Mathematics for Computer Scientists COMP1046
Tutorial 1 Solutions

1. Construct truth tables.

p	q	r	$q \vee r$	$p \to (q \vee r)$	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to r$	$(p \to (q \vee r)) \leftrightarrow ((p \wedge \neg q) \to r)$
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	F	T	T	F	T	T

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

2. Using tables of logical equivalences from the lectures:

   (1).

$$
\begin{array}{lll}
 & (p \to r) \vee (q \to r) & \\
\equiv & (\neg p \vee r) \vee (\neg q \vee r) & \text{by rule (20)} \\
\equiv & (\neg p \vee \neg q) \vee (r \vee r) & \text{by rules (8) and (10)} \\
\equiv & (\neg p \vee \neg q) \vee r & \text{by rule (5)} \\
\equiv & \neg(\neg p \vee \neg q) \to r & \text{by rule (22)} \\
\equiv & (p \wedge q) \to r & \text{by rule (15)}
\end{array}
$$

(2).

$$
\begin{aligned}
& \neg p \to (q \to r) \\
\equiv\ & p \vee (q \to r) && \text{by rule (20)} \\
\equiv\ & p \vee (\neg q \vee r) && \text{by rule (20)} \\
\equiv\ & \neg q \vee (p \vee r) && \text{by rules (8) and (10)} \\
\equiv\ & q \to (p \vee r) && \text{by rule (22)}
\end{aligned}
$$

(3). To be logically equivalent, two propositions must always take the same truth value.
Provide counterexample: $p \equiv F, q \equiv F, r \equiv F$, then $(p \to q) \to r$ is F and $p \to (q \to r)$ is T.
Therefore they are not logically equivalent.

3. This is one solution, others are possible:

   (a) $\neg \exists x (P(x) \wedge Q(x))$

   (b) $\forall x (Q(x) \to R(x))$

   (c) $\neg \exists x (P(x) \wedge R(x))$

   (d) No, because an individual c with $P(c), \neg Q(c), R(c)$ is true of (a) and (b) but makes (c) false.