AE1MCS: Tutorial 1
Part 2

Question 1

(1). Use quantifiers to express the statement that “There is a woman who has taken a flight on every airline in the world.”

Solution: Let $P(w, f)$ be “w has taken f” and $Q(f, a)$ be “f is a flight on a.” We can express the statement as

$$\exists w \forall a \exists f (P(w, f) \wedge Q(f, a)),$$

where the domains of discourse for w, f, and a consist of all the women in the world, all airplane flights, and all airlines, respectively.

The statement could also be expressed as

$$\exists w \forall a \exists f \, R(w, f, a),$$

where $R(w, f, a)$ is “w has taken f on a.” Although this is more compact, it somewhat obscures the relationships among the variables. Consequently, the first solution is usually preferable.

Question 1

(2). Use quantifiers to express the statement that "There does not exist a woman who has taken a flight on every airline in the world."

Solution: This statement is the negation of the statement "There is a woman who has taken a flight on every airline in the world" from Example 13. By Example 13, our statement can be expressed as $\neg\exists w \forall a \exists f (P(w, f) \wedge Q(f, a))$, where $P(w, f)$ is "w has taken f" and $Q(f, a)$ is "f is a flight on a." By successively applying De Morgan's laws for quantifiers in Table 2 of Section 1.4 to move the negation inside successive quantifiers and by applying De Morgan's law for negating a conjunction in the last step, we find that our statement is equivalent to each of this sequence of statements:

$$\begin{aligned}
\forall w \neg \forall a \exists f (P(w, f) \wedge Q(f, a)) &\equiv \forall w \exists a \neg \exists f (P(w, f) \wedge Q(f, a)) \\
&\equiv \forall w \exists a \forall f \neg (P(w, f) \wedge Q(f, a)) \\
&\equiv \forall w \exists a \forall f (\neg P(w, f) \vee \neg Q(f, a)).
\end{aligned}$$

This last statement states "For every woman there is an airline such that for all flights, this woman has not taken that flight or that flight is not on this airline." ◀

Question 2

Suppose that the domain of $Q(x, y, z)$ consists of triples (x, y, z) , where $x = 0, 1, or\ 2, y = 0\ or\ 1\ , and\ z = 0\ or\ 1$. Write out these propositions using disjunctions and conjunctions.

a) ∀y Q(0, y, 0)
b) $\exists x\ Q(x, 1{,}1)$
c) ∃z¬ $Q(0{,}0, z)$
d) $\exists x \neg\ Q(x, 0{,}1)$

Ans:

a) Q(0, 0, 0) ∧ Q(0, 1, 0)
b) $Q(0{,}1{,}1) \lor Q(1{,}1{,}1) \lor Q(2{,}1{,}1)$
c) $\neg Q(0{,}0{,}0) \lor \neg Q(0{,}0{,}1)$
d) $\neg\ Q(0{,}0{,}1) \lor \neg\ Q(1{,}0{,}1) \lor \neg Q(2{,}0{,}1)$

Question 3

Use rules of inference to show that if $\forall x\left(P(x) \rightarrow \left(Q(\mathrm{x}) \wedge S(x)\right)\right)$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

	Step	Reason
1.	$\forall x\,(P(x) \wedge R(x))$	Premise
2.	$P(a) \wedge R(a)$ for an arbitrary element a	Universal Instantiation from (1)
3.	$P(a)$	Simplification from (2)
4.	$R(a)$	Simplification from (2)
5.	$\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$	Premise
6.	$P(a) \rightarrow (Q(a) \wedge S(a))$	Universal Instantiation from (5)
7.	$Q(a) \wedge S(a)$	Modus ponens using (6) and (3)
8.	$S(a)$	Simplification from (7)
9.	$R(a) \wedge S(a)$	Conjunction using (4) and (8)
10.	$\forall x\,(R(x) \wedge S(x))$	Universal Generalization from (9)

Question 4

Use rules of inference to show that if $\forall x(P(x) \lor Q(x))$, $\forall x(\neg Q(x) \lor S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Step	Reason
1. $\exists x \neg P(x)$	Premise
2. $\neg P(c)$ for some element c	Existential Instantiation from (1)
3. $\forall x (P(x) \lor Q(x))$	Premise
4. $P(c) \lor Q(c)$	Universal Instantiation from (3)
5. $Q(c)$	Disjunctive syllogism using (4) and (2)
6. $\forall x (\neg Q(x) \lor S(x))$	Premise
7. $\neg Q(c) \lor S(c)$	Universal Instantiation from (6)
8. $S(c)$	Disjunctive syllogism using (7) and (5)
9. $\forall x (R(x) \to \neg S(x))$	Premise
10. $R(c) \to \neg S(c)$	Universal Instantiation from (9)
11. $S(c) \to \neg R(c)$	Contrapositive of (10)
12. $\neg R(c)$	Modus ponens using (11) and (8)
13. $\exists x \neg R(x)$	Existential Generalization from (12)

Question 5

Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable

Answer： Assume that this proposition is satisfiable. Using resolution on the first two clauses enables us to conclude $q \vee q$; in other words, we know that q has to be true. Using resolution on the last two clauses enables us to conclude $\neg q \vee \neg q$; in other words, we know that $\neg q$ has to be true. This is a contradiction. So this proposition is not satisfiable.