Useful information about Final exam paper

Structure of Final exam paper

Question No.	Marks	Topics covered
1	15	Functions, Modulus inequality, Quadratic, Logarithmic and exponential functions
2	15	Trigonometry, Remainder and Factor Theorems, Synthetic Division
3	15	Binomial Theorem, Generalised Binomial Theorem and applications, Numerical methods, Matrices
4	15	Partial fractions, Complex Numbers, Sequence and Series, Power series, method of differences

Notes:

1) This is a take-home-exam, which you must complete in 24-hours' time.
2) The exam paper will be available on module Moodle page at 9.30 am. on 6th January 2023.
3) Deadline for submission is: 9.30 am on 7th January 2023 (China Time).
4) Marks will be given for the best 3 answers.
   (i.e. you can attempt **ANY 3 out of 4** questions).
5) Total marks obtained will then be upscaled to 70%.
6) The final score will be calculated as: **Mid-sem exam (30%) + Final exam (70%)**.

Useful information about Final exam paper

Instructions:

1) You should write all **necessary steps** in your solutions.
2) It is expected that you will only use CELE approved calculator ($fx - 82$ series) for this exam. You will lose marks if because of use of other models of calculators, your numerical answer differs from our standardized marking scheme.
3) Formula Sheet will be attached to the question paper.
4) Please write your answers on a blank piece of paper. Alternatively, you may also use iPad/Tablet to write your answers.
5) Please complete **Academic Integrity Declaration** and create a single PDF file of all your answers to exam questions.
6) Name your file as: Your Student ID number_N036Final. For example: 20519999_N036Final.
7) Please upload this PDF file to submission drop-box on module Moodle page (available on the top of the Moodle page). Module Convenor will also email the link to the submission drop-box.
8) No excuses such as problems with internet connectivity, etc. will be entertained; so, you are suggested to submit your working well in advance before the deadline. Should you have any difficulty in uploading your file, please contact Module Convenor (Bamidele.Akinwolemiwa2@nottingham.edu.cn) **immediately** and follow their instructions.
9) This work must be completed on your own. Plagiarism and collusion are regarded as very serious academic offences and will be treated as such.

Seminar 11

In this seminar you will study:

- Arithmetic series
- Geometric series
- The Method of Differences

Arithmetic series

The sum of the first n terms of an A.P. is :

$$S_n = \frac{n}{2}[\,2a + (n-1)d\,]$$

where a is the first term and d is the common difference.

The sum of the first n terms of an A.P. is also given by

$$S_n = \frac{n}{2}[\,a + l\,]$$

where $l = a + (n-1)d =$ last term given in A.P.

Arithmetic series

Example: The sixth term of an A.P. is 23 and its twenty-second term is 39. Find the sixteenth term of the sequence and sum of its first 19 terms.

Solution:

$$\text{Sixth term is } 23 \quad \Rightarrow \quad a + 5d = 23 \qquad (1)$$

$$\text{Twenty-second term is } 39 \quad \Rightarrow \quad a + 21d = 39 \qquad (2)$$

$$(2) - (1) \ \text{ gives } 16d = 16 \ \therefore \ d = 1$$

$$\text{From} \quad (1) \ \ a + 5 \times 1 = 23 \ \therefore \ a = 18$$

$$\Rightarrow \ \text{Sixteenth term is} : a_{16} = a + 15d$$

$$\therefore a_{16} = 18 + 15 \times 1 = 33$$

$$\text{Sum of first nineteen terms is} : S_{19} = \frac{19}{2}\left[2 \times 18 + (18) \times 1\right]$$

$$= 19[27] = 513$$

Arithmetic series

1. The fourth and tenth terms of an A.P. are 33 and 81 respectively. Find the sum of its first 10 terms.

Answer: 450

2. The eighth term of an A.P. is 5 and the sum of its first fourteen terms is 49. Find a and d.

Answer: $a = -16 \quad d = 3$

3. For an A.P., $1, 3, 5, \ldots\ldots$, it is given that $S_n = 1521$. Find the value of n.

Answer: 39

4. The eight term of an A.P. is twice the fourth term, and its twentieth term is 40. Find the sum of its terms from the eighth to the twentieth inclusive

Answer: 364

Geometric series

The sum of the first n terms of a G.P. is :

$$S_n = \begin{cases} na & ; \quad r = 1 \\ a\left(\dfrac{1-r^n}{1-r}\right) & ; \quad r \neq 1 \end{cases}$$

where a is the first term and r is the common ratio.

The sum of infinite terms of a G.P. is $S = \dfrac{a}{1-r}$ (if $-1 < r < 1$)

Geometric series

Example 1: For a G.P. the second term is 6 and the fifth term is 48, find the sum of the first 10 terms.

Solution:

$$\text{Second term is } 6 \quad \Rightarrow \quad ar = 6 \qquad (1)$$

$$\text{Fifth term is } 48 \quad \Rightarrow \quad ar^4 = 48 \qquad (2)$$

$$(2) \div (1) \quad \text{gives } r^3 = 8 \;\Rightarrow\; r = 2$$

$$\text{From} \quad (1) \quad a \times 2 = 6 \;\Rightarrow\; a = 3$$

$$\text{Sum of first ten terms is : } S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$$

$$\boxed{\because S_n = \frac{a(r^n - 1)}{r - 1}}$$

$$= 3(1023) = 3069$$

Geometric series

Example 2: Express $3.123123\overline{123}$ as a vulgar fraction.

Solution:

$$3.123123\overline{123} = 3 + \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots$$

$$3.123123\overline{123} = 3 + \underbrace{\frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots}_{\text{sum of infinite terms of a GP}}$$

The first term of the GP is: $a = \dfrac{123}{10^3}$

The common ratio of the GP is: $r = \left(\dfrac{123}{10^6}\right) \div \left(\dfrac{123}{10^3}\right) = \dfrac{1}{10^3}$

$$\therefore \quad \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots = \frac{\dfrac{123}{10^3}}{1 - \dfrac{1}{10^3}}$$

$$= \frac{41}{333}$$

$$\because S = \frac{a}{1-r} \text{ if } |\, r \,| < 1$$

Geometric series

Example: Express $3.123123\overline{123}$ as a vulgar fraction.

Solution:

$$\Rightarrow \quad 3.123123\overline{123} = 3 + \frac{41}{333}$$

$$= \frac{1040}{333}$$

Geometric series

1. The third term of a G.P. is 4 and its fifth term is 8. If the sum of the first 10 terms is positive, find the sum.

Answer: $62(\sqrt{2}+1)$

2. Use the formula for the sum of infinite geometric series, to show that

$$20 - 10 + 5 - \frac{5}{2} + \ldots\ldots = \frac{40}{3}.$$

3. Express $1.031031\overline{031}$ as a vulgar fraction.

Answer: $\frac{1030}{999}$

4. Find the sum of the infinite geometric series:

$$\frac{1}{4} + \frac{1}{20} + \frac{1}{100} + \ldots\ldots$$

Answer: $\frac{5}{16}$

The formulae for the sum of power series

$$\underbrace{1+1+1+\ldots+1}_{n\text{ times}} = \sum_{1}^{n} 1 = n$$

$$1+2+3+\ldots+n = \sum_{1}^{n} n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\ldots n^2 = \sum_{1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+\ldots n^3 = \sum_{1}^{n} n^3 = \frac{n^2(n+1)^2}{4}$$

The formulae for the sum of power series

Example: Prove that $\sum_{1}^{n}(6n^2+4n-1) = n(n+2)(2n+1)$

Solution: $\sum_{1}^{n}(6n^2+4n-1) = 6\sum_{1}^{n}n^2 + 4\sum_{1}^{n}n - \sum_{1}^{n}1$

$$= 6\cdot\frac{n(n+1)(2n+1)}{6} + 4\cdot\frac{n(n+1)}{2} - n$$

$$= n(n+1)(2n+1) + 2n(n+1) - n$$

$$= n\,[\,(n+1)(2n+1) + 2(n+1) - 1\,]$$

$$= n\,[\,(n+1)(\underline{2n+1}) + (\underline{2n+1})\,]$$

$$= n(2n+1)[\,\underline{(n+1)+1}\,]$$

$$= n(2n+1)(n+2)$$

The formulae for the sum of power series

$$\sum_{1}^{n} 1 = n, \quad \sum_{1}^{n} n = \frac{n(n+1)}{2}, \quad \sum_{1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{1}^{n} n^3 = \frac{n^2(n+1)^2}{4}$$

1. Show that

$$\sum_{1}^{n} n(n+2) = \frac{n(n+1)(2n+7)}{6}$$

2. Show that

$$\sum_{1}^{n} n(n^2+1) = \frac{n(n+1)(n^2+n+2)}{4}$$

3. Show that

$$\sum_{1}^{n} (n+1)(n+2) = \frac{n(n^2+6n+11)}{3}$$

4. Show that

$$\sum_{1}^{n} (n-1)(n-3) = \frac{n(2n^2-9n+7)}{6}$$

The Method of differences

Example:

(i). Use the method of partial fractions to show that $\dfrac{1}{k(k+1)} = \dfrac{1}{k} - \dfrac{1}{k+1}$

(ii). Hence use the method of differences to show that $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

Solution:

(i). Let $\dfrac{1}{k(k+1)} = \dfrac{A}{k} + \dfrac{B}{k+1}$

$$\Rightarrow \quad 1 = A(k+1) + B(k)$$

Put $k = 0 \Rightarrow \; 1 = A(1)$

$$\therefore \; A = 1$$

Put $k = -1 \Rightarrow \; 1 = B(-1)$

$$\therefore \; B = -1$$

$$\Rightarrow \quad \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

The Method of differences

Solution:

$$(ii).\quad \sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$= \left(1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\cdots} + \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}} + \cancel{\frac{1}{n}} - \frac{1}{n+1} \right)$$

$$= 1 - \frac{1}{n+1}$$

$$= \frac{n}{n+1}$$

The Method of Differences

1. Show that

$$r^2 \cdot (r+1)^2 - (r-1)^2 \cdot r^2 = 4r^3$$

Hence prove that

$$\sum_1^n r^3 = \frac{n^2(n+1)^2}{4}$$

2. Express $\dfrac{1}{4r^2-1}$ as a sum of partial fractions.

Hence prove that

$$\sum_1^n \frac{1}{4r^2-1} = \frac{n}{(2n+1)}$$

3. Express $\dfrac{2}{(r+1)(r+3)}$ as a sum of partial fractions.

Hence prove that

$$\sum_1^n \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

4. Show that

$$\frac{1}{2}\left[r(r+1) - r(r-1)\right] = r$$

Hence prove that

$$\sum_1^n r = \frac{n(n+1)}{2}$$

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IN YOUR EXAMINATIONS