시계열 분석 및 응용

Assignment #6 (11, 14)

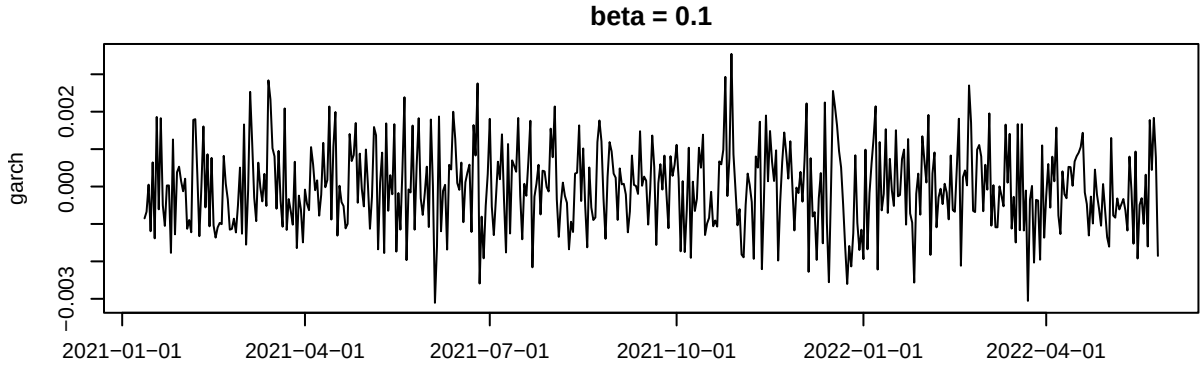
서울대학교 통계학과 2017-11362 박건도

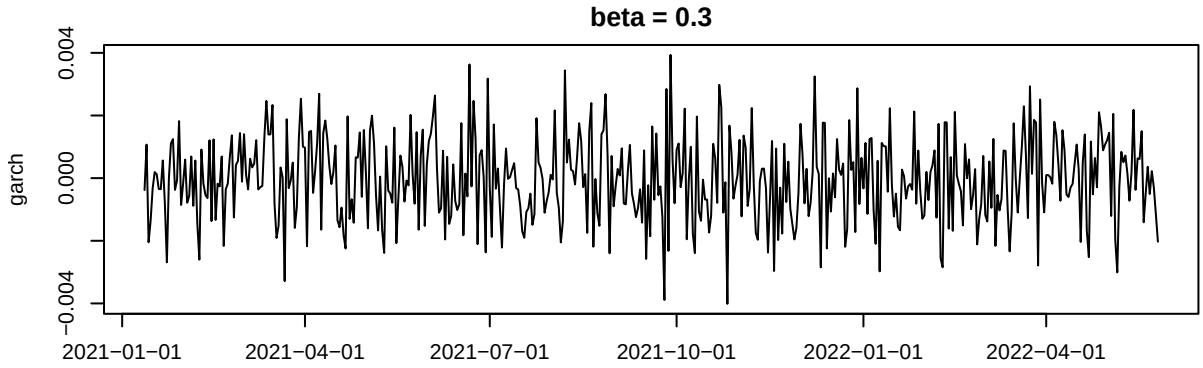
2022년 05월 27일

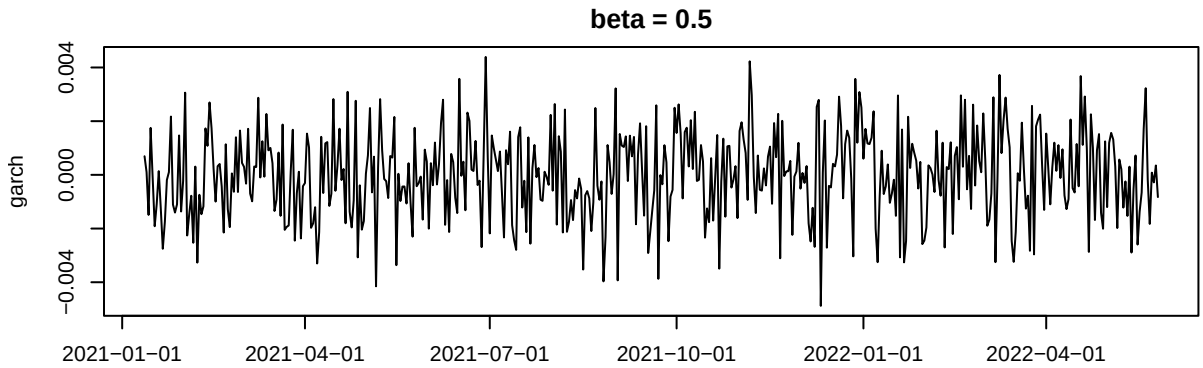
11. GARCH(1,1)

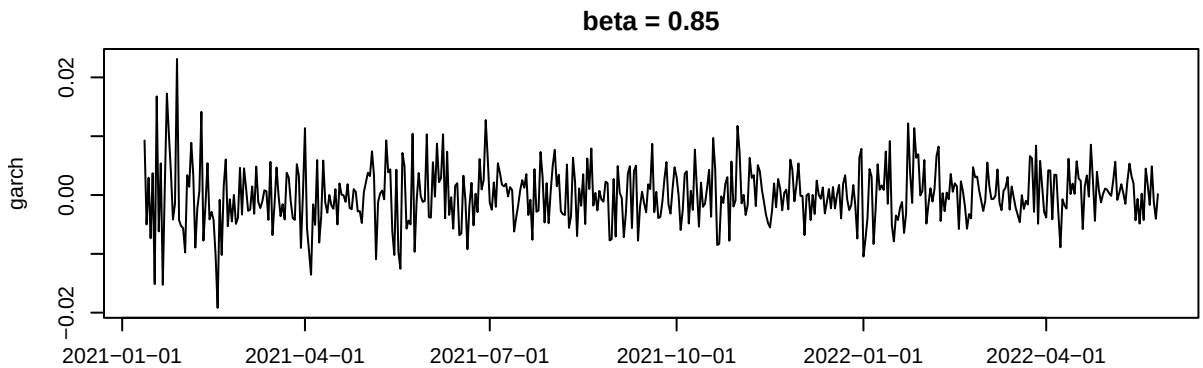
```
plot_garch <- function(omega, alpha, beta, cond){
  par(mfrow=c(4, 1), mai=c(0.3,0.5,0.3,0.3))
  for (b in beta){
    g_spec <- garchSpec(model=list(omega=omega, alpha=alpha, beta=b),
                        cond.dist=cond)
    data <- garchSim(spec=g_spec, n=500)
    plot(data, main=paste0("beta = ", b))
  }
}
```

```r
# omega = 1e-6, alpha = 0.1
plot_garch(1e-6, 0.1, c(0.1, 0.3, 0.5, 0.85), cond="norm")
```

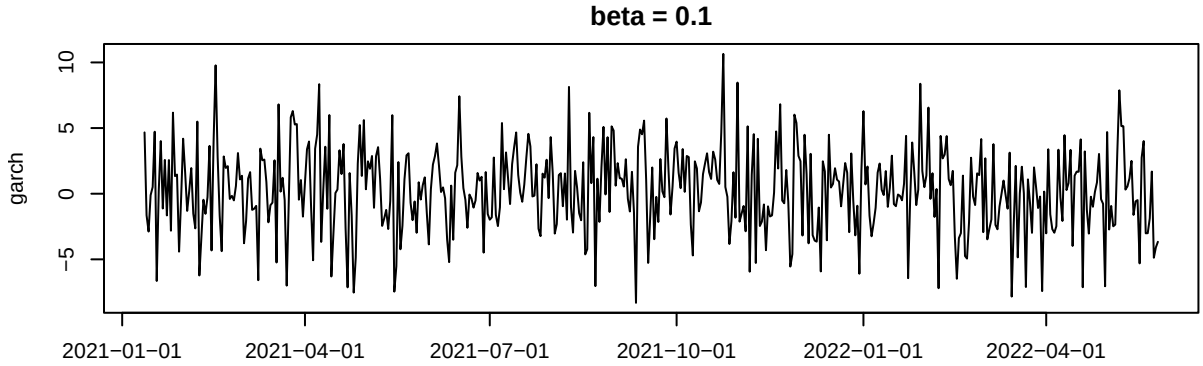


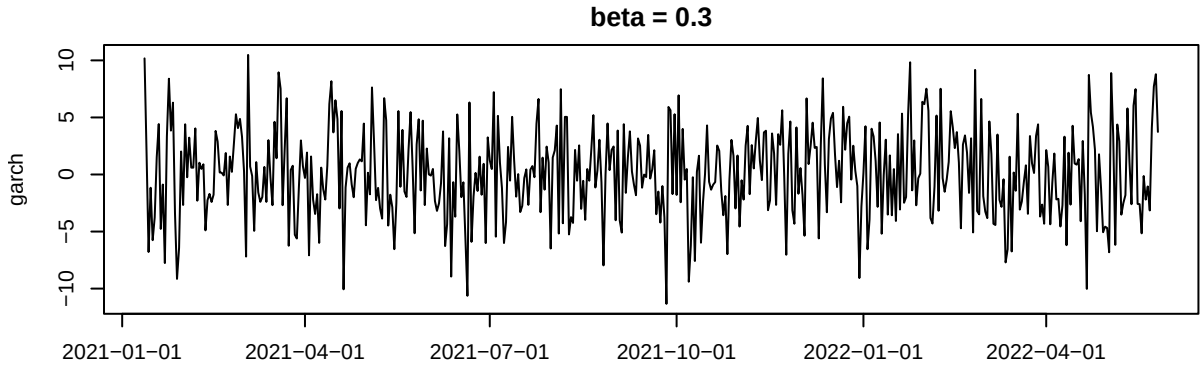


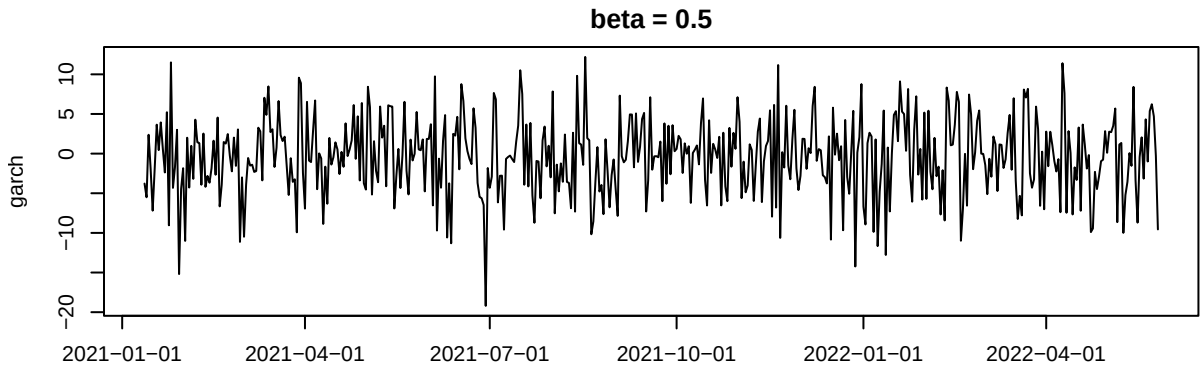


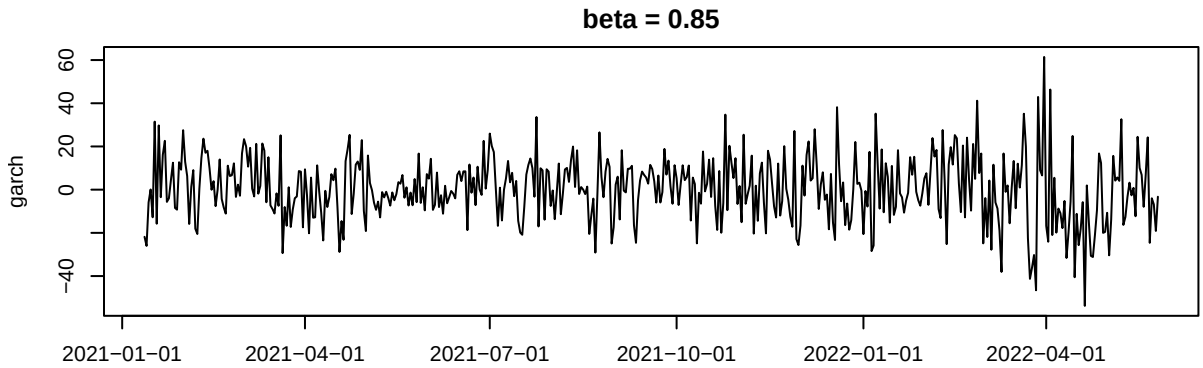


```r
# omega = 10, alpha = 0.1
plot_garch(10, 0.1, c(0.1, 0.3, 0.5, 0.85), cond="norm")
```

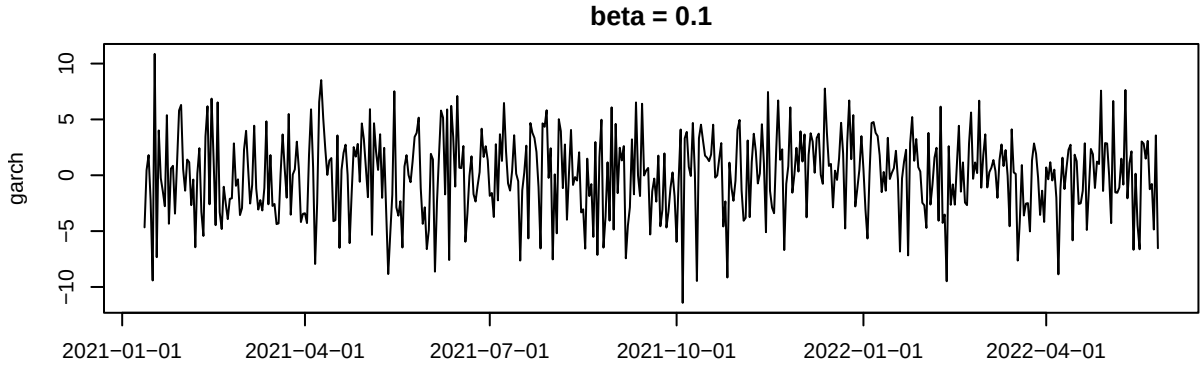


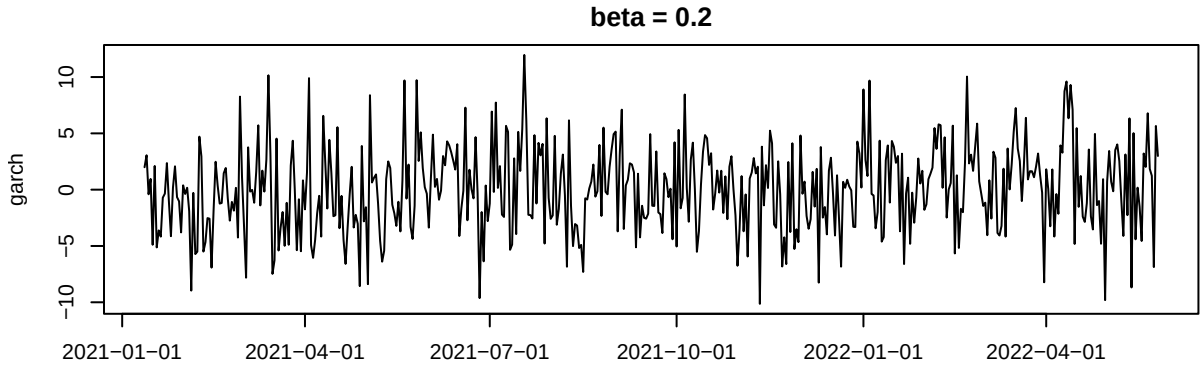


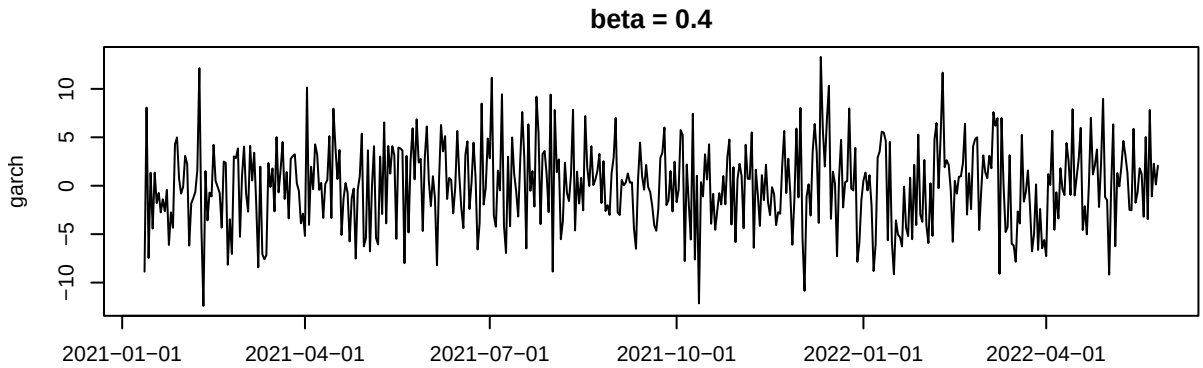


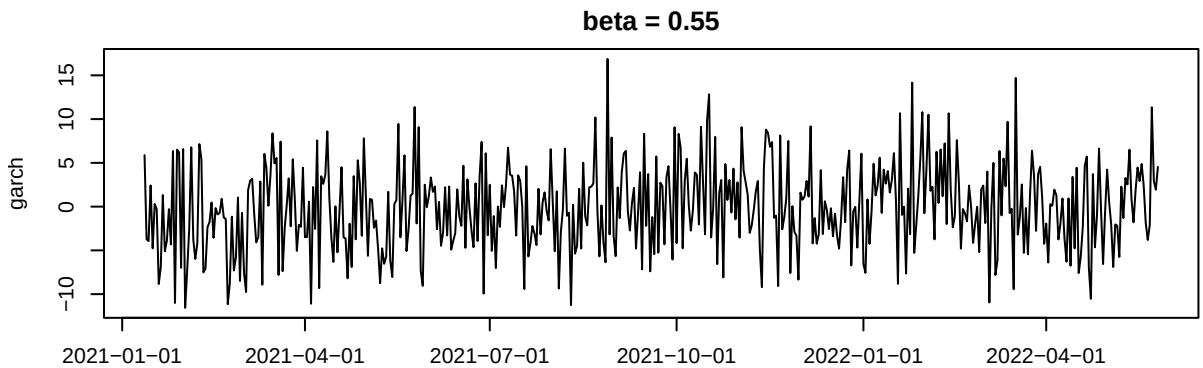


```r
# omega = 1e-6, alpha = 0.4
plot_garch(10, 0.1, c(0.1, 0.2, 0.4, 0.55), cond="norm")
```

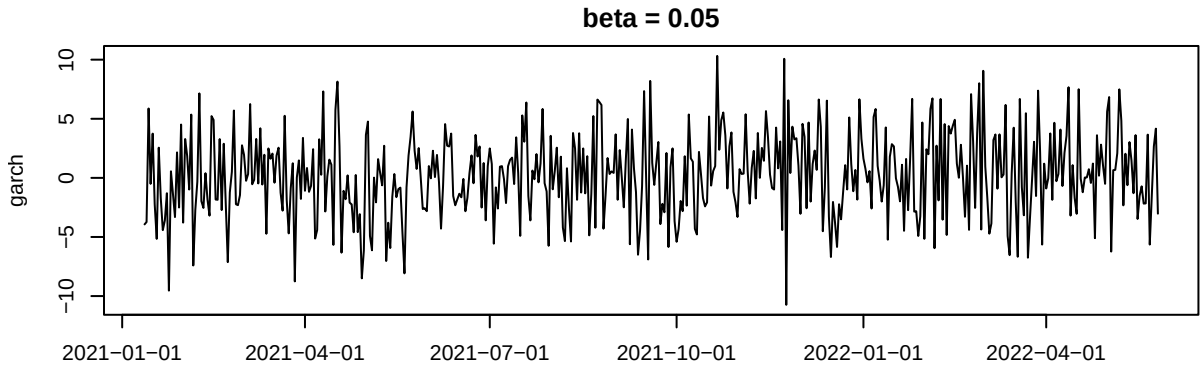


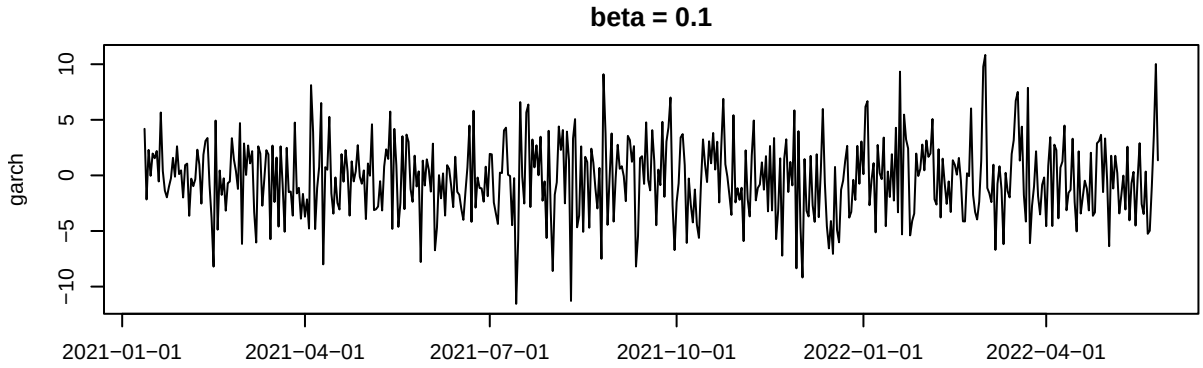


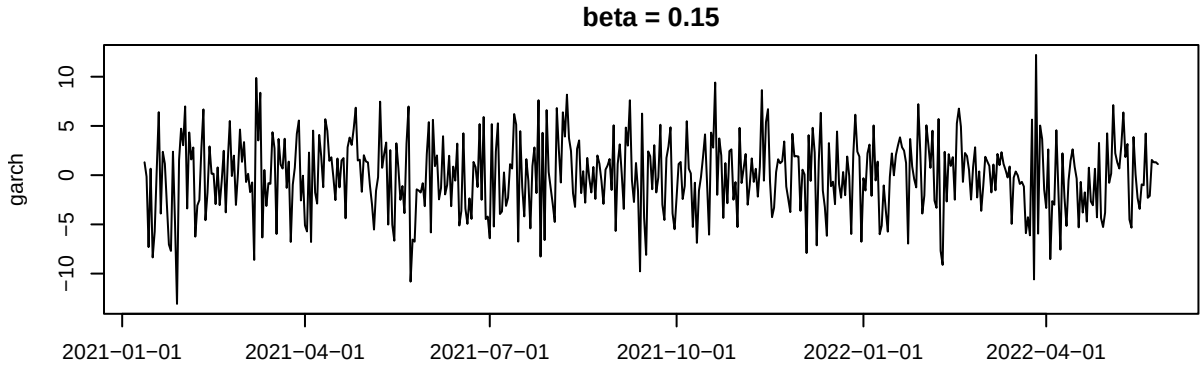


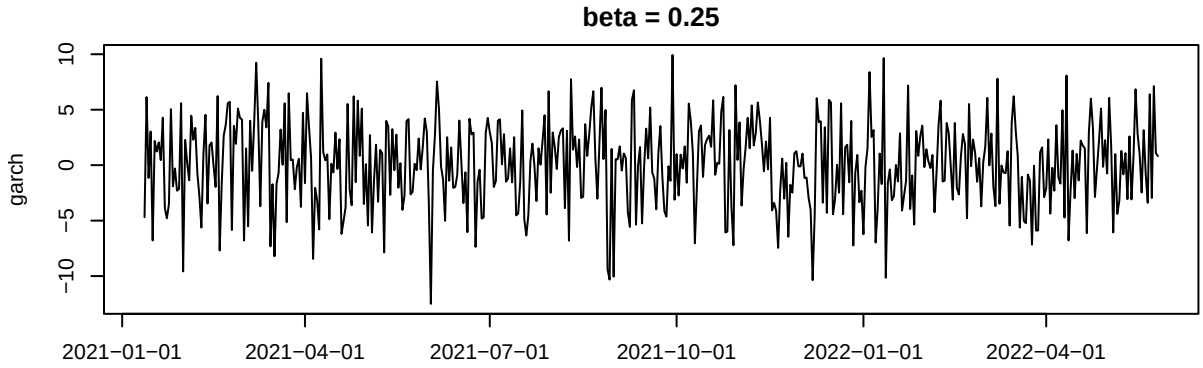


```r
# omega = 1e-6, alpha = 0.7
plot_garch(10, 0.1, c(0.05, 0.1, 0.15, 0.25), cond="norm")
```

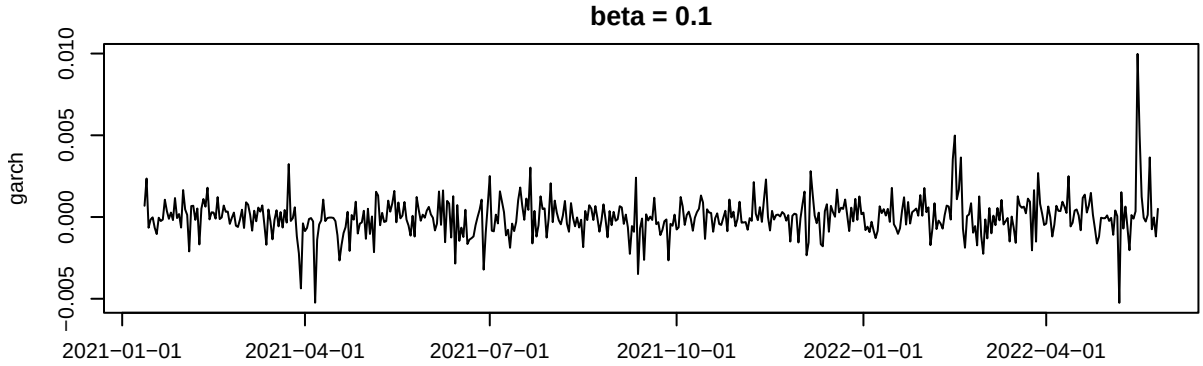


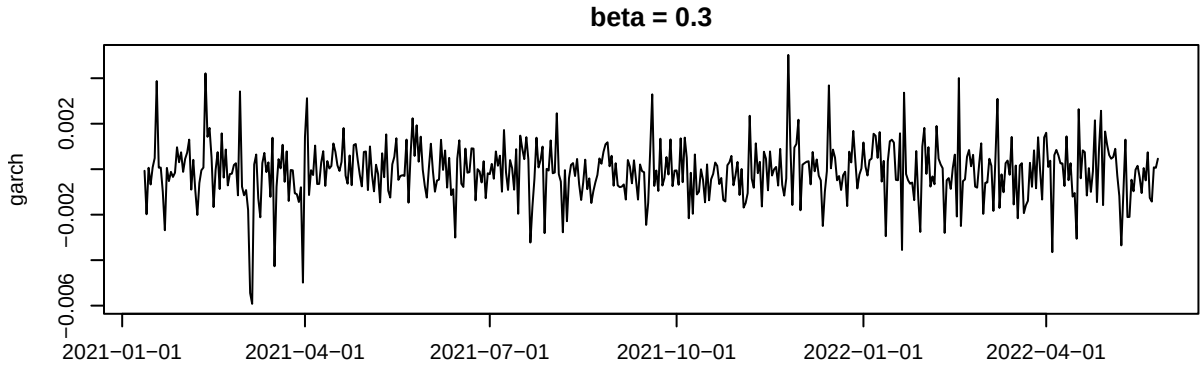


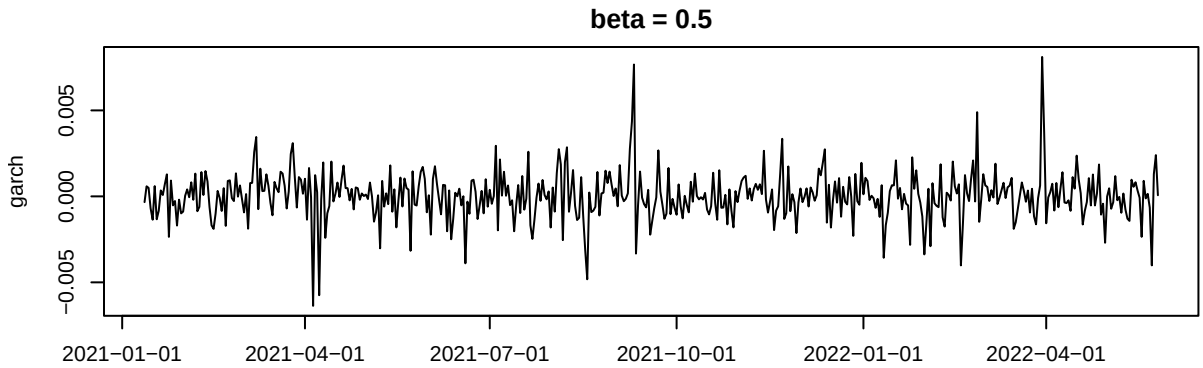


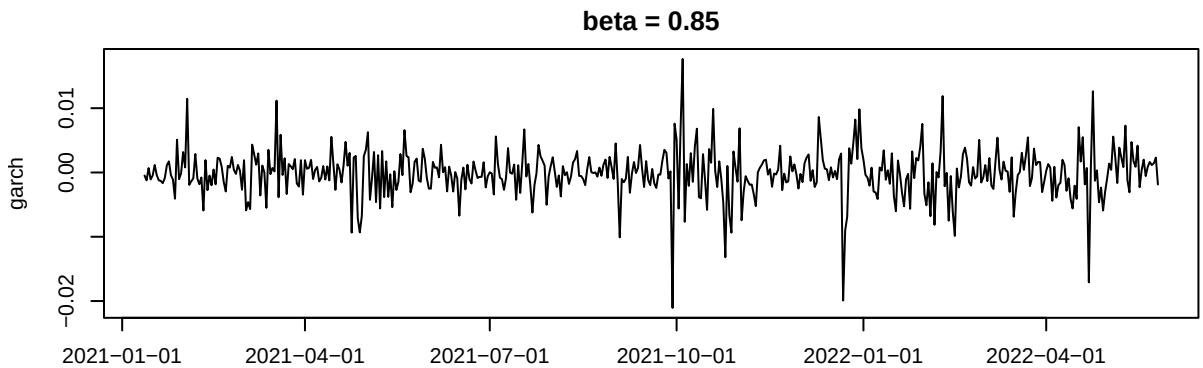


```r
# omega = 1e-6, alpha = 0.1
plot_garch(1e-6, 0.1, c(0.1, 0.3, 0.5, 0.85), cond="std")
```









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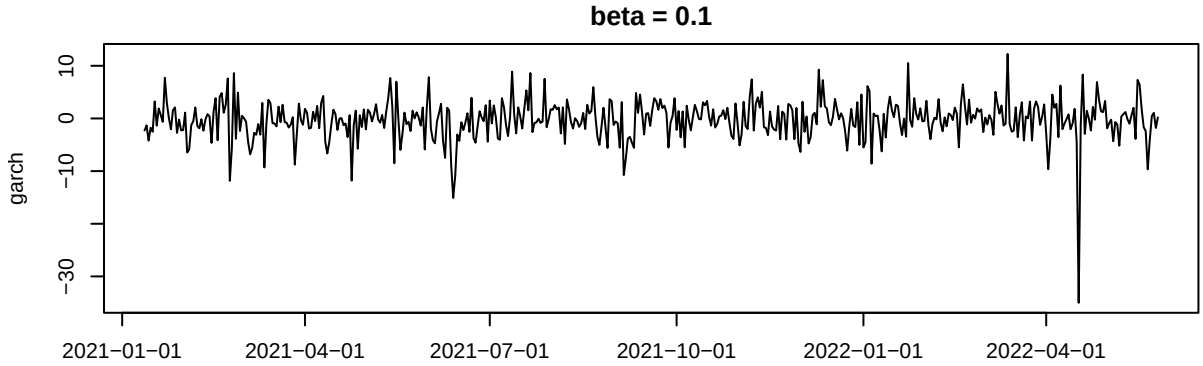

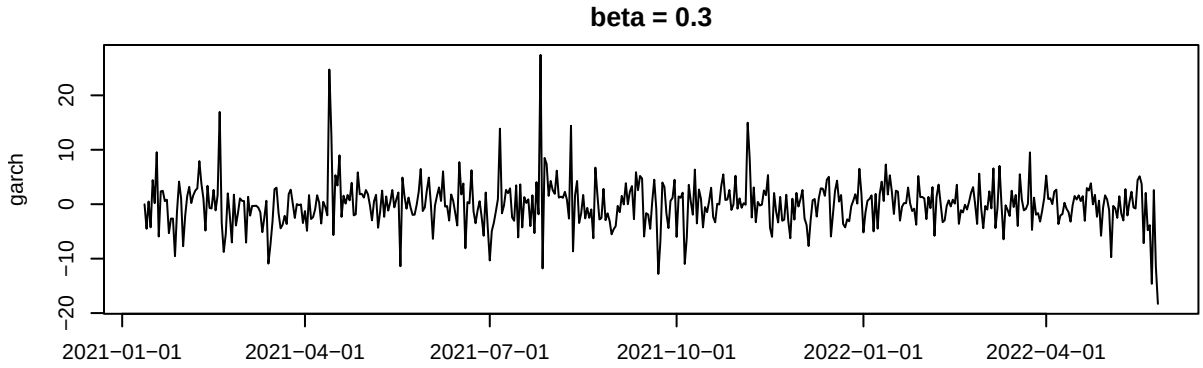

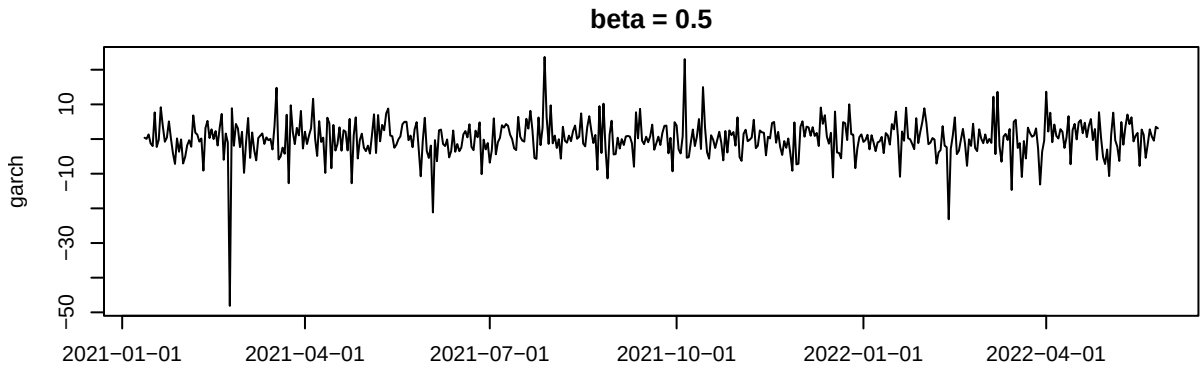

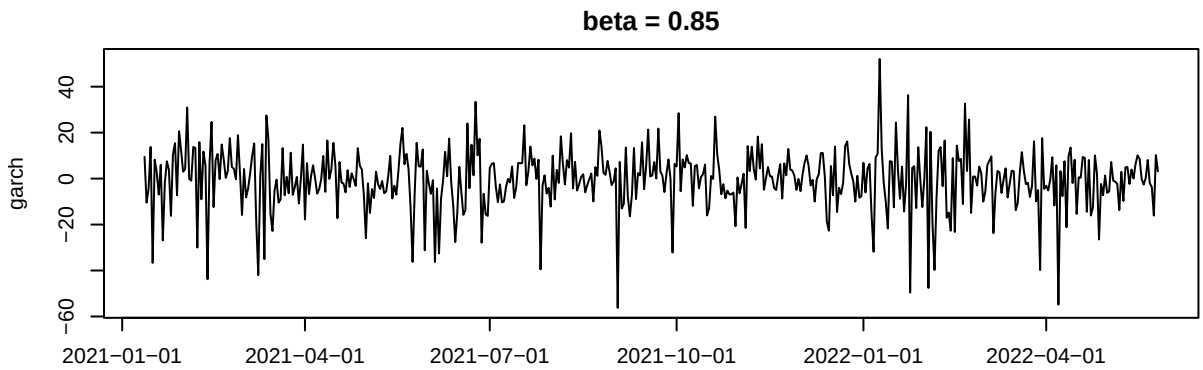

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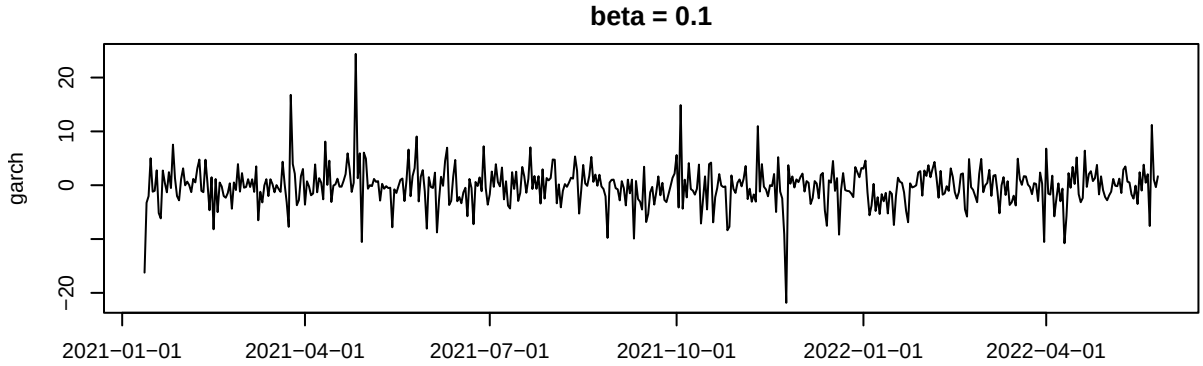
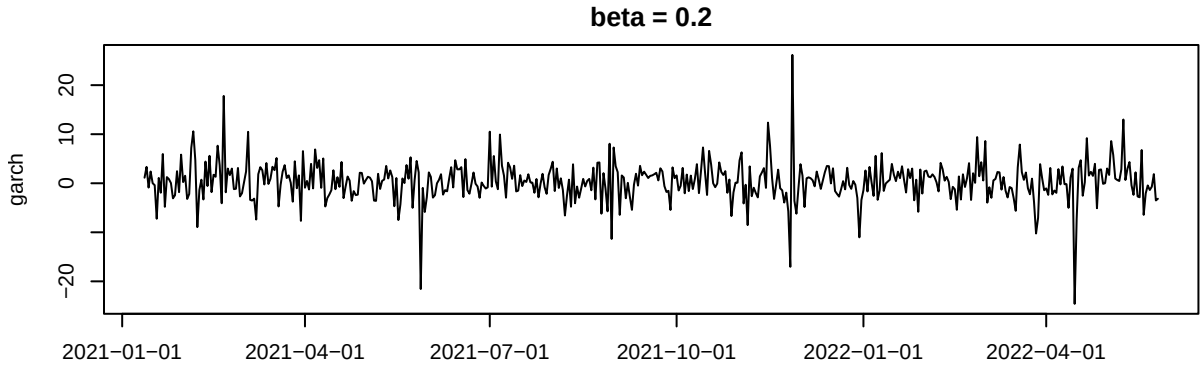
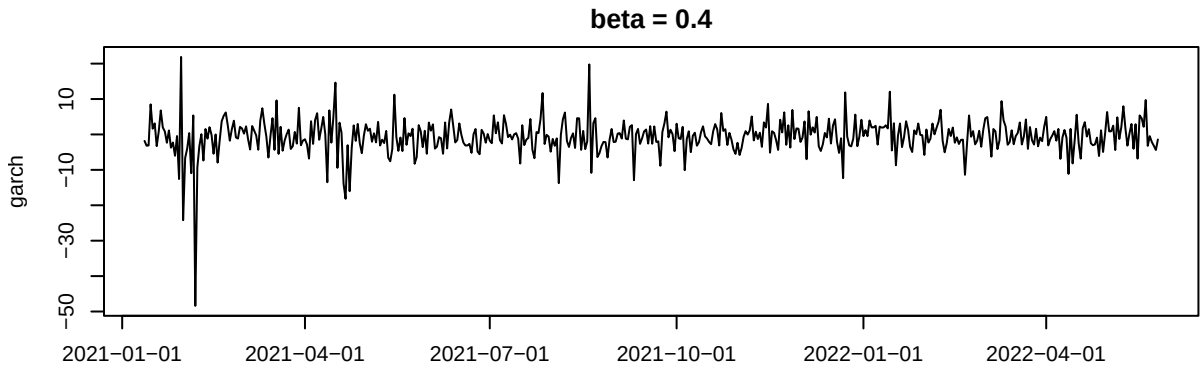
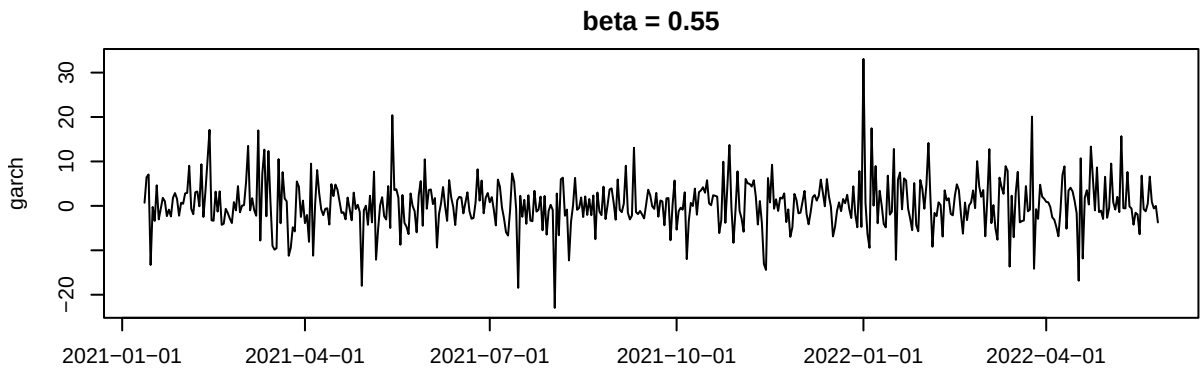
# omega = 1e-6, alpha = 0.4
plot_garch(10, 0.1, c(0.1, 0.2, 0.4, 0.55), cond="std")
```









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```

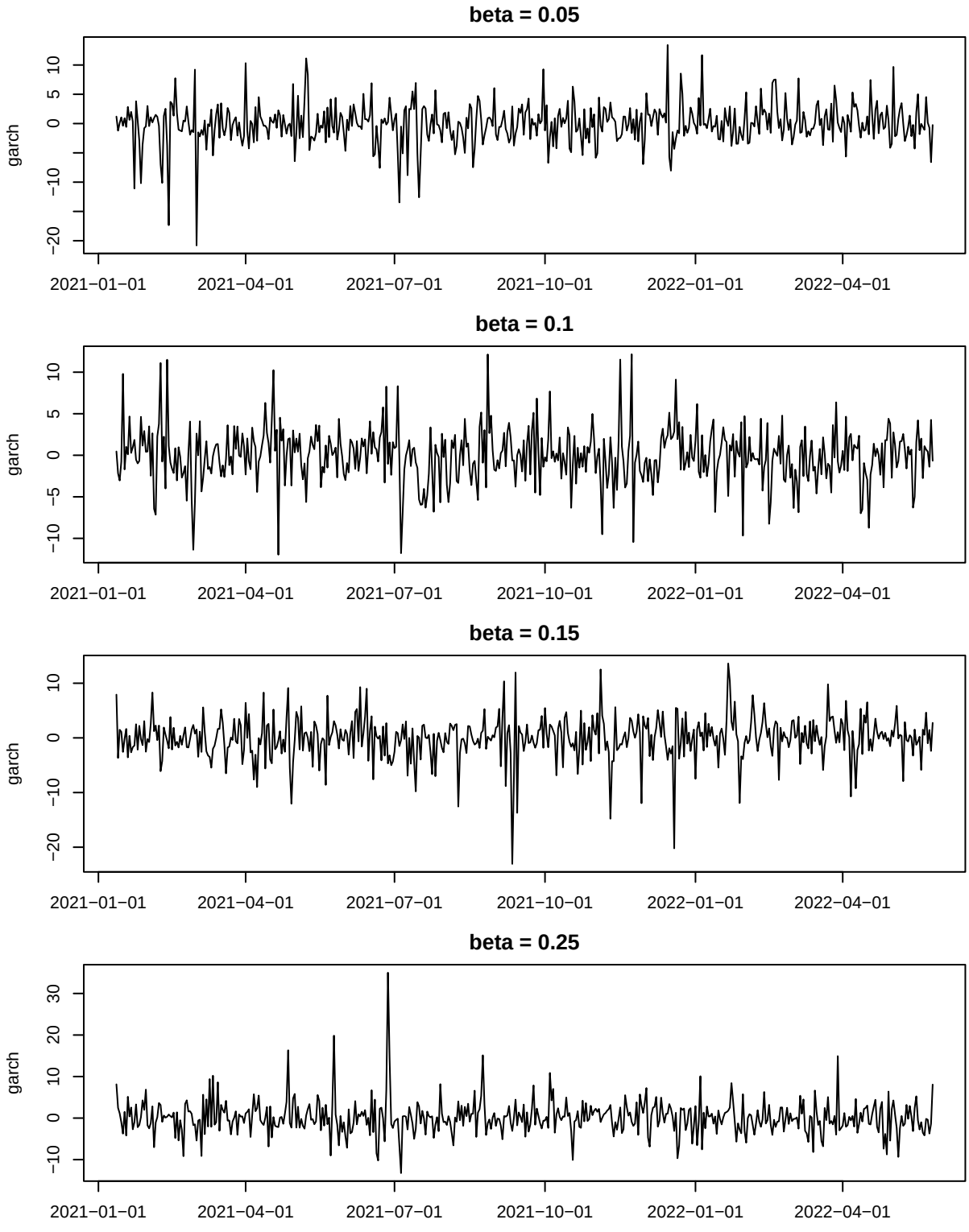

- ξ_t가 정규분포를 따를 때 보다 t분포를 따를 때 변동성이 더 크게 나타났다.
- ω, 즉 α_0이 크면 전체적인 변동 또한 커졌다.
- $\alpha + \beta$가 1에 가까워 질수록 분산이 크게 나타나는 형태를 보인다.
- α가 커질수록 변동성 또한 늘어난다.

14. ex_ch6_14

(1) GARCH(1,1) fitting

```
ex14_dat <- read.table('ex_ch6_14.txt', header=T)
ex14 <- ex14_dat$value
fit_garch <- garchFit(formula = ~garch(1, 1), data=ex14, trace=F)
summary(fit_garch)
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = ex14, trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x5637ea611340>
 [data = ex14]

Conditional Distribution:
 norm

Coefficient(s):
       mu      omega     alpha1      beta1
787.86498    0.50495    0.42963    0.60808

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.879e+02   4.844e-01  1626.55   <2e-16 ***
omega  5.049e-01         NaN      NaN      NaN
```

```
alpha1 4.296e-01   9.723e-03    44.19   <2e-16 ***
beta1  6.081e-01   5.232e-03   116.22   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -10874.44    normalized:  -5.965135

Description:
 Fri May 27 16:43:35 2022 by user:


Standardised Residuals Tests:
                               Statistic p-Value
 Jarque-Bera Test   R    Chi^2  413.8631  0
 Shapiro-Wilk Test  R    W      0.6410472 0
 Ljung-Box Test     R    Q(10)  16302.09  0
 Ljung-Box Test     R    Q(15)  23907.86  0
 Ljung-Box Test     R    Q(20)  31102.22  0
 Ljung-Box Test     R^2  Q(10)  1190.42   0
 Ljung-Box Test     R^2  Q(15)  1200.654  0
 Ljung-Box Test     R^2  Q(20)  1258.905  0
 LM Arch Test       R    TR^2   672.6502  0

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
11.93466 11.94674 11.93465 11.93912
```

(2) Change-point test

```r
### change point detection
# autocovariance function
cov_calc <- function(dat, h) {
  n <- length(dat)
  xmean <- mean(dat)
  x1 <- dat[1:(n-h)] - xmean
  x2 <- dat[(h+1):n] - xmean
  return(sum(x1 * x2/n))
}
```

```r
# long run variance
var_calc <- function(dat, max_h = 2^0.5*(log10(length(dat)))^2 ) {
  n <- length(dat)
  sd2_hat <- cov_calc(dat, 0)
  for(i in 1:max_h) {
    sd2_hat <- sd2_hat + 2*(1-i/n)*(cov_calc(dat, i)) # Bartlett kernel
  }
  return(sd2_hat)
}



CUSUM_calc <- function(dat) {
  ## return : maximum cusum test statistics and change point
  n <- length(dat)
  cusum <- abs((cumsum(dat) - (1:n)/n*sum(dat) ) / ( sqrt(n) * sqrt(var_calc(dat))))
  argmax <- which.max(cusum)
  if(max(cusum)>1.358) return(list("CUSUM_statistics" = max(cusum), "change_point"=argmax))
  else return(print("no change"))
}

CUSUM_calc(ex14)
```

```
$CUSUM_statistics
[1] 3.473803

$change_point
[1] 1051
```

유의수준 5%에서 평균의 변화가 없다는 귀무가설을 기각하고, 그 결론으로 change_point = 1051에서 평균의 변화가 있다는 결론을 내릴 수 있다. 즉, 1997/11/18에서 평균이 변한다고 할 수 있다.