The application of filters in image processing consists in a 2D convolution between the input image and the impulse response function of the LTE operator.

LTE operators are used as feature extractors in Convolutional Neural Networks (CNNs).

Given an input 2D signal $i(x, y)$, a 2D operator $Ti(x, y)$ is said to be linear if and only if

$$T\alpha i_1(x, y) + \beta i_2(x, y) = \alpha o_1(x, y) + \beta o_2(x, y)$$

with $o_1 = Ti_1$ and $o_2 = Ti_2$ and α, β are two constants. The operator is said to be **translation-equivariant** if and only if: $Ti(x - x_0, y - y_0) = o(x - x_0, y - y_0)$

If the operator is LTE, the output signal is given by the **convolution** between the input signal and the impulse response (point spread function) $h(x, y) = T\delta(x, y)$.

$$o(x, y) = i(x, y) * h(x, y) = Ti(x, y)$$

$$i(x, y) * h(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(\alpha, \beta)h(x - \alpha, y - \beta)\, d\alpha d\beta$$

Convolutions have some useful properties:

- **Associative property**: $f * (g * h) = (f * g) * h$ (useful because we can decompose kernels and obtain faster operations).

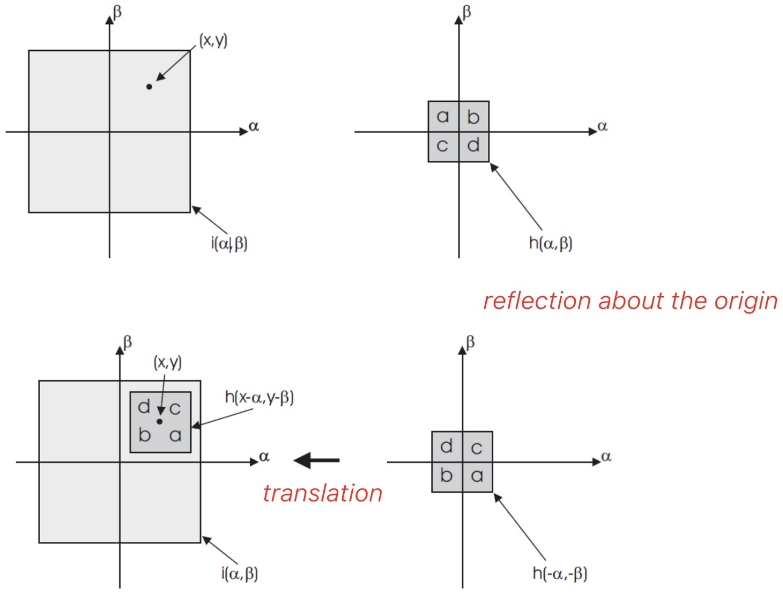


Figure 12: A graphical view of convolution

- **Commutative property**: $f * g = g * f$.
- **Distributive property**: $f * (g + h) = f * g + f * h$.
- **Convolution commutes with differentiation**: $(f * g)' = f' * g = f * g'$.

The correlation of signal $i(x, y)$ with respect to signal $h(x, y)$ is defined as:

$$i(x, y) \circ h(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} i(\alpha, \beta) h(x - \alpha, y - \beta) \, d\alpha d\beta$$

Correlation is not commutative, unlike the convolution.

2.2.3 Discrete convolution

Normal convolution is useful for signal theory, but we want to have a discrete convolution, where we use summations instead of integrals. The four convolution properties highlighted for the convolution hold for the discrete one too.

2.2.4 Practical implementation

CNNs learn flipped kernels. In image processing both the the input image and the kernel are stored into matrices of given finite sizes, with the image being much larger than the kernel. Conceptually, to obtain the output image we need to slide the kernel across the whole input image and compute the convolution at each pixel.

We have an issue on borders, since the kernel ”goes out” of the matrix of the image. We have two main options to solve this issue:

- `CROP`: common in image processing.
- `PAD`: preferred in CNNs. We can do zero-padding, replicate the first pixel many times, reflect the first k pixels (half of the kernel)...

Without padding convolutions shrink the images.

2.2.5 Mean filter

Mean filtering is the simplest and fastest way to denoise an image. It consists in replacing each pixel intensity by the average intensity overa chosen neighbourhood. According to signal processing theory, the Mean Filter carries out a low-pass filtering operation, which in image processing is also referred to as image smoothing. Smoothing is often aimed at image denoising, but sometimes is used to cancel out small-size unwanted details that might hinder the image analysis task. Linear filtering reduces noise but blurs the image, so we lose sharpness.

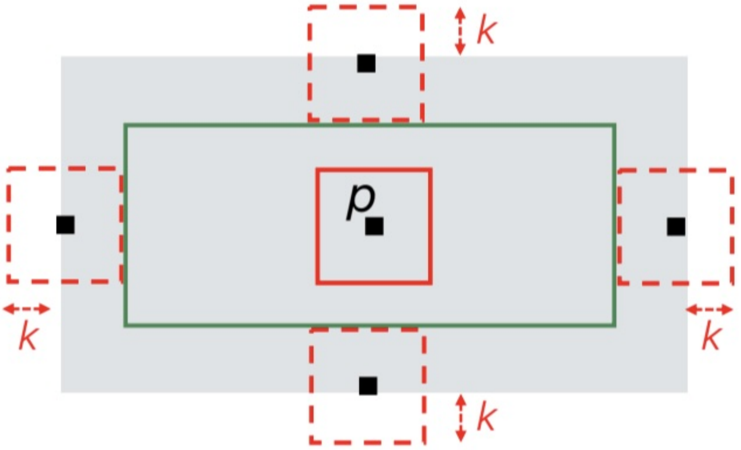


Figure 13: Issues with convolutions

2.2.6 Gaussian filter

Let's try to use a more sophisticated but still linear filter. The gaussian filter is an LTE operator whose impulse response is a 2D Gaussian function

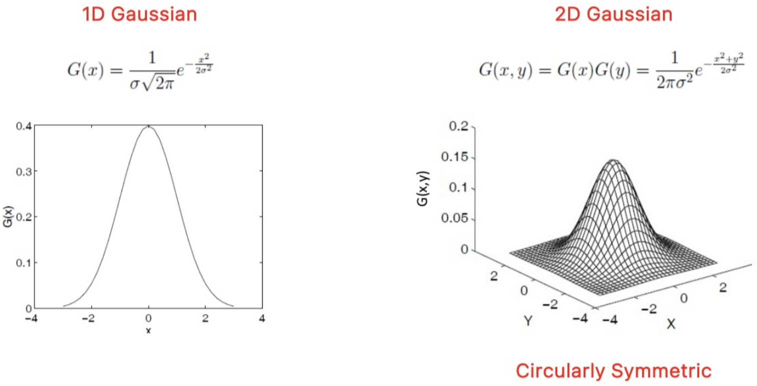


Figure 14: 1D and 2D Gaussian plot

The larger the size of the gaussian kernel, the more accurate will the discrete approximation be, but the computational cost grows with the filter size. We should then use larger sizes for filters with higher σ, smaller sizes whenever σ is smaller. A typical rule is to choose the size of the filter by capturing the interval $[-3\sigma, +3\sigma]$ since it captures 99% of the energy of the gaussian impulse. The Gaussian filter is the only one which has been mathematically proven of not altering image information but just semplification. To speedup the filtering we can apply the 2D gaussian by doing 2 1D gaussian filterings.

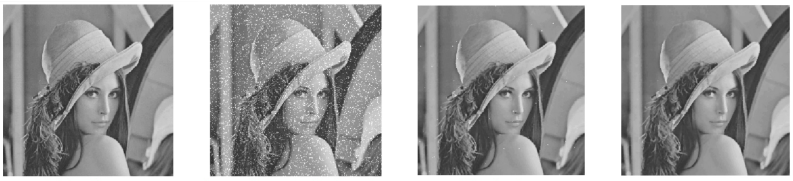
We also observe that the higher the σ, the higher is the smoothing caused by the filter. We can use this to remove details from the image.

2.2.7 Median filter

There is noise that Gaussian filters can't handle well, that is the salt and pepper noise. It's usually caused by image corruption (or broken pixels in the sensor). Linear filtering is ineffective and just blurs the image.

We can use a **non-linear** filter, where each pixel intensity is replaced by the median over a given neighbourhood. Median filtering counteracts impulse noise effectively, since outliers tend to fall at either the top or the bottom end of the sorted intensities. Median filtering tends to keep sharper edges than linear filters such as the Mean or the Gaussian.

The median filter can effectively denoise the image without introducing significant blur, yet, Gaussian-like noise, such as sensor noise, cannot be dealt with by the median, as this would require computing new noiseless intensities.



Original Image

Image Corrupted by impulse noise (+100, 5% of randomly picked pixels)

Filtering by a 3×3 Median

Filtering twice by a 3×3 Median

Figure 15: Example of the power of the median filter

2.2.8 Bilateral filter

The bilateral filter preserves edges while filtering out noise. It's an advanced non-linear filter to accomplish denoising of Gaussian-like noise without blurring the image. It's also called edge preserving smoothing.

$$O(p) = \sum_{q \in S} H(p,q) \cdot I_q \quad H(p,q) = \frac{1}{W(p)} G_{\sigma_s}(d_s(p,q)) G_{\sigma_r}(d_r(p,q))$$

- **spatial distance**: $d_s(p,q) = ||p-q||_2 = \sqrt{(u_p - u_q)^2 + (v_p - v_q)^2}$.
- **range (intensity) distance**: $d_r(I_p, I_q) = |I_p - I_q|$.
- **normalization factor**: $W(p) = \sum_{q \in S} G_{\sigma_s}(d_s(p,q)) G_{\sigma_r}(d_r(p,q))$

3 Advanced Topics in Deep Learning for Computer Vision