MCS Tutorial 6 Answer: Relations

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Relations

1. Consider a relation R on the set $\mathbb{Z}^+$ defined as

$$R = \{(x, y) | x + y \text{ is even}\}$$

Show whether R is reflexive, symmetric, antisymmetric and/or transitive.

2. Let R and S be the following relations:
   - R={(1,1), (1,2),(2,4),(3,2),(4,3)}
   - S={(1,0),(2,4),(3,1),(3,2),(4,1)}

   What is the composite of the relations R and S, $S^\circ R$?

3. Let R={(1,1), (2,4), (3,4),(4,2)}. Find the powers R^2, R^3, R^4,…

Try at home:-

$$R = \{(x, y) : x + y \text{ is odd}\}$$

$$R = \{(x, y) | x + y \text{ is even}\}$$

1. Answer:

- Since $x + x$ is even for any x, then $(x, x) \in R$ and R is **reflexive**.

- Since $x + y = y + x$, R is **symmetrical**.

- Since $4 + 2$ and $2 + 4$ are even, but $4 \neq 2$, R is **not antisymmetric**.

- Suppose $(x, y) \in R$ and $(y, z) \in R$.
  - Then, either both x and y are odd, or both are even.
  - If x and y are odd, then z must be odd $\Rightarrow$ $x + z$ is even $\Rightarrow$ $(x, z) \in R$.
  - If x and y are even, then z must be even $\Rightarrow$ $x + z$ is even $\Rightarrow$ $(x, z) \in R$.
  - Hence R is **transitive**.

Answer:

For every $(a,b) \in R$, $(b,c) \in S$ *forms* $(a,c) \in S \circ R$.

(a,b)	(b,c)	(a,c)
$(1,1)$	$(1,0)$	$(1,0)$
$(1,2)$	$(2,4)$	$(1,4)$
$(2,4)$	$(4,1)$	$(2,1)$
$(3,2)$	$(2,4)$	$(3,4)$
$(4,3)$	$(3,1)$	$(4,1)$
$(4,3)$	$(3,2)$	$(4,2)$

Therefore, $S \circ R = \{(1,0),(1,4),(2,1),(3,4),(4,1),(4,2)\}$.

Answer:

Find $R^2 = R \circ R$.

(a,b)	(b,c)	(a,c)
$(1,1)$	$(1,1)$	$(1,1)$
$(2,4)$	$(4,2)$	$(2,2)$
$(3,4)$	$(4,2)$	$(3,2)$
$(4,2)$	$(2,4)$	$(4,4)$

Then, $R^2 = \{(1,1),(2,2),(3,2),(4,4)\}$.

Find $R^3 = R^2 \circ R$. $(a,b) \in R$ *and* $(b,c) \in R^2$, *then* $(a,c) \in R^3$.

(a,b)	(b,c)	(a,c)
$(1,1)$	$(1,1)$	$(1,1)$
$(2,4)$	$(4,4)$	$(2,4)$
$(3,4)$	$(4,4)$	$(3,4)$
$(4,2)$	$(2,2)$	$(4,2)$

Then, $R^3 = \{(1,1),(2,4),(3,4),(4,2)\}$.

Find $R^4 = R^3 \circ R$. $(a,b) \in R$ *and* $(b,c) \in R^2$, *then* $(a,c) \in R^3$

(a,b)	(b,c)	(a,c)
$(1,1)$	$(1,1)$	$(1,1)$
$(2,4)$	$(4,2)$	$(2,2)$
$(3,4)$	$(4,2)$	$(3,2)$
$(4,2)$	$(2,4)$	$(4,4)$

Then, $R^4 = \{(1,1),(2,2),(3,2),(4,4)\}$.