Tutorial 8 Probability

1. Rolling two dice. Assume that we are using fair, regular dice (six-sided with values 1, 2, 3, 4, 5, 6 appearing equally likely). Furthermore, assume that all dice rolls are mutually independent events.

(a) You roll two dice and look at the sum of the faces that come up. What is the expected value of this sum? Express your answer as a real number.
$E(X+Y)=E(X)+E(Y)=7/2+7/2=7$

(b) Assuming that the two dice are independent, calculate the variance of their sum. Express your answer as a real number.
$Var(X+Y)=Var(X)+Var(Y)=2.917+2.917=5.834$

2.A famous basketball star is on trial for the murder of his girlfriend. One of the jurors does not have a preconceived opinion of the probability that the defendant is guilty; except the juror does think that if the State is prosecuting the basketball player, then the man is more likely to be guilty. This means the juror's initial notion of the probability of guilt has grown from 0.5 to 0.7, based upon the indictment.

During the trial, DNA evidence was presented to match blood evidence found at the murder scene with the defendant's blood. The DNA expert stated that her test was accurate with a reliability of 0.995, meaning that if the two samples of blood (one from the crime scene and one from the defendant) match, the test would detect this in 99.5% of the cases. Further, if the samples do not come from the same person, the test would declare a mismatch with the same probability.

a. The DNA expert then stated that there was no match between the two samples. Based upon this information, what will be the juror's revised probability that the defendant is guilty?

Solution:

Let $P(G)$ = Probability of Guilt;

Let $P(I)$ = Probability of Innocence;

Let $P(M)$ = Probability that DNA test finds a Match

Let $P(N)$ = Probability that DNA test finds No Match

We start with these known values:

$P(G) = .7$

$P(I) = .3$

$P(M|G) = .995$

$P(N|G) = .005$

$P(M|I) = .005$

$P(N|I) = .995$

We want to find $P(G|N)$, which equals $P(G \text{ and } N)/P(N)$

Using conditional probability, we can find the necessary values:

$P(G \text{ and } N) = P(N \text{ and } G) = P(N|G)P(G) = (.005)(.7) = .0035$

$P(N) = P(N \text{ and } G) + P(N \text{ and } I) = P(N|G)P(G) + P(N|I)P(I) = (.005)(.7) +$

$(.995)(.3) = .0035 + .2985 = .302$

$P(G|N) = .0035/.302 = .0116$