ME 425_525
HW # 3
Motion Control of a Flying Robot (Quadrotor)
Due: 24/11/2025, Time: 23:55

Consider the following dynamical model for a quadrotor written in a hybrid frame, where the first 3 equations are expressed in the inertial earth frame and the last 3 equations are written in the body frame.

$$
\begin{cases}
\ddot{X} &= (\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi)\,\dfrac{U_1}{m} \\
\ddot{Y} &= (-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi)\,\dfrac{U_1}{m} \\
\ddot{Z} &= -g + (\cos\theta\cos\phi)\,\dfrac{U_1}{m} \\
\dot{p} &= \dfrac{I_{YY} - I_{ZZ}}{I_{XX}}\,q\,r - \dfrac{J_{TP}}{I_{XX}}\,q\,\Omega + \dfrac{U_2}{I_{XX}} \\
\dot{q} &= \dfrac{I_{ZZ} - I_{XX}}{I_{YY}}\,p\,r + \dfrac{J_{TP}}{I_{YY}}\,p\,\Omega + \dfrac{U_3}{I_{YY}} \\
\dot{r} &= \dfrac{I_{XX} - I_{YY}}{I_{ZZ}}\,p\,q + \dfrac{U_4}{I_{ZZ}}
\end{cases}
$$

The second terms ($J_{TP}q\Omega/I_{XX}$ and $J_{TP}p\Omega/I_{YY}$) in the 4th and 5th equations, namely 'gyroscopic terms' are very small and therefore can be neglected.

Hover Control (4 DOF Control): Construct a Simulink model for the quadrotor using the above nonlinear motion equations. Design appropriate FF+FB (Feedforward+Feedback) controllers for the Vertical Take-Off and Landing (VTOL) as we developed in class. Assuming zero initial roll and pitch angles, i.e. quadrotor is on a flat surface and is not tilted, hover the quadrotor at arbitrary desired altitudes and plot flight data, i.e. altitude versus time and attitude angles (Euler angles) versus time, and control efforts (U_1, U_2, U_3 and U_4) vs time. If you start with small nonzero roll/pitch angles on the ground, i.e. quadrotor is initially tilted a little bit, can you hover the quadrotor at a desired altitude? Can you control it in the x-y plane? Comment on your results.

Trajectory Tracking Control (6 DOF Control): Using the position control method based on virtual control inputs, design FF+FB controllers to track smooth trajectories in 3D. For example, you may consider a scenario where the robot takes off from a ground spot, hovers at a desired altitude, then moves in a straight line, makes a circular loop, and finally lands. Provide flight data, i.e. position coordinates vs time, attitude angles (Euler angles) vs time, control efforts vs time, and comment on your results.

Note: You can select any reasonable mass and inertia values as your model parameters.