$$\frac{1}{x^2+3x+2} \quad \begin{array}{l} \deg(p(x)) = 1 \\ \deg(q(x)) = 2 \end{array}$$

$$\therefore \deg(p(x)) < \deg(q(x))$$

$$\frac{x^2+5}{x^2+1} \quad \deg(p(x)) = \deg(q(x))$$

$$\frac{1}{x^2+1} \quad \times$$

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} \quad \checkmark$$

$(x+a)^2$ Repeated linear factor

$$f(x) = \frac{p(x)}{q(x)} = \frac{1}{(x+a)(x+b)}$$

$$= \frac{A}{x+a} + \frac{B}{x+b}$$

(x^2+a) Non-repeated Quad. factor

$(x^2+a)^2$ Repeated Quad. factor

$$\frac{1}{(x+a)(x^2+b)} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b}$$

$$\frac{1}{(x^2+a)(x+b)} = \frac{Ax+B}{x^2+a} + \frac{C}{x+b)}$$

$$\frac{3x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$\frac{A(x-2)+B(x-1)}{\cancel{(x-1)}\cancel{(x-2)}} = \frac{3x}{\cancel{(x-1)}\cancel{(x-2)}}$$

STEP 1

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

STEP 2

$$\Rightarrow A(x+2) + B(x+1) = 1$$

$$x=-1 \Rightarrow A(-1+2) = 1 \Rightarrow \boxed{A=1}$$

$$x=-2 \Rightarrow B(-2+1) = 1 \Rightarrow \boxed{B=-1}$$

STEP 3

$$\therefore \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

STEP 1

$$\frac{3}{(x+1)(x^2+2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2}$$

STEP 2

$$A(x^2+2) + (Bx+C)(x+1) = 3$$

$$x=-1 \Rightarrow A(1+2) = 3 \Rightarrow \boxed{A=1}$$

$$Ax^2+2A + Bx^2+Bx+Cx+C = 3$$

Equate coefficients of x^2

$$A+B=0 \Rightarrow \boxed{B=-1}$$

Equate coefficient of x

$$B+C=0 \Rightarrow \boxed{C=1}$$

STEP 3

$$\therefore \frac{3}{(x+1)(x^2+2)} = \frac{1}{x+1} + \frac{(-x+1)}{x^2+2}$$

OR

$$\frac{3}{(x+1)(x^2+2)} = \frac{1}{x+1} - \frac{x}{x^2+2} + \frac{1}{x^2+2}$$

$$\frac{1}{(x+a)^2(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}$$

$$\frac{1}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

STEP 1

$$\frac{9}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$\frac{9}{\cancel{(x-1)^2(x+2)}} = \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{\cancel{(x-1)^2(x+2)}}$$

STEP 2

$$A(x-1)(x+2) + B(x+2) + C(x-1)^2 = 9$$

$$x = 1 \Rightarrow B(1+2) = 9 \Rightarrow \boxed{B = 3}$$

$$x = -2 \Rightarrow C(-2-1)^2 = 9 \Rightarrow \boxed{C = 1}$$

$$\Rightarrow A(x^2 + x - 2) + B(x+2) + C(x^2 - 2x + 1) = 9$$

Equating coefficients of (x^2)

$$\Rightarrow A + C = 0 \Rightarrow \boxed{A = -1}$$

STEP 3

$$\therefore \frac{9}{(x-1)^2(x+2)} = \frac{-1}{x-1} + \frac{3}{(x-1)^2} + \frac{1}{(x+2)}$$

Sequence

$$f : \mathbb{N} \to A$$

If $A = \mathbb{R}$ it is a sequence of real nos.

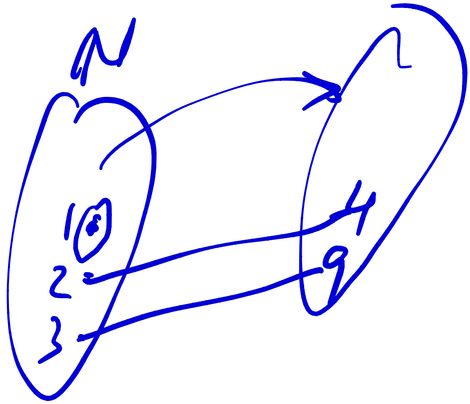
If $A = \mathbb{N}$, it - - - - natural nos.

If $A = \mathbb{Z}$, it - - - - integers.

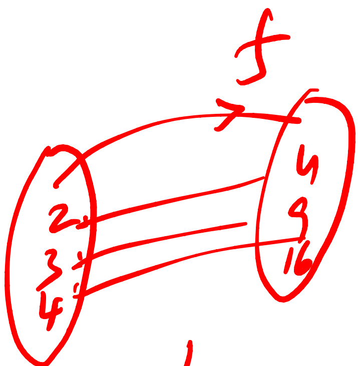
$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\cancel{A} \quad \mathbb{N} = \{0, 1, 2, \ldots\}$$

$$2^n \; ; \; n = 0, 1, 2 \ldots$$



$4, 9, 16, \ldots \rightarrow (n+1)^2 \; ; \; n = 1, 2, \ldots$



$\{a_n\}_{n=1}^{k}$

$\{a_n\}_1^{\infty}$

$\underbrace{a_1 , a_2 . \ldots a_k}_{\text{finite terms}}$

$a_1, a_2, \ldots$

Infinite sequence

$$\underbrace{\quad 3 \qquad 3 \qquad 3 \quad} \rightarrow \text{Diff. is constant } (d)$$

$$2, \; 5, \; 8, \; 11, \; \ldots$$

$$a_1, \; a_2, \; a_3 \; \ldots$$

$$a \quad a+d \quad a+2d \; \ldots \; \underline{a+(n-1)d}$$

a_n (n^{th} term of A.P.)

$$\therefore a_n = a + (n-1)d$$

first term

constant difference

Example

(7th) Seventh term = 19

$\Rightarrow a + 6d = 19$ — (1)

(18th) Eighteenth term = 41

$\Rightarrow a + 17d = 41$ — (2)

$\therefore$ (2) − (1) gives

$11d = 22 \Rightarrow \boxed{d = 2}$

$\therefore \; a + 6d = 19 \Rightarrow a = 19 - 12$

$\therefore \boxed{a = 7}$

$\therefore$ The first 7 terms are :

7, 9, 11, 13, 15, 17, 19.

$3 \quad 3 \quad 3 \quad 3$ ⇒ Ratio is constant = 3 (r)

$4, \; 12, \; 36, \; 108, \; 324$

$a, \; ar, \; ar^2, \; ar^3, \; \ldots \; a \cdot r^{n-1}$

n^{th} term

$$a_n = a \cdot r^{n-1}$$

a = First term

r = constant ratio

3^{rd} term $= 18 \Rightarrow a \cdot r^2 = 18$ —(1)

7^{th} term $= 1458 \Rightarrow ar^6 = 1458$ —(2)

(2) ÷ (1) gives

$$\frac{ar^6}{ar^2} = \frac{1458}{18}$$

$$\Rightarrow r^4 = 81 \Rightarrow r^2 = 9$$

$$\Rightarrow \boxed{r = \pm 3}$$

$$\therefore ar^2 = 18 \Rightarrow a = \frac{18}{9} \Rightarrow \boxed{a = 2}$$

∴ The GP is $\begin{cases} 2, 6, 18, 54, 162, 486, 1458 \\ \text{OR} \\ 2, -3, 18, -54, 162, -486, -1458 \end{cases}$

$2, 5, 8, 11, \ldots$ are in A.P.

$\Rightarrow \frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$ are in H.P.

Generalizing

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

are in H.P.

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ are in H.P.

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ are in G.P.

Fibonacci sequence

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ \ldots$$

$$f(0)=0$$

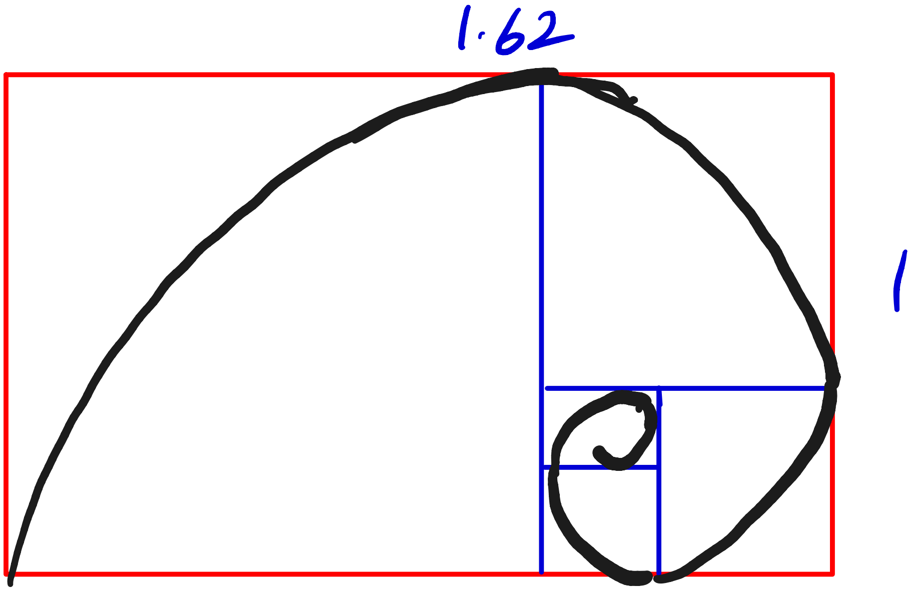
$$f(1)=1$$

$$f(n)=f(n-1)+f(n-2)\ ;\ n>1,\ n\in\mathbb{N}$$

$$f(1)=0$$

$$f(2)=1$$

$$f(n)=f(n-1)+f(n-2)\ ;\ n>2\ \ n\in\mathbb{N}.$$



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