Lecture 8

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Topics covered in this lecture session

1. The method of substitution for Definite Integrals.

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2. Integration by parts for Definite Integrals.

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2. Integration by parts for Definite Integrals.
3. Properties of Definite Integration.

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1. The method of substitution for Definite Integrals.
2. Integration by parts for Definite Integrals.
3. Properties of Definite Integration.
4. Applications of Integration (Area calculation).

The method of substitution

(for Definite Integrals)

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When using substitution, remember to change the limits of integration for the newly formed integral.

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x	1	2
t	1	4

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$$(i) \quad \int_{0}^{\pi/2} \frac{\cos x}{1+\sin^2 x} \, dx$$

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Integration by parts

(for Definite Integrals)

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$$\int_a^b u \cdot \frac{dv}{dx} \; dx = [u\,v]_a^b - \int_a^b v \cdot \frac{du}{dx} \; dx$$

Integration by parts

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Example 6: Evaluate $\displaystyle\int_0^1 x \cdot e^x \, dx$

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Let $u = x$

and $\dfrac{dv}{dx} = e^x$

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$$\therefore \; I = \left[\, x \cdot e^x \,\right]_0^1$$

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$$\therefore\; I = \left[\, x \cdot e^x \,\right]_0^1 - \int_0^1 e^x \cdot (1) \, dx$$

$$= (e - 0) - \left(e^1 - e^0\right) = 1$$

Integration by parts

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Properties of Definite Integration

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1. If $a \in D_f$, then $\displaystyle\int_a^a f(x)\ dx = 0$

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2. If f is integrable on $[\,a, b\,]$, then

$$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$

Properties of Definite Integration

3. If f is integrable on a closed interval containing three points a, b, and c, then

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where $f(x) = \begin{cases} \sin x & ; & 0 \le x \le \pi/2 \\ 1 & ; & \pi/2 \le x \le 5 \\ e^x - 5 & ; & 5 \le x \le 9 \end{cases}$

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$$I = \int_0^{\pi/2} \sin x\ dx$$

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$$I = \int_0^{\pi/2} \sin x\, dx + \int_{\pi/2}^{5} 1\, dx + \int_5^9 e^x - 5\, dx = e^5\left(e^4 - 1\right) - \frac{\pi}{2} - 14$$

Properties of Definite Integration

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Example 2: Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\ dx$

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\ dx$$

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$$\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$$

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Properties of Definite Integration

4. If f is integrable on $[0, a]$, then

$$\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$$

Example 2: Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\ dx$ (1)

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\ dx = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\ dx \qquad (2)$$

Properties of Definite Integration

(1) + (2) gives

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$$I + I = \int_{0}^{\pi/2} \frac{\sin x}{\sin x + \cos x}\; dx \; + \int_{0}^{\pi/2} \frac{\cos x}{\cos x + \sin x}\; dx$$

Properties of Definite Integration

(1) + (2) gives

$$I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \; dx \; + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} \; dx$$

$$\therefore \; 2\,I \; = \int_0^{\pi/2} 1 \;\; dx$$

Properties of Definite Integration

(1) + (2) gives

$$I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \; dx \; + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} \; dx$$

$$\therefore \; 2I \; = \int_0^{\pi/2} 1 \; dx$$

$$\therefore \; I \; = \frac{1}{2} \; \left[x \right]_0^{\pi/2}$$

Properties of Definite Integration

(1) + (2) gives

$$I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\ dx + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\ dx$$

$$\therefore\ 2\,I = \int_0^{\pi/2} 1\ dx$$

$$\therefore\ I = \frac{1}{2}\,[\,x\,]_0^{\pi/2} \Rightarrow I = \frac{\pi}{4}$$

Properties of Definite Integration

Example 3: Evaluate $$\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\,dx$$

Properties of Definite Integration

Example 3: Evaluate $$\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} \; dx$$

using the property

$$\int_0^a f(x) \; dx = \int_0^a f(a-x) \; dx$$

Properties of Definite Integration

Example 3: Evaluate $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\ dx$

$$I = \int_0^3 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{3-(3-x)}}\ dx$$

using the property

$$\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$$

Properties of Definite Integration

Example 3: Evaluate $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\ dx$

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Properties of Definite Integration

Example 3: Evaluate $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\ dx$ (1)

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$$\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx$$

(1) + (2) gives $2\,I = \int_0^3 1\ dx$

Properties of Definite Integration

Example 3: Evaluate $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\;dx$ (1)

$$I = \int_0^3 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{3-(3-x)}}\;dx$$

using the property

$$\int_0^a f(x)\;dx = \int_0^a f(a-x)\;dx$$

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(1) + (2) gives $\displaystyle 2\,I = \int_0^3 1\;dx \qquad \therefore\; I = \frac{1}{2}\left[\,x\,\right]_0^3$

Properties of Definite Integration

Example 3: Evaluate $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\ dx$ (1)

using the property

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(1) + (2) gives $\displaystyle 2I = \int_0^3 1\ dx \qquad \therefore\ I = \frac{1}{2}\left[x\right]_0^3 = \frac{3}{2}$

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Properties of Definite Integration

5. If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$$

6. If f is EVEN integrable on $[-a, a]$, then

$$\int_{-a}^a f(x)\ dx = 2\int_0^a f(x)\ dx$$

Properties of Definite Integration

7. If f is ODD integrable on $[-a, a]$, then

Properties of Definite Integration

7. If f is ODD integrable on $[-a, a]$, then

$$\int_{-a}^{a} f(x)\ dx = 0$$

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x}\, dx$

Properties of Definite Integration

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Let $f(x) = \dfrac{\sin x}{x^4 + x^2 + \cos x}$

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x}\, dx$

Let $f(x) = \dfrac{\sin x}{x^4 + x^2 + \cos x}$

$$\Rightarrow f(-x) = \frac{\sin(-x)}{(-x)^4 + (-x)^2 + \cos(-x)}$$

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x}\, dx$

Let $f(x) = \dfrac{\sin x}{x^4 + x^2 + \cos x}$

$$\Rightarrow \; f(-x) = \frac{\sin(-x)}{(-x)^4 + (-x)^2 + \cos(-x)} = \frac{-\sin x}{x^4 + x^2 + \cos x}$$

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x}\, dx$

Let $f(x) = \dfrac{\sin x}{x^4 + x^2 + \cos x}$

$$\Rightarrow f(-x) = \frac{\sin(-x)}{(-x)^4 + (-x)^2 + \cos(-x)} = \frac{-\sin x}{x^4 + x^2 + \cos x} = -f(x)$$

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x}\, dx$

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$\therefore\ f$ is odd function

Properties of Definite Integration

Example 4: Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin x}{x^4 + x^2 + \cos x} \, dx$

Let $f(x) = \dfrac{\sin x}{x^4 + x^2 + \cos x}$

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$$\therefore \ f \text{ is odd function } \Rightarrow \int_{-1}^{1} f(x)\, dx = 0$$

Applications of Integration

Applications of Integration

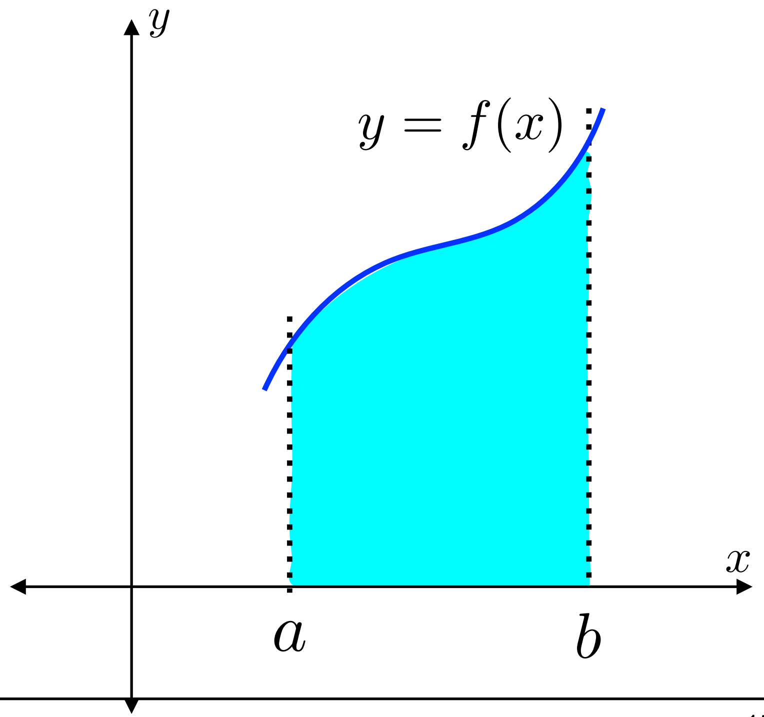
(**Area** calculation)

Applications of Integration

(**Area** calculation)

Result 1

The area of region bounded by the curve $y = f(x)$, lines $x = a$, $x = b$ and the X-axis is:



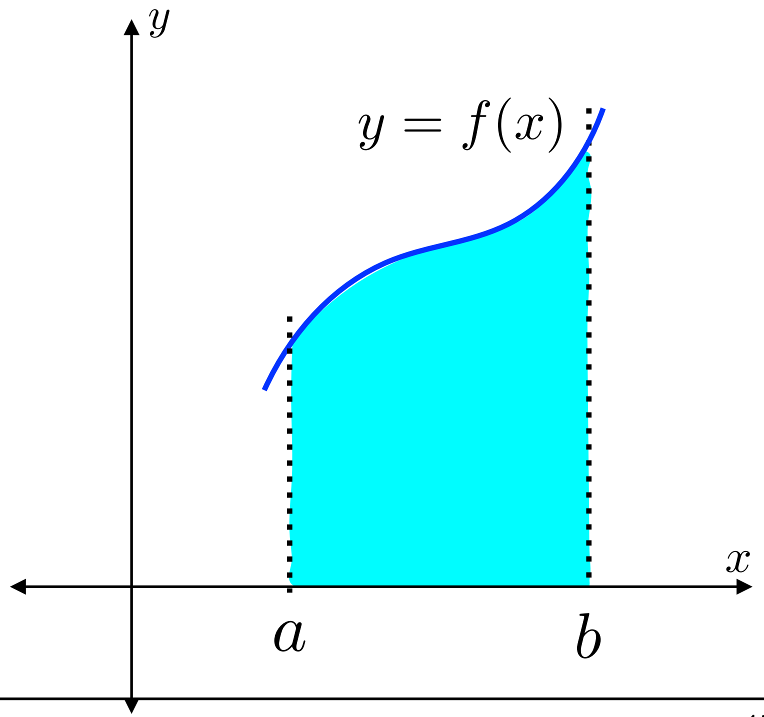
Applications of Integration

(Area calculation)

Result 1

The area of region bounded by the curve $y = f(x)$, lines $x = a$, $x = b$ and the X-axis is:

$$A = \int_a^b y \; dx = \int_a^b f(x) \; dx$$



Applications of Integration

(**Area** calculation)

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The area of region bounded by the curve $y = f(x)$, lines $x = a$, $x = b$ and the X-axis is:

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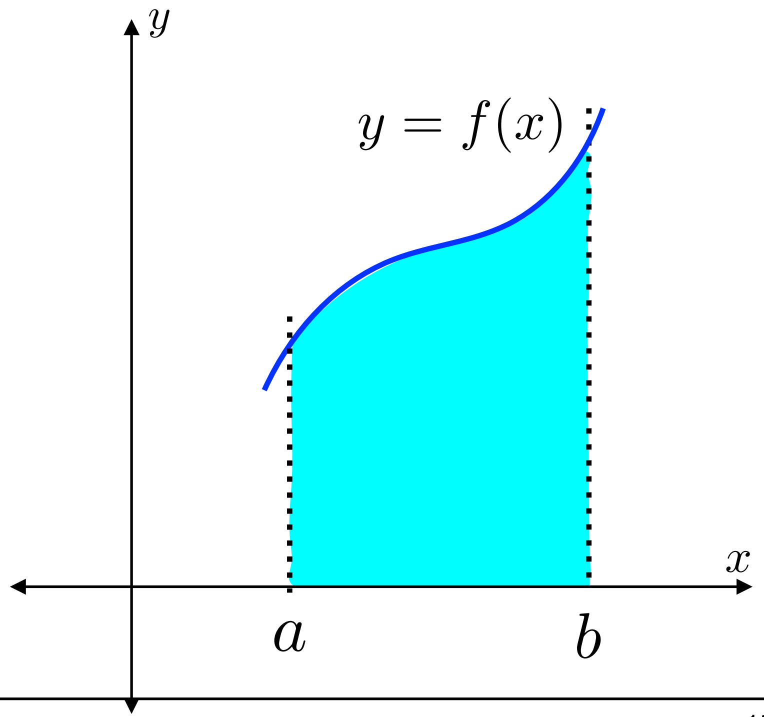

Applications of Integration

(Area calculation)

Example 7: Calculate the area of region bounded by the curve $y = \cos x$, lines $x = 0$, $x = \pi/2$ and the X-axis.

Applications of Integration

(Area calculation)

Example 7: Calculate the area of region bounded by the curve $y = \cos x$, lines $x = 0$, $x = \pi/2$ and the X-axis.

$$\text{Area, } A = \int_0^{\pi/2} \cos x \, dx$$

Applications of Integration

(**Area** calculation)

Example 7: Calculate the area of region bounded by the curve $y = \cos x$, lines $x = 0$, $x = \pi/2$ and the X-axis.

$$\text{Area, } A = \int_0^{\pi/2} \cos x \, dx \; = \left[\, \sin x \,\right]_0^{\pi/2}$$

Applications of Integration

(**Area** calculation)

Example 7: Calculate the area of region bounded by the curve $y = \cos x$, lines $x = 0$, $x = \pi/2$ and the X-axis.

$$\text{Area, } A = \int_0^{\pi/2} \cos x \, dx = \left[\sin x \right]_0^{\pi/2} = 1$$

Applications of Integration

(**Area** calculation)

Example 2: Calculate the area of region bounded by the curve $y = \cos x$ and the X-axis in $[\,0, \pi\,]$.

Applications of Integration

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Applications of Integration

(**Area** calculation)

Example 2: Calculate the area of region bounded by the curve $y = \cos x$ and the X-axis in $[\,0, \pi\,]$.

$$\text{Area, } A = \int_0^{\pi} \cos x \; dx = \left[\,\sin x\,\right]_0^{\pi}$$

Applications of Integration

(**Area** calculation)

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Applications of Integration

(Area calculation)

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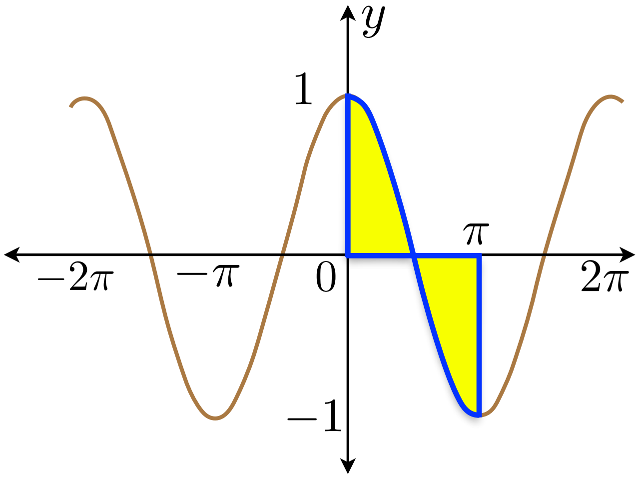

Applications of Integration

(**Area** calculation)

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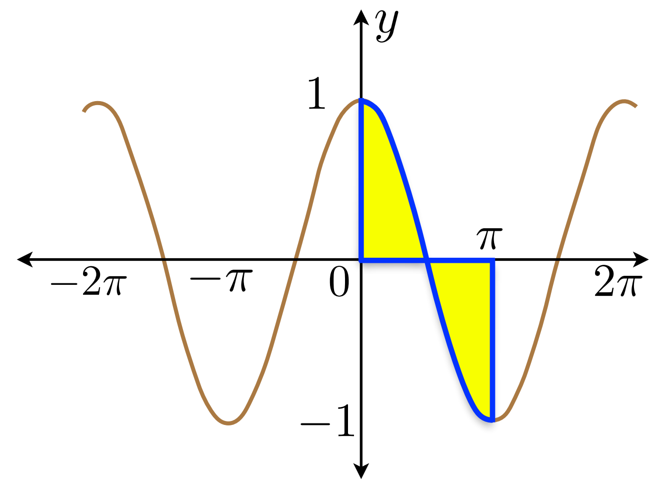


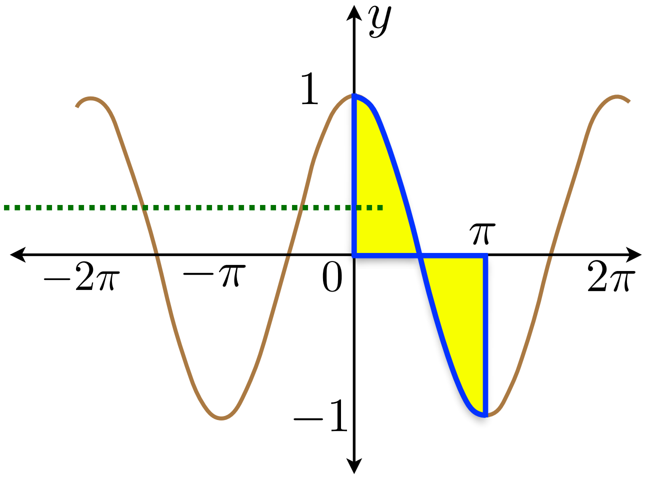
Applications of Integration

(**Area** calculation)

Example 2: Calculate the area of region bounded by the curve $y = \cos x$ and the X-axis in $[\,0, \pi\,]$.

$$\text{Area, } A = \cancel{\int_0^{\pi} \cos x \, dx = [\sin x]_0^{\pi} = 0}$$

$$\text{Area, } A = \boxed{2} \boxed{\int_0^{\pi/2} \cos x \, dx}$$



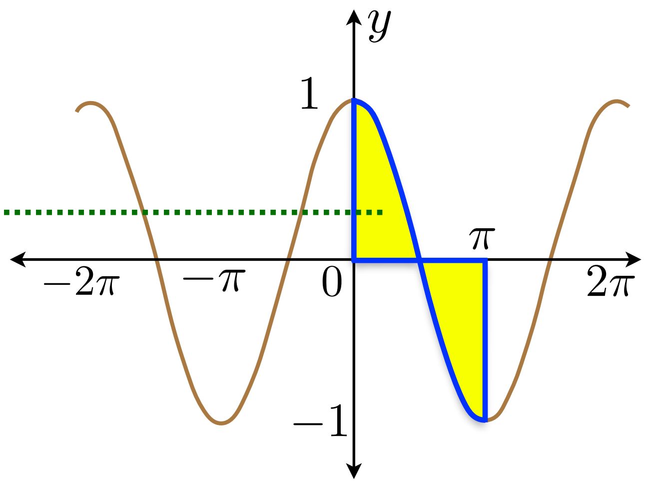
Applications of Integration

(**Area** calculation)

Example 2: Calculate the area of region bounded by the curve $y = \cos x$ and the X-axis in $[\,0, \pi\,]$.

$$\xcancel{\text{Area, } A = \int_0^{\pi} \cos x \; dx = [\sin x]_0^{\pi} = 0}$$

$$\text{Area, } A = 2 \int_0^{\pi/2} \cos x \; dx = 2 \; [\sin x]_0^{\pi/2}$$



Applications of Integration

(**Area** calculation)

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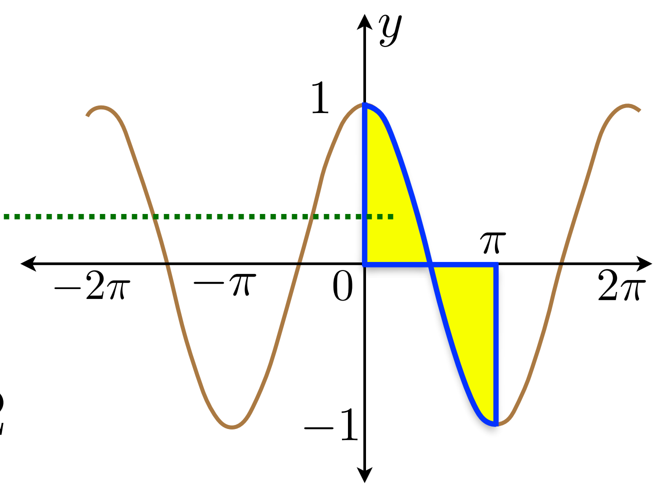
$$\text{Area, } A = 2 \int_0^{\pi/2} \cos x \, dx = 2 \, [\sin x]_0^{\pi/2} = 2$$



Applications of Integration

(**Area** calculation)

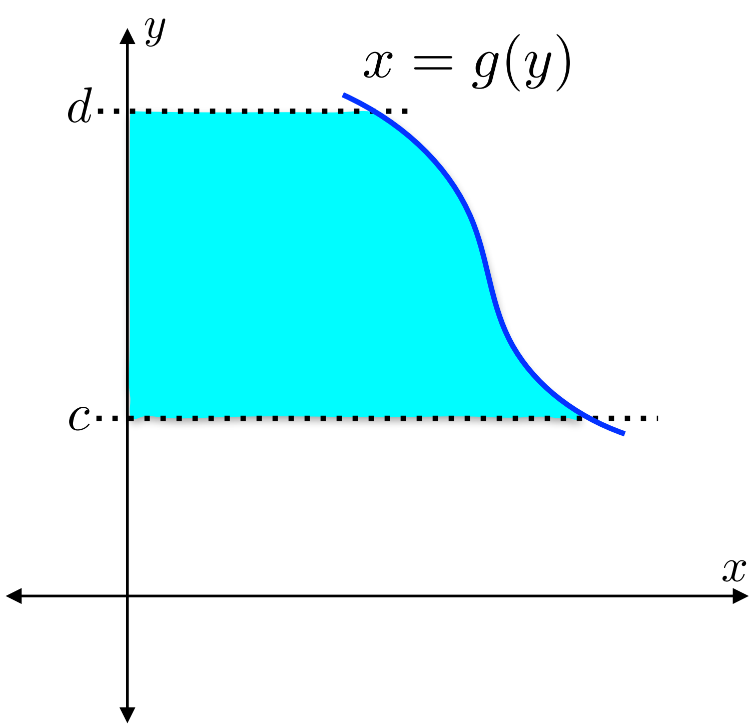
Result 2

Applications of Integration

(**Area** calculation)

Result 2

The area of region bounded by the curve $x = g(y)$, lines $y = c$, $y = d$ and the Y-axis is:



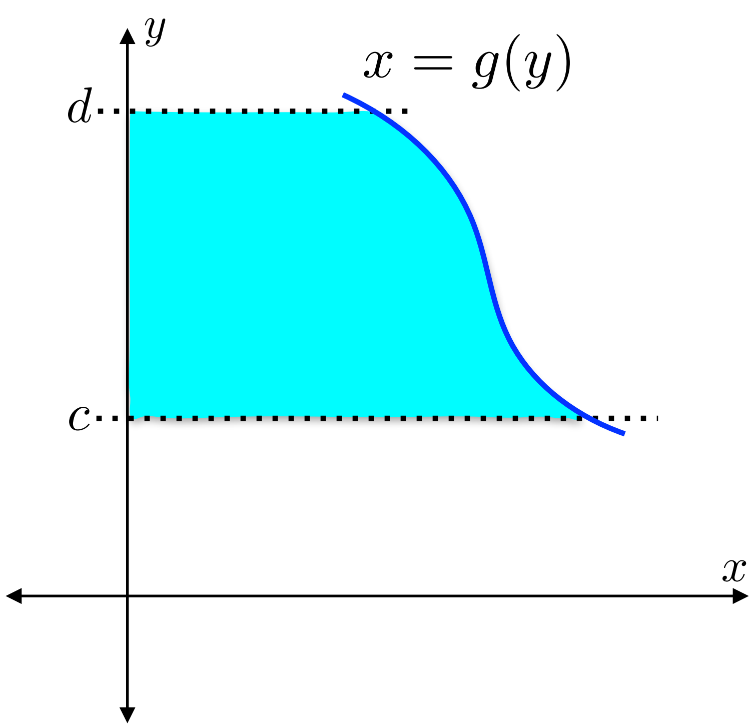
Applications of Integration

(Area calculation)

Result 2

The area of region bounded by the curve $x = g(y)$, lines $y = c$, $y = d$ and the Y-axis is:

$$A = \int_c^d x \ dy = \int_c^d g(y) \ dy$$



Applications of Integration

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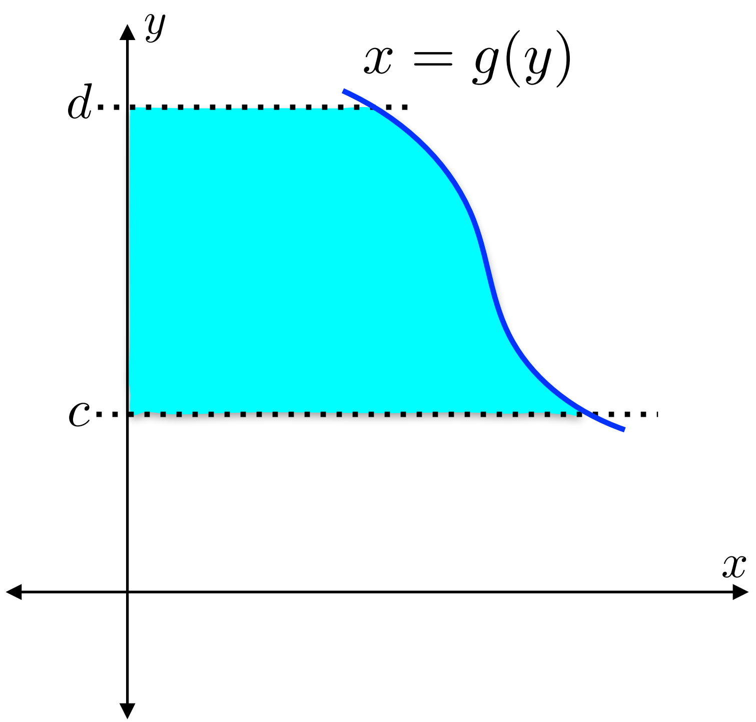


Applications of Integration

(**Area** calculation)

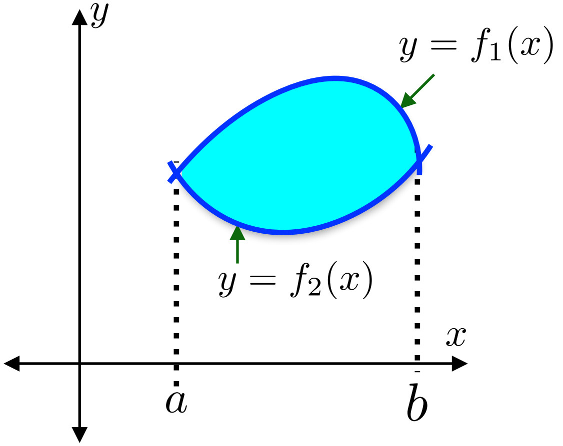
Result 3

Applications of Integration

(**Area** calculation)

Result 3

The area of region bounded by the curves $y = f_1(x)$, $y = f_2(x)$ and the X-axis is:



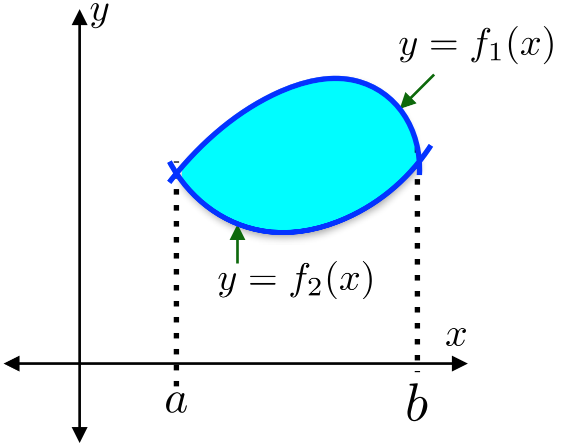
Applications of Integration

(**Area** calculation)

Result 3

The area of region bounded by the curves $y = f_1(x)$, $y = f_2(x)$ and the X-axis is:

$$A = \left| \int_a^b \left[f_1(x) - f_2(x) \right] \, dx \right|$$



Applications of Integration

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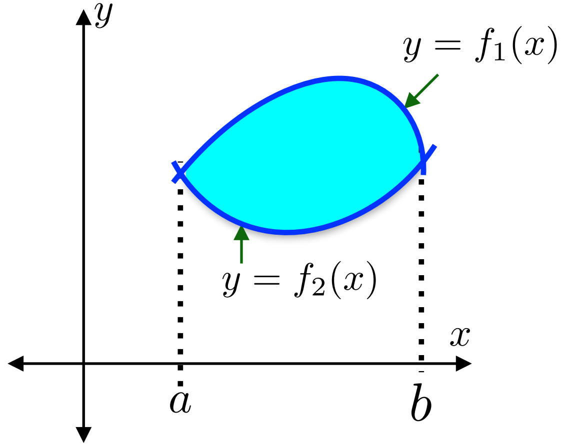

where a and b are x-coordinates of points of intersection of curves.

Applications of Integration

(**Area** calculation)

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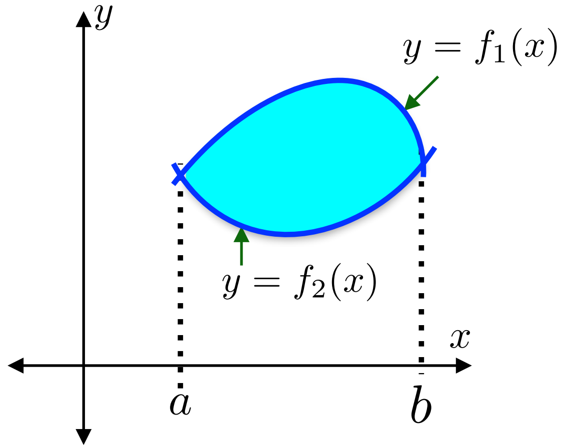


where a and b are x-coordinates of points of intersection of curves.

Applications of Integration

(**Area** calculation)

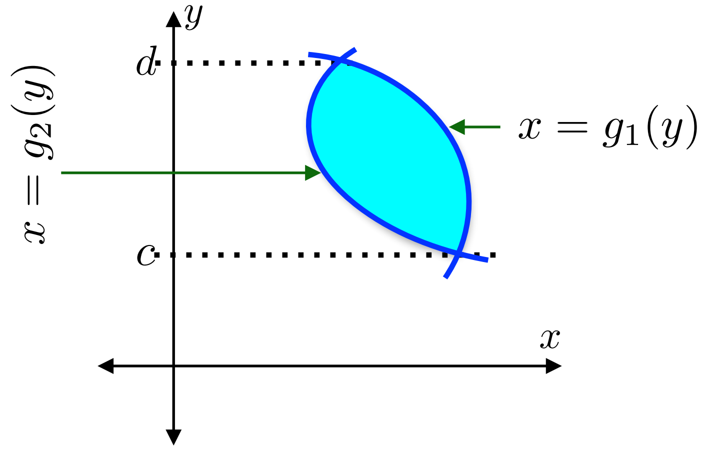
Result 4

Applications of Integration

(Area calculation)

Result 4

The area of region bounded by the curves $x = g_1(y)$, $x = g_2(y)$ and the Y-axis is:



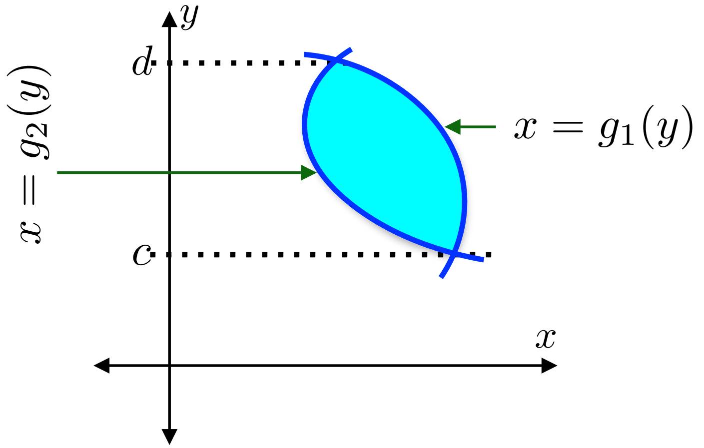
Applications of Integration

(Area calculation)

Result 4

The area of region bounded by the curves $x = g_1(y)$, $x = g_2(y)$ and the Y-axis is:

$$A = \left| \int_c^d \left[g_1(y) - g_2(y) \right] \, dy \right|$$



Applications of Integration

(**Area** calculation)

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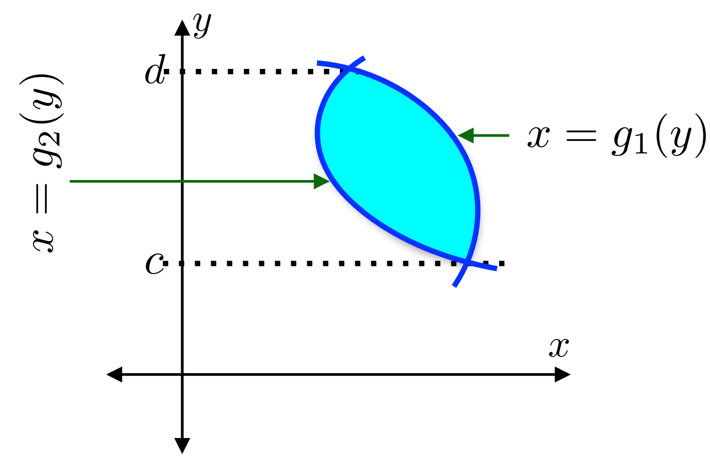

where c and d are y-coordinates of points of intersection of curves.

Applications of Integration

(**Area** calculation)

Result 4

The area of region bounded by the curves $x = g_1(y)$, $x = g_2(y)$ and the Y-axis is:

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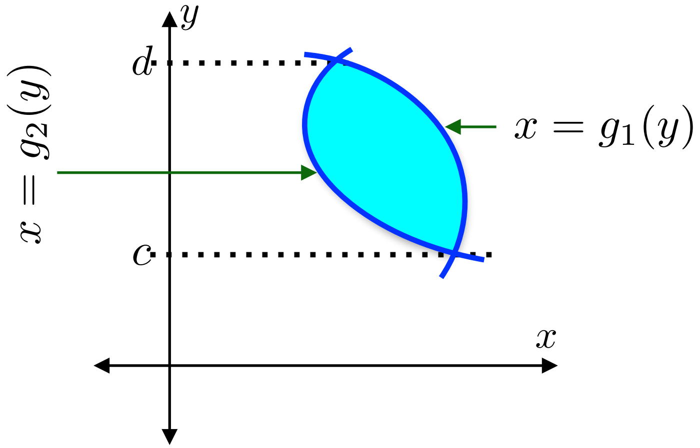

where c and d are y-coordinates of points of intersection of curves.

Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

Applications of Integration

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Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

$$\Rightarrow \; y^2 = y + 2$$

i.e. $y^2 - y - 2 = 0$

Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

$$\Rightarrow \; y^2 = y + 2$$

$$\text{i.e. } y^2 - y - 2 = 0$$

$$\Rightarrow \; (y+1)\cdot(y-2) = 0$$

Applications of Integration

(Area calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

$$\Rightarrow \; y^2 = y + 2$$

$$\text{i.e. } y^2 - y - 2 = 0$$

$$\Rightarrow \; (y+1)\cdot(y-2) = 0$$

$$\therefore \; y = -1 \text{ and } 2$$

are y-coordinates of points of intersection.

Applications of Integration

(Area calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

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$$\therefore \text{ Area, } A = \int_{-1}^{2} \left[\, (y+2) - y^2 \,\right] \, dx$$

Applications of Integration

(Area calculation)

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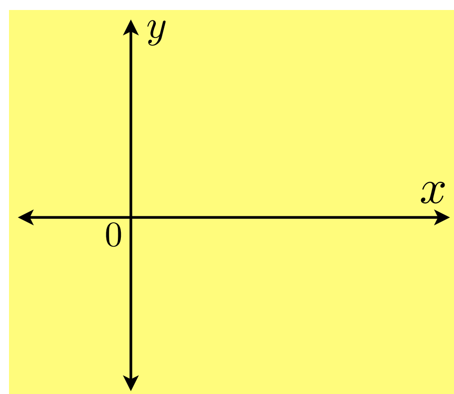

Applications of Integration

(Area calculation)

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Here, $y^2 = x$ and $x = y + 2$

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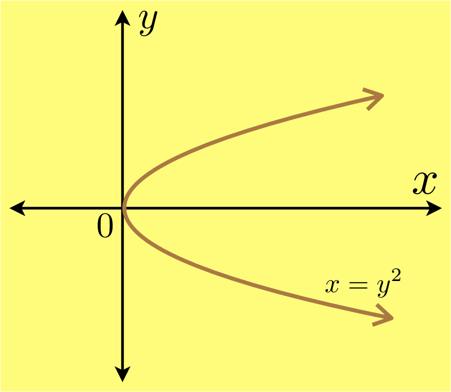

Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

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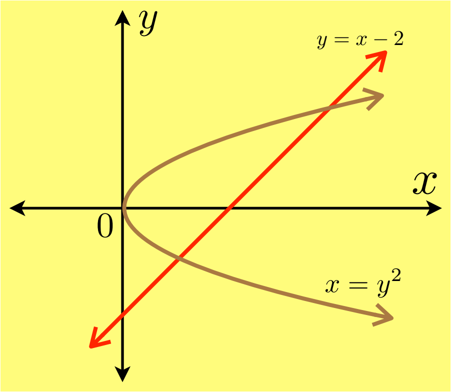
are y-coordinates of points of intersection.

$$\therefore \text{ Area, } A = \int_{-1}^{2} \left[(y+2) - y^2 \right] \, dx$$



Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

$$\Rightarrow \quad y^2 = y + 2$$

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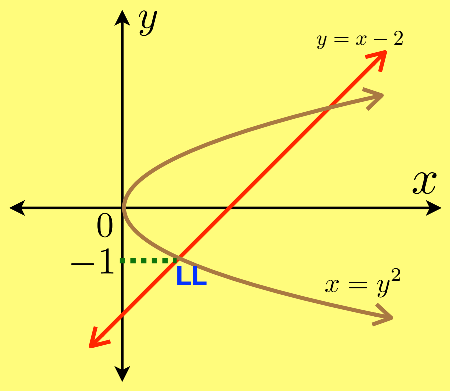

Applications of Integration

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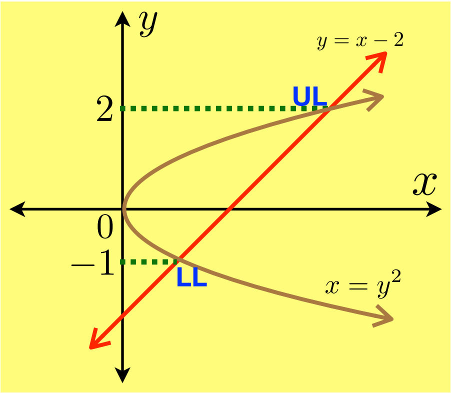

Applications of Integration
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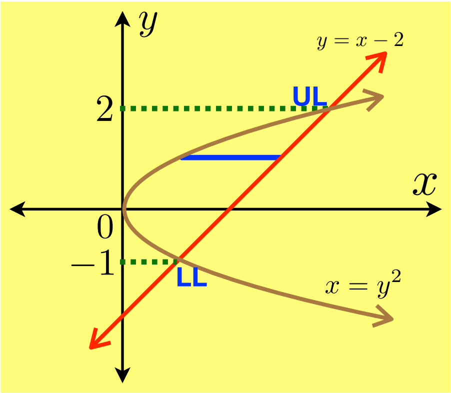

Applications of Integration

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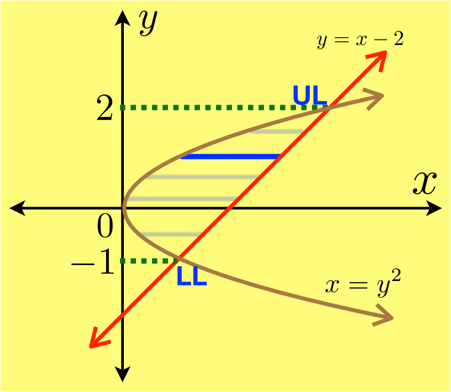

Applications of Integration

(Area calculation)

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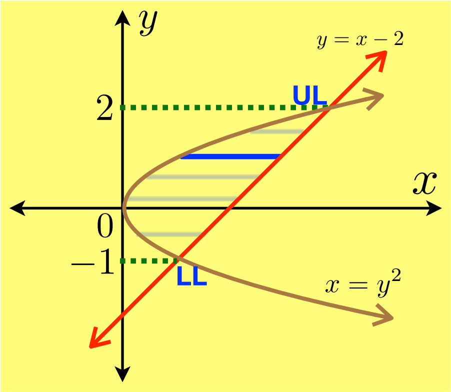
Right curve

$$\therefore \; \text{Area, } A = \int_{-1}^{2} \left[(y+2) - y^2 \right] \, dx$$



Applications of Integration

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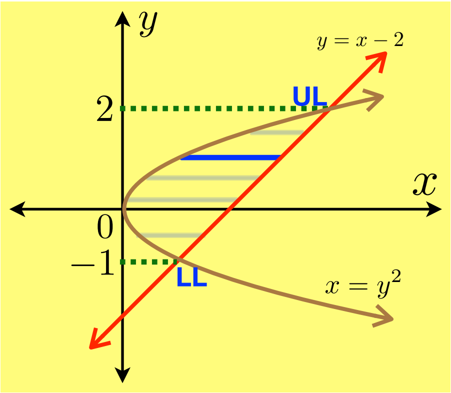
Right curve   Left curve

$$\therefore \text{ Area, } A = \int_{-1}^{2} \left[(y+2) - y^2 \right] \, dx$$



Applications of Integration

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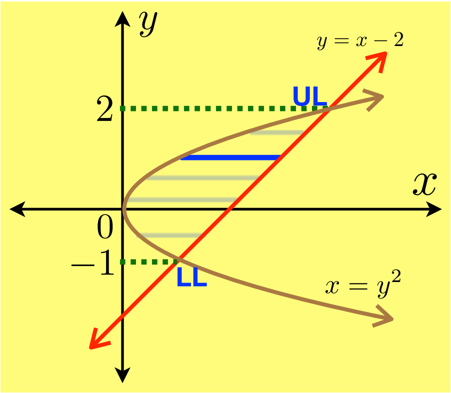


Right curve Left curve

$$\therefore \; \text{Area, } A = \int_{-1}^{2} \left[(y+2) - y^2 \right] \, dx$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^{2}$$

Applications of Integration

(**Area** calculation)

Example 9: Calculate the area of region enclosed by $x = y^2$ and $y = x - 2$.

Here, $y^2 = x$ and $x = y + 2$

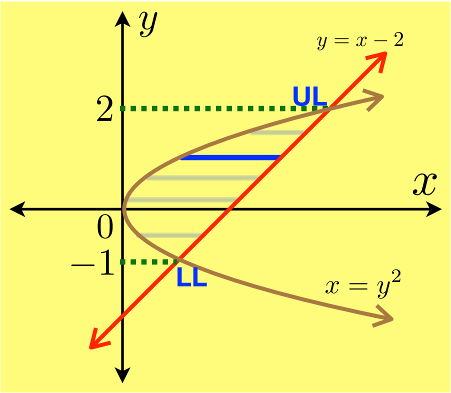
$$\Rightarrow \; y^2 = y + 2$$

$$\text{i.e. } y^2 - y - 2 = 0$$

$$\Rightarrow \; (y+1)\cdot(y-2) = 0$$

$$\therefore \; y = -1 \text{ and } 2$$

are y-coordinates of points of intersection.



Right curve Left curve

$$\therefore \text{ Area, } A = \int_{-1}^{2} \left[(y+2) - y^2 \right] \, dx$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^{2}$$

$$= \frac{9}{2}$$