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Lab 1 Worksheet

CELEN087

Instructions:

1. Create a blank LaTeX document called **Lab1Practice** to complete following questions.
2. Typeset the mathematical symbols/expressions/equations using suitable inline/display mode. You may use appropriate commands to create space for seperating questions.
3. Remember to save your LaTeX source code file **Lab1Practice.tex** before leaving the lab room.

Examples:

Typeset the following expressions/equations:

1. $$\tan x + \cot^2 x + \sin 2x + \cos^{-1} x$$
2. $$f'(x) = \frac{x^3 - 5x^{10}}{\sqrt{4 - x^2}}$$
3. $$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$
4. $$\lim_{x \to \infty} \left(\frac{\sin x}{x} \right)^2 = 0$$
5. The derivative of $u(x) \cdot v(x)$ is given by
$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$
6. If f is continuous on the interval $[a, b]$ and F is any antiderivative of f, then
$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Practice Questions:

1. Typeset the following expressions/equations:

(i) $e^{4-x} = 10$ (ii) $-1 \leq \sin\theta \leq 1$ (iii) $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

(iv) $\alpha = k\pi + \frac{\pi}{2}$ (v) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ (vi) $\ln \frac{1}{\sqrt{e}} + \log_2 16 = \frac{7}{2}$

(vii) $y' = \frac{dy}{dx}$ $(viii)$ $\sum n = \frac{n(n+1)}{2}$ (ix) $\int x^2\, dx = -\frac{1}{3}x^3 + C$

2. Typeset the following expressions/equations:

$$(i)\quad x_{n+1} = \frac{\sin x_n + 2 - x_n^2}{4} \qquad (ii)\quad (1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2$$

$$(iii)\quad r = 1.25\%r \Leftrightarrow \delta r = 0.0125r \qquad (vi)\quad \because x^{10} = 1024 \quad \therefore x = \pm 2$$

3. Typeset the following limit equations:

$$(i)\quad \lim_{x\to 2}\frac{x^2-4}{x-2} = 4 \qquad (ii)\quad \lim_{x\to -\infty} e^x = 0 \qquad (iii)\quad \lim_{t\to\infty}\frac{100t^{20}}{(\sqrt{e})^t} = 0$$

4. Typeset the following differential equations:

$$(i)\quad \frac{d}{dx}(\sec x) = \tan x \cdot \sec x \qquad (ii)\quad y = \sin(x^x) \Rightarrow \frac{dy}{dx} = \cos(x^x)\cdot x^x(1+\ln x)$$

5. Typeset the following integral equations:

$$(i)\quad \int f(t)\,dt = F(t) + C \qquad (ii)\quad \int_2^3 \frac{1}{x(\ln x)^2}\,dx = \frac{1}{\ln 2} - \frac{1}{\ln 3}$$

$$(iii)\quad \int_0^{\pi} \cos 3x\,dx = 0 \qquad (iv)\quad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

6. Typeset the following mathematical descriptions:

   (i) $y = \arccos x$ is the number in $[0, \pi]$ for which $\cos y = x$.

   (ii) The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1+h) - f(x_1)}{h}, \qquad h \neq 0.$$

   (iii) The series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

   converges to $\sin x$ for all x.

   (iv) $f(x, y, C) = 0$ is the general solution to the differential equation

$$F\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^n y}{dx^n}\right) = 0.$$

   (v) The limit $\lim_{x\to 0}\frac{x}{|x|}$ does not exist, because

$$\lim_{x\to 0^+}\frac{x}{|x|} = 1 \neq -1 = \lim_{x\to 0^-}\frac{x}{|x|}.$$

(Please note: Unfinished questions will be your homework after this lab session.)