My Presentation

Lab 4 work

XXX, 2051XXXX

March 12, 2023

Outline

First order derivative

Stationary Point

x_0 is a stationary point of $f(x)$ if $f'(x_0) = 0$.

Second order derivative

If x_0 is a stationary point of $y = f(x)$, then $f(x)$ may achieve its maximum/minimum value at $x = x_0$.

Second order derivative

If x_0 is a stationary point of $y = f(x)$, then $f(x)$ may achieve its maximum/minimum value at $x = x_0$.

Second Derivative Test

- If $f''(x_0) < 0$, then f has maximum value at $x = x_0$.
- If $f''(x_0) > 0$, then f has minimum value at $x = x_0$.

Question

Find the (local) **maximum/minimum** values of function

$$f(x) = 3x^4 - 20x^3 + 36x^2 - 15$$

Question

Find the (local) **maximum/minimum** values of function

$$f(x) = 3x^4 - 20x^3 + 36x^2 - 15$$

Solution:

$$\begin{aligned} f'(x) &= 12x^3 - 60x^2 + 72x && (1) \\ f''(x) &= 36x^2 - 120x + 72 && (2) \end{aligned}$$

$f'(x) = 0 \Rightarrow x = 0, 2, 3$ are stationary points of $f(x)$.

Cont'd

Below is a table for summarizing the test result:

$(x_0, f(x_0))$	$f''(x)\|_{x=x_0}$	classification
$(0, -15)$	> 0	point of minimum
$(2, 17)$	< 0	point of maximum
$(3, 12)$	> 0	point of minimum

Table: Second Derivative Test

Function plot

Below is a plot of $y = f(x)$ in **GeoGebra**:

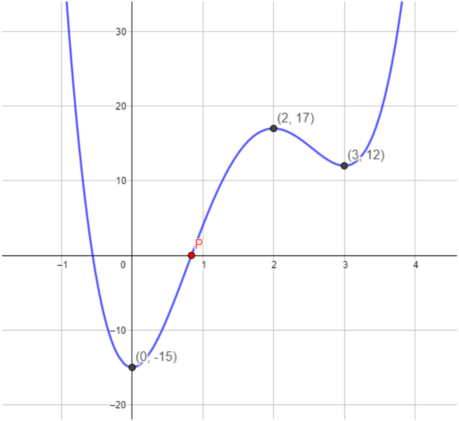

Figure: $f(x) = 3x^4 - 20x^3 + 36x^2 - 15$

N-R method

Use Newton-Raphson Method to find one apporximate root at P (correct to 5 d.p.), with $x_0 = 1$.

N-R method

Use Newton-Raphson Method to find one apporximate root at P (correct to 5 d.p.), with $x_0 = 1$.

N-R method

Use Newton-Raphson Method to find one apporximate root at P (correct to 5 d.p.), with $x_0 = 1$.

$$
\begin{aligned}
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} && (3) \\
&= \cdots \\
&= \frac{9x_n^4 - 40x_n^3 + 36x_n^2 + 15}{12x_n^3 - 60x_n^2 + 72x_n} && (4)
\end{aligned}
$$

Use Newton-Raphson Method to find one apporximate root at P (correct to 5 d.p.), with $x_0 = 1$.

$$
\begin{aligned}
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} && (3)\\
&= \cdots \\
&= \frac{9x_n^4 - 40x_n^3 + 36x_n^2 + 15}{12x_n^3 - 60x_n^2 + 72x_n} && (4)
\end{aligned}
$$

n	x_n
0	1.0000
1	0.8333
2	0.8384
3	0.8384

Use Newton-Raphson Method to find one apporximate root at P (correct to 5 d.p.), with $x_0 = 1$.

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \qquad (3) \\ &= \cdots \\ &= \frac{9x_n^4 - 40x_n^3 + 36x_n^2 + 15}{12x_n^3 - 60x_n^2 + 72x_n} \qquad (4) \end{aligned}$$

n	x_n
0	1.0000
1	0.8333
2	0.8384
3	0.8384

$\therefore$ the desired root $x^* = 0.8384$.

Remark

The iteration formula in previous frame is typeset by the following command lines:

```
\begin{eqnarray}
x_{n+1}&=& x_n-\frac{f(x_n)}{f'(x_n)}\\[1ex]
&=& \cdots \nonumber\\[1ex]
&=& \frac{9x_n^4-40x_n^3+36x_n^2+15}{12x_n^3-60x_n^2+72x_n}
\end{eqnarray}
```

Q&A

Thank you!

Any questions?