AE1MCS: Tutorial 4

1. Show that if a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$.

2.Prove that at least one of the real numbers a_1, a_2,…, a_n is greater than or equal to the average of these numbers.

3. Prove that if n is an integer, these four statements are equivalent: (1) n is even, (2) $n + 1$ is odd, (3) $3n + 1$ is odd, (4) $3n$ is even.

1. Show that if a and b are real numbers and a≠0, then there is a unique real number r such that ar+b=0.

Proof:

- First, $r = -\frac{b}{a}$ is a solution of ar+b=0 because $a\left(-\frac{b}{a}\right) + b = -b + b = 0$. Consequently, a real number r exists for which ar+b=0. (The existence part of the proof)

- Then, suppose s is a real number such that $as + b = 0$. Then $as + b = ar + b$. Subtracting b, we have $ar = as$. Divide both side by a, where a≠0, we see that $r = s$. This means if $s \neq r$, then $as + b \neq 0$. (The uniqueness part of the proof)

2.Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers.

Proof:

- Let us prove it by contradiction.
- Suppose that $a_1, a_2, \ldots, a_n$ are all less than A, where A is the average of these numbers.
- Then $a_1 + a_2 + \ldots + a_n < \mathrm{nA}$
- Divide both sides by n shows that $A = (a_1 + a_2 + \ldots + a_n)/\mathrm{n} < A$,

which is a contradiction.

3. Prove that if n is an integer, these four statements are equivalent: (1) n is even, (2) n+1 is odd, (3) 3n+1 is odd, (4) 3n is even.

Proof:

- Let us show that four statements are equivalent by showing that (1) implies (2), (2) implies (3), (3) implies (4), (4) implies (1).
- First, assume n is even. Hence there exists an integer k such that $n = 2k$ Then $n + 1 = 2k + 1$, so $n + 1$ is odd. This shows (1) implies (2).
- Second, assume $n + 1$ is odd. Hence there exists an integer k such that $n + 1 = 2k + 1$. Then 3n+1 $= 2n + (n + 1) = 2\mathrm{n} + 2\mathrm{k} + 1 = 2(\mathrm{n} + \mathrm{k}) + 1$, thus 3$n$+1 is odd and (2) implies (3).
- Third, suppose 3n+1 is odd. Hence there exists an integer k such that 3n+1 = $2k + 1$. Then $3n = (2k + 1) - 1 = 2k$, so $3n$ is even. (3) implies (4)
- Finally, suppose that n is not even. Then n is odd, so there exists an integer k such that $n = 2k$+1. $3n = 3(2k + 1) = 6k + 3 = 2(3k + 1) + 1$, so $3n$ is odd. By contraposition, (4) implies (1).

More Exercises in the Textbook

- Section 1.6
  - 3, 5, 7, 13, 15, 17-20, 23-29, 33, 34-35*
- Section 1.7
  - 13, 14, 16, 19-25, 34, 35, 38-40
- Section 1.8
  - 3, 4, 7, 15, 29-32
- Section 5.1
  - 3-17, 18, 19
- Section 5.2
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