Seminar 7

In this seminar you will study:

- The Binomial Theorem
- Applications of the Binomial Theorem in:
    - Approximation
    - Error Analysis

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R}, \ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1 + \frac{x}{2}\right)^4$ using the Binomial theorem.

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1 + \dfrac{x}{2}\right)^4$ using the Binomial theorem.

Solution:

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1+\frac{x}{2}\right)^4$ using the Binomial theorem.

Solution:

$$\left(1+\frac{x}{2}\right)^4 = 1 + \binom{4}{1}\cdot\frac{x}{2} + \binom{4}{2}\cdot\left(\frac{x}{2}\right)^2 + \binom{4}{3}\cdot\left(\frac{x}{2}\right)^3 + \binom{4}{4}\cdot\left(\frac{x}{2}\right)^4$$

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1+\frac{x}{2}\right)^4$ using the Binomial theorem.

Solution:

$$\left(1+\frac{x}{2}\right)^4 = 1 + \binom{4}{1} \cdot \frac{x}{2} + \binom{4}{2} \cdot \left(\frac{x}{2}\right)^2 + \binom{4}{3} \cdot \left(\frac{x}{2}\right)^3 + \binom{4}{4} \cdot \left(\frac{x}{2}\right)^4$$

$$= 1 + 4 \cdot \frac{x}{2} + 6 \cdot \left(\frac{x}{2}\right)^2 + 4 \cdot \left(\frac{x}{2}\right)^3 + 1 \cdot \left(\frac{x}{2}\right)^4$$

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1+\frac{x}{2}\right)^4$ using the Binomial theorem.

Solution:

$$\left(1+\frac{x}{2}\right)^4 = 1 + \binom{4}{1} \cdot \frac{x}{2} + \binom{4}{2} \cdot \left(\frac{x}{2}\right)^2 + \binom{4}{3} \cdot \left(\frac{x}{2}\right)^3 + \binom{4}{4} \cdot \left(\frac{x}{2}\right)^4$$

$$= 1 + 4 \cdot \frac{x}{2} + 6 \cdot \left(\frac{x}{2}\right)^2 + 4 \cdot \left(\frac{x}{2}\right)^3 + 1 \cdot \left(\frac{x}{2}\right)^4$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16}$$

The Binomial Theorem

The Binomial Theorem Formula 1: $x \in \mathbb{R},\ n \in \mathbb{N}$

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n$$

Example: Expand $\left(1+\frac{x}{2}\right)^4$ using the Binomial theorem.

Solution:

$$\left(1+\frac{x}{2}\right)^4 = 1 + \binom{4}{1} \cdot \frac{x}{2} + \binom{4}{2} \cdot \left(\frac{x}{2}\right)^2 + \binom{4}{3} \cdot \left(\frac{x}{2}\right)^3 + \binom{4}{4} \cdot \left(\frac{x}{2}\right)^4$$

$$= 1 + 4 \cdot \frac{x}{2} + 6 \cdot \left(\frac{x}{2}\right)^2 + 4 \cdot \left(\frac{x}{2}\right)^3 + 1 \cdot \left(\frac{x}{2}\right)^4$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16}$$

Note: final result of the expansion is a **polynomial**

The Binomial Theorem

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots\cdots + x^n \quad \text{where, } x \in \mathbb{R},\ n \in \mathbb{N}.$$

1. Expand $\left(1+\dfrac{x}{3}\right)^4$ using the binomial theorem.

2. Expand $\left(1-\dfrac{x}{2}\right)^3$ using the binomial theorem.

3. Expand $(1+2x)^4$ using the binomial theorem.

4. Expand $(1-x)^5$ using the binomial theorem.

The Binomial Theorem

$$(1+x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \cdots\cdots + x^n \quad \text{where, } x \in \mathbb{R},\ n \in \mathbb{N}.$$

1. Expand $\left(1+\dfrac{x}{3}\right)^4$ using the binomial theorem.

Answer: $1 + \dfrac{4x}{3} + \dfrac{2x^2}{3} + \dfrac{4x^3}{27} + \dfrac{x^4}{81}$

2. Expand $\left(1-\dfrac{x}{2}\right)^3$ using the binomial theorem.

Answer: $1 - \dfrac{3x}{2} + \dfrac{3x^2}{4} - \dfrac{x^3}{8}$

3. Expand $(1+2x)^4$ using the binomial theorem.

Answer: $1 + 8x + 24x^2 + 32x^3 + 16x^4$

4. Expand $(1-x)^5$ using the binomial theorem.

Answer: $1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\ b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\ b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3 + \dfrac{2}{x}\right)^4$ using the Binomial theorem.

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\, b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3 + \dfrac{2}{x}\right)^4$ using the Binomial theorem.

Solution: Here, $a = 3,\ b = \dfrac{2}{x},\ n = 4.$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\, b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3 + \dfrac{2}{x}\right)^4$ using the Binomial theorem.

Solution: Here, $a = 3,\ b = \dfrac{2}{x},\ n = 4.$

$$\therefore \left(3 + \frac{2}{x}\right)^4 = 3^4 + \binom{4}{1} \cdot 3^3 \cdot \frac{2}{x} + \binom{4}{2} \cdot 3^2 \cdot \left(\frac{2}{x}\right)^2 + \binom{4}{3} \cdot 3 \cdot \left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\ b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3+\dfrac{2}{x}\right)^4$ using the Binomial theorem.

Solution: Here, $a=3,\ b=\dfrac{2}{x},\ n=4.$

$$\therefore \left(3+\frac{2}{x}\right)^4 = 3^4 + \binom{4}{1} \cdot 3^3 \cdot \frac{2}{x} + \binom{4}{2} \cdot 3^2 \cdot \left(\frac{2}{x}\right)^2 + \binom{4}{3} \cdot 3 \cdot \left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$$

$$= 81 + 4 \cdot 27 \cdot \frac{2}{x} + 6 \cdot 9 \cdot \frac{4}{x^2} + 4 \cdot 3 \cdot \frac{8}{x^3} + \frac{16}{x^4}$$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\, b \in \mathbb{R},\;\; n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3 + \dfrac{2}{x}\right)^4$ using the Binomial theorem.

Solution: Here, $a = 3,\; b = \dfrac{2}{x},\;\; n = 4.$

$$\therefore \left(3 + \frac{2}{x}\right)^4 = 3^4 + \binom{4}{1} \cdot 3^3 \cdot \frac{2}{x} + \binom{4}{2} \cdot 3^2 \cdot \left(\frac{2}{x}\right)^2 + \binom{4}{3} \cdot 3 \cdot \left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$$

$$= 81 + 4 \cdot 27 \cdot \frac{2}{x} + 6 \cdot 9 \cdot \frac{4}{x^2} + 4 \cdot 3 \cdot \frac{8}{x^3} + \frac{16}{x^4}$$

$$= 81 + \frac{216}{x} + \frac{216}{x^2} + \frac{96}{x^3} + \frac{16}{x^4}$$

The Binomial Theorem

The Binomial Theorem Formula 2: $a,\ b \in \mathbb{R},\ \ n \in \mathbb{N}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \binom{n}{3} a^{n-3}\, b^3 + \cdots + b^n$$

Example: Expand $\left(3+\dfrac{2}{x}\right)^4$ using the Binomial theorem.

Solution: Here, $a = 3,\ b = \dfrac{2}{x},\ n = 4.$

$$\therefore \left(3+\frac{2}{x}\right)^4 = 3^4 + \binom{4}{1} \cdot 3^3 \cdot \frac{2}{x} + \binom{4}{2} \cdot 3^2 \cdot \left(\frac{2}{x}\right)^2 + \binom{4}{3} \cdot 3 \cdot \left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$$

$$= 81 + 4 \cdot 27 \cdot \frac{2}{x} + 6 \cdot 9 \cdot \frac{4}{x^2} + 4 \cdot 3 \cdot \frac{8}{x^3} + \frac{16}{x^4}$$

$$= 81 + \frac{216}{x} + \frac{216}{x^2} + \frac{96}{x^3} + \frac{16}{x^4}$$

Note: final result of the expansion is a **polynomial**

The Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \cdots + b^n \text{ where, } a,\, b \in \mathbb{R},\ n \in \mathbb{N}.$$

1. Expand $(5-2x)^4$ using the binomial theorem.

2. Expand $\left(\frac{x}{3}-2\right)^4$ using the binomial theorem.

3. Expand $\left(x^2-2\right)^4$ using the binomial theorem.

4. Expand $(4x-3)^4$ using the binomial theorem

The Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + b^n \text{ where, } a,\ b \in \mathbb{R},\ n \in \mathbb{N}.$$

1. Expand $(5-2x)^4$ using the binomial theorem.

Answer: $625 - 1000x + 600x^2 - 160x^3 + 16x^4$

2. Expand $\left(\frac{x}{3} - 2\right)^4$ using the binomial theorem.

Answer: $\frac{x^4}{81} - \frac{8x^3}{27} + \frac{8x^2}{3} - \frac{32x}{3} + 16$

3. Expand $\left(x^2 - 2\right)^4$ using the binomial theorem.

Answer: $x^8 - 8x^6 + 24x^4 - 32x^2 + 16$

4. Expand $(4x-3)^4$ using the binomial theorem

Answer: $256x^4 - 768x^3 + 864x^2 - 432x + 81$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \frac{2x}{5}\right)^5$.

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \dfrac{2x}{5}\right)^5$.

Solution:

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \dfrac{2x}{5}\right)^5$.

Solution:

$$\left(3 - \frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1} \cdot 3^4 \cdot \frac{-2x}{5} + \binom{5}{2} \cdot 3^3 \cdot \left(\frac{-2x}{5}\right)^2$$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \dfrac{2x}{5}\right)^5$.

Solution:

$$\left(3 - \frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1} \cdot 3^4 \cdot \frac{-2x}{5} + \binom{5}{2} \cdot 3^3 \cdot \left(\frac{-2x}{5}\right)^2$$

$$+ \binom{5}{3} \cdot 3^2 \cdot \left(\frac{-2x}{5}\right)^3 + \binom{5}{4} \cdot 3 \cdot \left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3-\dfrac{2x}{5}\right)^5$.

Solution:

$$\left(3-\frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1}\cdot 3^4 \cdot \frac{-2x}{5} + \binom{5}{2}\cdot 3^3\cdot\left(\frac{-2x}{5}\right)^2 + \boxed{\binom{5}{3}\cdot 3^2\cdot\left(\frac{-2x}{5}\right)^3} + \binom{5}{4}\cdot 3\cdot\left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

Thus, the term in x^3 is $\binom{5}{3}\cdot 3^2\cdot\left(\dfrac{-2}{5}\right)^3 x^3$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \dfrac{2x}{5}\right)^5$.

Solution:

$$\left(3 - \frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1} \cdot 3^4 \cdot \frac{-2x}{5} + \binom{5}{2} \cdot 3^3 \cdot \left(\frac{-2x}{5}\right)^2 + \boxed{\binom{5}{3} \cdot 3^2 \cdot \left(\frac{-2x}{5}\right)^3} + \binom{5}{4} \cdot 3 \cdot \left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

Thus, the term in x^3 is $\boxed{\binom{5}{3}} \cdot 3^2 \cdot \left(\dfrac{-2}{5}\right)^3 x^3$

$\therefore$ The coefficient of x^3 is $\boxed{10} \cdot 9 \cdot \dfrac{-8}{125}$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3 - \frac{2x}{5}\right)^5$.

Solution:

$$\left(3 - \frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1} \cdot 3^4 \cdot \frac{-2x}{5} + \binom{5}{2} \cdot 3^3 \cdot \left(\frac{-2x}{5}\right)^2$$

$$+ \boxed{\binom{5}{3} \cdot 3^2 \cdot \left(\frac{-2x}{5}\right)^3} + \binom{5}{4} \cdot 3 \cdot \left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

Thus, the term in x^3 is $\boxed{\binom{5}{3}} \cdot \boxed{3^2} \cdot \left(\frac{-2}{5}\right)^3 x^3$

$\therefore$ The coefficient of x^3 is $\boxed{10} \cdot \boxed{9} \cdot \frac{-8}{125}$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3-\dfrac{2x}{5}\right)^5$.

Solution:

$$\left(3-\frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1}\cdot 3^4\cdot\frac{-2x}{5} + \binom{5}{2}\cdot 3^3\cdot\left(\frac{-2x}{5}\right)^2 + \boxed{\binom{5}{3}\cdot 3^2\cdot\left(\frac{-2x}{5}\right)^3} + \binom{5}{4}\cdot 3\cdot\left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

Thus, the term in x^3 is $\boxed{\binom{5}{3}}\cdot\boxed{3^2}\cdot\boxed{\left(\frac{-2}{5}\right)^3}x^3$

$\therefore$ The coefficient of x^3 is $\boxed{10}\cdot\boxed{9}\cdot\boxed{\dfrac{-8}{125}}$

The Binomial Theorem: Finding the coefficient of x^n

Example: Find the coefficient of x^3 in the expansion of $\left(3-\frac{2x}{5}\right)^5$.

Solution:

$$\left(3-\frac{2x}{5}\right)^5 = 3^5 + \binom{5}{1}\cdot 3^4\cdot\frac{-2x}{5} + \binom{5}{2}\cdot 3^3\cdot\left(\frac{-2x}{5}\right)^2 + \boxed{\binom{5}{3}\cdot 3^2\cdot\left(\frac{-2x}{5}\right)^3} + \binom{5}{4}\cdot 3\cdot\left(\frac{-2x}{5}\right)^4 + \left(\frac{-2x}{5}\right)^5$$

Thus, the term in x^3 is $\boxed{\binom{5}{3}}\cdot\boxed{3^2}\cdot\boxed{\left(\frac{-2}{5}\right)^3}x^3$

$\therefore$ The coefficient of x^3 is $\boxed{10}\cdot\boxed{9}\cdot\boxed{\frac{-8}{125}} = -\frac{144}{25}$

The Binomial Theorem: Finding the coefficient of x^n

1. Find the coefficient of x^4

   in the expansion of $\left(2 - \frac{x}{5}\right)^6$.

2. Find the coefficient of x^{-3}

   in the expansion of $\left(3 - \frac{2}{x}\right)^5$.

3. Find the coefficient of x^3 in the

   expansion of $\left(2x - \frac{1}{x}\right)^5$.

4. Find the constant term (coefficient of x^0)

   in the expansion of $\left(\frac{1}{x} + 2x\right)^6$.

The Binomial Theorem: Finding the coefficient of x^n

1. Find the coefficient of x^4

   in the expansion of $\left(2 - \frac{x}{5}\right)^6$.

   Answer: $\frac{12}{125}$

2. Find the coefficient of x^{-3}

   in the expansion of $\left(3 - \frac{2}{x}\right)^5$.

   Answer: -720

3. Find the coefficient of x^3 in the

   expansion of $\left(2x - \frac{1}{x}\right)^5$.

   Answer: -80

4. Find the constant term (coefficient of x^0)

   in the expansion of $\left(\frac{1}{x} + 2x\right)^6$.

   Answer: 160

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\, n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\, n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $(|\,x\,| < 1)$, using the Generalised Binomial Theorem.

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x, n \in \mathbb{R}, \; |\, x \,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!} \cdot x^2 + \frac{n(n-1)(n-2)}{3\,!} \cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $\;(|\, x \,| < 1)$, using the Generalised Binomial Theorem.

Solution:

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\, n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $(|\,x\,| < 1)$, using the Generalised Binomial Theorem.

Solution:

$$(1+x)^{-3} = 1 + (-3)\cdot x + \frac{(-3)(-3-1)}{2\,!}\cdot x^2 + \frac{(-3)(-3-1)(-3-2)}{3\,!}\cdot x^3 + \cdots$$

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\, n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $(|\,x\,| < 1)$, using the Generalised Binomial Theorem.

Solution:

$$(1+x)^{-3} = 1 + (-3)\cdot x + \frac{(-3)(-3-1)}{2\,!}\cdot x^2 + \frac{(-3)(-3-1)(-3-2)}{3\,!}\cdot x^3 + \cdots$$

$$= 1 - 3x + \frac{(-3)(-4)}{2\cdot 1}\cdot x^2 + \frac{(-3)(-4)(-5)}{3\cdot 2\cdot 1}\cdot x^3 + \cdots$$

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\ n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $\quad (|\,x\,| < 1)$,
using the Generalised Binomial Theorem.

Solution:

$$(1+x)^{-3} = 1 + (-3)\cdot x + \frac{(-3)(-3-1)}{2\,!}\cdot x^2 + \frac{(-3)(-3-1)(-3-2)}{3\,!}\cdot x^3 + \cdots$$

$$= 1 - 3x + \frac{(-3)(-4)}{2\cdot 1}\cdot x^2 + \frac{(-3)(-4)(-5)}{3\cdot 2\cdot 1}\cdot x^3 + \cdots$$

$$= 1 - 3x + 6x^2 - 10x^3 + \cdots$$

The Generalised Binomial Theorem

The Generalised Binomial Theorem : $x,\, n \in \mathbb{R},\ |\,x\,| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

Example: Expand $(1+x)^{-3}$ up to the term with x^3, $(|\,x\,| < 1)$, using the Generalised Binomial Theorem.

Solution:

$$(1+x)^{-3} = 1 + (-3)\cdot x + \frac{(-3)(-3-1)}{2\,!}\cdot x^2 + \frac{(-3)(-3-1)(-3-2)}{3\,!}\cdot x^3 + \cdots$$

$$= 1 - 3x + \frac{(-3)(-4)}{2\cdot 1}\cdot x^2 + \frac{(-3)(-4)(-5)}{3\cdot 2\cdot 1}\cdot x^3 + \cdots$$

$$= 1 - 3x + 6x^2 - 10x^3 + \cdots$$

Note: final result of the expansion is an **infinite series**

The Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

where $x,\, n \in \mathbb{R},\ |\,x\,| < 1.$

1. Expand $(1+2x)^{\frac{5}{2}}$, $|\,x\,| < \frac{1}{2}$, up to the first four terms using the Generalized Binomial Theorem.

2. Expand $(1+x^2)^{-3}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

3. Expand $\dfrac{1}{\sqrt{1+x}}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

4. Expand $\dfrac{1}{(1-x)}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

The Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\cdot x^2 + \frac{n(n-1)(n-2)}{3\,!}\cdot x^3 + \cdots$$

where $x,\, n \in \mathbb{R},\ |\,x\,| < 1.$

1. Expand $(1+2x)^{\frac{5}{2}}$, $|\,x\,| < \frac{1}{2}$, up to the first four terms using the Generalized Binomial Theorem.

   Answer: $1 + 5x + \dfrac{15}{2}x^2 + \dfrac{5}{2}x^3 + \cdots$

2. Expand $(1+x^2)^{-3}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

   Answer: $1 - 3x^2 + 6x^4 - 10x^6 + \cdots$

3. Expand $\dfrac{1}{\sqrt{1+x}}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

   Answer: $1 - \dfrac{1}{2}x + \dfrac{3}{8}x^2 - \dfrac{5}{16}x^3 + \cdots$

4. Expand $\dfrac{1}{(1-x)}$, $|\,x\,| < 1$, up to the first four terms using the Generalized Binomial Theorem.

   Answer: $1 + x + x^2 + x^3 + \cdots$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1$,

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\, n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\dfrac{1}{1.05}$.

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\, n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\dfrac{1}{1.05}$.

Solution: $\dfrac{1}{1.05} = (1.05)^{-1}$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\, n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\dfrac{1}{1.05}$.

Solution: $\dfrac{1}{1.05} = (1.05)^{-1}$

$= (1+0.05)^{-1}$. Here $n = -1$, and $x = 0.05 \Rightarrow |\,x\,| < 1$.

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\frac{1}{1.05}$.

Solution: $\frac{1}{1.05} = (1.05)^{-1}$

$= (1+0.05)^{-1}$. Here $n = -1$, and $x = 0.05 \Rightarrow |\,x\,| < 1$.

$$\therefore (1+0.05)^{-1} = 1 + (-1) \times 0.05 + \frac{(-1) \times (-1-1)}{2\,!} \times 0.05^2 + \cdots$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\, n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\frac{1}{1.05}$.

Solution: $\frac{1}{1.05} = (1.05)^{-1}$

$= (1+0.05)^{-1}$. Here $n=-1$, and $x = 0.05 \Rightarrow |\,x\,| < 1$.

$$\therefore (1+0.05)^{-1} = 1 + (-1)\times 0.05 + \frac{(-1)\times(-1-1)}{2\,!}\times 0.05^2 + \cdots$$

$$= 1 - 0.05 + 0.05^2 + \cdots$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 1: Use the first three terms of the Generalized Binomial Theorem to find an approximate value of $\dfrac{1}{1.05}$.

Solution: $\dfrac{1}{1.05} = (1.05)^{-1}$

$= (1+0.05)^{-1}$. Here $n=-1$, and $x = 0.05 \Rightarrow |\,x\,| < 1$.

$$\therefore (1+0.05)^{-1} = 1 + (-1)\times 0.05 + \frac{(-1)\times(-1-1)}{2\,!}\times 0.05^2 + \cdots$$

$$= 1 - 0.05 + 0.05^2 + \cdots$$

$$\approx 0.9525$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Solution: $\sqrt[3]{0.99} = (1-0.01)^{\frac{1}{3}}$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Solution: $\sqrt[3]{0.99} = (1-0.01)^{\frac{1}{3}}$

$= [1+(-0.01)]^{\frac{1}{3}}$. Here $n = \frac{1}{3}$, and $x = -0.01 \Rightarrow |\,x\,| < 1.$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\, n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Solution: $\sqrt[3]{0.99} = (1-0.01)^{\frac{1}{3}}$

$= [1+(-0.01)]^{\frac{1}{3}}.$ Here $n = \frac{1}{3}$, and $x = -0.01 \Rightarrow |\,x\,| < 1.$

$$\therefore (1-0.01)^{\frac{1}{3}} = [1+(-0.01)]^{\frac{1}{3}} = 1 + \frac{1}{3}\times(-0.01) + \frac{\frac{1}{3}\times(\frac{1}{3}-1)}{2!}\times(-0.01)^2 + \cdots$$

Approximation using the Binomial Theorem

Approximation using the Binomial Theorem :

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Solution: $\sqrt[3]{0.99} = (1-0.01)^{\frac{1}{3}}$

$$= [1+(-0.01)]^{\frac{1}{3}}.\quad \text{Here } n = \tfrac{1}{3}, \text{ and } x = -0.01 \Rightarrow |\,x\,| < 1.$$

$$\therefore (1-0.01)^{\frac{1}{3}} = [1+(-0.01)]^{\frac{1}{3}} = 1 + \frac{1}{3}\times(-0.01) + \frac{\frac{1}{3}\times(\frac{1}{3}-1)}{2!}\times(-0.01)^2 + \cdots$$

$$= 1 - \frac{0.01}{3} - \frac{0.0001}{9} + \cdots$$

Approximation using the Binomial Theorem

<u>Approximation using the Binomial Theorem :</u>

Given $(1+x)^n$, where $x,\ n \in \mathbb{R},\ |\,x\,| < 1,$ apply the Generalised Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \cdots$$

Example 2: Use the first three terms of the Generalised Binomial Theorem to find an approximate value of $\sqrt[3]{0.99}$.

Solution: $\sqrt[3]{0.99} = (1-0.01)^{\frac{1}{3}}$

$= [1+(-0.01)]^{\frac{1}{3}}$. Here $n = \frac{1}{3}$, and $x = -0.01 \Rightarrow |\,x\,| < 1$.

$$\therefore (1-0.01)^{\frac{1}{3}} = [1+(-0.01)]^{\frac{1}{3}} = 1 + \frac{1}{3} \times (-0.01) + \frac{\frac{1}{3} \times (\frac{1}{3}-1)}{2!} \times (-0.01)^2 + \cdots$$

$$= 1 - \frac{0.01}{3} - \frac{0.0001}{9} + \cdots$$

$$\approx 0.9967$$

Approximation using the Binomial Theorem

1. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.05)^{-3}$.

2. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.01)^{-3}$.

3. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.04)^{-2}$.

4. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.01)^{-1}$.

Approximation using the Binomial Theorem

1. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.05)^{-3}$.

   Answer: 0.8638

2. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.01)^{-3}$.

   Answer: 0.9706

3. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.04)^{-2}$.

   Answer: 0.9245

4. Use the first four terms of the Generalised Binomial Theorem to find an approximate value of $(1.01)^{-1}$.

   Answer: 0.9901

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Application of the Binomial Theorem in Error Analysis

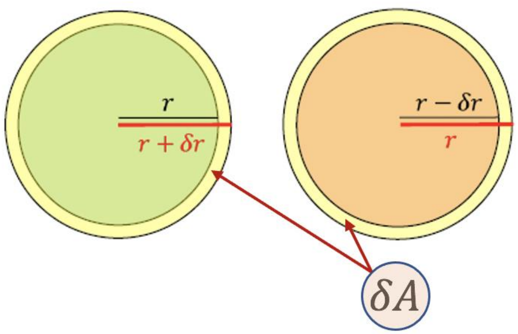
Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:

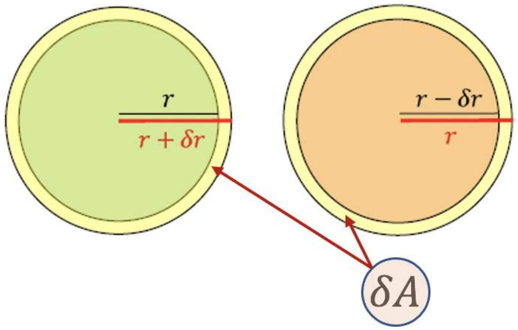


Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:

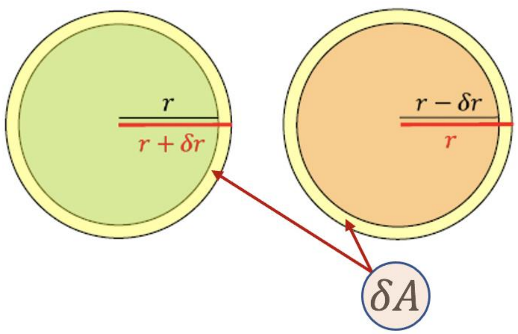
$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$



Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi r^2$.

Solution:



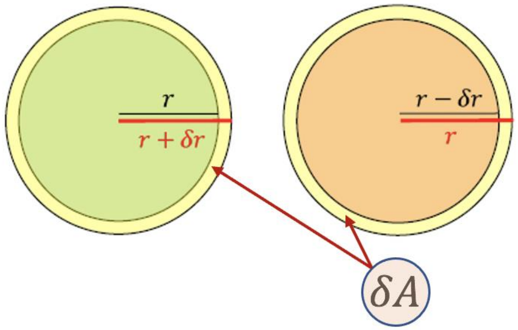
$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow\; A + \delta A = \pi(r + \delta r)^2$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

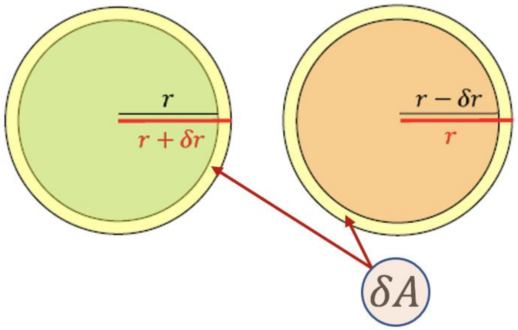
$$\Rightarrow\; A + \delta A = \pi(r + \delta r)^2$$

$$= \pi(r + 0.0125\, r)^2$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r. Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow\; A + \delta A = \pi(r + \delta r)^2$$

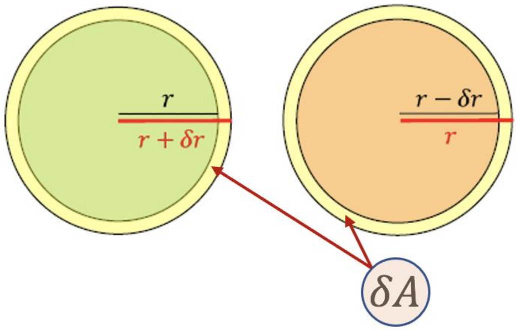
$$= \pi(r + 0.0125\, r)^2$$

$$= \pi r^2(1 + 0.0125)^2$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow\; A + \delta A = \pi(r + \delta r)^2$$

$$= \pi(r + 0.0125\, r)^2$$

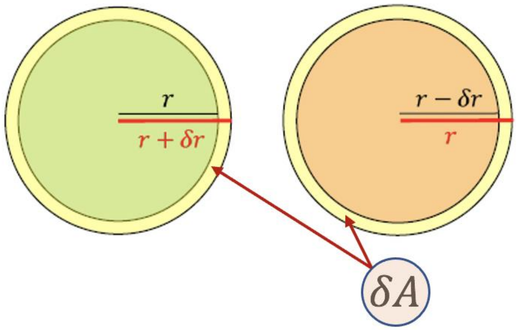
$$= \pi r^2(1 + 0.0125)^2$$

$$\approx A(1 + 2 \times 0.0125)$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:



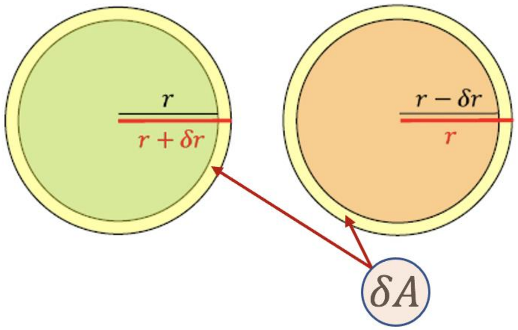
$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\begin{aligned}
\Rightarrow\; A + \delta A &= \pi(r + \delta r)^2 \\
&= \pi(r + 0.0125\, r)^2 \\
&= \pi r^2 (1 + 0.0125)^2 \\
&\approx A(1 + 2 \times 0.0125) \\
&= A + 0.025A
\end{aligned}$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi\, r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow \cancel{A} + \delta A = \pi(r + \delta r)^2$$

$$= \pi(r + 0.0125\, r)^2$$

$$= \pi r^2(1 + 0.0125)^2$$

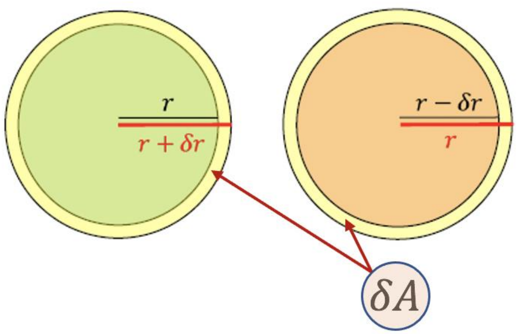
$$\approx A(1 + 2 \times 0.0125)$$

$$= \cancel{A} + 0.025 A$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow \cancel{A} + \delta A = \pi(r+\delta r)^2$$

$$= \pi(r + 0.0125\, r)^2$$

$$= \pi r^2 (1 + 0.0125)^2$$

$$\approx A(1 + 2 \times 0.0125)$$

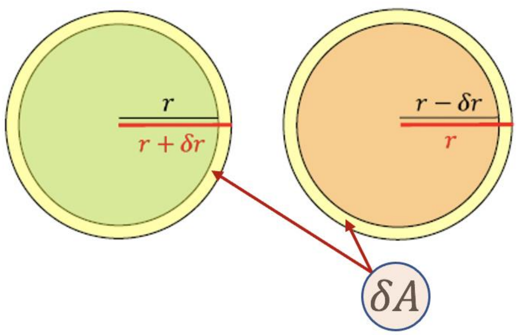
$$= \cancel{A} + 0.025A$$

$$\Rightarrow \delta A = 0.025A$$

Application of the Binomial Theorem in Error Analysis

Example: The radius r of a circle is measured with an error $\delta r = 1.25\%$ of r.
Use the approximation $(1+x)^n \approx 1 + nx$ to find the resulting error δA in the the calculated area. Area of a circle: $A = \pi r^2$.

Solution:



$$\delta r = 1.25\%\, r \;\Rightarrow\; \delta r = 0.0125\, r$$

$$\Rightarrow \cancel{A} + \delta A = \pi(r+\delta r)^2$$

$$= \pi(r + 0.0125\, r)^2$$

$$= \pi r^2 (1 + 0.0125)^2$$

$$\approx A(1 + 2 \times 0.0125)$$

$$= \cancel{A} + 0.025 A$$

$$\Rightarrow \delta A = 0.025 A$$

$\Rightarrow$ The error in the area is 2.5% of A

Application of the Binomial Theorem in Error Analysis

1. The diameter d of a circle is measured with an error of $\delta d = 1.5\%$ of d.
   Use $(1+x)^n \approx 1 + nx$ to find the resulting error, δA, in the calculated area.

2. The side l of a square is measured with an error of $\delta l = 1.2\%$ of l.
   Use $(1+x)^n \approx 1 + nx$ to find the resulting error δA, in the calculated area.

3. The radius r of the base of a right circular cone with fixed height h is measured with an error of $\delta r = 1.2\%$ of r.
   Use $(1+x)^n \approx 1 + nx$ to find the resulting error, δV, in the calculated volume.

4. The radius r of a sphere is measured with an error of $\delta r = 1.3\%$ of r. Use
   $(1+x)^n \approx 1 + nx + \dfrac{n(n-1)}{2}x^2$ to find the resulting error, δV, in the calculated volume.

Application of the Binomial Theorem in Error Analysis

1. The diameter d of a circle is measured with an error of $\delta d = 1.5\%$ of d.
   Use $(1+x)^n \approx 1+nx$ to find the resulting error, δA, in the calculated area.
   Area of a circle: $A = \pi r^2$

2. The side l of a square is measured with an error of $\delta l = 1.2\%$ of l.
   Use $(1+x)^n \approx 1+nx$ to find the resulting error δA, in the calculated area.
   Area of a square: $A = l^2$

3. The radius r of the base of a right circular cone with fixed height h is measured with an error of $\delta r = 1.2\%$ of r.
   Use $(1+x)^n \approx 1+nx$ to find the resulting error, δV, in the calculated volume.
   Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

4. The radius r of a sphere is measured with an error of $\delta r = 1.3\%$ of r. Use
   $(1+x)^n \approx 1+nx+\dfrac{n(n-1)}{2}x^2$ to find the resulting error, δV, in the calculated volume.
   Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Application of the Binomial Theorem in Error Analysis

1. The diameter d of a circle is measured with an error of $\delta d = 1.5\%$ of d.

   Use $(1+x)^n \approx 1 + nx$ to find the resulting error, δA, in the calculated area.

   Area of a circle: $A = \pi r^2$

   Answer: 3% of A

2. The side l of a square is measured with an error of $\delta l = 1.2\%$ of l.

   Use $(1+x)^n \approx 1 + nx$ to find the resulting error δA, in the calculated area.

   Area of a square: $A = l^2$

   Answer: 2.4% of A

3. The radius r of the base of a right circular cone with fixed height h is measured with an error of $\delta r = 1.2\%$ of r.

   Use $(1+x)^n \approx 1 + nx$ to find the resulting error, δV, in the calculated volume.

   Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

   Answer: 2.4% of V

4. The radius r of a sphere is measured with an error of $\delta r = 1.3\%$ of r. Use

   $$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2$$ to find the resulting error, δV, in the calculated volume.

   Volume of a sphere: $V = \frac{4}{3}\pi r^3$

   Answer: 3.95% of V

Homework Questions

1. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x)^{\frac{1}{2}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{2}$ in $1(i)$, find the approximate value of $\sqrt{2}$, give your answer in 4 d. p.

2. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x^3)^{\frac{1}{3}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{5}$ in $2(i)$, find the approximate value of $\sqrt[3]{124}$, give your answer in 4 d. p.

3. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x^4)^{\frac{1}{4}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{2}$ in $3(i)$, find the approximate value of $\sqrt[4]{15}$, give your answer in 4 d. p.

Homework Questions

1. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x)^{\frac{1}{2}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{2}$ in $1(i)$, find the approximate value of $\sqrt{2}$, give your answer in 4 d. p.

   Answer: $(i)\quad 1 - \dfrac{1}{2}x - \dfrac{1}{8}x^2 - \dfrac{1}{16}x^3 + \cdots \quad (ii)\quad 1.4219$

2. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x^3)^{\frac{1}{3}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{5}$ in $2(i)$, find the approximate value of $\sqrt[3]{124}$, give your answer in 4 d. p.

   Answer: $(i)\quad 1 - \dfrac{1}{3}x^3 - \dfrac{1}{9}x^6 - \dfrac{5}{81}x^9 + \cdots \quad (ii)\quad 4.9866$

3. (i) Use the first four terms of the Generalised Binomial Theorem to find the expansion of $(1-x^4)^{\frac{1}{4}}, |\,x\,| < 1$.

   (ii) By substituting $x = \frac{1}{2}$ in $3(i)$, find the approximate value of $\sqrt[4]{15}$, give your answer in 4 d. p.

   Answer: $(i)\quad 1 - \dfrac{1}{4}x^4 - \dfrac{3}{32}x^8 - \dfrac{7}{128}x^{12} + \cdots \quad (ii)\quad 1.9680$

THANKS FOR YOUR ATTENTION