report

July 13, 2024

1 Assignment 1, Heorhii Lopatin

The assignment consists of 7 chapters, so the solutions to each of them will be presented separately. The answers to the concrete questions will be written as comments to the code.

We will start by importing nessecary libraries.

```
[770]: import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import plotly.express as px
import itertools
import scipy.stats as st
import statsmodels.api as sm

[771]: # !pip install seaborn
import seaborn as sns
```

1.1 Task 1

Load the data, look at it and summarise it in two-three sentences

```
[772]: df = pd.read_csv('dane_projekt1.csv', sep=';') df
```

[772]:	id	waga	wzrost	plec	dzieci	wiek	dochod	oszczednosci	jednoos	\
0	52	78.93	176.20	2	5	59	979.01	673.17	0	
1	101	78.66	165.73	1	3	52	1043.36	543.89	0	
2	146	74.29	171.33	1	2	57	1227.69	773.20	0	
3	281	79.11	169.24	1	1	59	2356.74	1914.74	1	
4	167	79.23	177.78	2	1	48	1264.95	536.29	1	
	•••	•••					•••	•••		
305	136	77.59	170.41	2	3	62	1124.12	415.07	0	
306	13	72.39	163.27	1	1	61	701.51	44.23	0	
307	309	81.61	173.70	1	2	63	9557.08	3844.10	1	
308	173	77.24	175.21	2	1	60	1182.43	710.35	0	
309	171	75.77	161.41	1	4	61	1258.84	663.98	0	

miejsce wydatki_zyw

```
0
             3
                      194.96
             2
                      259.20
1
2
             1
                      244.41
             2
3
                      239.77
4
             3
                      291.05
            3
305
                      289.83
306
             2
                      282.16
             3
307
                   -10668.84
308
             3
                      246.36
309
             3
                      276.53
```

[310 rows x 11 columns]

Let's look at the columns of the given dataset.

The following is provided as a description of the dataset:

- id observation identifier, does not contain any additional information
- waga weight of respondent (in kg)
- wzrost height of respondent (in cm)
- plec gender of respondent's ID (1 "female", 2 "male")
- dzieci number of dependent children of the respondent (in persons)
- wiek age of respondent (in years)
- dochod- declared income of the respondent in the examined month (in bythalers)
- oszczedności declared savings of the respondent in the examined month (in bythalers; negative values denote that the total expenditure exceeded the income)
- jednoos household status (1 "single-person household", 0 "multi-person household")
- miejsce size of the place where the respondent lives (1 "up to 10,000 inhabitants", 2 "from 10,000 inhabitants to 100,000 inhabitants", 3 "above 100,000 inhabitants")
- wydatki_zyw declared expenditure on food by the respondent in the examined month (in bythalers).

The id column will not be used, so it should be dropped immediately.

```
[773]: df.drop(["id"], axis=1, inplace=True)
```

The columns are of the following types:

```
[774]: categorical = ["plec", "jednoos", "miejsce"]

quantitive = list(set(df.columns) - set(categorical))
```

```
[775]: df.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 310 entries, 0 to 309
Data columns (total 10 columns):
Column Non-Null Count Dtype

0	waga	310 non-null	float64
1	wzrost	310 non-null	float64
2	plec	310 non-null	int64
3	dzieci	310 non-null	int64
4	wiek	310 non-null	int64
5	dochod	310 non-null	float64
6	oszczednosci	310 non-null	float64
7	jednoos	310 non-null	int64
8	miejsce	310 non-null	int64
9	wydatki_zyw	310 non-null	float64
• .	03 .04(5)		

dtypes: float64(5), int64(5)

memory usage: 24.3 KB

We can deduce that there are 310 observetions and the dataset doesn't contain any null values.

Plec, jednoos, miejsce columns are categorical, while the rest is quantitive. Dzieci can also be treated as a categorical column because in the given dataset the number of children is between 0 and 5, so we could say that having x children puts you in a specific category, but it is a quantitive variable at the same time.

Next we will look at the description of each column.

[776]: df.describe()

[110].	ui . uot	,01100()					
[776]:		waga	wzrost	pled	dzieci	wiek	\
	count	310.000000	310.000000	310.000000	310.000000	310.000000	
	mean	79.814742	172.726774	1.458065	2.003226	56.241935	
	std	3.292902	6.906733	0.499044	1.352209	6.746653	
	min	68.890000	155.670000	1.000000	0.000000	37.000000	
	25%	77.430000	167.090000	1.000000	1.000000	52.000000	
	50%	79.890000	172.815000	1.000000	2.000000	56.000000	
	75%	82.112500	178.032500	2.000000	3.000000	61.000000	
	max	86.730000	190.790000	2.000000	5.000000	77.000000	
		dochoo	d oszczedno	sci jed	lnoos mie	jsce wydatl	ki_zyw
	count	310.000000	310.000	0000 310.00	00000 310.000	0000 310.0	000000
	mean	1519.658000	945.994	839 0.42	2.00	3226 12.3	383097
	std	1293.46432	787.601	212 0.49	0.738	9537 3232.3	382607
	min	645.270000	-204.240	0.00	00000 1.000	0000 -55640.	190000
	25%	966.807500	392.545	0.00	00000 1.000	0000 199.	757500
	50%	1190.840000	741.190	0.00	00000 2.000	0000 249.5	265000
	75%	1620.460000	1270.222	2500 1.00	00000 3.000	0000 283.5	267500
	max	17412.240000	5485.220	0000 1.00	00000 3.000	301.	690000

A few things to note:

Average for gender is 1.46 which implies that among the respondents 46% were females and 54% were males.

Average age of a respondent was 56 years, which means we can be certain that senior citizens were asked more frequently, assuming no country in the world has such high age average and the minimum for age was 37.

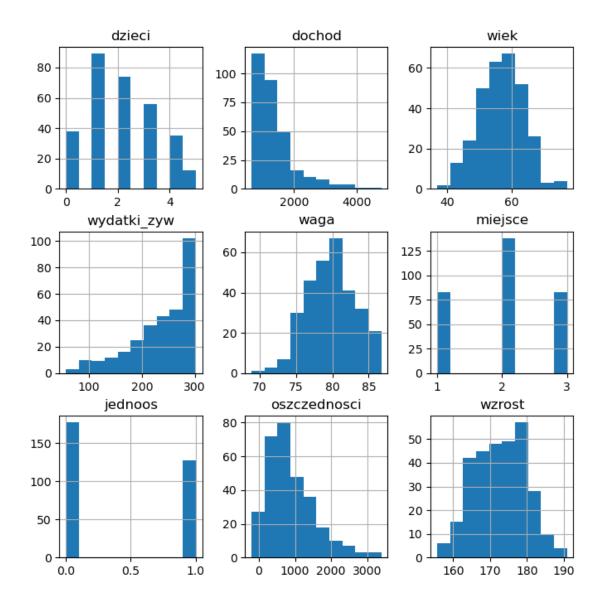
Minimum values for oszczedności and wydatki_zyw are negative. Negative savings mean that the expenses > income, while negative food expenses shouldn't apper in the dataset, hence we should remove those.

We'll look at how many people live in 1 person apartments with a certain amound of kids

```
[777]: | df = df[df["wydatki_zyw"] >= 0]
[778]: pd.crosstab(df["dzieci"], df["jednoos"], normalize=True,
                                                                margins=True)
[778]: jednoos
                      0
                                 1
                                        All
      dzieci
      0
               0.072368 0.052632
                                   0.125000
               0.174342 0.118421 0.292763
      1
      2
               0.128289 0.115132 0.243421
      3
               0.108553 0.075658 0.184211
      4
               0.065789 0.049342 0.115132
      5
               0.032895 0.006579 0.039474
      All
               0.582237
                         0.417763 1.000000
```

Interestingly enough, even with 3 or more kids a significant amount of adults live in single person households, which I find disturbing, to say the least.

Next we'll look at each column individually.



We can see that height, age and weight columns are visually somewhat close to normal distribution, which is something we could expect of a survey.

Food expenses and income columns are close to exponential distribution, while dzieci looks rather like a sample from Poisson distribution.

It should also be noted that more than a third of respondents expends about 300 on food. Perhaps it is related to the fact that most people spend the same % of their income on food, while most of respondents make 1000-2000.

1.2 Task 2

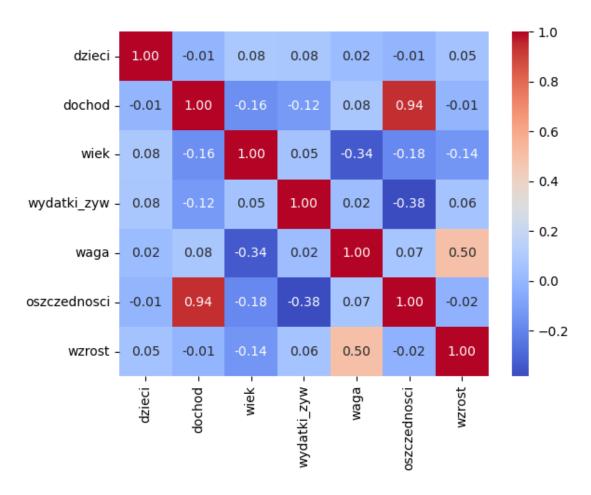
Check for correlations between variables. Calculate and present on a heatmap-type graph a reasonable correlation coefficient between quantitative variables, and examine the correlation of qualitative variables. Comment on the results, paying particular attention to issues of statistical significance

First, let's look at a correlation matrix between all variables.

```
[780]: numerical = list(
          set(df.columns)
          - set(
                categorical
          )
)

corr_matrix = df[numerical].corr()
sns.heatmap(corr_matrix, annot=True, cmap='coolwarm', fmt='.2f')
```

[780]: <Axes: >



We can see that weight, height and gender are correlated. This looks natural, as it biologically makes sense.

Note that age has negative correlation with almost every column. As it was previously noted, mostly elderly people were questioned, hence we can conclude that as people enter their 60s or 70s their weight, height, income and savings tend to decline.

Savings and income have a correlation coeffitient of 0.94. This implies that the amount a person is savig is directly impacted by how much they earn.

At the same time, savings are correlated with food expenses by -0.38, meaning that the more the person saves, the less they spend on food.

Next, we will look at correlation of categorical variables separately.

```
[781]: d = itertools.combinations(categorical, 2)
for a,b in d:
    contingency_table = pd.crosstab(df[a], df[b])
    chi2, p, dof, expected = st.chi2_contingency(contingency_table)
    print(f'{a} - {b}: p-value = {p}')
```

```
plec - jednoos: p-value = 0.7098649541109077
plec - miejsce: p-value = 0.22342423171950018
jednoos - miejsce: p-value = 0.7695667530897057
```

As we can see, the p-value is well above the significance level of 0.05(such will be the choice of the significance level for now), so we can't reject the null hypothesis, which in the context of chi2 contingency test states that the two variables are independent. In other words, the variables are not significantly correlated.

1.3 Task 3

Summarise the data with at least three different graphs

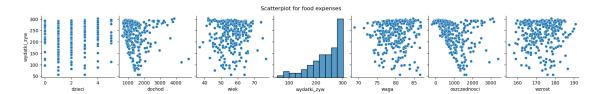
First we will look at a scatterplot for food expenses

```
[782]: ax = sns.pairplot(df, x_vars=quantitive, y_vars=["wydatki_zyw"])

ax.fig.suptitle("Scatterplot for food expenses", y=1.08)
```

```
/opt/homebrew/anaconda3/lib/python3.11/site-packages/seaborn/axisgrid.py:118:
UserWarning: The figure layout has changed to tight
  self._figure.tight_layout(*args, **kwargs)
```

```
[782]: Text(0.5, 1.08, 'Scatterplot for food expenses')
```



As it was mentioned before, nearly a third of respondants spends about 300 on food. It looks weird becouse their average income or weight seems to be the same. To find an explanation, let's first make some checks. We will look at the description of a group of people who spend less than average on food and those who spend more than average on food.

```
[783]: mean = df["wydatki_zyw"].mean()
       df_low_expenses = df[df["wydatki_zyw"] < mean][quantitive]</pre>
       df_high_expenses = df[df["wydatki_zyw"] >= mean][quantitive]
       df_low_expenses.drop(["wydatki_zyw"], axis=1, inplace=True)
       df_high_expenses.drop(["wydatki_zyw"], axis=1, inplace=True)
       print("Low expenses")
       print(df_low_expenses.describe())
       print("High expenses")
       print(df_high_expenses.describe())
      Low expenses
                  dzieci
                                dochod
                                                                 oszczednosci
                                              wiek
                                                           waga
              127.000000
                           127.000000
                                        127.000000
                                                     127.000000
                                                                    127.000000
      count
                          1426.469134
                1.881890
                                         55.543307
                                                      79.572047
                                                                   1112.777323
      mean
      std
                1.366342
                           616.415160
                                          7.024623
                                                       3.306505
                                                                    548.793985
      min
                0.000000
                           645.270000
                                         37.000000
                                                      72.330000
                                                                    232.550000
      25%
                1.000000
                          1015.450000
                                         51.000000
                                                      76.845000
                                                                    687.055000
      50%
                2.000000
                          1285.930000
                                         56.000000
                                                      79.830000
                                                                  1032.900000
      75%
                3.000000
                          1626.060000
                                         61.000000
                                                      81.725000
                                                                   1410.555000
                                                      86.570000
                5.000000
                          4780.150000
                                         77.000000
                                                                  3390.590000
      max
                  wzrost
      count
              127.000000
              171.996142
      mean
      std
                7.047841
      min
              155.670000
      25%
              166.605000
      50%
              171.590000
      75%
              177.460000
      max
              188.080000
      High expenses
                                dochod
                                              wiek
                                                                 oszczednosci
                  dzieci
                                                           waga
      count
              177.000000
                           177.000000
                                        177.000000
                                                     177.000000
                                                                    177.000000
                2.067797
                          1362.657627
                                         56.774011
                                                      79.996723
                                                                    726.677797
      mean
                1.321127
                           691.352353
                                          6.575160
                                                       3.299459
                                                                    701.226831
      std
      min
                0.000000
                           696.350000
                                         41.000000
                                                      68.890000
                                                                  -204.240000
      25%
                1.000000
                           913.270000
                                         53.000000
                                                      77.610000
                                                                    271.850000
      50%
                2.000000
                          1110.820000
                                                      79.900000
                                         57.000000
                                                                    496.020000
      75%
                3.000000
                          1535.140000
                                         61.000000
                                                      82.330000
                                                                    966.280000
                5.000000
                          3893.040000
                                         74.000000
                                                      86.730000
                                                                  3161.680000
      max
                  wzrost
      count
              177.000000
      mean
              173.386667
      std
                6.814054
      min
              159.040000
```

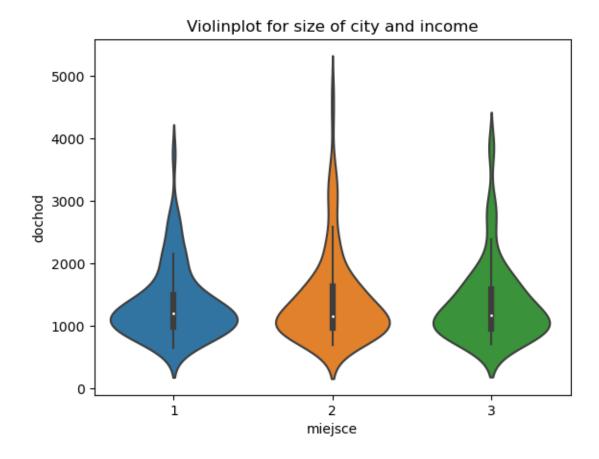
```
25% 167.820000
50% 173.770000
75% 178.720000
max 190.790000
```

Indeed, both groups have roughly the same average height, weight and age.

However the first group earns less and saves more. Later on in the analysis we will check for correlation between those variables. Perhaps conservative lifestyle is to blame for such division (e.g. we try not to spend a lot, so we don't spend a lot on food), or something else.

```
[784]: ax = sns.violinplot(data=df[df.dochod < 8000], y="dochod", x="miejsce") ax.set_title("Violinplot for size of city and income")
```

[784]: Text(0.5, 1.0, 'Violinplot for size of city and income')

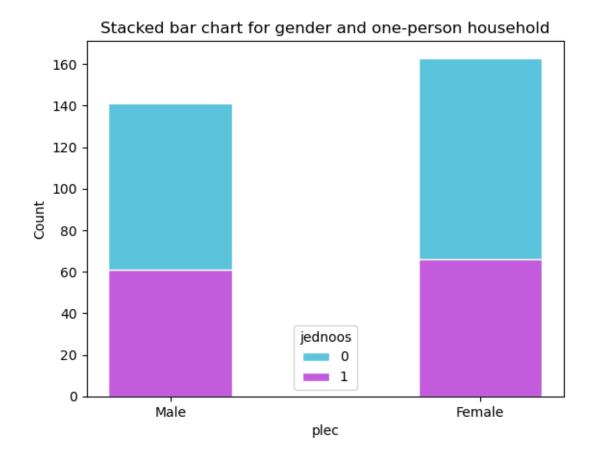


Note that the average for each group is the same, which is not something one would expect. Usually average income is biggest in large cities, as well as the gap between lower and upper class. However the plot shows that the most significant division is in the second group. Maybe farming is popular in bajtocja, or the data is not from 2024.

```
[785]: df_cp = df.copy()
df_cp["plec"] = df_cp["plec"].apply(lambda x: "Male" if x == 2 else "Female")

ax = sns.histplot(
    df_cp,
    x='plec',
    hue='jednoos',
    multiple='stack',
    palette=['#24b1d1', '#ae24d1'],
    edgecolor='white',
    shrink=0.4
)
ax.set_title("Stacked bar chart for gender and one-person household")
```

[785]: Text(0.5, 1.0, 'Stacked bar chart for gender and one-person household')

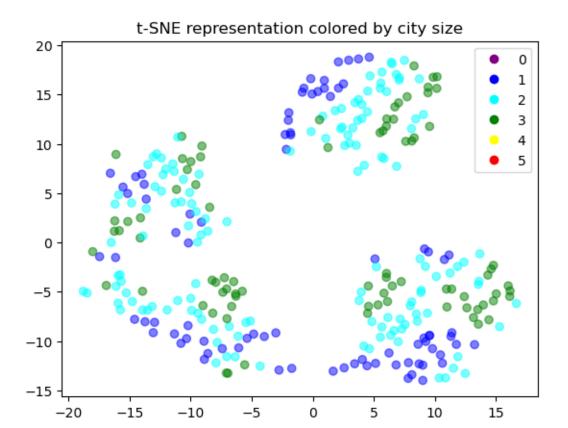


Percentage of people living in a one-person household is roughly the same for both genders, which is expected. It is surprising though that nearly 40% of respondents in both groups live alone.

Up next we will use a technique called t-distributed Stochastic Neighbor Embedding which helps represent multidimensional data on a plane preserving the distance between points.

```
[786]: from sklearn.manifold import TSNE
       from sklearn.preprocessing import StandardScaler
[787]: X = df.copy()
       scaler = StandardScaler()
       X_scaled = scaler.fit_transform(X)
[788]: \%\time
       tsne = TSNE(random_state=17)
       tsne_repr = tsne.fit_transform(X_scaled)
      CPU times: user 2.54 s, sys: 1.4 s, total: 3.94 s
      Wall time: 691 ms
[789]: colors = {0: "purple", 1: "blue", 2: "cyan", 3: "green", 4: "yellow", 5: "red"}
      markers = [plt.Line2D([0,0],[0,0],color=color, marker='o', linestyle='') for__
        ⇔color in colors.values()]
       plt.scatter(
           tsne_repr[:, 0],
           tsne_repr[:, 1],
           # c=df["jednoos"].map({0: "blue", 1: "orange"}),
           c=df["miejsce"].map(colors),
           alpha=0.5,
       plt.title("t-SNE representation colored by city size")
       plt.legend(markers, colors.keys(), numpoints=1)
```

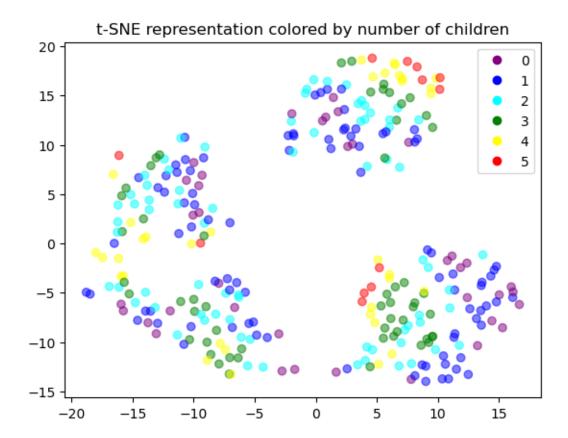
[789]: <matplotlib.legend.Legend at 0x32fec9a10>



Here we can see that the size of the city that the person lives in can be separated from other columns in the dataset.

```
[790]: colors = {0: "purple", 1: "blue", 2: "cyan", 3: "green", 4: "yellow", 5: "red"}
markers = [plt.Line2D([0,0],[0,0],color=color, marker='o', linestyle='') for_
color in colors.values()]
plt.scatter(
    tsne_repr[:, 0],
    tsne_repr[:, 1],
    # c=df["jednoos"].map({0: "blue", 1: "orange"}),
    c=df["dzieci"].map(colors),
    alpha=0.5,
)
plt.title("t-SNE representation colored by number of children")
plt.legend(markers, colors.keys(), numpoints=1)
```

[790]: <matplotlib.legend.Legend at 0x32fb85f50>



It is much harder, on the other hand, to separate respondents by number of children. This could also potentially mean that it would be hard to predict how many children a person has, based on other columns.

1.4 Task 4

Calculate two-sided confidence intervals at a confidence level of 1-=0.99 for the age variable for the following distribution parameters: - mean and standard deviation; - quartiles 1, 2 and 3.

Since the wiek variable follows normal distribution, we will use standart fromulas to calculate the confidence intervals.

```
[791]: wiek_mean = df["wiek"].mean()
    wiek_std = df["wiek"].std()

    confidence = 0.99
    alpha = 1 - confidence

[792]: z = st.norm.ppf(1 - alpha / 2)
    n = len(df["wiek"])
    eps = z * wiek_std / np.sqrt(n)
```

```
print("99% Confidence interval for age mean: ({:.8f}, {:.8f})".format(wiek_mean

→ eps, wiek_mean + eps))
```

99% Confidence interval for age mean: (55.25787117, 57.26186568)

```
[793]: wiek_mean = df["wiek"].mean()
wiek_std = df["wiek"].std()

n = len(df["wiek"]) - 1

t1 = st.chi2.ppf(1 - alpha / 2, df=n)
t2 = st.chi2.ppf(alpha / 2, df=n)
ns = n * (wiek_std ** 2)
v1 = np.sqrt(ns/t1)
v2 = np.sqrt(ns/t2)
print("99% Confidence interval for std: ({:.8f}, {:.8f})".format(v1, v2))
```

99% Confidence interval for std: (6.13639484, 7.56820760)

```
[794]: df["wiek"].describe()
```

```
[794]: count
                304.000000
                 56.259868
       mean
                  6.782444
       std
       min
                 37.000000
       25%
                 51.750000
       50%
                 56.500000
       75%
                 61.000000
                 77.000000
       max
       Name: wiek, dtype: float64
```

Since no formula for quantiles was provided on the labs, the one I found on the internet will be used.

```
print (f"Confidence interval for {q} quantile: ({wiek_mean - t1}, _{\sqcup} _{\hookrightarrow}{wiek_mean - t2})")
```

```
Confidence interval for 0.25 quantile: (50.67683231064825, 52.69352603166061)
Confidence interval for 0.5 quantile: (55.25152156054645, 57.26821528155881)
Confidence interval for 0.75 quantile: (59.82621081044465, 61.84290453145701)
```

To me the intervals seem to be authorised because we have more than 300 samples of the variable. In all of the cases, the actual paramater falls inside the confidence interval that we've calculated, which gives even more confidence. In each estimation we assume that the variable follows normal distribution, that samples are random and independent, and the size of the sample is large (300 should be more than enough), but is small relatively to the whole population.

1.5 Zadanie 5

Byteotian sociologists divide Byteotian society according to four wealth classes: - lower class (earned income below the 25th centile of the income distribution) - middle class (earned income equal to or above the 25th centile and below the 75th centile of the income distribution) - Upper middle class (attained income equal to or higher than the 75th percentile and lower than the 90th percentile of the income distribution) - Upper class (income attainment equal to or above the 90th percentile of the income distribution) Discuss and compare the variation in food expenditure across the above wealth classes

```
Lower Class food expenses: 258.6101315789473 +- 31.33304923956689
Middle Class food expenses: 231.22664473684208 +- 59.344930656526344
Upper middle Class food expenses: 201.49044444444448 +- 65.86981450888064
Upper Class food expenses: 249.86258064516124 +- 56.08326303914814
/var/folders/c5/l_7bjws53d7dxl4msb8qfgj80000gn/T/ipykernel_54998/1559054965.py:4
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
    df["klasa"] = df["dochod"].apply(lambda x: 1 if x < q25 else 2 if x < q75 else 3 if x < q90 else 4)
```

```
/var/folders/c5/1_7bjws53d7dx14msb8qfgj80000gn/T/ipykernel_54998/1559054965.py:1
0: SettingWithCopyWarning:
```

A value is trying to be set on a copy of a slice from a DataFrame

```
See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy df.drop(["klasa"], axis=1, inplace=True)
```

Surprisingly enough, food expenses are the highest in the lower wealth class. It is unexpected since higher wealth classes simply can afford to spend more on food. Although it's important to remember that the average age of a respondent is more than 50 years. Older people tend to go out less and spend most of their time home. It is possible that the average in the lower class is so high because food is something that contributes greatly to those people feeling happy.

1.6 Task 6

Answer the following research questions by conducting the best-fit statistical tests at a significance level of = 0, 01: - Are women characterised by higher savings values than men? - Is a lower proportion of food expenditure relative to income correlated with higher savings? - Is the average weight of women in the sample higher than 56 kg? and: - verify an additional (sensible) hypothesis of consistency with a specific parametric distribution for the selected variable (e.g. 'variable A has a Poisson distribution with parameter 1').

State the assumptions used and comment on whether they seem reasonable to you.

In order to answer the first question, conduct a Student's two sample t-test.

The null hypothesis is that means among both groups do not differ significantly, while the alternative hypothesis is the opposite.

```
[797]: m_savings = df[df["plec"] == 2]["oszczednosci"]
f_savings = df[df["plec"] == 1]["oszczednosci"]
t_stat, p_value = st.ttest_ind(f_savings, m_savings)
print(f"t-statistic: {t_stat}, p-value: {p_value}")
```

t-statistic: 0.4279557413869727, p-value: 0.6689883649475534

The p-value is above the chosen significance level, therefore we conclude that there is no significant difference in savings of men and women.

Next we will check for correlation between food expenditure relative to income and savings. We previously saw that there seems to be difference between people who spend less than average and more than average on food, so we expect that the correlation between the columns is significant. We will calculate person r value.

The null hypothesis is that there is no significant correlation and the alternative is, well, the opposite.

```
[798]: expenses_ratio = df["wydatki_zyw"] / df["dochod"] savings = df["oszczednosci"]
```

```
corr, p_value = st.pearsonr(expenses_ratio, savings)
print(f"Pearson correlation: {corr}, p-value: {p_value}")
```

Pearson correlation: -0.8882493471959096, p-value: 4.823720622546404e-104

The p value is less than the significance level, therefore we conclude that the null hypothesis is wrong, meaning the variables are strongly correlated.

The correlation number is -0.85 which tells us that the variables have strong negative correlation, meaning lower expenses mean greater savings. This means that the answer to the second question is positive.

To answer the third question we conduct a t-test once again, this time one-sided, however.

H_0 is that average female weight is less or equal to 56 kg, alternative is that it's greater.

```
[799]: f_weights = df[df["plec"] == 1]["waga"]
mu = 56

t_stat, p_value = st.ttest_1samp(f_weights, mu, alternative='greater')
print(f"t-statistic: {t_stat}, p-value: {p_value}")
```

t-statistic: 107.0286338796581, p-value: 1.5711163348632539e-152

The p-value is smaller than the the significane level, so the average women weight is certainly greater than 56 kg. This is unsurprising since mean weight is about 78 kg.

Lastly, let's check if the dzieci collumn follows poisson distribution.

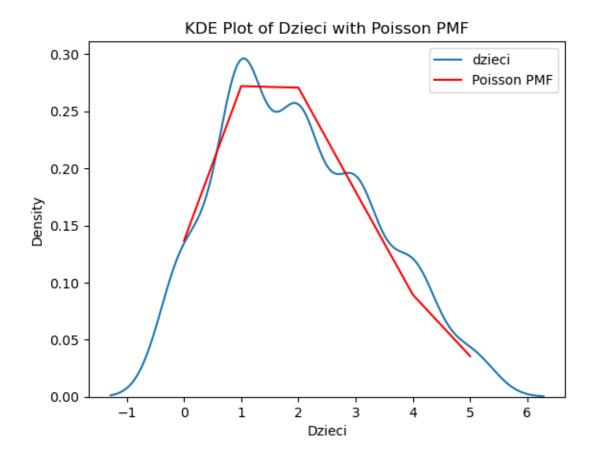
```
[800]: v = df["dzieci"]
print(f"Variance: {v.var()} Mean: {v.mean()}")
```

Variance: 1.7985821608476666 Mean: 1.9901315789473684

Variance and mean are relatively close, so it makes sense to check.

```
[801]: poisson_dist = st.poisson(v.mean())

x = np.arange(0, max(v) + 1)
sns.kdeplot(v, label='dzieci')
plt.plot(x, poisson_dist.pmf(x), 'r-', label='Poisson PMF')
plt.xlabel('Dzieci')
plt.ylabel('Density')
plt.title('KDE Plot of Dzieci with Poisson PMF')
plt.legend()
plt.show()
```



The plots look kinds the same too, so we will conduct a one sample Kolmogorov-Smirnov test, to determine if the data comes from poisson distribution. The significane level will remain at 0.01 and the zero hypothesis is that doesn't come from the poisson distribution.

```
[802]: st.kstest(v, 'poisson', args=(2, ))
```

[802]: KstestResult(statistic=0.28100584970983794, pvalue=1.0019448000405118e-21, statistic_location=1, statistic_sign=-1)

Clearly the p-value is smaller than the significance level and therefore we can conclude that the data comes from the poisson distribution with parameter 2.

1.7 Task 7

Run a test on the amount of food expenditure using variables from the base. Assume a significance level of = 0.01. To do this:

- Estimate an initial model containing all the variables from the original base (except id) and a constant, where the expenditure_zyw variable is the explanatory variable. Remember to decode the qualitative variables. (0.5pts)
- Comment on R2, tests of pooled and individual significance in the initial model. (1pt)

- Check that the preliminary model meets the assumptions of the Classical Linear Regression Model (KMRL). Pay particular attention to the issues of linearity of the functional form, homoskedasticity and lack of autocorrelation of the random component and the distribution of the random component. (2pts)
- Check whether there is a problem of imprecise collinearity (multicollinearity) in the initial model (0.5 points)
- Using an analysis of the outlier observations for the initial model, check whether the base contains errors. If you find suspicious observations, decide and justify what you do with them. (1pt)
- Improve the model so that it meets as many KMRL assumptions as possible. Describe the steps taken to obtain the 'best' model (4pkt). Hint: Consider different functional forms and transformations of the variables.
- Provide a quantitative interpretation of a selection of two individually significant coefficients in the 'best' model. Remember that a constant is not interpreted. Recommended choice of non-transformed variables. (1pt)
- What are the descriptive characteristics of individuals who are characterised by food expenditure belonging to the top 10% of food expenditure predictors in your 'best' model? Check and discuss (2pct).

```
[803]: data = pd.get_dummies(df, columns=categorical)
# data = df

X = data.drop(columns=['wydatki_zyw'])
y = data['wydatki_zyw']

X= sm.add_constant(X)
X = X.astype(float)
y = y.astype(float)
model = sm.OLS(y, X)
results = model.fit()

print(results.summary())
```

OLS Regression Results

						=======
Dep. Variable:		wydatki_zyw	R-square	ed:		0.679
Model:		OLS	Adj. R-s	squared:		0.668
Method:	I	east Squares	F-statis	stic:		61.90
Date:	Thu,	09 May 2024	Prob (F-	-statistic):		2.51e-66
Time:		23:19:14	Log-Like	elihood:	-1488.6	
No. Observations	No. Observations: 304 AIC:		AIC:			2999.
Df Residuals:		293	BIC:			3040.
Df Model:		10				
Covariance Type:		nonrobust				
	coef	std err	t	P> t	[0.025	0.975]
const	99.2794	39.923	2.487	0.013	20.707	177.852

waga	-0.4266	0.755	-0.565	0.572	-1.912	1.059
wzrost	-0.0961	0.410	-0.235	0.815	-0.903	0.710
dzieci	3.1627	1.433	2.207	0.028	0.342	5.983
wiek	-0.4059	0.307	-1.324	0.186	-1.009	0.197
dochod	0.1878	0.009	21.572	0.000	0.171	0.205
oszczednosci	-0.2120	0.009	-22.336	0.000	-0.231	-0.193
plec_1	46.2001	17.847	2.589	0.010	11.075	81.325
plec_2	53.0793	22.300	2.380	0.018	9.191	96.968
jednoos_0	46.0105	20.152	2.283	0.023	6.349	85.672
jednoos_1	53.2689	20.215	2.635	0.009	13.484	93.054
miejsce_1	34.7324	13.591	2.556	0.011	7.984	61.480
miejsce_2	29.3111	13.679	2.143	0.033	2.390	56.233
miejsce_3	35.2359	13.521	2.606	0.010	8.626	61.846
Omnibus:		223.700	Durbin-	======= Watson:		2.101
Prob(Omnibus):		0.000	Jarque-	Bera (JB):		3842.367
Skew:		-2.805	Prob(JB):		0.00
Kurtosis:		19.489	Cond. N	ο.		4.12e+19
==========				========	=======	

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 6.39e-31. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

R-squared value is 0.7 which means that 70% of wydatki_zyw variable can be explained thruogh the rest of the variables. In terms of individual variables, provided that the confidence level is 0.05, only a handful of variables contribute: const, dzieci, dochod, oszczedności, plec, miejsce.

Of those variables, the most significant ones seem to be dochod and oszcednosci, with t value over 20. We have already discussed why those columns are correlated, so this result could be expected. Number of children and the fact that a person lives alone also seem to contribute greatly. This makes sense because as a parent you have more people to take cared of, while living alone implies the opposite.

To check if the model satisfies the CLMR assumptions, they should first be at least listed:

- Linear trend,
- Independent residuals (lack of autocorrelation),
- Normally distributed residuals,
- Equal variance of residuals for all values of independent variables (homoscedasticity).

We will check them visually by creating and analyzing the following diagnostic plots:

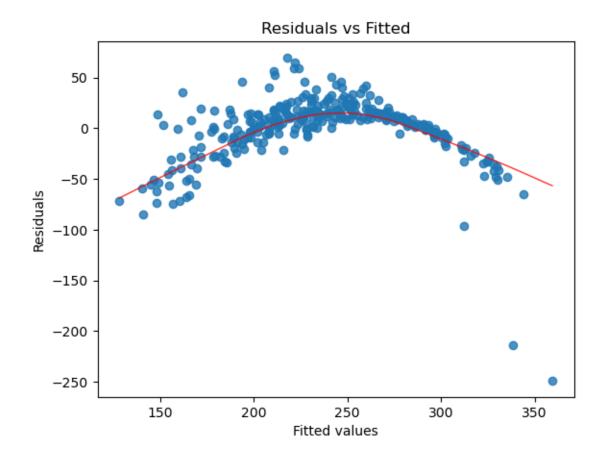
- The residual value vs the fitted value,
- The root square of the absolute value of standardized residuals vs the fitted value,
- The line plot of the residuals,
- Graphical analysis of the distribution of the residuals (histogram, boxplots, qq-plot).

(the text taken from exercises)

1.7.1 Linear functional form

```
[804]: # model values
       model_fitted_y = results.fittedvalues
       # model residuals
      model_residuals = results.resid
       # here we use matplotlib
       # with sns.residplot
       # we draw the scatterplot of residuals against the fitted values (scatter=True)
       # and we add a regression line
       plot_lm_1 = plt.figure()
       plot_lm_1.axes[0] = sns.regplot(x=model_fitted_y, y=model_residuals,
                                       scatter=True,
                                       ci=False,
                                       lowess=True,
                                       line_kws={'color': 'red', 'lw': 1, 'alpha': 0.
       ⇔8})
       plot_lm_1.axes[0].set_title('Residuals vs Fitted')
       plot_lm_1.axes[0].set_xlabel('Fitted values')
       plot_lm_1.axes[0].set_ylabel('Residuals')
```

```
[804]: Text(0, 0.5, 'Residuals')
```



As we can see, the trend is clearly not linear, therefore the linearity assumption is violated.

1.7.2 Independent residuals

```
[805]: indices = range(len(model_residuals))

# Plotting with Seaborn

plt.figure(figsize=(8, 6))

sns.lineplot(x=indices, y=model_residuals, linestyle='-')

plt.axhline(y=0, color='red', linestyle='--')

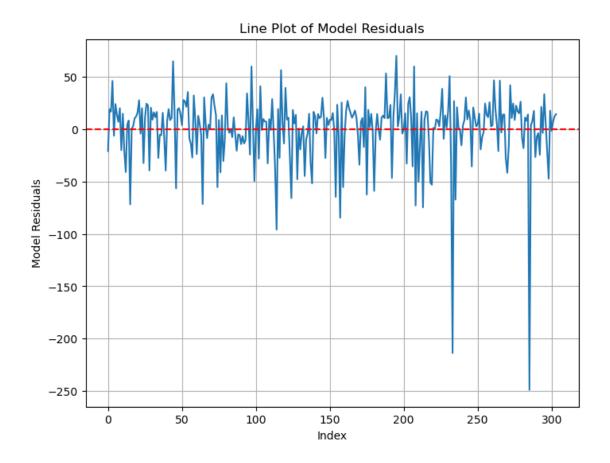
plt.title('Line Plot of Model Residuals')

plt.xlabel('Index')

plt.ylabel('Model Residuals')

plt.grid(True)

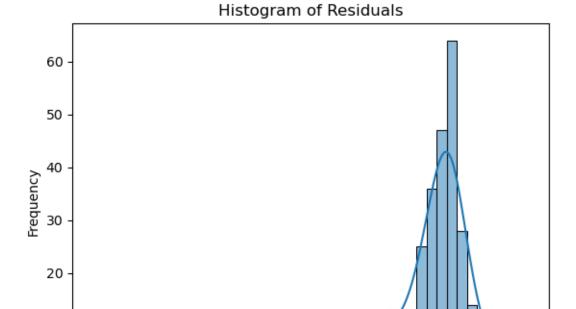
plt.show()
```



The residuals do seem to be independent, therefore the second assumption is not violated.

1.7.3 Normally distributed residuals

```
[806]: model_residuals = results.resid
sns.histplot(model_residuals, kde=True)
plt.title('Histogram of Residuals')
plt.xlabel('Residuals')
plt.ylabel('Frequency')
plt.show()
```



10

0

-250

-200

-150

There is a bump at -250, and the graph seems to be cutoff at 60. Even though it does remind normal distribution, it's not perfect. Therefore the assumption is "meh" - kinda holds but it's hard to say.

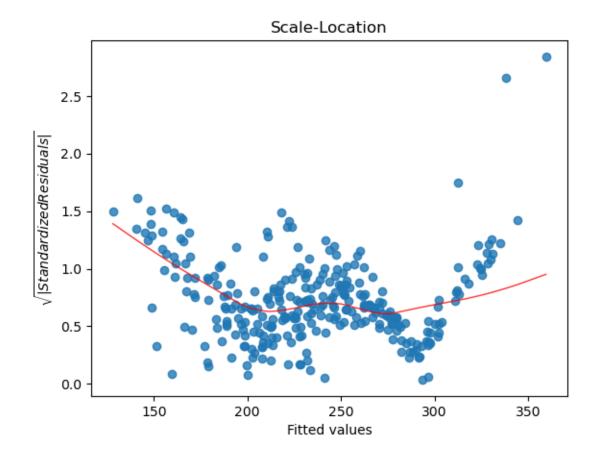
-100

Residuals

-50

0

[807]: Text(0, 0.5, '\$\\sqrt{|Standardized Residuals|}\$')



Clearly, the data is not homoscedastic, therefore the assumption is violated.

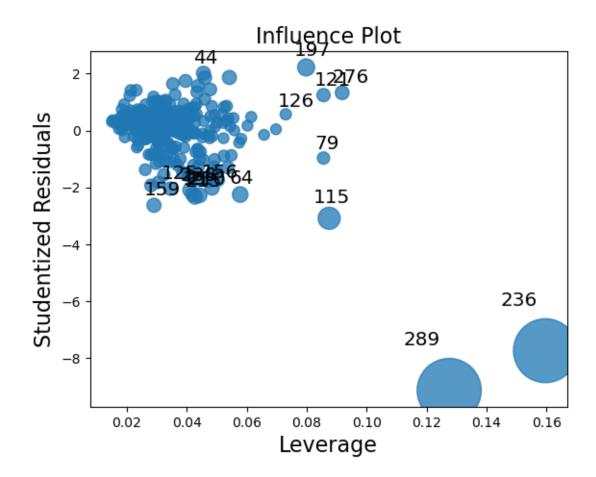
Next we will check if there is a problem with multicollinearity in the model and if there are influential observations.

print(vif_data) VIF feature 0 const 0.000000 1 1.731495 waga 2 wzrost 2.248774 3 dzieci 1.028401 4 wiek 1.203732 5 dochod 9.216374 6 oszczednosci 11.222733 7 plec_1 inf 8 plec_2 inf 9 jednoos_0 inf 10 jednoos_1 inf 11 miejsce_1 inf 12 miejsce_2 inf 13 miejsce_3 inf /opt/homebrew/anaconda3/lib/python3.11/sitepackages/statsmodels/regression/linear_model.py:1781: RuntimeWarning: divide by zero encountered in scalar divide return 1 - self.ssr/self.centered_tss /opt/homebrew/anaconda3/lib/python3.11/sitepackages/statsmodels/stats/outliers_influence.py:198: RuntimeWarning: divide by zero encountered in scalar divide vif = 1. / (1. - r_squared_i)

[809]: sm.graphics.influence_plot(results, criterion="cooks").show()

Since VIF is >10 for oszczedności, there are problems.

```
/var/folders/c5/l_7bjws53d7dxl4msb8qfgj80000gn/T/ipykernel_54998/3095779162.py:1
: UserWarning: Matplotlib is currently using
module://matplotlib_inline.backend_inline, which is a non-GUI backend, so cannot
show the figure.
sm.graphics.influence_plot(results, criterion="cooks").show()
```



The most influential observations are 289, 236 and 115. Judging by how far those observations are from the rest, it seems like those are errors. In this situation, the most sensible thing to do is to delete them.

```
[810]: df.drop([289,236, 115], inplace=True)
```

/var/folders/c5/1_7bjws53d7dxl4msb8qfgj80000gn/T/ipykernel_54998/4080861513.py:1
: SettingWithCopyWarning:

A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy df.drop([289,236, 115], inplace=True)

Now that we have removed the outliers, we can fit the model again and check the results, while also applying a transformation to the dependent variable.

```
[811]: from sklearn import preprocessing data = pd.get_dummies(df, columns=categorical)
```

```
X = data.drop(columns=['wydatki_zyw'])
y = data['wydatki_zyw']

X= sm.add_constant(X)
X = X.astype(float)
y = y.astype(float)
model = sm.OLS(y, X)
results = model.fit()

print(results.summary())
```

OLS Regression Results

______ Dep. Variable: wydatki_zyw R-squared: 0.850 Model: OLS Adj. R-squared: 0.845 Method: Least Squares F-statistic: 164.6 Thu, 09 May 2024 Prob (F-statistic): 2.98e-113 Date: Time: 23:19:15 Log-Likelihood: -1356.2No. Observations: 301 AIC: 2734. Df Residuals: 290 BIC: 2775.

Df Model: 10 Covariance Type: nonrobust

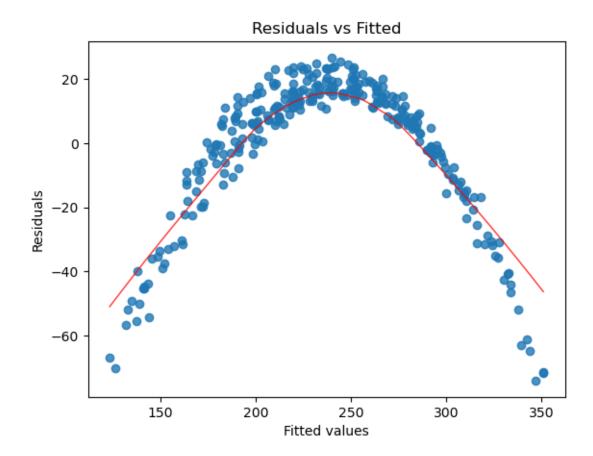
	coef	std err	t 	P> t	[0.025	0.975]
const	46.6787	27.187	1.717	0.087	-6.830	100.187
waga	-0.0965	0.512	-0.189	0.851	-1.103	0.910
wzrost	0.1219	0.279	0.437	0.662	-0.427	0.671
dzieci	1.0950	0.978	1.119	0.264	-0.830	3.020
wiek	-0.1838	0.209	-0.881	0.379	-0.594	0.227
dochod	0.2447	0.007	36.798	0.000	0.232	0.258
oszczednosci	-0.2433	0.007	-36.624	0.000	-0.256	-0.230
plec_1	22.0589	12.151	1.815	0.071	-1.857	45.975
plec_2	24.6199	15.188	1.621	0.106	-5.272	54.512
jednoos_0	27.0851	13.683	1.979	0.049	0.154	54.016
jednoos_1	19.5936	13.813	1.418	0.157	-7.594	46.781
miejsce_1	16.1978	9.259	1.749	0.081	-2.025	34.421
miejsce_2	14.7591	9.296	1.588	0.113	-3.537	33.055
miejsce_3	15.7218	9.218	1.706	0.089	-2.421	33.864
Omnibus:		73.074	 	======== Watson:		2.130
<pre>Prob(Omnibus):</pre>		0.000	Jarque-	Jarque-Bera (JB):		122.855
Skew:		-1.407	Prob(JB	Prob(JB):		2.10e-27
Kurtosis:		4.372	Cond. N	ο.		1.89e+19

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 2.81e-30. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

```
[812]: # model values
      model_fitted_y = results.fittedvalues
       # model residuals
       model_residuals = results.resid
       # here we use matplotlib
       # with sns.residplot
       # we draw the scatterplot of residuals against the fitted values (scatter=True)
       # and we add a regression line
       plot_lm_1 = plt.figure()
       plot_lm_1.axes[0] = sns.regplot(x=model_fitted_y, y=model_residuals,
                                       scatter=True,
                                       ci=False,
                                       lowess=True,
                                       line_kws={'color': 'red', 'lw': 1, 'alpha': 0.
       ⇔8})
       plot_lm_1.axes[0].set_title('Residuals vs Fitted')
       plot_lm_1.axes[0].set_xlabel('Fitted values')
       plot_lm_1.axes[0].set_ylabel('Residuals')
```

[812]: Text(0, 0.5, 'Residuals')



Residuals vs Fitted graph kinda reminds a parabola, so it seems natural to try to square some of the variables. Since coeffitients in dochod and oszedności are almost equal and have different, signs, it means that we should consider their difference as a separate variable Since dochod and oszczedności are the most important, we will try them.

Better, but it's not quite what we're looking for... At this point it's important to notice that the coeffitients reason food expenses are so related to income and savings is because food expenses are reletated to general expenses of a person. It might be beneficial to add an additional "expenses" column. After squaring it, this is what we get

```
[813]: from sklearn import preprocessing
  data = pd.get_dummies(df, columns=categorical)

X = data.drop(columns=['wydatki_zyw'])
# X["dochod"] = X["dochod"]**2
# X["oszczednosci"] = X["oszczednosci"]**2
X["wydatki"] = (X["dochod"] - X["oszczednosci"])**2

y = data['wydatki_zyw']

X= sm.add_constant(X)
```

```
X = X.astype(float)
y = y.astype(float)
model = sm.OLS(y, X)
results = model.fit()
print(results.summary())
```

OLS Regression Results								
Dep. Variable: Model: Method: Date: Time: No. Observation Df Residuals: Df Model: Covariance Type	Thu,	wydatki_zy OI .east Square 09 May 202 23:19:1 30 28 1 nonrobus	Adj. R- s F-stati 4 Prob (F 5 Log-Lik 1 AIC: 9 BIC:	-squared:		1.000 1.000 7.549e+04 0.00 -443.38 910.8 955.3		
	coef	std err	t	P> t	[0.025	0.975]		
const waga wzrost dzieci wiek dochod oszczednosci plec_1 plec_2 jednoos_0 jednoos_1 miejsce_1 miejsce_2 miejsce_3 wydatki	0.2849 0.3642 0.0112 1.5028 -0.2469 0.6983 -0.6983 0.2183 0.0666 0.2638 0.0211 0.0046 0.1272 0.1531 -0.0004	1.319 0.025 0.013 0.047 0.010 0.001 0.590 0.736 0.665 0.669 0.449 0.451 0.447	0.216 14.730 0.833 31.817 -24.519 526.280 -524.817 0.370 0.090 0.397 0.032 0.010 0.282 0.342 -352.367	0.829 0.000 0.405 0.000 0.000 0.000 0.712 0.928 0.692 0.975 0.992 0.778 0.732 0.000	-2.311 0.316 -0.015 1.410 -0.267 0.696 -0.701 -0.943 -1.383 -1.045 -1.296 -0.880 -0.760 -0.727 -0.000	2.881 0.413 0.038 1.596 -0.227 0.701 -0.696 1.379 1.516 1.572 1.338 0.889 1.014 1.033 -0.000		
Omnibus: 0.676 Prob(Omnibus): 0.713 Skew: 0.094 Kurtosis: 3.075			76 Durbin- .3 Jarque- 94 Prob(JE	-Watson: -Bera (JB): 3): Jo.		1.855 0.511 0.774 5.35e+21		

Notes:

^[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

^[2] The smallest eigenvalue is 1.36e-30. This might indicate that there are

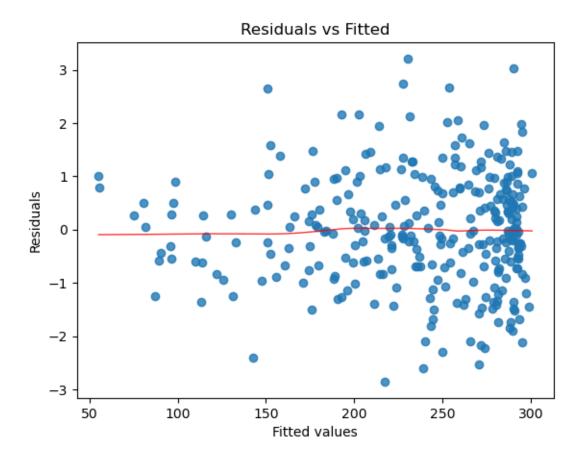
strong multicollinearity problems or that the design matrix is singular.

The R-squared value is now at 1.0, which is basically as good as it can get. We also see that waga, dzieci, wiek, dochod, oszczedności and wydatki have a p-value of 0, which implies that there some of the columns were more important than it was originally suspected.

The variable wydatki has a -0.0004 coeffitient, which means that the higher the expenses are, the lower the food expenses are. This is confirmed by some previous analysis on the wydatki_zyw variable, where it was shown that the more a person makes and the more saves, the smaller their food expenses are. At the same time, dochod and oszedności variables have the same coeffitients, proving that it is the delta that is important.

```
[814]: # model values
       model_fitted_y = results.fittedvalues
       # model residuals
       model_residuals = results.resid
       # here we use matplotlib
       # with sns.residplot
       # we draw the scatterplot of residuals against the fitted values (scatter=True)
       # and we add a regression line
       plot_lm_1 = plt.figure()
       plot_lm_1.axes[0] = sns.regplot(x=model_fitted_y, y=model_residuals,
                                       scatter=True,
                                       ci=False,
                                       lowess=True,
                                       line_kws={'color': 'red', 'lw': 1, 'alpha': 0.
        ⇔8})
       plot_lm_1.axes[0].set_title('Residuals vs Fitted')
       plot_lm_1.axes[0].set_xlabel('Fitted values')
       plot_lm_1.axes[0].set_ylabel('Residuals')
```

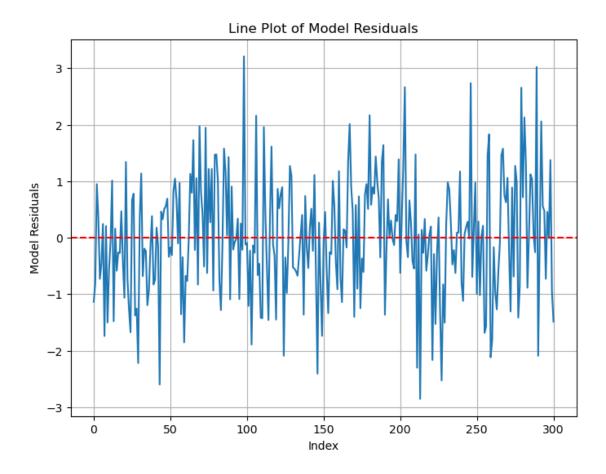
[814]: Text(0, 0.5, 'Residuals')



Linearity assumption holds

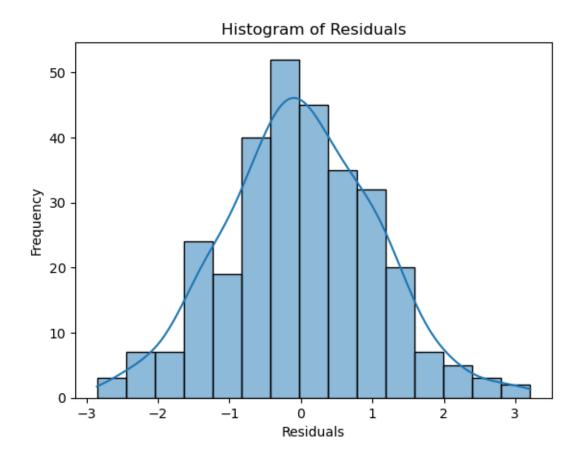
```
[815]: indices = range(len(model_residuals))

# Plotting with Seaborn
plt.figure(figsize=(8, 6))
sns.lineplot(x=indices, y=model_residuals, linestyle='-')
plt.axhline(y=0, color='red', linestyle='--')
plt.title('Line Plot of Model Residuals')
plt.xlabel('Index')
plt.ylabel('Model Residuals')
plt.grid(True)
plt.show()
```



The residuals are independent, the second assumption is fulfilled.

```
[816]: model_residuals = results.resid
    sns.histplot(model_residuals, kde=True)
    plt.title('Histogram of Residuals')
    plt.xlabel('Residuals')
    plt.ylabel('Frequency')
    plt.show()
```

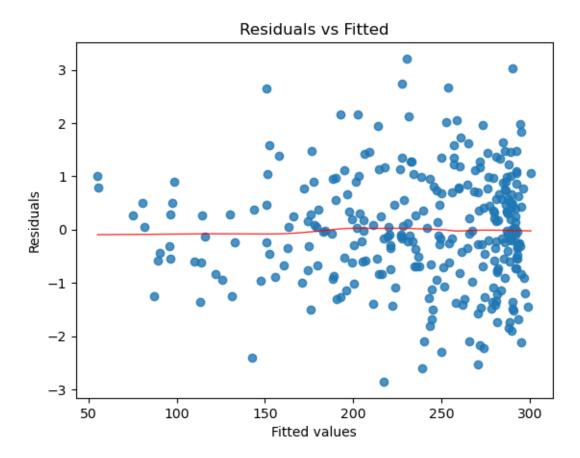


Residuals are normally distributed, the third assumption holds.

```
[817]: # model values
       model_fitted_y = results.fittedvalues
       # model residuals
       model_residuals = results.resid
       # here we use matplotlib
       # with sns.residplot
       # we draw the scatterplot of residuals against the fitted values (scatter=True)
       # and we add a regression line
       plot_lm_1 = plt.figure()
       plot_lm_1.axes[0] = sns.regplot(x=model_fitted_y, y=model_residuals,
                                        scatter=True,
                                        ci=False,
                                        lowess=True,
                                        line_kws={'color': 'red', 'lw': 1, 'alpha': 0.
        <del>4</del>8})
      plot_lm_1.axes[0].set_title('Residuals vs Fitted')
```

```
plot_lm_1.axes[0].set_xlabel('Fitted values')
plot_lm_1.axes[0].set_ylabel('Residuals')
```

[817]: Text(0, 0.5, 'Residuals')



The last assumption holds as well.

Lastly, we'll check for features of people who have the highest predicted wydatki_zyw values (top 10%)

```
[818]: model_fitted_y = results.fittedvalues
df["wydatki_zyw_pred"] = model_fitted_y

top10 = df[df["wydatki_zyw"] >= df["wydatki_zyw_pred"].quantile(0.9)]
df.drop(["wydatki_zyw_pred"], axis=1, inplace=True)
top10.drop(["wydatki_zyw_pred"], axis=1, inplace=True)
print("All food expenses:")
print(df.describe())
print("Top 10% of food expenses:")
print(top10.describe())
```

All food expenses: plec dzieci wiek waga wzrost 301.000000 301.000000 301.000000 301.000000 301.000000 count 79.809402 172.807608 1.465116 2.003322 56.259136 mean std 3.304593 6.960977 0.499612 1.340394 6.788664 min 68.890000 155.670000 1.000000 0.000000 37.000000 25% 77.400000 167.070000 1.000000 1.000000 51.000000 50% 79.900000 172.880000 1.000000 2.000000 57.000000 75% 82.140000 178.140000 2.000000 3.000000 61.000000 max86.730000 190.790000 2.000000 5.000000 77.000000 dochod oszczednosci jednoos miejsce wydatki_zyw 301.000000 301.000000 301.000000 301.000000 count 301.000000 1360.797276 867.673455 0.411960 2.000000 236.418771 mean std 596.720124 639.038726 0.493008 0.743864 56.686368 min 645.270000 -204.240000 0.000000 1.000000 56.270000 25% 965.440000 383.270000 0.000000 1.000000 204.340000 50% 1168.420000 714.850000 0.000000 2.000000 251.260000 75% 1215.550000 1594.150000 1.000000 3.000000 284.410000 3893.040000 3161.680000 1.000000 3.000000 301.690000 max Top 10% of food expenses: dzieci wiek dochod waga wzrost plec count 30.000000 30.000000 30.000000 30.000000 30.000000 30.000000 mean 81.275667 174.240667 1.633333 2.866667 53.066667 1535.883333 1.306043 std 2.282406 6.480367 0.490133 6.039715 975.916648 159.970000 1.000000 0.000000 753.680000 min 77.560000 42.000000 25% 80.187500 170.547500 1.000000 2.000000 48.000000 877.115000 50% 80.845000 174.615000 2.000000 3.000000 55.000000 1072.450000 75% 82.465000 179.430000 2.000000 4.000000 57.750000 2060.535000 86.060000 184.690000 2.000000 5.000000 65.000000 3893.040000 max oszczednosci jednoos miejsce wydatki_zyw 30.000000 30.000000 30.000000 30.000000 count 0.300000 761.765667 2.066667 294.076667 mean 984.462383 0.466092 std 0.784915 2.188081 min -95.950000 0.000000 1.000000 291.850000 25% 117.420000 0.000000 1.250000 292.607500 50% 325.600000 0.000000 2.000000 293.530000 75% 1351.660000 1.000000 3.000000 295.067500 3086.410000 1.000000 3.000000 301.690000 max

 $\label{lem:c5/l_7bjws53d7dxl4msb8qfgj80000gn/T/ipykernel_54998/325106759.py: 2: Setting With Copy Warning:$

A value is trying to be set on a copy of a slice from a DataFrame. Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy

df["wydatki_zyw_pred"] = model_fitted_y
/var/folders/c5/1_7bjws53d7dx14msb8qfgj80000gn/T/ipykernel_54998/325106759.py:5:
SettingWithCopyWarning:

A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy df.drop(["wydatki_zyw_pred"], axis=1, inplace=True)
/var/folders/c5/1_7bjws53d7dxl4msb8qfgj80000gn/T/ipykernel_54998/325106759.py:6:
SettingWithCopyWarning:

A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy top10.drop(["wydatki_zyw_pred"], axis=1, inplace=True)

Summing up, people with highest predicted food expenses: - more of them are males than on average (46% vs 63%) - they are on average 3 years younger - have 0.87 more kids on average - make about 190 more but save about 100 less - spend around 60 more on food on average