The University of Nottingham Ningbo China

Centre for English Language Education

AUTUMN SEMESTER 2017-2018 ANSWERS

SCIENCE A – Physics

Time allowed: **TWO HOURS**

Candidates may complete the front cover of the answer book and sign the attendance card.

Candidates must NOT start writing their answers until told to do so.

There are 6 questions. ATTEMPT ANY 4 QUESTIONS. Each question is worth 25 marks.

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception: those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination.
Subject-specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones, may be used.

DO NOT turn the examination paper over until instructed to do so.

INFORMATION FOR INVIGILATORS:
Please collect the examination paper and the answer booklets at the end of the exam.
A 15-minute warning should be announced before the end of the exam.

Constants:

$$k = 8.99 \times 10^{9}\mathrm{N \cdot m^2/C^2}$$
$$\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{C^2/N \cdot m^2}$$
$$\mathrm{g} = 9.80\ \mathrm{m}/s^2$$
$$\mu_0 = 4\pi \times 10^{-7}\mathrm{T \cdot m/A}$$

Q.1 (25 Marks)

Answer each of the following questions.

A. A stone is thrown vertically upward with a speed of 24.0 m/s.

(i) How fast is it moving when it reaches a height of 13.0 m?

(ii) How much time is required to reach this height?

(iii) Why are there two answers to (ii)?

Answer:

Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 24.0$ m/s, $a = -9.8$ m/s^2, and $y - y^0 = 13.0$ m.

(*i*) The velocity can be found from Eq, 2-12c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{\left(24.0\,\text{m/s}\right)^2 + 2\left(-9.80\,\text{m/s}^2\right)\left(13.0\text{ m}\right)} = \pm 17.9\,\text{m/s}$$

Thus the speed is $\boxed{\left|v\right| = 17.9\,\text{m/s}}$.

(*ii*) The time to reach that height can be found from Eq. 2-12b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2\left(24.0\,\text{m/s}\right)}{-9.80\,\text{m/s}^2}t + \frac{2\left(-13.0\text{ m}\right)}{-9.80\,\text{m/s}^2} = 0 \quad \rightarrow$$

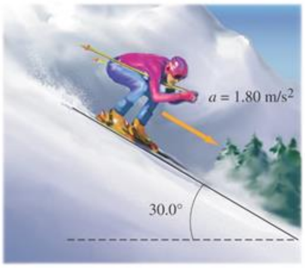
$$t^2 - 4.898t + 2.653 = 0 \quad \rightarrow \quad \boxed{t = 4.28\text{ s},\ 0.620\text{ s}}$$

(*iii*) There are two times at which the object reaches that height – once on the way up (t = 0.620 s), and once on the way down (t = 4.28 s).

B. A skier is accelerating down a 30.0° hill at 1.80 m/s^2, as shown in the figure below.

(i) What is the vertical component of her acceleration?

(ii) How long will it take her to reach the bottom of the hill, assuming she starts from rest and accelerates uniformly, if the elevation change is 325 m?



Answer:

Choose downward to be the positive *y* direction for this problem. Her acceleration is directed along the slope.

(*i*) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_y = a\sin\theta = (1.80\,\mathrm{m/s^2})\sin 30.0^\circ = \boxed{0.900\,\mathrm{m/s^2}}$.

(*ii*) The time to reach the bottom of the hill is calculated from the following equation, with a *y* displacement of

325 m, $v_{y0} = 0$, and $a_y = 0.900\,\mathrm{m/s^2}$.

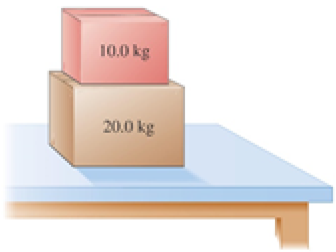
$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 325\ \mathrm{m} = 0 + 0 + \tfrac{1}{2}(0.900\,\mathrm{m/s^2})(t)^2 \;\rightarrow$$

$$t = \sqrt{\frac{2(325\ \mathrm{m})}{(0.900\,\mathrm{m/s^2})}} = \boxed{26.9\ \mathrm{s}}$$

C. A 20.0 kg box rests on a table,as shown in the figure below.

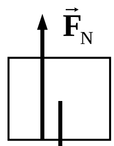
(i) What is the weight of the box and the normal force acting on it?

(ii) A 10.0 kg box is placed on top of the 20.0 kg box, also shown in the figure below. Determine the normal force that the table exerts on the 20.0 kg box and the normal force that the 20.0 kg box exerts on the 10.0 kg box.

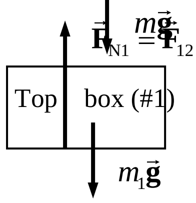


Answer:

(*i*) The 20.0 kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\ \mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{196\,\mathrm{N}}$. Since the box is at rest, the net force on the box must be 0, and so the normal force must also be $\boxed{196\,\mathrm{N}}$.



(*ii*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's third law. $\vec{\mathbf{F}}_{\mathrm{N2}}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must be 0. Write Newton's second law in the vertical direction for each box, taking the upward direction to be positive.

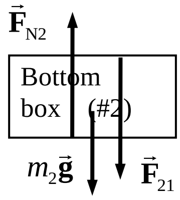


$$\sum F_1 = F_{\mathrm{N1}} - m_1 g = 0$$

$$F_{\mathrm{N1}} = m_1 g = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{98.0\ \mathrm{N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\mathrm{N2}} - F_{21} - m_2 g = 0$$

$$F_{\mathrm{N2}} = F_{21} + m_2 g = 98.0\ \mathrm{N} + (20.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{294\ \mathrm{N}}$$



D. The position of a small object is given by the equation $x = 5 + 5t$, where t is in seconds and x is in metres.

(i) Plot x as a function of t from $t = 0$ to $t = 3.0$ s.

Obvious; a straight line with an intercept of 5, and a slope value of plus 5.

(ii) What does the value for the slope of the line for $x = 5 + 5t$ mean?

Reference to the following:

- The rate of change of x with respect to t.
- For every increase in 1 second, there is a corresponding increase of 5 metres.

(iii) If the slope of the line in part (ii) is doubled, what will be the equation that now describes the position of the small object?

$$x = 5 + 10t$$

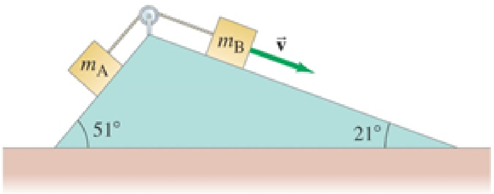
(iv) Evaluate the integral of the equation $x = 5 + 5t$ between $t = 0$ to $t = 3.0$ s? What does the integral mean in this context? Use appropriate graphs to explain your reasoning.

Obvious; hopefully ☺

Q.2 (25 Marks)

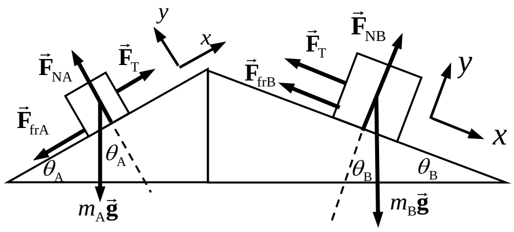
Answer each of the following questions.

A. Two masses m_A = 2.0 kg and m_B = 5.0 kg are on inclines and are connected together by a string as shown in the figure below. The coefficient of kinetic friction between each mass and its incline is μ_k = 0.30. If m_A moves up, and m_B moves down, determine their acceleration.



Answer:

We define the positive *x* direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two *x*-equations to find the acceleration.



Block A:

$$\sum F_{yA} = F_{NA} - m_A g\cos\theta_A = 0 \;\rightarrow\; F_{NA} = m_A g\cos\theta_A$$
$$\sum F_{xA} = F_T - m_A g\sin\theta - F_{frA} = m_A a$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g\cos\theta_B = 0 \;\rightarrow\; F_{NB} = m_B g\cos\theta_B$$
$$\sum F_{xB} = m_B g\sin\theta - F_{frB} - F_T = m_B a$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{fr} = \mu F_N$.

$$m_A a = F_T - m_A g\sin\theta_A - \mu_A m_A g\cos\theta_A \quad ; \quad m_B a = m_B g\sin\theta_B - \mu_B m_B g\cos\theta_B - F_T$$
$$m_A a + m_B a = F_T - m_A g\sin\theta_A - \mu_A m_A g\cos\theta_A + m_B g\sin\theta_B - \mu_B m_B g\cos\theta_B - F_T \;\rightarrow$$
$$a = g\left[\frac{-m_A(\sin\theta_A + \mu_A\cos\theta_A) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\ \mathrm{m/s^2})\left[\frac{-(2.0\ \mathrm{kg})(\sin 51^\circ + 0.30\cos 51^\circ) + (5.0\ \mathrm{kg})(\sin 21^\circ - 0.30\cos 21^\circ)}{(7.0\ \mathrm{kg})}\right]$$
$$= \boxed{-2.2\ \mathrm{m/s^2}}$$

B. A 46.0 kg crate, starting from rest, is pulled across a floor with a constant horizontal force of 225 N. For the first 11.0 m the floor is frictionless, and for the next 10.0 m the coefficient of friction is 0.20. What is the final speed of the crate after being pulled these 21.0 m?

Answer:

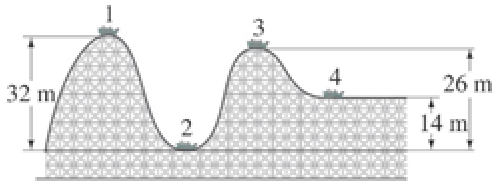
For the first part of the motion, the net force doing work is the 225 N force. For the second part of the motion, both the 225 N force and the force of friction do work. The friction force is the coefficient of friction times the normal force, and the normal force is equal to the weight.The work-energy theorem is then used to find the final speed.

$$W_{\text{total}} = W_1 + W_2 = F_{\text{pull}} d_1 \cos 0° + F_{\text{pull}} d_2 \cos 0° + F_f d_2 \cos 180° = \Delta K = \tfrac{1}{2} m\left(v_f^2 - v_i^2\right) \rightarrow$$

$$v_f = \sqrt{\frac{2\left[F_{\text{pull}}\left(d_1 + d_2\right) - \mu_k mgd_2\right]}{m}}$$

$$= \sqrt{\frac{2\left[(225\,\text{N})(21.0\,\text{m}) - (0.20)(46.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)(10.0\,\text{m})\right]}{(46.0\,\text{kg})}} = \boxed{13\,\text{m/s}}$$

C. Suppose the roller-coaster shown in the figure below passes point 1 with a speed of 1.70 m/s. If the average force of friction is equal to 0.23 of its weight, with what speed will it reach point 2? The distance traveled is 45.0 m.



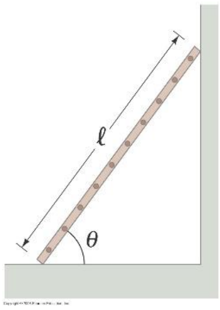
Answer:

Since there is a non-conservative force, apply energy conservation with the dissipative friction term. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational potential energy. We have $v_1 = 1.70\,\text{m/s}$, $y_1 = 32\,\text{m}$, and $y_2 = 0$. Solve for v_2. Note that the dissipated energy is given by $F_{\text{fr}} d = 0.23 mgd$.

$$\tfrac{1}{2} mv_1^2 + mgy_1 = \tfrac{1}{2} mv_2^2 + mgy_2 + 0.23mgd \rightarrow v_2 = \sqrt{-0.46gd + v_1^2 + 2gy_1}$$

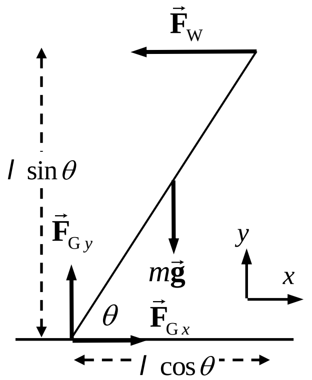
$$= \sqrt{-0.46\left(9.80\,\text{m/s}^2\right)(45.0\text{ m}) + (1.70\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(32\,\text{m})} = 20.67\,\text{m/s} \approx \boxed{21\,\text{m/s}}$$

D. A uniform ladder of mass *m* and length l leans at an angle θ against a frictionless wall, as shown in the figure below. If the coefficient of static friction between the ladder and the ground is μ_s, determine a formula for the minimum angle at which the ladder will not slip.



Answer:

Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.



$$\sum \tau = F_W l \sin\theta - mg\left(\tfrac{1}{2} l \cos\theta\right) = 0 \rightarrow F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{Gx} - F_W = 0 \rightarrow F_{Gx} = F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_y = F_{Gy} - mg = 0 \rightarrow F_{Gy} = mg$$

For the ladder to not slip, the force at the ground F_{Gx} must be less than or equal to the maximum force of static friction.

$$F_{Gx} \le \mu F_N = \mu F_{Gy} \rightarrow \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu mg \rightarrow \frac{1}{2\mu} \le \tan\theta \rightarrow \theta \ge \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Thus the minimum angle is $\boxed{\theta_{min} = \tan^{-1}\left(\frac{1}{2\mu}\right)}$.

Q.3 (25 Marks)

Answer each of the following questions.

A. A 130 kg astronaut (including space suit) acquires a speed of 2.50 m/s by pushing off with his legs from a 1700 kg space capsule.

(i) What is the change in speed of the space capsule?

(ii) If the push lasts 0.500 s, what is the average force exerted by each on the other? As the reference frame, use the position of the capsule before the push.

Answer:

(*i*) The momentum of the astronaut–space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_A = v_B = 0.$ We also have $v'_A = 2.50\,\text{m/s}.$

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = 0 = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_B = -v'_A \frac{m_A}{m_B} = -(2.50\,\text{m/s})\frac{130\text{ kg}}{1700\text{ kg}} = -0.1912\,\text{m/s} \approx \boxed{-0.19\,\text{m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

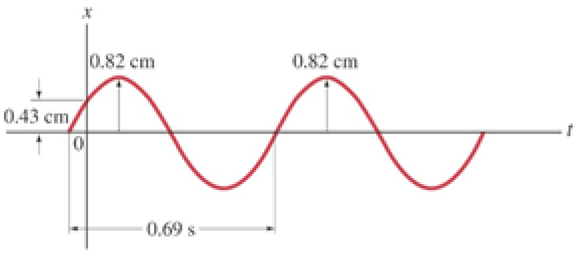
(*ii*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m(v'_A - v_A)}{\Delta t} = \frac{(130\text{ kg})(2.50\,\text{m/s} - 0)}{0.500\text{ s}} = \boxed{6.5\times10^2\,\text{N}}$$

B. The graph of displacement vs. time for a small mass *m* at the end of a spring is shown in the figure below. At *t* = 0, *x* = 0.43 cm.

(i) If *m* = 9.5 g, find the spring constant, *k.*

(ii) Write the equation for displacement *x* as a function of time.



Answer:

(*i*) From the graph, the period is 0.69 s. The period and the mass can be used to find the spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{0.0095\,\text{kg}}{(0.69\,\text{s})^2} = 0.7877\,\text{N/m} \approx \boxed{0.79\,\text{N/m}}$$

(*ii*) From the graph, the amplitude is 0.82 cm. The phase constant can be found from the initial conditions.

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right) = (0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t + \phi\right)$$

$$x(0) = (0.82\,\text{cm})\cos\phi = 0.43\text{cm} \rightarrow \phi = \cos^{-1}\frac{0.43}{0.82} = \pm 1.02\,\text{rad}$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$x = \boxed{(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t - 1.0\right)} \text{ or } (0.82\,\text{cm})\cos(9.1t - 1.0)$$

C. An object with mass 2.7 kg is executing simple harmonic motion, attached to a spring with spring constant k = 280 N/m. When the object is 0.020 m from its equilibrium position, it is moving with a speed of 0.55 m/s.

(i) Calculate the amplitude of the motion.

(ii) Calculate the maximum speed attained by the object.

Answer:

(*i*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \rightarrow$$

$$A = \sqrt{\frac{m}{k}v^2 + x^2} = \sqrt{\frac{2.7\text{ kg}}{280\,\text{N/m}}(0.55\,\text{m/s})^2 + (0.020\text{ m})^2} = 5.759\times10^{-2}\text{m} \approx \boxed{5.8\times10^{-2}\text{m}}$$

(*ii*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 \rightarrow$$

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = (5.759\times10^{-2}\text{m})\sqrt{\frac{280\,\text{N/m}}{2.7\text{ kg}}} = 0.5865\,\text{m/s} \approx \boxed{0.59\,\text{m/s}}$$

D. A fire hose exerts a force on the person holding it. This is because the water accelerates as it goes from the hose through the nozzle. How much force is required to hold a 7.0 cm diameter hose delivering 450 L/min through a 0.75 cm diameter nozzle?

Answer:

There is a forward force on the exiting water, and so by Newton's third law there is an equal force pushing backwards on the hose. To keep the hose stationary, you push forward on the hose, and so the hose pushes backwards on you. So the force on the exiting water is the same magnitude as the force on the person holding the hose. Use Newton's second law and the equation of continuity to find the force. Note that the 450 L/min flow rate is the volume of water being accelerated per unit time. Also, the flow rate is the product of the cross-sectional area of the moving fluid, times the speed of the fluid, and so

$$\frac{V}{t} = A_1 v_1 = A_2 v_2 .$$

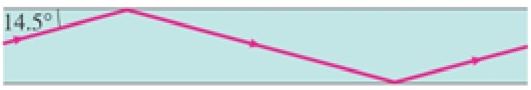
$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_2 - v_1}{t} = \rho\left(\frac{V}{t}\right)(v_2 - v_1) = \rho\left(\frac{V}{t}\right)\left(\frac{A_2 v_2}{A_2} - \frac{A_1 v_1}{A_1}\right) = \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$$= \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{\pi r_2^2} - \frac{1}{\pi r_1^2}\right)$$

$$= \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(\frac{450\ \mathrm{L}}{\mathrm{min}}\times\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\times\frac{1\ \mathrm{m}^3}{1000\ \mathrm{L}}\right)^2\left(\frac{1}{\pi\,\tfrac{1}{2}\left(0.75\times10^{-2}\ \mathrm{m}\right)^2} - \frac{1}{\pi\,\tfrac{1}{2}\left(7.0\times10^{-2}\ \mathrm{m}\right)^2}\right)$$

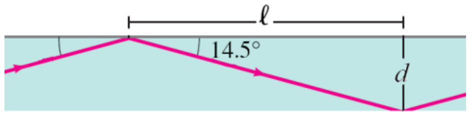
$$= 1259\ \mathrm{N} \approx \boxed{1300\ \mathrm{N}}$$

E. A ray of light, after entering a light fibre, reflects at an angle of 14.5° with the long axis of the fibre, as shown in the figure below. Calculate the distance along the axis of the fibre that the light ray travels between successive reflections off the sides of the fibre. Assume that the fibre has an index of refraction of 1.55 and is 1.40×10^{-4} m in diameter.



Answer:

The ray reflects at the same angle, so each segment makes a 14.5° angle with the side. We find the distance l between reflections from the definition of the tangent function.



$$\tan\theta = \frac{d}{\mathrm{l}} \rightarrow \mathrm{l} = \frac{d}{\tan\theta} = \frac{1.40\times10^{-4}\ \mathrm{m}}{\tan\ 14.5°} = \boxed{5.41\times10^{-4}\ \mathrm{m}}$$

Q.4 (25 Marks)

Answer each of the following questions.

A. Particles of charge +75, +48, and -85 µC are placed in a line as shown in the figure below. The centre one is 0.35 m from each of the others. Calculate the net force on each charge due to the other two.

Answer:

Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions

$$k = 8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\ .$$

$$\vec{\mathbf{F}}_{+75} = -k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(75\mu\mathrm{C})(85\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} = -147.2\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-150\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{+48} = k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(48\mu\mathrm{C})(85\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = 563.5\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{560\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{-85} = -k\frac{(85\mu\mathrm{C})(75\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} - k\frac{(85\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = -416.3\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-420\,\mathrm{N}\,\hat{\mathbf{i}}}$$

B. A flat square sheet of thin aluminum foil, 25 cm on a side, carries a uniformly distributed 275 ηC charge. What, approximately, is the electric field

(i) 1.0 cm above the centre of the sheet and

(ii) 15 m above the centre of the sheet?

Answer:

(*i*) When close to the sheet, we approximate it as an infinite sheet. We assume the charge is over both surfaces of the aluminum.

$$E = \frac{\sigma}{2\varepsilon_o} = \frac{\dfrac{275\times10^{-9}\,\mathrm{C}}{(0.25\,\mathrm{m})^2}}{2\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)} = \boxed{2.5\times10^5\ \mathrm{N/C},\ \text{away from the sheet}}$$

(*ii*) When far from the sheet, we approximate it as a point charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{275\times10^{-9}\,\mathrm{C}}{(15\,\mathrm{m})^2} = \boxed{11\,\mathrm{N/C},\ \text{away from the sheet}}$$

C. A 32 cm diameter conducting sphere is charged to 680 V relative to $V = 0$ at $r = \infty$.

(i) What is the surface charge density σ?

(ii) At what distance will the potential due to the sphere be only 25 V?

Answer:

(i) The potential at the surface of a charged sphere is

$$V_0 = \frac{Q}{4\pi\varepsilon_0 r_0} \quad \rightarrow \quad Q = 4\pi\varepsilon_0 r_0 V_0 \quad \rightarrow$$

$$\sigma = \frac{Q}{\text{Area}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\varepsilon_0}{r_0} = \frac{(680\,\text{V})(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)}{(0.16\,\text{m})} = 3.761\times10^{-8}\,\text{C/m}^2$$

$$\approx \boxed{3.8\times10^{-8}\,\text{C/m}^2}$$

(ii) The potential away from the surface of a charged sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi\varepsilon_0 r} = \frac{r_0 V_0}{r} \quad \rightarrow \quad r = \frac{r_0 V_0}{V} = \frac{(0.16\,\text{m})(680\,\text{V})}{(25\,\text{V})} = 4.352\,\text{m} \approx \boxed{4.4\,\text{m}}$$

D. An electric field of 4.80×10^5 V/m is desired between two parallel plates, each of area 21.0 cm^2 and separated by 0.250 cm of air. What charge must be on each plate?

Answer:

We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use the equations below

$$Q = CV = \varepsilon_0 \frac{A}{d}(Ed) = \varepsilon_0 AE = (8.85\times10^{-12}\,\text{F/m})(21.0\times10^{-4}\,\text{m}^2)(4.80\times10^5\ \text{V/m})$$

$$= \boxed{8.92\times10^{-9}\,\text{C}}$$

E. How much energy must a 28 V battery expend to charge a 0.45 µF and a 0.20 µF capacitor fully when they are placed
(i) in parallel; and
(ii) in series?
(iii) How much charge flows from the battery in each case?

Answer:

(*i*) Using the equations below

$$U_{\text{parallel}} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(C_1 + C_2)V^2 = \tfrac{1}{2}\left(0.65\times10^{-6}\,\text{F}\right)\left(28\,\text{V}\right)^2 = 2.548\times10^{-4}\,\text{J} \approx \boxed{2.5\times10^{-4}\,\text{J}}$$

(*ii*) Using the equations below

$$U_{\text{series}} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2 = \tfrac{1}{2}\left(\frac{\left(0.45\times10^{-6}\,\text{F}\right)\left(0.20\times10^{-6}\,\text{F}\right)}{0.65\times10^{-6}\,\text{F}}\right)\left(28\,\text{V}\right)^2$$

$$= 5.428\times10^{-5}\,\text{J} \approx \boxed{5.4\times10^{-5}\,\text{J}}$$

(*iii*) The charge can be found from the following

$$U = \tfrac{1}{2}QV \;\rightarrow\; Q = \frac{2U}{V} \;\rightarrow\; Q_{\text{parallel}} = \frac{2\left(2.548\times10^{-4}\,\text{J}\right)}{28\,\text{V}} = \boxed{1.8\times10^{-5}\,\text{C}}$$

$$Q_{\text{series}} = \frac{2\left(5.428\times10^{-5}\,\text{J}\right)}{28\,\text{V}} = \boxed{3.9\times10^{-6}\,\text{C}}$$

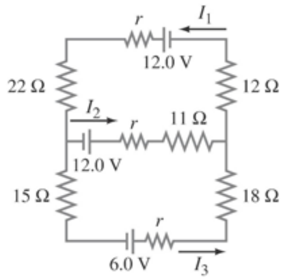
Q.5 (25 Marks)

Answer each of the following questions.

A. The figure below is considered a complex circuit.

(i) Determine the currents I_1, I_2, and I_3. You can assume that the internal resistance of each battery is $r = 1.0\ \Omega$.

(ii) What is the terminal voltage of the 6.0 V battery?



Answer:

(*i*) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise. We add series resistances.

$$12.0\,\text{V} - I_2(12\Omega) + 12.0\,\text{V} - I_1(35\Omega) = 0 \;\rightarrow\; 24 = 35I_1 + 12I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(12\Omega) - 6.0\,\text{V} + I_3(34\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 35I_1 + 12I_2 = 35(I_2 + I_3) + 12I_2 = 47I_2 + 35I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for I_3.

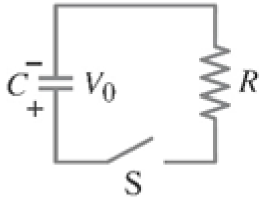
$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 47I_2 + 35I_3 = 47\left(\frac{6 + 34I_3}{12}\right) + 35I_3 \;\rightarrow$$

$$I_3 = \boxed{2.97\,\text{mA}} \;;\; I_2 = \frac{6 + 34I_3}{12} = \boxed{0.508\,\text{A}} \;;\; I_1 = I_2 + I_3 = \boxed{0.511\,\text{A}}$$

(*ii*) The terminal voltage of the 6.0-V battery is $6.0\,\text{V} - I_3 r = 6.0\,\text{V} - (2.97 \times 10^{-3}\,\text{A})(1.0\Omega)$

$= 5.997\,\text{V} \approx \boxed{6.0\,\text{V}}$.

B. The *RC* circuit in the figure below has R = 8.7 kΩ and C = 3.0 μF. The capacitor is at voltage V_0 at t = 0, when the switch is closed. How long does it take the capacitor to discharge to 0.10% of its initial voltage?



Answer:

The voltage of the discharging capacitor is given by $V_C = V_0 e^{-t/RC}$. The capacitor voltage is to be $0.0010V_0$.

$$V_C = V_0 e^{-t/RC} \rightarrow 0.0010V_0 = V_0 e^{-t/RC} \rightarrow 0.0010 = e^{-t/RC} \rightarrow \ln(0.010) = -\frac{t}{RC} \rightarrow$$

$$t = -RC\ln(0.010) = -(8.7\times10^3\,\Omega)(3.0\times10^{-6}\,\mathrm{F})\ln(0.0010) = \boxed{0.18\,\mathrm{s}}$$

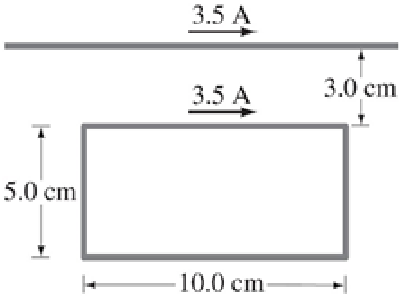
C. An electron is projected vertically upward with a speed of 1.70×10^6 m/s into a uniform magnetic field of 0.480 T that is directed horizontally away from the observer. What is the radius of motion? The mass of an electron is 9.11×10^{-31}kg, and the charge on an electron is 1.66×10^{-19} C.

Answer:

The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{max} = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(1.70\times10^6\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(0.480\,\mathrm{T})} = \boxed{2.02\times10^{-5}\,\mathrm{m}}$$

D. A rectangular loop of wire is placed next to a straight wire, as shown in the figure below. There is a current of 3.5 A in both wires. Determine the magnitude and direction of the net force on the loop.



Answer:

The magnetic field at the loop due to the long wire is into the page, and can be calculated .The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}} l_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}} l_{\text{far}} = \frac{\mu_0}{2\pi} I_1 I_2 l \left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}}\right)$$

$$= \frac{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}}{2\pi}(3.5\,\text{A})^2(0.100\,\text{m})\left(\frac{1}{0.030\,\text{m}} - \frac{1}{0.080\,\text{m}}\right) = \boxed{5.1\times10^{-6}\,\text{N, towards wire}}$$

Q.6 (25 Marks)

Answer each of the following questions.

A. A 420 turn solenoid, 25 cm long, has a diameter of 2.5 cm. A 15 turn coil is wound tightly around the centre of the solenoid. If the current in the solenoid increases uniformly from 0 to 5.0 A in 0.60 s, what will be the induced emf in the short coil during this time? The value for the permeability of free space (μ_0) is $4\pi \times 10^{-7} \mathrm{T \cdot m/A}$.

Answer:

The average emf induced in the short coil is given by the "difference" version of Eq. 29-2b. *N* is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the field created by the solenoid. The field in a solenoid is given by Eq. 28-4, $B = \mu_0 I N_{\text{solenoid}} / l_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathcal{E}| = \frac{N_{\text{short}} A_{\text{short}} \Delta B_{\text{short}}}{\Delta t} = \frac{N_{\text{short}} A_{\text{short}} \Delta\left(\frac{\mu_0 I N_{\text{solenoid}}}{l_{\text{solenoid}}}\right)}{\Delta t} = \frac{\mu_0 N_{\text{short}} N_{\text{solenoid}} A_{\text{short}}}{l_{\text{solenoid}}} \frac{\Delta I_{\text{solenoid}}}{\Delta t}$$

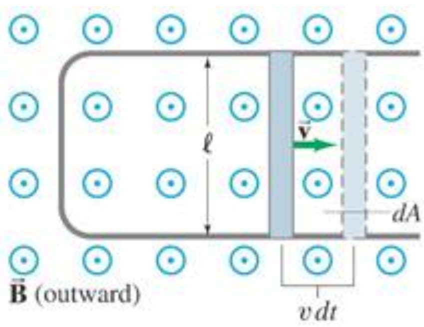
$$= \frac{(4\pi \times 10^{-7} \mathrm{T \cdot m/A})(15)(420)\pi(0.0125\,\mathrm{m})^2}{(0.25\,\mathrm{m})} \frac{(5.0\,\mathrm{A})}{(0.60\,\mathrm{s})} = \boxed{1.3 \times 10^{-4}}$$

B. In the figure below, the rod moves to the right with a speed of 1.3 m/s and has a resistance of 2.5 Ω. The rail separation is l = 25.0 cm. The magnetic field is 0.35 T, and the resistance of the U-shaped conductor is 25.0 Ω at a given instant. Calculate

(i) the induced emf

(ii) the current in the U-shaped conductor, and

(iii) the external force needed to keep the rod's velocity constant at that instant.



Answer:

(*i*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from below

$$\mathcal{E} = Blv = (0.35\mathrm{T})(0.250\,\mathrm{m})(1.3\,\mathrm{m/s}) = 0.1138\,\mathrm{V} \approx \boxed{0.11\,\mathrm{V}}$$

(*ii*) Find the induced current from Ohm's law, using the **total** resistance.

$$I = \frac{\varepsilon}{R} = \frac{0.1138\,\text{V}}{25.0\,\Omega + 2.5\,\Omega} = 4.138\times10^{-3}\,\text{A} \approx \boxed{4.1\,\text{mA}}$$

(*iii*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by

$$F = I l B = \left(4.138\times10^{-3}\,\text{A}\right)\left(0.250\,\text{m}\right)\left(0.35\,\text{T}\right) = 3.621\times10^{-4}\,\text{N} \approx \boxed{0.36\,\text{mN}}$$

C. A 250 loop circular armature coil with a diameter of 10.0 cm rotates at 120 rev/s in a uniform magnetic field of strength 0.45 T. What is the rms voltage output of the generator? What would you do to the rotation frequency in order to double the rms voltage output?

Answer:

Rms voltage is found from the peak induced emf. Peak induced emf is calculated from below

$$\varepsilon_{\text{peak}} = NB\omega A \quad \rightarrow$$

$$V_{\text{rms}} = \frac{\varepsilon_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(250)(0.45\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.050\,\text{m})^2}{\sqrt{2}}$$
$$= 471.1\,\text{V} \approx \boxed{470\,\text{V}}$$

To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s.

D. The back emf in a motor is 85 V when the motor is operating at 1100 rpm. How would you change the motor's magnetic field if you wanted to reduce the back emf to 75 V when the motor was running at 2300 rpm?

Answer:

The magnitude of the back emf is proportional to both the rotation speed and the magnetic field, from the equation below. Thus $\frac{\varepsilon}{B\omega}$ is constant.

$$\frac{\varepsilon_1}{B_1\omega_1} = \frac{\varepsilon_2}{B_2\omega_2} \quad \rightarrow \quad B_2 = \frac{\varepsilon_2}{\omega_2}\frac{B_1\omega_1}{\varepsilon_1} = \frac{(75\,\text{V})}{(2300\,\text{rpm})}\frac{B_1(1100\,\text{rpm})}{(85\,\text{V})} = 0.42B_1$$

So $\boxed{\text{reduce the magnetic field to 42\% of its original value}}$.

E. A 35 mH inductor with 2.0 kΩ resistance is connected in series to a 26-μF capacitor and a 60-Hz, 45-V (rms) source. Calculate

(i) the rms current,

(ii) the phase angle, and

(iii) the power dissipated in this circuit.

Answer:

(*i*) The current is found from the voltage and impedance. The impedance is given by

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(2.0\,\Omega)^2 + \left[2\pi(60\,\text{Hz})(0.035\,\text{H}) - \frac{1}{2\pi(60\,\text{Hz})(26\times10^{-6}\,\text{F})}\right]^2} =$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{45\,\text{V}}{88.85\,\Omega} = 0.5065\,\text{A} \approx \boxed{0.51\,\text{A}}$$

(*ii*) Use Eq. 30-29a to find the phase angle.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \frac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi(60\,\text{Hz})(0.035\,\text{H}) - \frac{1}{2\pi(60\,\text{Hz})(26\times10^{-6}\,\text{F})}}{2.0\,\Omega} = \tan^{-1}\frac{-88.83\,\Omega}{2.0\,\Omega} = \boxed{-88^\circ}$$

(*ii*) The power dissipated is given by $P = I_{\text{rms}}^2 R = (0.5065\,\text{A})^2 (2.0\,\Omega) = \boxed{0.51\,\text{W}}$