CELEN036 Lecture 11 (online) - Dec 2022

$$\{ S_1, S_2, \ldots\ldots S_n \ldots\ldots \}$$

SERIES

$$\sum_{1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

Counter

Sequence

$\{a_1 \quad a_2 \ldots \ldots a_n \ldots \ldots\}$

$\Downarrow$

Corresponding

SERIES $\{S_1, S_2 \ldots \ldots S_n, \ldots \ldots\}$

Where $S_1 = a_1$

$S_2 = a_1 + a_2$

$\vdots$

$S_n = a_1 + \ldots + a_n$

$$S_n = \sum_{1}^{n} a_k = \cancel{a_1} + \cancel{a_2} + \cancel{\cdots} + a_n$$

$$S_{n-1} = \sum_{1}^{n-1} a_k = \cancel{a_1} + \cancel{a_2} + \cdots + \cancel{a_{n-1}}$$

diff.

$$S_n - S_{n-1} = a_n$$

$$\therefore \boxed{a_n = S_n - S_{n-1}}$$

$$a,\ a+d,\ a+2d,\ a+3d, \ldots$$

$$\ldots a+(n-1)d$$

$$\therefore S_n = a+(a+d)+(a+2d)+\ldots + [a+(n-1)d]$$

$S_n = \begin{cases} \text{sum of first } n \text{ terms of A.P.} \\ n^{th} \text{ term of Arithmetic series} \end{cases}$

Example:

Eight term $= 8^{th}$ term $= a_8$

$= a + 7d = 23$ — ①

Twenty fourth term $= a_{24}$

$= a + 23d = 103$ — ②

② − ① gives

$16d = 80 \Rightarrow d = 5$

$\therefore\ a = 23 - 7d = 23 - 35$

$\Rightarrow a = -12$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\therefore S_{30} = \frac{30}{2}\left[2a+(30-1)d\right]$$

$$= 15\left[-24+29 \times 5\right]$$

$$= \underline{1815}.$$

G. sequence is

$$a, ar, ar^2, \ldots\ldots ar^{n-1}, \ldots$$

$$\therefore S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

If $r = 1$

$$\Rightarrow S_n = a + a + a + \ldots\ldots + a$$
(n times)

$$\underline{S_n = na}$$

Ex.

$r = \frac{1}{3}$, $S_4 = 150$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$\Rightarrow 150 = a\left[\frac{1-\left(\frac{1}{3}\right)^4}{1-\left(\frac{1}{3}\right)}\right]$$

When $n=4$

$$\Rightarrow 150 = \frac{a\left(\frac{81-1}{81}\right)}{\left(\frac{3-1}{3}\right)}$$

$$= a \cdot \frac{\cancel{80}^{\,40}}{\cancel{81}_{\,27}} \cdot \frac{\cancel{3}}{\cancel{2}}$$

$$\therefore a = \frac{150 \cdot 27}{40} = \frac{405}{4}.$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

What if $|r|<1$

$\Rightarrow |r|^n \to 0$ as $n\to\infty$

n tends to infinity

$$\therefore \lim_{n\to\infty} S_n = \frac{a}{1-r}\left[1-\lim_{n\to\infty} r^n\right]$$

$$= \frac{a}{1-r}(1-0)$$

$$\Downarrow$$

$$S = \frac{a}{1-r}$$

Example

$\frac{1}{5} \qquad \frac{1}{5} \qquad \frac{1}{5}$

$$5 + 1 + \frac{1}{5} + \frac{1}{25} + \text{----}$$

$\therefore$ This is an Infinite G.Series

with $r = \frac{1}{5} < 1$

$$S = \frac{a}{1-r} = \frac{5}{1-\left(\frac{1}{5}\right)} = \frac{25}{4}$$

$$= 6.25$$

Ex. 2

Vulgar fraction $\rightarrow \left(p/q\right)$

$p, q \in \mathbb{Z},$

$q \neq 0.$

Express $1.2222\ldots$

OR $1.2\dot{2}$ OR $1.\overline{22}$

as a vulgar fraction.

$$1.2222\ldots$$

$$= 1 + 0.2 \xleftarrow{1/10} + 0.02 \xleftarrow{1/10} + 0.002 + \ldots$$

$$= 1 + \left(\frac{a}{1-r}\right)$$

where $a = 0.2$
$r = 1/10$

$$= 1 + \frac{(2/10)}{1 - 1/10} = 1 + \frac{2}{9} = \boxed{\frac{11}{9}}$$

$$1.\overline{123} = 1.123123123123\ldots$$

$$= 1 + 0.123 \xleftarrow{0.001} + 0.000123 \xleftarrow{0.001} + 0.000000123 + \ldots$$

$$= 1 + \left(\frac{a}{1-r}\right)$$ where $a = 0.123$, $r = 0.001$

$$= 1 + \left(\frac{123/1000}{1 - 1/1000}\right)$$

$$= 1 + \frac{123}{999} = \frac{1122}{999}$$

$$\sum_{1}^{n} k \quad \text{or} \quad \sum_{1}^{n} n \quad \text{or} \quad \sum n = \frac{n(n+1)}{2}$$

$$\sum n^3 = \frac{n^2(n+1)^2}{4}$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

$$= \left[\sum n\right]^2$$

$$\text{Sum} = \sum n^{th} \text{ term}$$

$$\underset{\uparrow}{1} + (\underbrace{1+2}) + (\underbrace{1+2+3}) + \cdots$$

1^{st} term 2^{nd} term 3^{rd} term

$$\text{Sum} = \sum n^{th} \text{ term}$$

$$= \sum (1+2+3+\cdots+n)$$

$$= \sum (\sum n)$$

$$= \sum \left[\frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2}\left[\sum n^2 + \sum n\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}\left[(2n+1) + 3\right]$$

$$= \frac{n(n+1)(2n+4)}{12}$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$\sum_{1}^{n} \frac{r}{(r+1)!}$$

$$= \sum_{1}^{n} \left[\frac{1}{r!} - \frac{1}{(r+1)!} \right]$$