Lecture 11

Topics covered in this lecture session

1. Series
    - Partial sums and the sigma notation
    - Arithmetic series
    - Geometric series
    - Sum of infinite geometric series
2. Power Series
3. Method of differences.

Structure of Final exam paper

Question No.	Marks	Topics covered
1	15	Functions, Modulus inequality, Quadratic, Logarithmic and exponential functions
2	15	Trigonometry, Remainder and Factor Theorems, Synthetic Division
3	15	Binomial Theorem, Generalised Binomial Theorem and applications, Numerical methods, Matrices
4	15	Partial fractions, Complex Numbers, Sequence and Series, Power series, method of differences

Useful information about Final exam paper

Notes:

1) This is a take-home-exam, which you must complete in 24-hours' time.
2) The exam paper will be available on module Moodle page at 9.30 am. on 6th January 2023.
3) Deadline for submission is: **9.30 am on 7th January 2023** (China Time).
4) Marks will be given for the best 3 answers. (i.e. you can attempt **ANY 3 out of 4** questions).
5) Total marks obtained will then be upscaled to 70%.
6) The final score will be calculated as: **Mid-semester exam (30%) + Final exam (70%)**.

Useful information about Final exam paper

Instructions:

1) You should write all **necessary steps** in your solutions.
2) It is expected that you will only use CELE approved calculator ($fx-82$ series) for this exam. You will lose marks if because of use of other models of calculators, your numerical answer differs from our standardized marking scheme.
3) Formula Sheet will be attached to the question paper.
4) Please write your answers on a blank piece of paper. Alternatively, you may also use iPad/Tablet to write your answers.
5) Please complete the coursework submission form (downloadable from module Moodle page) and create a **single** PDF file of all your answers to exam questions with completed submission form on the top.
6) Name your file as: Your Student ID number_N036Final. For example: 20519999_N036Final.
7) Please upload this PDF file to submission drop-box on module Moodle page (available on the top of the Moodle page). Module Convenor will also email the link to the submission drop-box.
8) No excuses such as problems with internet connectivity, etc. will be entertained; so, you are suggested to submit your working well in advance before the deadline. Should you have any difficulty in uploading your file, please contact Module Convenor (Bamidele.Akinwolemiwa2@nottingham.edu.cn) **immediately** and follow their instructions.
9) This work must be completed on your own. Plagiarism and collusion are regarded as very serious academic offences and will be treated as such.

Partial Sums

Let $a_1,\ a_2,\ a_3, ...$ be a given sequence.

Define the sums:
$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \\ S_n &= a_1 + a_2 + a_3 + ... + a_n \end{aligned}$$

The sums defined as above are called partial sums.

Sigma notation

$$\sum_{k=1}^{n} a_k = \sum_{1}^{n} a_n = a_1 + a_2 + a_3 + ... + a_n$$

$$\sum_{1}^{\infty} a_n = a_1 + a_2 + a_3 + ... + a_n + ...$$

Using sigma notation, the partial sums for sequence $\{a_n\}$ is:

$$S_n = \sum_{k=1}^{n} a_k$$

Sigma notation

Some examples of use of sigma notation:

$$\sum_{1}^{5} r^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$1 - x + x^2 - x^3 + ... = \sum_{n=0}^{\infty} (-1)^n\, x^n$$

$$2 - 4 + 8 - 16 + ... + 128 = \sum_{n=1}^{7} (-1)^{n+1}\, 2^n$$

Series

A series $\{S_n\}$ is a sequence whose terms are partial sums of terms of a given sequence $\{a_n\}$.

e.g. if a given sequence is 2, 4, 6, 8, 10, ...

then the corresponding (associated) series is:

$$2, \quad 2+4, \quad 2+4+6, \quad 2+4+6+8, \quad .....$$

i.e. $2, \quad 6, \quad 12, \quad 20, \quad .....$

Series

n^{th} term of series $\{S_n\}$ can be obtained from sequence $\{a_n\}$, using

$$S_n = \sum_{k=1}^{n} a_k$$

On the other hand,

$$\begin{aligned} S_n - S_{n-1} &= (\, a_1 + a_2 + \dots + a_{n-1} + a_n \,) \\ &\qquad - (\, a_1 + a_2 + \dots + a_{n-1} \,) \\ &= a_n \end{aligned}$$

$$\therefore \boxed{a_n = S_n - S_{n-1}}$$

Arithmetic Series

Consider the sum S_n of the first n terms of an A.P.

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \dots\dots + [a + (n-2)d] + [a + (n-1)d]$$

Writing $l = a + (n-1)\,d$

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \;\dots\dots + (l-d) + l$$

Reversing the sum

$$S_n = l + (l-d) + (l-2d) + (l-3d) + \;\dots\dots + (a+d) + a$$

Adding

$$2S_n = \underbrace{(a+l) + (a+l) + (a+l) + (a+l) + \;\dots\dots + (a+l) + (a+l)}_{\text{n times}}$$

Arithmetic Series

$$\therefore\; 2S_n = n\,(a+l) \;\Rightarrow\; S_n = \frac{n}{2}\,(a+l) \;=\frac{n}{2}\,[\,a + a + (n-1)\,d\,]$$

Thus, the sum of the first n terms of an A.P. is:

$$\boxed{S_n = \frac{n}{2}\,[\,2a + (n-1)\,d\,]}$$

Example:

The eighth term of an A.P. is 23 and its 24th term is 103.

Find the sum of its first 30 terms.

Geometric Series

The sum of the first n terms of an A.P. is:

$$\boxed{S_n = \begin{cases} na & ; \quad r = 1 \\ a\left(\dfrac{1-r^n}{1-r}\right) & ; \quad r \neq 1 \end{cases}}$$

Example:

If $r = \dfrac{1}{3}, \quad S_4 = 150,$ find the first term a.

Sum of Infinite Geometric Series

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \quad ; \quad r \neq 1$$

If $|r| < 1$ then, $\lim\limits_{n\to\infty} r^n = 0$

$$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[a\left(\frac{1-r^n}{1-r}\right)\right] = \frac{a}{1-r} \Rightarrow \boxed{S = \frac{a}{1-r}}$$

Example: Find the sum of the infinite geometric series:

$$5 + 1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + ...$$

Harmonic Series

The harmonic series is the divergent infinite series

$$\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ...$$

Power Series

If $k \in \mathbb{N}$, the series:

$$1^k + 2^k + 3^k + ... + n^k = \sum_{n=1}^{n} n^k$$

is called the Power Series.

- When $k = 1$,

$$1 + 2 + 3 + ... + n = \sum_{n=1}^{n} n = \frac{n(n+1)}{2}$$

- When $k = 2$,

$$1^2 + 2^2 + 3^2 + ... + n^2 = \sum_{n=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

Power Series

- When $k = 3$,

$$1^3 + 2^3 + 3^3 + ... + n^3 = \sum_{n=1}^{n} n^3 = \frac{n^2(n+1)^2}{4}$$

Example: Prove that

$$\sum n^3 = \left(\sum n\right)^2$$

Power Series

Find the sum: $1 + (1+2) + (1+2+3) + ...$ (up to n terms)

Solution:

$$\boxed{\text{Sum} \;=\; \sum n^{th} \text{ term}}$$

$$\therefore \text{Sum} = \sum (1+2+3+ .... + n)$$

$$= \sum \left(\sum n \right) = \sum \frac{n(n+1)}{2}$$

Power Series

$$\therefore \text{Sum} = \frac{1}{2} \sum (n^2 + n)$$

$$= \frac{1}{2} \left(\sum n^2 + \sum n \right)$$

$$= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

$$= \frac{n(n+1)(n+2)}{6}$$ (upon simplification)

Method of differences

Find the sum: $\displaystyle\sum_{1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right)$

Solution:

$$= \left(\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\dots} + \cancel{\frac{1}{n}} - \frac{1}{n+1} \right)$$

$$= 1 - \frac{1}{n+1}$$

Method of differences

Find the sum: $\displaystyle\sum_{1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$

Solution:

$$= \lim_{n\to\infty} \left(\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\dots} + \cancel{\frac{1}{n}} - \frac{1}{n+1} \right)$$

$$= \lim_{n\to\infty} \left(1 - \frac{1}{n+1} \right) = 1 - 0 = 1$$

Method of differences

Example: Given $\dfrac{r}{(r+1)\,!} = \dfrac{1}{r\,!} - \dfrac{1}{(r+1)\,!}$.

Use the method of differences, to find the sum $\displaystyle\sum_{1}^{n} \frac{r}{(r+1)\,!}$.

Solution: $\displaystyle\sum_{1}^{n} \frac{r}{(r+1)\,!} = \sum_{1}^{n} \left[\frac{1}{r\,!} - \frac{1}{(r+1)\,!}\right]$

$$= \frac{1}{1\,!} - \cancel{\frac{1}{2\,!}} + \cancel{\frac{1}{2\,!}} - \cancel{\frac{1}{3\,!}} + \cancel{\frac{1}{3\,!}} - \cancel{\frac{1}{4\,!}} + \cancel{\cdots} + \cancel{\frac{1}{(n-1)\,!}} - \cancel{\frac{1}{n\,!}} + \cancel{\frac{1}{n\,!}} - \frac{1}{(n+1)\,!}$$

$$= \frac{1}{1\,!} - \frac{1}{(n+1)\,!} \quad = 1 - \frac{1}{(n+1)\,!}$$

Best wishes for your

FINAL exams…

~~ from ~~

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