

COMP1046 Mathematics for Computer Scientists: Exercises for Lecture 3

Work on these problems with reference to the definitions given in Lecture 3.

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 0 & -1 & 2 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 2 & 3 \end{pmatrix}$.

- Q1. What is the submatrix of \mathbf{B} when the 2nd row and the 2nd and 4th columns are cancelled?

ANSWER: $\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$

- Q2. Compute the minor for this submatrix.

ANSWER:

Minor is $\det \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} = 3 \times 2 - (0 \times -1) = 6$ using the formula $ad - bc$ given on Slide 5.

- Q3. What is the complement submatrix of $a_{2,3}$ from \mathbf{A} ?

ANSWER: Cancel row 2 and column 3: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

- Q4. Compute the complement minor of $a_{2,3}$ from \mathbf{A} .

ANSWER: Complement minor is $M_{2,3} = \det \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 3 \times 3 - 0 = 9$.

- Q5. Compute the cofactor of $a_{2,3}$ from \mathbf{A} .

ANSWER: Cofactor is $A_{2,3} = (-1)^{2+3}M_{2,3} = -9$.

- Q6. Compute the adjugate matrix $\text{adj}\mathbf{A}$.

ANSWER: Replace each element by its cofactor, each computed using the same method used in Q3-Q5, then take the transpose:

$$\text{adj}\mathbf{A} = \begin{pmatrix} 7 & 0 & 0 \\ 3 & 6 & -9 \\ -2 & 3 & 6 \end{pmatrix}^T = \begin{pmatrix} 7 & 3 & -2 \\ 0 & 6 & 3 \\ 0 & -9 & 6 \end{pmatrix}$$

Q7. Compute $\mathbf{A}(\text{adj}\mathbf{A})$.

ANSWER:

$$\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} 7 & 3 & -2 \\ 0 & 6 & 3 \\ 0 & -9 & 6 \end{pmatrix} = \begin{pmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{pmatrix}$$

Q8. What do you think the value on the leading diagonal is?

ANSWER: The determinant $\det\mathbf{A}$.