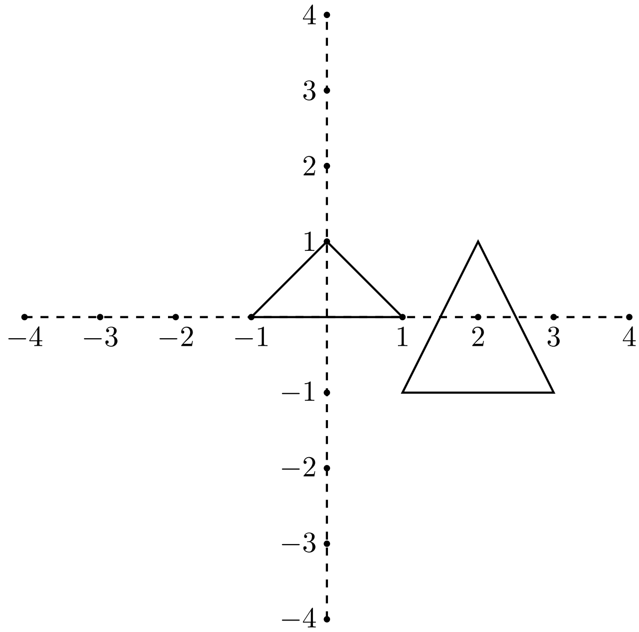
COMP1046 Tutorial 5 : Geometric Mappings

Anthony Bellotti

Consider the following geometric shapes:



Call the smaller triangle on the left, triangle $T1$.
Call the larger triangle on the right, triangle $T2$.

1. What is the 3×3 matrix that represents the geometric mapping from $T1$ to $T2$?

Answer:

This is a vertical scaling by 2 followed by a translation by $(2, -1)$. This is represented as

$$\begin{pmatrix} \mathbf{M} & \mathbf{t} \\ 0 & 1 \end{pmatrix}.$$

where the scaling transform $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and translation $\mathbf{t} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
Hence the geometric mapping is

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. Apply the translation $(0, 1)$ to $T2$, followed by the geometric mapping given by

$$\mathbf{S} = \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Draw the resulting shape on the grid and call it $T3$.

Answer:

For each point in T2, add $(0, 1)$ then take product with $\mathbf{S}$ to the left. For example, for the bottom left point $(1, -1)$ in $T2$:

(a) add $(0, 1)$ gives $(1, 0)$;

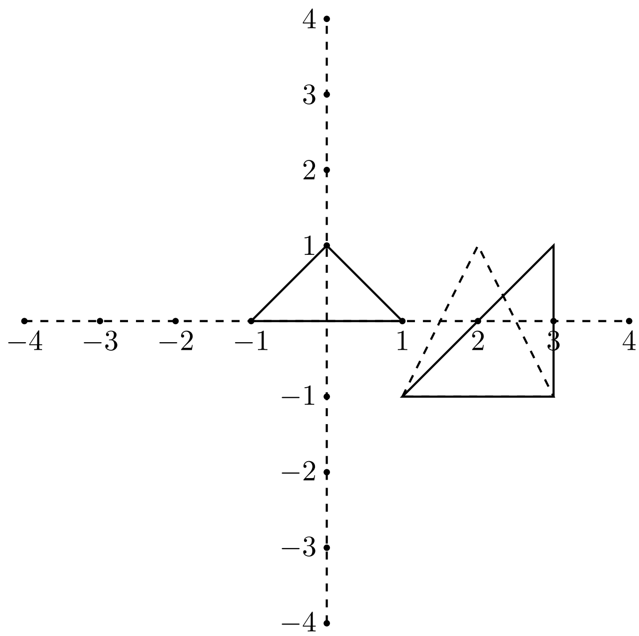
(b) convert to homogeneous coordinates by including a third coordinate with constant value 1: $(1, 0, 1)$;

(c) find product

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

so the mapped point in $T3$ is $(1, -1)$.

Repeating this procedure for all points gives:



3. What type of geometric mapping is $\mathbf{S}$? That is: is it a scaling, vertical or horizontal reflection, rotation, vertical or horizontal shear or translation, or a combination of these?

   Answer:
   It is a horizontal shear with a translation.

4. Express the geometric mapping from $T1$ to $T3$ by a single 3×3 matrix.

   Answer:
   The translation from $T2$ can be expressed as

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

   Now take the product

$$\begin{aligned} \mathbf{STA} &= \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

   and this gives the geometric mapping from $T1$ to $T3$.