Random_Walk

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0.1 Code:

Here we simulate the random walk in n-dimensions. First we check the possibility of returning to origin
a given number of steps. Then we estimate the probability of return in given number of trials.

```
In [1]: import numpy as np
        from matplotlib import pyplot as plt

        def random_walk(dimension, steps):
            """
            Function to determine if the return to origin is possible
            within given number of steps.
            """
            cood = [0 for _ in range(dimension)] # origin coordinate
            for _ in range(steps):
                cood[np.random.randint(dimension)] += np.random.choice([1,-1], p=[0.5, 0.5]) # add +1/-
                if not any(cood): # check iterable
                    #print(cood)
                    return True
            return False

        """
        To get the probability we repeat the random walk for given number of trials.
        Fraction return to zero is the probability.
        """

        def rw_prob(dimension, steps, trials):
            """
            Probability of returning to zero
            """
            return np.mean([random_walk(dimension, steps) for _ in range(trials)])
```

0.1.1 Plottong the probability vs # of steps

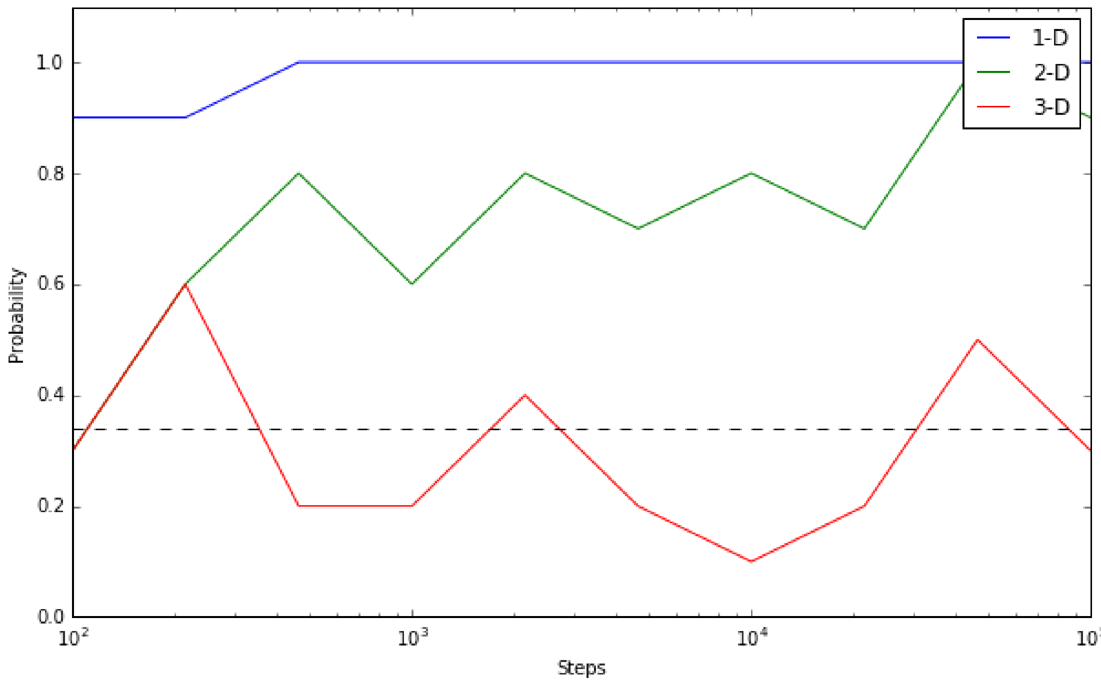
```
In [2]: %matplotlib inline

        plt.figure(figsize=(10,6))
        steps = np.logspace(2,5,10, dtype= int)
        dims = [1, 2, 3]

        for dim in dims:
            prob = [rw_prob(dim, _, 10) for _ in steps]
            plt.plot(steps, prob, label = '{0}-D'.format(dim))
```

```
plt.plot(steps, [0.34]* len(steps), '--', c='k') # dash line is 0.34
plt.xlabel('Steps')
plt.ylabel('Probability')
plt.ylim(0,1.1)
plt.xscale('log')
plt.legend()
```

```
Out[2]: <matplotlib.legend.Legend at 0x1058097f0>
```



0.1.2 Verification

The dependance of return probability on dimension was explained by George Polya. He proved that simple random walk is recurrent in dimension d = 1, 2 and transient in dimension d $\geq$ 3. i.e. it is certain that return to origin for d = 1,2, but probability is less than one for d $\geq$ 3. In fact for 3-D probability is ˜ 0.34.

Our solution is in agreement with 1-D and 3-D solution, and for 2-D I would expect that the probability should approach 1 for higher number of steps.

ref: http://mathworld.wolfram.com/PolyasRandomWalkConstants.html http://websites.math.leidenuniv.nl/probability/lecturenotes/RandomWalks.pdf