# 1 Syntax

| **Phase 0** ($e_0$ pre-expansion, $e$ post-expansion) | **Phase 1** (macro definitions) |
|---|---|
| $e_0 ::= x_0 \mid \lambda x_0.\, e_1 \mid e_0\ e_0 \mid \text{\$splice}\ e_1 \mid \text{\$let-macro}\ x_1 = e_1 \text{ in } e_0$ | $e_1 ::= x_1 \mid \lambda x_1.\, e_1 \mid e_1\ e_1 \mid \text{dia}(e_0) \mid \text{let-dia}\ x_0 = e_0 \text{ in } e_1$ |
| $e ::= x \mid \lambda x.\, e \mid e\ e$ | |
| $\tau_0, \tau ::= \tau \to \tau$ | $\tau_1 ::= \tau_1 \to \tau_1 \mid \Diamond \tau_0$ |
| $v ::= \lambda x_0.\, e_0$ | $v_1 ::= \lambda x_1.\, e_1 \mid \text{dia}(e)$ |

# 2 Typing Rules

$\boxed{\Delta; \Gamma \vdash_0 e_0 : \tau_0}$ $\quad$ $\boxed{\Gamma \vdash e : \tau}$ $\quad$ $\boxed{\Delta; \Gamma \vdash_1 e_1 : \tau_1}$

**Lambda calculus fragment**

$$\frac{x_0 : \tau_0 \in \Gamma}{\Delta; \Gamma \vdash_0 x_0 : \tau_0} \qquad \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{x_1 : \tau_1 \in \Delta}{\Delta; \Gamma \vdash_1 x_1 : \tau_1}$$

$$\frac{\Delta; \Gamma, x_0 : \tau_0^{\text{in}} \vdash_0 e_0 : \tau_0^{\text{out}}}{\Delta; \Gamma \vdash_0 \lambda x_0.\, e_0 : \tau_0^{\text{in}} \to \tau_0^{\text{out}}} \qquad \frac{\Delta; \Gamma, x : \tau^{\text{in}} \vdash e : \tau^{\text{out}}}{\Delta; \Gamma \vdash \lambda x.\, e : \tau^{\text{in}} \to \tau^{\text{out}}} \qquad \frac{\Delta, x_1 : \tau_1^{\text{in}}; \Gamma \vdash_0 e_1 : \tau_1^{\text{out}}}{\Delta; \Gamma \vdash_1 \lambda x_1.\, e_1 : \tau_1^{\text{in}} \to \tau_1^{\text{out}}}$$

$$\frac{\Delta; \Gamma \vdash_0 e_0^{\text{fun}} : \tau_0^{\text{in}} \to \tau_0^{\text{out}} \quad \Delta; \Gamma \vdash_0 e_0^{\text{in}} : \tau_0^{\text{in}}}{\Delta; \Gamma \vdash_0 e_0^{\text{fun}}\ e_0^{\text{in}} : \tau_0^{\text{out}}} \qquad \frac{\Delta; \Gamma \vdash e^{\text{fun}} : \tau^{\text{in}} \to \tau^{\text{out}} \quad \Delta; \Gamma \vdash e^{\text{in}} : \tau^{\text{in}}}{\Delta; \Gamma \vdash e^{\text{fun}}\ e^{\text{in}} : \tau^{\text{out}}} \qquad \frac{\Delta; \Gamma \vdash_1 e_1^{\text{fun}} : \tau_1^{\text{in}} \to \tau_1^{\text{out}} \quad \Delta; \Gamma \vdash_1 e_1^{\text{in}} : \tau_1^{\text{in}}}{\Delta; \Gamma \vdash_1 e_1^{\text{fun}}\ e_1^{\text{in}} : \tau_1^{\text{out}}}$$

**Modal fragment**

$$\frac{\Delta; \Gamma \vdash_1 e_1 : \tau_1 \quad \Delta, x_1 : \tau_1; \Gamma \vdash_0 e_0 : \tau_0}{\Delta; \Gamma \vdash_0 \text{\$let-macro}\ x_1 = e_1 \text{ in } e_0 : \tau_0} \qquad \frac{\Delta; \Gamma \vdash_1 e_1^{\text{x}} : \Diamond \tau_0 \quad \Delta; \Gamma, x_0 : \tau_0 \vdash_1 e_1^{\text{body}} : \tau_1}{\Delta; \Gamma \vdash_1 \text{let-dia}\ x_0 = e_1^{\text{x}} \text{ in } e_1^{\text{body}} : \tau_1}$$

$$\frac{\Delta; \Gamma \vdash_1 e_1 : \Diamond \tau_0}{\Delta; \Gamma \vdash_0 \text{\$splice}\ e_1 : \tau_0} \qquad \frac{\Delta; \Gamma \vdash_0 e_0 : \tau_0}{\Delta; \Gamma \vdash_1 \text{dia}(e_0) : \Diamond \tau_0}$$

## 3 Big-Steps Operational Semantics

$\boxed{e_0 \Downarrow_0 v}$

$$\frac{e_0 \Downarrow_E e \qquad e \Downarrow v}{e_0 \Downarrow_0 v}$$

$\boxed{e \Downarrow v}$ $\boxed{e_1 \Downarrow_1 v_1}$

**Lambda calculus fragment**

$$\frac{}{\lambda x.\, e \Downarrow \lambda x.\, e} \qquad \frac{}{\lambda x_1.\, e_1 \Downarrow_1 \lambda x_1.\, e_1}$$

$$\frac{e^{\text{fun}} \Downarrow \lambda x.\, e^{\text{out}} \qquad e^{\text{in}} \Downarrow v^{\text{in}} \qquad e^{\text{out}}[v^{\text{in}}/x] \Downarrow v^{\text{out}}}{e^{\text{fun}}\ e^{\text{in}} \Downarrow v^{\text{out}}}$$

$$\frac{e_1^{\text{fun}} \Downarrow_1 \lambda x_1.\, e_1^{\text{out}} \qquad e_1^{\text{in}} \Downarrow_1 v_1^{\text{in}} \qquad e_1^{\text{out}}[v_1^{\text{in}}/x_1] \Downarrow_1 v_1^{\text{out}}}{e_1^{\text{fun}}\ e_1^{\text{in}} \Downarrow_1 v_1^{\text{out}}}$$

**Modal fragment**

$$\frac{e_1^{\text{x}} \Downarrow_1 \text{dia}(e) \qquad e_1^{\text{body}}[e/x_0] \Downarrow_1 v_1}{\text{let-dia } x_0 = e_1^{\text{x}} \text{ in } e_1^{\text{body}} \Downarrow_1 v_1}$$

$$\frac{e_0 \Downarrow_{\text{E}} e}{\text{dia}(e_0) \Downarrow_1 \text{dia}(e)}$$

**Expansion**

$\boxed{e_0 \Downarrow_{\text{E}} e}$

**Lambda calculus fragment**

$$\frac{}{x_0 \Downarrow_{\text{E}} x} \qquad \frac{e_0 \Downarrow_{\text{E}} e}{\lambda x_0.\, e_0 \Downarrow_{\text{E}} \lambda x.\, e} \qquad \frac{e_0^{\text{fun}} \Downarrow_{\text{E}} e^{\text{fun}} \qquad e_0^{\text{in}} \Downarrow_{\text{E}} e^{\text{in}}}{e_0^{\text{fun}}\ e_0^{\text{in}} \Downarrow_{\text{E}} e^{\text{fun}}\ e^{\text{in}}}$$

**Modal fragment**

$$\frac{e_1 \Downarrow_1 v_1 \qquad e_0[v_1/x_1] \Downarrow_{\text{E}} e}{\text{\$let-macro } x_1 = e_1 \text{ in } e_0 \Downarrow_0 e} \qquad \frac{e_1 \Downarrow_1 \text{dia}(e)}{\text{\$splice } e_1 \Downarrow_{\text{E}} e}$$