시계열 분석 및 응용

Assignment #2 (6, 7, 8, 10, 12)

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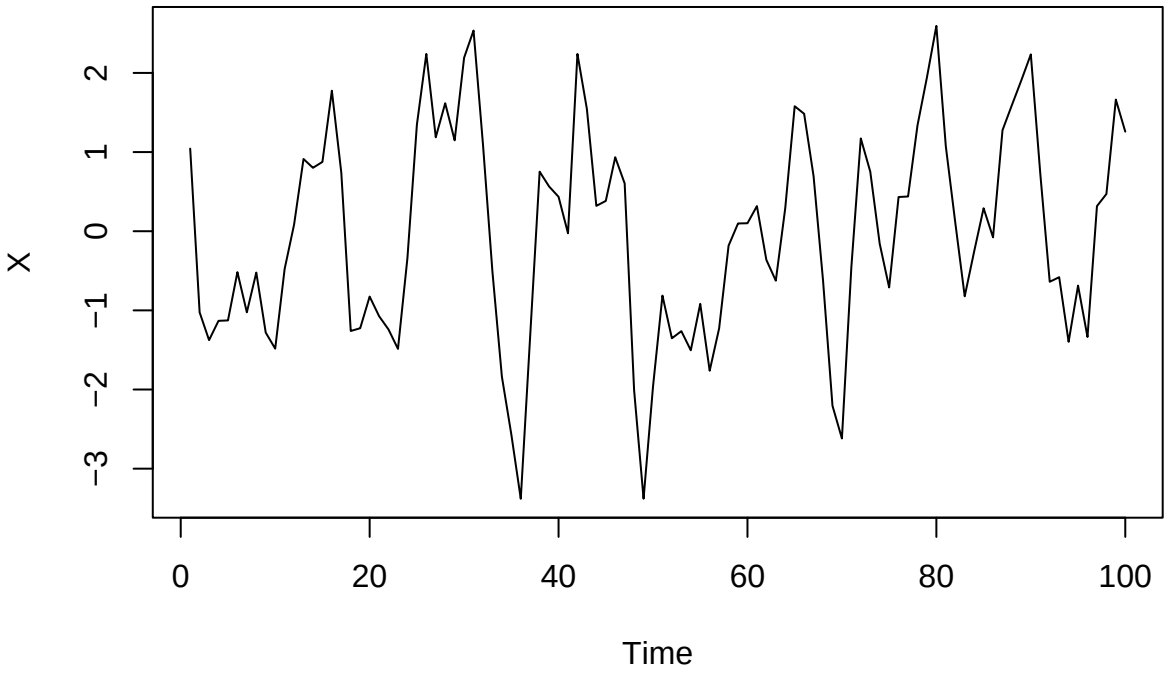
6. ARMA(1,1)

정상 시계열 $X_t = \phi X_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$ 에서 $\phi = 0.5, \theta = 0.5$인 모형을 생각하자. 여기서 $\mu = 0$으로 뒀다.

```
X <- arima.sim(n=100, list(ar=c(0.5), ma=c(0.5)))
plot(X)
```



$\hat{\phi}, \hat{\theta}$를 구하자.

```
X.fit <- arima(X, order=c(1,0,1), include.mean=F)
X.fit
```

```
Call:
arima(x = X, order = c(1, 0, 1), include.mean = F)
```

```
Coefficients:
         ar1     ma1
      0.5593  0.4111
s.e.  0.0987  0.0971

sigma^2 estimated as 0.735:  log likelihood = -126.99,  aic = 259.97
```

$\hat{\phi} = 0.5593, \hat{\theta} = 0.4111.$

$\hat{X}(101), \cdots, \hat{X}(110)$을 구하면 아래와 같다.

```
X.predicted <- forecast(X.fit, h=10)
X.predicted
```

```
    Point Forecast      Lo 80    Hi 80     Lo 95    Hi 95
101    0.578129616 -0.5205757 1.676835 -1.102195 2.258454
102    0.323360689 -1.2076441 1.854366 -2.018109 2.664831
103    0.180862790 -1.4621895 1.823915 -2.331969 2.693694
104    0.101160561 -1.5754077 1.777729 -2.462929 2.665250
105    0.056581341 -1.6303354 1.743498 -2.523335 2.636498
106    0.031647196 -1.6584939 1.721788 -2.553201 2.616495
107    0.017700977 -1.6734476 1.708850 -2.568688 2.604090
108    0.009900548 -1.6815631 1.701364 -2.576970 2.596771
109    0.005537596 -1.6860246 1.697100 -2.581483 2.592559
110    0.003097300 -1.6884957 1.694690 -2.583971 2.590166
```

7. AR(1)

$X_t = 0.7X_{t-1} + \epsilon_t.$

(a) $\epsilon_t \sim iid\, N(0, 1)$.

```
X <- arima.sim(list(ar=c(0.7)), n=100)
X.fit_ML <- arima(X, order=c(1, 0, 0), include.mean=F, method="ML")
X.fit_ML
```

```
Call:
arima(x = X, order = c(1, 0, 0), include.mean = F, method = "ML")

Coefficients:
         ar1
```

```
      0.7156
s.e.  0.0682

sigma^2 estimated as 0.9327:  log likelihood = -138.77,  aic = 281.54
```

```
X.fit_CSS <- arima(X, order=c(1, 0, 0), include.mean=F, method="CSS")
X.fit_CSS
```

```
Call:
arima(x = X, order = c(1, 0, 0), include.mean = F, method = "CSS")

Coefficients:
         ar1
      0.7225
s.e.  0.0692

sigma^2 estimated as 0.9418:  part log likelihood = -138.9
```

(b) $\epsilon_t \sim iid\, t(4)$.

```
rt4 <- function(n) rt(n, 4)
X <- arima.sim(list(ar=c(0.7)), n=100, rand.gen=rt4)
X.fit_ML <- arima(X, order=c(1, 0, 0), include.mean=F, method="ML")
X.fit_ML
```

```
Call:
arima(x = X, order = c(1, 0, 0), include.mean = F, method = "ML")

Coefficients:
         ar1
      0.7323
s.e.  0.0678

sigma^2 estimated as 3.063:  log likelihood = -198.25,  aic = 400.51
```

```
X.fit_CSS <- arima(X, order=c(1, 0, 0), include.mean=F, method="CSS")
X.fit_CSS
```

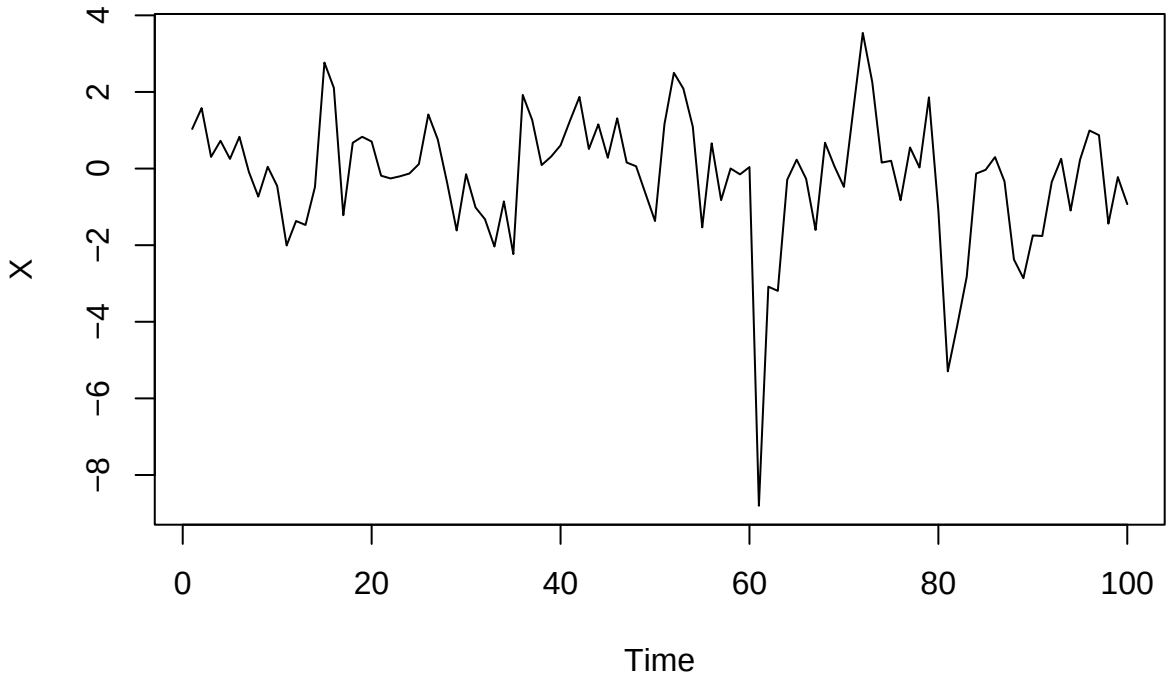
```
Call:
arima(x = X, order = c(1, 0, 0), include.mean = F, method = "CSS")

Coefficients:
```

```
         ar1
      0.7275
s.e.  0.0679

sigma^2 estimated as 3.043:  part log likelihood = -197.54
```

8. AR(2)

$X_t = 0.5X_{t-1} + 0.1X_{t-2} + \epsilon_t$

```
X <- arima.sim(list(ar=c(0.5, 0.1)), n=100, rand.gen=rt4)
plot(X)
```



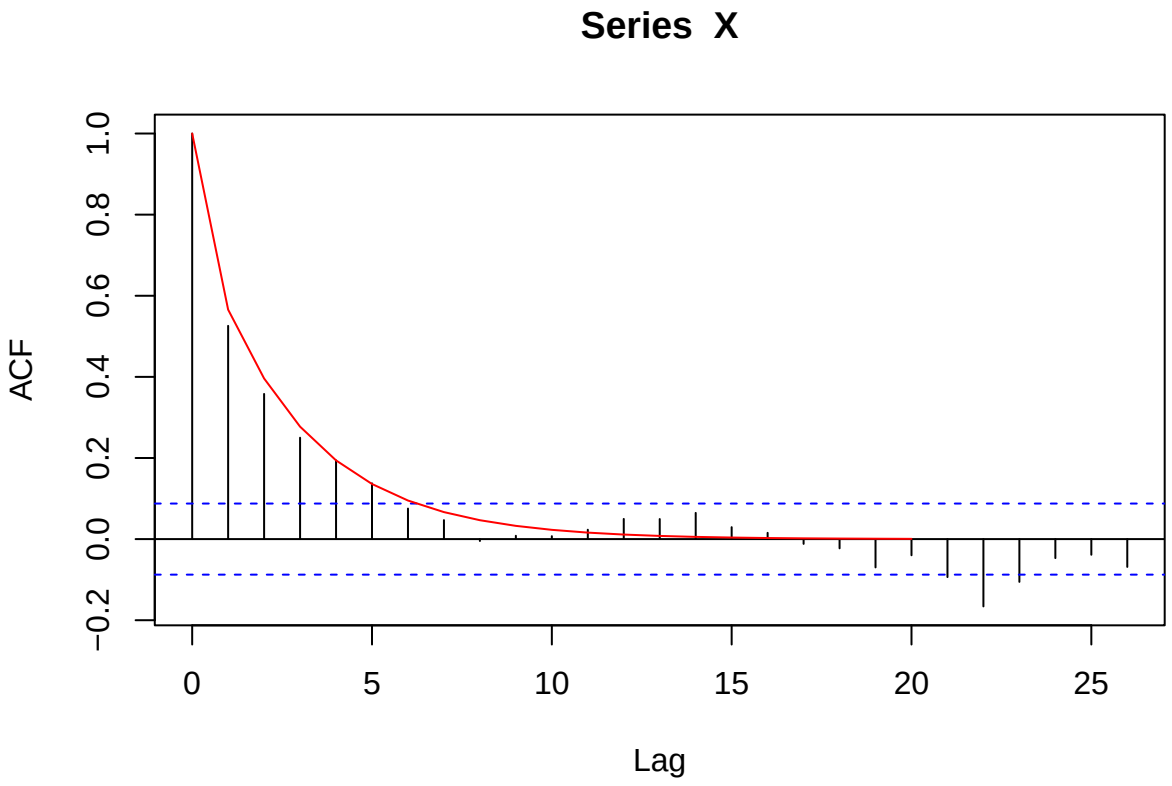

```
X.fit <- arima(X, order=c(2, 0, 0), include.mean=F)

# 95% confidence interval
confint(X.fit)
```

```
          2.5 %    97.5 %
ar1  0.2783471 0.6685124
ar2 -0.2088294 0.1816439
```

10. ARMA(1, 1)

$X_t = 0.7X_{t-1} - 0.2\epsilon_{t-1} + \epsilon_t.$

```
X <- arima.sim(list(ar=c(0.7), ma=c(-0.2)), n=500)
acf(X)
lines(0:20, c(1, 215/266*(0.7)^(1:20)), col='red')
```

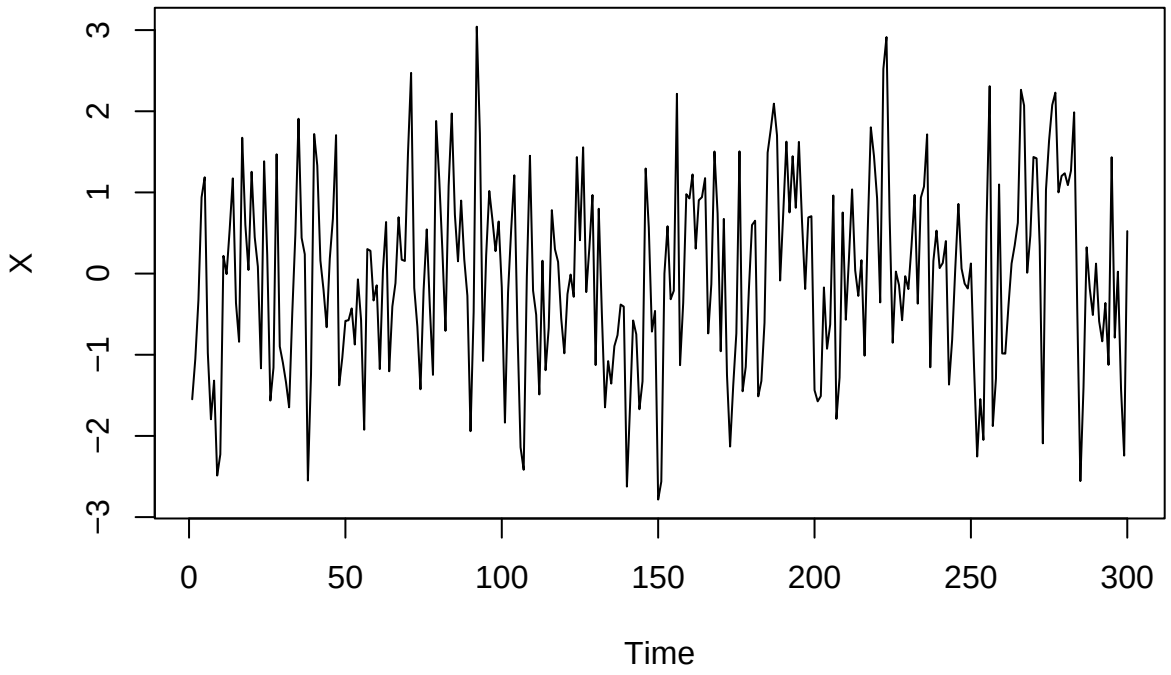


red line이 ideal한 ACF이고, black line은 sampling을 통해 구한 ACF이다.

12. MA(1)

$X_t = 0.5\epsilon_{t-1} + \epsilon_t$.

```
X <- arima.sim(list(ma=c(0.5)), n=300)
plot(X)
```




```
X.fit <- arima(X, order=c(0, 0, 1), include.mean=F)
X.fit
```

```
Call:
arima(x = X, order = c(0, 0, 1), include.mean = F)

Coefficients:
         ma1
      0.4739
s.e.  0.0547

sigma^2 estimated as 1.109:  log likelihood = -441.35,  aic = 886.71
```

```
X.predicted <- forecast(X.fit, h=10)
X.predicted
```

```
    Point Forecast     Lo 80    Hi 80     Lo 95    Hi 95
301      0.5580523 -0.791661 1.907766 -1.506156 2.622261
302      0.0000000 -1.493624 1.493624 -2.284302 2.284302
303      0.0000000 -1.493624 1.493624 -2.284302 2.284302
304      0.0000000 -1.493624 1.493624 -2.284302 2.284302
305      0.0000000 -1.493624 1.493624 -2.284302 2.284302
306      0.0000000 -1.493624 1.493624 -2.284302 2.284302
307      0.0000000 -1.493624 1.493624 -2.284302 2.284302
308      0.0000000 -1.493624 1.493624 -2.284302 2.284302
```

```
309      0.0000000 -1.493624 1.493624 -2.284302 2.284302
310      0.0000000 -1.493624 1.493624 -2.284302 2.284302
```