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Price and quality competition

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Abstract This study considers an oligopoly model with simultaneous price and quality choice. Ex-ante homogeneous sellers compete by offering products at one of two quality levels. The consumers have heterogeneous tastes for quality: for some consumers it is efficient to buy a high quality product, while for others it is efficient to buy a low quality product. In the symmetric equilibrium firms use mixed strategies that randomize both price and quality, and obtain strictly positive profits. This framework highlights trade-offs which determine the impact of consumer protection policy in the form of quality standards.

Keywords Oligopoly · Price and quality competition · Quality standards

JEL Classification L13 · L15 · L50

1 Introduction

In some professional service markets, the providers (e.g., consultants, lawyers, architects) set quality levels simultaneously with their price quotations when competing for clients' custom. The firms have potential to provide the same service at different quality levels. However, often a firm's offer is not preceded by a negotiation process and there is little transparency regarding the alternative qualities that could have been provided (and could have been closer to a client's actual needs). The rivals' quality

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and price choices are at best disclosed after the submission of the bids.[1] Similarly, in markets where the products are bundled with service terms such as business software, the firms' offers can be regarded as simultaneous price-quality bids. In these examples clients' preferences for the services are likely to be different. Building on these observations, this study proposes an oligopoly model where sellers simultaneously compete in quality and price for buyers with heterogeneous tastes for quality and analyzes the equilibrium outcome.

More specifically, the analysis shows that in the symmetric equilibrium of the proposed price-quality competition model, the firms randomize on both prices and qualities. Price and quality dispersion emerges from competition of ex-ante identical sellers in the provision of a homogeneous product. Some features of the proposed model are the following. Sellers can choose between two levels of quality. All consumers value both qualities, but it is efficient for some consumers to buy a high-quality product, and for others to buy a low-quality product.[2] The sellers know the valuations for either quality and their distribution in the population, but they cannot distinguish among buyers and, therefore, cannot price discriminate. The sellers offer their products at only one of two quality levels.[3]

In consumer product markets (e.g., groceries, household supplies), some observed quality differences stem from packaging, labelling, availability of information, add-ons, or expert/celebrity endorsements. For example, some products indicate an improved recipe, added vitamin C, or are labelled as healthy living options. Such quality improvements most often do not call for a long-term decision. Firms can relatively easily change the packaging, slightly improve a recipe, or arrange for an endorsement and rivals are unlikely to observe the internal price-quality decision before making their own choices. Anecdotal evidence suggests that in these markets there is much variation in firms' price-quality offers. Also, consumers are likely to differ in their willingness to pay for quality. The price-quality competition model predicts both price and quality dispersion in the symmetric equilibrium and seems consistent with the patterns observed in these markets.[4]

In the symmetric equilibrium, there is a monotonic relationship between prices and qualities. Low-quality is always associated with lower prices, and high-quality with higher prices. At equilibrium, there is a positive probability that any one firm is the sole provider of a given quality and, even though it faces some competition from the other quality, it can charge a price in excess of marginal cost. In effect, the symmetric equilibrium leads to positive expected profits for the firms. The difference between the highest (lowest) price for a high-quality product and lowest (highest) price for a

[1] There is typically no renegotiation process and the firms cannot subsequently alter their prices. Also, for a given project, there are rarely dynamic considerations involved in the bidding so that firms essentially play a one-shot game.

[2] That is, high-end consumers' marginal valuation of the high-quality product exceeds its cost, while the remaining (low-end) consumers' marginal valuation of the high-quality product is below its cost.

[3] The equilibrium characterized in this research is consistent with a market in which the sellers first decide whether to offer only one or both qualities, but to put up a menu of qualities they incur a positive cost.

[4] Note that since the seminal work of Varian (1980), mixed strategy equilibria have been linked to both intertemporal and cross-sectional variation. For a review of the literature on price dispersion, see Baye et al. (2006).

low-quality product is equal to the difference in high-end (low-end) consumers' valuation for the high and low-quality products.

Due to the fact that high-end (low-end) consumers can eventually shift to a different quality, the highest price at which a high-quality (low-quality) is offered is strictly lower than high-end (low-end) consumers' valuation of the high-quality (low-quality). Low-end consumers obtain a positive net surplus if they purchase a low-quality and, for a nontrivial range of parameters, this is also the case when they buy a high-quality. High-end consumers are left with a positive net surplus regardless of the quality they consume. This contrasts with most price dispersion models where prices equal to consumers' willingness to pay for the good are charged with positive probability. This is the case in Varian (1980) and Rosenthal (1980), for instance, where homogeneous sellers compete for consumers with identical preferences who differ in their search costs. Some buyers have infinite search costs and shop at random, while the others have zero search costs and purchase from the lowest price seller. The expected profit of a firm equals the monopoly profit on its locked-in group (i.e., the corresponding share of random shoppers).[5]

Armstrong and Chen (2009) analyze price and quality competition in oligopoly. In their model, like in the current one, a high-quality is associated with high prices and a low-quality with lower prices. However, they consider consumers with homogeneous tastes for quality who differ in their attentiveness to quality (a low-quality is worthless and would not be produced if there were no inattentiveness). Other models of price and quality competition consider consumer heterogeneity, but focus on perfectly competitive markets. In Wolinsky (1983) where the consumers may differ in their taste for quality and receive noisy signals of a seller's quality, a separating equilibrium where prices fully reveal quality exists under certain conditions. Buyers with homogenous quality tastes might still differ in their knowledge of product quality: while some are fully aware of quality, others are not. Along this line, Cooper and Ross (1984) allow uninformed consumers to have rational expectations on the price-quality relationship. They show that, with U-shaped average cost functions, there exists a rational expectations equilibrium with dispersion in qualities, but not in prices.

The price-quality competition model offers an oligopoly framework for the analysis of consumer protection policy in the form of quality standards (QS).[6] The impact of a QS in this setting is driven by the trade-off between increasing competition and offering consumers the quality they desire, and it pins down the potentially perverse effects of quality regulation. Unlike previous studies of QS's under oligopoly price competition, this exercise shows that the intervention might reduce both total welfare and consumer surplus. However, depending on the parameter values, a QS might also boost both welfare and consumer surplus, or harm welfare while benefiting the consumers.

The next section presents the model and the symmetric mixed strategy equilibrium when the market is fully covered. Section 3 discusses the impact of a quality standard, while Sect. 4 concludes. The proofs missing from the text and the

[5] Note that most of the price dispersion literature has focused on one dimensional (price) variation. In recent years, a few studies analyze two-dimensional dispersion. This work, like Armstrong and Chen (2009), deals with mixed strategy equilibria in prices and qualities.

[6] Public bodies or professional associations have used QS's in professional service markets to improve performance. See, the OFT (2001) and EC (2004, 2005) reports on competition in professions.

characterization of the symmetric equilibrium when the market is not covered are relegated to the Appendix.

2 A model of price and quality competition

2.1 The framework

Consider a market where $N \geq 2$ identical suppliers can offer an otherwise homogeneous product at two quality levels, a high one (q_H) and a low one (q_L). The constant marginal cost of producing the high-quality is $c > 0$, while the one of producing the low-quality is normalized to zero. Sellers simultaneously choose prices and qualities. Each firm offers only one quality level. There is a unit mass of consumers, each demanding one unit of the product. A fraction $1-\alpha$ of the consumers are willing to pay θ_1 for the low-quality and θ_3 for the high-quality, while a fraction α of the consumers has willingness to pay for the low and high-quality equal to θ_2 and θ_4, respectively. I assume that $\theta_1 > 0$, $\theta_3 > c$ and $\theta_i > \theta_j$ for $i > j$, and refer to the consumers with a lower (higher) valuation for either quality as "low-end" ("high-end"). Assume that it is efficient for low-end consumers to buy a low-quality and for the high-end consumers to buy a high-quality product, that is $\theta_3 - \theta_1 < c < \theta_4 - \theta_2$. However, at equilibrium consumers will purchase the quality which provides the best deal. Note that, in this model, the consumers' valuations for the products are consistent with Mussa–Rosen preferences.

The consumers are able to compare all available products both in terms of price and quality before they purchase.[7] Firms know consumers' valuations for either quality and the market composition, but cannot price discriminate. When $\alpha = 0$ (or $\alpha = 1$), firms supply the efficient quality q_L (or q_H), compete a la Bertrand, and make zero profits. The remainder of the paper focuses on $\alpha \in (0, 1)$.

Lemma 1 *For $\alpha \in (0, 1)$, a) there is no symmetric pure strategy equilibrium; and b) for $N \geq 4$, there is a family of asymmetric pure strategy equilibria, where at least two firms choose each quality, low-quality is offered at $p = 0$ and high-quality at $p = c$, and all firms make zero profits. Total welfare and consumer surplus are given by*

$$(\theta_4 - c)\alpha + \theta_1(1 - \alpha).$$

Armstrong and Chen (2009) present similar results in a model of consumer inattentiveness with price-quality competition. There consumers have homogeneous tastes for quality, but a fraction of them do not observe (or correctly assess) quality and (wrongly) believe that all products are of the same (high) quality.[8] In the symmetric equilibrium of their model firms mix on both qualities and prices. To exploit

[7] In consumer good markets, quality differences that stem from labelling (e.g., improved recipe), packaging, add-ons (widget with purchase) can be easily assessed by buyers. In the case of professional services, providers' bids spell out how a project would be carried out allowing the clients to evaluate the quality.

[8] Besides from the "boundedly rational" one, an alternative "rational" interpretation of their model is that some consumers do not mind consuming the low-quality product.

consumers' inattentiveness, firms provide a useless low-quality product with a positive probability. In contrast, in the current model firms face fully rational consumers and the symmetric mixed strategy equilibrium discussed below is related to heterogeneity in consumers' tastes.

Note that a sequential version of this model with two firms has a pure strategy equilibrium (unique up to the identity of the firms) which predicts maximum quality differentiation. In this sense, the consideration of simultaneous move underlies the inexistence of pure strategy equilibria in duopoly.

2.2 The symmetric mixed-strategy equilibrium

For any $N \geq 2$, there exists a symmetric equilibrium in which firms choose both prices and qualities stochastically and make positive profits. This section focuses on a situation in which the market is fully covered in equilibrium (i.e., all consumers make a purchase). Appendix A presents a necessary condition for the market to be fully covered for an arbitrary number of firms. However, for expositional simplicity, this section assumes that the following sufficient condition holds:

$$\alpha \leq \frac{\theta_2 + \theta_3 - \theta_4}{\theta_3 - c}. \tag{1}$$

This condition guarantees that the highest price charged by the firms does not exceed θ_3 so that, regardless of the price draw, all consumers buy the product.[9]

I prove the existence of the symmetric equilibrium by construction. When (1) holds, the firms randomize on prices over the interval $S = [p_0, p_1] \cup [p_2, p_4]$ where $p_0 > 0$, $p_2 > c$, $p_1 \leq \theta_1$ and $p_4 < \theta_4$.[10] Let $F(p)$ denominate the equilibrium cumulative price distribution function defined on S. A low-quality product is offered whenever $p \leq p_1$ and a high quality one if $p \geq p_2 > p_1$. Then, the probability of offering a low-quality product is $P = F(p_1) = F(p_2)$. The last equality follows from the fact that prices in (p_1, p_2) are not assigned positive density in equilibrium.

The boundary prices p_2 and p_4 satisfy:

$$p_2 = p_1 + \theta_3 - \theta_1 \quad \text{and} \quad p_4 = p_0 + \theta_4 - \theta_2. \tag{2}$$

The firms mix over a disconnected support (with two cutoff points $p_1 < p_2$) because consumers have heterogeneous preferences for quality. The difference between the lowest price at which a high-quality is offered (p_2) and the highest price at which the low-quality is offered (p_1) is exactly equal to low-end consumers' marginal valuation for quality (that is, the difference between low-end consumers' valuation for the high-quality and their valuation for the low-quality, $\theta_3 - \theta_1$). At the same

[9] A necessary condition for (1) to hold is $\theta_3 > \theta_4 - \theta_2$. Finally, note that if $\theta_3 \leq \theta_4 - \theta_2$, then $p_4 > \theta_3$ and with a positive probability low-end consumers are excluded from the market. The analysis of the case when the market is not fully covered is presented in Appendix B.

[10] When the market is covered in the symmetric equilibrium, the only relevant boundary prices are p_0, p_1, p_2 and p_4. In the uncovered market analysis, an additional boundary price (p_3) comes into play (see Appendix B).

time, the difference between the highest price at which a high-quality is offered (p_4) and the lowest price at which the low-quality product is offered (p_0) is determined by high-end consumers' marginal valuation for quality (that is, the difference between high-end consumers' valuation for the high-quality and their valuation for the low-quality product, $\theta_4 - \theta_2$).[11]

To characterize the symmetric mixed strategy equilibrium, it is necessary to identify the boundary prices (p_0, p_1, p_2 and p_4) and the equilibrium cdf, $F(p)$. This is done in continuation by making use of the constant profit conditions. In the mixed strategy equilibrium a firm is indifferent between any two prices which are assigned positive density (i.e., that form part of the support $S = [p_0, p_1] \cup [p_2, p_4]$).

Step 1 In equilibrium a firm is indifferent between p_0 and p_4.

The expected profit of a firm at price p_0 is

$$\pi(p_0) = p_0[1-\alpha+\alpha(P+1-F(p_0+\theta_4-\theta_2))^{N-1}] = p_0[1-\alpha+\alpha P^{N-1}]. \tag{3}$$

At this price only a low-quality is offered. Low-end consumers buy for sure at price p_0: a low-quality is never sold at a lower price, and a high-quality (for which they are willing to pay at most θ_3) provides them with a lower surplus even when it is sold at its lowest possible price (p_2). High-end consumers buy at p_0 only if all firms supply a low-quality. Observe that even if the high-quality is provided at its maximal price (p_4) still high-end consumers are indifferent between purchasing the high-quality and buying a low-quality at p_0, its lowest possible price (this happens because $\theta_2 - p_0 = \theta_4 - p_4$). The last equality in expression (3) follows from the fact that $F(p_0+\theta_4-\theta_2) = F(p_4) = 1$. This must be the case because if all firms had an atom at p_4, then an individual firm would be strictly better off moving mass to $p_4 - \epsilon$ for some small $\epsilon > 0$.[12]

The expected profit at price p_4 is given by

$$\pi(p_4) = p_4\alpha P^{N-1} = (p_0+\theta_4-\theta_2-c)\alpha P^{N-1}. \tag{4}$$

At price p_4 only a high-quality is offered. When $p_4 \leq \theta_3$, low-end consumers would buy at p_4 with probability $(1 - F(p_4))^{N-1}$. This is the probability that all other suppliers price above p_4 and is equal to zero.

Then, for a firm to be indifferent between p_0 and p_4, in equilibrium it must hold that $\pi(p_0) = \pi(p_4)$. Using (3) and (4), it follows that

$$p_0 = (\theta_4-\theta_2-c)\frac{\alpha}{1-\alpha}P^{N-1}. \tag{5}$$

The constant profit condition allows to pin down p_0 and, using (2), also p_4 as functions of P, the probability of offering a low quality.

[11] In Armstrong and Chen (2009), the firms mix over a connected price support with only one cutoff point because consumers have the same valuations for quality ($\theta_1 = \theta_2 = 0$ and $\theta_3 = \theta_4 > 0$). A low quality is offered at prices below the cutoff to take advantage of inattentive consumers.

[12] This would result in a jump up in demand and only a negligible loss due to the lower price.

Step 2 In equilibrium a firm is indifferent between p_1 and p_2.

Note that low-end consumers are indifferent between buying a low-quality at price p_1 and a high-quality at price p_2 [see (2)]. In effect, they weakly prefer a low-quality at price $p \leq p_1$ to a high-quality. High-end consumers buy a low-quality at price p_1 only if no firm supplies a high-quality and all low-quality suppliers price above p_1. But, this happens with probability zero. Formally, the expected profit of a firm charging price p_1 is:

$$\pi(p_1) = p_1[(1-\alpha)(1-F(p_1))^{N-1} + \alpha(P - F(p_1))^{N-1}] = p_1(1-\alpha)(1-P)^{N-1}.$$

By the previous argument low-end consumers buy at p_2 only if no firm supplies a low-quality. High-end consumers buy for sure at price p_2: this is the best high-quality deal they can get and it provides a higher net surplus than the best possible low-quality deal ($\theta_2 - p_0 < \theta_4 - (p_1 + \theta_3 - \theta_1) \Leftrightarrow p_2 < p_4$). Then, the expected profit of a firm at price p_2 is:

$$\begin{aligned} \pi(p_2) &= (p_2 - c)[\alpha + (1-\alpha)(1-F(p_2))^{N-1}] \\ &= (p_1 + \theta_3 - \theta_1 - c)[\alpha + (1-\alpha)(1-P)^{N-1}]. \end{aligned}$$

For a firm to be indifferent between p_1 and p_2, in equilibrium it must hold that $\pi(p_1) = \pi(p_2)$. This condition defines

$$p_1 = (\theta_1 - \theta_3 + c)\left[1 + \frac{1-\alpha}{\alpha}(1-P)^{N-1}\right]. \tag{6}$$

In effect, the constant profit condition allows to pin down p_1 and, using (2), also p_3 as functions of P.

Step 3 In equilibrium a firm is indifferent between p_0 and p_2.

Using the equilibrium requirement that $\pi(p_0) = \pi(p_2)$, together with (5) and (6), it follows that

$$\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{\theta_1 - \theta_3 + c}{\theta_4 - \theta_2 - c}\left[\frac{\alpha + (1-\alpha)(1-P)^{N-1}}{(1-\alpha) + \alpha P^{N-1}}\right] = \left(\frac{P}{1-P}\right)^{N-1}. \tag{7}$$

Expression (7) implicitly defines the probability of choosing a low-quality ($P = F(p_1) = F(p_2)$) which is well defined.[13]

Step 4 Indifference between any two prices in $S = [p_0, p_1] \cup [p_2, p_4]$.

Note first that by (3) and (5) the equilibrium profit of a firm is

$$\pi_E = (\theta_4 - \theta_2 - c)\left(1 + \frac{\alpha}{1-\alpha}P^{N-1}\right)\alpha P^{N-1}, \tag{8}$$

where P is defined by (7).

[13] The RHS of (7) is increasing in P and ranges from 0 to ∞, while the LHS is positive and decreasing in P, such that there is unique solution in $[0, 1]$.

At a price $p \in [p_0, p_1]$, a firm offers a low quality and its expected profit is given by

$$\pi(p) = p[(1-\alpha)(1-F(p))^{N-1} + \alpha(P-F(p))^{N-1}],$$

whereas at a price $p \in [p_2, p_4]$, a firm offers a high quality and its expected profit is given by

$$\pi(p) = (p-c)[\alpha(P+1-F(p))^{N-1} + (1-\alpha)(1-F(p))^{N-1}].$$

As the constant profit condition holds at all prices $p \in [p_0, p_1] \cup [p_2, p_4]$, it follows that the atomless price cdf $F(p)$ is defined implicitly by

$$p[(1-\alpha)(1-F(p))^{N-1} + \alpha(P-F(p))^{N-1}] = \pi_E \quad \text{for } p_0 \le p \le p_1 \quad \text{and} \tag{9}$$

$$(p-c)[\alpha(P+1-F(p))^{N-1} + (1-\alpha)(1-F(p))^{N-1}] = \pi_E \quad \text{for } p_2 \le p \le p_4. \tag{10}$$

Note that the two additive terms on the left hand sides of (9) and (10) capture the offsetting influences which underlie the constant profit conditions. At a boundary price, one of these terms has shrunk to zero and cannot decrease anymore. In effect, the equal profit condition does not hold at prices outside the support $S = [p_0, p_1] \cup [p_2, p_4]$.

Finally, (9) and (10) guarantee that a firm is indifferent between offering a low quality at prices in $[p_0, p_1]$ and offering a high quality at prices in $[p_2, p_4]$.

Steps 1–4 are sufficient to characterize the symmetric equilibrium.

Proposition 1 *If* (1) *holds and* $N \ge 2$, *there exists a symmetric mixed strategy equilibrium where firms randomize on prices and qualities. Firms choose prices with support* $S = [p_0, p_1] \cup [p_2, p_4]$. *The boundary prices* p_0, p_1, p_2 *and* p_4 *are defined by* (2), (5), *and* (6). *The atomless price cdf* $F(p)$ *is defined implicitly by* (9) *and* (10). $P = F(p_1) = F(p_2)$ *is the probability of choosing a low-quality and is defined by* (7). *A low-quality product is associated with prices in* $[p_0, p_1]$ *and a high-quality product with prices in* $[p_2, p_4]$.

In this symmetric mixed strategy equilibrium, firms offer both a low-quality (at a relatively low price) and a high-quality (at a higher price) with positive probability. As a result of this randomization, with a positive probability, each firm is the sole provider of a given quality and the sellers are able to sustain positive profits. The mixed strategy equilibrium depends on demand heterogeneity: for some consumers it is efficient to buy a low-quality, while for others it is efficient to buy a high-quality. As expected in a model where consumers can correctly assess product quality, at equilibrium there is a monotonic relationship between price and quality.

In the covered market equilibrium presented in Proposition 1, high-end consumers buy a high-quality product if at least one firm supplies it (which happens with probability $1 - P^N$) and buy a low-quality product only if all firms offer a low-quality. Low-end consumers buy a low-quality product if at least one firm supplies it [which happens with probability $1 - (1-P)^N$] and buy a high-quality product only if all firms offer high-quality products. Then, welfare is given by

$$(\theta_4-c)\alpha(1-P^N)+\theta_2\alpha P^N+\theta_1(1-\alpha)[1-(1-P)^N]+(\theta_3-c)(1-\alpha)(1-P)^N.$$

Aggregate profits follow from (8). Consumer surplus equals total welfare minus aggregate profits. Algebraic manipulations lead to the following result.

Corollary 1 *If* (1) *holds, at equilibrium, the expected profit of a firm is given by* (8). *Total welfare is*

$$(\theta_4-c)\alpha+\theta_1(1-\alpha)-(\theta_4-\theta_2-c)\alpha P^N-(c-\theta_3+\theta_1)(1-\alpha)(1-P)^N, \quad (11)$$

and consumer surplus is given by

$$(\theta_4-c)\alpha+\theta_1(1-\alpha)-(\theta_4-\theta_2-c)\alpha P^{N-1}\left[P+N\left(1+\frac{\alpha}{1-\alpha}P^{N-1}\right)\right]$$
$$-(c-\theta_3+\theta_1)(1-\alpha)(1-P)^N, \quad (12)$$

where P is implicitly defined by (7).

The source of welfare and consumer surplus loss in the covered market symmetric equilibrium in Proposition 1 comes from the fact that with a positive probability consumers buy an inefficient quality. With probability one, all consumers buy and obtain a positive net surplus. But, when all firms offer a low-quality (i.e., with probability P^N) high-end consumers obtain a gross surplus of θ_2, lower than the incremental surplus of (θ_4-c) which is the first best. Likewise, when all firms offer a high-quality (i.e., with probability $(1-P)^N$) low-end consumers buy an inefficiently high-quality. Notice that the first two terms in expressions (11) and (12) give the first best outcome and the last two terms are negative. The term in square brackets in (12) captures the consumer surplus loss due to pricing above marginal cost.

When the market is not covered (that is, for $p_4>\theta_3$),[14] the symmetric equilibrium introduces a second source of inefficiency. In that case, low-end consumers are excluded from the market with a positive probability (i.e., $(1-F(\theta_3))^N>0$) and obtain zero surplus. Proposition 3 in Appendix B presents the symmetric equilibrium when the market is not covered.[15]

The following example illustrates the type of mixed strategy equilibrium presented in Proposition 1.[16]

Example 1 Let $N=2$, $\alpha=.5$, $\theta_1=6$, $\theta_2=8$, $c=10$, $\theta_3=14$ and $\theta_4=20$. Using the results in Proposition 1, at equilibrium, $p_0=1$, $p_1=3$, $p_2=11$, $p_4=13$ and $P=.5$. The price cdf is

[14] Recall that a sufficient condition to be in this region is $\theta_3<\theta_4-\theta_2$.

[15] Notice that the type of equilibrium which applies depends both on the degree of consumer heterogeneity and on the number of firms. When $N\to\infty$, $p_0\to 0$, and it is possible to have $p_4=\theta_4-\theta_2+p_0>\theta_3$ for $N<N_0$ and $\theta_4-\theta_2<\theta_3$ for $N\geq N_0$ for some $N_0\geq 3$. For this reason, the necessary condition for an uncovered market symmetric equilibrium to exist depends on N. (Appendix A presents a necessary condition for a *covered market* symmetric equilibrium to exist for an arbitrary number of firms.)

[16] Example 2 in Appendix B illustrates the symmetric equilibrium when the market is not fully covered.

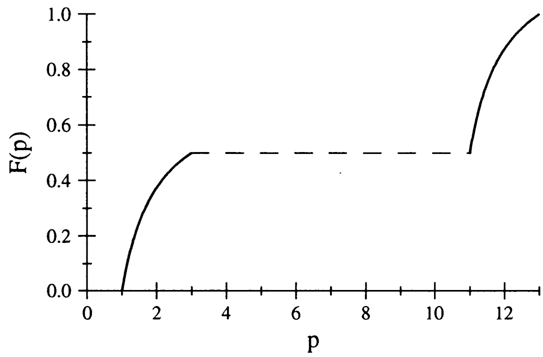

Fig. 1 The price cdf in Example 1

$$F(p) = \begin{cases} .75 - \frac{.75}{p} & \text{for } p \in [1, 3] \\ 1.25 - \frac{.75}{p-10} & \text{for } p \in [11, 13] \end{cases}.$$

Firms offer a low-quality at price $p \in [1, 3]$ and offer a high-quality at $p \in [11, 13]$. The equilibrium profit is .75. Consumer surplus is 6 and total welfare is 7.5. Figure 1 illustrates the mixed strategy equilibrium in this case.

2.3 Large oligopolies

First notice that when $N \to \infty$, the probability to choose a low-quality is defined by

$$\left(\frac{1-\alpha}{\alpha}\right)\frac{\theta_1 - \theta_3 + c}{\theta_4 - \theta_2 - c} = \left(\frac{P}{1-P}\right)^{N-1} \Leftrightarrow P = \frac{\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{\theta_1-\theta_3+c}{\theta_4-\theta_2-c}\right]^{1/N-1}}{1+\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{\theta_1-\theta_3+c}{\theta_4-\theta_2-c}\right]^{1/N-1}}.$$

Then, $\lim_{N\to\infty} P = 1/2$. From (2), (5) and (6), it follows that $p_0 \to 0$ and $p_2 \to c$ as $N \to \infty$, such that the equilibrium cdf in Proposition 1 converges to a discrete distribution which assigns probability 1/2 to 0 and probability 1/2 to c. In addition, a firm's profit and total industry profits both converge to zero when the market is nearly competitive. The outcome of the symmetric mixed strategy equilibrium converges to the asymmetric pure strategy equilibrium of the game presented in Lemma 1: total welfare and consumer surplus in the limit are equal to $(\theta_4 - c)\alpha + \theta_1(1-\alpha)$ (see Corollary 1).

2.4 Discussion

By assumption, in this model each firm chooses only one quality level. In a different set-up where firms can offer a menu of (both) qualities at a cost and first decide whether to provide one or both quality levels, only the equilibrium presented in Proposition 1 could still apply. If firms offered both qualities at different prices, in a symmetric equilibrium, they would end up competing a la Bertrand in two separate markets and be unable to cover the cost of offering a menu. For the equilibrium in Proposition 1 to survive, a firm should not have unilateral incentives to deviate by offering both qualities at different prices. It can be checked that such deviation is unprofitable if the cost of offering the menu exceeds $p_0(1-\alpha)$. Similarly, if firms faced some fixed costs of entry, only the mixed strategy equilibrium could support entry.

The existence of a mixed strategy equilibrium does not depend on having masses of different consumer types. In a duopoly version of the model with a continuum of consumer types, numerical simulations confirm the existence of an equilibrium where firms mix on quality. This intuition also follows from the fact that in the sequential duopoly model with a continuum of types—aside from the pure strategy equilibrium (with maximum quality differentiation)—there are equilibria where firms mix over qualities, too (see Wang and Yang 2001).

Besancenot and Vranceanu (2004) consider a model of price and quality competition where high-end consumers are "experts" and value only the high-quality product (eventually, they care only for the extra features provided by a high-quality). A variant of my model with "expert" consumers (where $\theta_2 = 0$) has a qualitatively similar symmetric mixed strategy equilibrium where firms mix on prices in the support $[p_0, p_1] \cup [p_2, \theta_4]$.

3 Consumer protection policy: quality standards

The price-quality competition model can be used to study the impact of quality standards (QS) in imperfectly competitive markets. Ronnen (1991) and Crampes and Hollander (1995) built on the decrease in price brought about by a QS under duopoly price competition and concluded that the policy always boosts welfare.[17] These models consider sequential quality and price competition. With fixed quality costs, Ronnen (1991) shows that a QS benefits consumers as it fosters price competition by limiting vertical differentiation. In Crampes and Hollander (1995) where the quality cost is variable, the policy might harm consumers if the increase in the low-quality due to the QS triggers a significant increase in the high-quality. In such case, the price increase due to higher costs offsets the competitive effect.[18] In both models, the firms choose deterministic asymmetric quality levels in the free market, and a QS benefits the low-quality supplier and harms the high-quality one. In the current model, a QS harms all firms.

In contrast to these studies, in my oligopoly model with exogenous quality levels, the intervention might reduce welfare. The impact of a QS depends on the trade-off between increasing competition and catering to consumers with diverse tastes. With imperfect competition and heterogeneous preferences for quality, a QS benefits some consumers at the expense of others. The overall effect of the policy on welfare and

[17] In the case of oligopoly *quantity* competition, Valletti (2000) shows that QS's unambiguously decrease welfare.

[18] Note also that in competitive markets a QS has no price effect. There a QS might be used due to information asymmetry and it might harm some consumer groups (see Armstrong 2008 for a review).

consumer surplus depends on the composition of the market and on the relative efficiency gain of a quality match.

In this model, a relevant QS leads to Bertrand competition in the symmetric equilibrium.[19] Then, under a relevant QS, welfare and consumer surplus are both given by $W_{QS} = CS_{QS} = (\theta_4 - c)\alpha + (\theta_3 - c)(1 - \alpha)$. A straightforward comparison of the welfare levels in the symmetric free-market equilibrium (see Corollary 1) and under a QS leads to the following result.

Proposition 2 *If* (1) *holds and* $N \geq 2$, *a quality standard reduces welfare iff*

$$\alpha P^N < b(1-\alpha)[1-(1-P)^N]$$

and it reduces consumer surplus iff

$$\alpha P^{N-1}\left[P + N\left(1 + \frac{\alpha}{1-\alpha}P^{N-1}\right)\right] < b(1-\alpha)[1-(1-P)^N],$$

where P *is defined by* (7) *and* $b = (\theta_1 - \theta_3 + c)/(\theta_4 - \theta_2 - c)$.

As N goes to infinity, the LHS of both inequalities in Proposition 2 converges to zero while the RHS converges to $(c - \theta_3 + \theta_1)(1-\alpha) > 0$. So, in nearly competitive markets, a QS harms both consumer surplus and total welfare. The reason is that, in these markets, competition eliminates the price distortion (recall that $p_0 \to 0$ and $p_2 \to c$ as $N \to \infty$) and the intervention only restricts consumers' choice.

Unlike the limit case, in more concentrated markets the impact of a QS is not clear-cut. For instance, in duopoly markets, a QS might decrease both welfare and consumer surplus, increase them both, or harm welfare while increasing consumer surplus. For $N = 2$, it is possible to write the conditions in Proposition 2 in terms of the parameters as P has a closed-form solution.[20] Figure 2 presents the effects of QS's on consumer surplus (CS) and welfare (W) for different (α, b) combinations when $N = 2$. The impact of a QS stems from the trade-off between the (positive) competitive effect and the (negative) effect of less product diversity.

When there are few high-end consumers (α is small), the probability that all firms choose a low-quality is higher. Then, a QS is more likely to make a difference to the high-end consumers. If however the relative efficiency gain of quality match to the low-end consumers [captured by $b = (\theta_1 - \theta_3 + c)/(\theta_4 - \theta_2 - c)$] is large enough, the negative effect of a QS on product diversity offsets the positive competitive effect. In this region both welfare and consumer surplus decrease (see Fig. 2). In contrast, for low α and low b the harm from a quality mismatch to low end consumers is offset by the positive price effect (the crossed line area). As α increases, the price effect of a QS benefits consumer surplus and the quality mismatch effect harms welfare (the vertical line area).

[19] Note that a relevant QS lies between the lowest and the highest available quality levels.

[20] See expression (14) in Appendix A. Note also that for an arbitrary number of firms, there is no closed-form solution for P.

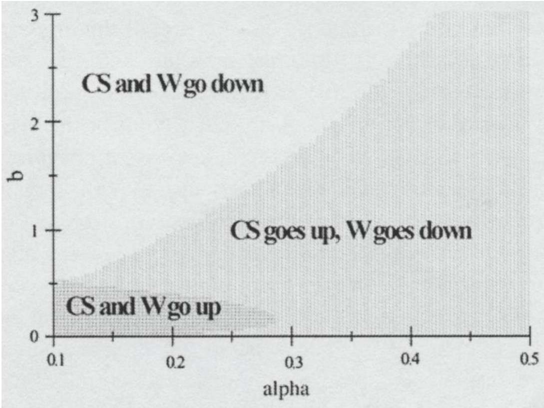

Fig. 2 The effect of a QS on CS and W when $N = 2$ and $b = (\theta_1 - \theta_3 + c)/(\theta_4 - \theta_2 - c)$

The possibly undesired effects of intervention in this model come from the existence of a critical mass of consumers whose valuation of the incremental quality imposed by the QS does not cover its cost.[21] The policy increases competition, but it also harms consumers if they place low value on quality.[22] Recent evidence on consumers' inattention to fine print terms might make consumer protection policies such as QS's more appealing. It is therefore useful to understand the effects of QS's in different settings. Market outcomes in the presence of inattention (see Armstrong and Chen 2009) might resemble those driven by heterogeneity in consumers' tastes for quality. Yet, while a QS meant to protect the inattentives is obviously beneficial in the former case, the impact is not clear-cut in the latter case.

4 Conclusions

In consumer good markets, the firms occasionally offer quality updates that stem from packaging, labelling, endorsements, or maintenance service terms. These features can be easily altered as they do not require a long-term investment. Some professional service providers compete by submitting price-quality bids to their clients, so that firms do not observe rivals' quality choices before making their own decisions. To analyze such markets, this paper proposes an oligopoly model of simultaneous price and quality competition. The sellers are ex-ante identical and each firm chooses only one of two quality levels. The consumers have heterogeneous tastes for quality: it is efficient for some consumers to buy a high-quality, while for others it is efficient to buy a low-quality product.

[21] Appendix B focuses on the uncovered market equilibrium and shows that a QS might still decrease welfare and consumer surplus, even when it improves consumers' participation in the market. (See Example 3.)

[22] With perfect competition and information asymmetry, Leland (1979) derives conditions for a QS to increase welfare. While the negative impact of a QS in his setting is the same as here, the positive effect is different as it stems from correcting the "lemons problem".

The study shows that in the symmetric equilibrium of the model, the firms mix on both price and quality and there is a monotonic relationship between the two. With positive probability each firm is the sole provider of a given quality and, therefore, the firms obtain strictly positive profits in the symmetric equilibrium. The framework predicts dispersion in prices and qualities and matches casual observations in consumer good markets. An analysis of quality standards in this setting captures (undesired) effects of the intervention that were neglected by previous models of imperfect competition. The policy increases price competition, but reduces product diversity and, for nontrivial parameter ranges, it might harm both welfare and consumer surplus.

This model can also be related to studies of endogenous vertical differentiation (see Tirole 1988; Anderson et al. 1992). The work in this area focuses on models where quality is a long-run variable and the firms compete sequentially on quality and price. The standard duopoly model with perfect substitutes predicts maximal quality differentiation and deterministic pricing.[23] In contrast, a model with simultaneous price and quality choices is more suitable in professional service or consumer good markets where quality choices are not observable and do not require long-term investments.

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5 Appendix

Proof of Lemma 1 (a) If all firms choose high (low) quality, they compete a la Bertrand and make zero profits. Then, a unilateral deviation to a low (high) quality and price p is profitable as it generates strictly positive profits equal to $(1-\alpha)\varepsilon$ (or, $\alpha(\varepsilon - c)$) whenever $0 < p < c - (\theta_3 - \theta_1)$ (or, $c < p < \theta_4 - \theta_2$). Hence, there is no symmetric pure strategy equilibrium. (b) If at least two firms offer a low-quality and at least two firms offer a high-quality, all firms compete a la Bertrand, make zero profits and there is no profitable unilateral deviation. In this equilibrium, all low-end consumers buy a low-quality at $p = 0$ and all high-end consumers buy a high-quality at $p = c$. □

5.1 Appendix A: Covered market equilibrium

5.1.1 Conditions for the existence of a symmetric covered market equilibrium

For expositional simplicity, the main text assumes that (1) holds. This is a sufficient condition for the market to be covered in the symmetric equilibrium. Let us now derive (1) and present a more complex necessary condition for an arbitrary number of firms.

[23] With perfect substitutes, quality differentiation relaxes price competition in the product market. With imperfect substitutes, Ma and Burgess (1993) show that quality differentiation intensifies price competition. Ishibashi (2001) also employs a model with imperfect substitutes to study strategic delegation under quality competition.

First note that the market is covered in the symmetric equilibrium if $p_4 \leq \theta_3$, where p_4 is the upper bound of the support of the equilibrium pricing cdf. (Clearly, if $p_4 > \theta_3$ there is a positive probability that low-end consumers are excluded from the market. This happens if all firm offer high-quality products at prices higher than low-end consumers' valuation of the high-quality product, θ_3.) By (2) and (5),

$$p_4 = (\theta_4 - \theta_2 - c)\frac{\alpha}{1-\alpha}P^{N-1} + \theta_4 - \theta_2. \tag{13}$$

Using the fact that $P < 1$ [see (7)], the requirement that $p_4 \leq \theta_3$ leads to the sufficient condition (1):

$$\alpha \leq \frac{\theta_2 + \theta_3 - \theta_4}{\theta_3 - c}.$$

To obtain a necessary condition for the market to be covered in the symmetric equilibrium *for an arbitrary number of firms*, note that $P(N)$ is decreasing [see (7)], so that $p_4(N)$ is decreasing. It follows that $p_4(N) \leq p_4(2)$. Using (13), $p_4(2) = (\theta_4 - \theta_2 - c)\frac{\alpha}{1-\alpha}P(2) + \theta_4 - \theta_2$, where $P(2)$ follows from (7):

$$P(2) = \frac{b(1-\alpha)^2(2-\alpha)+\alpha^2(1-\alpha)-\sqrt{[b(1-\alpha)^2(2-\alpha)+\alpha^2(1-\alpha)]^2-4b(1-\alpha)^2[b(1-\alpha)^3-\alpha^3]}}{2[b(1-\alpha)^3-\alpha^3]}, \tag{14}$$

where $b = (\theta_1 - \theta_3 + c)/(\theta_4 - \theta_2 - c)$. Then, $p_4 \leq \theta_3$ for any $N \geq 2$ iff

$$\frac{b(1-\alpha)(2-\alpha)\alpha + \alpha^3 - \alpha\sqrt{[b(1-\alpha)(2-\alpha)+\alpha^2]^2 - 4b[b(1-\alpha)^3 - \alpha^3]}}{2[b(1-\alpha)^3 - \alpha^3]} \leq \frac{\theta_3 - \theta_4 + \theta_2}{\theta_4 - \theta_2 - c}.$$

Proof of Proposition 1 For $F(p)$ defined by (9) and (10) to be a well-defined cdf, it has to be continuous and increasing on S. Let us first consider prices in $[p_0, p_1]$. By (9), F is increasing in p. I show in continuation that $F(p_0) = 0$. Notice that (9) evaluated at p_0 gives

$$p_0[(1-\alpha)(1-F(p_0))^{N-1} + \alpha(P - F(p_0))^{N-1}] = p_1(1-\alpha)(1-P)^{N-1}$$

and using (5) and (6) it becomes

$$\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{\theta_1-\theta_3+c}{\theta_4-\theta_2-c}\left[\frac{\alpha+(1-\alpha)(1-P)^{N-1}}{(1-\alpha)(1-F(p_0))^{N-1}+\alpha(P-F(p_0))^{N-1}}\right] = \left(\frac{P}{1-P}\right)^{N-1}.$$

Using (7), it follows that $F(p_0) = 0$.

Let us consider prices in $[p_2, \theta_3] \cap [p_2, p_4]$. By (10), F is increasing in p. I show in continuation that $F(p_2) = P$. Notice that (10) evaluated at p_2 gives

$$\begin{aligned}&(p_2-c)[\alpha(P+1-F(p_2))^{N-1}+(1-\alpha)(1-F(p_2))^{N-1}]\\&\quad=(p_0+\theta_4-\theta_2-c)\alpha P^{N-1}\Leftrightarrow\\&(p_1+\theta_3-\theta_1-c)[\alpha(P+1-F(p_2))^{N-1}+(1-\alpha)(1-F(p_2))^{N-1}]\\&\quad=(p_0+\theta_4-\theta_2-c)\alpha P^{N-1}\end{aligned}$$

and, using (5) and (6), the expression becomes

$$\left(\frac{1-\alpha}{\alpha}\right)^2\frac{\theta_1-\theta_3+c}{\theta_4-\theta_2-c}\left[\frac{\alpha(P+1-F(p_2))^{N-1}+(1-\alpha)(1-F(p_2))^{N-1}}{1-\alpha+\alpha P^{N-1}}\right]$$
$$=\left(\frac{P}{1-P}\right)^{N-1}. \tag{15}$$

Using (7), it follows that $F(p_2)=P=F(p_1)$. In addition, notice that if $p_4=p_0+\theta_4-\theta_2\in[p_2,\theta_3]$, then (10) evaluated at p_4 gives

$$(p_4-c)[\alpha(P+1-F(p_4))^{N-1}+(1-\alpha)(1-F(p_4))^{N-1}]=(p_4-c)\alpha P^{N-1}$$

and it follows that $F(p_4)=1$.

The boundary price p_1 should not exceed θ_1. As $p_4\le\theta_3$, from $\pi(p_2)=\pi(p_4)$, it follows that

$$p_1=\theta_1-\theta_3+c+(p_0+\theta_4-\theta_2-c)\frac{\alpha P^{N-1}}{\alpha+(1-\alpha)(1-P)^{N-1}}\le\theta_1-\theta_3+p_4\le\theta_1.$$

For the strategies presented in Proposition 1 to be indeed an equilibrium, there should be no profitable unilateral deviation. The relevant deviations to be considered in this case are the following: (i) offer a low-quality at $p\in(p_1,\theta_1]$; (ii) offer a high-quality at $p\in[c,p_2]$; and (iii) offer a high-quality at $p\in(p_4,\theta_4]$.

Notice that deviations with a high-quality on $[p_0,c]$ are not profitable because the price is below marginal cost and deviations with a low-quality in $(\theta_2,\theta_4]$ are not profitable because the price exceeds the willingness to pay for low-quality. Note also that deviations with a low-quality to some price $p\in(\theta_1,\theta_2)\cap(p_1,\theta_2)$ are not profitable: Low-end consumers cannot afford the low-quality. High-end consumers buy only if a high-quality is not offered at this price (i.e., $F(\theta_2)\le F(p_2)$). Then, they purchase at p with probability $(P-F(p)+1-F(p+\theta_4-\theta_2))^{N-1}=0$. The last equality follows from the fact that, in this range, $p+\theta_4-\theta_2>p_0+\theta_4-\theta_2=p_4$ so that $F(p+\theta_4-\theta_2)=1$ and $F(p)=P$.

(i) The deviator offers a low-quality at $p\in(p_1,\theta_1]$. Low-end consumers buy at p only if all other firms price above $\theta_3-\theta_1+p$. High-end consumers buy at p with probability $(P-F(p)+1-F(p+\theta_4-\theta_2))^{N-1}=0$. Then, deviator's profit is

$$\pi_D(p)=p(1-\alpha)(1-F(p+\theta_3-\theta_1))^{N-1}.$$

As $p\in(p_1,\theta_1]$, then $p+\theta-\theta_L\in(p_1+\theta_3-\theta_1,\theta_3]=(p_2,\theta_3]$. Hence, $F(p+\theta_3-\theta_1)$ is defined by

$$(p + \theta_3 - \theta_1 - c)[\alpha(1 - F(p + \theta_3 - \theta_1) + P)^{N-1} \\ +(1 - \alpha)(1 - F(p + \theta_3 - \theta_1))^{N-1}] = \pi_E. \tag{16}$$

Notice that, by continuity, $\pi_D(p_1) = p_1(1 - \alpha)(1 - F(p_2))^{N-1} = p_1(1 - \alpha)(1 - P)^{N-1} = \pi_E$. Using (16), deviator's price and profits are given respectively by

$$p = c + \theta_1 - \theta_3 + \frac{\pi_E}{[\alpha(1 - F(p + \theta_3 - \theta_1) + P)^{N-1} + (1 - \alpha)(1 - F(p + \theta_3 - \theta_1))^{N-1}]} \quad \text{and}$$

$$\pi_D(F) = \left\{c + \theta_1 - \theta_3 + \frac{\pi_E}{[\alpha(1 - F + P)^{N-1} + (1 - \alpha)(1 - F)^{N-1}]}\right\}(1 - \alpha)(1 - F)^{N-1}.$$

For $\theta_3 - c < \theta_1$, $\partial \pi_D(F)/\partial F < 0$ such that $\pi_D(p) \le \pi_D(p_1) = \pi_E$. Consequently, this is not a profitable deviation.

(ii) The deviator offers a high-quality at $p \in [c, p_2)$. Then, his expected profit is

$$\pi_D(p) = (p - c)[\alpha + (1 - \alpha)(1 - F(p + \theta_1 - \theta_3))^{N-1}].$$

Clearly, all high-end consumers buy at p. Low-end consumers buy only if all other firms price above $p + \theta_1 - \theta_3$. As $p \in [c, p_2)$, $(p + \theta_1 - \theta_3) \in [c + \theta_1 - \theta_3, p_1)$.

First let us consider deviations to p such that $p + \theta_1 - \theta_3 \in [p_0, p_1)$. Then, $F(p + \theta_1 - \theta_3)$ is defined by

$$(p + \theta_1 - \theta_3)[(1 - \alpha)(1 - F(p + \theta_1 - \theta_3))^{N-1} + \alpha(P - F(p + \theta_1 - \theta_3))^{N-1}] \\ = p_1(1 - \alpha)(1 - P)^{N-1}.$$

It follows that the deviator's price and its profit are respectively:

$$p = \theta_3 - \theta_1 + \frac{p_1(1 - \alpha)(1 - P)^{N-1}}{(1 - \alpha)(1 - F)^{N-1} + \alpha(P - F)^{N-1}} > c \quad \text{and}$$

$$\pi_D(F) = \left\{\theta_3 - \theta_1 - c + \frac{p_1(1 - \alpha)(1 - P)^{N-1}}{(1 - \alpha)(1 - F)^{N-1} + \alpha(P - F)^{N-1}}\right\} \\ \times[\alpha + (1 - \alpha)(1 - F)^{N-1}].$$

It can be shown that $\partial \pi_D(F)/\partial F > 0$ (recall that $\theta_3 - \theta_1 - c < 0$), and then for all $p \in [p_0, p_1)$, $\pi_D(p) \le \pi_D(p_2)$. But, $\pi_D(p_2) = (p_2 - c)[\alpha + (1 - \alpha)(1 - P)^{N-1}] = \pi_E$. Consequently, this is not a profitable deviation.

If $c + \theta_1 - \theta_3 < p_0$ for some N, then for any $p \in [c + \theta_1 - \theta_3, p_0]$, it holds that $F(p + \theta_1 - \theta_3) = 0$. It follows that $\partial \pi_D(p)/\partial p > 0$ and $\pi_D(p) \le \pi_D(p_0) \le \pi_E$ as the previous argument applies. (Notice however that as $N \to \infty$, $p_0 \to 0$, and $c + \theta_1 - \theta_3 > 0$.)

(iii) The deviator offers a high-quality at $p \in (p_4, \theta_4]$. Low-end consumers do not buy in this case (they strictly prefer a low quality, or a high quality at a lower price), and high-end consumers buy only if deviator's deal is the best available. For deviations with $p \in [p_4, p_1 + \theta_4 - \theta_2]$, deviator's profit is

$$\pi_D(p) = (p - c)\alpha(P - F(p + \theta_2 - \theta_4))^{N-1}.$$

For $p \in (p_4, p_1 + \theta_4 - \theta_2]$, note that as $p + \theta_2 - \theta_4 \in (p_0, p_1]$ then $F(p + \theta_2 - \theta_4)$ is defined by

$$(p + \theta_2 - \theta_4)[(1 - \alpha)(1 - F(p + \theta_2 - \theta_4))^{N-1} \\ + \alpha(P - F(p + \theta_2 - \theta_4))^{N-1}] = \pi_E.$$

Hence, deviator's price and its profit can be written respectively as:

$$p = \theta_4 - \theta_2 + c + \frac{\pi_E}{(1 - \alpha)(1 - F)^{N-1} + \alpha(P - F)^{N-1}} \quad \text{and}$$
$$\pi_D(F) = \left\{\theta_4 - \theta_2 + \frac{\pi_E}{(1 - \alpha)(1 - F)^{N-1} + \alpha(P - F)^{N-1}}\right\} \alpha(P - F)^{N-1}.$$

It can be shown that $\partial \pi_D(F)/\partial F < 0$ (recall that $\theta_4 - \theta_2 > 0$), and then in the relevant range $\pi_D(p) \leq \pi_D(p_4)$. Notice that $\pi_D(p_4) = (p_0 + \theta_4 - \theta_2 - c)\,\alpha P^{N-1} = \pi_E$. In effect, there is no gain from such deviation. For deviations with $p \in (p_1 + \theta_4 - \theta_2, \theta_4]$, neither low-end nor high-end consumers purchase, so clearly there is no gain from such deviation.

Finally, notice that, as the deviations above are not profitable, firms cannot gain from mixing on quality either. □

5.2 Appendix B: Uncovered market equilibrium

In this part, I present the symmetric mixed strategy equilibrium in the uncovered market case. A sufficient condition for the market to be uncovered is $\theta_4 - \theta_2 > \theta_3$. In this case the support of the price cdf is $[p_0, p_1] \cup [p_2, \theta_3] \cup [p_3, p_4]$. Notice first that expressions (2)–(7) still apply. To pin down the equilibrium cdf's it is necessary to identify the boundary price p_3 and the probability to price below θ_3, that is $F(\theta_3) < 1$. Using (7) and (8), p_3 and $F(\theta_3)$ are implicitly defined by $\pi(\theta_3) = \pi_E$ and $\pi(p_3) = \pi_E$ where:

$$\pi(\theta_3) = (\theta_3 - c)[\alpha(P + 1 - F(\theta_3))^{N-1} + (1 - \alpha)(1 - F(\theta_3))^{N-1}] \quad \text{and} \\ \pi(p_3) = (p_3 - c)\alpha(P + 1 - F(\theta_3))^{N-1}. \tag{17}$$

Low-end consumers purchase a high-quality at θ_3 (which leaves them with zero net surplus) only if this is the lowest price in the market and they no longer purchase it at p_3 because their valuation of a high-quality is lower. High-end consumers obtain a higher net surplus from the high-quality sold at θ_3 than from the best possible low-quality deal, that is $\theta_4 - \theta_3 > \theta_2 - p_0 \Leftrightarrow \theta_4 - \theta_2 + p_0 > \theta_3$. Similarly, high-end consumers obtain a higher net surplus from the high-quality sold at p_3 than from the best possible low-quality deal, that is $\theta_4 - p_3 > \theta_2 - p_0 \Leftrightarrow \theta_4 - \theta_2 + p_0 = p_4 > p_3$. Hence, they buy at θ_3 or at p_3 if all other firms offer a low-quality or charge a higher price.

Proposition 3 *For $N \geq 2$ and $\theta_4 - \theta_2 > \theta_3$, there exists a symmetric mixed strategy equilibrium where firms randomize on prices and qualities. Firms choose prices with support $S = [p_0, p_1] \cup [p_2, \theta_3] \cup [p_3, p_4]$. The boundary prices p_0, p_1, p_2, p_3 and p_4 are defined by* (2), (5), (6) *and* (17). *The atomless pricing cdf $F(p)$ is defined implicitly by*

$$\begin{aligned} p[(1-\alpha)(1-F(p))^{N-1} + \alpha(P-F(p))^{N-1}] = \pi_E \quad \text{for } p_0 \leq p \leq p_1; \\ (p-c)[\alpha(P+1-F(p))^{N-1} + (1-\alpha)(1-F(p))^{N-1}] \\ = \pi_E \quad \text{for } p_2 \leq p \leq \theta_3; \quad \text{and} \\ (p-c)\alpha(P+1-F(p))^{N-1} = \pi_E \quad \text{for } p_3 \leq p \leq p_4. \end{aligned} \tag{18}$$

where the probability of offering a low-quality $P = F(p_1) = F(p_2)$, the boundary price p_3, and the equilibrium profit π_E are defined by (7), (8) *and* (17). *A low-quality is associated with prices in $[p_0, p_1]$ and a high-quality with prices in $[p_2, \theta_3] \cup [p_3, p_4]$.*

Proof As most of the arguments overlap with those presented in the proof of Proposition 1, I focus here on the additional arguments required for the uncovered market case.

(a) *Pricing cdf analysis* Let us consider prices in $[p_3, p_4]$ which are assigned positive density if $p_4 > \theta_3$. By (18), F is increasing in p. I show in continuation that $F(p_3) = F(\theta_3)$. Notice first that from (17), the boundary price p_3 is given by

$$p_3 = c + \frac{(\theta_3 - c)[\alpha(P+1-F(\theta_3))^{N-1} + (1-\alpha)(1-F(\theta_3))^{N-1}]}{\alpha(P+1-F(\theta_3))^{N-1}}. \tag{19}$$

If we evaluate (18) at p_3 and substitute (19), then it becomes

$$\begin{aligned} &\frac{(\theta_3-c)[\alpha(P+1-F(\theta_3))^{N-1} + (1-\alpha)(1-F(\theta_3))^{N-1}]}{\alpha(P+1-F(\theta_3))^{N-1}}\alpha(P+1-F(p_3))^{N-1} \\ &= (p_0 + \theta_4 - \theta_2 - c)\alpha P^{N-1}. \end{aligned}$$

But, as indifference between θ_3 and p_4 requires

$$(\theta_3-c)[\alpha(P+1-F(\theta_3))^{N-1}+(1-\alpha)(1-F(\theta_3))^{N-1}]=(p_0+\theta_4-\theta_2-c)\alpha P^{N-1}$$

it follows that $F(p_3) = F(\theta_3)$.

(b) *Boundary prices* As $p_4 > \theta_3$, it follows from $\pi(p_2) = \pi(\theta_3)$ that

$$p_1 = \theta_1 - \theta_3 + c + (\theta_3 - c)\frac{\alpha(P+1-F(\theta_3))^{N-1} + (1-\alpha)(1-F(\theta_3))^{N-1}}{\alpha + (1-\alpha)(1-P)^{N-1}} \leq \theta_1.$$

Also, using (4) and (17), it can also be shown that $p_3 \in (\theta_3, p_4)$.

(c) *Deviations* Two additional deviations need to be considered in this case. Note that they replace iii) in the Proof of Proposition 1. First, suppose that the deviator offers a high-quality at $p \in (\theta_3, p_3)$. Low-end consumers do not buy here, so deviator's profit is

$$\pi_D(p) = (p - c)\alpha(P + 1 - F(\theta_3))^{N-1} < (p_3 - c)\alpha(P + 1 - F(\theta_3))^{N-1} = \pi_E.$$

Such deviation is not profitable. Finally, suppose that the deviator offers a high-quality at $p \in (p_4, \theta_4)$. Low-end consumers do not buy here, so deviator's profit is

$$\pi_D(p) = (p - c)\alpha(P - F(p + \theta_2 - \theta_4))^{N-1}.$$

When $p \in (p_4, \theta_4)$, then $p + \theta_2 - \theta_4 \in (p_0, \theta_2)$ so that $F(p + \theta_2 - \theta_4)$ is defined by

$$(p+\theta_2-\theta_4)[(1-\alpha)(1-F(p+\theta_2-\theta_4))^{N-1}+\alpha(P-F(p+\theta_2-\theta_4))^{N-1}]=\pi_E.$$

Hence, deviator's price and its profit can be written respectively as:

$$p = \theta_4 - \theta_2 + c + \frac{\pi_E}{(1-\alpha)(1-F)^{N-1} + \alpha(P-F)^{N-1}} \quad \text{and}$$
$$\pi_D(F) = \left\{\theta_4 - \theta_2 + \frac{\pi_E}{(1-\alpha)(1-F)^{N-1} + \alpha(P-F)^{N-1}}\right\}\alpha(P-F)^{N-1}.$$

It can be shown that $\partial\pi_D(F)/\partial F < 0$ (recall that $\theta_4 - \theta_2 > 0$), and then in the relevant range $\pi_D(p) \leq \pi_D(p_4)$. Notice that $\pi_D(p_4) = (p_0 + \theta_4 - \theta_2 - c)\,\alpha P^{N-1} = \pi_E$. In effect, there is no gain from such deviation, either. □

The following example illustrates the type of mixed strategy equilibrium presented in Proposition 3.

Example 2 Let $N = 2$, $\alpha = .5$, $\theta_1 = 6$, $\theta_2 = 7$, $c = 10$, $\theta_3 = 14$ and $\theta_4 = 23$. Using the results in Proposition 3, at equilibrium, $p_0 = 1.82$, $p_1 = 3.39$, $p_2 = 11.39$, $p_3 = 15.29$, $p_4 = 17.82$, $P = .30$ and $F(\theta_3) = .86$. The price cdf is

$$F(p) = \begin{cases} .65 - \frac{1.183}{p} & \text{for } p \in [1.82, 3.39] \\ 1.15 - \frac{1.183}{p-10} & \text{for } p \in [11.39, 14] \\ 1.3 - \frac{2.366}{p-10)} & \text{for } p \in [15.29, 17.82] \end{cases}.$$

Each firm's equilibrium profit is 1.183. Consumer surplus is 6.33 and total welfare is 8.69. Figure 3 illustrates the mixed strategy equilibrium in this case.

As before, expected profit of a firm is given by $\pi_E = (\theta_4 - \theta_2 - c)(1 + \frac{\alpha}{1-\alpha}P^{N-1})\alpha P^{N-1}$. If $p_4 > \theta_3$, a quality standard is welfare decreasing whenever

$$(\theta_4 - \theta_2 - c)\alpha P^N + (\theta_3 - c)\,(1-\alpha)(1 - F(\theta_3))^N$$
$$< (\theta_3 - \theta_1 - c)\,(1-\alpha)[(1-P)^N - 1].$$

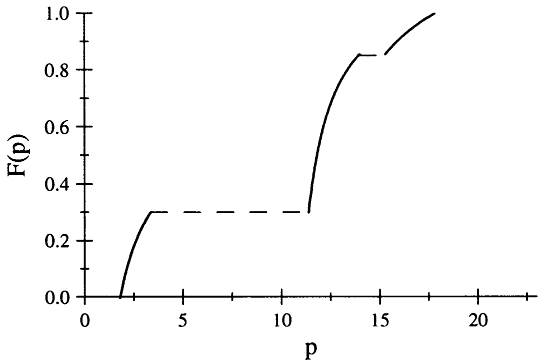

Fig. 3 The price cdf in Example 2

In the equilibrium presented in Proposition 3, high-end consumers buy a high-quality if at least one firm supplies it (which happens with probability $1 - P^N$), and low-end consumers buy a low-quality if at least one firm supplies it (which happens with probability $1 - (1 - P)^N$). Then, only if all firms offer a low-quality, high-end consumers buy a low-quality. And, low-end consumers buy a high-quality only if all firms offer a price in the interval $[p_2, \theta_3]$. Then, welfare is given by:

$$(\theta_4 - c)\alpha(1 - P^N) + \theta_2\alpha P^N + \theta_1(1 - \alpha)[1 - (1 - P)^N]$$
$$+(\theta_3 - c)(1 - \alpha)[(1 - P)^N - (1 - F(\theta_3))^N].$$

Consumer surplus equals total welfare minus aggregate profits.

Welfare and consumer surplus under a relevant QS are given by $W_{QS} = CS_{QS} = (\theta_4 - c)\alpha + (\theta_3 - c)(1 - \alpha)$. When $p_4 > \theta_3$, a QS might decrease both consumer surplus and welfare (see Example 3); raise both consumer surplus and welfare (see Example 4); or decrease welfare and raise consumer surplus (see Example 2 and notice that welfare and consumer surplus under a QS is 8.5).

Example 3 Let $N = 2$, $\alpha = .1$, $\theta_1 = 7$, $\theta_2 = 8$, $c = 10$, $\theta_3 = 14$ and $\theta_4 = 23$. Under a QS, welfare and consumer surplus equal $W_{QS} = CS_{QS} = 4.9$, while at the symmetric equilibrium the free market creates total welfare $W = 7.16$ and consumer surplus $CS = 6.17$. Then, a QS reduces total welfare by 2.26 and consumer surplus by 1.27. (In this case, $p_4 = 15.5 > \theta_3 = 14$.)

Example 4 Let $\alpha = .5$, $\theta_1 = 6$, $\theta_2 = 7$, $c = 10$, $\theta_3 = 14$ and $\theta_4 = 30$. Under a QS, welfare and consumer surplus equal $W_{QS} = CS_{QS} = 12$, while at the symmetric equilibrium the free market creates total welfare $W = 11.96$ and consumer surplus of $CS = 9.02$. Then, a QS raises total welfare by .04 and consumer surplus by 2.98.

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