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1. Understand the basic concepts
   a. List all the binary relations on the set {0,1}.
   b. List the reflexive relations on the set {0,1}.
   c. List the irreflexive relations on the set {0,1}.
   d. List the symmetric relations on the set {0,1}.
   e. List the transitive relations on the set {0,1}.
   f. List the antisymmetric relations on the set {0,1}.
   g. List the asymmetric relations on the set {0,1}.
   h. List the relations on the set {0,1} that are reflexive and symmetric.
   i. List the relations on the set {0,1} that are neither reflexive nor irreflexive.

2. In the questions below suppose R and S are relations on $\{a, b, c, d\}$, where $R = \{(a, b), (a, d), (b, c), (c, c), (d, a)\}$ and $S = \{(a, c), (b, d), (d, a)\}$.
   a. Construct R^2 .
   b. Construct R^3 .
   c. Construct S^2
   d. Construct S^3
   e. Construct $R \circ S$.
   f. Construct $S \circ R$.

3. Proof
   a. Show that the relation R on a set A is symmetric if and only if $R = R^{-1}$, where R^{-1} is the inverse relation

   b. Show that the relation R on a set A is reflexive if and only if the inverse relation $R = R^{-1}$ is reflexive

   c. Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all positive integers n.

   d. Let R be a reflexive relation on a set A. Show that R^n is reflexive for all positive integers n.

1. How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers k, k + 1, k + 2, in the correct order
   a. where these consecutive integers can perhaps be separated by other integers in the permutation?
   b. where they are in consecutive positions in the permutation?

2. A circular r-permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

   a. Find a formula for the number of circular r-permutations of n people.
   b. Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

3. There are six runners in the 100-yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the

runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)

4. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

5. How many numbers must be selected from the set {1, 2, 3, 4, 5, 6} to guarantee that at least one pair of these numbers add up to 7?
6. How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16?

7. Use mathematical induction to prove the sum rule for m tasks from the sum rule for two tasks.
8. Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters ∗, >, <, !, +, and =.
    a. How many different passwords are available for this computer system?
    b. How many of these passwords contain at least one occurrence of at least one of the six special characters?
    c. Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password