# AE1MCS: Mathematics for Computer Scientists

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# Reading

Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Edition, 2013.

- Chapter 7, Section 7.1 An Introduction to Discrete Probability

# Discrete Probability

- Combination and probability theory share common origins (analyzing gambling games).
- The theory of probability now plays an essential role in a wide variety of disciplines (e.g. the study of genetics).
- In computer science,
  - Probability theory plays an important role in the study of the complexity of algorithms.
  - Probabilistic algorithms vs. deterministic algorithms.
  - Probability theory can help us answer questions that involve uncertainty.
  - ...

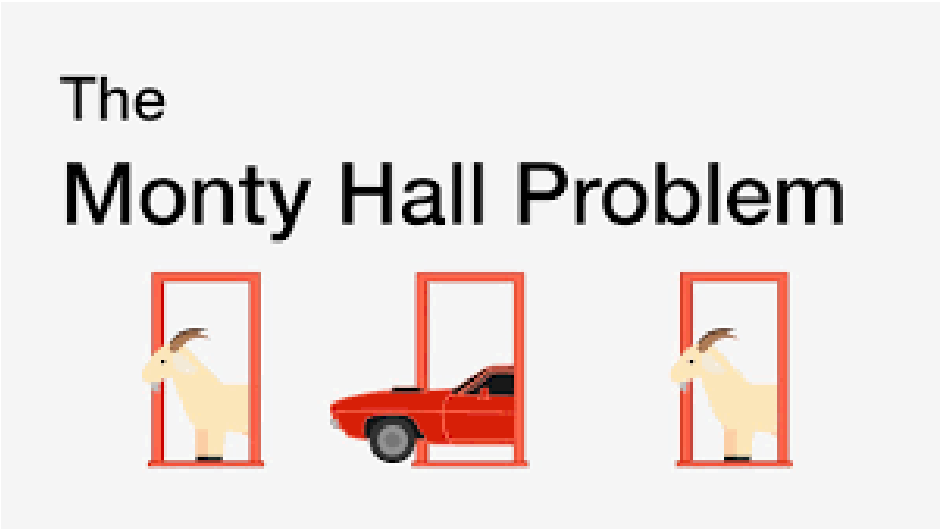
# Content

- Probability of an Event
- Probabilities of Complements and Unions of Events

# Monty Hall Three-Door Puzzle

You are asked to select one of three doors to open; the large prize is behind one of the three doors and the other two doors are losers. Once you select a door, the game show host, who knows what is be behind each door:

- whether or not you selected the winning door, he opens one of the other two doors that he knows is a losing door (selecting at random if both are losing doors).
- Then he asks you whether you would like to switch doors.



# Finite Probability

Laplace's definition of the probability of an event with **finitely many, equally likely, possible outcomes** is as follows.

## Definition

If $S$ is a finite nonempty sample space of equally likely outcomes, and $E$ is an event, that is, a subset of $S$, then the probability of $E$ is

$$p(E) = \frac{|E|}{|S|}$$

- An **experiment** is a procedure that yields one of a given set of possible outcomes.
- The **sample space** of the experiment is the set of possible outcomes.
- An **event** is a subset of the sample space.

# Probability of an Event

In the eighteenth century, the French mathematician Laplace, who also studied gambling, defined **the probability of an event as the number of successful outcomes divided by the number of possible outcomes**.

# Example: Poker 1

Find the probability that a hand of five cards in poker contains four cards of one kind.

- A deck of cards contains 52 cards.
- There are 13 different kinds of cards, with four cards of each kind.
- These kinds are twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, kings, and aces.
- There are 4 suits: spades, clubs, hearts, and diamonds, each containing 13 cards.

# Example: Poker 2

What is the probability that a poker hand contains a full house, that is, three of one kind and two of another kind?

# Probabilities of Complements and Unions of Events

**Theorem**

*Let $E$ be an event in a sample space $S$. The probability of the event $\overline{E} = S - E$, the complementary event of $E$, is given by*

$$p(\overline{E}) = 1 - p(E)$$

**Theorem**

*Let $E_1$ and $E_2$ be events in the sample space $S$. Then*

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

How to prove them?

# Example

- A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0?
- What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5?

# Example 1

A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0?

# Example 2

What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by *either* 2 or 5?

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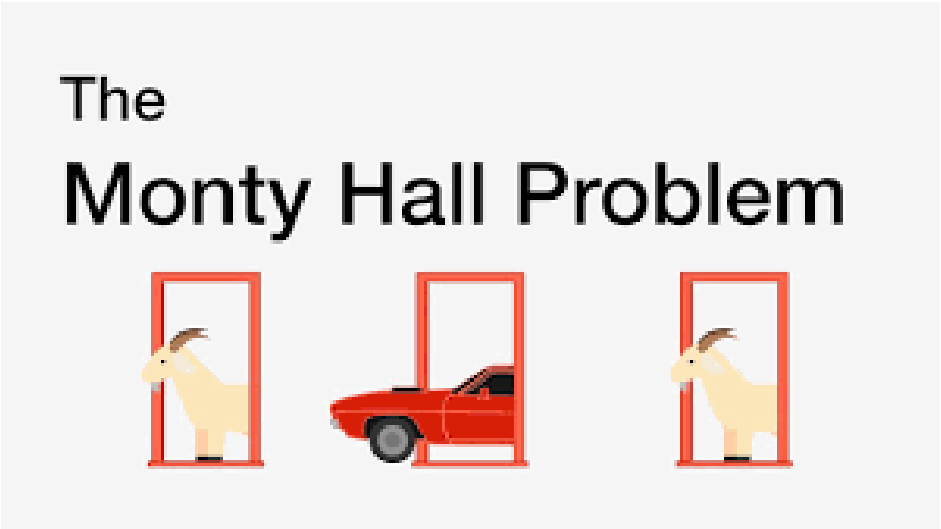

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