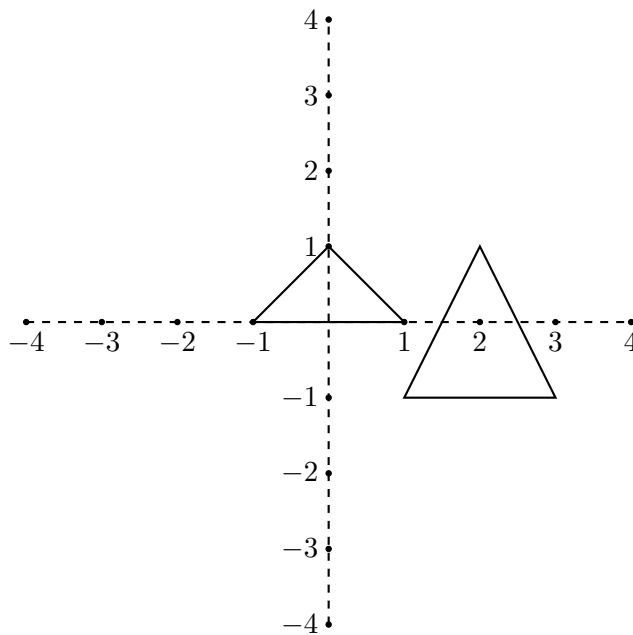


COMP1046 Tutorial 5 : Geometric Mappings

Anthony Bellotti

Consider the following geometric shapes:



Call the smaller triangle on the left, triangle $T1$.

Call the larger triangle on the right, triangle $T2$.

1. What is the 3×3 matrix that represents the geometric mapping from $T1$ to $T2$?

Answer:

This is a vertical scaling by 2 followed by a translation by $(2, -1)$. This is represented as

$$\begin{pmatrix} \mathbf{M} & \mathbf{t} \\ 0 & 1 \end{pmatrix}.$$

where the scaling transform $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and translation $\mathbf{t} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Hence the geometric mapping is

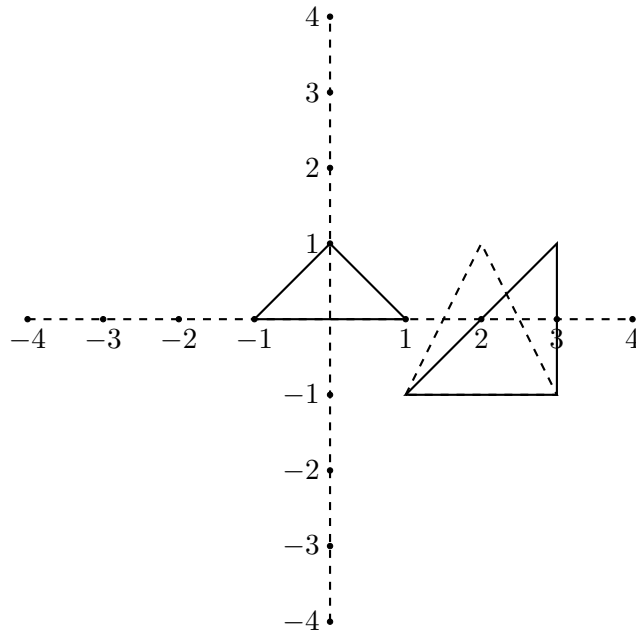
$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. Apply the translation $(0, 1)$ to $T2$, followed by the geometric mapping given by

$$\mathbf{S} = \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Draw the resulting shape on the grid and call it $T3$.

Answer:



3. What type of geometric mapping is \mathbf{S} ? That is: is it a scaling, vertical or horizontal reflection, rotation, vertical or horizontal shear or translation, or a combination of these?

Answer:

It is a horizontal shear with a translation.

4. Express the geometric mapping from $T1$ to $T3$ by a single 3×3 matrix.

Answer:

The translation from $T2$ can be expressed as

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Now take the product

$$\begin{aligned}
 \mathbf{STA} &= \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}
 \end{aligned}$$

and this gives the geometric mapping from $T1$ to $T3$.