CS2030 Lecture 10

Functional Programming Concepts

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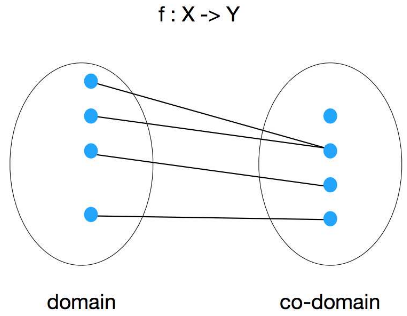
Lecture Outline and Learning Outcomes

- Understand the concepts of *referential transparency* and *no-side-effects* in **pure functions**
- Know how to perform **function composition**
- Appreciate how **currying** supports *partial evaluation*
- Understand how side effects can be handled within *contexts* represented as **functors** or **monads**
- Awareness of the *laws of functors and monads*
- Appreciate that *object-oriented programming* and *functional programming* are *complementary* techniques

Function

- A *function* is a mapping from a set of inputs X (domain) to a set of outputs Y (co-domain), $f : X \rightarrow Y$.
  - Every input in the domain maps to exactly one output
  - Multiple inputs can map to the same output
  - Not all values in the co-domain are mapped



Pure Function

- A *pure function* is a function that
  - takes in arguments and returns a deterministic value
  - has no other *side effects*
- Examples of side effects:
  - Modifying external state
  - Program input and output
  - Throwing exceptions
- The absence of side-effects is a necessary condition for *referential transparency*
  - any expression can be replaced by its resulting value, without changing the property of the program

Pure Function

- Exercise:
  - Are the following functions pure?

```java
int p(int x, int y) {
    return x + y;
}

int q(int x, int y) {
    return x / y;
}

void r(List<Integer> queue, int i) {
    queue.add(i);
}

int s(int i) {
    return this.x + i;
}
```


Higher Order Functions

- Functions are first-class citizens
  - Higher-order functions can take in other functions

```
jshell> Function<Integer,Integer> f = x -> x + 1
f ==> $Lambda$16/0x00000008000b7840@5e3a8624

jshell> Function<Integer,Integer> g = x -> Math.abs(x) * 10
g ==> $Lambda$17/0x00000008000b7c40@604ed9f0

jshell> f.apply(2)
$.. ==> 3

jshell> int sumList(List<Integer> list, Function<Integer,Integer> f) {
   ...> int sum = 0;
   ...> for (Integer item : list) { sum += f.apply(item); }
   ...> return sum; }
|  created method sumList(List<Integer>,Function<Integer,Integer>)

jshell> sumList(List.of(1, -2, 3), f)
$.. ==> 5

jshell> sumList(List.of(1, -2, 3), g)
$.. ==> 60
```


Function Composition

- Function composition: $(g \circ f)(x) = g(f(x))$

```
jshell> Function<String, Integer> f = str -> str.length()
f ==> $Lambda$14/731395981@475530b9

jshell> Function<Integer, Circle> g = x -> new Circle(x)
g ==> $Lambda$15/650023597@4c70fda8
```

- `Function<T,R>` has a default `andThen` method:

```java
default <V> Function<T,V> andThen(
        Function<? super R, ? extends V> after)
```

```
jshell> f.andThen(g).apply("abc")
$.. ==> Circle with radius: 3.0
```

- `Function<T,R>` has an alternative default `compose` method:

```java
default <V> Function<V,R> compose(
        Function<? super V, ? extends T> before)
```

```
jshell> g.compose(f).apply("abc")
$.. ==> Circle with radius: 3.0
```


Function With Multiple Arguments

- Consider the following:

```
jshell> BinaryOperator<Integer> f = (x,y) -> x + y
f ==> $Lambda$14/1268650975@2b98378d

jshell> f.apply(1, 2)
$.. ==> 3
```

- We can achieve the same with just `Function<T,R>`

```
jshell> Function<Integer, Function<Integer,Integer>> f = new Function<>() {
   ...>     @Override
   ...>     public Function<Integer,Integer> apply(Integer x) {
   ...>        return new Function<Integer,Integer>() {
   ...>            @Override
   ...>            public Integer apply(Integer y) {
   ...>                return x + y;
   ...>            }
   ...>        };
   ...>     }
   ...> }
f ==> 1@2b98378d

jshell> f.apply(1).apply(2)
$.. ==> 3
```

Currying

- The lambda expression `(x, y) -> x + y` can be re-expressed as `x -> (y -> x + y)` or simply, `x -> y -> x + y`

```
jshell> Function<Integer, Function<Integer,Integer>> f = x -> y -> x + y
f ==> $Lambda$14/486898233@26be92ad

jshell> f.apply(1).apply(2)
$.. ==> 3
```

- This is known as **currying**, and it gives us a way to handle lambdas of an arbitrary number of arguments
- Currying supports *partial evaluation*
  - E.g. partially evaluating `f` for increment:

```
jshell> Function<Integer,Integer> inc = f.apply(1)
inc ==> $Lambda$15/575593575@46d56d67

jshell> inc.apply(10)
$.. ==> 11
```


Pure Functions.. or *Pure Fantasy?*

- Side-effects are a necessary evil
- Handle side-effects within a *context*, e.g.
  - `Maybe`/`Optional` handles the context of missing values
  - `ImList` handles the context of list processing
  - `Stream` handles the context of loops (and parallel) processing
  - *etc.*
- Values wrapped within contexts can be represented by **Functors** (with `map`) or **Monads** (with `flatMap`)
- In the following slides, assume `Functor<T>` and `Monad<T>` are generic interfaces with specific methods to be implemented

Functor

- Functor has a method:

  `<R> Functor<R> map(Function<T,R> f)` $\boxed{\texttt{c}} \xRightarrow{x \to f(x)} \boxed{\texttt{f(c)}}$

- A functor must obey the two *functor laws*:
  - **Identity**: if mapping over an identity function $x \to x$, then resulting functor should be unchanged:

    `functor.map(x -> x)` $\equiv$ `functor` $\boxed{\texttt{c}} \xRightarrow{x \to x} \boxed{\texttt{c}}$

  - **Associative**: if mapping over $g \circ h$, then the resulting functor should be the same as mapping over h then g

    `functor.map(g.compose(h))` $\equiv$ `functor.map(f).map(g)`

    $$\boxed{\texttt{c}} \xRightarrow{g \circ h} \boxed{\texttt{g(h(c))}} \equiv \boxed{\texttt{c}} \xRightarrow{h} \boxed{\texttt{h(c)}} \xRightarrow{g} \boxed{\texttt{g(h(c))}}$$

Functor

- `Optional` is a functor with `map`

```
jshell> Optional<String> opt1 = Optional.of("abc")
opt1 ==> Optional[abc]

jshell> Optional<String> opt0 = Optional.empty()
opt0 ==> Optional.empty

jshell> opt1.map(x -> x).equals(opt1) // identity
$3 ==> true

jshell> opt0.map(x -> x).equals(opt0) // identity
$4 ==> true

jshell> Function<String,Integer> h = x -> x.length()
h ==> $Lambda$16/1282473384@224edc67

jshell> Function<Integer,Integer> g = x -> x * 10
g ==> $Lambda$17/1188392295@d8355a8

jshell> opt1.map(g.compose(h)).equals(opt1.map(h).map(g)) // associative
$.. ==> true

jshell> opt0.map(g.compose(h)).equals(opt0.map(h).map(g)) // associative
$.. ==> true
```

Monad

- Monad with the following methods:
  - `Monad<T> of(T value)`, that creates the Monad
  - `<R> Monad<R> flatMap(Function<T,Monad<R>> f)`

$$\boxed{\mathsf{c}} \xRightarrow{x \to \boxed{\mathsf{f(x)}}} \boxed{\mathsf{f(c)}}$$

- Just like functor laws, there are monad laws
- In the following slide, suppose
  - `Monad.of(x)` gives $\boxed{\mathsf{x}}$, i.e. wraps x within a context
  - `monad` is a constant represented by $\boxed{\mathsf{c}}$, i.e. some fixed value wrapped within a context

Monad

- **Right identity**: `monad.flatMap(x -> Monad.of(x))` $\equiv$ `monad`

$$\boxed{\mathsf{c}} \xRightarrow{x \to \boxed{\mathsf{x}}} \boxed{\mathsf{c}}$$

- **Left identity**: `Monad.of(x).flatMap(f)` $\equiv$ `f.apply(x)`

$$\boxed{\mathsf{x}} \xRightarrow{x \to \boxed{\mathsf{f(x)}}} \boxed{\mathsf{f(x)}} \equiv x \to \boxed{\mathsf{f(x)}}$$

- **Associative**

`monad.flatMap(h).flatMap(g)` $\equiv$ `monad.flatMap(x -> h.apply(x).flatMap(g))`

$$\boxed{\mathsf{c}} \xRightarrow{x \to \boxed{\mathsf{h(x)}}} \boxed{\mathsf{h(c)}} \xRightarrow{x \to \boxed{\mathsf{g(x)}}} \boxed{\mathsf{g(h(c))}} \equiv \boxed{\mathsf{c}} \xRightarrow{x -> \boxed{\mathsf{g(h(x))}}} \boxed{\mathsf{g(h(c))}}$$

Note the composition $g \circ h$ is expressed as `x -> h.apply(x).flatMap(g)`

Monad

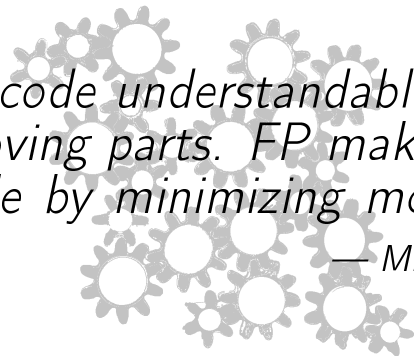
- `Optional` is also a Monad

```
jshell> Function<String,Optional<Integer>> f = x -> Optional.of(x.length())
f ==> $Lambda$19/1529306539@61832929

jshell> Optional.of("abc").flatMap(f).equals(f.apply("abc")) // left identity
$.. ==> true

jshell> Function<String,Optional<Integer>> e = x -> Optional.empty()
e ==> $Lambda$20/1582797472@26653222

jshell> Optional.of("abc").flatMap(e).equals(e.apply("abc")) // left identity
$.. ==> true

jshell> Optional<String> opt = Optional.of("monad")
opt ==> Optional[monad]

jshell> opt.flatMap(x -> Optional.of(x)).equals(opt) // right identity
$.. ==> true

jshell> opt = Optional.empty()
opt ==> Optional.empty

jshell> opt.flatMap(x -> Optional.of(x)).equals(opt) // right identity
$.. ==> true
```


OOP and FP Are Complementary

- *continued...*

```
jshell> Function<Integer,Optional<Integer>> g = x -> Optional.of(x * 10)
g ==> $Lambda$22/670035812@6f7fd0e6

jshell> opt.flatMap(f).flatMap(g).equals(
   ...>    opt.flatMap(x -> f.apply(x).flatMap(g))) // associative
$.. ==> true
```



OO makes code understandable by encapsulating moving parts. FP makes code understandable by minimizing moving parts.

— Michael Feathers