

$$\log(n!) = \underbrace{\log 1 + \log 2 + \log 3 + \dots + \log n}_{p \cdot n/2 \leq n} \leq \underbrace{\log n + \log n + \dots + \log n}_n = n \log n \in O(n \log n)$$

$$\begin{aligned} \log(n!) &= \log 1 + \log 2 + \dots + \log n \geq \underbrace{\log(n/2) + \log(n/2+1) + \dots + \log n}_{p \cdot n/2 \geq n/2} \\ &\geq \underbrace{\log(n/2) + \log(n/2) + \dots + \log(n/2)}_{p \cdot n/2 \geq n/2} = n/2 \cdot \log(n/2) \\ &\in \Omega(n \log n) \end{aligned}$$