# analysis_2-24

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```
library(ggplot2)
library(coda)
library(runjags)
library(fastDummies)
data = data.frame(read.csv("CEdata.csv",header=TRUE))
```

The original slides has scale(), but I removed it to better interpret the results.

```
data$log_TotalExpSTD <- log(data$Expenditure)
data$log_TotalIncomeSTD <- log(data$Income)
```

Create the binary columns and rows for each categorical variable:

```
data$Rural = fastDummies::dummy_cols(data$UrbanRural)[,names(fastDummies::dummy_cols(data$UrbanRural))==
data$Race_Black = fastDummies::dummy_cols(data$Race)[,names(fastDummies::dummy_cols(data$Race)) == ".da
data$Race_NA = fastDummies::dummy_cols(data$Race)[,names(fastDummies::dummy_cols(data$Race)) == ".data_3
data$Race_Asian = fastDummies::dummy_cols(data$Race)[,names(fastDummies::dummy_cols(data$Race)) == ".da
data$Race_PI = fastDummies::dummy_cols(data$Race)[,names(fastDummies::dummy_cols(data$Race)) == ".data_5
data$Race_M = fastDummies::dummy_cols(data$Race)[,names(fastDummies::dummy_cols(data$Race)) == ".data_6
```

Same parameters from slides:

```
modelString <-"
model {
## sampling
for (i in 1:N){
y[i] ~ dnorm(beta0 + beta1*x_income[i] + beta2*x_rural[i] +
beta3*x_race_B[i] + beta4*x_race_N[i] +
beta5*x_race_A[i] + beta6*x_race_P[i] +
beta7*x_race_M[i], invsigma2)
}
## priors
beta0 ~ dnorm(mu0, g0)
beta1 ~ dnorm(mu1, g1)
beta2 ~ dnorm(mu2, g2)
beta3 ~ dnorm(mu3, g3)
beta4 ~ dnorm(mu4, g4)
beta5 ~ dnorm(mu5, g5)
beta6 ~ dnorm(mu6, g6)
beta7 ~ dnorm(mu7, g7)
invsigma2 ~ dgamma(a, b)
sigma <- sqrt(pow(invsigma2, -1))
}
"
```

```
y = as.vector(data$log_TotalExpSTD)
x_income = as.vector(data$log_TotalIncomeSTD)
x_rural = as.vector(data$Rural)
x_race_B = as.vector(data$Race_Black)
x_race_N = as.vector(data$Race_NA)
```

```r
x_race_A = as.vector(data$Race_Asian)
x_race_P = as.vector(data$Race_PI)
x_race_M = as.vector(data$Race_M)
N = length(y)
```

Same parameter values from slides:

```r
the_data <- list("y" = y, "x_income" = x_income,
                 "x_rural" = x_rural, "x_race_B" = x_race_B,
                 "x_race_N" = x_race_N, "x_race_A" = x_race_A,
                 "x_race_P" = x_race_P, "x_race_M" = x_race_M,
                 "N" = N,
                 "mu0" = 0, "g0" = 1, "mu1" = 0, "g1" = 1,
                 "mu2" = 0, "g2" = 1, "mu3" = 0, "g3" = 1,
                 "mu4" = 0, "g4" = 1, "mu5" = 0, "g5" = 1,
                 "mu6" = 0, "g6" = 1, "mu7" = 0, "g7" = 1,
                 "a" = 1, "b" = 1)
```

```r
initsfunction <- function(chain){
  .RNG.seed <- c(1,2)[chain]
  .RNG.name <- c("base::Super-Duper",
                 "base::Wichmann-Hill")[chain]
  return(list(.RNG.seed=.RNG.seed,
              .RNG.name=.RNG.name))
}
```

Thinning of 50 is needed, otherwise beta0 and beta1 have very high lag

```r
posterior_MLR <- run.jags(modelString,
                      n.chains = 1,
                      data = the_data,
                      monitor = c("beta0", "beta1", "beta2",
                                  "beta3", "beta4", "beta5",
                                  "beta6", "beta7", "sigma"),
                      adapt = 1000,
                      burnin = 5000,
                      sample = 5000,
                      thin = 50,
                      inits = initsfunction)
```

```
## Loading required namespace: rjags

## Compiling rjags model...
## Calling the simulation using the rjags method...
## Note: the model did not require adaptation
## Burning in the model for 5000 iterations...
## Running the model for 250000 iterations...
## Simulation complete
## Calculating summary statistics...

## Warning: Convergence cannot be assessed with only 1 chain

## Finished running the simulation
```

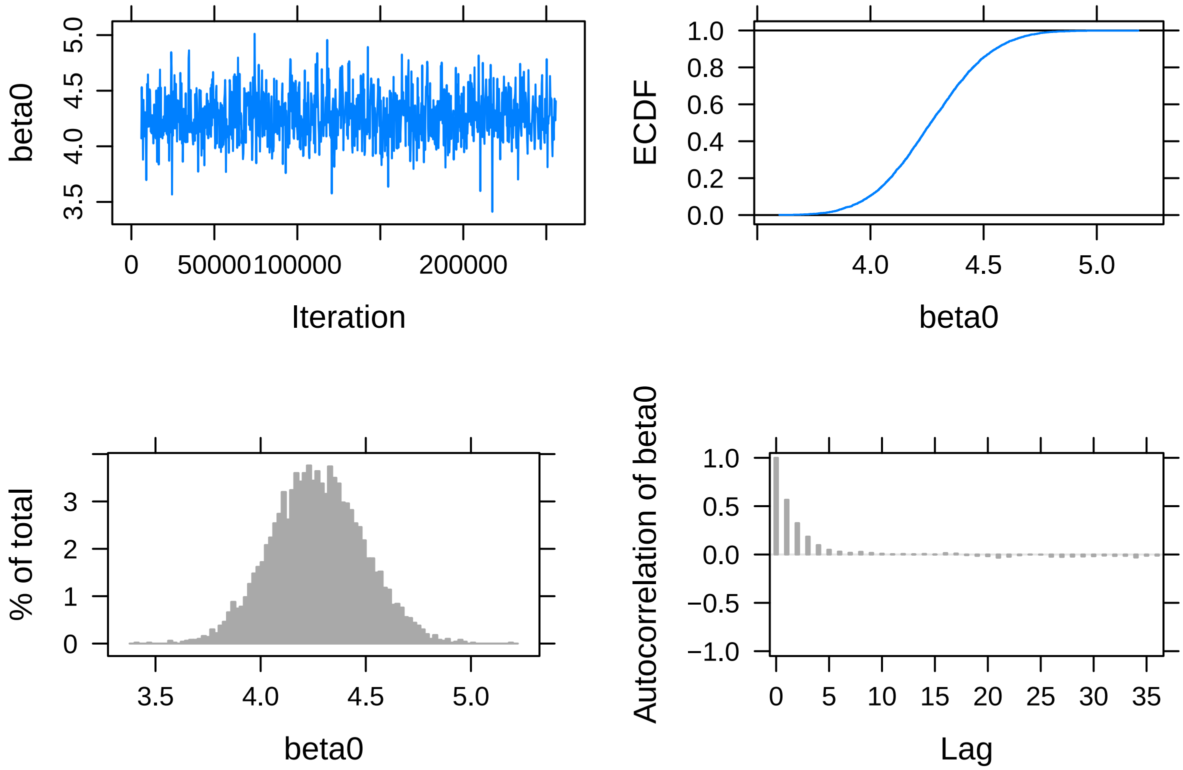
```r
summary(posterior_MLR)
```

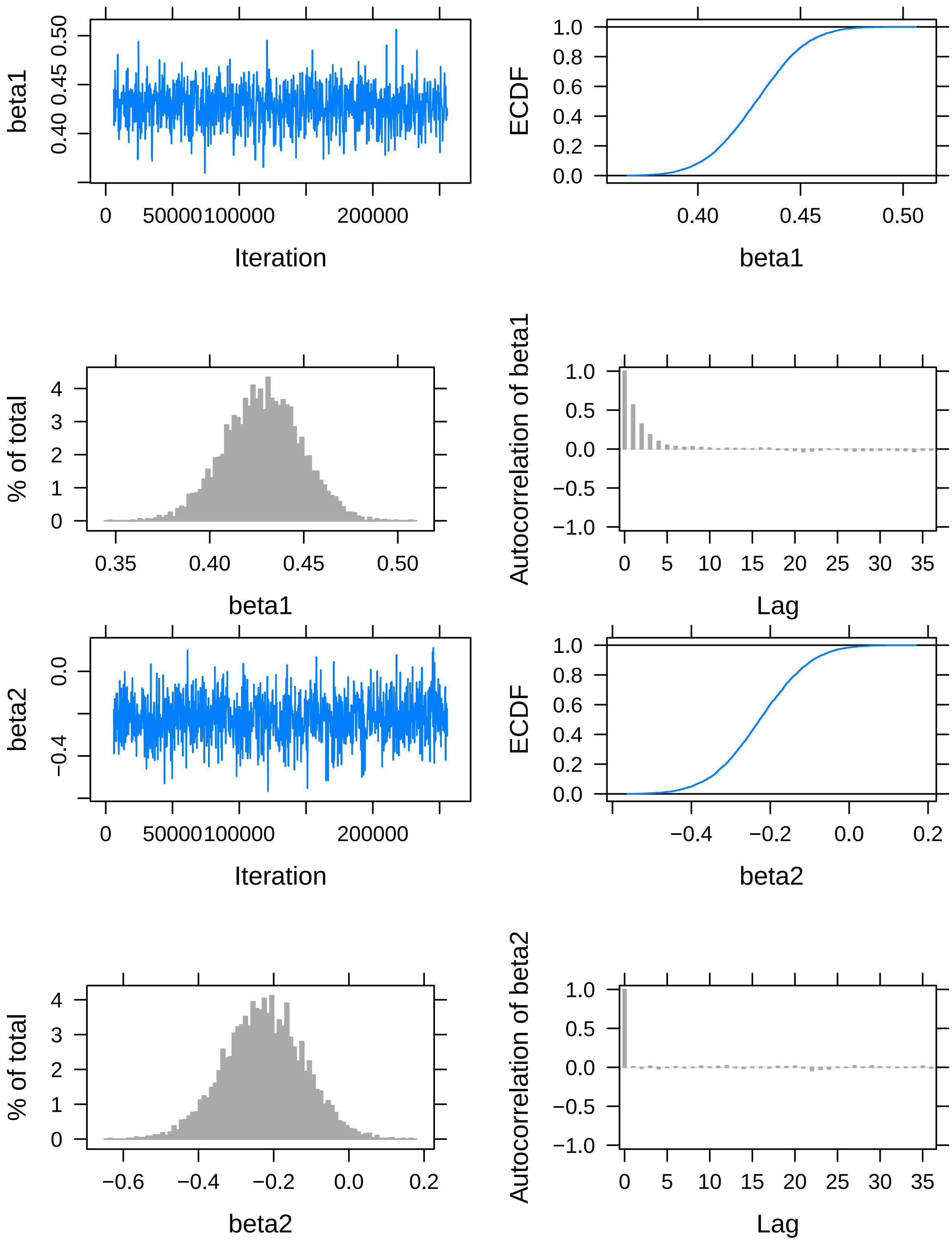
```
##          Lower95      Median     Upper95         Mean         SD Mode
## beta0  3.8401288  4.265856654  4.68625457  4.270064662 0.21777079   NA
```

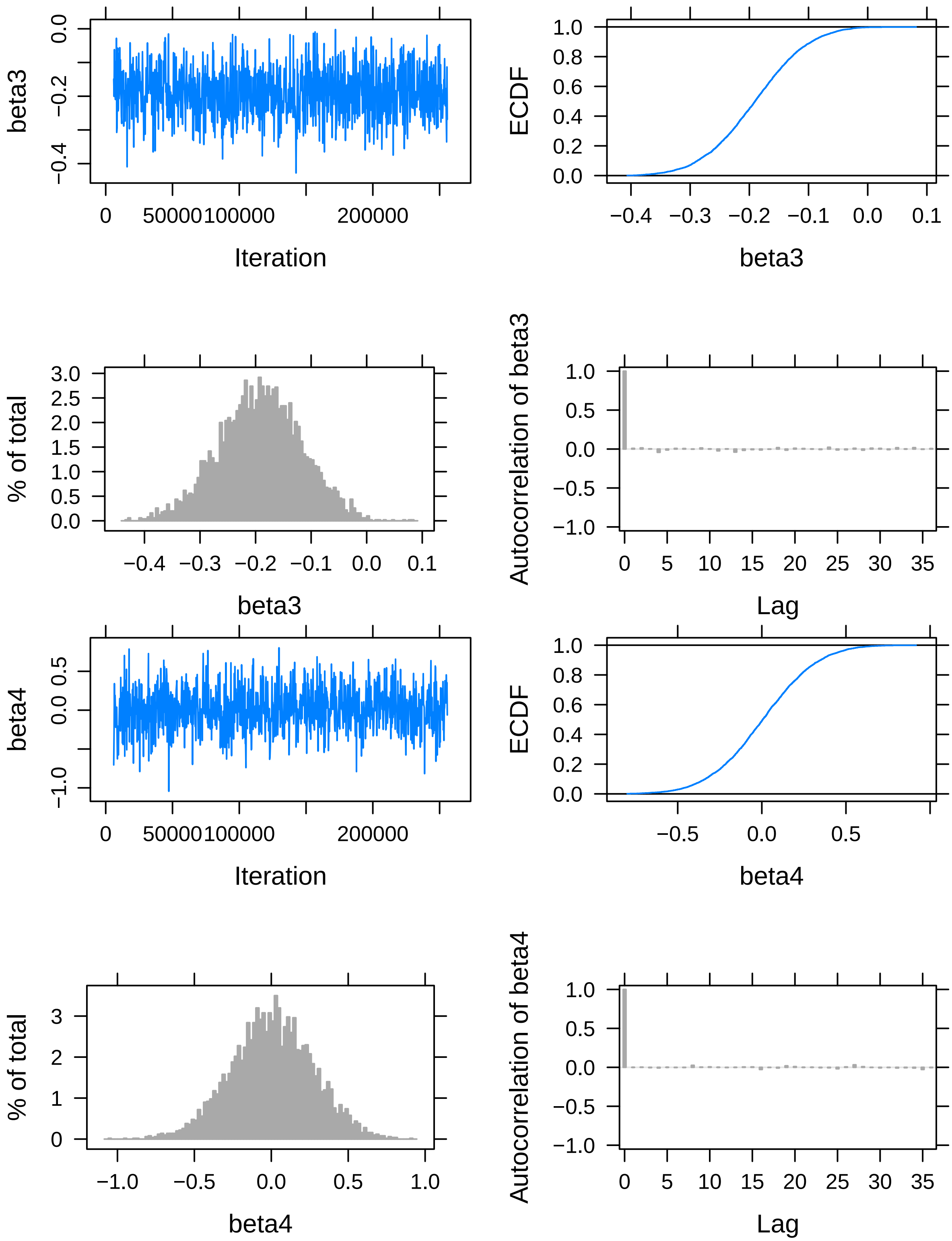
```
## beta1  0.3882419  0.428382748  0.46702186  0.428186927 0.02024773   NA
## beta2 -0.4324816 -0.227295748 -0.02303789 -0.226517253 0.10523080   NA
## beta3 -0.3306556 -0.190482478 -0.04140575 -0.191212993 0.07415444   NA
## beta4 -0.5206527  0.004840372  0.51719407  0.004623109 0.26616225   NA
## beta5 -0.0510838  0.162814921  0.40645797  0.162083182 0.11830143   NA
## beta6 -0.4595443  0.083013003  0.68580148  0.082703016 0.28996472   NA
## beta7 -0.2892273  0.040916578  0.39024470  0.042723529 0.17484430   NA
## sigma  0.6904107  0.721247329  0.75457051  0.721355983 0.01634697   NA
##              MCerr MC%ofSD SSeff        AC.500 psrf
## beta0 0.0058531144     2.7  1384  8.671215e-03   NA
## beta1 0.0005445194     2.7  1383  1.118156e-02   NA
## beta2 0.0014881883     1.4  5000  3.940920e-03   NA
## beta3 0.0010321668     1.4  5161  1.701377e-03   NA
## beta4 0.0037641027     1.4  5000  6.253260e-03   NA
## beta5 0.0017356791     1.5  4646  3.048365e-02   NA
## beta6 0.0041007204     1.4  5000  9.247818e-03   NA
## beta7 0.0024726717     1.4  5000  1.516543e-02   NA
## sigma 0.0002311811     1.4  5000 -1.238722e-05   NA
```

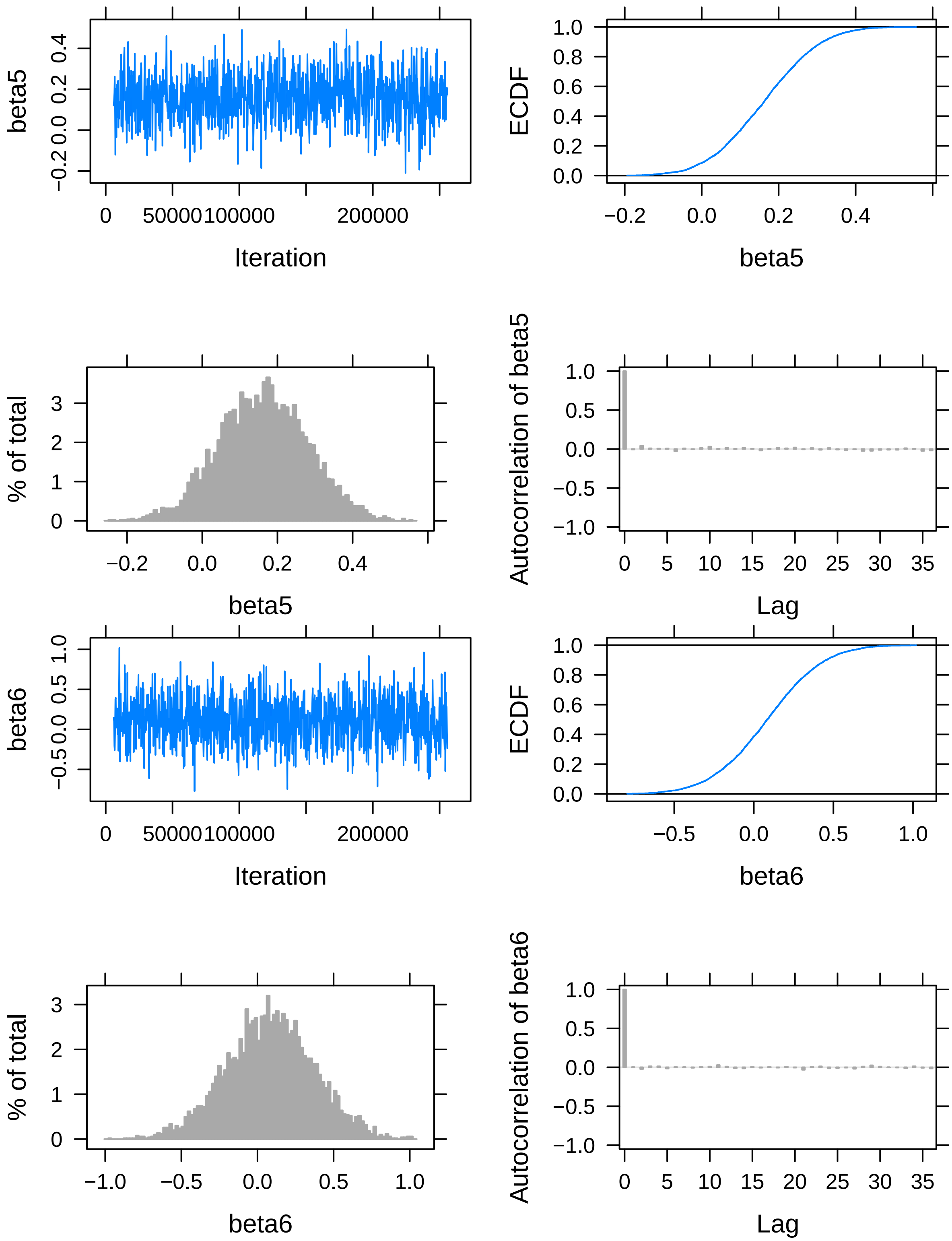
```r
plot(posterior_MLR)
```

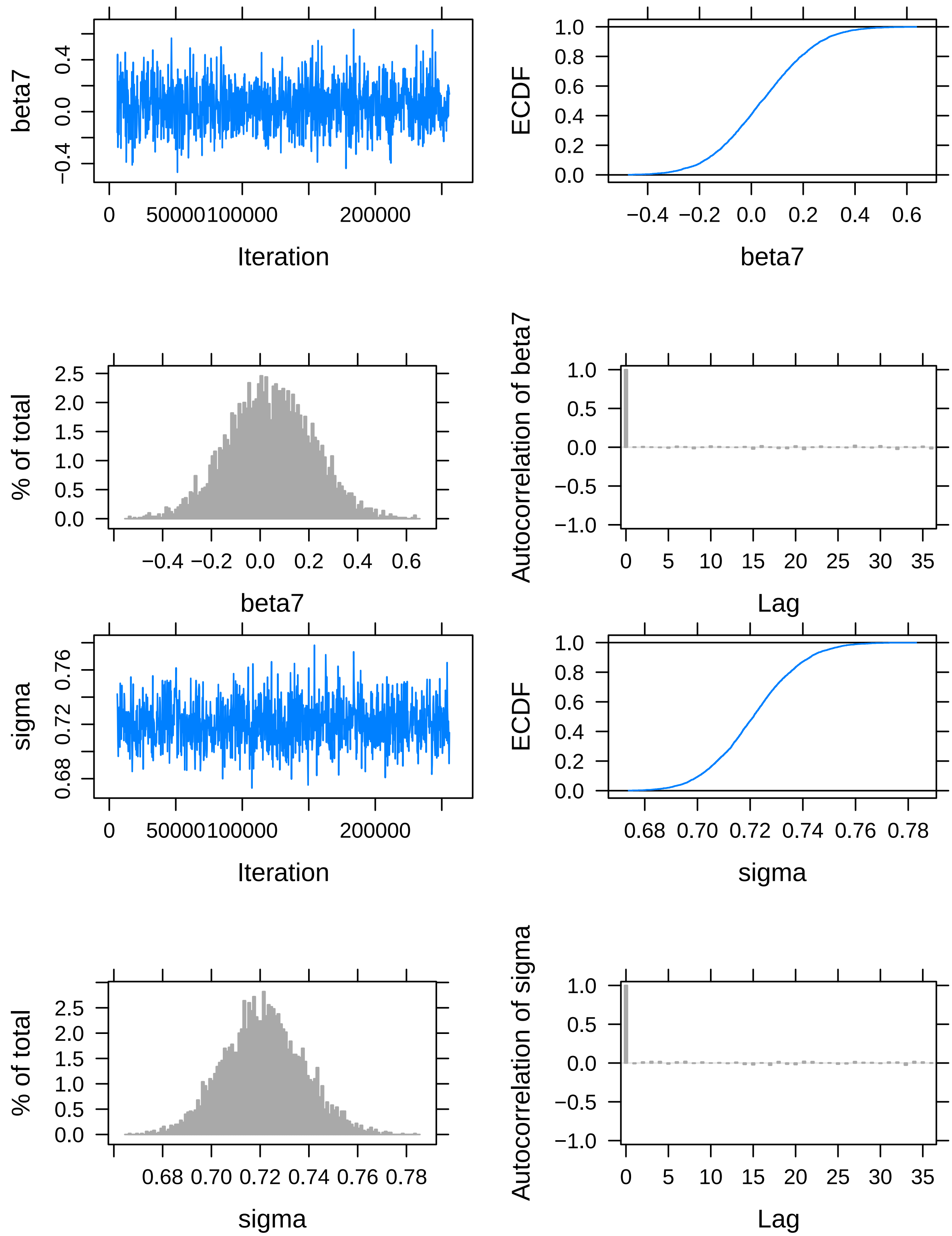
```
## Generating plots...
```

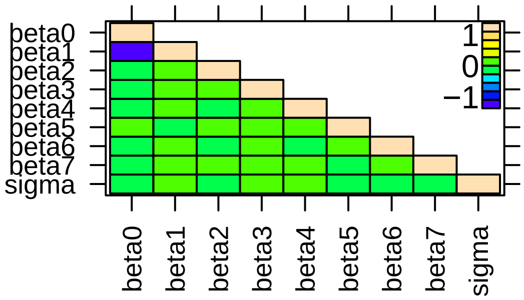



beta1
0.40 0.45 0.50
0 50000100000 200000
Iteration
ECDF
1.0 0.8 0.6 0.4 0.2 0.0
0.40 0.45 0.50
beta1
% of total
4 3 2 1 0
0.35 0.40 0.45 0.50
beta1
Autocorrelation of beta1
1.0 0.5 0.0 −0.5 −1.0
0 5 10 15 20 25 30 35
Lag
beta2
0.0 −0.4
0 50000100000 200000
Iteration
ECDF
1.0 0.8 0.6 0.4 0.2 0.0
−0.4 −0.2 0.0 0.2
beta2
% of total
4 3 2 1 0
−0.6 −0.4 −0.2 0.0 0.2
beta2
Autocorrelation of beta2
1.0 0.5 0.0 −0.5 −1.0
0 5 10 15 20 25 30 35
Lag


beta3
Iteration
0
50000100000
200000
0.0
−0.2
−0.4
ECDF
beta3
1.0
0.8
0.6
0.4
0.2
0.0
−0.4
−0.3
−0.2
−0.1
0.0
0.1
% of total
beta3
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Autocorrelation of beta3
Lag
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
30
35
beta4
Iteration
0.5
0.0
−1.0
ECDF
beta4
−0.5
0.0
0.5
% of total
beta4
3
2
1
0
−1.0
−0.5
0.0
0.5
1.0
Autocorrelation of beta4
Lag


beta5
Iteration
ECDF
beta5
% of total
beta5
Autocorrelation of beta5
Lag
beta6
Iteration
ECDF
beta6
% of total
beta6
Autocorrelation of beta6
Lag


beta7
Iteration
ECDF
beta7
% of total
beta7
Autocorrelation of beta7
Lag
sigma
Iteration
ECDF
sigma
% of total
sigma
Autocorrelation of sigma
Lag




```
post <- as.mcmc(posterior_MLR)
```

```
synthesize <- function(X, index, n){
  mean_Y <- post[index, "beta0"] + X$x_income * post[index, "beta1"] + X$x_rural * post[index, "beta2"]
  synthetic_Y <- rnorm(n,mean_Y, post[index,"sigma"])
  data.frame(X$x_income, synthetic_Y)
}
```

## Generating m = 20 synthetic values

```
n <- dim(data)[1]
m <- 20
synthetic_m <- vector("list",m)
params <- data.frame(x_income, x_rural, x_race_B, x_race_N, x_race_A, x_race_P, x_race_M)
for (l in 1:m){
  synthetic_one <- synthesize(params,4980+l,n)
  names(synthetic_one) <- c("OriginalIncome", "logIncome_syn")
  synthetic_m[[l]] <- synthetic_one
}
```

## Analysis-specific utility measures

```
orig_mean <- mean(synthetic_m[[l]]$OriginalIncome)
orig_median <- median(synthetic_m[[l]]$OriginalIncome)
orig_variance <- var(synthetic_m[[l]]$OriginalIncome)
print(lm(data$log_TotalExpSTD ~ synthetic_m[[l]]$OriginalIncome))
```

```
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$OriginalIncome)
##
## Coefficients:
##                     (Intercept)  synthetic_m[[l]]$OriginalIncome
##                          4.3219                           0.4211
```

```
mean <- c()
median <- c()
variance <- c()

print("=====================================")
```

```
## [1] "===================================="
```

```r
for (l in 1:m) {
  mean[l] = mean(synthetic_m[[l]]$logIncome_syn)
  variance[l] = var(synthetic_m[[l]]$logIncome_syn)
  median[l] = median(synthetic_m[[l]]$logIncome_syn)
  print(lm(data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn))

}
```

```
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                    (Intercept)  synthetic_m[[l]]$logIncome_syn
##                         6.0441                          0.3114
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                    (Intercept)  synthetic_m[[l]]$logIncome_syn
##                         6.0261                          0.3157
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                    (Intercept)  synthetic_m[[l]]$logIncome_syn
##                         6.1399                          0.3027
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                    (Intercept)  synthetic_m[[l]]$logIncome_syn
##                         5.8109                          0.3384
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                    (Intercept)  synthetic_m[[l]]$logIncome_syn
##                         5.7633                          0.3431
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
```

```
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.9170                          0.3256
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.6414                          0.3604
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.7851                          0.3397
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.6685                          0.3533
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        6.1405                          0.3015
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.7832                          0.3448
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                   (Intercept)  synthetic_m[[l]]$logIncome_syn
##                        5.9312                          0.3248
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
```

```
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       5.9897                          0.3182
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       5.8456                          0.3354
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       6.1835                          0.2956
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       5.6236                          0.3596
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       6.2848                          0.2834
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       6.1029                          0.3054
##
##
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                  (Intercept)  synthetic_m[[l]]$logIncome_syn
##                       5.9889                          0.3178
##
##
```

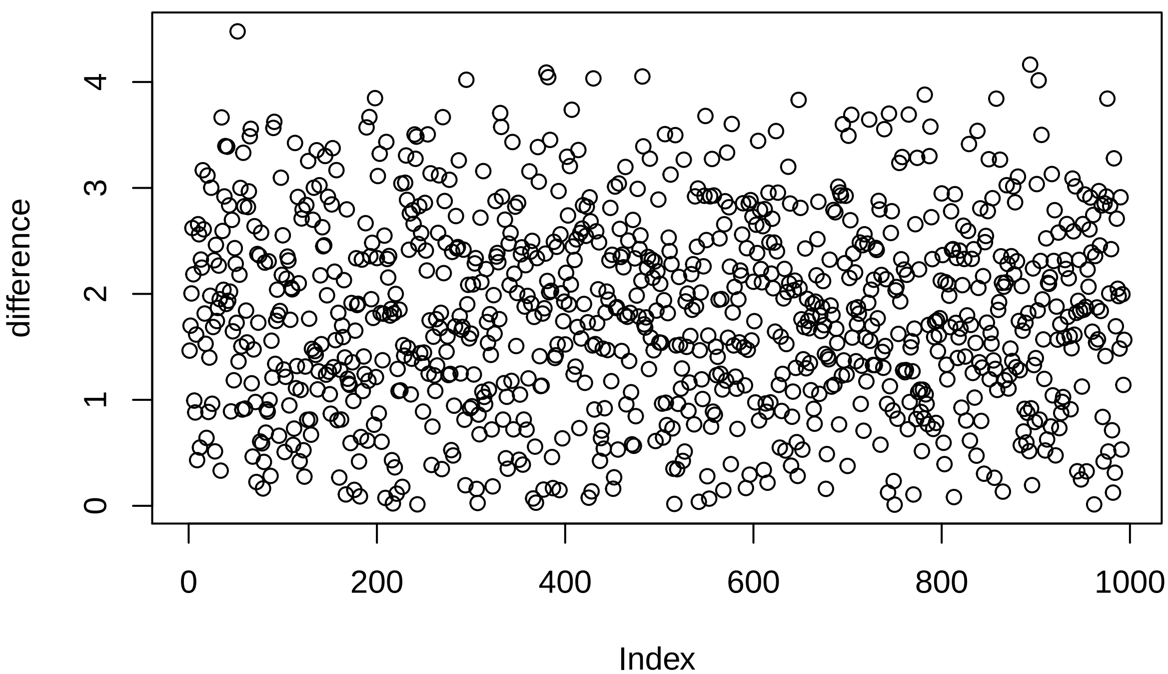
```
## Call:
## lm(formula = data$log_TotalExpSTD ~ synthetic_m[[l]]$logIncome_syn)
##
## Coefficients:
##                     (Intercept)  synthetic_m[[l]]$logIncome_syn
##                          6.0513                          0.3104
```

```
q_m <- sum(mean) / m
b_m <- sum((mean - q_m)^2 / (m-1))
u_m <- sum(variance) / m


L_s = q_m - (u_m^2) * 3
U_s = q_m + (u_m^2) * 3

L_o = orig_mean - (orig_variance^2) * 3
U_o = orig_mean + (orig_variance^2) * 3

L_i = max(L_s,L_o)
U_i = min(U_s,U_o)

I = ((U_i - L_i) / (2*(U_o - L_o))) + ((U_i - L_i)/ (2*(U_s - L_s)))
print(I)
```

```
## [1] 0.6822071
```

## My own identification disclosure risk measure

```
difference = abs(synthetic_one$OriginalIncome - synthetic_one$logIncome_syn)
plot(difference)
```



```r
sum(difference) / 994
```

```
## [1] 1.820398
```

```r
max_sep = 0.01
individual = 1

cluster_orig = 0
cluster_syn = 0

for(i in 1:993){
  if (abs(synthetic_one$logIncome_syn[i] - synthetic_one$logIncome_syn[individual]) < max_sep){
      cluster_syn = cluster_syn + 1
  }
  if (abs(synthetic_one$OriginalIncome[i] - synthetic_one$OriginalIncome[individual]) < max_sep){
      cluster_orig = cluster_orig + 1
  }
}
print(cluster_orig)
```

```
## [1] 9
```

```r
print(cluster_syn)
```

```
## [1] 2
```

## AR Calculation Example

```r
CEdata_org <- data[, 1:4]
CEdata_syn <- as.data.frame(cbind(CEdata_org[, "UrbanRural"],
                                  exp(synthetic_one
                                      [, "logIncome_syn"]),
                                  cbind(CEdata_org
                                        [, c("Race",
                                             "Expenditure")])))
names(CEdata_syn) <- c("UrbanRural", "Income",
                       "Race", "Expenditure")
```

```r
CEdata_org$LogIncome <- round(log(CEdata_org$Income),
                              digits = 1)
CEdata_org$LogExpenditure <- round(log(CEdata_org$Expenditure),
                                   digits = 1)
CEdata_syn$LogIncome <- round(log(CEdata_syn$Income),
                              digits = 1)
CEdata_syn$LogExpenditure <- round(log(CEdata_syn$Expenditure),
                                   digits = 1)
```

```r
H <- 50
beta0_draws <- post[1:H, "beta0"]
beta1_draws <- post[1:H, "beta1"]
sigma_draws <- post[1:H, "sigma"]
```

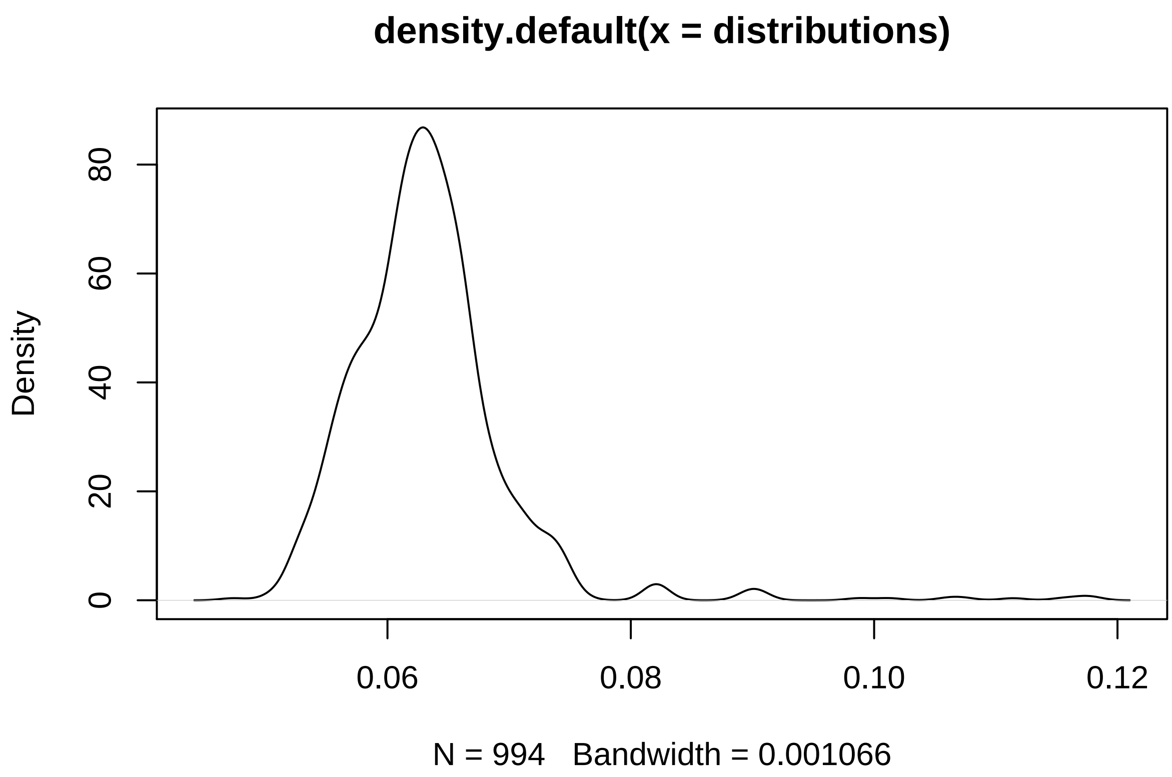
```r
compute_logsumexp <- function(log_vector){
  log_vector_max <- max(log_vector)
```

```
  exp_vector <- exp(log_vector - log_vector_max)
  sum_exp <- sum(exp_vector)
  log_sum_exp <- log(sum_exp) + log_vector_max
  return(log_sum_exp)
}
```

```
ARCalculation <- function(i){
  y_i <- CEdata_org$LogIncome[i]
  y_i_guesses <- seq((y_i - 2.5), (y_i + 2.5), 0.5)
  X_i <- CEdata_syn$LogExpenditure[i]
  G <- length(y_i_guesses)

  CU_i_logZ_all <- rep(NA, G)
  for (g in 1:G){
    q_sum_H <- sum((dnorm(y_i_guesses[g],
                          mean = (beta0_draws + beta1_draws * X_i),
                          sd = sigma_draws)) /
              (dnorm(y_i, mean = (beta0_draws + beta1_draws * X_i),
                     sd = sigma_draws)))
    log_pq_h_all <- rep(NA, H)
    for (h in 1:H){
      log_p_h <- sum(log(dnorm(CEdata_syn$LogIncome,
                               mean = (beta0_draws[h] + beta1_draws[h] *
                                         CEdata_syn$LogExpenditure),
                               sd = sigma_draws[h])))
      log_q_h <- log(((dnorm(y_i_guesses[g],
                             mean = (beta0_draws[h] + beta1_draws[h] * X_i),
                             sd = sigma_draws[h])) /
              (dnorm(y_i, mean = (beta0_draws[h] + beta1_draws[h] * X_i),
                     sd = sigma_draws[h]))) / q_sum_H)
      log_pq_h_all[h] <- log_p_h + log_q_h
    }
    CU_i_logZ_all[g] <- compute_logsumexp(log_pq_h_all)
  }

  prob <- exp(CU_i_logZ_all - max(CU_i_logZ_all)) /
    sum(exp(CU_i_logZ_all - max(CU_i_logZ_all)))
  outcome <- as.data.frame(cbind(y_i_guesses, prob))
  names(outcome) <- c("guess", "probability")

  return(outcome[outcome[order(outcome$probability, decreasing = TRUE), ]$guess == y_i,])
  }
```

```
distributions <- c(994)
for(i in 1:994){
  distributions[i] = ARCalculation(i)$probability
}
```

```
plot(density(distributions))
```


density.default(x = distributions)
Density
0
20
40
60
80
0.06
0.08
0.10
0.12
N = 994 Bandwidth = 0.001066