Useful information about Final exam paper

Structure of Final exam paper

Question No.	Marks	Topics covered
1	15	Functions, Modulus inequality, Quadratic, Logarithmic and exponential functions
2	15	Trigonometry, Remainder and Factor Theorems, Synthetic Division
3	15	Binomial Theorem, Generalised Binomial Theorem and applications, Numerical methods, Matrices
4	15	Partial fractions, Complex Numbers, Sequence and Series, Power series, method of differences

Notes:

1) This is a take-home-exam, which you must complete in 24-hours' time.
2) The exam paper will be available on module Moodle page at 9.30 am. on 6th January 2023.
3) Deadline for submission is: 9.30 am on 7th January 2023 (China Time).
4) Marks will be given for the best 3 answers.
(i.e. you can attempt **ANY 3 out of 4** questions).
5) Total marks obtained will then be upscaled to 70%.
6) The final score will be calculated as: **Mid-sem exam (30%) + Final exam (70%)**.

Useful information about Final exam paper

Instructions:

1) You should write all **necessary steps** in your solutions.
2) It is expected that you will only use CELE approved calculator ($fx-82$ series) for this exam. You will lose marks if because of use of other models of calculators, your numerical answer differs from our standardized marking scheme.
3) Formula Sheet will be attached to the question paper.
4) Please write your answers on a blank piece of paper. Alternatively, you may also use iPad/Tablet to write your answers.
5) Please complete the coursework submission form (downloadable from module Moodle page) and create a **single** PDF file of all your answers to exam questions with completed submission form on the top.
6) Name your file as: Your Student ID number_N036Final. For example: 20519999_N036Final.
7) Please upload this PDF file to submission drop-box on module Moodle page (available on the top of the Moodle page). Module Convenor will also email the link to the submission drop-box.
8) No excuses such as problems with internet connectivity, etc. will be entertained; so, you are suggested to submit your working well in advance before the deadline. Should you have any difficulty in uploading your file, please contact Module Convenor (Bamidele.Akinwolemiwa2@nottingham.edu.cn) **immediately** and follow their instructions.
9) This work must be completed on your own. Plagiarism and collusion are regarded as very serious academic offences and will be treated as such.

SEM

Please take a few minutes to complete the Student Evaluation of Module

- This is an evaluation of the module CELEN036
- There are 5 questions and opportunity for you to give some comments
- Scan the QR code below, and select the module CELEN036

The SEM does not require a PIN

Seminar 10

In this seminar you will study:

- The method of partial fractions
- Arithmetic progressions (AP)
- Geometric progressions (GP)

The method of partial fractions

Non-repeated linear factors

$$\frac{1}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Non-repeated quadratic factors

$$\frac{1}{(x^2+a)(x+b)} = \frac{Ax+B}{x^2+a} + \frac{C}{x+b}$$

Repeated linear factors

$$\frac{1}{(x+a)^2(x+b)} = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{x+b}$$

In all these types, the constants A and B or A, B and C are to be determined.

The method of partial fractions

Non repeated linear factors

$$\frac{1}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\Rightarrow\ A\,(x+b) + B\,(x+a) = 1$$

Put $x = -a$ to find the value of A

and then

put $x = -b$ to find the value of B.

Non repeated quadratic factor

$$\frac{1}{(x^2+a)(x+b)} = \frac{Ax+B}{(x^2+a)} + \frac{C}{(x+b)}$$

$$\Rightarrow\ (Ax+B)\,(x+b) + C\,(x^2+a) = 1$$

Put $x = -b$ to find the value of C

and then

equate the terms in x^2 or x

or constants, to find A and B.

The method of partial fractions

Example: Express $\dfrac{x^2+1}{(x-1)^2(x+1)}$ as a sum of partial fractions.

This is in the form of a repeated linear factor:

$$\therefore \quad \frac{x^2+1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

Express the following rational fractions as a sum of partial fractions

1. $\dfrac{9x}{(x+5)(x-4)}$

Answer: $\dfrac{9x}{(x+5)(x-4)} = \dfrac{5}{x+5} + \dfrac{4}{x-4}$

2. $\dfrac{17}{(2x-3)(x+7)}$

Answer: $\dfrac{17}{(2x-3)(x+7)} = \dfrac{2}{2x-3} - \dfrac{1}{x+7}$

3. $\dfrac{17}{(3x-2)(x+5)}$

Answer: $\dfrac{17}{(3x-2)(x+5)} = \dfrac{3}{3x-2} - \dfrac{1}{x+5}$

4. $\dfrac{7x}{(x-2)(x+5)}$

Answer: $\dfrac{7x}{(x-2)(x+5)} = \dfrac{2}{x-2} + \dfrac{5}{x+5}$

Express the following rational fractions as a sum of partial fractions

1. $\dfrac{21}{(x^2+5)(x-4)}$

Answer: $\dfrac{21}{(x^2+5)(x-4)} = \dfrac{-x-4}{x^2+5} + \dfrac{1}{x-4}$

2. $\dfrac{100}{(x-3)^2(x+7)}$

Answer:
$\dfrac{100}{(x-3)^2(x+7)} = \dfrac{1}{x+7} - \dfrac{1}{x-3} + \dfrac{10}{(x-3)^2}$

3. $\dfrac{1}{(x-2)(x^2-5)}$

Answer: $\dfrac{1}{(x-2)(x^2-5)} = \dfrac{x+2}{x^2-5} - \dfrac{1}{x-2}$

4. $\dfrac{49}{(x-2)(x+5)^2}$

Answer:
$\dfrac{49}{(x-2)(x+5)^2} = \dfrac{1}{x-2} - \dfrac{1}{x+5} - \dfrac{7}{(x+5)^2}$

Arithmetic progressions (AP)

An arithmetic sequence (arithmetic progression) is given by

$$\underset{\downarrow}{a},\quad \underset{\downarrow}{a+d},\quad \underset{\downarrow}{a+2d},\quad \underset{\downarrow}{a+3d},\quad \ldots\ldots\ldots,\quad \underset{\downarrow}{a+(n-1)d}$$

1^{st} Term, 2^{nd} Term, 3^{rd} Term, 4^{th} Term, ……, n^{th} Term

$\therefore$ the n^{th} term of an A.P. is $\boxed{a_n = a + (n-1)d}$

a is the first term of an A.P.

d is the common difference

Arithmetic progressions (AP)

Example: For an AP, the third term is 8 and the sixteenth term is 47.
Find the first term a and the common difference d. Hence, write the first seven terms of the AP.

Solution:

$$\text{Third term is } 8 \quad \Rightarrow \quad a + 2d = 8 \qquad (1)$$

$$\text{Sixteenth term is } 47 \quad \Rightarrow \quad a + 15d = 47 \qquad (2)$$

$(2) - (1)$ gives $39 = 13d \;\; \therefore \;\; d = 3$

From (1) $a + 2 \times 3 = 8 \;\; \therefore \;\; a = 2$

Thus, $a = 2$, and $d = 3$.

$\Rightarrow$ The first seven terms of the AP are: $2, 5, 8, 11, 14, 17, 20 \ldots$

Arithmetic progressions (AP)

1. Find the twelfth term of the AP:

   $2, 6, 10, \ldots$

   Answer: 46

2. In the AP: $2, 6, 10, \ldots$

   what term is the number 106.

   Answer: twenty-seventh, i.e. a_{27}

3. Find the fourteenth term of the AP:

   $3, \frac{7}{3}, \frac{5}{3}, \ldots$

   Answer: $-\frac{17}{3}$

4. Find the tenth term of the AP:

   $2345.78,\ 2967.54,\ 3589.30, \ldots$

   Answer: 7941.62

Arithmetic progressions (AP)

1. For an AP, the sixth term is 20 and the eleventh term is 40.
   Find the fifteenth term.

   Answer: 56

2. For an AP, the seventh term is 20 and the twelfth term is 40.
   Find the eighteenth term.

   Answer: 64

3. For an AP, the ninth term is 38 and the nineteenth term is 138.
   Find the twentieth term.

   Answer: 148

4. For an AP, the fourth term is 10 and the fourteenth term is 40.
   Find the tenth term.

   Answer: 28

Geometric progressions (GP)

A geometric sequence (Geometric progression) is given by

$$a, \quad ar, \quad ar^2, \quad ar^3, \quad \dots \dots \dots, \quad ar^{n-1}$$

1^{st} Term, 2^{nd} Term, 3^{rd} Term, 4^{th} Term, n^{th} Term

$\therefore$ the n^{th} term of the G.P. is $\boxed{a_n = ar^{n-1}}$

a is the first term of the G.P.

r is the common ratio.

Geometric progressions (GP)

Example: For a GP, the third term is 400 and the seventh term is $250,000$. Find the first term a and the common ratio r. Hence, write the first seven terms of the GP.

Solution:

$$\text{Third term is } 400 \quad \Rightarrow \quad ar^2 = 400 \qquad (1)$$

$$\text{Seventh term is } 250,000 \quad \Rightarrow \quad ar^6 = 250,000 \qquad (2)$$

$$(2) \div (1) \text{ gives } r^4 = 625 \Rightarrow (r^2)^2 = (25)^2$$

$$\Rightarrow \; r^2 = 25$$

$$\therefore \; r = \pm 5$$

$$\text{From } (1) \; ar^2 = 400 \Rightarrow a \times 25 = 400$$

$$\therefore \; a = 16$$

$$\text{Thus, } \quad a = 16, \quad \text{and} \quad r = \pm 5.$$

Geometric progressions (GP)

Example: For a GP, the third term is 400 and the seventh term is $250,000$.
Find the first term a and the common ratio r. Hence, write the first seven terms of the GP.

Solution:

$\Rightarrow$ with $r = 5$ the first seven terms of the GP are:

$$16,\ 80,\ 400,\ 2000,\ 10,000,\ 50,000,\ 250,000\ldots$$

$\Rightarrow$ with $r = -5$ the first seven terms of the GP are:

$$16,\ -80,\ 400,\ -2000,\ 10,000,\ -50,000,\ 250,000\ldots$$

Geometric progressions (GP)

1. Find the seventh term of the GP:

   $2,\ 4,\ 8,\ 16, \ldots$

   Answer: 128

2. Find the ninth term of the GP:

   $2,\ 2\sqrt{3},\ 6, \ldots$

   Answer: 162

3. Find the fifth term of the GP:

   $2,\ -10,\ 50,\ -250 \ldots$

   Answer: 1250

4. Find the fifth term of the GP:

   $1000,\ 1060,\ 1123.60, \ldots$

   Answer: 1262.48

Geometric progressions (GP)

1. For a GP, the second term is 3402 and the seventh term is 14. Find a and r.

Answer: $a = 10206, \quad r = \frac{1}{3}$

2. For a GP, the second term is 20 and the fourth term is 320. Find a and r.

Answer: $a = 5, \quad r = 4,$
or $a = -5, \quad r = -4$

3. For a GP, the fourth term is 8 and the tenth term is 512. Find a and r.

Answer: $a = 1, \quad r = 2,$
or $a = -1, \quad r = -2$

4. For a GP, the second term is 20 and the fifth term is 160. Find a and r.

Answer: $a = 10, \quad r = 2$

THANKS FOR YOUR ATTENTION