Block removability analysis considering the influence of adjacent blocks

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Abstract

Great efforts are dedicated to study the removability of rock blocks in the block theory. However, the failure of a single block may affect the removability of its adjacent blocks and several non-removable blocks may be combined to form a removable block. These phenomena are not adequately captured in the classical key block theory. This paper proposed a method to investigate the influence of each block to the removability its adjacent blocks. The block system is constructed using the element-block assembling approach and the outer boundaries of each complex-block are explicitly determined. The adjacency relationship between different blocks are represented using an undirected graph, which also helps to determine the nested and interlocked blocks that should be regarded as an integrity in the analysis process. The set of blocks that may fall after the failure of each key block is determined using a breadth-first searching process, which is driven by a queue on the basis of the adjacency graph. The proposed method can be used as an extension of the classical block theory and the results can help to derive more realistic and reasonable decisions in the engineering process. Several examples are given to validate and demonstrate the proposed method.

1 Introduction

The stability of rock mass is mainly determined by the discrete blocks enclosed by intersected discontinuities. Over the past decades, block theory has been developed to analyze the stability of discrete rock masses and has been successfully applied to various engineering projects. Numerous contributions have also been made to enriched the block theory in different aspects ZHENG et al. (2013); Wang et al. (2013); Xiao-ming and Yin-he (2015); Zheng et al. (2015a).

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In the originally developed block theory, the analysis of each block is performed independently, i.e., the adjacent blocks of a given block B are assumed to be fixed when analyzing the removability and stability of B (Goodman and Shi 1985). However, due to the complexity of the geometry arrangements, the destabilization of a key block may affect the removability of its adjacent blocks. Specifically, when analyzing a block system, some key blocks are relatively "independent", i.e., the failure of these blocks will not affect the adjacent blocks, while other key blocks may trigger a massive falling, i.e., the failure of these blocks may liberate their adjacent blocks that are previously restrained. Obviously, the second category of key blocks is more "critical" when considering the stability of the entire rock masses because the failure of these blocks may impact a larger region.

This phenomenon is further demonstrated using a 2-dimensional (2D) example as shown in fig.1. Several finite-sized fractures are contained in a slope region (fig.1a) and 43 blocks are formed by these fractures (fig.1b). As can be determined using the block theory, blocks B7 and B9 are the key blocks around the excavation surfaces and the block B9 should be given more attention because it has a larger volume. However, as can be seen from fig.1, the failure of B7 can destabilize its adjacent blocks and thus may lead to the failure of the shaded region (fig.1c), while the failure of B9 will not affect other blocks (fig.1d). The volume of the blocks that may fall after block B7 is much larger than block B9 and thus B7 should be more "critical" than B9.

Several developments of the classical block theory are carried out to study the stability of blocks under the influence of their adjacent blocks. Wibowo (1997) investigated the secondary key-blocks by evaluating the originally non-removable blocks surrounding the primary key-block. Bafghi and Verdel (2003) developed a key-group method to investigate groups of collapsible blocks and preformed an iterative and progressive stability analysis. Further developments extended this method to consider the mechanical parameters uncertainties (Bafghi and Verdel 2004), intragroup forces (Bafghi and Verdel 2005) and three dimensional situations (Noroozi et al. 2012). However, the fractures considered in these models are all infinitely extended and thus the blocks are all convex shaped. In practical engineering, finite-sized fractures exist and the blocks formed by these fractures may present arbitrary shapes. More realistic results will be derived if the complex shapes of blocks are considered.

Few studies are carried out to analyze the removability of blocks that are influenced by complex shaped blocks, i.e., the blocks that are formed by finite sized fractures. Fu and Ma (2014) analyzed the support design of rock blocks by considering the interactions of adjacent batches of key blocks. Zhang (2015) described a method for block progressive failure analysis. However, despite the great significance and achievements of the above mentioned methods. The emphasis of these models mainly focuses on investigating different batches of key blocks. The influence of each key block to the removability of its adjacent blocks are not identified and analyzed. Moreover, analyzing the interactions between complex blocks, such as determining nested or interlocked blocks, are also omitted.

This paper proposed a method to investigate the influence of each key block to the removability of its adjacent blocks. The complex-blocks enclosed by finite-sized fractures are identified using the element-block assembling approach and the boundary surfaces of each block are explicitly constructed. The adjacency relationship between blocks is described using an undirected graph, which can also be used to retrieved the fixed surfaces of each block and track the block falling process. An algorithm based on the vector method is implemented to analyze the stability of individual blocks and the block sets that should be regarded as an integrity, i.e., the nested and interlocked blocks, are determined and merged before the removable analysis. The blocks that will fall after the failure of each key block is determined using a breadth-first searching process based on the adjacency graph. The proposed method can be used as an extension of the classical block theory and the determined accompanying set associated with each key block can be employed as new criteria to analyze the stability of rock mass. Several examples are given to demonstrate this method.

2 Explicit representation of blocks

Most discrete models of rock masses start with an explicit representation of block systems, in which the positions and shapes of each block are determined. However, besides the geometry of individual blocks, the adjacency relationship between discrete blocks is also important in analyzing the influence of each key block. This section introduced the data structure used to represent the block geometries and detailed the method to obtain and record the adjacency relationship between blocks Xia et al. (2015); Zheng et al. (2015b,c). Some concepts from the graph theory that are involved in the analysis process are also briefly reviewed.

2.1 Block geometry

Generally, most numerical models are established to investigate a portion of rock masses around the excavation because it is usually ineffective, if not impossible, to analysis the infinite rock mass. Extra emphasis should be given to the blocks formed around the excavation because these blocks are the causes of the instability of rock masses. In this study, the modeling domain is approximated using several convex subdomains and a flag field is allocated to mark the type of each boundary surface of subdomains. Specifically, the surfaces that describe the shape of the excavation are marked as excavation surfaces and the surfaces that separate the modeling domain with the infinite rock masses are defined as fixed surfaces, which are usually selected far enough from the excavation so that the blocks can be adequately modeled.

An algorithm developed based on the element-block assembling approach is used to identify the shapes and positions of discrete blocks. Both planar and curved fractures can be modeled using this algorithm and the resulting blocks are represented as assemblages of convex element-blocks. The procedures of the block identification method used in this study are detailed in Yu et al. (2009). Further developments of this method are performed to handle robustness issues Xiao-ming and Yin-he (2015); Xia et al. (2016); Zheng et al. (2016).

The identified blocks are classified based on the adjacency relationship to different surfaces. Specifically, the excavated blocks are defined as the blocks that are adjacent to the excavation surfaces, i.e., block B1 is an excavated block if and only if the outer boundaries of B1 are overlap with an excavation surface. Denote the set of all the excavation surfaces as Sef, then we have:

in which denote all the boundary surfaces of block B1. Similarly, block B2 is defined as a fixed block if and only if B2 is adjacent to the fixed surfaces:

in which Sff is the set of all the fixed surfaces. In this study, the fixed blocks are regarded as the extension of the infinite rock mass during the analysis. The blocks that are neither adjacent to the excavated surfaces nor the fixed surfaces are defined as unexposed blocks, i.e., B3 is an unexposed block if and only if:

The removability and stability of each block are mainly determined by the outer boundaries of blocks, which are not explicitly represented in the element-block assembling approach because each complex-block is represented as an assemblage of element-blocks. A method is developed to construct the boundary structures of each complex-block. Specifically, the outer boundary polygons of each assemblage of element-blocks are determined by eliminating the common parts of the adjacent element-blocks' boundaries. Each resulting polygon is represented using its vertex chains in 2D coordinates along with a matrix characterizing the coordinate transformation from 2D to 3D. Under this representation scheme, the polygons may present arbitrary shapes and the predicates that whether two coplanar polygons are overlapped (such as the predicates shown in eq.1, eq.2 and eq.3) can be easily calculated.

During the removability analysis, all the constructed boundary polygons are recorded in a list Lb and the explicit description of the outer boundaries of each block B is achieved by maintaining an index list Lp(B) for B: each index in Lp(B) refers to a polygon in Lb and all the polygons that are determined by Lp(B) constitute the outer boundaries of B. The outward normal vector of each boundary polygon in Lb is also explicitly recorded, so that the identically boundary polygons that are shared by adjacent blocks can be distinguished. This representation method helps to construct algorithms that are more concise and efficient. Note that although some additional memories are consumed to record opposite polygons with identical shapes, the overall order of magnitude of the space complexity will not be affected. Moreover, the costs of space efficiency will be redeemed by the fast development of computer hardware.

2.2 Block Adjacency

In this study, the adjacency relationship between blocks are determined based on the geometry data of blocks and recorded in an undirected graph. Two blocks B1 and B2 are defined as adjacent (or connected) to each other if and only if their outer boundaries are overlapped:

i.e., there exist at least one index pair (I_1, I_2), such that $I1 \in L_p(B_1)$, $I2 \in L_p(B_2)$ and P(I1) overlap with P(I2), in which P(I1) and P(I2) are the boundary polygons in Lb that are associated with index I1 and I2, respectively. The predicate that whether two blocks are adjacent can be calculated by traversing all the boundary polygons pairs of B1 and B2.

The adjacency relationship in a block system can be interpreted as an undirected graph G(V, E), which is defined as a set V of nodes together with a set E of undirected edges. Each edge in E connects 2 elements in V. In this study, each block contained in the block system is regarded as a node in V and an edge e = (n1, n2) is contained in E if and only if the blocks corresponding to n1 and n2 are adjacent to each other. The constructed undirected graph G is defined as the adjacency graph of the block system. The adjacency list data structure is used to represent G(V, E). Specifically, every node in V is recorded in a list and for each node $n \in V$, a list is maintained to record the nodes that are adjacent to n. In this way, the basic operations involved in investigating the influence of each key block, such as retrieving the adjacent blocks of a given block or modifying the graph structure to reflect the influence of block fallings, can be performed efficiently.

The representation scheme introduced in this section is demonstrated using an examples. An adjacency graph is constructed for the 2D block systems as shown in fig.1, i.e., each block contained in the block system is regarded as a node and 2 nodes are connected by an edge if and only if the corresponding blocks are adjacent. The constructed graph is shown in fig.2.

3 Analyzing the block system

This study aims at determining the influence of each key block to the removability of its adjacent blocks. As most developed methods (Goodman and Shi 1985; Warburton 1981), the movement of blocks is limited to translation in this study and the blocks are assumed to be rigid. This section detailed the method used to analyze the removability of blocks considering the influence of their adjacent blocks. The adjacency graph introduced in section 2 helps to conduct this analysis.

3.1 Block Removability

Generally, the removability of a given block B can be determined using either the vector method (Warburton 1981) or the stereographic projection method (Goodman and Shi 1985; Zhang and Kulatilake 2003). In these methods, the surrounding blocks of B are assumed to be fixed. In this study, a modified vector method is used to determine the removability of a single block and a concept of geometrical removability is introduced Zheng et al. (2019b,a).

The removability of a single block B is determined based on the constraints proposed on B. Denote m as the vector representing the moving direction of the block B. There are 2 sets of constraints restricting m: (1) the fixed boundaries of B and (2) the resultant driving force R acting on B. Specifically, each fixed boundary of B will prevent any movement directed toward it and the direction of m must coincide with R. Assume the normal vectors of the fixed boundaries of B are ni (i = 1, 2, . . . , N) (ni points toward B), then the constraints on m can be represented as:

$nim \geq 0$

$Rm \geq 0$

If there exist a nonzero vector m, s.t. m meets the constraints represented by eq.5 and eq.6 then B is removable, otherwise B is nonremovable. The existence of such vector can be determined within the time complexity of O(N2) by traversing all the extreme points of the convex feasible region. Fig.5 gives an example of several blocks with the gravity being the only driving force. Moreover, if there exist a nonzero vector m', s.t. m' meets the constraints represented by eq.5, then B is defined as geometrical removable. This concept is used to determine interlocked blocks in section 3.3. Apparently, a removable block must be a geometrical removable block, but not versa, in particular, the block shown in fig.5c is a geometrical removable block but not a removable block if the gravity is considered to be the only driving force.

4 Conclusion

A method is proposed to analyze the influence of each key block in a block system formed by finite sized fractures. The block structures are explicitly described and there is no limit to the shape and complexity of the blocks. The removability of blocks is analyzed considering the influence of its adjacent blocks and an algorithm is proposed to determine the accompanying set associated with each key block based on a breadth-first search process.

The concept of undirected graph is used to represent the adjacency relations between blocks and the constructed adjacency graph helps to determine the nested and interlocked block sets contained in the block system. The updated arrangement of the block system after the failure of some blocks can be tracked using the adjacency graph.

A queue is used to drive the search process of the accompanying set associated with each key block and the results can be used to estimate the influence of each key block. The determined accompanying sets can also be employed to analyze the stability of rock masses as an extension of the classical block theory.

Note that the present study is established based on an explicitly represented block system. Probabilistic methods can be used to investigate the uncertainties in the rock mass model and a large number of realizations of discrete fracture network are required to reflect the statistical nature of the fractures. It also worth noting that more sophisticated mechanical analysis of the accompanying sets is necessary to obtain more realistic results of the rock mass stability. Future work should consider the inaccuracy introduced by the measured data and the mechanical reactions between adjacent blocks.

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