Addis Ababa Science and Technology University

Final Exam: Probability and Statistics

Name:_____ **Id.**_____ signature:_____
Department _____ **Section**_____

Time allowed: 3:00

General Instruction:

- Write your name and Id
- Check that the exam paper contains three parts on 9 pages including tables
- Write clearly and neatly; unreadable answer will not be marked
- You can use the back pages of the exam paper for rough work but you are not allowed to use your paper.
- Using scientific calculator is allowed.
- Switch of your mobile before you start exam.
- Any cheating, facilitating for cheating & disturbance in exam class will result to cancellation.

Good luck!!!

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$\lambda = 3$ $\quad \frac{\lambda^{-x} e^{-1}}{x!} =$

Part I: write the most simplified answers on the blank space (1 points for each blank space)

1. If probability of P(A)=0.3, P(B)=0.7 and P(AnB)=0.3 then $P(A^c \text{ n } B^c)$ 0.3
2. Let A and B be two events defined on a sample space S and P(A)=0.3, P(B)=0.4, and then
   a) If A and B are independent, then P(A/B)= PA = 0.3
   b) If A and B are mutually exclusive, then P(A U B)= ~~0.7~~ PA + PB = 0.7
3. Let A and B be events such that $P(A \cup B) = 0.75, P(A \cap B) = 0.25$ *and* $P(A') = 0.6$, P(B)= 0.4
4. If A and B are two events such that $P(A \cup B) = \frac{3}{4}, P(A \cap B) = 0.25$ *and* $P(A^C) = \frac{2}{3}$ *and then* $P(A^C/B)$= 5/8
5. If the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$.
   a. What is the probability that no accidents will occur? $e^{-3} = 1/e^3$
   b. What is the variance of this random variable? $\lambda = 3$

Part II: Briefly explain the following question

1. What are the main objectives of measure of variation? (2points)

2. State the central limit theorem. (2 points)

3. Give four types of probability sampling techniques and discuss each of them (2 points)

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Part III: workout; show clearly and neatly all necessary steps on the space provided

1. 120 employees of a certain factory are given a performance test and are divided in to two groups as those with good performance(G) and those with Poor performance(P) the result is given below. (2 points)

	Good performance (G)	Poor performance (P)	Total
Male	60	20	80
female	25	15	40
Total	85	35	120

a) What is the probability of a person to be male given that it has a good performance?

b) What is the probability of a person to have a poor performance?

$P(\text{male} \mid GP) = \frac{P(\text{male} \cap PGP)}{P(GP)} = \frac{\frac{60}{120}}{\frac{85}{120}} = \frac{60}{85}$

Poor performance $= \frac{35}{120}$

2. *Suppose that 3% of TOSHIBA computers produced by TOSHIBA Company are defective. If a random sample of 12 computers are chosen, Find the probability that* ***(4points)***

   i. None of the computers selected are defective.

   ii. At least 3 of the computers are none defective.

   iii. Find the mean and variance the number of defective computers.

$P(D) = 3\% \rightarrow$ 12 COMPUTER $=$ Sucess $= 3\%$

$x = 0 \quad \binom{n}{x} (p)^x (q)^{n-x} = \binom{12}{x} (0.3)^x (0.97)^{12-x}$

$x = 0 = \binom{12}{0} (0.03)^0 (0.97)^{12} =$

At least 3 of them mean $X \geq 3 = 1 - (P1)(P2)(P3)P($

$-[\binom{12}{0}(0.03)^0(0.97)^{12} + \binom{12}{1}(0.03)^1(0.97)^{11} + \binom{12}{2}(0.03)^2(0.97)^{10} + \binom{12}{3}(0$

$\text{Mean} = n \cdot p$

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3. The probability that a cellular phone company NOKIA sells X number of new phone contracts per day is shown below. **(3points)**

X	4	5	6	8	10
P(X)	0.4	0.3	0.1	0.15	.05

i. What is the probability that they will sell 6 or more contracts three days in a row?

ii. Find the mean, variance, and standard deviation for this probability distribution.

I $P(x) = 0.1 + 0.15 + 0.05 = 0.3$

mean $= \Sigma x P(x) = 4 \times 0.4 + 5 \times 0.3 + 6 \times 0.1 + 8 \times 0.15 + 10 \times 0.05$

$= 1.6 + 1.5 + 0.6 + 1.2 + 0.5$

5.4

Variance $= \Sigma x^2 P(x) = 16 \times 0.4 + 25 \times 0.3 + 36 \times 0.1 + 64 \times 0.15 + 100 \times 0.05$

4. The time taken to complete a certain standard exam for deriving license by trainees is normally distributed with a mean of 120 minutes and a standard deviation of 10 minutes. If we randomly select one trainee, Calculate the probability that he/she will complete the exam

a) Between 110 and 130 minutes

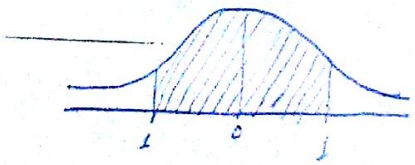
b) after 145 minutes

c) before 105 minutes

(3 Points)

a) $Z = \frac{x-\mu}{\sigma} = \frac{110-120}{10} = -1$

$Z = \frac{130-120}{10} = 1$



$P(-1 < z < 1) = 2(P(0 < z < 1))$

$= 2(0.3413)$

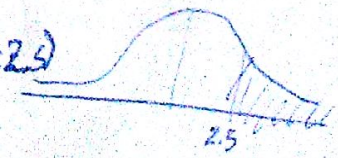
$= 0.6826$

b) $z = \frac{145-120}{10} = 2.5$

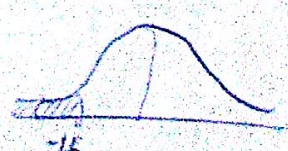
$P(z > 2.5) = 0.5 - P(0 < z < 2.5)$

$= 0.5 - 0.4938$

$= 0.0062$



c) $z = \frac{105-120}{10} = -1.5$ $P(z < -1.5) = 0.5 - P(0 > z > -1.5)$



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4. The diameter of electric cable, say X, is assumed to be a continuous random variable with pdf:

$$f(x) = \begin{cases} ke^{-x/3} & x>0 \\ 0 & Otherwise \end{cases}$$

a) Determine K (1 point)
b) Find E(X) and Var(X) (4 point)
c) Calculate var(KX) (1 point)
d) Var(k+X) (1 point)

a) $\int_0^\infty k e^{-x/3} dx = 1 = \frac{k e^{-x/3}}{-1/3} \Big|_0^\infty = 1$

$-3k e^{-x/3} \Big|_0^\infty = 1 = [0 + 3k] = 1 \quad k = 1/3$

B) E(x) & V(x) $E(x) = \int_0^\infty x f(x) = \int_0^\infty x \frac{1}{3} e^{-x/3} dx$

$\frac{1}{3}\int_0^\infty x e^{-x/3} dx =$

$x \to e^{-1/3x}$
$1 \to -3e^{-1/3x}$

$-3x e^{-1/3x} - 9e^{-1/3x} \Big|_0^\infty = [0 - (-9)] = 9 = 3$

Variance $= \int_0^\infty x^2 f(x) dx + \mu^2 = \frac{1}{3}\int_0^\infty x^2 e^{-x/3} dx$

$x^2 \to e^{-1/3x}$
$2x \to -3e^{-1/3x}$
$2 \to 9e^{-1/3x}$
$0 \to -27e^{-1/3x}$

$\frac{1}{3}\left[-3x^2 e^{-1/3x} - 18x e^{-1/3x} - 54e^{-1/3x}\right]_0^\infty [0 + 18]$

Var = $18 - 9 = 9$

c) $Var(kx) = k^2 V(x) = k^2 \cdot 9 = 9k^2$

D) $Var(k+x) = Var\ k + Var\ x = 9$

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6. A cocoa packaging machine fills bags so that the bag contents have a standard deviation of 3.5 g. Weights of contents of bags are normally distributed. If a random sample of 20 bags gives a mean of 102.0 g, construct 99% confidence interval for the mean weight of the population (i.e., all bags)? *(3 points)*

Given: $\sigma = 3.5$ $n = 20$ $\bar{x} = 102.0$ $1-\alpha = 0.99$

$\alpha = 0.99 - 1 \Rightarrow \alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$

$= P\left(-z_{\alpha/2} < z < z_{\frac{\alpha}{2}}\right)$

$= P(-z_{0.005} < z < z_{0.005})$

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\left(\bar{x} - z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}\right)$

$\left(102 - z_{\alpha/2} \times \frac{3.5}{\sqrt{20}} < \mu < 102 + z_{\alpha/2} \times \frac{3.5}{\sqrt{20}}\right)$

7. The strength of steel wire made by an existing process is normally distributed with a mean of 1250. A batch of wire is made by a new process, and a random sample consisting of 25 measurements gives an average strength of 1312 with a standard deviation of 150. Is there evidence at the 1% level of significance that the new process gives a larger mean strength than the old? (5 points)

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8. An architect wants to determine the relationship between the heights (in feet) of a building and the number of floors in the building. The data for a sample of 10 buildings in Pittsburgh are shown.

floors (x)	64	54	40	31	45	38	42	41	37	40
Height (y)	841	725	635	616	615	582	535	520	511	485

a) Draw the scatter diagram and comment on the type of relationship.*(2 points)*

b) Calculate intercept (a) and slope (b), then fit the regression line.*(4points)*

c) Predict the height of building if the number of floors (x) =44. *(1 point)*

d) Find the correlation coefficient (r) and give your interpretation based on your result.*(3 points)*

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Standard Normal Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499
3.1	0.499	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998

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