COMP 1046: Mathematics for Computer Scientists

Tutorial: Functions Answer Sheet

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Functions

1. Show that the function $f(x) = |x|$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of non-negative real numbers, the resulting function is invertible.

   Answer:

For a function to be invertible, it needs to be bijective. Therefore, we need to check if this is one-to-one (injective) and onto (surjective).

(a) Injective: No:

For some $x_1 \neq x_2$, say $x_1=-x_2$, we have $f(x_1)=f(x_2)$, by definition of injective, the function is not injective.

If the domain is restricted to the set of nonnegative real numbers, f(x) is injective because

Assume $f(x_1)=f(x_2)$, we have $x_1= x_2$

Therefore, on the restricted domain f(x) is injective.

(b).Surjective

For some element $b \in rng(f)$, that $b=|a|$, with $a \in R$, b must be positive. Thus, the range is the set of all nonnegative real numbers. Because the range and codomain are the same, we can conclude that f is surjective.

(c) Bijective: No, because it is not injective. Though, on the restricted domain, it is bijective because it is both injective and surjective.

d) Invertible: Again, only on the restricted domain.