

CODING PROJECT REPORT

IMPERIAL COLLEGE LONDON

EARTH SCIENCE AND ENGINEERING

Fluid Flow in Porous Media

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1 Introduction:

The interaction between fluids and porous media is a complex matter, choreographed by the competing forces of capillarity, gravity, and viscosity. This interaction is not only central to the earth's subsurface processes but also pivotal in industrial applications, particularly in the management of hydrocarbon reservoirs and gas storage sites, as well as carbon sequestration and hydrogen storage and usage. The present report examines three distinct yet interrelated scenarios of fluid displacement in porous media, each with its unique set of conditions and implications for subsurface engineering.

2 The Cases:

In this report we will be working with 3 cases:

- The first case explores the primary drainage process where carbon dioxide (CO₂), a non-wetting phase, is injected into an aquifer, displacing the wetting phase without any spontaneous imbibition. This scenario is emblematic of CO₂ sequestration efforts, where understanding the advective flow and percolation-like advance at the pore scale is vital for predicting the migration and ultimate fate of CO₂ in subsurface environments.
- The second case shifts focus to the displacement of hydrogen, a non-wetting phase, by water in a storage site, epitomizing secondary imbibition. The withdrawal of hydrogen from storage necessitates a comprehensive study of water encroachment, which can lead to hydrogen trapping. This case is crucial for energy storage applications and involves both advective flow and spontaneous imbibition, requiring a nuanced understanding of percolation dynamics and fluid trapping mechanisms.
- The third case presents a mixed-wet system, where water displaces CO₂ in a depleted oil field. The intricate interaction between water, CO₂, and residual oil presents a complex scenario of mixed imbibition and drainage processes. This situation is particularly relevant to enhanced oil recovery (EOR) operations and carbon capture and storage (CCS) strategies. The mixed-wet nature of the system necessitates a detailed examination of both advective flow and spontaneous imbibition to optimize fluid displacement and maximize recovery.

3 Advective Flow:

In this study, we investigate fluid displacement in porous media by examining saturation profiles, relative permeabilities, capillary pressures, and their impact on fractional flow. Our analysis is underpinned by established models for relative permeability and capillary pressure, which are functions of saturation and described by the following expressions:

- Effective saturation S_e is defined as:

$$S_e = \frac{S_1 - S_{1i}}{1 - S_{2r} - S_{1i}} \quad (1)$$

where S_1 is the saturation of phase 1, S_{1i} is the irreducible saturation of phase 1, and S_{2r} is the residual saturation of phase 2.

- Relative permeabilities for phases 1 and 2, k_{r1} and k_{r2} , are modeled as:

$$k_{r1} = k_{r1}^{max} S_e^a \quad (2)$$

$$k_{r2} = k_{r2}^{max} (1 - S_e)^b \quad (3)$$

where a and b are fitting parameters that reflect the flow characteristics in the porous medium.

3.1 Procedure

In order to calculate **analytically** the value of vd in regards to the fluid's saturation, we discretize the saturation range from S_{1i} to $1 - S_{2r}$ into 10,000 points to construct a high-resolution saturation profile. For each point, we calculate the relative permeabilities and capillary pressure using the equations shown above, which lead to the computation of fractional flow f_1 and finally the dimensionless velocity v_D .

As for the **numerical** method, here is the procedure we will be following: The advection equation, which models the transport of a substance within a flow field, is given by:

$$\frac{\partial S_1}{\partial t_D} + \frac{df_1}{dS_1} \frac{\partial S_1}{\partial x_D} = 0 \quad (4)$$

To solve this equation numerically, we employ the finite difference method, discretizing the domain into a grid with time steps $\Delta t_D = 0.001$ and spatial steps $\Delta x_D = 0.01$. The derivatives are approximated as follows:

The centered difference is selected for its higher accuracy, capturing changes in both directions around a point. Substituting these into the advection equation and rearranging for $S_{n+1,j}$, we obtain an explicit scheme for the next time step:

$$S_{n+1,j} = S_{n,j} - \frac{\Delta t_D}{2\Delta x_D} \left(\frac{df_1}{dS_1} \right)_{n,j} (S_{n,j+1} - S_{n,j-1}) \quad (5)$$

In this scheme, $\left(\frac{df_1}{dS_1} \right)_{n,j}$ represents the derivative of fractional flow with respect to saturation, evaluated at the current time step and spatial position. Boundary conditions are applied at the first and last grid points to complete the solution.

```
def plot_vd(self):
    self.calculate_fractional_flow()
    # Numerically calculate the derivative of f1 with respect to S
    self.vd = np.gradient(self.f1, self.S)
    shock_index = np.argmax(self.S == self.shock_saturation)
    new_vd = np.copy(self.vd)
    new_vd[:shock_index] = np.max(new_vd)
    # Plot Vd vs. S
    plt.figure(figsize=(6, 4))
    plt.plot(new_vd, self.S, color='g', label='Vd')
    num_vd = self.calculate_vd_numerically()
```

Figure 1: Example Snippet Code for Numerical Use.

3.2 Plots for CO2 Injection into an Aquifer

3.2.1 Relative Permeability Plot

The following figure is the plot for relative permeability plot. As we can see it makes sense considering that this is the case of an injection of a non-wetting phase that displaces a wetting phase.

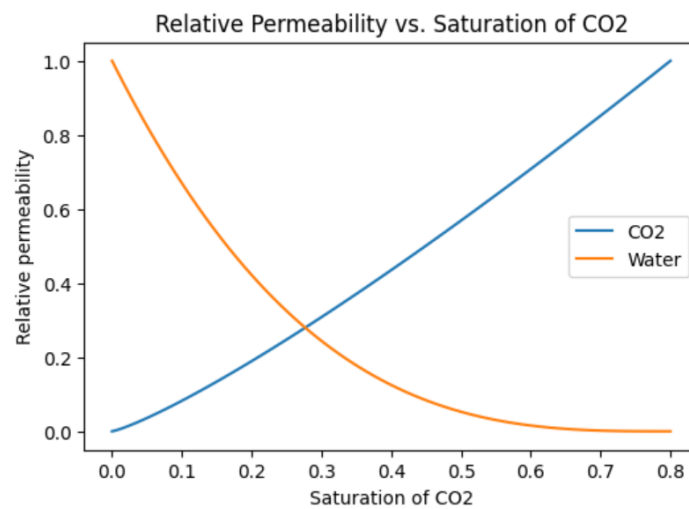


Figure 2: Plot for relative permeability.

3.2.2 Fractional Flow Plot

We observe a uniform movement and flow of the fluid as the saturation of the CO2 increases, showing minimal to no shock.

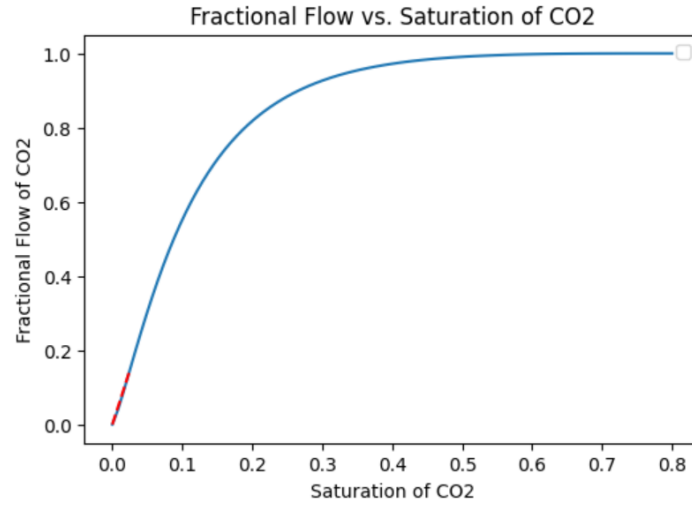


Figure 3: Plot for relative permeability.

3.2.3 Plotting v_D against Saturation

We notice a minimal shock happening when the saturation of CO2 is at a point equaling 0.048. Also, when we plot the numerical and the analytical values together we can see that they are matching.

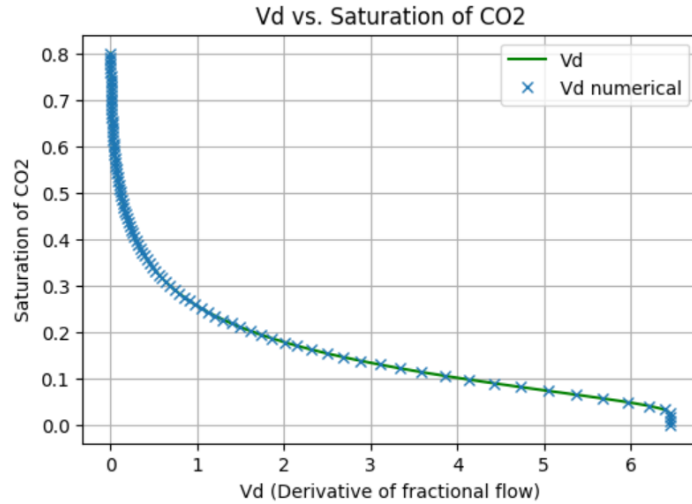


Figure 4: Plot for relative permeability.

3.3 Plots for Water Displacing Hydrogen

3.3.1 Relative Permeability Plot

The following figure is the plot for relative permeability plot. As we can see it makes sense considering that this is the case of an injection of a wetting phase that displaces a non-wetting phase, which is the hydrogen.

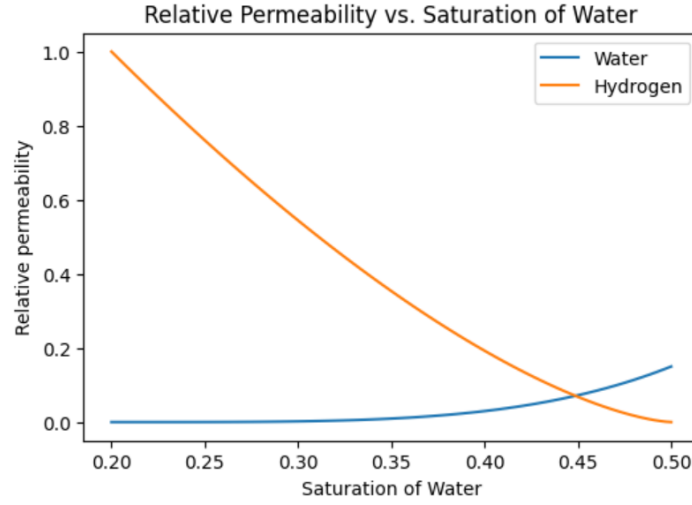


Figure 5: Plot for relative permeability.

3.3.2 Fractional Flow Plot

We can clearly see a strong shock occurring, causing the flow to drastically increase once the saturation of water reaches a certain level and causing massive fluid movement. We have to be careful because this makes it complicated to retrieve the stored fluid.

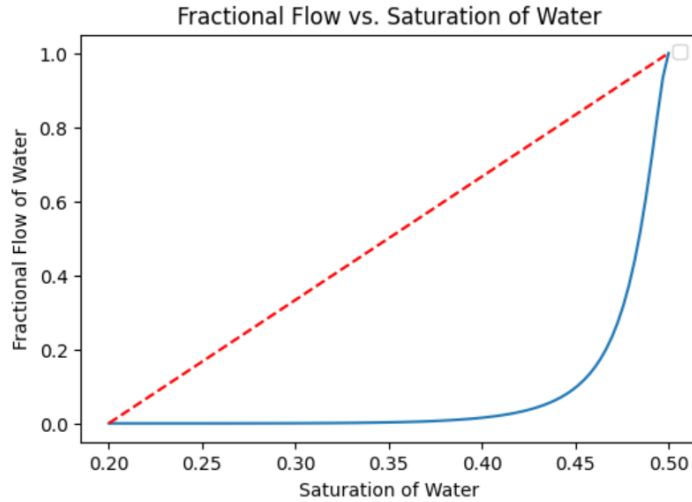


Figure 6: Plot for relative permeability.

3.3.3 Plotting vD against Saturation

At a value of $\frac{1}{1-S_{2r}-S_{1i}}$ there's a discontinuous change in water saturation changing from initial saturation to 1-residual saturation. We can also see that the analytical values match the ones we found numerically.

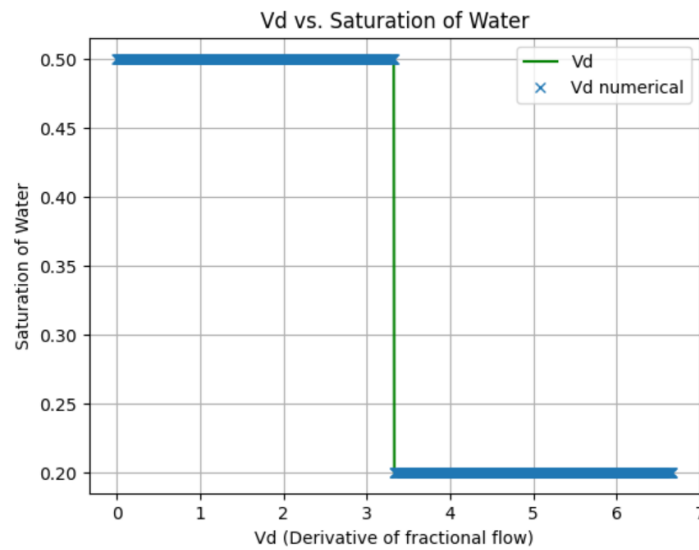


Figure 7: Plot for relative permeability.

3.4 Plots for Water displacing CO2

3.4.1 Relative Permeability Plot

The following figure is the plot for relative permeability plot. This is a case of a mixed wet system, with a combination of imbibition and drainage.

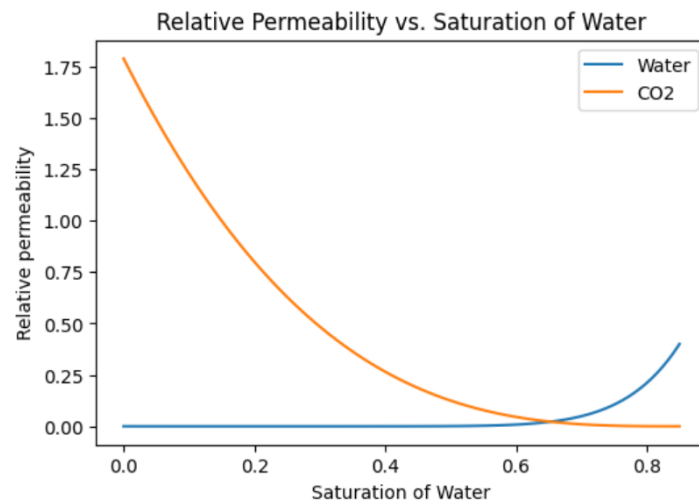


Figure 8: Plot for relative permeability.

3.4.2 Fractional Flow Plot

The red line is the line where the slope between the initial and current saturation is equal to the tangent or the derivative at that same point.

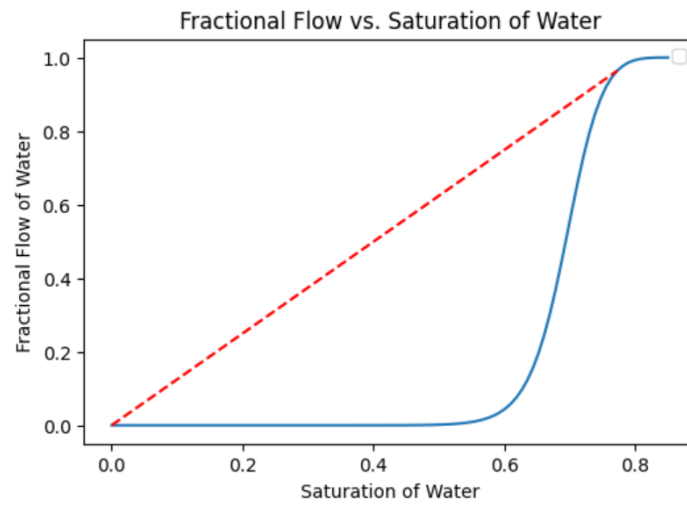


Figure 9: Plot for relative permeability.

3.4.3 Plotting v_D against Saturation

At that value of saturation that the red line is pointing towards, in the previous figure, there's a discontinuous jump in saturation. The analytical solution is matching our calculations.

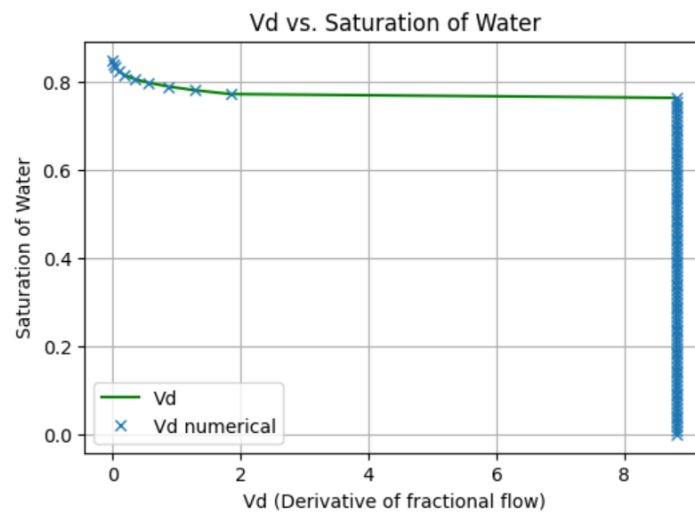


Figure 10: Plot for relative permeability.

4 Study of Spontaneous Imbibition:

In cases where a wetting phase is displacing a non-wetting phase, or in mixed-wet systems, there will be spontaneous imbibition. We will try to calculate, again, both numerically and analytically and try to plot it and explain what is happening.

4.1 Procedure:

Analytically, our objective is to determine the imbibition fractional flow function $F_1(S_1)$. This task is framed by a complex integral equation that describes the relationship between fractional flow and fluid saturation within a porous medium.

The integral equation is expressed as:

$$F_1(S_1) = 1 - \frac{\phi}{2c^2} \int_{S_{1i}}^{S_1^*} \frac{(\beta - S_{1i})D(\beta)}{F_1(\beta)} d\beta$$

Here, D represents the diffusivity term, which is critical for understanding the spontaneous flow behavior and is defined by:

$$D = -\frac{k_{r1}k_{r2}}{\mu_1k_{r2} + \mu_2k_{r1}} K \frac{dP_c}{dS_1}$$

This diffusivity term, D , is further linked to the capillary pressure P_c , which is a function of saturation, given by the relationship:

$$P_c = P_{c_{\max}} \frac{\left(\frac{S_1^*}{S_{1i}}\right)^{-c} - \left(\frac{S_1}{S_{1i}}\right)^{-c}}{\left(\frac{S_1^*}{S_{1i}}\right)^{-c} - 1}; \quad P_c \geq 0$$

To circumvent the direct solution of the integral containing $F_1(\beta)$, an iterative method is employed. We begin with an initial guess for F_1 , as suggested:

$$F_{1_{\text{initial}}} = \frac{S_1 - S_{1i}}{S_1^* - S_{1i}}$$

The terms S_{1i} and S_1^* denote the initial and the saturation at which the capillary pressure becomes zero (the fluid cannot spontaneously imbibe anymore), respectively. With this initial guess, we proceed to solve for c^2 using the equation:

$$c^2 = \frac{\phi}{2} \int_{S_{1i}}^{S_1^*} \frac{(\beta - S_{1i})D(\beta)}{F_1(\beta)} d\beta$$

The iterative process involves substituting the guess into the right-hand side of the integral equation and solving for a new estimate of F_1 . This estimate is then used in the next iteration, and the process is repeated until the change in F_1 between successive iterations falls below a specified convergence criterion.

To do this, we have used python and especially sympy to perform the complex differentiation and integration.

```

def calculate_fractional_flow(self):
    self.calculate_diffusivity()
    # Initial guess for F1
    self.F1 = (self.S - self.S1i) / (self.S1_star - self.S1i)
    # This is where the integration happens
    self.c_squared = (self.porosity / 2) * np.trapz(((self.S - self.S1i) / self.F1) * self.D, self.S)
    # Iterative solution
    for iteration in range(4):
        integral = np.zeros_like(self.S)

```

Figure 11: Example Code for Imbibition

4.2 Plotting

Once F_1 is obtained, it is differentiated with respect to S_1 to find the relationship between ω and S_1 , where $\omega = x/\sqrt{t}$ is the dimensionless variable representing the normalized position in the medium over time. Finally, we will plot ω against S_1 to visualize the saturation profile as a function of this dimensionless distance-time parameter.

4.2.1 Water displaces hydrogen in a storage site:

We can probably see the behaviour of the fluid in this case, as the dimensionless unit varies from 0 to about $1.25 \cdot 10^{-3}$. However the other values seemingly do not match or make sense. The most logical conclusion is that this non-sense result should be attributed to division by zero. Also, it's quite difficult to solve a differential function when you find that there are many recurrent terms in different differentiation/integration.



Figure 12: Water displacing hydrogen plot. We can notice the evolution of w which is the dimensionless value that normalizes the distance with the square root of the time.

4.2.2 Water displacing CO2 in a depleted oil-field:

Similarly here, the dimensionless unit varies from 0 to about $5 \cdot 10^{-5}$. Again, the numerical results do not make sense, probably due to some null division somewhere.

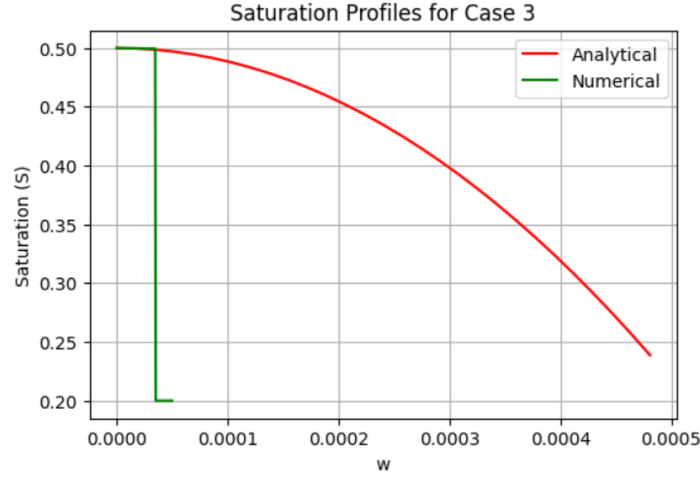


Figure 13: Water displacing CO2 in a depleted oil-field. The w which is the dimensionless value that normalizes the distance with the square root of the time.

4.2.3 Plotting both together:

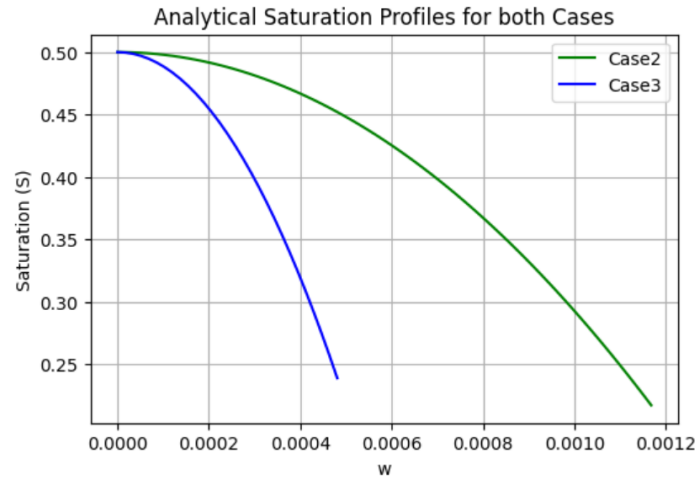


Figure 14: Plotting both Case 2 and Case 3 together.

4.2.4 Physical Interpretation:

The shape of the curve can provide insights into the characteristics of the porous medium and the fluids involved. A steeper initial slope could indicate strong initial capillary forces. A long tail at higher w values suggests that imbibition continues at these levels but at a much slower rate, which could maybe be due to lower permeability further from the source or the diminishing driving force as the saturation front progresses.

4.3 Discussion:

To recapitulate, the spontaneous imbibition process is governed by a partial differential equation characterizing the diffusive-like behavior of fluid flow in porous media. The driving force behind this movement is encapsulated by the capillary dispersion coefficient, denoted as D . This coefficient is a function of the relative permeabilities, absolute permeability, and the derivative of the capillary pressure ($\frac{dP_c}{dS_1}$).

Boundary conditions are set with an initial saturation S_{1i} , and a critical saturation S_1^* , at which point capillary pressure drops to zero, ceasing spontaneous diffusion. A key variable, $\omega = \frac{x}{\sqrt{t}}$, scales the distance by the square root of time, indicative of diffusive systems. The focus shifts to solving for the imbibition rate Q_1 , presumed proportional to $\frac{1}{\sqrt{t}}$, and determining the constant C , which signifies the rate at which the wetting phase enters the medium at the inlet.

Central to the analysis is the fractional flow function F_1 , ranging from 0 to 1, symbolizing the ratio of the wetting phase flow rate to its maximum at the inlet.

Then we look at an iterative strategy for resolving the integral equation for F_1 , starting with an initial linear approximation and refining to obtain the saturation profile as a function of ω . These profiles illuminate the imbibition patterns, showing rapid movement in water-wet media with almost front-like progression, whereas in mixed-wet media the process is markedly slower. Through this iterative, semi-analytic method, solutions can be derived that precisely depict the spontaneous imbibition process.

5 Conclusion:

In summary, this project has investigated the dynamics of fluid movement within porous media, focusing on spontaneous imbibition and advective flow processes. Through a combination of analytical and numerical methods, we developed an understanding of how CO₂ and water displace each other and interact with hydrocarbon and hydrogen in various contexts.

Both analytical approaches and numerical were used to solve and find values that are able to be plotted. Once we calculated these values, the project's findings were illustrated by plotting the dimensionless variable w against fluid saturation, highlighting the interplay between capillary forces and advective transport mechanisms. These insights are crucial for optimizing recovery in hydrocarbon reservoirs and managing subsurface fluid storage, especially carbon sequestration and hydrogen storage.

Overall, the study has provided valuable contributions to our understanding of multi-phase flow in Geo-energy applications, demonstrating the nuanced balance of forces that govern fluid behavior in subsurface environments.