Topic: Electric Field and Flux Density

Key Points:

Now, let's proceed to the next topic.

Topic: Divergence Theorem

Key Points:

1. **Definition**: The divergence theorem, also known as Gauss's theorem, relates the flux of a vector field through a closed surface to the divergence of the field within the volume enclosed by that surface.

2. **Mathematical Formulation**: Mathematically, the theorem states that for a vector field F and a closed surface *S* enclosing a volume *V*:

   $$\oint_S \mathbf{F} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{F} \, dV$$

   where $d\mathbf{S}$ is the outward normal vector to the surface.

3. **Applications**: The divergence theorem is widely used in physics and engineering, particularly in electromagnetism, fluid dynamics, and heat transfer, to convert volume integrals into surface integrals, making calculations easier.

4. **Physical Interpretation**: The theorem implies that the total flux leaving a volume through its boundary is equal to the total amount of the source (or sink) of the field within that volume.

5. **Conditions for Application**: The divergence theorem applies to vector fields that are continuously differentiable over the volume and its boundary, ensuring that the surface integral can be computed easily.

6. **Examples**: Common applications include calculating electric flux through a closed surface surrounding a charge distribution or analyzing fluid flow across surfaces.

MCQ Questions

1. **What does the divergence theorem relate?**
   - A) Volume integral to line integral
   - B) Surface integral to line integral
   - C) Surface integral to volume integral
   - D) Volume integral to scalar quantity

   Answer: C) Surface integral to volume integral
   Explanation: The divergence theorem relates the surface integral of a vector field to the volume integral of its divergence.

2. **For which type of fields can the divergence theorem be applied?**
   - A) Only conservative fields
   - B) Only solenoidal fields
   - C) Continuously differentiable fields
   - D) Discontinuous fields

   Answer: C) Continuously differentiable fields
   Explanation: The divergence theorem requires the vector field to be continuously differentiable over the volume and its boundary.

3. **In the context of the divergence theorem, what does the term $d\mathbf{S}$ represent?**
   - A) Area element
   - B) Volume element
   - C) Outward normal vector to the surface

- D) Scalar field

Answer: C) Outward normal vector to the surface
Explanation: dS denotes the outward normal vector to the surface, representing the direction of the flux.

4. **If ∇⋅F=0 throughout a volume, what can be inferred about the flux through the surface enclosing that volume?**

    - A) It is positive
    - B) It is negative
    - C) It is zero
    - D) It is infinite

    Answer: C) It is zero
    Explanation: If the divergence is zero, it indicates that there are no net sources or sinks, resulting in zero net flux through the surface.

5. **Calculate the surface integral of F=(x2,y2,z2) over a sphere of radius 1 centered at the origin using the divergence theorem.**

    - A) 34π
    - B) 4π
    - C) 32π
    - D) 0

    Answer: B) 4π
    Explanation: The divergence ∇⋅F=3. The volume of the sphere is 34 π(1)3=34π. Thus, the flux is 3⋅34π=4π.

6. **What is the total flux through a closed surface surrounding a point charge of Q? (Use ε0 for permittivity)**

    - A) ε0Q
    - B) ε0Q2

- C) 0
- D) $4\pi Q$

Answer: A) $\varepsilon 0Q$
Explanation: By applying Gauss's law, the total electric flux through the closed surface is equal to $\varepsilon 0Q$.

Next topic coming up!

Topic: Electric Potential

Key Points:

1. **Definition**: Electric potential (V) at a point is defined as the work done per unit charge in bringing a positive test charge from infinity to that point in an electric field, without any acceleration.

2. **Formula**: Mathematically, the electric potential is given by:

   $V=-\int \infty r \mathrm{E}\cdot d\mathrm{r}$

   where E is the electric field and dr is the differential displacement vector.

3. **Units**: The SI unit of electric potential is the volt (V), which is equivalent to one joule per coulomb (J/C).

4. **Potential Difference**: The difference in electric potential between two points is known as the potential difference or voltage (ΔV), which determines the energy available to move charges between those points.

5. **Relation to Electric Field**: The electric field is the negative gradient of electric potential:

$$E = -\nabla V$$

This relationship shows how potential varies in space.

6. **Equipotential Surfaces**: Surfaces on which the electric potential is constant are called equipotential surfaces. No work is required to move a charge along these surfaces, and they are always perpendicular to electric field lines.

MCQ Questions

1. **What is the SI unit of electric potential?**
   - A) Ampere
   - B) Joule
   - C) Volt
   - D) Coulomb

   Answer: C) Volt
   Explanation: The SI unit of electric potential is the volt, which is defined as one joule per coulomb.

2. **What does electric potential represent?**
   - A) Energy per unit charge
   - B) Charge per unit volume
   - C) Force per unit charge
   - D) Voltage drop across a conductor

   Answer: A) Energy per unit charge
   Explanation: Electric potential indicates the amount of work done to move a unit positive charge from a reference point to a specific point in an electric field.

3. **The relationship between electric field and electric potential can be expressed as:**

    - A) $E = V$
    - B) $E = \nabla V$
    - C) $E = -\nabla V$
    - D) $V = \nabla \cdot E$

    Answer: C) $E = -\nabla V$
    Explanation: The electric field is defined as the negative gradient of the electric potential.

4. **What is an equipotential surface?**

    - A) A surface where the electric field is zero
    - B) A surface where the electric potential is constant
    - C) A surface where current flows
    - D) A surface perpendicular to electric field lines

    Answer: B) A surface where the electric potential is constant
    Explanation: Equipotential surfaces are those where the electric potential remains the same, meaning no work is needed to move charges along these surfaces.

5. **If the electric potential at a point is 5V and a positive charge of 2C is moved to a point where the potential is 10V, what is the work done?**

    - A) 10 J
    - B) 20 J
    - C) 5 J
    - D) 15 J

    Answer: B) 10 J

Explanation: The work done is given by $W=q\Delta V=2C\cdot(10V-5V)=2C\cdot 5V=10J$.

6. **Calculate the electric potential V at a distance of 2m from a point charge of $Q=1\mu C$ in a vacuum. (Use $k=9\times 109$N m2/C2)**
   - A) $4.5V$
   - B) $2.25V$
   - C) $4.5\times 10-3V$
   - D) $18V$

   Answer: A) $4.5V$
   Explanation: The potential is given by $V=k\cdot rQ=9\times 109\cdot 21\times 10-6=4.5V$.

Next topic!

Topic: Potential Gradient

Key Points:

1. **Definition**: The potential gradient is a vector quantity that represents the rate of change of electric potential with respect to distance. It indicates how the electric potential varies in space.

2. **Mathematical Expression**: Mathematically, the potential gradient is given by:

   $$E=-\nabla V$$

   where E is the electric field vector and V is the electric potential.

3. **Physical Interpretation**: The potential gradient signifies the direction and magnitude of the electric field. A steep potential

gradient indicates a strong electric field.

4. **Units**: The units of potential gradient are volts per meter (V/m), which are equivalent to newtons per coulomb (N/C).

5. **Relation to Force**: The force on a charge in an electric field can be expressed in terms of the potential gradient, where $F=q\cdot E$.

6. **Applications**: The concept of potential gradient is crucial in understanding capacitors, electric circuits, and field theory, helping in predicting the behavior of electric charges in an electric field.

MCQ Questions

1. **What does the potential gradient indicate?**

   - A) Rate of charge flow
   - B) Rate of change of electric potential
   - C) Electric field strength
   - D) Work done on a charge

   Answer: B) Rate of change of electric potential
   Explanation: The potential gradient measures how quickly the electric potential changes with distance in a given direction.

2. **What is the relationship between electric field E and potential gradient ∇V?**

   - A) $\mathrm{E}=\nabla V$
   - B) $\mathrm{E}=-\nabla V$
   - C) $\mathrm{E}=0$
   - D) $\mathrm{E}=V$

   Answer: B) $\mathrm{E}=-\nabla V$

Explanation: The electric field is the negative gradient of electric potential, indicating the direction of decreasing potential.

3. **What are the units of potential gradient?**

    - A) Joules
    - B) Volts per meter
    - C) Newtons
    - D) Coulombs

    Answer: B) Volts per meter
    Explanation: The potential gradient is measured in volts per meter (V/m), which corresponds to the change in potential over a distance.

4. **In a region where the electric potential is constant, the potential gradient is:**

    - A) Positive
    - B) Negative
    - C) Zero
    - D) Undefined

    Answer: C) Zero
    Explanation: If the electric potential is constant, there is no change in potential, resulting in a potential gradient of zero.

5. **If the electric potential V varies with distance x as $V=5x2+3$, what is the potential gradient E at $x=1m$?**

    - A) 10 V/m
    - B) 5 V/m
    - C) 0 V/m
    - D) 15 V/m

    Answer: A) 10 V/m

Explanation: The potential gradient $E=-dxdV=-dxd(5x2+3)=-10x$. At $x=1m$, $E=-10\times1=-10$V/m.

6. **Calculate the potential gradient if the electric potential at a point is 20V and changes to 10V over a distance of 5m.**
   - A) 2 V/m
   - B) 1 V/m
   - C) 0.5 V/m
   - D) 4 V/m

   Answer: D) 2 V/m
   Explanation: The potential gradient is given by $\Delta V/\Delta d=(20V-10V)/5m=10V/5m=2V/m$.

Now, let's move on to the next topic.

Topic: Energy Density in Electrostatic Field

Key Points:

1. **Definition**: Energy density in an electrostatic field refers to the amount of energy stored per unit volume in the electric field. It is a measure of how much energy is available in a given volume due to the electric field.

2. **Formula**: The energy density u in an electric field is given by:

   $u=21\varepsilon E2$

   where ε is the permittivity of the medium and E is the magnitude of the electric field.

3. **Units**: The SI unit of energy density is joules per cubic meter (J/m^3),

which indicates the energy stored in a volume of space.

4. **Physical Significance**: Energy density provides insight into how electric fields can store energy, which is critical in applications like capacitors, where energy storage is essential.

5. **Applications**: Understanding energy density is vital for designing devices that utilize electric fields, such as capacitors, transformers, and energy storage systems.

6. **Comparison with Magnetic Energy Density**: Similar to electric fields, magnetic fields also store energy, expressed as:

   $u=21\mu B2$

   where B is the magnetic field and μ is the permeability.

MCQ Questions

1. **What is the formula for energy density in an electrostatic field?**

   - A) $u=\varepsilon E2$
   - B) $u=21\varepsilon E2$
   - C) $u=21\mu B2$
   - D) $u=21E2$

   Answer: B) $u=21\varepsilon E2$
   Explanation: The energy density in an electrostatic field is defined as half the product of permittivity and the square of the electric field strength.

2. **What does energy density measure in an electric field?**

   - A) Charge per unit volume
   - B) Force per unit volume

- C) Energy per unit volume
- D) Electric field strength

Answer: C) Energy per unit volume
Explanation: Energy density quantifies the amount of energy stored within a specific volume of the electric field.

3. **What is the SI unit of energy density?**

   - A) Joules
   - B) Joules per meter
   - C) Joules per cubic meter
   - D) Newtons

   Answer: C) Joules per cubic meter
   Explanation: The SI unit of energy density is joules per cubic meter (J/m^3), representing energy stored in a volume.

4. **The energy density in an electric field increases with which of the following?**

   - A) Decreasing electric field strength
   - B) Increasing electric field strength
   - C) Decreasing permittivity
   - D) Increasing charge density

   Answer: B) Increasing electric field strength
   Explanation: Energy density increases with the square of the electric field strength, indicating that stronger fields store more energy.

5. **Calculate the energy density in a vacuum if the electric field strength is E=100V/m. (Use ε0=8.85×10−12F/m)**

   - A) 0.044J/m3
   - B) 0.0885J/m3

- C) 0.001J/m3
- D) 0.0000885J/m3

Answer: A) 0.044J/m3
Explanation: The energy density is calculated as $u=21\varepsilon0E2=21(8.85\times10-12)(100)2=0.044J/m3$.

6. **In which applications is understanding energy density in electrostatic fields crucial?**

   - A) Transistors
   - B) Capacitors
   - C) Inductors
   - D) Resistors

   Answer: B) Capacitors
   Explanation: Understanding energy density is vital for the design and functioning of capacitors, where energy is stored in an electric field.

Let's continue with the next topic!