Introduction to computational statistics

LMS January 2026

* Descriptive statistics

* Foundations of probability

* Hypothesis testing

* Bayesian statistics

LMS statistics and hypothesis testing

Chapter 1. Descriptive statistics

* Prediction and inference
* Population and sampling
* True parameters and statistical estimators

Chapter 2. Foundations of probability

* Probability and random events
* Discrete probability; Bernoulli, Uniform, Binomial, Poisson
* Continuous probability; Gaussian, exponential, Uniform.

Chapter 3. Hypothesis testing (I)

* the law of large numbers
* the central limit theorem
* Confidence intervals and critical regions.

Chapter 4. Hypothesis testing (II)

* The Fisher, Pearson, Neyman approach.
* Some examples: t-test, F-test, χ^2-test
* Parametric vs non parametric tests
* Error types in hypothesis testing (*) Bayesian statistics.

2.1 Random events and probability

* Random events ("stochastic"; from στοχαστικός, Something whose output we don't know

* Probability : number $\in [0,1]$ quantifying certainty / "surprise".

  Example : tossing coins (H, T)

  $P(H) = 0 \rightarrow$ Certain I will never get H

  $P(H) = 1 \rightarrow$ " always get H

  $0 < P(H) < 1 \rightarrow \exists$ level of uncertainty / "surprise"

* Unitarity : the sum of probabilities for all possible outcomes x_i must add up to 1.

$$\sum_{\forall x_i} P(x_i) = 1$$

Example : tossing coins (H, T appear 1/2 times)

$$P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1 \checkmark$$

Example : rolling dice (1, 2, ..., 6 appear 1/6 times)

$$P(1) + P(2) + \ldots + P(6) = \frac{1}{6} + \frac{1}{6} + \ldots + \frac{1}{6} = 1 \checkmark$$

* Cardano, Bernoulli, Laplace (1500s, 1700s)
  First formulations of probability, games and chance

* Richard von Mises; "Probability, statistics and truth" 1928
  Frequentist definition of probability

* Andrey Kolmogorov; "Foundations of Probability" 1933
  Prob. as positive number, unitarity

* Claude Shannon; "Mathematical theory of communication" 1948
  Probability, information as surprise

2.2 Discrete probability distributions

* Discrete: number of possible outcomes is a finite number. $\begin{cases} \text{coins} \\ \text{dice} \\ \text{counting} \end{cases}$

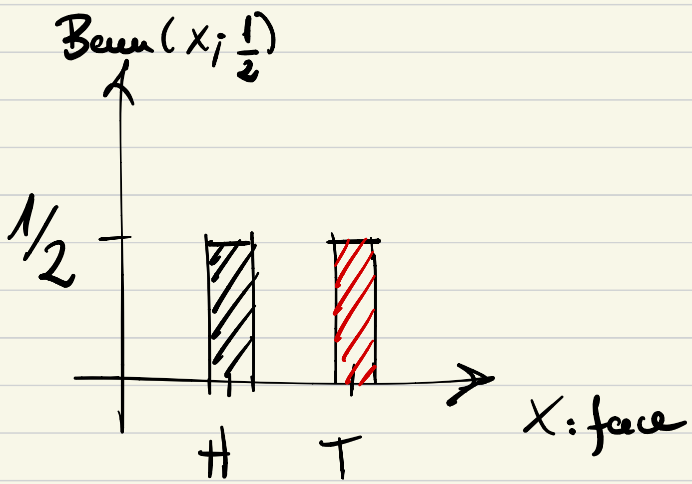
i) Bernoulli distribution

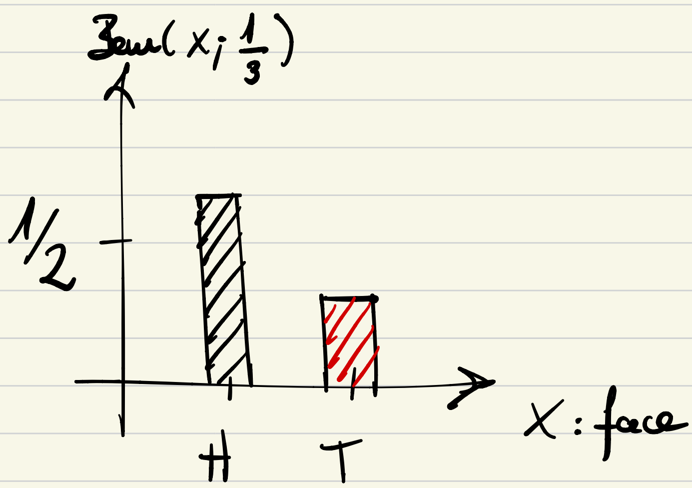
"A random variable x follows a Bernoulli distribution if $\exists$ 2 possible outcomes, with prob. p for success, $1-p$ for failure"

$$\text{Bern}(x;p) = \begin{cases} p & \text{if } x = \text{success} \\ 1-p & \text{if } x = \text{failure} \end{cases}$$

Example: tossing fair coin ($X = H/T$; $p = 1/2$)

Example: tossing biased coin ($X = H/T$; $p = 1/3$)





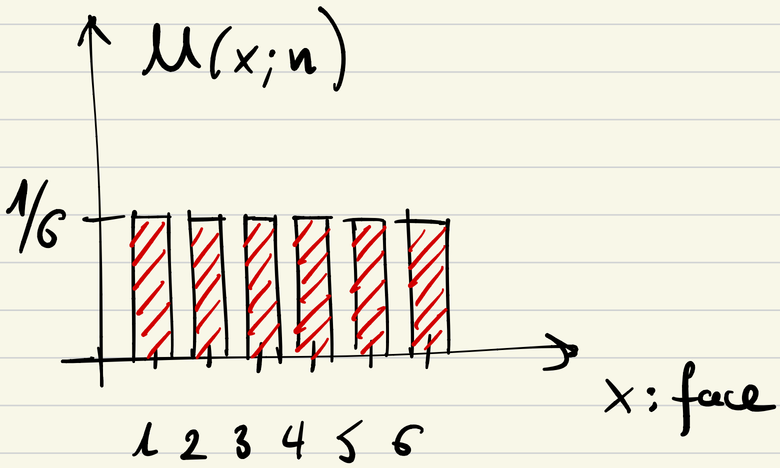
* Bernoulli; "Ars conjectandi" 1713 ("the art of conjecturing")

i) Uniform distribution

"A random variable x follows a uniform distribution if $\forall$ possible outcomes $(x_1, \dots, x_n)$ $P(x_i) = \frac{1}{n}$"

$$\mathcal{U}(x; n) = \frac{1}{n} \quad ; \quad \forall x_i$$

Example : rolling fair dice ($x \in (1, 2, \dots, 6)$; $p = 1/6$)



* Cardano ; "Liber de ludo aleae" 1663 (Book on games of chance)

i) Binomial distribution

"Probability of observing x successes in n attempts, if the prob. of each individual success is p

$$\left\{ \underline{B(x; n, p) = \binom{n}{x} p^{x} (1-p)^{n-x}} \right\}$$

x : number of successes
n : number of trials
p : probability each success

Example: Probability of 5 times H tossing 10 times a coin

$$B\left(5; 10, \frac{1}{2}\right) = \binom{10}{5}\left(\frac{1}{2}\right)^{5}\left(1-\frac{1}{2}\right)^{(10-5)} = 0.246$$

Exercise

Example: Probability of 3 times a 6 in 10 dice

$$B\left(3; 10, \frac{1}{6}\right) = \binom{10}{3}\left(\frac{1}{6}\right)^{3}\left(1-\frac{1}{6}\right)^{(10-3)} = 0.155$$

Exercise

Example: Probability of passing a test (A, B, C) answering randomly.

$$B\left(5; 10, \frac{1}{3}\right) = \binom{10}{5}\left(\frac{1}{3}\right)^{5}\left(1-\frac{1}{3}\right)^{(10-5)} = 0.137$$

Exercise

$*$ What happens in the case $x = n$?

Example : probability of tossing 10 Heads out of 10

$$B(10; 10, \tfrac{1}{2}) = \cancel{\binom{10}{10}}^{1} \left(\frac{1}{2}\right)^{10} \left(\cancel{1-\frac{1}{2}}\right)^{0 \to 1}$$

$$= 1 \cdot \left(\frac{1}{2}\right)^{10} \cdot 1 = \left(\frac{1}{2}\right)^{10} \checkmark$$

$*$ General : recover the individual probability p, raised to the number of attempts n.

$$B(n; n, p) = \underbrace{\binom{n}{n}}_{1} p^n \underbrace{(1-p)^0}_{1} = \underline{p^n}$$

* What if I ask the probability of less/more than some x ?

i) Prob of obtaining 5 or less times H in 10 coin tosses

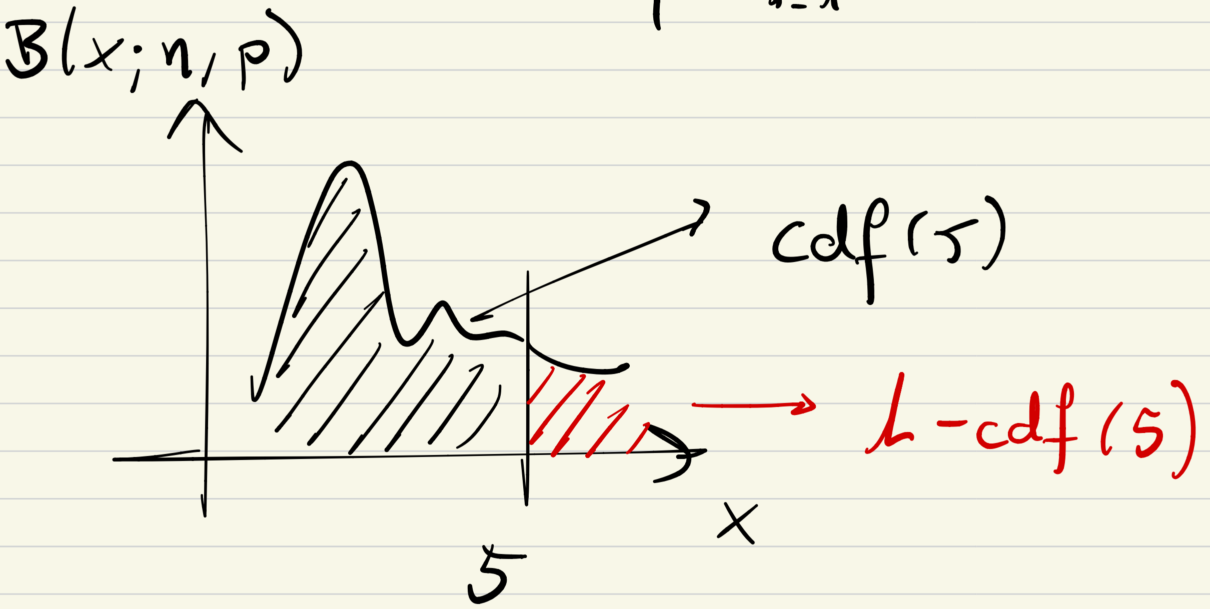
$$B\left(x\le 5; 10, \tfrac{1}{2}\right) = B\left(0;10,\tfrac{1}{2}\right)+B\left(1;10,\tfrac{1}{2}\right)+\dots+B\left(5;10,\tfrac{1}{2}\right)$$

$$= \underbrace{\sum_{x=1}^{5} B\left(x;10,\tfrac{1}{2}\right)}_{\text{Cumulative distribution function cdf.}} = cdf(5)$$

ii) Prob. of obtaining more than 5 H in 10 coin tosses

$$B\left(x>5;10,\tfrac{1}{2}\right) = 1 - B\left(x\le 5;10,\tfrac{1}{2}\right) = 1-cdf(5)$$

Unitarity $\sum_{i=1}^{n} P(x_i) = 1$



iii) Poisson distribution

"Probability of observing x events in a time interval, given an observed historical average λ"

$$\left\{ P(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \right.$$

x : number of times I observe

λ : observed average.

Example : Probability of observing 3 cancer patients, with $\lambda = 5$.

$$P(3;5) = \frac{e^{-5}\,5^{3}}{3!} = 0.140$$

(*) Exercise

Example : Probability of observing 5 or less patients in same λ.

$$P(x \leq 5; 5) = P(1) + P(2) + \ldots + P(5)$$

$$= \underbrace{\sum_{x=1}^{5} P(x;\lambda=5)}_{\text{Cumulative distribution function cdf}} = 0.616$$

(*) Exercise

Example: Probability of more than 5 patients

$$P(x>5, \lambda=5) \underset{\uparrow \text{ linearity}}{=} 1 - cdf(5) = 0.384$$

(*) Exercise.

2.3) Continuous distributions

* Continuous : amount of possible outcomes is infinite / uncountable

Coin : 2 outcomes $(H, T) \rightarrow p = \frac{1}{2}$

Dice : 6 outcomes $(1,2,3,4,5,6) \rightarrow p = \frac{1}{6}$

$\left.\begin{array}{l}\text{Discrete cases "Mass dist."} \\ \text{Frequentist definition ✓}\end{array}\right.$

Continuous variable (T, h, conc.) : ∞ outcomes $\longrightarrow p = \frac{1}{\infty} = 0$ WTF

↳ Frequentist approach does not work

Need a new mathematical object "Density distribution"

i) Discrete case

Probability $P \in [0, 1]$

Unitarity $\sum_{\forall x_i} P(x_i) = 1$

ii) Continuous case

Density $f(x)$

Unitarity $\int_{-\infty}^{\infty} dx\, f(x) = 1$

* Gaussian

Summary ; Foundations of probability

i) Discrete case : A random variable can have a countable / discrete number of outcomes x_i

Probability $P \in [0,1]$

Unitarity $\sum_{\forall x_i} P(x_i) = 1$

* Bernoulli events $\mathrm{Bern}(x_i, p)$
* Uniform events $U(x_i, n)$
* Binomial events $B(x_i ; n, p)$
* Poisson events $P(x_i ; \lambda)$

ii) Continuous case : A random variable can take any value in a continuous range $x \in \mathbb{R}$

Density $f(x)$

Unitarity $\int_{-\infty}^{\infty} f(x)\, dx = 1$

* Gaussian distribution
* Exponential distribution
* Continuous uniform