Mid-Semester Examination

Wednesday 16—November from 15:00 to 16:00.

Confirm your Exam room from the email sent by CPSO

Questions in the mid-semester examinations will cover all the topics

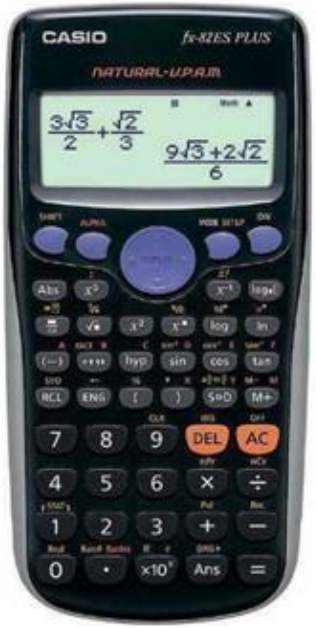
treated in **Seminar 1 to 5**

Mid-Semester Examination

Only permissible calculators are allowed in the examination

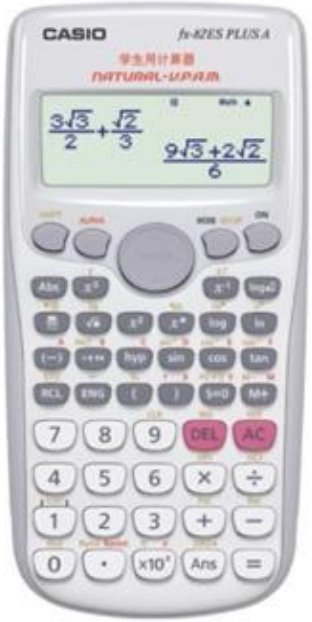
Permissible calculator are the $fx-82$ series

$fx-82ES\ PLUS$



$fx-82ES\ PLUS\ 2nd\ edition$

$fx-82ES\ PLUS\ A$



Mid-semester Examination: Important Information

Date: Wednesday, 16–November 2022

1. Time: 15:00 to 16:00 (Make sure you confirm your examination room from the email sent to you by CPSO).

2. The examination is for 60 minutes. Examination comprises of 20 Short-answer questions (total marks 60), and you are to answer **all** questions.

3. Weighting: 30% toward the module marks.

4. Examination booklet is provided, and you should write your answers and workings on the box provided for each question in the booklet. Make sure to write your **Student ID** (e.g. 2051xxxx) and **Seminar Group** (e.g. A-29) correctly on the space provided in the booklet.

Mid-semester Examination: Important Information

Date: Wednesday, 16–November 2022

5. A set of useful formulae is included in Page 2 of the exam booklet.

6. Last page of the exam booklet is blank, and you may use this for rough work. **Note:** no piece of paper (for rough work) is allowed in the examination.

7. Only permissible calculators ($fx - 82$) series, are allowed.

Mid-semester Examination: Important Information

Date: Wednesday, 16–November 2022

Mid-semester examination will cover topics treated in Seminar 1 to 5

Study Hint: Study Lecture 1 to 5, Seminar 1 to 5, Problem Sheet 1 to 5, and related exercises from the reference textbooks.

Review the sample mid-semester examination questions on Moodle.

Use the office hours to discuss your questions with tutors.

Independent Learning Week (ILW)

Week 9 w/c 7—November 2022

CELE has named the week commencing on Monday 7th November as your Independent Learning Week.

You will have **no lectures and seminars** throughout the Independent Learning Week, but you will be given some learning activities to engage in.

The goal is for you to use this week to reflect on what learning independently outside the classroom means to you, sharpen your study skills, and reinforce your preparation moving toward the end of the semester. To help you:

1) Group and Individual tasks will be posted on Moodle on Friday 4th November. Information about your group (numbering 4 to 5 students in the same seminar group) will also be made available on Moodle.

2) Classrooms will remain booked at the usual times in case you need a place where to meet with other students to work on the module's activities.

3) Tutors will remain available during office hours (check Moodle for their updated time slots).

Seminar 5

In this seminar you will study:

- Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms
- The Remainder Theorem
- The Factor Theorem
- The method of synthetic division
  - Finding quotients and remainders
  - Solving polynomial equations

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

Example: Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $r\cos(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Also, find the period and range of $f(x)$, and sketch the curve of $f(x)$.

Solution:

$$f(x) = \boxed{\sqrt{3}\sin x} + \boxed{\cos x} \equiv r\cos(x-\theta) = \boxed{r\cos x\cos\theta} + \boxed{r\sin x\sin\theta}$$

$$\Rightarrow \quad \begin{cases} r\cos\theta = 1 \\ r\sin\theta = \sqrt{3} \end{cases} \quad \Rightarrow \quad r = 2$$

$$\Rightarrow \quad \begin{cases} \cos\theta = \dfrac{1}{2} \\ \sin\theta = \dfrac{\sqrt{3}}{2} \end{cases} \quad \Rightarrow \quad \theta = \frac{\pi}{3} \qquad \boxed{\because \theta \in \left(0, \frac{\pi}{2}\right)}$$

Thus, $f(x) = \sqrt{3}\sin x + \cos x = 2\cos\left(x - \frac{\pi}{3}\right)$

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

Example: Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $r\cos(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

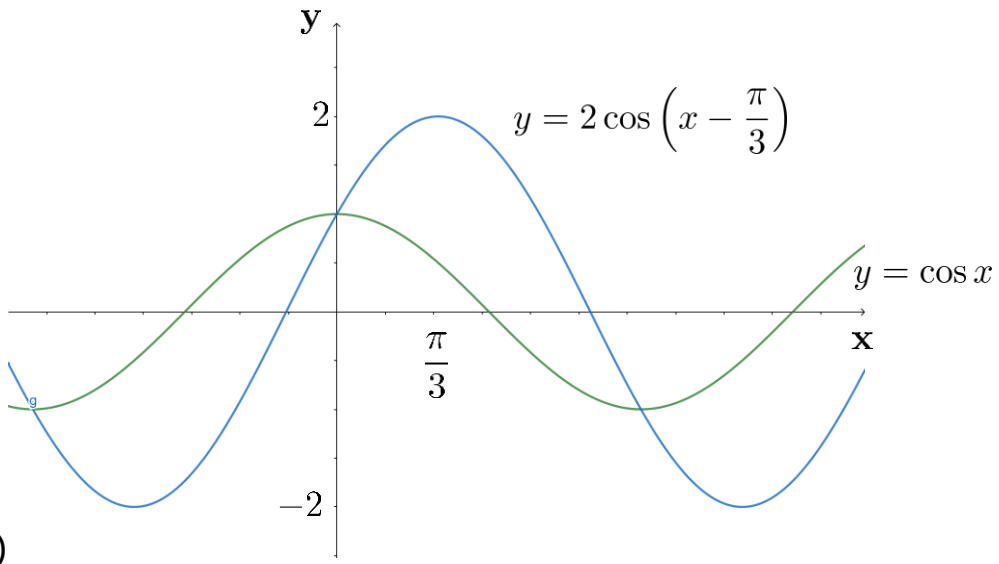
Also, find the period and range of $f(x)$, and sketch the curve of $f(x)$.

Solution:

$$f(x) = \sqrt{3}\sin x + \cos x = 2\cos\left(x - \frac{\pi}{3}\right)$$

Period of f is $\frac{2\pi}{|1|} = 2\pi$

Range of f is $[-2, 2]$



Click the Moodle Link for:
Video on calculator use (Trigonometry)

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

(i). Express $f(x) = \cos x - \sqrt{3}\sin x$ in the form $r\cos(x+\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = 2\cos\left(x + \frac{\pi}{3}\right)$

(ii). Express $f(x) = \sqrt{3}\sin x - \cos x$ in the form $r\sin(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = 2\sin\left(x - \frac{\pi}{6}\right)$

(iii). Express $f(x) = 3\sin x + 4\cos x$ in the form $r\cos(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = 5\cos(x - 0.6435)$

(iv). Express $f(x) = 4\cos x - 3\sin x$ in the form $r\cos(x-\theta)$, where $\theta \in \left(-\frac{\pi}{2}, 0\right)$.

Answer: $f(x) = 5\cos(x + 0.6435)$

The Remainder theorem and the Factor theorem

The Remainder theorem

If a polynomial $p(x)$ is divided by $(x - c)$, then the remainder is $p(c)$.

The Factor theorem

A polynomial $p(x)$ has a factor $(x - c)$ if and only if $p(c) = 0$.

The Remainder theorem and the Factor theorem

(i). If $p(x) = x^4 - 2x^2 + 1$ is divided by $(x + 1)$, find the remainder.

Answer: 0

(ii). Find the value of k if $(x + 1)$ is a factor of $p(x) = k^2x^3 - 7kx - 10$.

Answer: 2 or 5

(iii). Find the value of k if $(x + k)$ is a factor of $p(x) = x^2 + 2x + 1$.

Answer: 1

(iv). The remainder when $p(x) = ax^2 + 5x + 9$ is divided by $(x - 2)$ is 3. Find the value of a.

Answer: -4

The method of Synthetic division

Example: Use the method of synthetic division to find the quotient $q(x)$ and the remainder r if $p(x) = 2x^4 + 5x^3 - 2x - 8$ is divided by $(x + 3)$.

$+ 0x^2$

Here $s(x) = x + 3 = x - c$

$\Rightarrow c = -3$

	2	5	0	-2	-8
-3	↓	-6	3	-9	33
	2	-1	3	-11	25

Quotient $q(x)$ $= 2x^3 - x^2 + 3x - 11$ Remainder $r = 25$

Use the method of Synthetic division to find $q(x)$ and r, when p(x) is divided by s(x).

$(i).\quad p(x) = x^4 - 3x^3 + 4x + 7$

$s(x) = x - 2$

Answer: $q(x) = x^3 - x^2 - 2x$, $r = 7$

$(ii).\quad p(x) = x^3 - 3x^2 + 4x + 7$

$s(x) = x - 3$

Answer: $q(x) = x^2 + 4$, $r = 19$

$(iii).\quad p(x) = x^3 + 2x^2 - 4x - 2$

$s(x) = x + 3$

Answer: $q(x) = x^2 - x - 1$, $r = 1$

$(iv).\quad p(x) = x^3 - 3x^2 + 8x - 1$

$s(x) = x + 1$

Answer: $q(x) = x^2 - 4x + 12$, $r = -13$

Solving polynomial equations

Result

Let $p(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0$ be a polynomial with integer coefficients (i.e. $c_i \in \mathbb{Z}$, for $i = 0, 1, \cdots, n$). If m is an integer zero of $p(x)$, then m is a divisor of the constant term c_0.

For example, if $p(x) = x^3 - 27$, 3 is an integer zero of $p(x)$, (i.e. $m = 3$), and 3 is a divisor of -27, (since $c_0 = -27$).

Solving polynomial equations

Example: Solve $p(x) = x^3 - 2x^2 - 9x + 18 = 0$

Solution:

Possible zeros are $\pm 1, \pm 2 \ \pm 3, \pm 6, \pm 9, \pm 18.$

Try $\pm 1, \pm 2 \ \pm 3.$

Note: exam question on solving cubic equation will have at least one of these integers as a zero of $p(x)$.

$p(1) = 8 \Rightarrow p(x) \neq 0 \ \ \therefore \ \ 1$ is not a zero of $p(x)$

$p(-1) = 24 \Rightarrow p(x) \neq 0 \ \ \therefore \ \ -1$ is not a zero of $p(x)$

$p(2) = 0 \Rightarrow p(x) = 0 \ \ \therefore \ \ 2$ is a zero of $p(x)$

$\Rightarrow (x - 2)$ is one of the factors of $p(x)$

Use the method of synthetic division to find the other factor.

Solving polynomial equations

Example: Solve $p(x) = x^3 - 2x^2 - 9x + 18 = 0$

Here $s(x) = x - 2 = x - c$

$\Rightarrow \boxed{c = 2}$

	1	−2	−9	18
2	↓	2	0	−18
	1	0	−9	0

Note: remainder $r = 0 \because s(x) = x - 2$ is a factor of $p(x) = x^3 - 2x^2 - 9x + 18$.

Thus, the other factor is $\boxed{x^2 - 9}$

$$\therefore\ p(x) = (x-2)\cdot(x^2-9)$$

$$= (x-2)\cdot(x-3)\cdot(x+3)$$

$$\therefore\ p(x) = 0 \Rightarrow (x-2)\cdot(x-3)\cdot(x+3) = 0$$

$$\Rightarrow\ x = 2 \text{ or } x = 3 \text{ or } x = -3$$

Solving cubic equations

(i). Solve $p(x) = 2x^3 - 9x^2 + 7x + 6 = 0$.

Answer: $x = 2,\ 3,$ or $-\frac{1}{2}$

(ii). Solve $p(x) = 2x^3 - 3x^2 - 11x + 6 = 0$.

Answer: $x = -2,\ 3,$ or $\frac{1}{2}$

(iii). Solve $p(x) = 3x^3 - 2x^2 - 17x - 12 = 0$.

Answer: $x = -1,\ 3,$ or $-\frac{4}{3}$

(iv). Solve $p(x) = x^3 + 6x^2 + 11x + 6 = 0$.

Answer: $x = -1,\ -2,$ or -3

Solving cubic equations

(i). Solve $p(x) = 6x^3 - 11x^2 - 14x + 24 = 0$.

Answer: $x = 2, \quad -\frac{3}{2}, \quad \text{or} \quad \frac{4}{3}$

(ii). Solve $p(x) = 12x^3 + 19x^2 - 12x - 4 = 0$.

Answer: $x = -2, \quad -\frac{1}{4}, \quad \text{or} \quad \frac{2}{3}$

(iii). Solve $p(x) = 8x^3 - 30x^2 + 19x - 3 = 0$.

Answer: $x = 3, \quad \frac{1}{4}, \quad \text{or} \quad \frac{1}{2}$

(iv). Solve $p(x) = 6x^3 + 19x^2 + x - 6 = 0$.

Answer: $x = -3, \quad -\frac{2}{3}, \quad \text{or} \quad \frac{1}{2}$

Additional review exercises:

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

(i). Express $f(x) = \cos x + 3\sin x$ in the form $r\cos(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = \sqrt{10}\cos(x - 1.249)$

(ii). Express $f(x) = 4\sin x - 3\cos x$ in the form $r\sin(x-\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = 5\sin(x - 0.6435)$

(iii). Express $f(x) = \cos 2x - \sin 2x$ in the form $r\cos(2x+\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = \sqrt{2}\cos\left(2x + \frac{\pi}{4}\right)$

(iv). Express $f(x) = 2\cos 3x + 5\sin 3x$ in the form $r\sin(3x+\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Answer: $f(x) = \sqrt{29}\sin(3x + 0.3805)$

THANKS FOR YOUR ATTENTION