

# 1 Staged Diamond Calculus

## 1.1 Syntax

Phase 0 ( $e_0$ pre-expansion, $e$ post-expansion)	Phase 1 (macro definitions)
$e_0 ::= x_0 \mid \lambda x_0. e_0 \mid e_0 e_0 \mid \$splice\ e_1$ $\mid \$let\text{-}macro\ x_1 = e_1\ in\ e_0$ $e ::= x \mid \lambda x. e \mid e\ e$ $\tau_0, \tau ::= \tau \rightarrow \tau$ $v ::= \lambda x. e$ $\Gamma_0 ::= \cdot \mid \Gamma_0, x_0 : \tau_0$ $\Gamma ::= \cdot \mid \Gamma, x : \tau$	$e_1 ::= x_1 \mid \lambda x_1. e_1 \mid e_1\ e_1 \mid dia(e_0)$ $\mid let\text{-}dia\ x_0 = e_1\ in\ e_1$ $\tau_1 ::= \tau_1 \rightarrow \tau_1 \mid \Diamond \tau_0$ $v_1 ::= \lambda x_1. e_1 \mid dia(e)$ $\Gamma_1 ::= \cdot \mid \Gamma_1, x_1 : \tau_1$

## 2 Typing Rules

$$\boxed{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\boxed{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \tau_1}$$

### Lambda calculus fragment

$$\frac{x_0 : \tau_0 \in \Gamma_0}{\Gamma_1; \Gamma_0 \vdash_0 x_0 : \tau_0} \quad \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \frac{x_1 : \tau_1 \in \Gamma_1}{\Gamma_1; \Gamma_0 \vdash_1 x_1 : \tau_1}$$

$$\frac{\Gamma_1; \Gamma_0, x_0 : \tau_0^{\text{in}} \vdash_0 e_0 : \tau_0^{\text{out}}}{\Gamma_1; \Gamma_0 \vdash_0 \lambda x_0. e_0 : \tau_0^{\text{in}} \rightarrow \tau_0^{\text{out}}} \quad \frac{\Gamma, x : \tau^{\text{in}} \vdash e : \tau^{\text{out}}}{\Gamma \vdash \lambda x. e : \tau^{\text{in}} \rightarrow \tau^{\text{out}}} \quad \frac{\Gamma_1, x_1 : \tau_1^{\text{in}}; \Gamma_0 \vdash_0 e_1 : \tau_1^{\text{out}}}{\Gamma_1; \Gamma_0 \vdash_1 \lambda x_1. e_1 : \tau_1^{\text{in}} \rightarrow \tau_1^{\text{out}}}$$

$$\frac{\Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{fun}} : \tau_0^{\text{in}} \rightarrow \tau_0^{\text{out}} \quad \Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{in}} : \tau_0^{\text{in}}}{\Gamma_1; \Gamma_0 \vdash_0 e_0^{\text{fun}}\ e_0^{\text{in}} : \tau_0^{\text{out}}} \quad \frac{\Gamma \vdash e^{\text{fun}} : \tau^{\text{in}} \rightarrow \tau^{\text{out}} \quad \Gamma \vdash e^{\text{in}} : \tau^{\text{in}}}{\Gamma \vdash e^{\text{fun}}\ e^{\text{in}} : \tau^{\text{out}}} \quad \frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{fun}} : \tau_1^{\text{in}} \rightarrow \tau_1^{\text{out}} \quad \Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{in}} : \tau_1^{\text{in}}}{\Gamma_1; \Gamma_0 \vdash_1 e_1^{\text{fun}}\ e_1^{\text{in}} : \tau_1^{\text{out}}}$$

### Modal fragment

$$\frac{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \tau_1 \quad \Gamma_1, x_1 : \tau_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_0 \$let\text{-}macro\ x_1 = e_1\ in\ e_0 : \tau_0}$$

$$\frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^x : \Diamond \tau_0 \quad \Gamma_1; \Gamma_0, x_0 : \tau_0 \vdash_1 e_1^{\text{body}} : \tau_1}{\Gamma_1; \Gamma_0 \vdash_1 let\text{-}dia\ x_0 = e_1^x\ in\ e_1^{\text{body}} : \tau_1}$$

$$\frac{\Gamma_1; \Gamma_0 \vdash_1 e_1 : \Diamond \tau_0}{\Gamma_1; \Gamma_0 \vdash_0 \$splice\ e_1 : \tau_0}$$

$$\frac{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_1 dia(e_0) : \Diamond \tau_0}$$

### 3 Big-Steps Operational Semantics

$e_0 \Downarrow_0 v$	$e \Downarrow v$	$e_1 \Downarrow_1 v_1$
<b>Lambda calculus fragment</b>		
$\frac{e_0 \Downarrow_E e \quad e \Downarrow v}{e_0 \Downarrow_0 v}$	$\frac{}{\lambda x. e \Downarrow \lambda x. e}$ $\frac{e^{\text{fun}} \Downarrow \lambda x. e^{\text{out}} \quad e^{\text{in}} \Downarrow v^{\text{in}}}{e^{\text{out}}[x := v^{\text{in}}] \Downarrow v^{\text{out}}}$ $\frac{}{e^{\text{fun}} e^{\text{in}} \Downarrow v^{\text{out}}}$	$\frac{}{\lambda x_1. e_1 \Downarrow_1 \lambda x_1. e_1}$ $\frac{e_1^{\text{fun}} \Downarrow_1 \lambda x_1. e_1^{\text{out}} \quad e_1^{\text{in}} \Downarrow_1 v_1^{\text{in}}}{e_1^{\text{out}}[x_1 := v_1^{\text{in}}] \Downarrow_1 v_1^{\text{out}}}$ $\frac{}{e_1^{\text{fun}} e_1^{\text{in}} \Downarrow_1 v_1^{\text{out}}}$
<b>Modal fragment</b>		
$\frac{e_1^x \Downarrow_1 \text{dia}(e) \quad e_1^{\text{body}}[x_0 := e] \Downarrow_1 v_1}{\text{let-dia } x_0 = e_1^x \text{ in } e_1^{\text{body}} \Downarrow_1 v_1}$ $\frac{e_0 \Downarrow_E e}{\text{dia}(e_0) \Downarrow_1 \text{dia}(e)}$		

#### Expansion

$$e_0 \Downarrow_E e$$

<b>Lambda calculus fragment</b>		
$\frac{}{x_0 \Downarrow_E x}$	$\frac{e_0 \Downarrow_E e}{\lambda x_0. e_0 \Downarrow_E \lambda x. e}$	$\frac{e_0^{\text{fun}} \Downarrow_E e^{\text{fun}} \quad e_0^{\text{in}} \Downarrow_E e^{\text{in}}}{e_0^{\text{fun}} e_0^{\text{in}} \Downarrow_E e^{\text{fun}} e^{\text{in}}}$
<b>Modal fragment</b>		
$\frac{e_1 \Downarrow_1 v_1 \quad e_0[x_1 := v_1] \Downarrow_E e}{\text{\$let-macro } x_1 = e_1 \text{ in } e_0 \Downarrow_0 e}$	$\frac{e_1 \Downarrow_1 \text{dia}(e)}{\text{\$splice } e_1 \Downarrow_E e}$	

## 4 Staged Box Calculus

$$\boxed{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e : \tau}$$

**Lambda calculus fragment**

$$\frac{x : \tau \in \Gamma_1}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash x : \tau}$$

$$\frac{\Delta; \Gamma_n; \dots; \Gamma_1, x : \tau^{\text{in}} \vdash e : \tau^{\text{out}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \lambda x. e : \tau^{\text{in}} \rightarrow \tau^{\text{out}}} \quad \frac{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{fun}} : \tau^{\text{in}} \rightarrow \tau^{\text{out}} \quad \Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{in}} : \tau^{\text{in}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{fun}} e^{\text{in}} : \tau^{\text{out}}}$$

**Modal fragment**

$$\frac{x : \tau \in \Delta}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash x : \tau}$$

$$\frac{\Delta; \Gamma_n; \dots; \Gamma_1; \cdot \vdash e : \tau}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \text{box}(e) : \Box \tau} \quad \frac{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash e^x : \Box \tau^x \quad \Delta, x : \tau^x; \Gamma_n; \dots; \Gamma_1 \vdash e^{\text{body}} : \tau^{\text{body}}}{\Delta; \Gamma_n; \dots; \Gamma_1 \vdash \text{let-box } x = e^x \text{ in } e^{\text{body}} : \tau^{\text{body}}}$$

### 4.1 Comparison

$$\frac{\Delta; \Gamma_1; \cdot \vdash e : \tau}{\Delta; \Gamma_1 \vdash \text{box}(e) : \Box \tau} \quad \frac{\Gamma_1; \Gamma_0 \vdash_0 e_0 : \tau_0}{\Gamma_1; \Gamma_0 \vdash_1 \text{dia}(e_0) : \Diamond \tau_0}$$

$$\frac{\Delta; \Gamma_1 \vdash e^x : \Box \tau^x \quad \Delta, x : \tau^x; \Gamma_1 \vdash e^{\text{body}} : \tau^{\text{body}}}{\Delta; \Gamma_1 \vdash \text{let-box } x = e^x \text{ in } e^{\text{body}} : \tau^{\text{body}}} \quad \frac{\Gamma_1; \Gamma_0 \vdash_1 e_1^x : \Diamond \tau_0 \quad \Gamma_1; \Gamma_0, x_0 : \tau_0 \vdash_1 e_1^{\text{body}} : \tau_1}{\Gamma_1; \Gamma_0 \vdash_1 \text{let-dia } x_0 = e_1^x \text{ in } e_1^{\text{body}} : \tau_1}$$