Foundation Algebra for Physical Sciences & Engineering

Module Code: CELEN036

Credits: 10

Semester: Autumn

Dr. Pragnesh Gajjar
Senior Tutor in Science & Engineering, CELE.

Introduction and Welcome

Dr Pragnesh Gajjar
Senior Tutor in Maths & Computer Science

Dr Bamidele Akinwolemiwa
Maths TEam Leader & Module Convenor

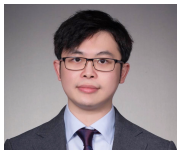
Dr Abdulrauf Shaikh
Seminar Tutor

Dr Chenfei Zhang
Seminar Tutor

GTA
Seminar Tutor

Dr Hanwen Zhang
Seminar Tutor

Dr Chenyang Due
Seminar Tutor



Mr Shen Wang
Seminar Tutor

Dr. Pei Zhang
Seminar Tutor

Mr Abdulhalim Saeed
Seminar Tutor

Maths Modules

The Preliminary Year Mathematics Course consists of two modules:

1. **CELEN036 (Foundation Algebra for Physical Sciences and Engineering) in the Autumn Semester.**
2. CELEN037 (Foundation Calculus & Mathematical Techniques) in the Spring Semester.

In order to progress, you MUST pass both these modules.

Module Content

This module consists of following topics:

- Functions
  - Modulus Inequalities
  - Quadratic Equations
  - Exponential and Logarithmic Functions
- Trigonometry
- Remainder and Factor Theorems
  - Polynomial factorisation
- Numerical Methods
  - Bisection method
  - Fixed-point iteration method
- Binomial Theorem
  - Applications in approximation and error estimation
- Matrices
  - Algebra of matrices
  - Applications in solving systems of linear equations
- Complex numbers
  - Algebra of Complex numbers
  - Polar form
- Partial fractions
- Sequences and Series

Module aims & Learning outcomes

- Provide students with the mathematical knowledge and fluency in algebraic techniques essential for analysing basic problems in engineering or sciences.
- Develop basic mathematical skills in algebra and trigonometry.
- Develop mathematical techniques and their application to problem solving.
- Communicate mathematical arguments using standard terminology.
- Use of e-learning and self-study skills.

Teaching methods and support

- **Lectures**: 2 hours per week (weeks 2-3, 5-8, 10-14)
- **Seminars**: 2 hours per week (weeks 2-3, 5-8, 10-14)
  (practice problems based on methods covered during the week's lecture)
- Independent Learning Week (**ILW**): w/c. 7 Nov 2022.
- **Office hours**: Provisional (TBC)

Day	Time	Room	Day	Time	Room
Monday	11 am - 12 noon	TB 120	Monday	1 pm - 2 pm	TB 417
Tuesday	11 am - 12 noon	TB 412	Tuesday	2 pm - 3 pm	Trent 381
Wednesday	11 am - 12 noon	TB 405	Thursday	1 pm - 2 pm	TB 323
Thursday	1 pm - 2 pm	TB 417	Thursday	3 pm - 4 pm	PB 218
Friday	3 pm - 4 pm	TB 410	Friday	4 pm - 5 pm	TB 417

Learning resources moodle.Nottingham

What can you find on module Moodle page?

- Module Booklet (all essential module information)
- Tutors' Information (when and where to contact tutors for help on difficulties)
- Learning resources (lecture slides, seminar slides, lecture videos, and more)
- Assessment information (weighting of assessments, style of exam papers)
- Sample exam papers and solutions (as a feedback)
- Further reading information (textbooks)

Module Assessment

Type of assessment	When?	Information
Mid semester exam (**30%** weighting)	Wednesday **16 November 2022**	• Duration: 1 hour. • Exam consists of short-answer questions. • Relevant formulae included in questions. • Permissible Calculators allowed.
Final written exam (**70%** weighting)	During University Exam period	• Duration: 1 hour 30 minutes. • Exam consists of 7 equally weighted questions. • Formula sheet will be provided. • **Permissible Calculators** are allowed.

Permissible calculators

Casio $fx-82$ series

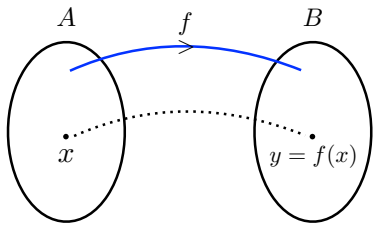
Lecture 1

Topics covered in this lecture session

1. Functions
   - Definitions (Function, Range, Domain, etc.)
   - Standard functions and their graphs
   - Sketching graphs of functions:
     - Some standard curves
     - Sketching other functions using translations, stretching, compression properties
   - Composition of functions
   - Inverse functions
2. Inequalities

Function

- A function f is a rule that associates a **unique** output with **each** input.

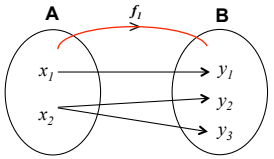


x is Input — **Independent variable**

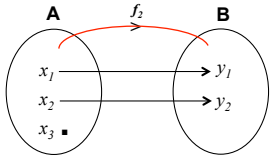
y is Output — **Dependent variable**

- All x in A must have exactly one mapped value in B.
- All x in A must be mapped.

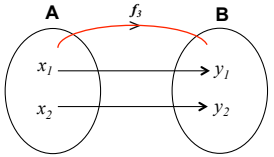
Which of the following mappings are functions?



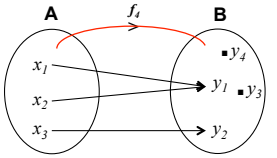
f_1 is not a function because the element x_2 of A is **NOT** mapped uniquely.



f_2 is not a function because the element x_3 of A is **NOT** mapped.



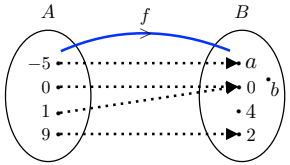
f_3 is a function (Type: One-one & onto)



f_4 is a function (Type: Many one & into).

Functions can be represented by 5 common methods:

Using Venn-diagrams



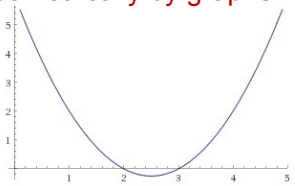
Numerically by tables

x	0	1	2	3
$y = f(x)$	3	4	-1	6

Algebraically by formulas

$$y = f(x) = x^2 - 5x + 6$$

Geometrically by graphs



Verbally (i.e. described in words)

e.g. Newton's law of Universal Gravitation.

The gravitational force of attraction between two bodies in the Universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

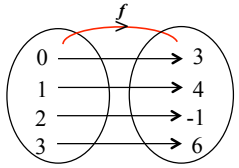
Domain and Range of a function

Domain is the set of allowable inputs (i.e. x values)

Range is the set of outputs (i.e. y values) when x varies over the domain.

e.g. For the function f defined by

x	0	1	2	3
$y = f(x)$	3	4	-1	6



Domain of f is $D_f = \{0, 1, 2, 3\}$

Range of f is $R_f = \{-1, 3, 4, 6\}$

Standard functions

Polynomial Function

- A polynomial in x is a function f that can be expressed as a sum of **finitely** many terms of the form ax^n, where a is constant and n is a non-negative integer.
- Its general form is $P(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$
- The degree (or order) of the polynomial is defined as the highest power of x that occurs in a polynomial.

e.g. $4x^5 - 15x^4 + 7x^3 + x$ is a polynomial of degree 5.

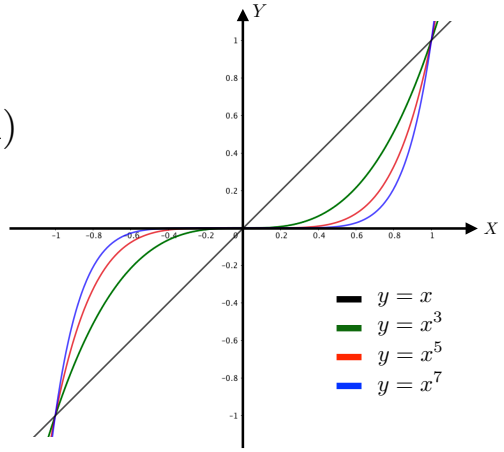
Polynomial functions

Degree	Name	General Form	Example
0	constant	c	9
1	Linear	$ax + b$	$2x + 3$
2	Quadratic	$ax^2 + bx + c$	$5x^2 - 2x + 3$
3	Cubic	$ax^3 + bx^2 + cx + d$	$2x^3 - 2x + 3$
4	Quartic	$ax^4 + bx^3 + cx^2 + dx + e$	$3x^4 + 4x^3 - x^2 + 2x + 7$
5	Quintic	$ax^5 + bx^4 + cx^3 + dx^2 + ex + f$	$x^5 - 2x^2 + 3x + 9$

Graphs of polynomial functions

Graphs of

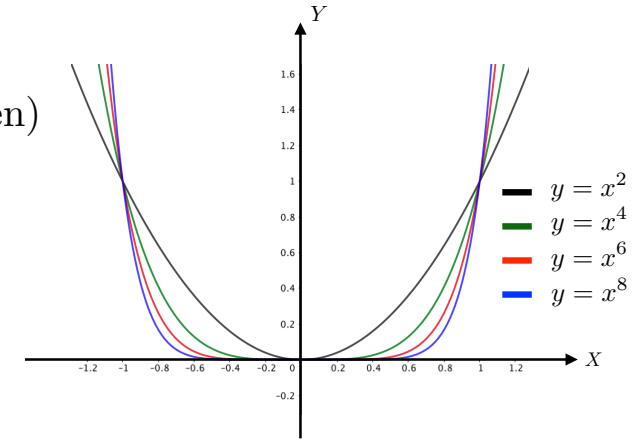
$x^n \quad (n \text{ is odd})$



Graphs of polynomial functions

Graphs of

$x^n \quad (n \text{ is even})$

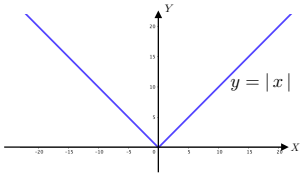


Standard functions

Modulus Function

The modulus function for $x \in \mathbb{R}$ is defined by

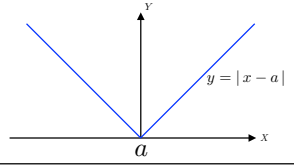
$$|x| = \begin{cases} x & ; \quad x \geq 0 \\ -x & ; \quad x < 0. \end{cases}$$



e.g. $|5| = 5$ and $|-5| = -(-5) = 5$

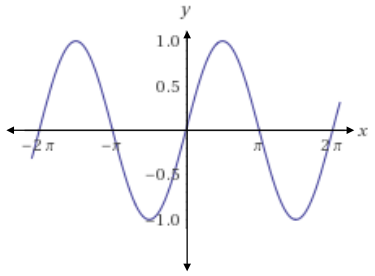
In general,

$$|x - a| = \begin{cases} x - a & ; \quad x \geq a \\ a - x & ; \quad x < a \end{cases}$$

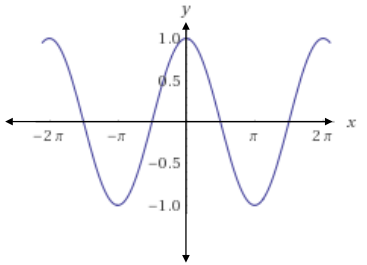


Standard functions

Trigonometric Functions



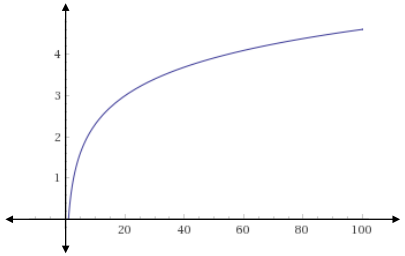
$f(x) = y = \sin x$



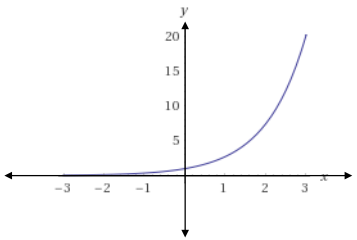
$g(x) = y = \cos x$

Standard functions

Logarithmic and exponential functions



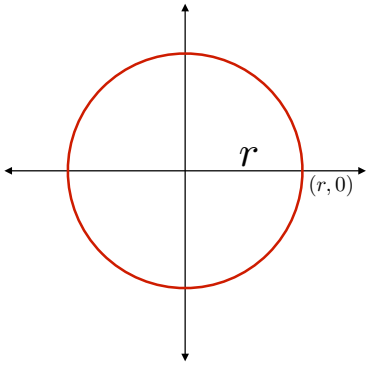
$p(x) = y = \ln x$



$q(x) = y = e^x$

Standard curves

1. Circle with centre at origin



Its equation is:

$$x^2 + y^2 = r^2$$

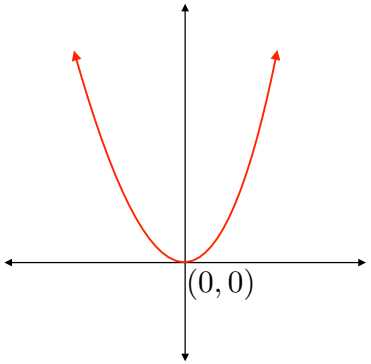
This equation is a combination of two functions

$f_1(x) = y = +\sqrt{r^2 - x^2}$ upper semicircle

and $f_2(x) = y = -\sqrt{r^2 - x^2}$ lower semicircle

Standard curves

2. Parabola



Its equation is:

$$y = x^2$$

What type of function is:

$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^2$?

Answer: Many-one and into.

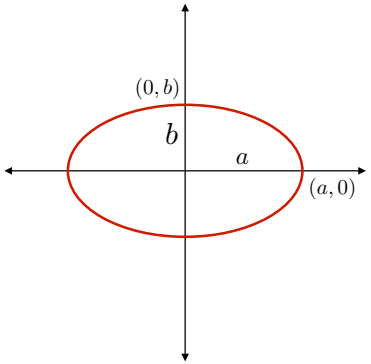
What type of function is:

$f : \mathbb{R} \to \mathbb{R}^+ \cup \{0\}, \quad f(x) = x^2$?

Answer: Many-one and onto.

Standard curves

3. Ellipse



Its equation is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

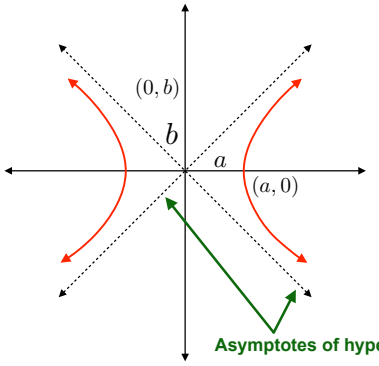
This equation is a combination of two functions

$f_1(x) = y = +\frac{b}{a}\sqrt{a^2 - x^2}$ upper semi-ellipse

and $f_2(x) = y = -\frac{b}{a}\sqrt{a^2 - x^2}$ lower semi-ellipse

Standard curves

4. Hyperbola



Its equation is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

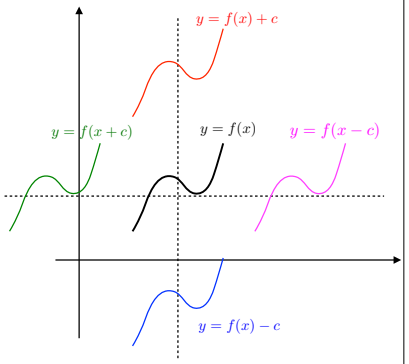
If asymptotes are at right angle, the curve is called rectangular hyperbola.

Its equation is: $x^2 - y^2 = a^2$

Sketching graphs of functions (Translations)

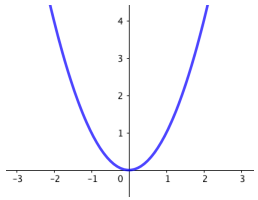
For $c > 0$, to obtain the graph of

1) $y = f(x) + c$, shift the graph of $y = f(x)$ by a distance of c units upward.
2) $y = f(x) - c$, shift the graph of $y = f(x)$ by a distance of c units downward.
3) $y = f(x - c)$, shift the graph of $y = f(x)$ by a distance of c units to the right.
4) $y = f(x + c)$, shift the graph of $y = f(x)$ by a distance of c units to the left.

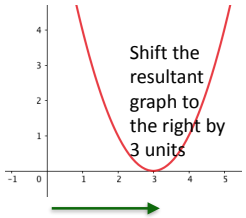


Sketching graphs of functions

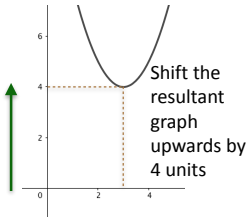
Example: Sketch the graph of: $y = f(x) = (x - 3)^2 + 4$



Graph of $y = x^2$



Graph of $y = (x - 3)^2$

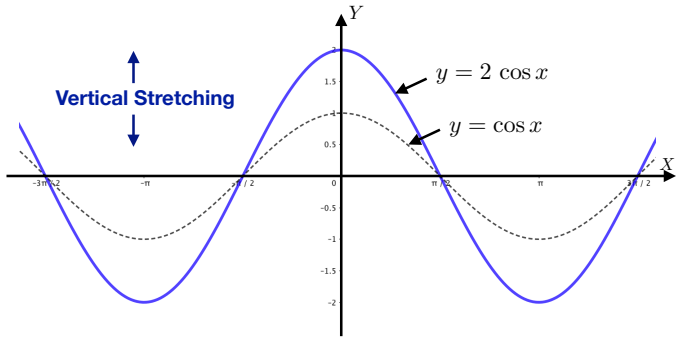


Graph of $y = (x - 3)^2 + 4$

Sketching graphs of functions

Stretches and Compressions

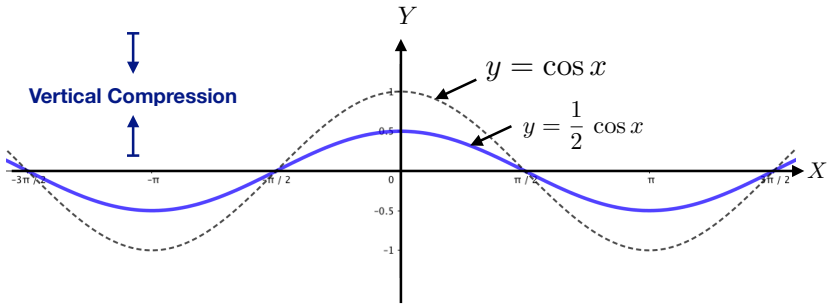
1. $f(x)$ and $k \cdot f(x)$ $(k > 1)$



Sketching graphs of functions

Stretches and Compressions

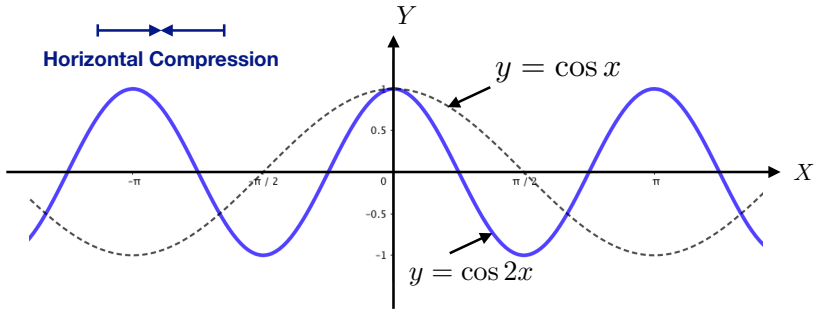
2. $f(x)$ and $k \cdot f(x)$ $(k < 1)$



Sketching graphs of functions

Stretches and Compressions

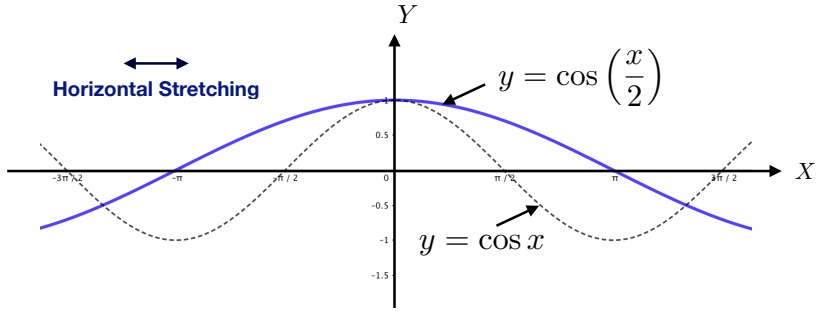
3. $f(x)$ and $f(kx)$ $(k > 1)$



Sketching graphs of functions

Stretches and Compressions

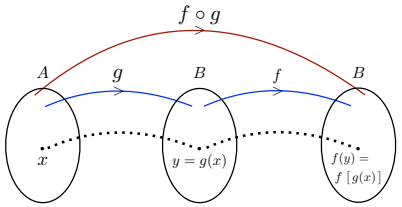
4. $f(x)$ and $f(kx)$ $(k < 1)$



Composition of functions

The composition of functions f and g is defined by

$$(f \circ g)(x) = f\,(\,g(x)\,)$$



e.g. $f(x) = x^2 + 3, \quad g(x) = \sqrt{x},$ then

$$\begin{aligned}(f \circ g)(x) &= f\,(g(x)) \\ &= f\,(\sqrt{x}\,) \\ &= (\sqrt{x}\,)^2 + 3 \\ &= x + 3.\end{aligned}$$

and

$$\begin{aligned}(g \circ f)(x) &= g\,(f(x)) \\ &= g\,(x^2 + 3) \\ &= \sqrt{x^2 + 3}\end{aligned}$$

Composition of functions

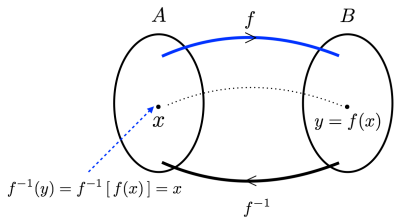
Example: Given $f(x) = 2x + 3$ and $g(x) = x^2 - k$.

Find k if $(f \circ g)(1) = 3$.

$$(f \circ g)(1) = 3 \quad \Rightarrow \quad f(g(1)) = 3$$
$$\Rightarrow \quad f(1^2 - k) = 3$$
$$\Rightarrow \quad 2(1 - k) + 3 = 3$$
$$\Rightarrow \quad k = 1$$

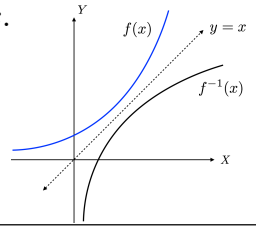
Inverse function

- The inverse function performs the opposite operation to the function f.



$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

- Inverse function only exists for functions that are one-one and onto.
- The graph of $f^{-1}(x)$ is a reflection of $f(x)$ in the line $y = x$.



Inverse function

Example: Given function $f : \mathbb{R}^+ \cup \{0\} \to \{y \in \mathbb{R} \,/\, y \geq 2\}$,

$f(x) = 3x^2 + 2$, find $f^{-1}(x)$.

- Step 1 Let $y = f(x) = 3x^2 + 2$
- Step 2 Express x as a function of y.

$$\Rightarrow 3x^2 = y - 2 \quad \Rightarrow x = \sqrt{\frac{y-2}{3}}$$

- Step 3 $f^{-1}(x)$ is obtained by replacing y by x on the **RHS**. $\therefore \; f^{-1}(x) = \sqrt{\frac{x-2}{3}}$

Inequalities

Inequality	Meaning
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b

Properties of inequalities

(1) $a > b \Leftrightarrow a + c > b + c \quad ; \quad c \in \mathbb{R}$

(2) $a > b \Leftrightarrow ac > bc \quad ; \quad c > 0$

i.e. Inequality will NOT change if both sides are multiplied by a positive number.

(3) $a > b \Leftrightarrow ac < bc \quad ; \quad c < 0$

i.e. Inequality will change if both sides are multiplied by a negative number.

Modulus Inequality

(4) $|x - a| < b \Leftrightarrow a - b < x < a + b$

Properties of inequalities

Modulus Inequality

Solve $|4x - 3| \geq 5$

$\Rightarrow \quad \pm (4x - 3) \geq 5$

$\Rightarrow \quad (4x - 3) \geq 5 \quad$ **or** $\quad -(4x - 3) \geq 5$

$\Rightarrow \quad 4x \geq 8 \qquad \Rightarrow \quad -4x + 3 \geq 5$

$\Rightarrow \quad x \geq 2 \qquad \Rightarrow \quad -4x \geq 2$

$\Rightarrow \quad x \leq -\frac{1}{2}$

Note the change in the inequality

Thus, $x \leq -\frac{1}{2}$ **or** $x \geq 2$. i.e. $x \in \mathbb{R} - \left(-\frac{1}{2}, 2\right)$