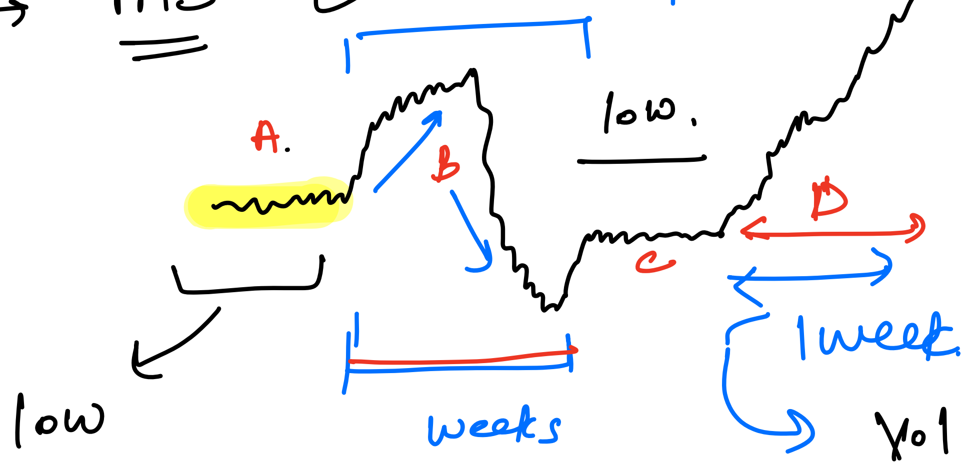
Volatility Modeling → Quant Project

1) ARCH
2) GARCH
3) EWMA

What is Volatility → Fluctuation in the stock price

stock → MS [Volatility of stock is high]



Vol of stock

- stock price fluctuating a lot → Vol of stock is high

A, C → Just fluctuating } → Vol is low

B, D → fluctuating a lot (up & down)
→ Vol is high

Why do we need Volatility Modeling ⇒ ??

- VM helps us to forecast the uncertainity or risk in the financial market
- Plays a very imp role in derivative pricing, asset allocation, risk management etc

- Derivative Pricing - Vol
- Market Risk → Vol

Future

Past performance

S_1 | S_2

Portfolio.

Performance
Beta
Sortino
⋮

when you invest →

hold sell

Sell

S_1

Vol. Modelling → how much the stock will fluctuate

Vol = 20%. ⇒ $100 → 100×(1−0.2) $80] sad
100(1+0.2) $120] happy

Volatility is Not Constant

- Derivative Models → Vol is const ✗
  - → Black scholes Model
  - → Binomial Tree model

- In real markets, volatility changes over time.
- During times of market stress (2007-08 financial crisis, 2020 covid) volatility spikes ⇒ good model helps the financial institution prepare well in Advance.

$\boxed{\text{Vol} \rightarrow +\text{ve}}$

Vol → -20% ✗
20%

100 → 80, 120

- Banks & hedge funds uses volatility forecast to calculate Value at Risk no

VaR → market
→ potential loss

stock data → Vol model → Future Vol → VaR Model.

in your portfolio

- There is an entire market for volatility product:
  a) VIX future
  b) VIX Options
  c) Volatility ETF

- Vol. Modeling is used by quant researchers, hedge fund, risk team, central banks.

GARCH

EWMA

1)

:

- Vol: How much the stock price will fluctuate

- In real market → imp how vol will change.

✓ [- During market stress → financial instit^n]

- Vol. Based product ⇒ ✓] ⇒

VaR ⇒ potential loss.

↳ S_1 | S_2 ⇒ how much can be losses

Vol model → how much fluctuation in your stocks.

Volatility Models → 1) ARCH 2) GARCH 3) EWMA

① ARCH Model.

• ARCH stands for Auto regressive Conditional Heteroskedasticity.

AR

linear regression

C

H

Lets breaks.

variance is changing

AR : It means today's value depends on past value

C : modeling something that changes based on new information

H : changing variance ⇒ variance is not

→ Linear regression → residual is homoskedastic → Constant.

ARCH help us to model volatility in the financial data.

Why do we need ARCH Model ??

→ In real market, we see volatility clustering – big moves are followed by big moves.

Today | Tomorrow

Big News → Vol. in stock price → Higher

General form of ARCH(q)

$$\underset{t}{\sigma_t^2} = \alpha_0 + \underset{t-1}{\alpha_1 \varepsilon_{t-1}^2} + \underset{t-2}{\alpha_2 \varepsilon_{t-2}^2} + \dots \alpha_q \varepsilon_{t-q}^2$$

σ_t^2 = Vol. at time 't'

α_0 = Base level of volatility

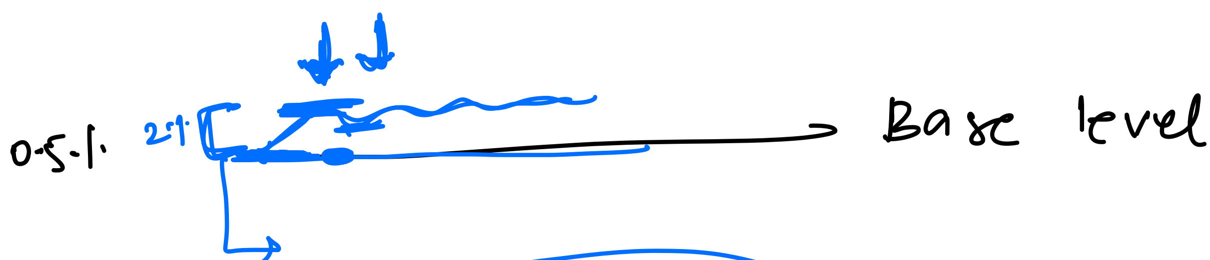
α_1 = How much yesterday shock affect today volatility.

- ε_{t-1}^2 = Shock

→ You expect stock → to move 0.5% today.

→ But Actual it moves → 2%

Shock = 1.5% movement of the stock. → $\varepsilon_{t-1} = 1.5\%$



Base level = 0.5

Act Vol = 2%

Error = 1.5%

Yesterday

Today Vol = 0.5% + $\alpha_1 \cdot 1.5\%$

- Tomorrow Vol = 0.5% + α_1 (1.5%)

  0.5% → get this number

  1.5% → Today

→ ARCH model → stock data = α_0

Base level of vol

⇒ [stocks data] → return [↕] = $\ln\left(\frac{S_t}{S_{t-1}}\right)$ or $\frac{S_t}{S_{t-1}} - 1$

→ Vol = std. dev. s (↕) = $\boxed{0.5\%}$

ⓐ Expected 0.5% → Actual 2% ⇒ Error 1.5%

Exp 2%

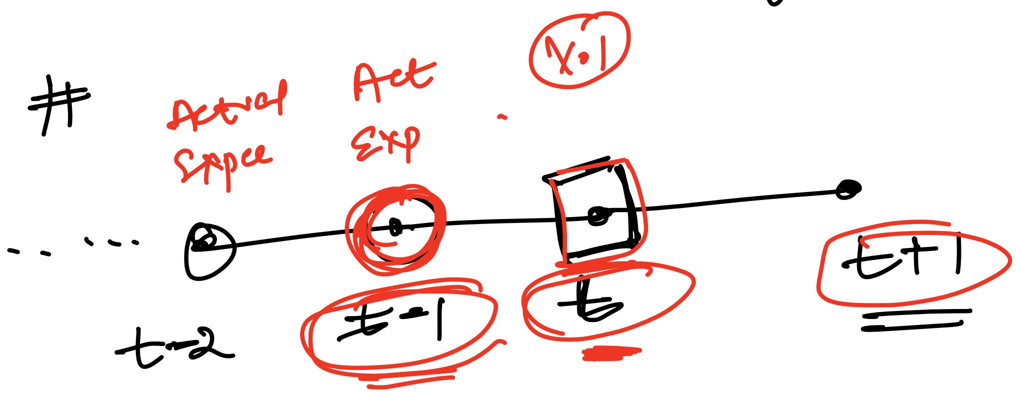
ARCH σ_t^2 = 0.5 + α_1 1.5% → higher Vol.

Actual 0.5

ⓑ Actual vol is lower

→ ε_{t-1}^2 ⇒ we don't care if the surprise is good or bad, we only care how big it is.

#



ARCH (1)
ARCH (2)
↳ (3)

- Stock at time t ⇒ Does it carry all the info of past
- $\boxed{t-1}$

GARCH

GARCH stands for a Generalized Auto-Regressive Conditional Heteroskedasticity

Generalized G : A better version of ARCH model
Auto Regressive AR : Uses past value
Conditional C : Changes depends on past data

Heteroskedasticity H: ↳ variance is not const
↳ it changes over time

Eg: If vol is const

$\sigma = 10\%$

Up

100 — t = 0

100 × (1.1) — t = 1

100 × (1.1)^2 — t = 2

100 × (1.1)^3 — t = 3

⋮

100
100 (0.9)
100 × (0.9)(0.9)
100 (0.9)(0.9)(0.9)

t = 1 = 1.1
t = 2 = 1.2
t = 3 = 1.4

(100 + 10%) → 100 + 10% ⇒ 110% ⇒ 1.1
(100 − 10%) = 90% ⇒ 0.9

→ GARCH is just like ARCH, but a little smarter.

GARCH

→ ARCH model only uses past squared errors (ε^2_{t-1}) to estimate todays volatility.

→ But in real life:

- Suprises (ARCH).
- Past Volatility ⟹ GARCH.

= Basel level vol + past shock + past vol → GARCH

→ [Tomorrow's Vol ⟶ α_0. + Today's Vol + Today shock]
①

ARCH → ①

GARCH → ① + ②

GARCH.(p,q).

$$\underbrace{\sigma_t^2}_{\text{Vol}} = \underbrace{\alpha_0}_{\text{Basel level of Vol}} + \underbrace{\alpha_1 \underbrace{\varepsilon_{t-1}^2}_{\text{Yest. shock}}}_{\text{ARCH}} + \underbrace{\beta_1 \underbrace{\sigma_{t-1}^2}_{\text{Yesterday Vol.}}}_{\text{Additional parameter}}$$

ARCH. vs GARCH

→ ⎣___⎦ + past volatility.

$\sigma^2_{t-1} = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 \sigma^2_{t-1}$ ARCH

→ ARCH model only uses past shock.
GARCH model uses past shock & past volatility

- α_0, α_1, B_1 → historical data

- shock → A-P
- Vol ~~PCY~~ → Garch model → modeled Vol

→ Python

2020 |——| 2025 ~~April~~ April → Train our model

May Actual Vol

2020 → 2025
2000 → 2025

2016 – 2025 → ✓
2000 – 2025 ✓

MLE

↳ Calibration technique. ⟹ α_0 =, α_1 =, β_1 =

↳ maximum likelihood estimator

~~Act~~ closed Model Vol

α_1 = 0 to 1
β_1 = 0 to 1
→ $\alpha_1 = 0.5$, $\beta_1 = 0.2$ ✓

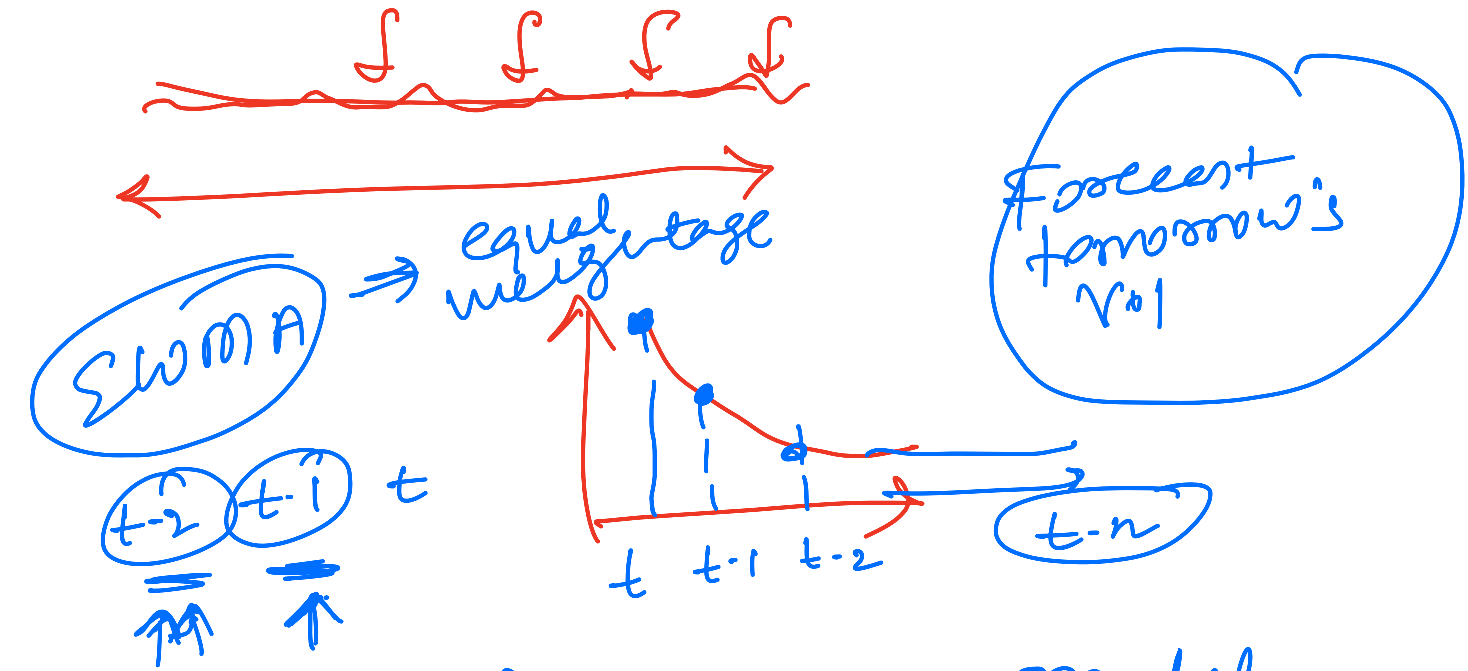
③ EWMA → Exponentially Weighted Moving Average.

✓

recent

t - 5   t - 4   t - 3   t - 2   t - 1 $\boxed{t}$

1) High Vol Environment → Covid.
2) Low Vol Environment



→ EWMA is a simple model.

→ It gives more weightage to recent observation & less weightage to older observation.

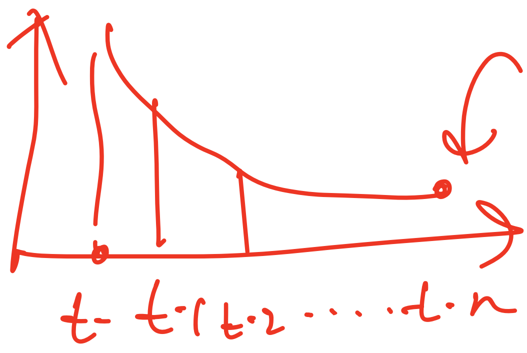
Why EWMA Model

- Vol isn't const
- Sometimes recent behaviour is more imp than the older history

Eg 2000 — 2025 → it depend

- EWMA captures by exponentially

decaying the weights



weights will drops

older history

Formula

$$\boxed{\sigma_t^2 = (1-\lambda)\,\varepsilon_{t-1}^2 + \lambda\,\sigma_{t-1}^2}$$

σ_t^2 = Today's forecasted Vol.

ε_{t-1}^2 = Shock.

σ_{t-1}^2 = Yesterday forecasted vol.

λ = Decay factor. → 0.94

#

	λ
Daily Vol	0.94
Weekly Vol	0.97

Long memory | 1 → longer period
Short memory | 0 → forgets quickly.

Analogy

- You are on a Beach.
- The most recent waves matters the most to guess the next wave
- But you don't ignore the past completly
- You just give less & less weights to older waves.

ARCH vs GARCH vs EWMA

ARCH	GARCH	EWMA
• Autoregressive conditional Heteroskedastic	• Generalized ARCH	• Exponentially weighted moving Avg

	• Vol depends on past shock	• Vol depends on past shock + past vol	• Vol depend on exponentially decaying past shock + vol
	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$	$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda \sigma_{t-1}^2$
Memory of past volatility	No	Yes	Yes.
Weights on the recent shocks	→ Equal weight	→ vary α_1, β	→ Exponential decrease
Captures Vol clustering	Yes	Yes	Yes
	Not used this at all	→ standard model most of the cases Fails	→ Vol is higher