Lecture 3

Topics covered in this lecture session

1. Trigonometric functions.
2. More about Trigonometric functions.
3. Solving Trigonometric equations.

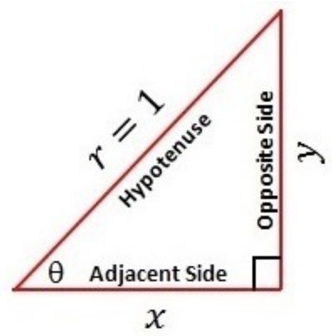
Trigonometric functions

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r} = \frac{x}{1} = x$$

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r} = \frac{y}{1} = y$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cos\theta \neq 0$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \quad ; \quad \sin\theta \neq 0$$



$$\sec\theta = \frac{1}{\cos\theta} \quad ; \quad \cos\theta \neq 0$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad ; \quad \sin\theta \neq 0$$

Trigonometric identities

The basic trigonometric identities are:

$$\cos^2\theta + \sin^2\theta = 1 \qquad (1)$$

$$1 + \tan^2\theta = \sec^2\theta \quad ; \quad \cos\theta \neq 0$$

obtained by dividing (1) by $\cos^2\theta$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta \quad ; \quad \sin\theta \neq 0$$

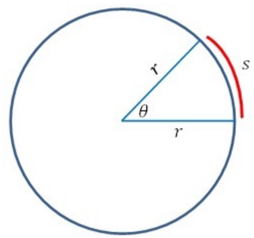
obtained by dividing (1) by $\sin^2\theta$

Conversion (degree ↔ radians)

By definition, the length of the enclosed arc (s) is equal to the radius (r) multiplied by the magnitude of the angle (θ) in radians.

$$s = r\,\theta \quad \Rightarrow \quad \theta = \frac{s}{r}$$

$\therefore$ For one complete revolution (360^o), the magnitude in radians is



$$360^\circ = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$

$$\Rightarrow \boxed{\pi = 180^\circ}$$

An important relation to convert degrees to radians and vice-versa.

Conversion (degree ↔ radians)

$$\boxed{\text{angle in radians} = \text{angle in degrees} \times \left(\frac{\pi}{180°}\right)}$$

$$\boxed{\text{angle in degrees} = \text{angle in radians} \times \left(\frac{180°}{\pi}\right)}$$

$$45° = 45° \times \left(\frac{\pi}{180°}\right) = \frac{\pi}{4}\ \text{radians}$$

$$270° = 270° \times \left(\frac{\pi}{180°}\right) = \frac{3\pi}{2}\ \text{radians}$$

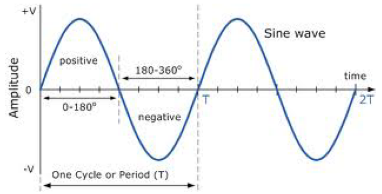
$$\frac{\pi}{6}\ \text{radians} = \left(\frac{180°}{\pi}\right) \times \frac{\pi}{6} = 30°$$

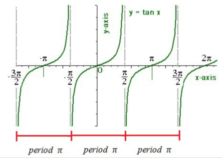
$$\frac{5\pi}{12}\ \text{radians} = \left(\frac{180°}{\pi}\right) \times \frac{5\pi}{12} = 75°$$

Periodic functions

If $f(x + p) = f(x)$, the function f is called periodic and p is defined as its period. The smallest positive value of p is called the Principal period of f.

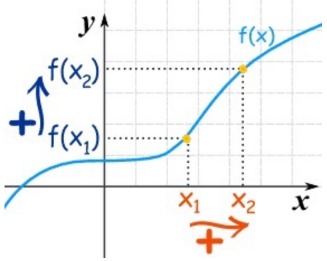
Trigonometric function	Principal Period
cos	2π
sin	
sec	
cosec	
tan	π
cot	



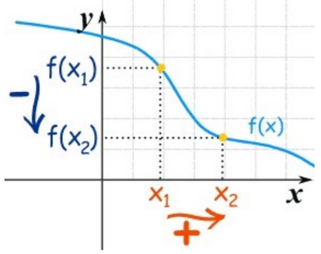


Increasing and Decreasing functions

If $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$, then the function f is said to be an increasing (↑) function.



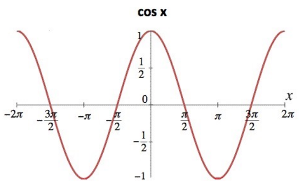
If $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$, then the function f is said to be a decreasing (↓) function.

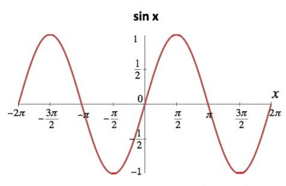


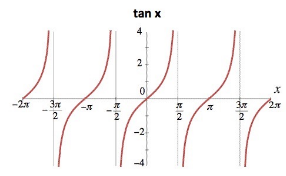
Increasing and Decreasing functions

Quadrant	1	2	3	4
cos	↓	↓	↑	↑
sin	↑	↓	↓	↑
tan	↑	↑	↑	↑

Quadrant	1	2	3	4
sec	↑	↑	↓	↓
cosec	↓	↑	↑	↓
cot	↓	↓	↓	↓



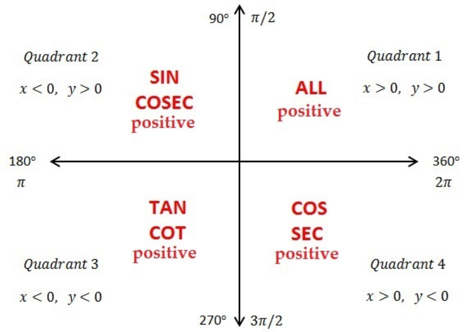




Signs of Trigonometric functions in the quadrants

$$\cos\theta = \frac{x}{r} = x$$

$$\sin\theta = \frac{y}{r} = y$$



Example If $\tan\theta = \dfrac{-3}{4}$; $\dfrac{3\pi}{2} \leq \theta \leq 2\pi$, find $\cos\theta$ and $\sin\theta$.

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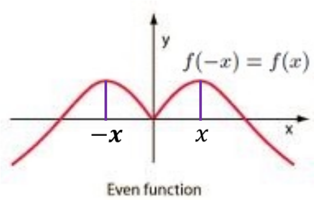
Even and Odd Trigonometric functions

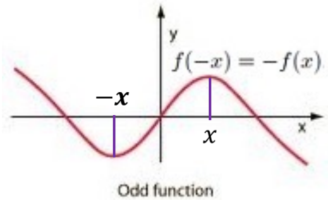
The function f is said to be an even function if $f(-x) = f(x)$



e.g. $\cos(-\theta) = \cos\theta$

$\Rightarrow$ cos is an even function.

The function f is said to be an odd function if $f(-x) = -f(x)$



e.g. $\sin(-\theta) = -\sin\theta$

$\Rightarrow$ sin is an odd function.

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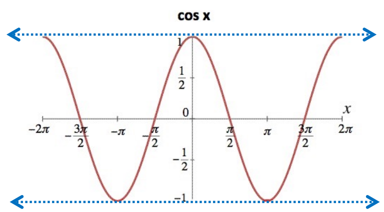
Range of Trigonometric functions

From the graph of cosine function, it is clear that

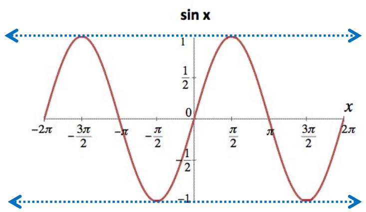
$$-1 \leq \cos\theta \leq 1$$



$\therefore$ Range of cos function is $[-1, 1]$

Similarly,

$$-1 \leq \sin\theta \leq 1$$



$\therefore$ Range of sin function is $[-1, 1]$

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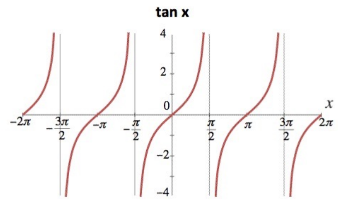
Range of Trigonometric functions

Also, $\tan\theta \in \mathbb{R}$

$\cot\theta \in \mathbb{R}$

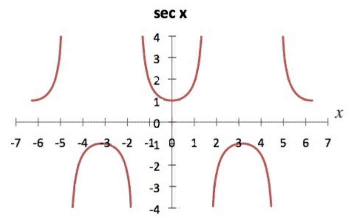


$\therefore$ Range of tan function is $\mathbb{R}$.

Range of cot function is $\mathbb{R}$.

And, $\sec\theta \leq -1$ **or** $\sec\theta \geq 1$

$\mathrm{cosec}\,\theta \leq -1$ **or** $\mathrm{cosec}\,\theta \geq 1$



$\therefore$ Range of sec function is $\mathbb{R} - (-1, 1)$.

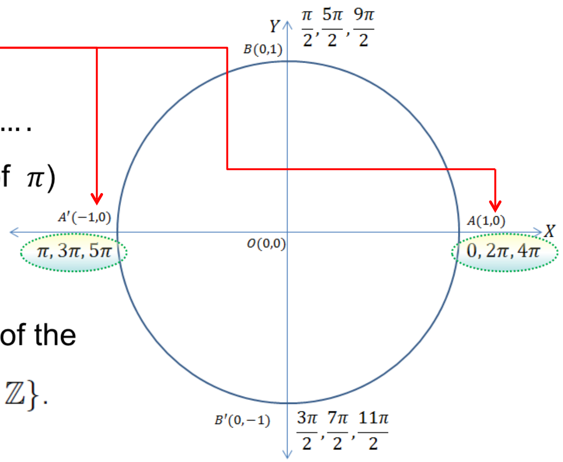
Range of cosec function is $\mathbb{R} - (-1, 1)$.

Sets of Zeros of Trigonometric functions

Consider $\sin\theta = 0$

Then, $\theta = 0, \pi, 2\pi, 3\pi, \ldots.$

(i.e. multiples of π)



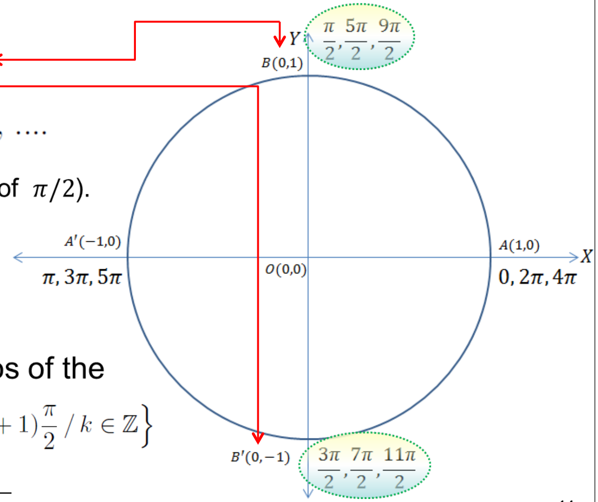
Thus, the set of zeros of the sin function $\{k\pi \,/\, k \in \mathbb{Z}\}$.

Sets of Zeros of Trigonometric functions

Similarly, $\cos\theta = 0$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots.$$

(i.e. odd multiples of $\pi/2$).



Thus, the set of zeros of the $\cos$ function is $\left\{(2k+1)\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$

Note…

Function	**Domain**	**Range**	**Set of zeros**	**Period**
cos	$\mathbb{R}$	$[-1,1]$	$\left\{(2k+1)\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$	2π
sin	$\mathbb{R}$	$[-1,1]$	$\{k\pi \,/\, k \in \mathbb{Z}\}$	2π
tan	$\mathbb{R} - \left\{(2k+1)\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$	$\mathbb{R}$	$\{k\pi \,/\, k \in \mathbb{Z}\}$	π
sec	$\mathbb{R} - \left\{(2k+1)\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$	$\mathbb{R} - (-1,1)$	ϕ	2π
cosec	$\mathbb{R} - \{k\pi \,/\, k \in \mathbb{Z}\}$	$\mathbb{R} - (-1,1)$	ϕ	2π
cot	$\mathbb{R} - \{k\pi \,/\, k \in \mathbb{Z}\}$	$\mathbb{R}$	$\left\{(2k+1)\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$	π

Solving Trigonometric equations

A trigonometric equation is an equation containing one or more trigonometric functions of the variable, say θ.

Solving for θ means finding the values of θ (in given interval) which makes the trigonometric equation true.

e.g. The solution of $\cos\theta = \frac{1}{2}$ in $\left(0, \frac{\pi}{2}\right)$ is $\frac{\pi}{3}$ radian

whereas its solution in $(0, 2\pi)$ is

$$\frac{\pi}{3} \quad \text{or} \quad \left(2\pi - \frac{\pi}{3}\right) = \frac{5\pi}{3} \text{ radians.}$$