Tutorial 1 - Matrices

COMP1046 - Maths for Computer Scientists

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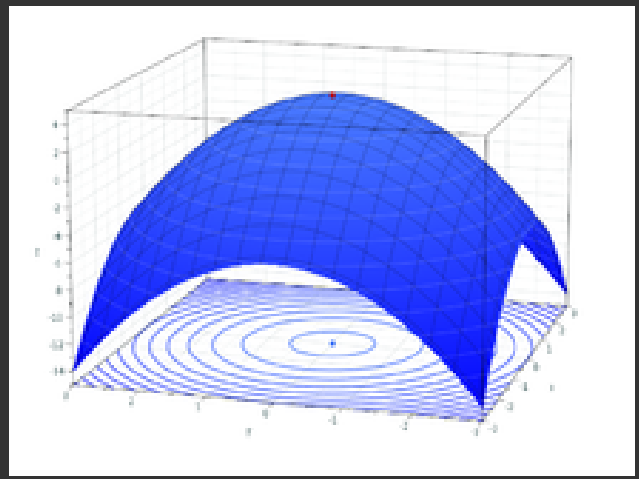

Exercise

Use course material in Lectures 2 to 4 to answer these questions.

Let $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 0 & -2 \\ 1 & 3 & 1 \end{pmatrix}$.

1. Based on $\mathbf{A}$, what is $\mathbf{a_2}$ and $\mathbf{a^1}$?
2. Compute $2\mathbf{B} + \mathbf{A}^T$.
3. Suppose $\mathbf{A} + \mathbf{D} = \mathbf{0}$. Compute $\mathbf{D}$.
4. Compute $\mathbf{AB}$.
5. Compute $\mathbf{BA}$.

Exercise

Let $\mathbf{C} = \begin{pmatrix} 2 & -1 & 0 & 2 \\ 4 & 0 & 1 & -1 \\ 1 & 0 & 2 & -2 \end{pmatrix}$

6. What is the submatrix of **C** when the 1st row and 2nd and 3rd columns are cancelled?
7. What is the minor for this submatrix?
8. Compute the complement minor $M_{1,3}$ and cofactor $A_{1,3}$ of **AB**.
9. Compute det(**BA**) and $(\mathbf{BA})^{-1}$.
10. Confirm that your answer is correct by taking the product of **BA** with its inverse.

Exercise

Let $\mathbf{D} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 4 & 0 \\ -4 & 9 & -1 \end{pmatrix}$ and $\mathbf{E} = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 6 & -2 \\ -3 & -9 & 3 \end{pmatrix}$.

11. Compute $\det(\mathbf{D})$.
12. Compute $\mathbf{D}^{-1}$, or explain if it cannot be computed.
13. Compute the ranks of $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{E}$.
14. Suppose that matrix $\mathbf{X}$ has an inverse $\mathbf{X}^{-1}$. Prove that the inverse of $\mathbf{X}^{\mathrm{T}}$ is $\left(\mathbf{X}^{-1}\right)^{\mathrm{T}}$.
    Hint: You should use a property of the matrix product from Lecture 2.