$$= \Sigma_{W_1 W_1 | w_2} + \Sigma_{B_1 B_1}\left(\frac{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}\right)$$

$$+ \frac{\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{W_2 W_1}}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}$$

$$- \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}\left(\frac{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}\right)$$

$$+ \frac{\Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}$$

$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}\left(\frac{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}\right)$$

$$+ 2\frac{\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}}{1 + \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2}$$

$$= \Sigma_{W_1 W_1 | w_2} + \Sigma_{B_1 B_1} \sigma_{H|y_2}^2 + \Sigma_{B_1 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$+ \Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{W_2 W_1} \sigma_{H|y_2}^2$$

$$- \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2 - \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$+ \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$+ 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2$$

$$= \Sigma_{W_1 W_1 | w_2} + \tau_1 \tau_1^T \sigma_{H|y_2}^2 + \Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{W_2 W_1} \sigma_{H|y_2}^2$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_1^T \sigma_{H|y_2}^2 + \Sigma_{B_1 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$- \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2 - \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$+ \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 \sigma_{H|y_2}^2$$
$$+ 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \sigma_{H|y_2}^2.$$

The top two lines of the previous equivalence are what we need, that is, they are the conditional covariance of the proposed model. This means that all the lower terms in the previous equivalence must cancel out. Rearranging the bottom six terms and dropping the positive $\sigma_{H|y_2}^2$, we get

$$\Sigma_{B_1 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2 - \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}$$
$$+ \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}$$
$$- \Sigma_{B_1 B_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2$$
$$+ 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_2^T \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1}$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \Sigma_{B_2 B_1} \tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2$$

.

Noting that $\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2$ is a constant, and expanding each $\Sigma_{B_j B_k} = \tau_j \tau_k^T$, this becomes

$$\tau_1 \tau_1^T (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) - \tau_1 (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) \tau_1^T$$
$$+ \tau_1 (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2)(\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) \tau_1^T$$
$$- \tau_1 (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) \tau_1^T (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2)$$
$$+ 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) \tau_1^T$$
$$- 2\Sigma_{W_1 W_2} \Sigma_{W_2 W_2}^{-1} \tau_2 \tau_1^T (\tau_2^T \Sigma_{W_2 W_2}^{-1} \tau_2) = 0.$$

Thus, we have proven the equivalence of the proposed conditioning method's mean and covariance with that given by the conditional MVN equations.

APPENDIX B
Mixed native and nonnative station data updating

Suppose that we have a grid of K points for which we are trying to estimate the ground motions Y given ground motions $Y_D = y_D$ at a set of L stations. Suppose that the ground motion of interest is a particular ground-motion measure (peak ground acceleration [PGA], for example). Also, suppose that the station data can be divided into $M + 1$ ground-motion measures, of which the ground-motion measure corresponding to Y is one of them. We can divide the station data into the native (PGA, call it Y_N) and nonnative data (not PGA, call them Y_{NN}) by partitioning Y_D such that,

$$Y_D = \begin{bmatrix} Y_N \\ - \\ Y_{NN} \end{bmatrix}. \tag{B1}$$

This note does not assume that the between-event residuals for different ground-motion measures are perfectly correlated; hence, there is a different normalized between-event residual for each different measure. We say that

$$Y = \mu_Y + W_Y + \tau_Y H_N, \tag{B2}$$

$$Y_N = \mu_{Y_N} + W_N + \tau_N H_N. \tag{B3}$$

We can define $Y_{NN} = [Y_1^T | \ldots | Y_M^T]^T$ partitioned into the M different ground-motion measures, in which

$$Y_i = \mu_{Y_i} + W_i + \tau_i H_i, \, i = 1, \ldots, M, \tag{B4}$$

and we define

$$Y_D = \mu_{Y_D} + W_D + T_D H_D, \tag{B5}$$

in which
$$\mu_{Y_D} = \begin{bmatrix} \mu_{Y_N} \\ - \\ \mu_{Y_1} \\ - \\ \vdots \\ - \\ \mu_{Y_M} \end{bmatrix}, W_D = \begin{bmatrix} W_N \\ - \\ W_1 \\ - \\ \vdots \\ - \\ W_M \end{bmatrix}, H_D = \begin{bmatrix} H_N \\ H_1 \\ \vdots \\ H_M \end{bmatrix},$$

$$
T_D = \begin{bmatrix} \boldsymbol{\tau}_N & | & 0 & | & \ldots & | & 0 \\ - & - & - & - & - & - & - \\ 0 & | & \boldsymbol{\tau}_1 & | & \ddots & | & \vdots \\ - & - & - & - & - & - & - \\ \vdots & | & \ddots & | & \ddots & | & 0 \\ - & - & - & - & - & - & - \\ 0 & | & \ldots & | & 0 & | & \boldsymbol{\tau}_M \end{bmatrix}, \text{ and } T_D\mathbf{H_D} = \begin{bmatrix} \boldsymbol{\tau}_N H_N \\ - \\ \boldsymbol{\tau}_1 H_1 \\ - \\ \vdots \\ - \\ \boldsymbol{\tau}_M H_M \end{bmatrix}
$$

(T_D is a lower diagonal partitioned matrix called capital tau).

Then, we can use the law of total probability to get

$$
f_{Y|Y_D}(y|y_D) = \int_{\mathbf{H_D}} f_{Y|Y_D\mathbf{H_D}}(y|y_D,\boldsymbol{\eta_D}) f_{\mathbf{H_D}|Y_D}(\boldsymbol{\eta_D}|y_D) d\boldsymbol{\eta_D},
$$
(B6)

in which $\int_{\mathbf{H_D}}() d\boldsymbol{\eta_D} = \int_{H_N} \int_{H_1} \ldots \int_{H_M}() d\eta_M \ldots d\eta_1 d\eta_N$. This once again splits the density into the between- and within-event portions.

Determining $f_{\mathbf{H_D}|Y_D}(\boldsymbol{\eta_D}|y_D)$:
Using the Bayes equation, we get

$$
f_{\mathbf{H_D}|Y_D}(\boldsymbol{\eta_D}|y_D) \propto f_{Y_D|\mathbf{H_D}}(y_D|\boldsymbol{\eta_D}) f_{\mathbf{H_D}}(\boldsymbol{\eta_D}).
$$
(B7)

Now $\mathbf{H_D} \sim \text{MVN}(0, \Sigma_{\mathbf{H_D H_D}})$, in which $\Sigma_{\mathbf{H_D H_D}}$ is the cross-intensity measure correlations for the $M+1$ intensity measures present. Given the residual bias vector $\mathbf{H_D} = \boldsymbol{\eta_D}$, we have $Y_D|(\mathbf{H_D} = \boldsymbol{\eta_D}) \sim \text{MVN}(\boldsymbol{\mu}_{Y_D} + T_D\boldsymbol{\eta_D}\Sigma_{W_D W_D})$. We can then determine the distribution of $\mathbf{H_D}|(Y_D = y_D)$ using the completing the square method for matrices:

$$
f_{\mathbf{H_D}|Y_D}(\boldsymbol{\eta_D}|y_D)
$$
$$
\propto \exp\left[-\frac{1}{2}(y_D - \boldsymbol{\mu}_{Y_D} - T_D\boldsymbol{\eta_D})^T \Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D} - T_D\boldsymbol{\eta_D})\right]
$$
$$
\times \exp[-\frac{1}{2}\boldsymbol{\eta_D}^T \Sigma_{\mathbf{H_D H_D}}^{-1}\boldsymbol{\eta_D}]
$$
$$
= \exp\left[-\frac{1}{2}\left((y_D - \boldsymbol{\mu}_{Y_D})^T \Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D})\right.\right.
$$
$$
+ \boldsymbol{\eta_D}^T T_D^T \Sigma_{W_D W_D}^{-1} T_D\boldsymbol{\eta_D} - 2(y_D - \boldsymbol{\mu}_{Y_D})^T \Sigma_{W_D W_D}^{-1} T_D\boldsymbol{\eta_D}
$$
$$
\left.\left.+ \boldsymbol{\eta_D}^T \Sigma_{\mathbf{H_D H_D}}^{-1}\boldsymbol{\eta_D}\right)\right]
$$
$$
\propto \exp\left[-\frac{1}{2}(\boldsymbol{\eta_D}^T (T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{\mathbf{H_D H_D}}^{-1})\boldsymbol{\eta_D}\right.
$$
$$
\left.- 2(y_D - \boldsymbol{\mu}_{Y_D})^T \Sigma_{W_D W_D}^{-1} T_D\boldsymbol{\eta_D}\right].
$$

Let $M = T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{\mathbf{H_D H_D}}^{-1}$ and $b = T_D^T \Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D})$. Then, we have

$$
f_{\mathbf{H_D}|Y_D}(\boldsymbol{\eta_D}|y_D) \propto \exp[-\frac{1}{2}(\boldsymbol{\eta_D}^T M \boldsymbol{\eta_D} - 2b^T \boldsymbol{\eta_D})]
$$
$$
= \exp[-\frac{1}{2}(\boldsymbol{\eta_D}^T M \boldsymbol{\eta_D} - 2b^T M^{-1} M \boldsymbol{\eta_D})]
$$
$$
= \exp[-\frac{1}{2}((\boldsymbol{\eta_D} - M^{-1}b)^T M(\boldsymbol{\eta_D} - M^{-1}b) - b^T M^{-1}b)]
$$
$$
\propto \exp[-\frac{1}{2}(\boldsymbol{\eta_D} - M^{-1}b)^T M(\boldsymbol{\eta_D} - M^{-1}b)].
$$

By inspection this means that $\boldsymbol{\mu}_{\mathbf{H_D}|y_D} = M^{-1}b$ and $\Sigma_{\mathbf{H_D H_D}|y_D} = M^{-1}$, or

$$
\boldsymbol{\mu}_{\mathbf{H_D}|y_D} = (T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{\mathbf{H_D H_D}}^{-1})^{-1} T_D^T \Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D}),
$$
(B8)

$$
\Sigma_{\mathbf{H_D H_D}|y_D} = (T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{\mathbf{H_D H_D}}^{-1})^{-1}.
$$
(B9)

Determining $f_{Y|Y_D,\mathbf{H_D}}(y|y_D,\boldsymbol{\eta_D})$:
Now $f_{Y|Y_D,\mathbf{H_D}}(y|y_D,\boldsymbol{\eta_D}) = f_{(\boldsymbol{\mu}_Y + W_Y + \tau_Y\eta_N)|W_D,\mathbf{H_D}}(y|w_2,\boldsymbol{\eta_D})$ using equation (B2) and the fact that being given $Y_D, \mathbf{H_D}$ is equivalent to being given $W_D, \mathbf{H_D}$ (using equation B5), this then reduces to finding the distribution of W_Y given $W_D = w_D = y_D\boldsymbol{\mu}_{Y_D} - T_D\boldsymbol{\eta_D}$. Using the conditional MVN equations (4) and (5), this implies that

$$
Y|(Y_D = y_D, \mathbf{H_D} = \boldsymbol{\eta_D}) \sim \text{MVN}(\boldsymbol{\mu}_{Y|y_D,\boldsymbol{\eta_D}}, \Sigma_{YY|y_D,\boldsymbol{\eta_D}}), \quad \text{(B10)}
$$

with

$$
\boldsymbol{\mu}_{Y|y_D,\boldsymbol{\eta_D}} = \boldsymbol{\mu}_Y + \boldsymbol{\tau}_Y\eta_N + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D} - T_D\boldsymbol{\eta_D}),
$$
(B11)

$$
\Sigma_{YY|y_D,\boldsymbol{\eta_D}} = \Sigma_{W_Y W_Y} - \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}\Sigma_{W_D W_Y}.
$$
(B12)

Determining $f_{Y|Y_D}(y|y_D)$:
By introducing the $K \times (M+1)$ matrix

$$
T_{Y0} = \begin{bmatrix} \tau_{Y_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{YK} & 0 & \cdots & 0 \end{bmatrix},
$$

we can rewrite

$$
\boldsymbol{\mu}_{Y|y_D,\boldsymbol{\eta_D}} = \boldsymbol{\mu}_Y + T_{Y0}\boldsymbol{\eta_D} + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D} - T_D\boldsymbol{\eta_D}).
$$

We proceed by following the same steps as in the main portion of this article. We first define the matrix $C = T_{Y0} - \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D$. Then again, we can rewrite

$$
\boldsymbol{\mu}_{Y|y_D,\boldsymbol{\eta_D}} = \boldsymbol{\mu}_Y + C\boldsymbol{\eta_D} + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \boldsymbol{\mu}_{Y_D}).
$$

We recast the problem into the following random variables $V = CH_D|(Y_D = y_D)$, $v = C\eta_D$, and $U = Y|(Y_D = y_D, H_D = \eta_D) - v$. Then, we have

$$V \sim \text{MVN}(\mu_V = C\mu_{H_D|y_D}, \Sigma_{VV} = C\Sigma_{H_D H_D|y_D} C^T), \quad (B13)$$

$$U \sim \text{MVN}(\mu_U = \mu_Y + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \mu_{Y_D}),$$
$$\Sigma_{YY} = \Sigma_{W_Y W_Y|w_D}). \quad (B14)$$

Then, we can expand $f_{Y|Y_D}(y|y_D)$ in terms of V instead of H_D:

$$f_{Y|Y_D}(y|y_D) = \int_V f_{Y|Y_D, V}(y|y_D, v) f_{V|Y_D}(v|y_D) dv$$
$$= \int_V f_{U+V|Y_D, V}(y|y_D, v) f_{V|Y_D}(v|y_D) dv$$
$$= \int_V f_{U+V|V}(y|v) f_V(v) dv,$$

in which we have used the fact that being given V is equivalent to being given H_D and noting that both U and V are already conditioned on Y_D. This implies that $Y|(Y_D = y_D) = U + V$, and because U and V are independent MVN random variables, we get

$$Y|(Y_D = y_D) \sim \text{MVN}(\mu_{Y|y_D}, \Sigma_{YY|y_D}), \quad (B15)$$

with

$$\mu_{Y|y_D} = \mu_U + \mu_V$$
$$= \mu_Y + C\mu_{H_D|y_D} + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \mu_{Y_D})\Sigma_{YY|y_D}$$
$$= \Sigma_{UU} + \Sigma_{VV} = \Sigma_{W_Y W_Y|w_D} + C\Sigma_{H_D H_D|y_D} C^T.$$

In summary, we have

$$\mu_{Y|y_D} = \mu_Y + \tau_Y \mu_{H_N|y_D} + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}$$
$$\times (y_D - \mu_{Y_D} - T_D\mu_{H_D|y_D}), \quad (B16)$$

$$\Sigma_{YY|y_D} = \Sigma_{W_Y W_Y|w_D} + C\Sigma_{H_D H_D|y_D} C^T, \quad (B17)$$

in which

$$\mu_{H_D|y_D} = (T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{H_D H_D}^{-1})^{-1} T_D^T \Sigma_{W_D W_D}^{-1}(y_D - \mu_{Y_D}), \quad (B18)$$

$$\mu_{H_N|y_D} = [\mu_{H_D|y_D}]_1, \quad (B19)$$

$$\Sigma_{H_D H_D|y_D} = (T_D^T \Sigma_{W_D W_D}^{-1} T_D + \Sigma_{H_D H_D}^{-1})^{-1}, \quad (B20)$$

$$\Sigma_{W_Y W_Y|w_D} = \Sigma_{W_Y W_Y} - \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}\Sigma_{W_D W_Y}, \quad (B21)$$

$$C = T_{Y0} - \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D. \quad (B22)$$

If no native data are present, then the results hold as described in equations (B16)–(B22), except

$$T_D = \begin{bmatrix} 0 & | & \tau_1 & | & 0 & | & \cdots & | & 0 \\ - & - & - & - & - & - & - & - & - \\ 0 & | & 0 & | & \tau_2 & | & \ddots & | & \vdots \\ - & - & - & - & - & - & - & - & - \\ \vdots & | & \vdots & | & \ddots & | & \ddots & | & 0 \\ - & - & - & - & - & - & - & - & - \\ 0 & | & 0 & | & \cdots & | & 0 & | & \tau_M \end{bmatrix} = \begin{bmatrix} 0 & | & T_{NN} \end{bmatrix},$$

and $y_D, \mu_{Y_D}, \Sigma_{W_D W_D}$, and $\Sigma_{W_Y W_D}$ become $y_{NN}, \mu_{Y_{NN}}, \Sigma_{W_{NN} W_{NN}}$, and $\Sigma_{W_Y W_{NN}}$, respectively.

Comparing with the main result of this article:
From this article, we had

$$\mu_{Y_1|y_2} = \mu_{Y_1} + \mu_{B_1|y_2} + \Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}(y_2 - \mu_{y_2} - \mu_{B_2|y_2}), \quad (19a)$$

which looks very similar to equation (B16) if we note that $\mu_{B_Y|y_D} = \tau_Y \mu_{H_N|y_D}$ and $\mu_{B_D|y_D} = T_D\mu_{H_D|y_D}$, we get

$$\mu_{Y|y_D} = \mu_Y + \mu_{B_Y|y_D} + \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(y_D - \mu_{Y_D} - \mu_{B_D|y_D}).$$

Similarly, from this article, we had

$$\Sigma_{Y_1 Y_1|y_2} = \Sigma_{W_1 W_1|w_2} + cc^T \sigma_{H|y_2}^2$$
$$= \Sigma_{W_1 W_1|w_2} + (\tau_1 - \Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}\tau_2)$$
$$\times (\tau_1 - \Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}\tau_2)^T \sigma_{H|y_2}^2$$
$$= \Sigma_{W_1 W_1|w_2} + \tau_1 \tau_1^T \sigma_{H|y_2}^2 + \Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}(\tau_2 \tau_2^T \sigma_{H|y_2}^2)$$
$$\times \Sigma_{W_2 W_2}^{-1}\Sigma_{W_2 W_1} - 2\Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}(\tau_2 \tau_1 \sigma_{H|y_2}^2)$$
$$= \Sigma_{W_1 W_1|w_2} + \Sigma_{B_1 B_1|y_2} + \Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}\Sigma_{B_2 B_2|y_2}$$
$$\times \Sigma_{W_2 W_2}^{-1}\Sigma_{W_2 W_1} - 2\Sigma_{W_1 W_2}\Sigma_{W_2 W_2}^{-1}\Sigma_{B_2 B_1|y_2}, \quad (20a)$$

which looks very similar to equation (B17) if we expand it

$$\Sigma_{YY|y_D} = \Sigma_{W_Y W_Y|w_D} + C\Sigma_{\mathbf{H_D H_D}|y_D} C^T$$

$$= \Sigma_{W_Y W_Y|w_D}$$
$$+ (T_{Y0} - \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D)\Sigma_{\mathbf{H_D H_D}|y_D}(T_{Y0}$$
$$- \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D)^T$$

$$= \Sigma_{W_Y W_Y|w_D} + T_{Y0}\Sigma_{\mathbf{H_D H_D}|y_D} T_{Y0}^T$$
$$+ \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_D^T\Sigma_{W_D W_D}^{-1}\Sigma_{W_D W_Y}$$
$$- 2\Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1} T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_{Y0}^T$$

$$= \Sigma_{W_Y W_Y|w_D} + \boldsymbol{\tau}_Y\boldsymbol{\tau}_Y^T\sigma_{\mathrm{H}_N|y_D}^2$$
$$+ \Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_D^T)\Sigma_{W_D W_D}^{-1}\Sigma_{W_D W_Y}$$
$$- 2\Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}(T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_{Y0}^T).$$

Noting that $\Sigma_{B_Y B_Y|y_D} = \boldsymbol{\tau}_Y\boldsymbol{\tau}_Y^T\sigma_{\mathrm{H}_N|y_D}^2, \Sigma_{B_D B_D|y_D} = T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_D^T$ and $\Sigma_{B_D B_Y|y_D} = T_D\Sigma_{\mathbf{H_D H_D}|y_D} T_{Y0}^T$, we get

$$\Sigma_{YY|y_D} = \Sigma_{W_Y W_Y|w_D} + \Sigma_{B_Y B_Y|y_D}$$
$$+ \Sigma_{W_D W_D}\Sigma_{W_D W_D}^{-1}\Sigma_{B_D B_D|y_D}\Sigma_{W_D W_D}^{-1}\Sigma_{W_D W_Y}$$
$$- 2\Sigma_{W_Y W_D}\Sigma_{W_D W_D}^{-1}\Sigma_{B_D B_Y|y_D}.$$

Looking at equation (B17), we still have separated conditional within- and between-event processes. However, unlike the main result of this article, the between-event covariance matrix $C\Sigma_{\mathbf{H_D H_D}|y_D} C^T$ is no longer perfectly correlated, because it cannot be expressed as an outer product of vectors. Nonetheless, simulating this between-event process only involves simulating MVN realizations of $\Sigma_{\mathbf{H_D H_D}|y_D}$ and then scaling them appropriately by C.