Tutorial 4 - Linear Mappings
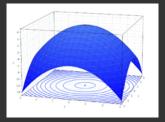
COMP1046 - Maths for Computer Scientists

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Consider the set

$$E = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid 3x_1 = x_2 + x_3 + x_4\}$$

and the linear mapping $f : E \to \mathbb{R}^4$,

$$
f(x_1, x_2, x_3, x_4) = (\ \ 3x_1 + x_3 + 2x_4,
$$
$$
2x_1 - x_2 + 2x_3 + x_4,
$$
$$
x_1 + x_2 - x_3 + x_4,
$$
$$
4x_1 + x_2 + 3x_4 \ \ ).
$$

1. Show that $(E, +, .)$ is a vector space, where the internal and external composition laws are the usual real number addition and scalar product.

2. Construct a basis for $(E, +, .)$.

3. Compute $\dim(E)$.

4. Compute $\ker(f)$.

5. Compute $\dim(\ker(f))$.

6. Compute $\operatorname{Im}(f)$.

7. Compute $\dim(\operatorname{Im}(f))$ based on your result for $\operatorname{Im}(f)$.

8. Are your results from question parts 5 and 7 confirmed by the Rank-Nullity Theorem?