$$ax^2+bx+c=0$$

$$\Delta = b^2-4ac < 0$$

$$\underbrace{(-2)^2=4,\ (5)^2=25,\ (-10)^2=100}_{\geq 0}$$

$$\left(\sqrt{-1}\right)^2 = -1 < 0 \quad \checkmark$$

$$\sqrt{-7} = \sqrt{(-1)\times 7}$$
$$= \sqrt{-1} \times \sqrt{7}$$

$$\sqrt{-1} = i \qquad j$$

$$\sqrt{-7} = \sqrt{-1} \cdot \sqrt{7}$$

$$= \sqrt{7}\, i$$

$$x^2 + x + 1 = 0$$

$$\Delta = 1 - 4 = -3 < 0$$

$$\therefore x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\, i}{2}$$

$$\left(\frac{-1+\sqrt{3}\,i}{2}\right) \qquad \left(\frac{-1-\sqrt{3}\,i}{2}\right)$$

Cartesian form $\qquad a \pm ib$

$$i=\sqrt{-1} \Rightarrow i^2=-1$$

$$\left[(1+i)^2\right]^5 = \left[\cancel{1}+\cancel{i^2}+2i\right]^5$$

$$= [2i]^5$$

$$= 32 i^5$$

Imaginary part

$$Z = a + ib = x + iy$$

Real part

$a, b \in \mathbb{R}$ $x, y \in \mathbb{R}$

$\&$ $i = \sqrt{-1}$.

$Re(Z) = a =$ real part of Z

$Im(Z) = b =$ Imaginary part of Z

Complex Nos. ($\mathbb{C}$)
$(a+ib)$

- Real nos. ($\mathbb{R}$)
  - Rationals ($\mathbb{Q}$)
    - Integers ($\mathbb{Z}$)
      - Negative Integers
      - 0
      - Positive Integers ($\mathbb{N}$)
        - Prime nos.
        - 1
        - Composite Nos.
    - Fractions (p/q ; $q \neq 0$)
  - Irrationals ($e, \pi, \sqrt{2}, \ldots$)
- Imaginary numbers ($i, \sqrt{-7}, \ldots$)

$z_1 = x_1 + i y_1$

$z_2 = x_2 + i y_2$

Then $z_1 = z_2 \iff \left.\begin{array}{l} x_1 = x_2 \\ \text{and } y_1 = y_2 \end{array}\right\}$

$$0 = 0 + i(0)$$

$z_1 = 2 + 3i$

$z_2 = 4 - 5i$

$$z_1 \cdot z_2 = (2+3i)(4-5i)$$

$$= 8 - 10i + 12i - 15i^2$$

$$= 8 + 2i + 15 = \underline{(23) + i(2)}$$

Quadratic Eq^n: 1 root is complex

$\Rightarrow$ 2nd root is complex

Cubic eq^n: 1 root is complex

$\Rightarrow$ 2nd root is complex

3rd root must be REAL

1st complex root | 2nd complex root

$z = (a + ib)$ | $\bar{z} = a - ib$

$$\bar{\bar{z}} = \overline{(a - ib)} = a + ib = z$$

$$\begin{cases} z_1 = 2+3i \\ z_2 = 4+5i \end{cases}$$

Find $\dfrac{z_1}{z_2}$

$$\frac{z_1}{z_2} = \left(\frac{2+3i}{4+5i}\right)\left(\frac{4-5i}{4-5i}\right)$$

$$= \frac{23+2i}{4^2 - 5^2 i^2}$$

$$= \frac{23+2i}{41}$$

$$= \left(\frac{23}{41}\right) + i\left(\frac{2}{41}\right) \checkmark$$

$$\sqrt{5-12i} = \underbrace{\text{Complex No.}}$$

$$\Rightarrow \sqrt{5-12i} = (a+ib)$$

Squaring

$$\Rightarrow \underline{5}\underline{-12i} = \underline{a^2} + \underline{2ab}\,i + \underline{i^2 b^2}$$

$a = 3$ & $b = -2$

$$\Rightarrow \sqrt{5-12i} = 3-2i$$

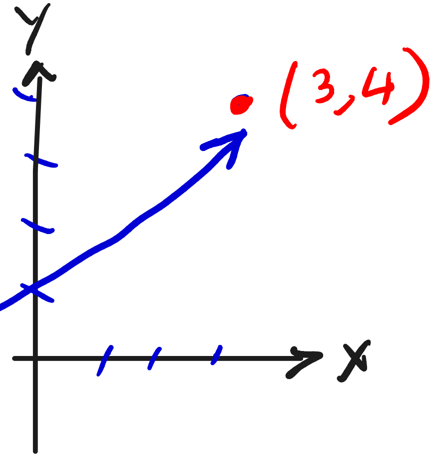
$a = -3$ & $b = 2$

OR

or $-3+2i$

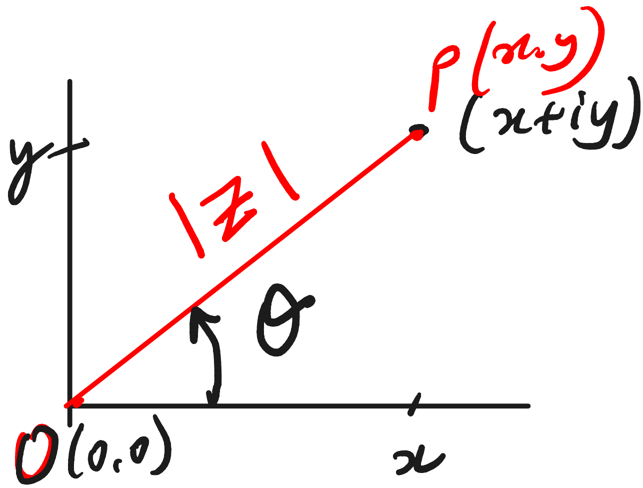
$(3,4) \in \mathbb{R}$

$z = 3+4i \in \mathbb{C}$



Argand diagram

$z = x+iy \Rightarrow$



Modulus of complex numbers

$$r = |z| = \sqrt{x^2+y^2}$$

$$|z_1| \sim |z_2|$$

$\sim$ sim

positive sum

eg. $|z_1| = 3$

& $|z_2| = 5$

Then $|z_1| \sim |z_2|$

$= 3 \sim 5$

$= 2$

OR $|z_1| \sim |z_2|$

$$= \Big|\, |z_1| - |z_2| \,\Big|$$

$$\left|\frac{2-3i}{4+\sqrt{2}i}\right| = \frac{|2-3i|}{|4+\sqrt{2}i|}$$

$$= \frac{\sqrt{2^2+(-3)^2}}{\sqrt{4^2+(\sqrt{2})^2}}$$

$$= \sqrt{\frac{13}{18}}$$

<u>Example</u> Given $z_1 = 2+3i$

$z_2 = 4-3i$

$z_3 = 12+5i$

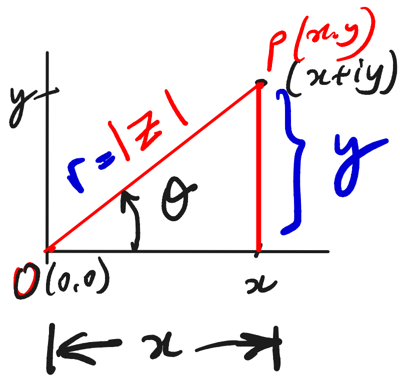
Find $\left|\frac{z_1^2 \cdot \overline{z_2}}{z_3 \cdot \overline{z_1}}\right| = \frac{|z_1|^{\cancel{2}}\,|z_2|}{|z_3|\cdot|z_1|}$

$$|z| = |\bar{z}|$$ ✓

$$|z| = |a+ib| = \sqrt{a^2+b^2}$$

$$|\bar{z}| = |a-ib| = \sqrt{a^2+(-b)^2} = \sqrt{a^2+b^2}$$

$$z = x + iy$$



$$\therefore z = r\cos\theta + i\cdot r\sin\theta$$

$$\Rightarrow z = r\left[\cos\theta + i\sin\theta\right]$$

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

<u>Polar form</u>

where $r = |z|$

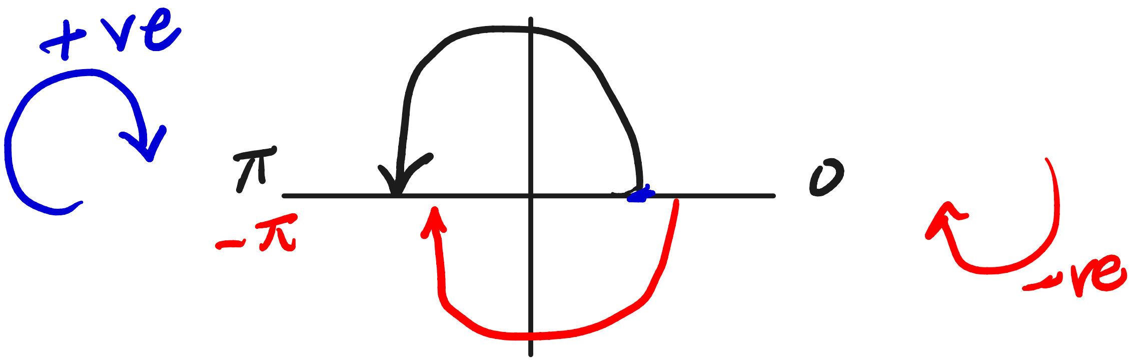
$\theta = ?$

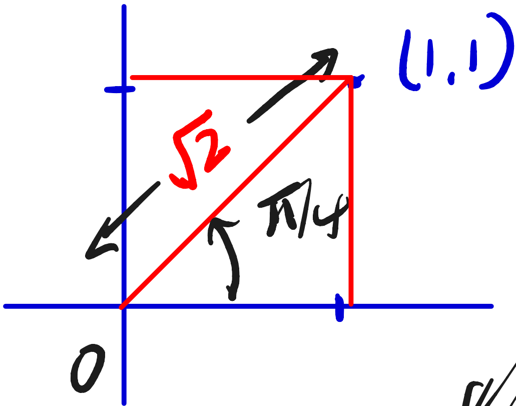
(argument of a complex no.)

$Arg(z)$

$arg(z) \rightarrow$ Principal argument of z

if $\theta \in (-\pi, \pi]$





$z_1 = 1 + i$

$x = 1$ $\quad$ $y = 1$

$r = \sqrt{1+1} = \sqrt{2}$

$$z = r(\cos\theta + i\sin\theta)$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$$

$\cos\theta = x/r$

$\sin\theta = y/r$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1} y/x$

$\theta = \pi - \tan^{-1}\left|\frac{y}{x}\right|$

> 0

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

> 0

π

$-\pi$

$\theta = -\pi + \tan^{-1}\left|\frac{y}{x}\right|$

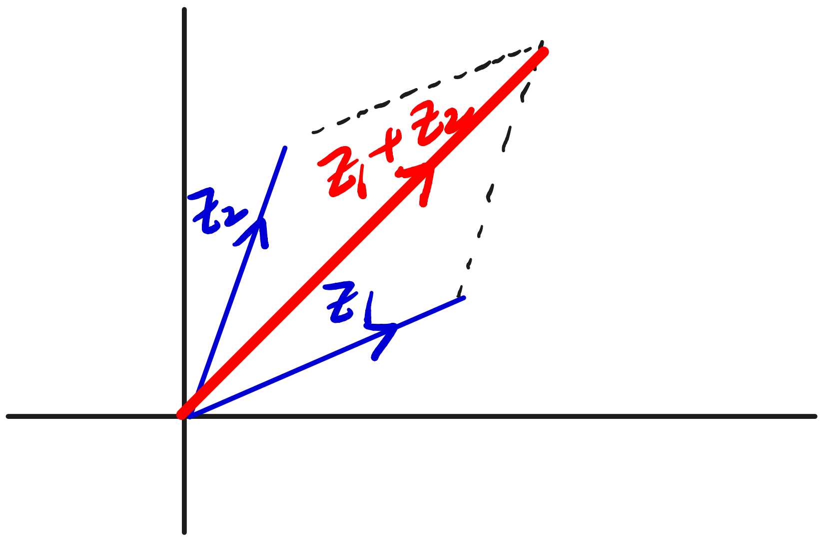
< 0

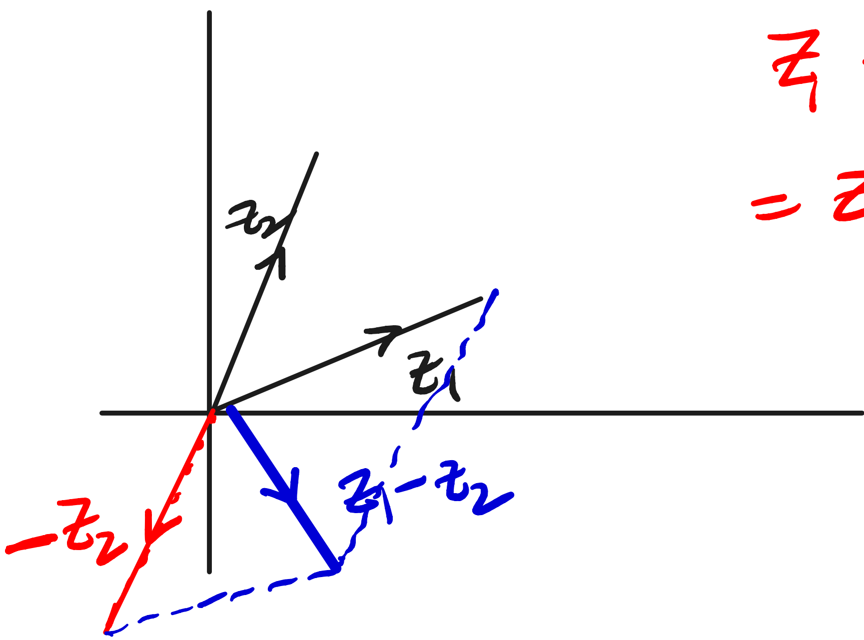
$\theta = -\tan^{-1}\left|\frac{y}{x}\right|$

< 0

Ans: Must be in RADIAN mode

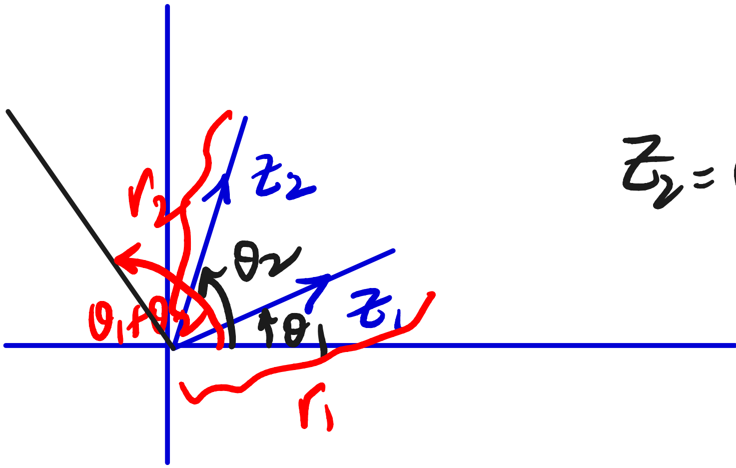
SHIFT MODE 4





$z_1 - z_2$

$= z_1 + (-z_2)$



$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$