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Ceng 111 – Fall 2021
Week 2a

Digital Computation

Credit: Some slides are from the “Invitation to Computer Science” book by G. M. Schneider, J. L. Gersting and some from the “Digital Design” book by M. M. Mano and M. D. Ciletti.

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What is Computer Science?

- All of the following concepts are incomplete and do not capture the richness and diversity of this exciting field:
  - computers,
  - programming languages,
  - software applications, and uses.

From "Invitation to Computer Science"

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Computer Science is the study of algorithms (= methods)

including:

1. their formal and mathematical properties
2. their hardware realizations
3. their linguistic realizations
4. their applications

From "Invitation to Computer Science"

Measure the height of a tall building with a barometer

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- What would be your answer?
- One student answered:
  - "I would tie the barometer to a rope, hang it down from the top of the building to the bottom and measure the length of the rope!"
  - Of course, the instructor rejects the answer since it doesn't include any "physics"

Check the following for two different versions of the `legend':
http://www.snopes.com/college/exam/barometer.asp

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Analyze/Compare Algorithms
= Pros and Cons of Algorithms =

What are the disadvantages of these?

1. Drop the barometer from the top, and measure the time it takes to reach the ground.
2. Make a pendulum and time its period wrt. the top and the bottom of the building.
3. Walk down the stairs marking "barometer units" on the wall.
4. Measure its shadow and the buildings shadow. Workout the height of the building from barometer's height.

Cons:

1. You lose the barometer; it breaks.
2. You need a loooong rope and how long that rope is going to be depends on the answer to the question.
3. It takes too long. It is too tiring.
4. What if there is no sun?

Where does the word ‘algorithm’ come from?

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- From a Persian mathematician, astronomer and geographer: Mohammed ibn-Musa al-Khwarizmi
- “Algorithmi” is the latin form of his name
- He contributed to science by
  - Decimal positional number system (e.g., $32 = 10^1 x3 + 10^0 x2$)
  - Presented the first systematic solutions to linear and quadratic equations
- In fact, the word “Algebra” comes from one of his operators (al-jabr: subtracting a number from both sides of an equation) for solving equations

Mohammed ibn-Musa al-Khwarizmi (780-850)

Source: Wikipedia

A formal definition of algorithm

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- "Starting from an initial state and initial input (perhaps empty), the instructions describe a computation that, when executed, will proceed through a finite number of well-defined successive states, eventually producing "output" and terminating at a final ending state."

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Valid Operations in Algorithms

- **Sequential** – simple well-defined task, usually declarative sentence.
- **Conditional**- "ask a question and select the next operation on the basis of the answer to the question – usually an "if-then" or "if then else"
- **Iterative**- "looping" instructions – repeat a set of instructions

From "Invitation to Computer Science"

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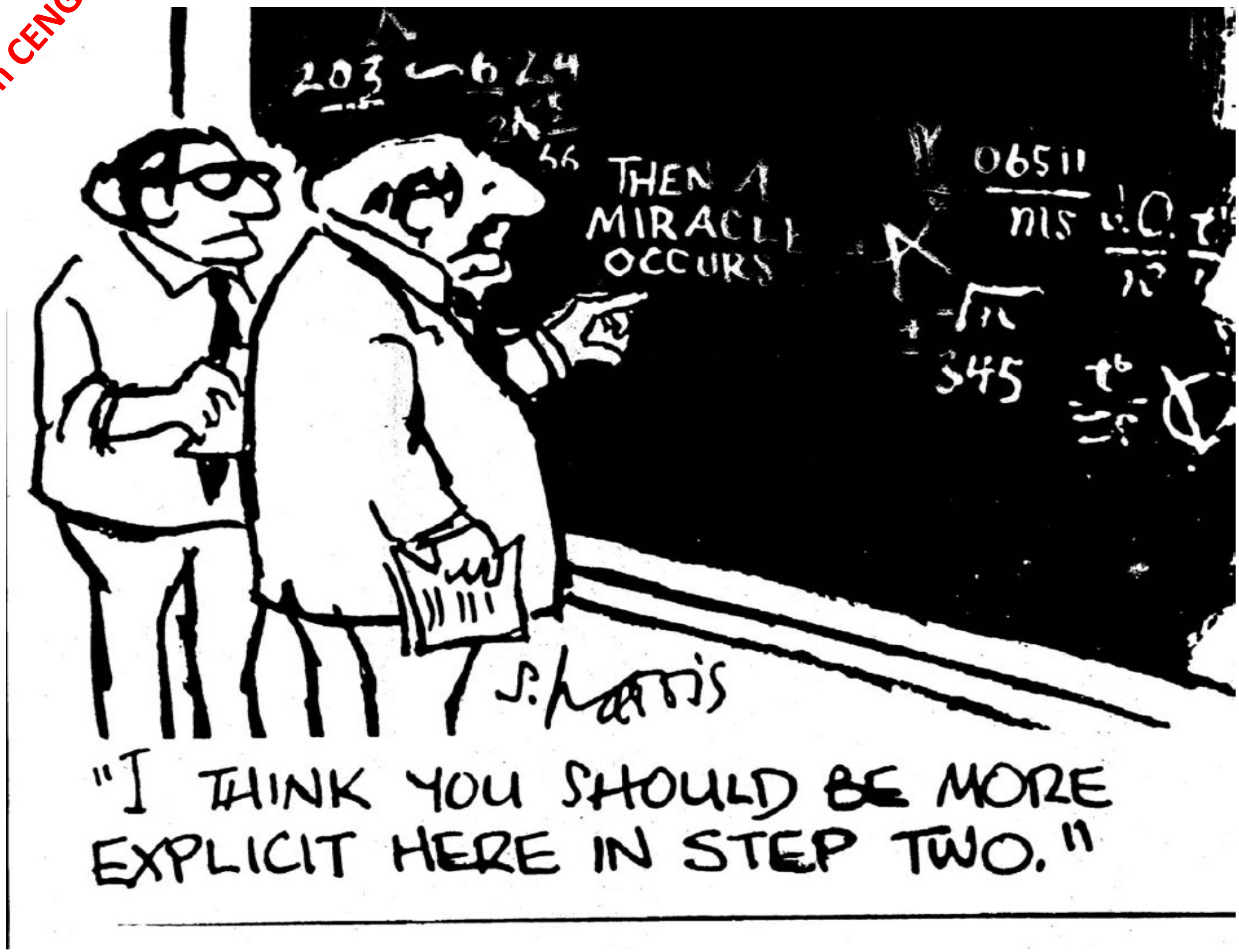

From "Invitation to Computer Science"

How to represent algorithms

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- Pseudo-code

Algorithm for Adding Two *m*-Digit Numbers

Given: $m \geq 1$ and two positive numbers each containing m digits, $a_{m-1}\ a_{m-2}, \ldots a_0$ and $b_{m-1}\ b_{m-2}, \ldots b_0$
Wanted: $c_m c_{m-1}\ c_{m-2} \ldots c_0$, where $c_m c_{m-1}\ c_{m-2} \ldots c_0 = (a_{m-1}\ a_{m-2} \ldots a_0) + (b_{m-1}\ b_{m-2} \ldots b_0)$

Algorithm:

Step 1 Set the value of *carry* to 0.
Step 2 Set the value of *i* to 0.
Step 3 While the value of *i* is less than or equal to $m - 1$, repeat the instructions in steps 4 through 6.
Step 4 Add the two digits a_i and b_i to the current value of *carry* to get c_i.
Step 5 If $c_i \geq 10$, then reset c_i to $(c_i - 10)$ and reset the value of *carry* to 1; otherwise, set the new value of *carry* to 0.
Step 6 Add 1 to *i*, effectively moving one column to the left.
Step 7 Set c_m to the value of *carry*.
Step 8 Print out the final answer, $c_m\ c_{m-1}\ c_{m-2} \cdots c_0$.
Step 9 Stop.

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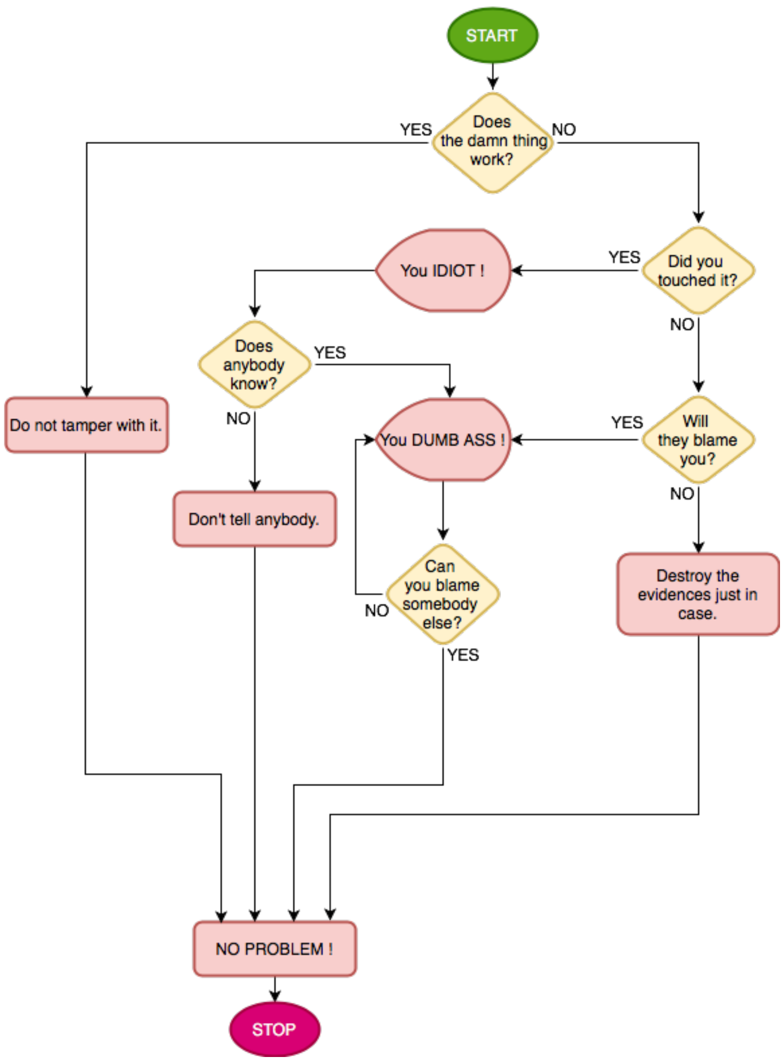
How to represent algorithms

- Flow-charts



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“Computation”

- Digital vs. analog computation
- Sequential vs. parallel computation
- Batch vs. interactive computation
- Evolutionary, molecular, quantum computation
- “Physical computation” / “Digital Physics”
  - ‘The whole universe is itself a computation’

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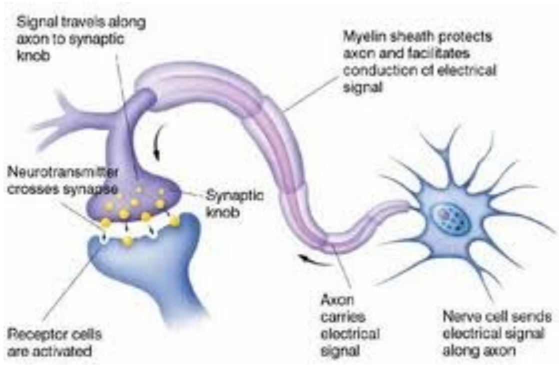
Computation in our brain

- Highly-connected network of neurons.
- How many neurons?
  - Approx. 10^{11} neurons and 10^{14} synapses.
- How do they transmit information?
  - Using nothing else than charged molecules.



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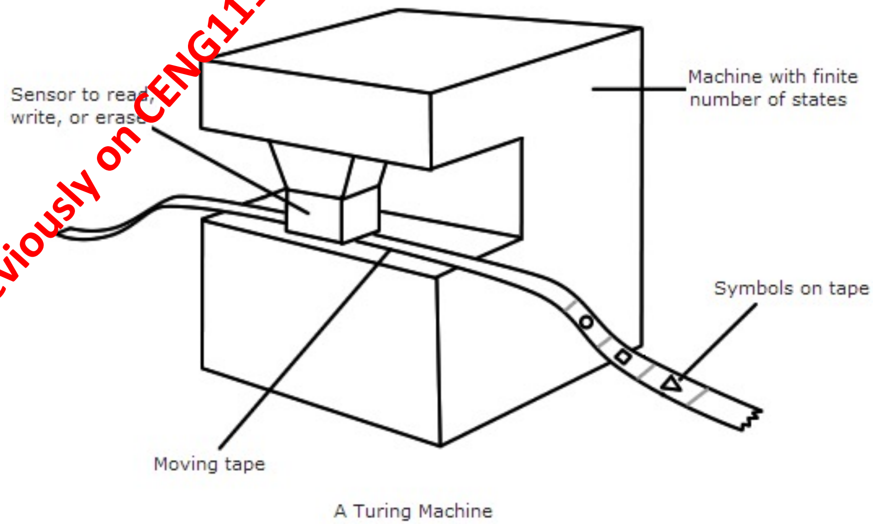
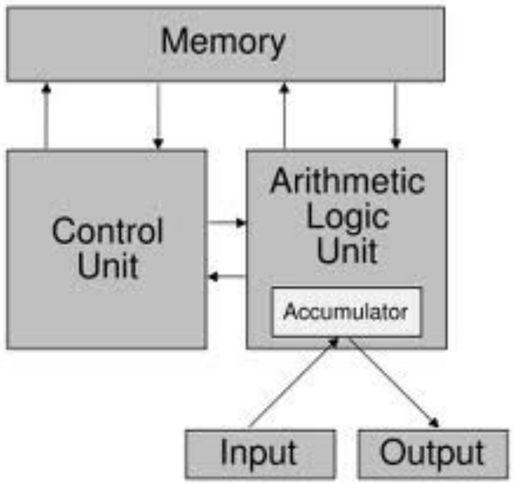


Turing Machine



Von Neumann Architecture

DIGITAL COMPUTATION

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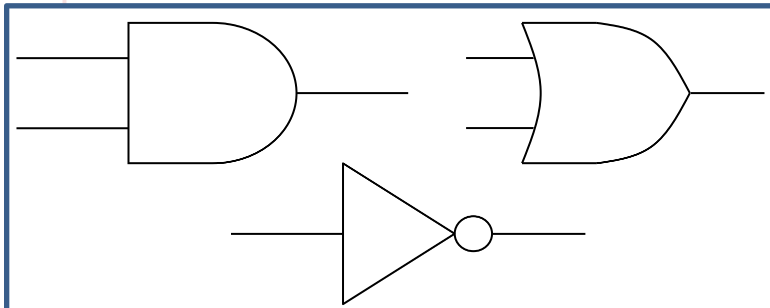
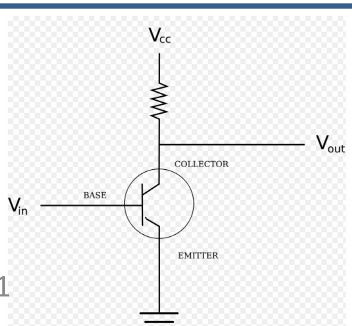
A computer

Devices



Gates



Transistors

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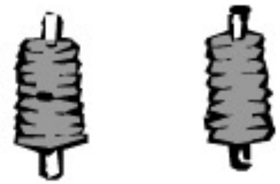
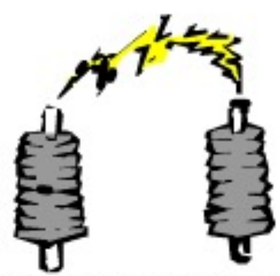
Everything in a PC is Binary … well, almost …

States of a Bit			
0	 2+2=5 FALSE	 OFF	 LOW VOLTAGE
1	 2+2=4 TRUE	 ON	 HIGH VOLTAGE

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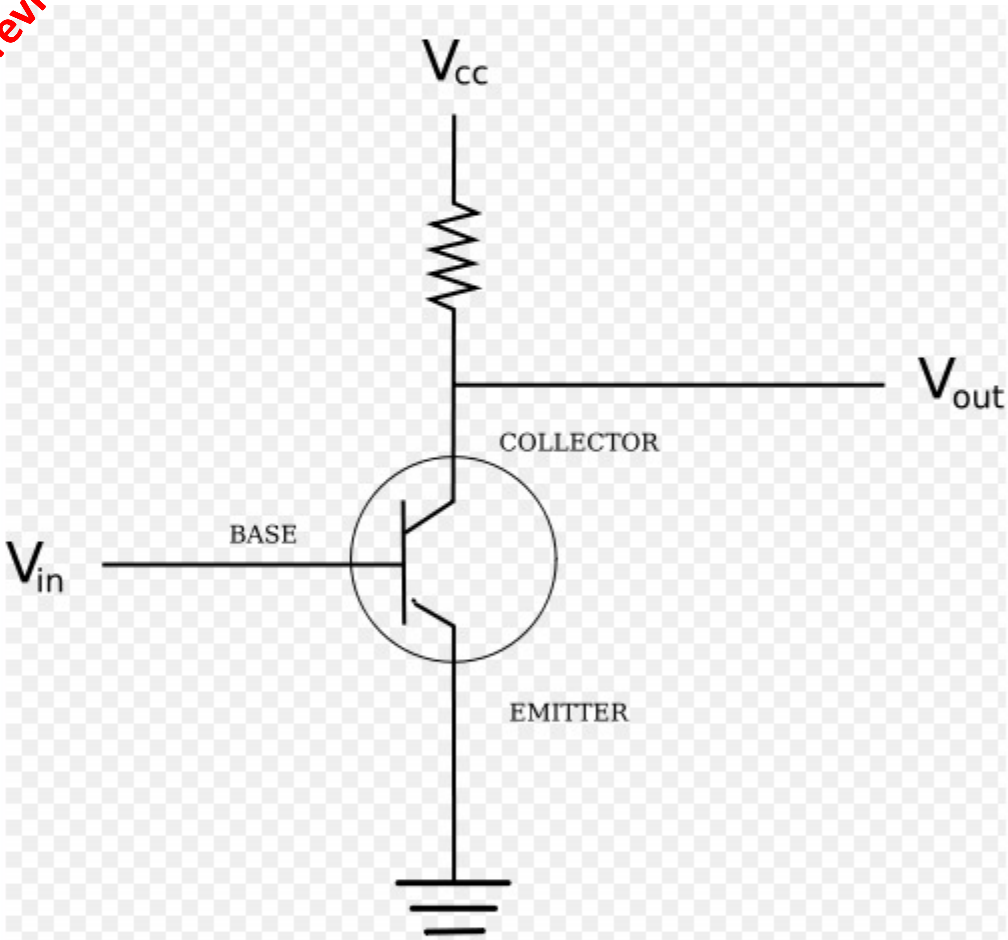
A transistor



This circuit functions as a switch. In other words, based on the *control* voltage, the circuit either passes Vin to output or not.

AND gate

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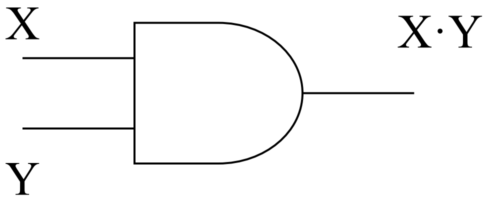
X	Y	X·Y
0	0	0
0	1	0
1	0	0
1	1	1



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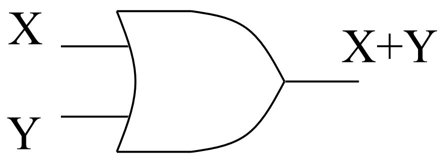
OR Gate

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1



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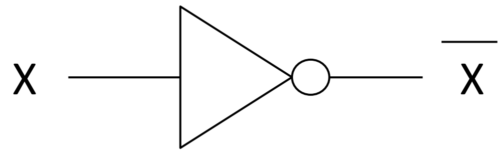
NOT Gate

X	$\overline{X}$
0	1
1	0



Today

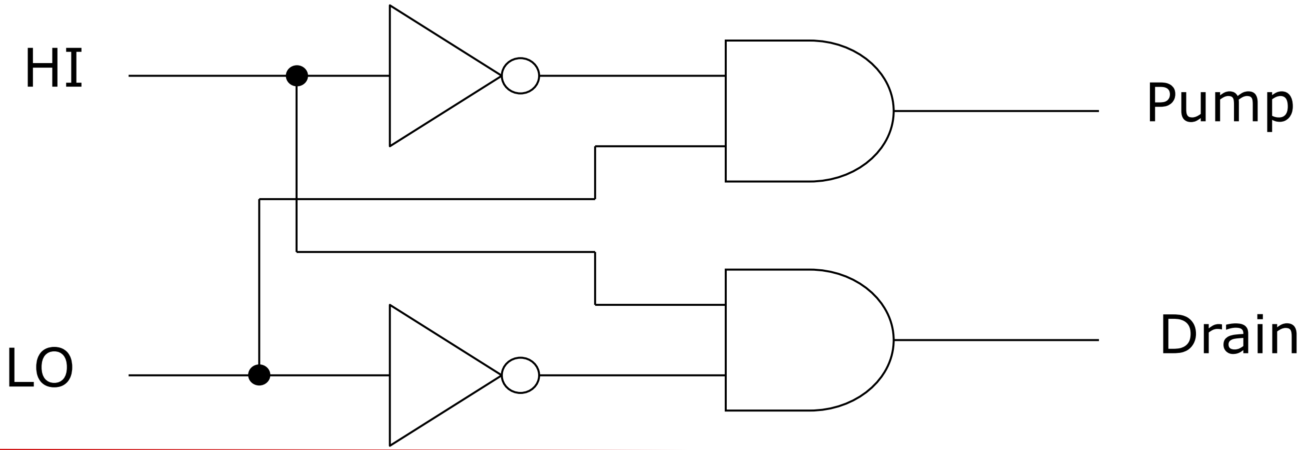
- Digital Computation

An example problem: Water Tank

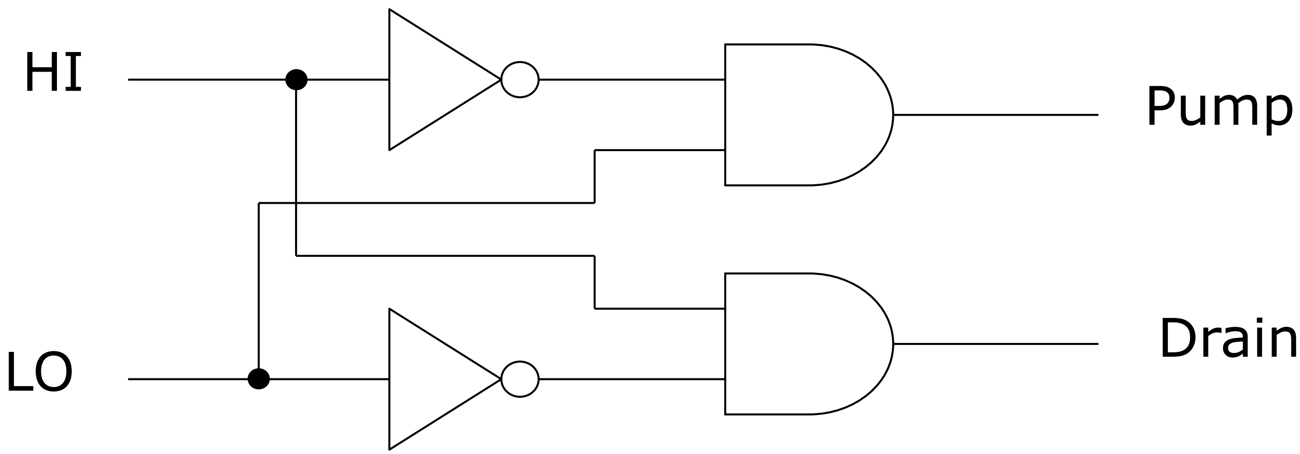
Truth Table Representation

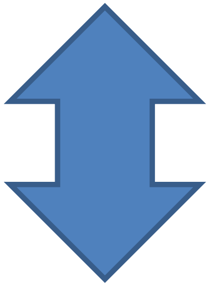
HI	LO	Pump	Drain	
0	0	0	0	Tank level is OK
0	1	**1**	0	Low level, pump more in
1	0	0	**1**	High level, drain some out
1	1	x	x	Inputs cannot occur

Schematic Representation



Boolean Logic/Algebra





Pump = HI’.LO
Drain = HI.LO’

Boolean formula describing the circuit.

The binary addition

$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 10 \end{array}$$

Question (Binary notation) : 111010 + 11011 = ?

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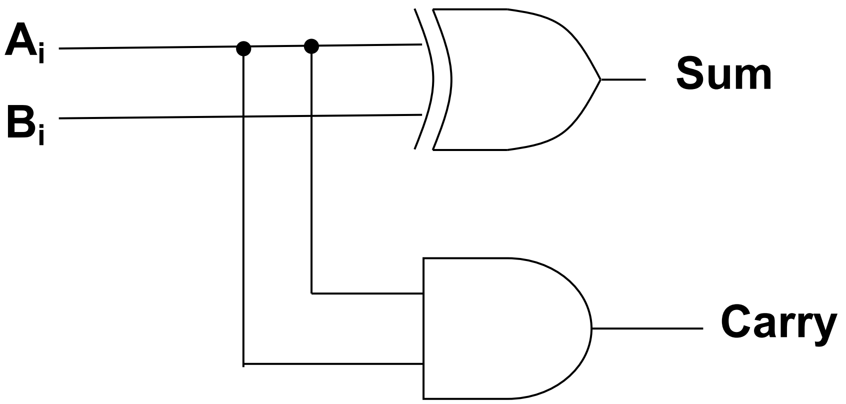
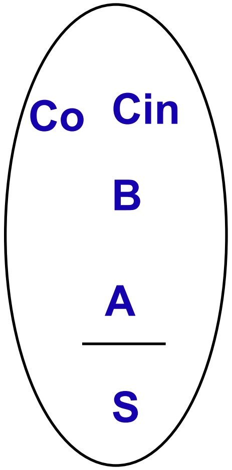
1-bit Half-adder

A_i	B_i	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



1-bit full-adder

```
     1
   0 0 1 1
+  0 0 1 0
  ---------
   0 1 0 1
```



A	B	CI	S	CO
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

N-bit Adder