Early Module Feedback (EMF)

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Foundation Algebra for Physical Sciences & Engineering (CELEN036 UNNC) (AUC1 22-23)

Module Introduction

The module aims to provide students with the mathematical knowledge and fluency in algebraic techniques essential for analysing basic problems in engineering or sciences. Key elements are the development of basic mathematical skills in algebra and trigonometry, and algebraic mathematical techniques and their application to problem solving.

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Available from 9:00 am Monday 17-Oct to 5:00 pm Friday 21-Oct

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Your comments are useful to us, because it helps us to improve the delivery of this module and to respond to any queries you may have.

Early Module Feedback (EMF)

Respond to 4 questions and make relevant comments about the module

1. The module content was of sufficient quality to assist my learning on this module

2. Module materials were clear about what was expected of me

3. I was given sufficient opportunity to contact my teachers/faculty on this module

4. The overall experience of studying this module has contributed to my learning

5. In your opinion, what is working well on the module so far? If there are any suggestions for the remaining weeks on the module, please also leave your comments here.

Seminar 3

In this seminar you will study:

- Trigonometric Identities
- Converting angles: from degrees to radians and vice-versa
- Finding range and period of trigonometric functions
- Finding values of trigonometric function

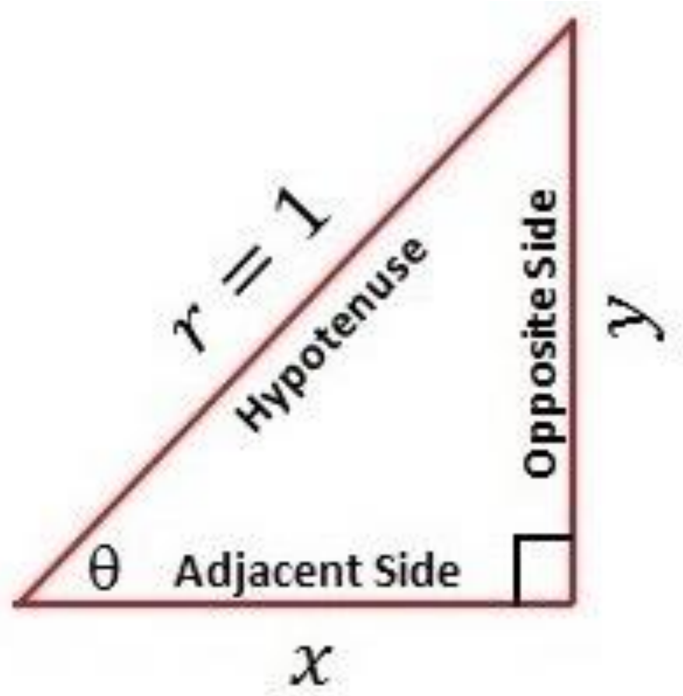
Trigonometric functions

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r} = \frac{x}{1} = x$$

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r} = \frac{y}{1} = y$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cos\theta \neq 0$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \quad ; \quad \sin\theta \neq 0$$



$$\sec\theta = \frac{1}{\cos\theta} \quad ; \quad \cos\theta \neq 0$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad ; \quad \sin\theta \neq 0$$

Trigonometric identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta \quad ; \quad \cos\theta \neq 0$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\,\theta \quad ; \quad \sin\theta \neq 0$$

Trigonometric identities

Example 1: Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$

Trigonometric identities

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Solution:

$$\text{LHS} = \tan^2\theta - \sin^2\theta$$

Trigonometric identities

Example 1: Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$

Solution:

$$\begin{aligned} \text{LHS} &= \tan^2\theta - \sin^2\theta \\ &= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \end{aligned}$$

Trigonometric identities

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Solution:

$$\begin{aligned}
\text{LHS} &= \tan^2\theta - \sin^2\theta \\
&= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \\
&= \frac{\sin^2\theta - \sin^2\theta \cdot \cos^2\theta}{\cos^2\theta}
\end{aligned}$$

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&= \frac{\sin^2\theta - \sin^2\theta \cdot \cos^2\theta}{\cos^2\theta} \\
&= \frac{\sin^2\theta}{\cos^2\theta} \cdot (1 - \cos^2\theta)
\end{aligned}$$

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&= \frac{\sin^2\theta}{\cos^2\theta} \cdot (\sin^2\theta)
\end{aligned}$$

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&= \frac{\sin^2\theta}{\cos^2\theta} \cdot (\sin^2\theta) \\
&= \tan^2\theta \cdot \sin^2\theta = \text{RHS}
\end{aligned}$$

Trigonometric identities

Example 2: Prove that $$\frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1} = \sec^2\theta$$

Trigonometric identities

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Solution:

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Trigonometric identities

Example 2: Prove that $\dfrac{1+\cot^2\theta}{\operatorname{cosec}^2\theta-1}=\sec^2\theta$

Solution:

$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta-1}$$

$$= \frac{1+\dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta}-1}$$

Trigonometric identities

Example 2: Prove that $\dfrac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1} = \sec^2\theta$

Solution:

$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1}$$

$$= \frac{1+\dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta} - 1}$$

$$= \frac{\dfrac{\sin^2\theta+\cos^2\theta}{\color{red}{\sin^2\theta}}}{\dfrac{1-\sin^2\theta}{\color{red}{\sin^2\theta}}}$$

Trigonometric identities

Example 2: Prove that $$\frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta-1}=\sec^2\theta$$

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$$= \frac{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}{\dfrac{1-\sin^2\theta}{\sin^2\theta}}$$

$$= \frac{1}{(1-\sin^2\theta)}$$

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$$= \frac{1}{(1-\sin^2\theta)} \quad = \frac{1}{\cos^2\theta}$$

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$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1}$$

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$$= \frac{1}{(1-\sin^2\theta)} \quad = \frac{1}{\cos^2\theta}$$

$$= \sec^2\theta = \text{RHS}$$

Trigonometric identities

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Alternative method

$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta-1}$$

Trigonometric identities

Example 2: Prove that $\dfrac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1} = \sec^2\theta$

Solution:

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<u>Alternative method</u>

$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1}$$

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<u>Alternative method</u>

$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1}$$

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$$= \frac{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta} - 1}$$

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$$\text{LHS} = \frac{1+\cot^2\theta}{\operatorname{cosec}^2\theta - 1}$$

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$$= \frac{\left(\dfrac{1}{\sin^2\theta}\right)}{\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)}$$

$$= \frac{1}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} = \sec^2\theta = \text{RHS}$$

Trigonometric identities

(i). Find the value of

$$\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} - \frac{1}{\tan^2\theta} - \frac{1}{\cot^2\theta} - \frac{1}{\sec^2\theta} - \frac{1}{\operatorname{cosec}^2\theta}$$

(ii). Verify that

$$\cos^4\theta + \sin^4\theta = 1 - 2\cos^2\theta \cdot \sin^2\theta$$

(iii). Prove that

$$\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 2\operatorname{cosec} x$$

(iv). Find the value of

$$(1+\cot^2\theta)\cdot(1-\cos^2\theta)$$

Trigonometric identities

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Answer: 1

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(iv). Find the value of

$$(1+\cot^2\theta)\cdot(1-\cos^2\theta)$$

Answer: 1

Trigonometric identities

(vi). Prove that

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

(vii). Prove that

$$\frac{\cos\theta + 1}{\tan^2\theta} = \frac{\cos\theta}{\sec\theta - 1}$$

$(viii)$. Prove that

$$\frac{\tan^2 x - 1}{\sec^2 x} = \frac{\tan x - \cot x}{\tan x + \cot x}$$

(ix). Prove that

$$\frac{\cos^2 x - 1}{1+\cos^2 x} = 1 - 2\sin^2 x$$

Conversion Formulae

- Degrees to Radians

$$\text{angle in radians} = \text{angle in degrees} \times \left(\frac{\pi}{180^\circ}\right)$$

- Radians to Degree

$$\text{angle in degrees} = \text{angle in radians} \times \left(\frac{180^\circ}{\pi}\right)$$

Conversion Formulae

(i). Convert 225° to radians.

(ii). Convert $\dfrac{13\pi}{12}$ to degrees.

(iii). Convert $\dfrac{5\pi}{3}$ to degrees.

(iv). Convert 315° to radians.

Conversion Formulae

(i). Convert 225° to radians.

Answer: $\frac{5\pi}{4}$

(ii). Convert $\frac{13\pi}{12}$ to degrees.

Answer: 195°

(iii). Convert $\frac{5\pi}{3}$ to degrees.

Answer: 300°

(iv). Convert 315° to radians.

Answer: $\frac{7\pi}{4}$

The range of Trigonometric functions

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- The range of $\sin$ and $\cos$ functions is: $[-1, 1]$.

  i.e. $-1 \leq \cos\theta \leq 1$ and $-1 \leq \sin\theta \leq 1, \quad \theta \in \mathbb{R}$

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- The range of $\sec$ and cosec functions is: $\mathbb{R} - (-1, 1)$.

i.e. $\sec\theta \leq -1$ or $\sec\theta \geq 1, \quad \theta \neq (2k+1)\dfrac{\pi}{2}, \ k \in \mathbb{Z}$

and $\operatorname{cosec}\theta \leq -1$ or $\operatorname{cosec}\theta \geq 1, \quad \theta \neq k\pi, \ k \in \mathbb{Z}$

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  and $\quad \operatorname{cosec}\theta \leq -1 \quad$ or $\quad \operatorname{cosec}\theta \geq 1, \quad \theta \neq k\pi, \; k \in \mathbb{Z}$

- The range of $\tan$ and $\cot$ functions is: $\mathbb{R}$.

  i.e. $\quad \tan\theta \in (-\infty, +\infty), \quad \theta \neq (2k+1)\dfrac{\pi}{2}, \; k \in \mathbb{Z}$

  and $\quad \cot\theta \in (-\infty, +\infty), \quad \theta \neq k\pi, \; k \in \mathbb{Z}$

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Example: Find the range of $f(x) = 5 - 3\sin(4x - 7)$

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For $f(x) = 5 - 3\sin(4x - 7)$, the angle θ is $4x - 7$.

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Example: Find the range of $f(x) = 5 - 3\sin(4x - 7)$

Solution:

For any $\theta \in \mathbb{R}$, $\quad -1 \leq \sin\theta \leq 1.$

For $f(x) = 5 - 3\sin(4x - 7)$, the angle θ is $4x - 7$.

$$\Rightarrow \quad -1 \leq \sin(4x - 7) \leq 1$$

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Solution:

For any $\theta \in \mathbb{R}$, $-1 \leq \sin\theta \leq 1$.

For $f(x) = 5 - 3\sin(4x - 7)$, the angle θ is $4x - 7$.

$$\Rightarrow \quad -1 \leq \sin(4x - 7) \leq 1$$

$$\Rightarrow \quad -1 \times (-3) \leq \sin(4x - 7) \times (-3) \leq 1 \times (-3)$$

Multiply the inequality through by (-3)

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Multiply the inequality through by (-3)

$\Rightarrow \quad 3 \geq -3\sin(4x - 7) \geq -3$

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For any $\theta \in \mathbb{R}$, $\;-1 \le \sin\theta \le 1$.

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$\Rightarrow \quad 3 \ge -3\sin(4x - 7) \ge -3$

$\Rightarrow \quad -3 \le -3\sin(4x - 7) \le 3$

$\Rightarrow \quad -3 + (5) \le -3\sin(4x - 7) + (5) \le 3 + (5)$ Add (5) to the inequality

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$$\Rightarrow \quad -3 + (5) \leq -3\sin(4x - 7) + (5) \leq 3 + (5)$$

Add (5) to the inequality

$$\Rightarrow \quad 2 \leq 5 - 3\sin(4x - 7) \leq 8$$

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Multiply the inequality through by (-3)

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$$\Rightarrow \quad 2 \le f(x) \le 8$$

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$\Rightarrow \quad -3 \leq -3\sin(4x - 7) \leq 3$

$\Rightarrow \quad -3 + (5) \leq -3\sin(4x - 7) + (5) \leq 3 + (5)$ Add (5) to the inequality

$\Rightarrow \quad 2 \leq 5 - 3\sin(4x - 7) \leq 8$

$\Rightarrow \quad 2 \leq f(x) \leq 8 \quad \Rightarrow$ The range of $f : R_f = [2, 8]$

Find the range of the following trignonometric functions.

(i). $f(x) = 2\sin(3x+5)$

(ii). $f(x) = 5\cos(x+4) - 3$

(iii). $f(x) = 4\sec(x+3)$

(iv). $f(x) = 3\sin(2x+7) + 1$

Find the range of the following trignonometric functions.

(i). $f(x) = 2\sin(3x+5)$

Answer: $[-2, 2]$

(ii). $f(x) = 5\cos(x+4) - 3$

Answer: $[-8, 2]$

(iii). $f(x) = 4\sec(x+3)$

Answer: $\mathbb{R} - (-4, 4)$

(iv). $f(x) = 3\sin(2x+7) + 1$

Answer: $[-2, 4]$

The period of Trigonometric functions

- The period (principal period) of $aT_1(bx + c) + d$ is $\dfrac{2\pi}{|b|}$,

where T_1 is the trigonometric function: $\sin$, $\cos$, cosec, or $\sec$.

- The period (principal period) of $aT_2(bx + c) + d$ is $\dfrac{\pi}{|b|}$,

where T_2 is the trigonometric function: $\tan$ or $\cot$.

Find the period of the following trignonometric functions.

$(i).\ \ f(x) = 2\sin(3x+5) - 4$

$(ii).\ \ f(x) = 3\cos(9-4x) + 1$

$(iii).\ \ f(x) = 4\sec(x+3) - 1$

$(iv).\ \ f(x) = 5\tan(7-2x)$

Find the period of the following trignonometric functions.

$(i).\ \ f(x) = 2\sin(3x+5) - 4$

Answer: $\frac{2\pi}{3}$

$(ii).\ \ f(x) = 3\cos(9-4x) + 1$

Answer: $\frac{\pi}{2}$

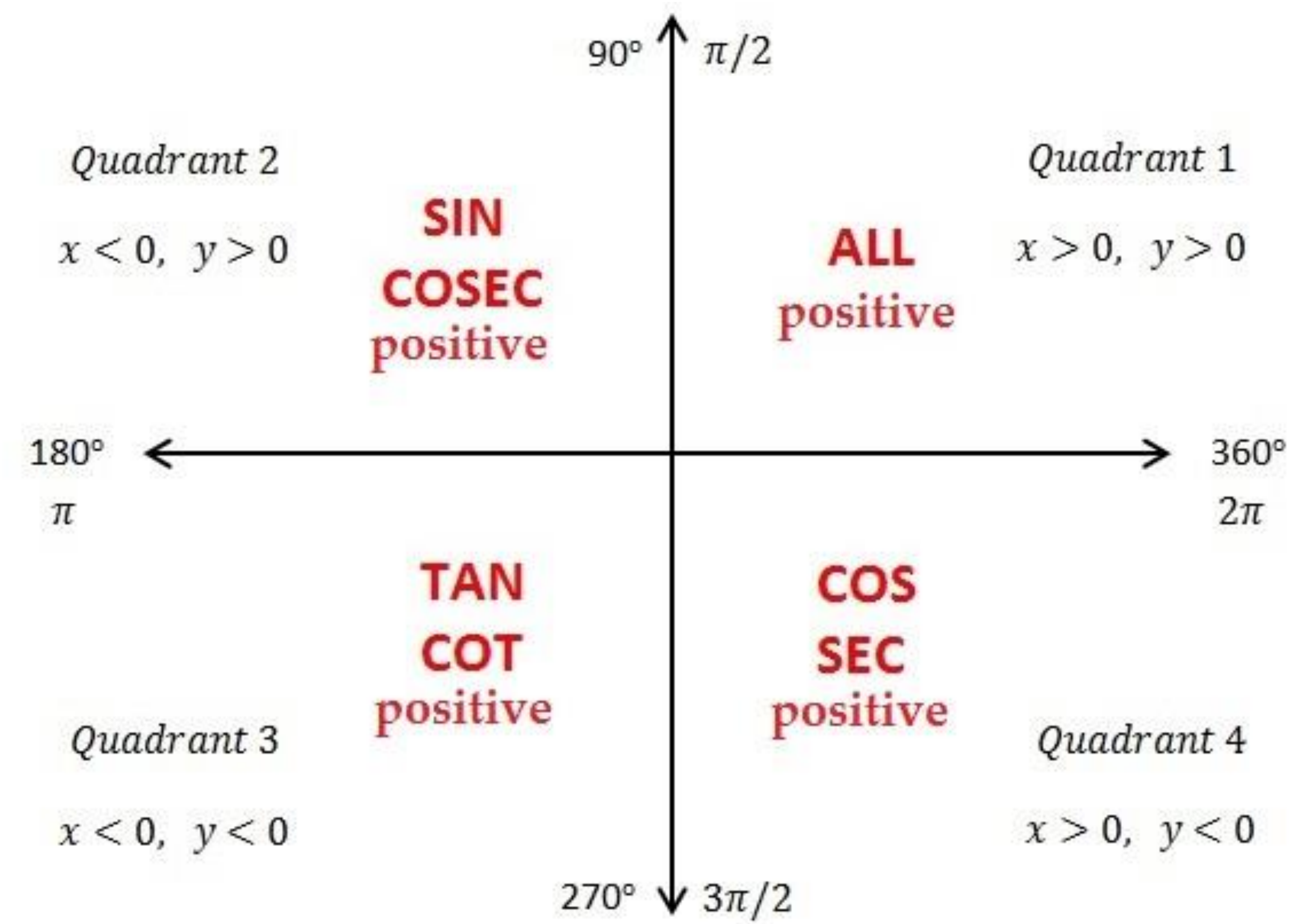
$(iii).\ \ f(x) = 4\sec(x+3) - 1$

Answer: 2π

$(iv).\ \ f(x) = 5\tan(7-2x)$

Answer: $\frac{\pi}{2}$

Signs of Trigonometric functions in the quadrants



Finding values of Trigonometric functions

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Example: If $\cot\theta = -\dfrac{9}{40}$, find $\cos\theta + \sin\theta$, where $\dfrac{3\pi}{2} < \theta < 2\pi$.

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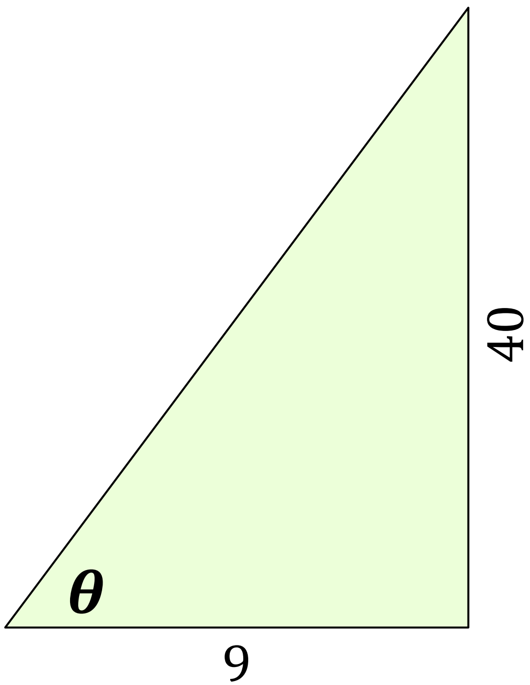

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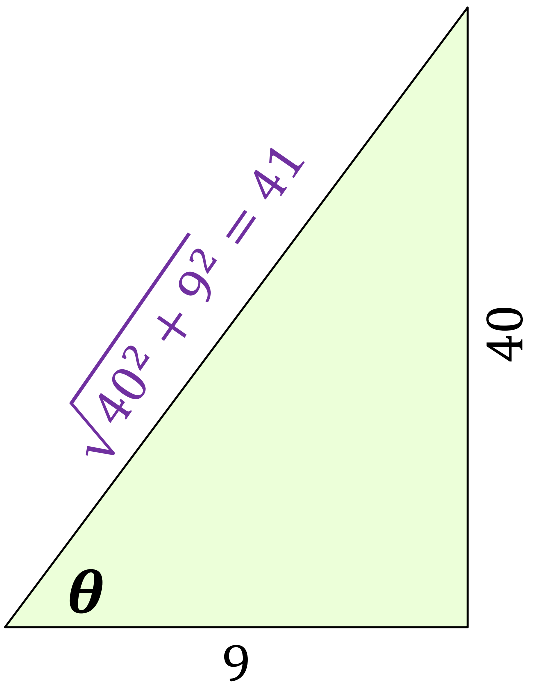

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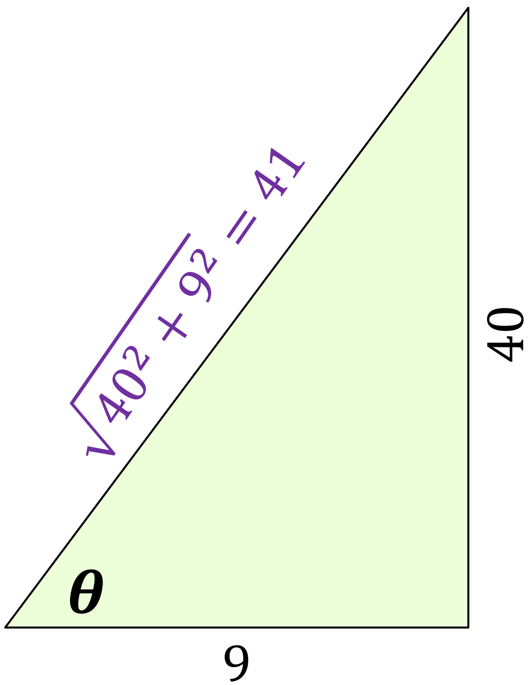

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Since $\quad \dfrac{3\pi}{2} < \theta < 2\pi$

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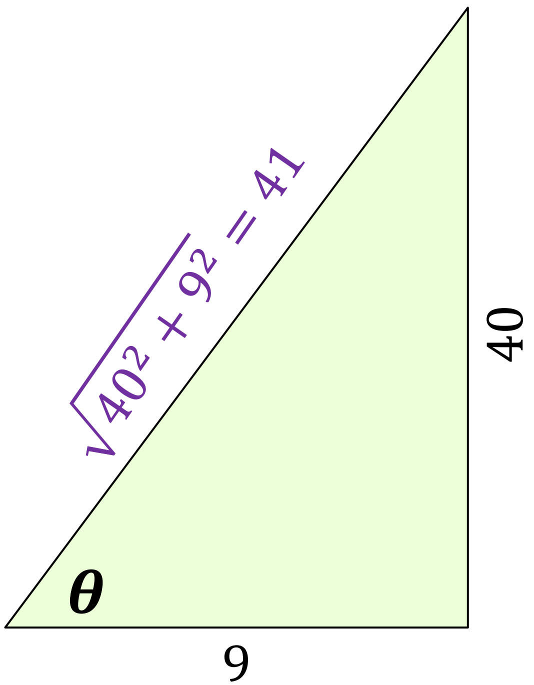

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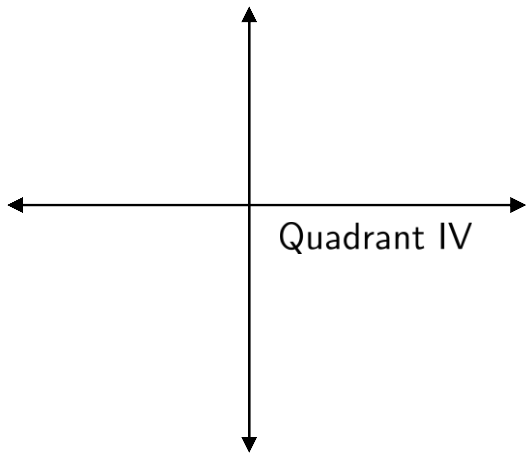

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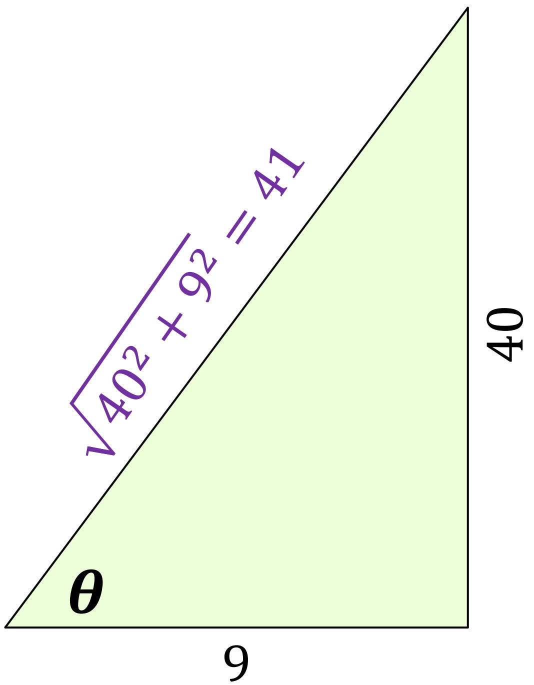

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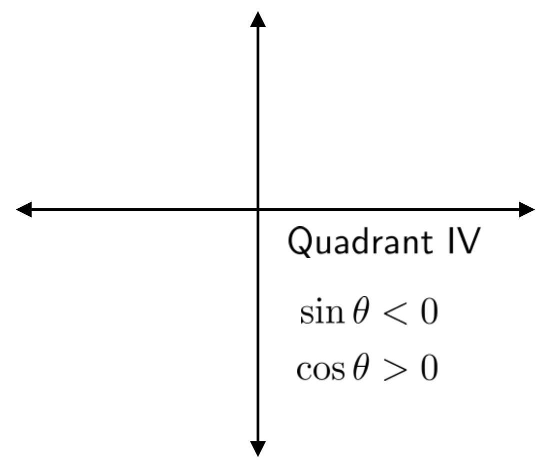

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Example: If $\cot\theta = -\dfrac{9}{40}$, find $\cos\theta + \sin\theta$, where $\dfrac{3\pi}{2} < \theta < 2\pi$.

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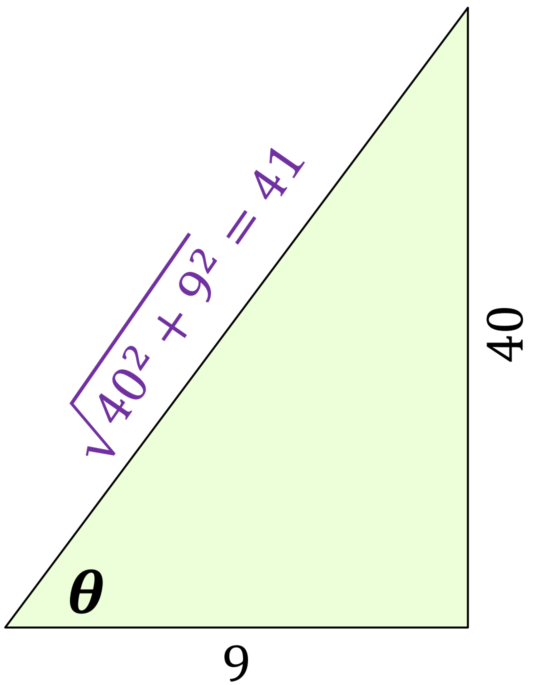

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Since $\dfrac{3\pi}{2} < \theta < 2\pi$

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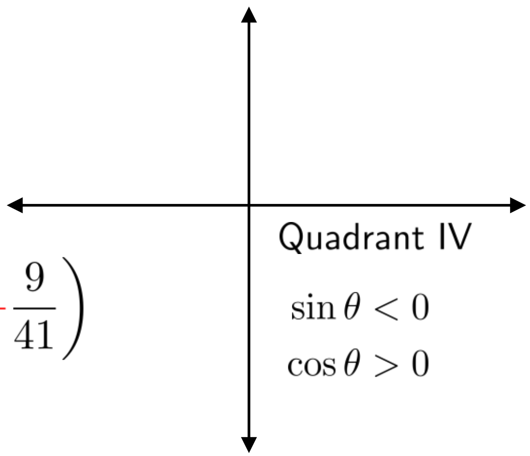
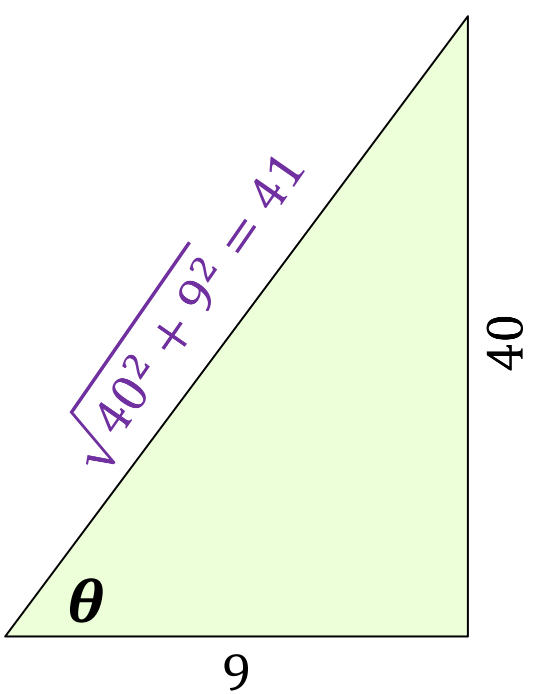


$$\sin\theta + \cos\theta = \left(-\frac{40}{41}\right) + \left(+\frac{9}{41}\right)$$

Finding values of Trigonometric functions

Example: If $\cot\theta = -\dfrac{9}{40}$, find $\cos\theta + \sin\theta$, where $\dfrac{3\pi}{2} < \theta < 2\pi$.

Solution:



$$\cot\theta = -\frac{9}{40} \quad \Rightarrow \quad \tan\theta = -\frac{40}{9}$$

Since $\quad \dfrac{3\pi}{2} < \theta < 2\pi$

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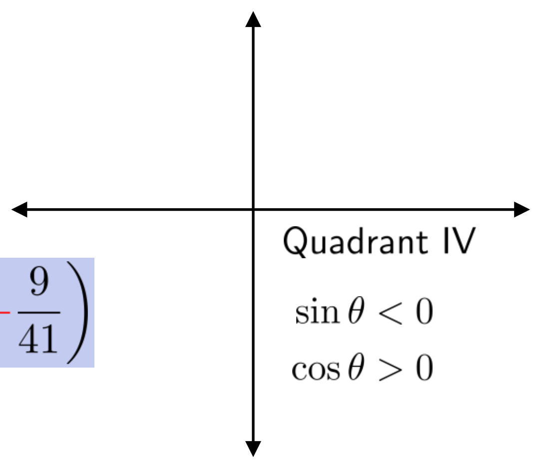

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Finding values of Trigonometric functions

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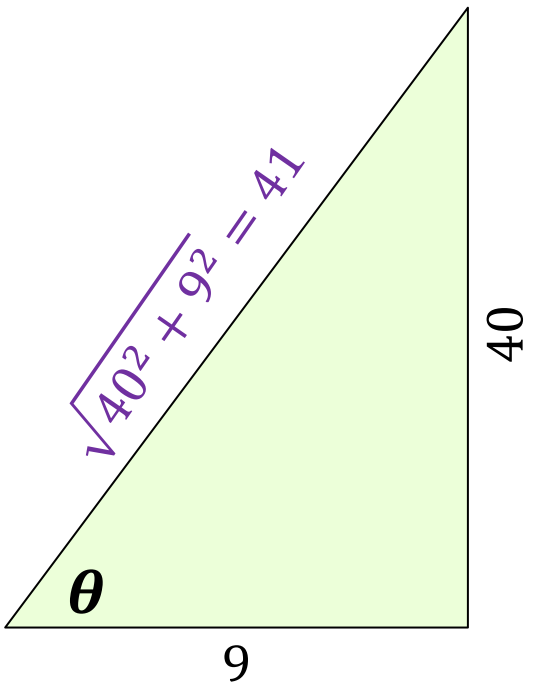

$$\cot\theta = -\frac{9}{40} \quad \Rightarrow \quad \tan\theta = -\frac{40}{9}$$

Since $\quad \dfrac{3\pi}{2} < \theta < 2\pi$

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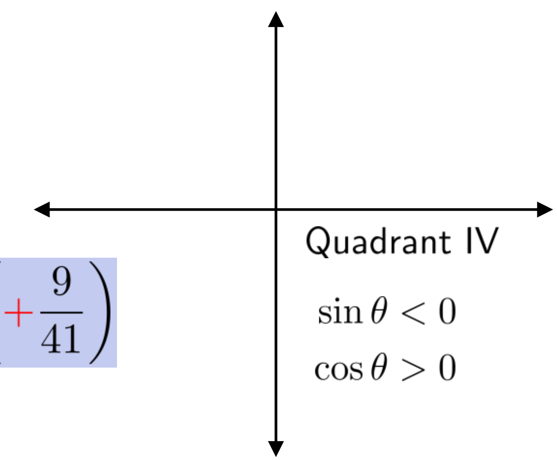

$$\sin\theta + \cos\theta = \left(-\frac{40}{41}\right) + \left(+\frac{9}{41}\right)$$

$$= -\frac{31}{41}$$

Find $\sin\theta$ for each of the following.

(i). $\cos\theta = \dfrac{3}{4}, \quad 0 < \theta < \dfrac{\pi}{2}.$	(ii). $\cot\theta = \dfrac{4}{3}, \quad \dfrac{\pi}{2} < \theta < \pi.$
(iii). $\tan\theta = \dfrac{2}{\sqrt{21}}, \quad \pi < \theta < \dfrac{3\pi}{2}.$	(iv). $\sec\theta = \dfrac{11}{4}, \quad \dfrac{3\pi}{2} < \theta < 2\pi.$

Find $\sin\theta$ for each of the following.

(i). $\cos\theta = \dfrac{3}{4}, \quad 0 < \theta < \dfrac{\pi}{2}.$

Answer: $\dfrac{\sqrt{7}}{4}$

(ii). $\cot\theta = \dfrac{4}{3}, \quad \dfrac{\pi}{2} < \theta < \pi.$

Answer: $\dfrac{3}{5}$

(iii). $\tan\theta = \dfrac{2}{\sqrt{21}}, \quad \pi < \theta < \dfrac{3\pi}{2}.$

Answer: $-\dfrac{2}{5}$

(iv). $\sec\theta = \dfrac{11}{4}, \quad \dfrac{3\pi}{2} < \theta < 2\pi.$

Answer: $-\dfrac{\sqrt{105}}{11}$

Finding values of Trigonometric functions

(i). Given $\tan\theta = \dfrac{12}{5}$, $\pi < \theta < \dfrac{3\pi}{2}$,

find $\operatorname{cosec}\theta - \sec\theta$.

(ii). Given $\tan\theta = -\dfrac{4}{3}$, $\dfrac{3\pi}{2} < \theta < 2\pi$,

prove that prove that $\sin^2\theta + \cos^2\theta = 1$.

(iii). Given $\cos\theta = \dfrac{12}{13}$, $0 < \theta < \dfrac{\pi}{2}$,

find $\sec\theta + \tan\theta$.

(iv). Given $\cos\theta = -\dfrac{15}{17}$, $\pi < \theta < \dfrac{3\pi}{2}$,

find $\sin\theta + \tan\theta$.

Finding values of Trigonometric functions

(i). Given $\tan\theta = \dfrac{12}{5}$, $\pi < \theta < \dfrac{3\pi}{2}$,

find $\operatorname{cosec}\theta - \sec\theta$.

Answer: $\dfrac{91}{60}$

(ii). Given $\tan\theta = -\dfrac{4}{3}$, $\dfrac{3\pi}{2} < \theta < 2\pi$,

prove that prove that $\sin^2\theta + \cos^2\theta = 1$.

(iii). Given $\cos\theta = \dfrac{12}{13}$, $0 < \theta < \dfrac{\pi}{2}$,

find $\sec\theta + \tan\theta$.

Answer: $\dfrac{3}{2}$

(iv). Given $\cos\theta = -\dfrac{15}{17}$, $\pi < \theta < \dfrac{3\pi}{2}$,

find $\sin\theta + \tan\theta$.

Answer: $\dfrac{16}{255}$

THANKS FOR YOUR ATTENTION