COURSEWORK SUBMISSION FORM

TO THE STUDENT: Please complete the boxes and submit the form together with your essay/coursework.

Name (s)	**ID Number (s)**	**Date Handed In**
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Module Title Introduction to Algorithms		**Module Convenor** Chenfei Zhang
Coursework Title Coursework Assignment, Semester 1, 2022-2023		**Module Code** CELEN086

Compulsory

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The deadline for handing in coursework is 4pm.

Signature (s)

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Date & Time received **Late: YES / NO**	**Penalty**	**Extenuating Circumstances** **YES / NO**	**Evidence Attached** **YES / NO**

Question 1

(a)

Arg 1 = n

Arg 2 = n-1

Arg 3 = n

Arg 4 = m-1

(b)

Algorithm: sqrt(n)

Requires: One positive integer n;

Returns: One integer = $\lfloor\sqrt{n}\rfloor$

```
1.  return sqrtHelper(n, n-1)
```

Algorithm: sqrtHelper(n,m)

Requires: Two positive integers n, m;

Returns: One integer = $\lfloor\sqrt{n}\rfloor$

```
1. if m*m > n
2.      return sqrtHelper(n, m-1)
3. else
4.      return m
5. endif
```

I think the two algorithms have the same efficiency. Since the Line 1 is reversed, the if-else is also reversed. So it could not bring any change to the efficiency.

For example: sqrt(5)

For algorithm in (a):

```
return sqrtHelper(5, 4)
        4*4 <= 5     False
        return sqrtHelper(5, 3)
                3*3 <= 5     False
                return sqrtHelper(5, 2)
                        2*2 <= 5     True
```

return 2

return 2

For algorithm in (b):

return sqrtHelper(5, 4)

4*4 > 5 True

return sqrtHelper(5, 3)

3*3 > 5 True

return sqrtHelper(5, 2)

2*2 > 5 False

return 2

return 2

(c)

Algorithm: isPrime(n)

Requires: One positive integer n;

Returns: True if n is a prime number

1. return primeHelper(n,sqrt(n))

Algorithm: primeHelper(a, b)

Requires: Two positive integers a, b;

Returns: True if a is a prime number

1. if b == 1
2. return True;
3. else if a % b == 0
4. return False
5. else
6. return primeHelper(a, b-1)
7. endif

isPrime(41):

return primeHelper(41,6)

b == 1 False

a % b == 0 False

return primeHelper(41,5)

```
                b == 1        False
                a % b == 0          False
                        return primeHelper(41,4)
                        b == 1        False
                        a % b == 0          False
                                return primeHelper(41,3)
                                b == 1        False
                                a % b == 0          False
                                        return primeHelper(41,2)
                                        b == 1        False
                                        a % b == 0          False
                                                return primeHelper(41,1)
                                                b == 1 True
                                                        return True
        return True
```

Question 2

(a)

Algorithm: delete(x, L)

Requires: One existing element x, list L

Returns: List L without x

1. return deleteHelper(x, [], L)

Algorithm: deleteHelper(x, L1,L2)

Requires: One existing element x, list L1, L2

Returns: List L without x

1. if x == head(L2)
2. return cons(L1, tail(L2))
3. else
4. return deleteHelper(x, cons(L1, head(L2)), tail(L2))
5. endif

(b)

Algorithm: selectionSort(L)

Requires: list L

Returns: Sorted list L

1. return sortHelper(L, [])

Algorithm: sortHelper(L1, L2)

Requires: list L1, L2

Returns: Sorted list L

1. if tail(L1) == []
2. return cons (head(L1), L2)
3. else
4. L2 = cons (maxlist(L1), L2)
5. return sortHelper(delete(maxlist(L1)), L2)
6. endif

(c)

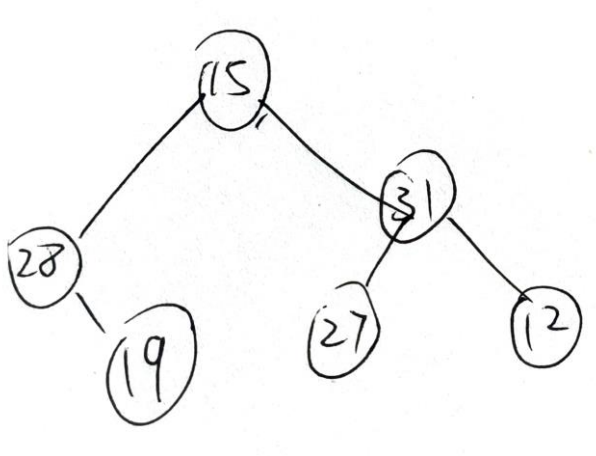
L = [4, 7, 6, 3, 5, 1]

```
return sortHelper(L, [])
        tail(L1) == []        False
        L2 = cons (7, [])
        return sortHelper([4, 6, 3, 5, 1], [7])
                tail(L1) == []        False
                L2 = cons (6, [7])
                return sortHelper([4, 3, 5, 1], [6, 7])
                        tail(L1) == []        False
                        L2 = cons (5, [6, 7])
                        return sortHelper([4, 3, 1], [5, 6, 7])
                                tail(L1) == []        False
                                L2 = cons (4, [5, 6, 7])
                                return sortHelper([3, 1], [4, 5, 6, 7])
                                        tail(L1) == []        False
                                        L2 = cons (3, [4, 5, 6, 7])
                                        return sortHelper([1], [3, 4, 5, 6, 7])
                                                tail(L1) == []        True
                                                return cons (1, [3, 4, 5, 6, 7])
```

return [1, 3, 4, 5, 6, 7]

Question 3

(a)



[19, 28, 27, 12, 31, 15]

(b)

Algorithm: average (T)

Requires: a binary tree (T)

Returns: the average of all values stored in the tree

1. return (averageList(1, traversal(T)))

Algorithm: traversal(T)

Requires: a binary tree (T), an empty list L

Returns: a list L

1. if isLeaf(T)
2. return (cons(root(T), L)
3. else
4. cons(traversal(left(T)), L)
5. cons(traversal(right(T)), L)
6. cons(root(T)), L)
7. return L
8. endif

Algorithm: averageList (m, L)

Requires: a list L, length of list m

Returns: average of list

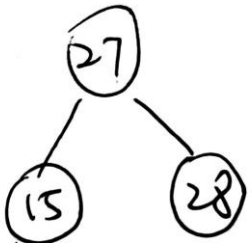
1. if tail(L) == []
2. return (head(L) / m)
3. else
4. return (averageList (m+1, cons(head(L) + head(tail(L)), tail(tail(L)))))
5. endif

(c)

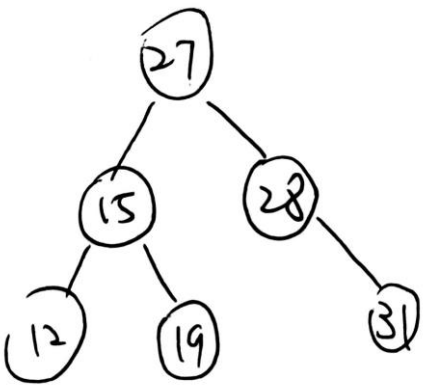
1. sort the list: [12, 15, 19, 27, 28, 31]

2. Find the middle element, and store it in root node. (27)

3. For left/right sub-lists, find the middle elements and store them in the left/right child nodes on next level.



6. repeat this process until all elements are stored in the tree.



Algorithm: count (x, T)

Requires: a binary tree T

Returns: number of elements greater than x

1. L = traversal(T)
2. return countHelper(0, L)

Algorithm: countHelper(x, L)

Requires: a list L, x

Returns: number of elements greater than x

```
1. if header(L) > x
2.     x = x+1
3. Endif
4. if tail(L) ==[]
5.     return x
6. else
7.     return countHelper(x, tail(L))
8. endif
```

Tracing:

```
return countHelper(0, [19, 28, 27, 12, 31, 15])
        19 > 18       true
                X = 1
        tail(L) ==[]  false
                return countHelper(1, [28, 27, 12, 31, 15])
                28 > 18       true
                        X = 2
                tail(L) ==[]  false
                        return countHelper(2, [27, 12, 31, 15])
                        27 > 18       true
                                X = 3
                        tail(L) ==[]  false
                                return countHelper(3, [12, 31, 15])
                                12 > 18       false
                                        X = 3
                                tail(L) ==[]  false
                                        return countHelper(3, [31, 15])
                                        12 > 18       true
                                                X = 4
                                        tail(L) ==[]  false
                                                return countHelper(4, [15])
                                                15 > 18 false
```

X = 4
tail(L) ==[] true
return 4
return 4

Question 4

(a)

Algorithm: bin2dec(L)
Requires: a list L
Returns: value in decimal number system

1. return decHelper(length(L) - 1, L, 0)

Algorithm: decHelper(x, L, n)
Requires:x, n, a list L
Returns: value n in decimal number system

1. if x == 0
2. return n + head(L)
3. else
4. return decHelper(x-1, tail(L), n + head(L)*(2^n))
5. endif

(b)

return decHelper(5, [1, 1, 0, 1, 0, 1], 0)
X == 0 false
return decHelper(4, [1, 0, 1, 0, 1], 32)
X == 0 false
return decHelper(3, [0, 1, 0, 1], 48)
X == 0 false
return decHelper(2, [1, 0, 1], 48)
X == 0 false
return decHelper(1, [0, 1], 52)
X == 0 false
return decHelper(0, [1], 52)

X == 0 true

return 53

return 53

(c)

Algorithm: dec2bin(n)

Requires: a number n

Returns: a list L

1. let a = largestPower(n, 1)
2. return reverse(binHelper(a, [], n))

Algorithm: binHelper(a, L, n)

Requires: a number n, a, L

Returns: a list L

1. if a == 0 && n == 1
2. return cons(1, L)
3. else if a == 0 && n == 0
4. return cons(0, L)
5. else if 2 ^ a <= n
6. return binHelper(a-1, cons(1, L), n – 2^(a–1))
7. else
8. return binHelper(a-1, cons(0, L), n)
9. endif

Algorithm: largestPower(n, a)

Requires: numbers n, a

Returns: largest Power

1. if 2 ^ a < n
2. return largestPower(n, a+1)
3. else
4. return a – 1
5. endif

Tracing:

```
a = 4
return reverse(binHelper(4, [], 19))
        a == 0 && n == 1          false
        a == 0 && n == 0          false
        2 ^ 4 <= 19         true
                return binHelper(3, [1], 3)
                a == 0 && n == 1          false
                a == 0 && n == 0          false
                2 ^ 3 <= 3          false
                        return binHelper(2, [0, 1], 3)
                        a == 0 && n == 1          false
                        a == 0 && n == 0          false
                        2 ^ 2 <= 3          false
                                return binHelper(1, [0, 0, 1], 3)
                                a == 0 && n == 1          false
                                a == 0 && n == 0          false
                                2 ^ 1 <= 3          true
                                        return binHelper(0, [1, 0, 0, 1], 1)
                                        a == 0 && n == 1          true
                                                return [1, 1, 0, 0, 1]
return [1, 0, 0, 1, 1]
```