# 시계열분석 및 실습 HW#1 2017-11362 박건도

1. (1) strictly stationary $\not\Rightarrow$ stationary

$X_1 \sim f(x) = \frac{1}{\pi}\frac{1}{1+x^2} \quad (x \in \mathbb{R})$

$X_1 = X_2 = \cdots = X_t = \cdots$

$\Rightarrow P(X_{t_1} \le x_1, \ldots, X_{t_k} \le x_k) = P(X_1 \le \min(x_1, \cdots, x_k))$

$= P(X_{t_1+h} \le x_1, \cdots, X_{t_k+h} \le x_k), \ \forall t, h$ (strictly stationary)

$\mathbb{E}X_t = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x}{1+x^2}dx = \frac{1}{\pi}\int_1^{\infty} \frac{1}{t}dt = \infty$ : does not exist. (not stationary)

(2) stationary $\not\Rightarrow$ strictly stationary

$X_0 \sim Exp(1), \ X_1 \sim Poisson(1), \ X_0 \perp X_1$

$$\begin{cases} X_0 = X_2 = X_4 = \cdots \\ X_1 = X_3 = X_5 = \cdots \end{cases}$$

$\mathbb{E}X_t = 1, \ \mathbb{E}X_t^2 = 2 < \infty, \ \forall t$

$$Cov(X_t, X_{t+h}) = \begin{cases} Cov(X_0, X_1), & h \text{ is odd} \\ Cov(X_0, X_0), & h \text{ is even}, t \text{ is even} \\ Cov(X_1, X_1), & h \text{ is even}, t \text{ is odd} \end{cases} = \begin{cases} 0, & h \text{ is odd} \\ 1, & h \text{ is even} \end{cases}$$

$= Cov(X_0, X_h), \ \forall t, h$ (stationary)

$P(X_0 \le 1, X_1 \le 0) = P(X_0 \le 1)P(X_1 = 0) = \int_0^1 e^{-x}dx \cdot \frac{1}{e} = (1 - \frac{1}{e})\frac{1}{e}$

$P(X_1 \le 1, X_2 \le 0) = P(X_1 \le 1)P(X_2 = 0) = 0$ $\quad \neq$ (not strictly stationary)

2. $X_t = \varepsilon_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim iid(0,1)$

$\gamma(h) = Cov(X_t, X_{t+h}) = Cov(\varepsilon_t + \varepsilon_{t+1}, \varepsilon_{t+h} + \varepsilon_{t+h+1})$

$$= \begin{cases} Cov(\varepsilon_t + \varepsilon_{t+1}, \varepsilon_t, \varepsilon_{t+1}), & h=0 \\ Cov(\varepsilon_t + \varepsilon_{t+1}, \varepsilon_{t+1}, \varepsilon_{t+2}), & h=1 \\ Cov(\varepsilon_t + \varepsilon_{t+1}, \varepsilon_{t+2+h'}, \varepsilon_{t+3+h'}), & h \ge 2 \ (h' = h-2 \ge 0) \end{cases}$$

$$= \begin{cases} 2, & h=0 \\ 1, & h=1 \\ 0, & h \ge 2 \end{cases}$$

Thm 1.2.6 $\Rightarrow \gamma(-h) = \gamma(h)$.

3. $(X,Y) \sim N_2(\mu, \Sigma)$. $\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$

$X|Y \sim N(\mu_x + \Sigma_{12}\Sigma_{22}^{-1}(Y-\mu_y), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$

$\Rightarrow \mathbb{E}(X|Y) = \mu_x + \Sigma_{12}\Sigma_{22}^{-1}(Y-\mu_y)$

$= \Sigma_{12}\Sigma_{22}^{-1} Y + \mu_x - \Sigma_{12}\Sigma_{22}^{-1}\mu_y$

$= aY + b$ $(a = \Sigma_{12}\Sigma_{22}^{-1},\ b = \mu_x - \Sigma_{12}\Sigma_{22}^{-1}\mu_y)$

4. $\mathbb{E}(X_t | X_{t-1}) = \mu + Cov(X_t, X_{t-1}) \cdot Var(X_{t-1})^{-1}(X_{t-1} - \mu)$

$= \mu + \frac{\gamma(1)}{\gamma(0)}(X_{t-1} - \mu)$

5. $\{X_t\} \perp \{Y_t\}$.

$\{X_t\}, \{Y_t\}$ : stationary $\Rightarrow \begin{cases} \mathbb{E}X_t = \mu_x,\ \mathbb{E}Y_t = \mu_y,\ \mathbb{E}X_t^2 < \infty,\ \mathbb{E}Y_t^2 < \infty,\ \forall t \\ \gamma_X(h) = Cov(X_t, X_{t+h}) = Cov(X_0, X_h),\ \gamma_Y(h) = Cov(Y_t, Y_{t+h}) = Cov(Y_0, Y_h) \end{cases}$

$\mathbb{E}(X_t + Y_t) = \mu_x + \mu_y$, $\forall t$

$\mathbb{E}X_tY_t = 0 \Rightarrow \mathbb{E}(X_t+Y_t)^2 = \mathbb{E}(X_t^2 + Y_t^2 + 2X_tY_t) = \mathbb{E}X_t^2 + \mathbb{E}Y_t^2 < \infty$, $\forall t$

$Cov(X_t+Y_t, X_{t+h}+Y_{t+h}) = Cov(X_t, X_{t+h}) + Cov(Y_t, Y_{t+h}) + \underbrace{Cov(X_t, Y_{t+h}) + Cov(Y_t, X_{t+h})}_{=0}$

$= \gamma_X(h) + \gamma_Y(h)$

$\Rightarrow \gamma_{X+Y}(h) = \gamma_X(h) + \gamma_Y(h)$.

$\therefore \{X_t + Y_t\}$ : stationary.

6. $Var(X + tY) \geq 0$, $\forall t \in \mathbb{R}$

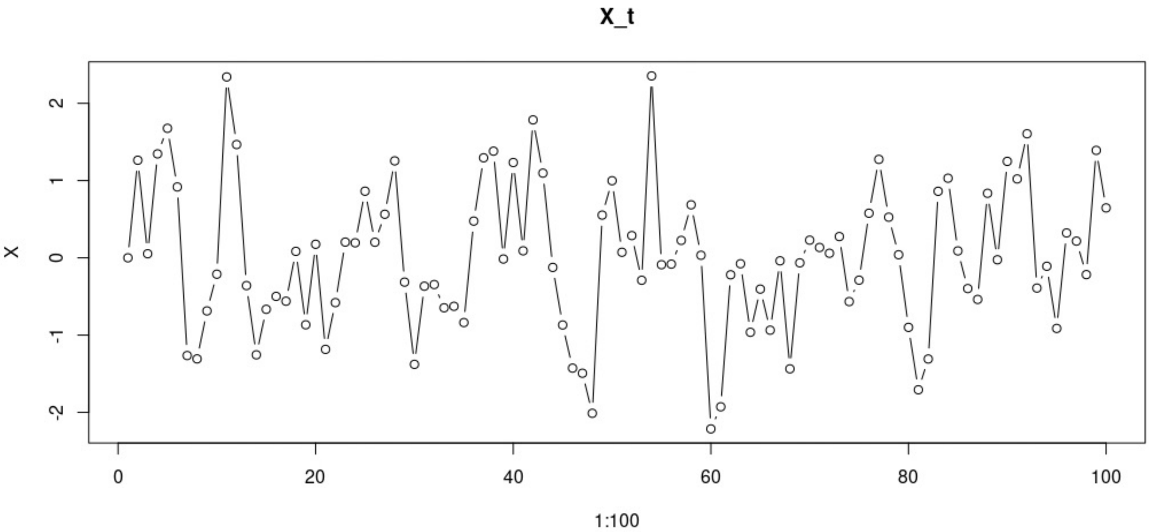
$\Rightarrow Var(X) + t^2 Var(Y) + 2t\, Cov(X,Y) \geq 0$, $\forall t \in \mathbb{R}$

$D/4 = Cov(X,Y)^2 - Var(X)Var(Y) \leq 0$.

$\Rightarrow |Cov(X,Y)| \leq \sqrt{Var(X)Var(Y)}$

$\therefore |Cov(X_t, X_{t+h})| \leq Var(X_t)^{1/2} Var(X_{t+h})^{1/2}$

7.

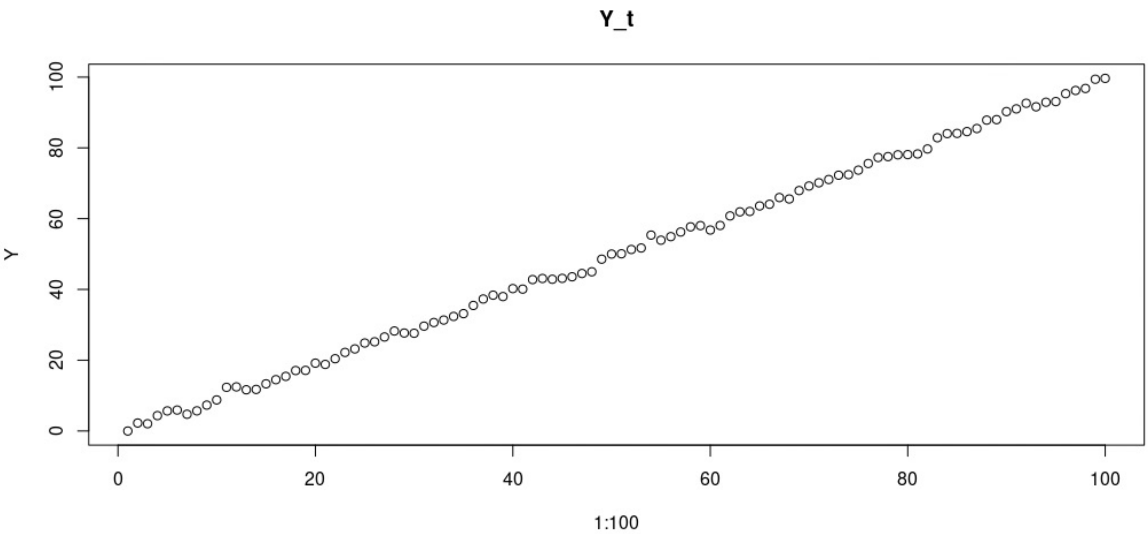



$\mathbb{E}X_t = 0$

$\mathbb{E}X_t^2 < \infty$

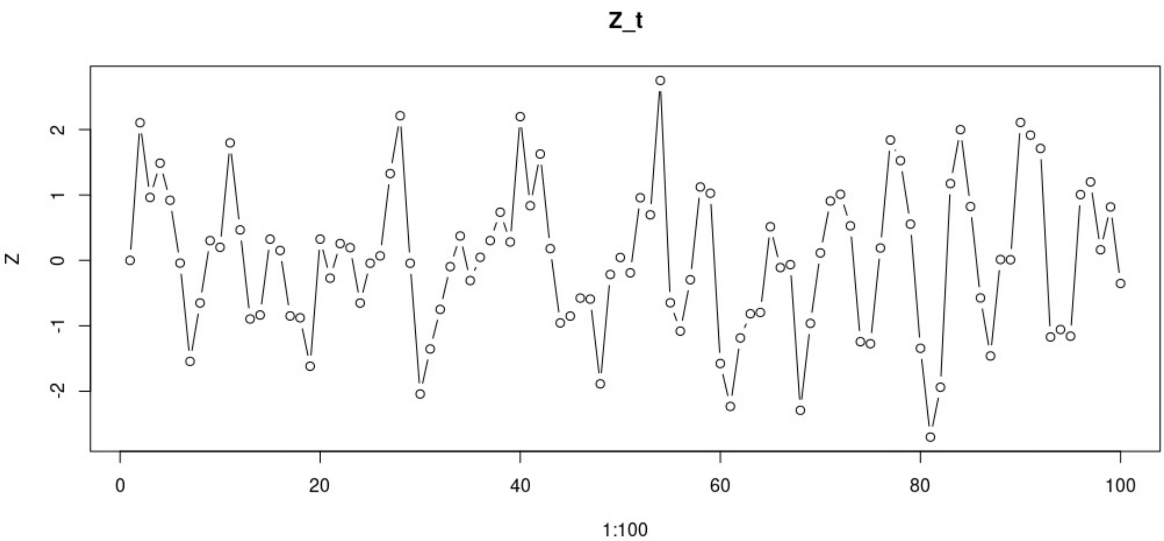
$Cov(X_t, X_{t+h})$
$\parallel$
$Cov(X_0, X_h)$

$\Rightarrow$ stationary




$\mathbb{E}Y_t = t$

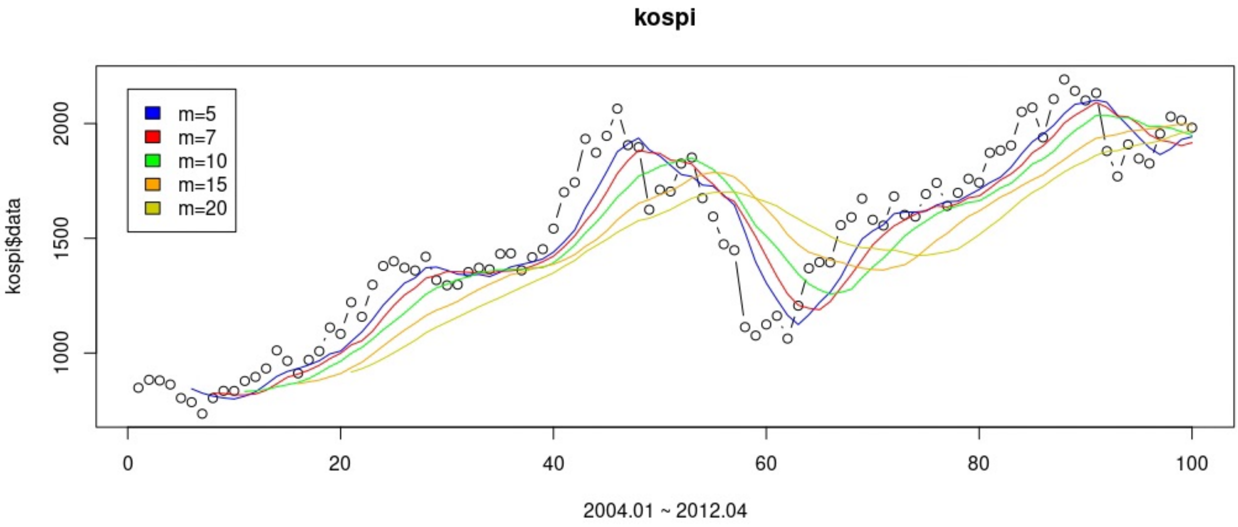
$\Rightarrow$ not stationary



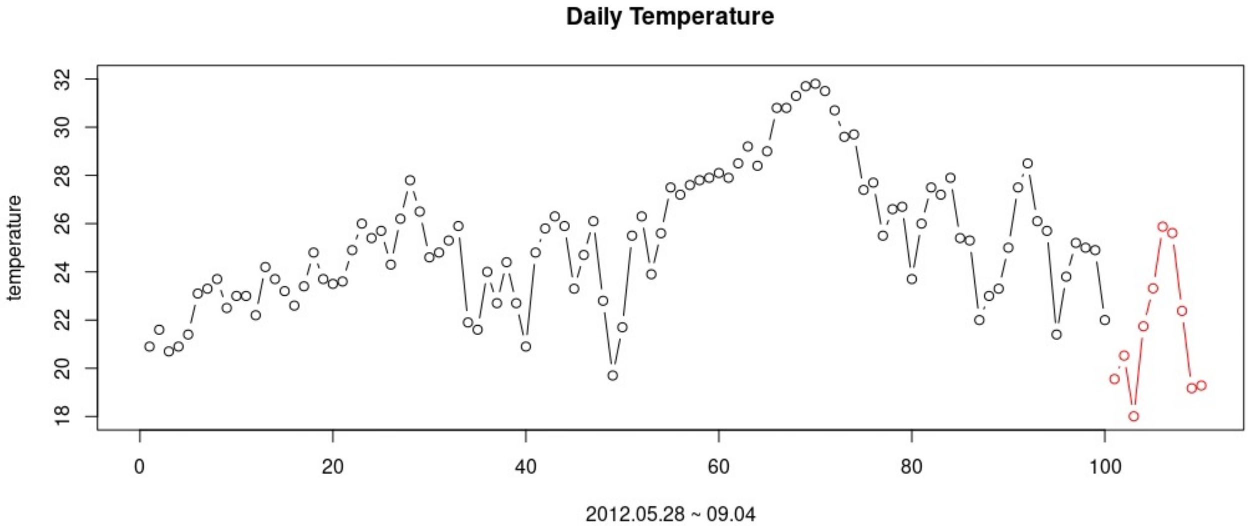

$\mathbb{E}Z_t = \sin t$

$\Rightarrow$ not stationary

8.




9.

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|---|---|---|---|---|
| $a_1 = 0.966$ | $a_5 = 0.286$ | $a_9 = -0.348$ | $a_{13} = -0.164$ | $a_{17} = -0.516$ |
| $a_2 = -0.204$ | $a_6 = -0.270$ | $a_{10} = 0.193$ | $a_{14} = 0.525$ | $a_{18} = 0.259$ |
| $a_3 = 0.256$ | $a_7 = 0.038$ | $a_{11} = -0.395$ | $a_{15} = -0.155$ | $a_{19} = -0.147$ |
| $a_4 = -0.221$ | $a_8 = 0.424$ | $a_{12} = 0.276$ | $a_{16} = 0.002$ | $a_{20} = 0.191$ |

| | | | | |
|---|---|---|---|---|
| $\hat{X}_{101} = 19.55$ | $\hat{X}_{102} = 20.52$ | $\hat{X}_{103} = 18.01$ | $\hat{X}_{104} = 21.74$ | $\hat{X}_{105} = 23.31$ |
| $\hat{X}_{106} = 25.87$ | $\hat{X}_{107} = 25.61$ | $\hat{X}_{108} = 22.38$ | $\hat{X}_{109} = 19.17$ | $\hat{X}_{110} = 19.28$ |