

Tutorial Answer

Counting

1. Recall that a bit string is an ordered list of characters using only the digits 0 and 1.
 - a) How many bit strings of length ten are there?
 - b) How many bit strings of length ten have exactly three 1s?
 - c) How many bit strings of length ten have exactly three 1s and none of these 1s are adjacent to each other?

2. You have a combination lock with four digits (0 to 9). You set the lock so that you do not use the same digit more than once.
 - a) How many ways are there to set the lock?
 - b) If you additionally do not use four consecutive digits (ie increasing by 1 in each place; eg 1,2,3,4 or 3,4,5,6), how many ways do you have for setting the lock?
 - c) You now think that you will **also** allow for **any** four-digits made from the digits 0 to 4. How many ways are there to set the lock now?



Answers: Question 1

- a) There are ten positions in the string where each position must be filled with one of two possibilities. Thus, there are $2^{10}=1024$ ways.
- b) There are $C(10,3)=120$ ways to designate places to choose the positions for the 1s in the string. Fill the remaining with 0s.
- c) Start with a string of seven 0s. There are eight places to put the 1s so that no two of the 1s are adjacent. We can put those three 1s into the eight places in $C(8,3)=56$ different ways.

Answers: Question 2

- a) Four digits, order matters: $P(10,4)=5040$.
- b) There are 7 ways to get consecutive digits (ie 0,1,2,3 to 6,7,8,9), so $5040-7 = 5033$ ways.
- c) Do this using the subtraction rule:
 - Number of ways to get any four digits from 0 to 4: using product rule: $5^4=625$.
 - Overlap is the number of ways that 0 to 4 can be allocated without repeat: $P(5,4)=120$, minus the 2 ways to get consecutive digits with 0 to 4 = 118.
 - Now use the subtraction rule: $5033+625-118=5540$.

Further practice and homework

Many exercises in Rosen's textbook:-

- For Counting, try exercises in Section 6.1;
- For Permutations and Combinations, try exercises in Section 6.3.