2016-10-18. AEI MCS

3. Use Rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Step	Reason
1. $\exists x \neg P(x)$	Premise
2. $\neg P(c)$ for some element c	Existential Instantiation from (1)
3. $\forall x(P(x) \vee Q(x))$	Premise
4. $P(c) \vee Q(c)$	Universal Instantiation from (3)
5. $Q(c)$	Disjunctive syllogism using (4) and (2)
6. $\forall x(\neg Q(x) \vee S(x))$	Premise
7. $\neg Q(c) \vee S(c)$	Universal Instantiation from (6)
8. $S(c)$	Disjunctive syllogism using (7) and (5)
9. $\forall x(R(x) \to \neg S(x))$	Premise
10. $R(c) \to \neg S(c)$	Universal Instantiation from (9)
11. $S(c) \to \neg R(c)$	Contrapositive of (10)
12. $\neg R(c)$	Modus ponens using (11) and (8)
13. $\exists x \neg R(x)$	Existential Generalization from (12)

2. Prove that if $\forall x(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

Step	Reason
1. $\forall x(P(x) \wedge R(x))$	Premise
2. $P(a) \wedge R(a)$ for an arbitrary element a	Universal Instantiation from (1)
3. $P(a)$	Simplification from (2)
4. $R(a)$	Simplification from (2)
5. $\forall x(P(x) \to (Q(x) \wedge S(x)))$	Premise
6. $P(a) \to (Q(a) \wedge S(a))$	Universal Instantiation from (5)
7. $Q(a) \wedge S(a)$	Modus ponens using (6) and (3)
8. $S(a)$	Simplification from (7)
9. $R(a) \wedge S(a)$	Conjunction using (4) and (8)
10. $\forall x(R(x) \wedge S(x))$	Universal Generalization from (9)