

8.5 Two-dimensional transformation

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Total Number of Topics: 10

Topic 1: Two-dimensional Translation

Key Points:

- Definition:** Two-dimensional translation refers to the movement of an object in a two-dimensional space without altering its shape, size, or orientation. It involves shifting the object by a specified distance along the x and y axes.
- Mathematical Representation:** The translation of a point $((x, y))$ by a distance $((dx, dy))$ can be expressed mathematically as: $[T(x, y) = (x + dx, y + dy)]$ Here, (T) is the transformation applied to the point.
- Transformation Matrix:** The translation can also be represented using a transformation matrix in homogeneous coordinates: $[\begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}]$ This allows for the application of translation alongside other transformations, such as rotation and scaling.
- Applications:** Translation is widely used in computer graphics, animation, and robotics, where the position of objects needs to be changed without affecting their other properties.

MCQ Questions:

- What is the result of translating a point (2, 3) by (1, -1)?
 - A) (3, 2)
 - B) (1, 4)
 - C) (2, 2)
 - D) (3, 4)

Answer: A) (3, 2)

Explanation: The new coordinates are calculated as $((2+1, 3-1) = (3, 2))$.

- Which of the following matrices represents a translation of (dx, dy)?
 - A) $[\begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}]$
 - B) $[\begin{bmatrix} dx & 0 & 0 \\ 0 & dy & 0 \\ 0 & 0 & 1 \end{bmatrix}]$
 - C) $[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ dx & dy & 1 \end{bmatrix}]$
 - D) $[\begin{bmatrix} 1 & dy & 0 \\ dx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}]$

Answer: A) $[\begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}]$

Explanation: This matrix correctly applies translation to a point in homogeneous coordinates.

3. Which of the following transformations preserves the shape and size of an object?

- A) Scaling
- B) Rotation
- C) Translation
- D) Shearing

Answer: C) Translation

Explanation: Translation only changes the position of the object without affecting its dimensions or angles.

4. If a point (1, 1) is translated by (-2, 3), what are the new coordinates?

- A) (-1, 4)
- B) (3, -2)
- C) (1, 4)
- D) (3, 1)

Answer: A) (-1, 4)

Explanation: The calculation yields $((1-2, 1+3) = (-1, 4))$.

5. The translation of a rectangle from its original position to a new position is primarily used in which field?

- A) Quantum Mechanics
- B) Computer Graphics
- C) Fluid Dynamics
- D) Structural Engineering

Answer: B) Computer Graphics

Explanation: In computer graphics, objects are frequently translated for rendering scenes.

6. Which of the following best describes the effect of translating an object in 2D space?

- A) Rotation about a fixed point
- B) Changing the size of the object
- C) Moving the object without deformation
- D) Reflecting the object across an axis

Answer: C) Moving the object without deformation

Explanation: Translation moves the object without changing its properties.

7. In the context of computer graphics, translation is usually represented in which coordinate system?

- A) Polar Coordinates
- B) Cartesian Coordinates
- C) Spherical Coordinates
- D) Cylindrical Coordinates

Answer: B) Cartesian Coordinates

Explanation: Translation in 2D typically uses Cartesian coordinates.

8. A translation operation can be represented as a combination of which of the following transformations?

- A) Scaling and Rotation
- B) Rotation and Shear
- C) Only Scaling
- D) None of the above

Answer: D) None of the above

Explanation: Translation is a distinct operation and cannot be combined with others without affecting its nature.

Topic 2: Rotation

Key Points:

1. **Definition:** Rotation in 2D refers to the circular movement of a point or object around a fixed point, known as the pivot or rotation center, typically the origin (0, 0).
2. **Mathematical Representation:** The rotation of a point $((x, y))$ by an angle (θ) can be expressed using the following transformation equations: $[x' = x \cos(\theta) - y \sin(\theta)] [y' = x \sin(\theta) + y \cos(\theta)]$ Here, $((x', y'))$ are the new coordinates after rotation.
3. **Transformation Matrix:** The rotation can be expressed using a rotation matrix: $[R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}]$ This matrix can be used in conjunction with the translation matrix in homogeneous coordinates.
4. **Counter-Clockwise Convention:** Positive angles are typically considered counter-clockwise in a standard Cartesian coordinate system, while negative angles represent clockwise rotation.

MCQ Questions:

1. If a point (2, 3) is rotated 90 degrees counter-clockwise around the origin, what are the new coordinates?

- A) (-3, 2)
- B) (3, -2)
- C) (2, -3)
- D) (3, 2)

Answer: A) (-3, 2)

Explanation: The calculation for a 90-degree rotation yields ((-3, 2)).

2. The rotation matrix for an angle (θ) is given by which of the following forms?

- A) $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$
- B) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
- C) $\begin{bmatrix} -\sin(\theta) & \cos(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
- D) $\begin{bmatrix} \sin(\theta) & \cos(\theta) \\ \cos(\theta) & -\sin(\theta) \end{bmatrix}$

Answer: B) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

Explanation: This is the correct form for the rotation matrix in 2D.

3. What will be the result of rotating the point (1, 0) by 180 degrees?

- A) (1, 0)
- B) (0, -1)
- C) (-1, 0)
- D) (0, 1)

Answer: C) (-1, 0)

Explanation: Rotating by 180 degrees transforms the point to ((-1, 0)).

4. Which transformation alters the position of a shape but preserves its orientation?

- A) Shearing
- B)

Scaling

- C) Rotation
- D) Reflection

Answer: C) Rotation

Explanation: Rotation changes the position but retains the shape's orientation.

5. If a point is rotated around a point that is not the origin, what additional step is necessary?

- A) No additional step
- B) Translate the point to the origin first

- C) Scale the point

- D) Shear the point

Answer: B) Translate the point to the origin first

Explanation: You need to translate to the origin, rotate, and then translate back.

6. Which of the following angles corresponds to a clockwise rotation of 90 degrees?

- A) -90 degrees

- B) 90 degrees

- C) 180 degrees

- D) 270 degrees

Answer: A) -90 degrees

Explanation: Clockwise rotation is represented by negative angles.

7. The effect of rotating an object by 360 degrees results in which of the following?

- A) No change

- B) Scaling of the object

- C) Reflection of the object

- D) Shearing of the object

Answer: A) No change

Explanation: A full rotation brings the object back to its original position.

8. When rotating a shape about the origin, the shape's center of rotation is at which coordinate?

- A) (0, 0)

- B) (1, 1)

- C) (a, b)

- D) (x, y)

Answer: A) (0, 0)

Explanation: The origin is the typical center of rotation in 2D transformations.

Topic 3: Scaling

Key Points:

1. **Definition:** Scaling is the transformation that alters the size of an object in two-dimensional space by a scaling factor along the x and y axes, effectively enlarging or shrinking the object.

2. **Mathematical Representation:** The scaling of a point $((x, y))$ by factors (sx) and (sy) can be expressed as: $[S(x, y) = (sx \cdot x, sy \cdot y)]$ Here, (sx) and (sy) are the scaling factors for the x and y axes, respectively.
3. **Transformation Matrix:** Scaling can be represented using a scaling matrix in homogeneous coordinates: $[S = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix}]$ This matrix can be combined with other transformations such as translation and rotation.
4. **Uniform vs Non-Uniform Scaling:** Uniform scaling occurs when both scaling factors are the same (e.g., $(sx = sy)$), while non-uniform scaling results in different factors for the x and y axes, which can distort the shape.

MCQ Questions:

1. What happens to an object when it is uniformly scaled by a factor of 2?

- A) Its shape is distorted
- B) Its dimensions are doubled
- C) It is rotated
- D) It is translated

Answer: B) Its dimensions are doubled

Explanation: Uniform scaling increases both dimensions equally.

2. Which of the following scaling factors will shrink an object?

- A) Greater than 1
- B) Equal to 1
- C) Less than 1
- D) Negative

Answer: C) Less than 1

Explanation: Scaling factors less than 1 reduce the size of the object.

3. A point $(4, 5)$ is scaled by factors $(sx = 3)$ and $(sy = 2)$. What are the new coordinates?

- A) $(12, 10)$
- B) $(8, 10)$
- C) $(12, 15)$
- D) $(4, 2)$

Answer: A) $(12, 10)$

Explanation: The new coordinates are calculated as $((3 \cdot 4, 2 \cdot 5) = (12, 10))$.

4. Which of the following matrices represents a non-uniform scaling transformation?

- A) $[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}]$

- B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- C) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- D) $\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer: C) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Explanation: This matrix applies different scaling factors to x and y, resulting in non-uniform scaling.

5. Which type of scaling maintains the aspect ratio of an object?

- A) Non-uniform scaling
- B) Uniform scaling
- C) Reflection
- D) Translation

Answer: B) Uniform scaling

Explanation: Uniform scaling uses the same factor for both axes, preserving the aspect ratio.

6. When a shape is scaled by a factor of -1, what is the result?

- A) The shape is mirrored across the origin
- B) The shape is rotated 180 degrees
- C) The shape is shrunk to zero
- D) The shape remains unchanged

Answer: A) The shape is mirrored across the origin

Explanation: Negative scaling inverts the shape across the origin.

7. The scaling operation is often used in which field?

- A) Structural Engineering
- B) Graphics Design
- C) Aerodynamics
- D) Chemistry

Answer: B) Graphics Design

Explanation: In graphics design, scaling is frequently used to adjust the size of images and shapes.

8. Which of the following does not affect the center of scaling?

- A) Uniform Scaling
- B) Non-Uniform Scaling

- C) Scaling about a fixed point

- D) Scaling about the origin

Answer: D) Scaling about the origin

Explanation: Scaling about the origin keeps the center fixed at (0,0).

Topic 4: Reflection

Key Points:

1. **Definition:** Reflection is a transformation that flips an object over a specified line, known as the line of reflection, creating a mirror image of the original object.
2. **Types of Reflection:** The most common types of reflection in 2D space include reflection across the x-axis, y-axis, and lines such as $(y = x)$ or $(y = -x)$.
3. **Mathematical Representation:** The reflection of a point $((x, y))$ across the x-axis is given by: $[R_x(x, y) = (x, -y)]$ Across the y-axis: $[R_y(x, y) = (-x, y)]$ Reflection across the line $(y = x)$ is given by: $[R_{y=x}(x, y) = (y, x)]$
4. **Transformation Matrix:** The reflection transformation can be represented using matrices. For reflection across the x-axis: $[R_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}]$ For reflection across the y-axis: $[R_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}]$

MCQ Questions:

1. What is the result of reflecting the point (2, 3) across the x-axis?

- A) (2, -3)
- B) (-2, 3)
- C) (2, 3)
- D) (-2, -3)

Answer: A) (2, -3)

Explanation: The y-coordinate changes sign, resulting in (2, -3).

2. The reflection matrix across the y-axis is:

- A) $[\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}]$

$[\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}]$

- B) $[\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}]$
- C) $[\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}]$

- D) $\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$

Answer: B) $\begin{bmatrix} -1 & 0 & 0 & 1 \end{bmatrix}$

Explanation: This matrix inverts the x-coordinate for reflection across the y-axis.

3. If a point (1, 1) is reflected across the line ($y = x$), what are the new coordinates?

- A) (1, 1)
- B) (1, -1)
- C) (-1, 1)
- D) (1, 1)

Answer: A) (1, 1)

Explanation: The point remains the same since it lies on the line ($y = x$).

4. Which transformation changes the orientation of an object?

- A) Translation
- B) Rotation
- C) Scaling
- D) Reflection

Answer: D) Reflection

Explanation: Reflection creates a mirror image, thus changing the orientation.

5. Which line is used for reflection to create a mirror image of an object?

- A) Any arbitrary line
- B) The x-axis only
- C) The y-axis only
- D) The line of symmetry only

Answer: A) Any arbitrary line

Explanation: Reflection can occur across any specified line.

6. Reflecting a triangle across the x-axis results in which of the following?

- A) The triangle retains its shape and orientation
- B) The triangle becomes larger
- C) The triangle is inverted along the x-axis
- D) The triangle is rotated

Answer: C) The triangle is inverted along the x-axis

Explanation: Reflection flips the triangle over the x-axis.

7. If a shape is reflected across two intersecting lines, what is the result?

- A) The shape is rotated
- B) The shape is translated
- C) The shape is scaled
- D) The shape is unchanged

Answer: A) The shape is rotated

Explanation: Reflecting across two intersecting lines results in a rotation.

8. Which of the following is true about the properties of reflection?

- A) It preserves distances but not angles
- B) It preserves angles but not distances
- C) It preserves both distances and angles
- D) It alters both distances and angles

Answer: C) It preserves both distances and angles

Explanation: Reflection maintains congruency between the original and reflected shape.

Topic 5: Shear Transformation

Key Points:

1. **Definition:** Shear transformation distorts the shape of an object by shifting its points in a specific direction, resulting in a transformation that slides one part of the object relative to another.
2. **Types of Shear:** Shearing can occur in various directions: horizontal shear (along the x-axis) and vertical shear (along the y-axis). In horizontal shear, points are moved parallel to the x-axis, while in vertical shear, they are moved parallel to the y-axis.
3. **Mathematical Representation:** The shear transformation for horizontal shear can be expressed as: $[S(x, y) = (x + sh \cdot y, y)]$ where (sh) is the shear factor along the x-axis. For vertical shear: $[S(x, y) = (x, y + sv \cdot x)]$ where (sv) is the shear factor along the y-axis.
4. **Transformation Matrix:** The shear transformation can be represented using matrices. For horizontal shear: $[S_{\{h\}} = \begin{bmatrix} 1 & sh \\ 0 & 1 \end{bmatrix}]$ For vertical shear: $[S_{\{v\}} = \begin{bmatrix} 1 & 0 \\ sv & 1 \end{bmatrix}]$

MCQ Questions:

1. What effect does a horizontal shear transformation have on an object?
 - A) It rotates the object
 - B) It scales the object uniformly
 - C) It distorts the shape along the x-axis

- D) It translates the object

Answer: C) It distorts the shape along the x-axis

Explanation: Horizontal shear shifts points in the x-direction based on their y-coordinates.

2. The matrix representation for vertical shear with shear factor (sv) is:

- A) $\begin{bmatrix} 1 & 0 \\ sv & 1 \end{bmatrix}$
- B) $\begin{bmatrix} 1 & sv \\ 0 & 1 \end{bmatrix}$
- C) $\begin{bmatrix} sv & 0 \\ 0 & 1 \end{bmatrix}$
- D) $\begin{bmatrix} 1 & 0 \\ 0 & sv \end{bmatrix}$

Answer: A) $\begin{bmatrix} 1 & 0 \\ sv & 1 \end{bmatrix}$

Explanation: This matrix correctly applies vertical shear to the points.

3. If a point (2, 3) is subjected to a shear transformation with a shear factor of 2 along the x-axis, what are the new coordinates?

- A) (2, 3)
- B) (8, 3)
- C) (8, 3)
- D) (2, 6)

Answer: C) (8, 3)

Explanation: The calculation yields $(2 + 2 \cdot 3, 3) = (8, 3)$.

4. Which transformation alters the shape of an object without changing its area?

- A) Scaling
- B) Rotation
- C) Shear
- D) Reflection

Answer: C) Shear

Explanation: Shearing distorts the shape but keeps the area constant.

5. In which scenarios is shear transformation commonly used?

- A) Architectural design and graphics
- B) Data compression
- C) Color grading in images
- D) Audio processing

Answer: A) Architectural design and graphics

Explanation: Shear transformation is often applied in these fields to create perspective effects.

6. A vertical shear with a shear factor of 3 will do what to the coordinates of the point (2, 1)?

- A) (2, 1)
- B) (2, 5)
- C) (5, 1)
- D) (5, 3)

Answer: B) (2, 5)

Explanation: The new coordinates are calculated as $(2, 1 + 3 \cdot 2) = (2, 5)$.

7. Shearing can be best described as:

- A) A rigid transformation
- B) A non-rigid transformation
- C) A rotational transformation
- D) A scaling transformation

Answer: B) A non-rigid transformation

Explanation: Shearing is non-rigid as it alters the shape of the object.

8. If the shear factor for horizontal shear is negative, what will be the effect?

- A) The shape is distorted in the opposite direction
- B) The shape becomes smaller
- C) The shape rotates
- D) The shape remains unchanged

Answer: A) The shape is distorted in the opposite direction

Explanation: A negative shear factor shifts points in the opposite direction.

Topic 6: 2D Composite Transformation

Key Points:

1. **Definition:** A 2D composite transformation is the combination of multiple 2D transformations (like translation, rotation, scaling, reflection, and shearing) into a single transformation that can be applied to an object.
2. **Matrix Multiplication:** Composite transformations are typically achieved by multiplying the individual transformation matrices. The order of multiplication is significant and affects the final result due to the non-commutative nature of matrix multiplication.
3. **Homogeneous Coordinates:** To perform composite transformations, homogeneous coordinates are used. This allows for the representation of transformations, including translation, in matrix form, enabling the

combination of transformations through matrix multiplication.

4. **Transformation Order:** The order in which transformations are applied matters. For example, performing scaling before rotation will yield a different result than rotating first and then scaling.

MCQ Questions:

1. What is the primary advantage of using composite transformations?

- A) It simplifies calculations
- B) It reduces memory usage
- C) It allows for greater flexibility in transformations
- D) It increases computational complexity

Answer: A) It simplifies calculations

Explanation: Composite transformations allow for multiple transformations to be combined into a single operation.

2. In composite transformations, which operation is applied last in the order?

- A) Translation
- B) Scaling
- C) Rotation
- D) Shear

Answer: D) Shear

Explanation: The last transformation in the multiplication sequence is applied last.

3. If the transformation matrices (A) and (B) are multiplied, what does the resulting matrix represent?

- A) The transformation represented by (A) followed by (B)
- B) The transformation represented by (B) followed by (A)
- C) A new transformation unrelated to (A) and (B)
- D) A scaled version of (A)

Answer: A) The transformation represented by (A) followed by (B)

Explanation: The resulting matrix encapsulates the effect of both transformations in sequence.

4. Why is it important to use homogeneous coordinates in 2D transformations?

- A) They allow for representation of 3D transformations
- B) They simplify matrix operations and include translation
- C) They are easier to compute

- D) They require less memory

Answer: B) They simplify matrix operations and include translation

Explanation: Homogeneous coordinates make it possible to represent translations alongside other transformations as matrix operations.

5. If you first rotate a point and then translate it, how does this differ from translating first and then rotating?

- A) The results are always the same
- B) The results can be very different due to the order of operations
- C) The second operation negates the first
- D) There is no difference, only the coordinates change

Answer: B) The results can be very different due to the order of operations

Explanation: The order of transformations affects the final position and orientation of the point.

6. What will be the result of multiplying a transformation matrix by the identity matrix?

- A) The transformation matrix itself remains unchanged
- B) The result will be a zero matrix
- C) The transformation will not occur
- D) The identity matrix will be scaled

Answer: A) The transformation matrix itself remains unchanged

Explanation: Multiplying by the identity matrix has no effect on the original matrix.

7. When combining transformations, which of the following is NOT a valid transformation?

- A) Rotation
- B) Reflection
- C) Translation
- D) Compression

Answer: D) Compression

Explanation: Compression is not a standard transformation; scaling achieves similar results.

8. In 2D composite transformations, if two transformation matrices (M_1) and (M_2) are combined, what form does the resulting matrix take?

- A) A scalar value
- B) A matrix representing a single transformation
- C) A vector of transformed points

- D) An array of transformation types

Answer: B) A matrix representing a single transformation

Explanation: The combined matrix encapsulates the effects of both transformations.
