

# AE1MCS: Mathematics for Computer Scientists

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Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Edition, 2013.

- Chapter 7, Section 7.1 An Introduction to Discrete Probability

# Discrete Probability

- Combinatorics and probability theory share common origins (analyzing gambling games).
- The theory of probability now plays an essential role in a wide variety of disciplines (e.g. the study of genetics).
- In computer science,
  - Probability theory plays an important role in the study of the complexity of algorithms.
  - Probabilistic algorithms vs. deterministic algorithms.
  - Probability theory can help us answer questions that involve uncertainty.
  - ...

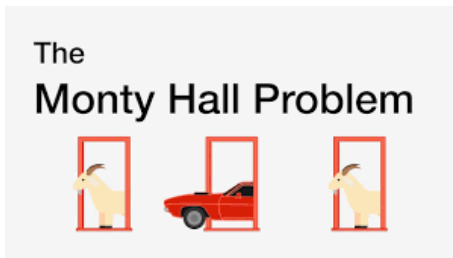
# Content

- Probability of an Event
- Probabilities of Complements and Unions of Events

# Monty Hall Three-Door Puzzle

You are asked to select one of three doors to open; the large prize is behind one of the three doors and the other two doors are losers. Once you select a door, the game show host, who knows what is behind each door, opens one of the other two doors that he knows is a losing door (selecting at random if both are losing doors).

- whether or not you selected the winning door, he opens one of the other two doors that he knows is a losing door (selecting at random if both are losing doors).
- Then he asks you whether you would like to switch doors.



# Finite Probability

Laplace's definition of the probability of an event with **finitely many, equally likely, possible outcomes** is as follows.

## Definition

If  $S$  is a finite nonempty sample space of equally likely outcomes, and  $E$  is an event, that is, a subset of  $S$ , then the probability of  $E$  is

$$p(E) = \frac{|E|}{|S|}$$

- An **experiment** is a procedure that yields one of a given set of possible outcomes.
- The **sample space** of the experiment is the set of possible outcomes.
- An **event** is a subset of the sample space.

# Probability of an Event

In the eighteenth century, the French mathematician Laplace, who also studied gambling, defined **the probability of an event as the number of successful outcomes divided by the number of possible outcomes.**

## Example

A box contains 4 blue balls and 5 red balls. What is the probability that a ball chosen at random from the box is blue?



# Example: Lottery

In a lottery, players win a large prize when they pick four digits that match, in the correct order, four digits selected by a random mechanical process. A smaller prize is won if only three digits are matched.

- What is the probability that a player wins the large prize?
- What is the probability that a player wins the small prize?

# Example: Lottery (Answer)

## Example: Poker 1

Find the probability that a hand of five cards in poker contains four cards of one kind.

- A deck of cards contains 52 cards.
- There are 13 different kinds of cards, with four cards of each kind.
- These kinds are twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, kings, and aces.
- There are 4 suits: spades, clubs, hearts, and diamonds, each containing 13 cards.

## Example: Poker 2

What is the probability that a poker hand contains a full house, that is, three of one kind and two of another kind?

# Probabilities of Complements and Unions of Events

## Theorem

*Let  $E$  be an event in a sample space  $S$ . The probability of the event  $\bar{E} = S - E$ , the complementary event of  $E$ , is given by*

$$p(\bar{E}) = 1 - p(E)$$

## Theorem

*Let  $E_1$  and  $E_2$  be events in the sample space  $S$ . Then*

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

How to prove them?

# Example

- A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0?
- What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5?

# Example 1

A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0?

## Example 2

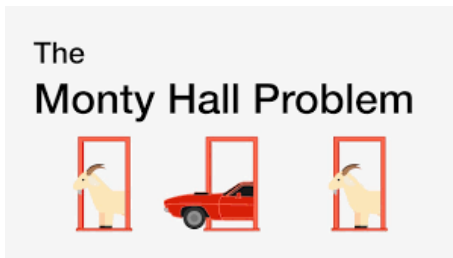
What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by *either* 2 or 5?



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