

# 1 Staged Diamond Calculus

## 1.1 Syntax

<b>Phase 0</b> $(e_0 \text{ pre-expansion,}$ $e \text{ post-expansion})$	<b>Phase 1</b> $(\text{macro definitions})$
$e_0 ::= x_0 \mid \lambda x_0. e_0 \mid e_0 e_0 \mid \$\text{splice } e_1$ $\quad \mid \$\text{let-macro } x_1 = e_1 \text{ in } e_0$	$e_1 ::= x_1 \mid \lambda x_1. e_1 \mid e_1 e_1 \mid \text{dia}(e_0)$ $\quad \mid \text{let-dia } x_0 = e_1 \text{ in } e_1$
$e ::= x \mid \lambda x. e \mid e e$	
$\tau_0, \tau ::= \tau \rightarrow \tau$	$\tau_1 ::= \tau_1 \rightarrow \tau_1 \mid \Diamond \tau_0$
$v ::= \lambda x. e$	$v_1 ::= \lambda x_1. e_1 \mid \text{dia}(e)$
$\Gamma_0 ::= \cdot \mid \Gamma_0, x_0 : \tau_0$	$\Gamma_1 ::= \cdot \mid \Gamma_1, x_1 : \tau_1$
$\Gamma ::= \cdot \mid \Gamma, x : \tau$	

## 2 Typing Rules

$$\boxed{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\boxed{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \tau_1}$$

### Lambda calculus fragment

$$\begin{array}{ccc}
\frac{x_0 : \tau_0 \in \Gamma_0}{\Gamma_1; \Gamma_0 \vdash_0 x_0 : \tau_0} & \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} & \frac{x_1 : \tau_1 \in \Gamma_1}{\Gamma_1; \Gamma_0 \vdash_1 x_1 : \tau_1} \\
\\
\frac{\Gamma_1; \Gamma_0, x_0 : \tau_0^{\text{in}} \vdash_0 e_0 : \tau_0^{\text{out}}}{\Gamma_1; \Gamma_0 \vdash_0 \lambda x_0. e_0 : \tau_0^{\text{in}} \rightarrow \tau_0^{\text{out}}} & \frac{\Gamma_1; \Gamma, x : \tau^{\text{in}} \vdash e : \tau^{\text{out}}}{\Gamma_1; \Gamma \vdash \lambda x. e : \tau^{\text{in}} \rightarrow \tau^{\text{out}}} & \frac{\Gamma_1, x_1 : \tau_1^{\text{in}}, \Gamma_0 \vdash_0 e_1 : \tau_1^{\text{out}}}{\Gamma_1; \Gamma_0 \vdash_1 \lambda x_1. e_1 : \tau_1^{\text{in}} \rightarrow \tau_1^{\text{out}}} \\
\\
\frac{\Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{fun}} : \tau_0^{\text{in}} \rightarrow \tau_0^{\text{out}} \quad \Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{in}} : \tau_0^{\text{in}}}{\Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{fun}} e_0^{\text{in}} : \tau_0^{\text{out}}} & \frac{\Gamma_1; \Gamma \vdash e^{\text{fun}} : \tau^{\text{in}} \rightarrow \tau^{\text{out}} \quad \Gamma_1; \Gamma \vdash e^{\text{in}} : \tau^{\text{in}}}{\Gamma_1; \Gamma \vdash e^{\text{fun}} e^{\text{in}} : \tau^{\text{out}}} & \frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{fun}} : \tau_1^{\text{in}} \rightarrow \tau_1^{\text{out}} \quad \Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{in}} : \tau_1^{\text{in}}}{\Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{fun}} e_1^{\text{in}} : \tau_1^{\text{out}}}
\end{array}$$

### Modal fragment

$$\begin{array}{ccc}
\frac{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \tau_1 \quad \Gamma_1, x_1 : \tau_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_0 \$\text{let-macro } x_1 = e_1 \text{ in } e_0 : \tau_0} & \frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^x : \Diamond \tau_0 \quad \Gamma_1; \Gamma_0, x_0 : \tau_0 \vdash_1 e_1^{\text{body}} : \tau_1}{\Gamma_1; \Gamma_0 \vdash_1 \text{let-dia } x_0 = e_1^x \text{ in } e_1^{\text{body}} : \tau_1} & \\
\\
\frac{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \Diamond \tau_0}{\Gamma_1; \Gamma_0 \vdash_0 \$\text{splice } e_1 : \tau_0} & \frac{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_1 \text{dia}(e_0) : \Diamond \tau_0} & 
\end{array}$$

### 3 Big-Steps Operational Semantics

$e_0 \Downarrow_0 v$	$e \Downarrow v$	$e_1 \Downarrow_1 v_1$
<b>Lambda calculus fragment</b>		
$\frac{e_0 \Downarrow_E e \quad e \Downarrow v}{e_0 \Downarrow_0 v}$	$\frac{}{\lambda x. e \Downarrow \lambda x. e}$ $\frac{e^{\text{fun}} \Downarrow \lambda x. e^{\text{out}} \quad e^{\text{in}} \Downarrow v^{\text{in}} \quad e^{\text{out}}[v^{\text{in}}/x] \Downarrow v^{\text{out}}}{e^{\text{fun}} e^{\text{in}} \Downarrow v^{\text{out}}}$	$\frac{}{\lambda x_1. e_1 \Downarrow_1 \lambda x_1. e_1}$ $\frac{e_1^{\text{fun}} \Downarrow_1 \lambda x_1. e_1^{\text{out}} \quad e_1^{\text{in}} \Downarrow_1 v_1^{\text{in}} \quad e_1^{\text{out}}[v_1^{\text{in}}/x_1] \Downarrow_1 v_1^{\text{out}}}{e_1^{\text{fun}} e_1^{\text{in}} \Downarrow_1 v_1^{\text{out}}}$
<b>Modal fragment</b>		
$\frac{e_1^x \Downarrow_1 \text{dia}(e) \quad e_1^{\text{body}}[e/x_0] \Downarrow_1 v_1}{\text{let-dia } x_0 = e_1^x \text{ in } e_1^{\text{body}} \Downarrow_1 v_1}$ $\frac{e_0 \Downarrow_E e}{\text{dia}(e_0) \Downarrow_1 \text{dia}(e)}$		

**Expansion**

$$e_0 \Downarrow_E e$$

**Lambda calculus fragment**

$\frac{}{x_0 \Downarrow_E x}$	$\frac{e_0 \Downarrow_E e}{\lambda x_0. e_0 \Downarrow_E \lambda x. e}$	$\frac{e_0^{\text{fun}} \Downarrow_E e^{\text{fun}} \quad e_0^{\text{in}} \Downarrow_E e^{\text{in}}}{e_0^{\text{fun}} e_0^{\text{in}} \Downarrow_E e^{\text{fun}} e^{\text{in}}}$
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**Modal fragment**

$\frac{e_1 \Downarrow_1 v_1 \quad e_0[v_1/x_1] \Downarrow_E e}{\text{\$let-macro } x_1 = e_1 \text{ in } e_0 \Downarrow_0 e}$	$\frac{e_1 \Downarrow_1 \text{dia}(e)}{\text{\$splice } e_1 \Downarrow_E e}$
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## 4 Staged Box Type Theory

$$\boxed{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e : \tau}$$

**Lambda calculus fragment**

$$\frac{x : \tau \in \Gamma_1}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash x : \tau}$$

$$\frac{\Delta; \Gamma_n; \dots; \Gamma_1, x : \tau^{\text{in}} \vdash e : \tau^{\text{out}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \lambda x. e : \tau^{\text{in}} \rightarrow \tau^{\text{out}}} \quad \frac{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{fun}} : \tau^{\text{in}} \rightarrow \tau^{\text{out}} \quad \Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{in}} : \tau^{\text{in}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{fun}} e^{\text{in}} : \tau^{\text{out}}}$$

**Modal fragment**

$$\frac{x : \tau \in \Delta}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash x : \tau}$$

$$\frac{\Delta; \Gamma_n; \dots; \Gamma_1; \cdot \vdash e : \tau}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \text{box}(e) : \Box \tau} \quad \frac{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^x : \Box \tau^x \quad \Delta, x : \tau^x; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{body}} : \tau^{\text{body}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \text{let-box } x = e^x \text{ in } e^{\text{body}} : \tau^{\text{body}}}$$

### 4.1 Comparison

$$\frac{\Delta; \Gamma_1; \cdot \vdash e : \tau}{\Delta; \Gamma_1 \vdash \text{box}(e) : \Box \tau} \quad \frac{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_1 \text{dia}(e_0) : \Diamond \tau_0}$$

$$\frac{\Delta; \Gamma_1 \vdash e^x : \Box \tau^x \quad \Delta, x : \tau^x; \Gamma_1 \vdash e^{\text{body}} : \tau^{\text{body}}}{\Delta; \Gamma_1 \vdash \text{let-box } x = e^x \text{ in } e^{\text{body}} : \tau^{\text{body}}} \quad \frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^x : \Diamond \tau_0 \quad \Gamma_1; \Gamma_0, x_0 : \tau_0 \vdash_1 e_1^{\text{body}} : \tau_1}{\Gamma_1; \Gamma_0 \vdash_1 \text{let-dia } x_0 = e_1^x \text{ in } e_1^{\text{body}} : \tau_1}$$