

Probabilistic
Graphical
Models



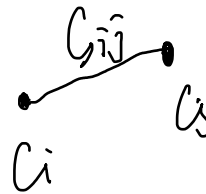
Inference

Message Passing

Cluster Graph Properties

Cluster Graphs

- Undirected graph such that:
 - nodes are clusters $\mathcal{C}_i \subseteq \{X_1, \dots, X_n\}$
 - edge between \mathcal{C}_i and \mathcal{C}_j associated with sepset $\mathcal{S}_{i,j}$ $\subseteq \mathcal{C}_i \cap \mathcal{C}_j$



小量/质

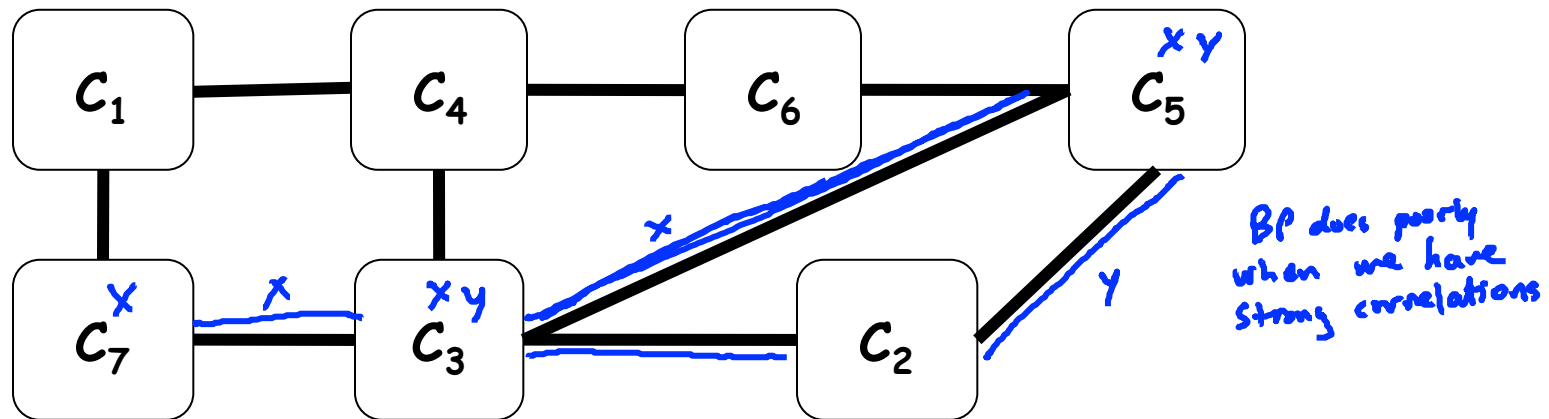
Family Preservation

- Given set of factors Φ , we assign each ϕ_k to a cluster $\mathcal{C}_{\alpha(k)}$ s.t. $\text{Scope}[\phi_k] \subseteq \mathcal{C}_{\alpha(k)}$
cluster can accommodate ϕ_k
- For each factor $\phi_k \in \Phi$, there exists a cluster \mathcal{C}_i s.t. $\text{Scope}[\phi_k] \subseteq \mathcal{C}_i$ *← accommodates ϕ_k*

11.8/12.2.

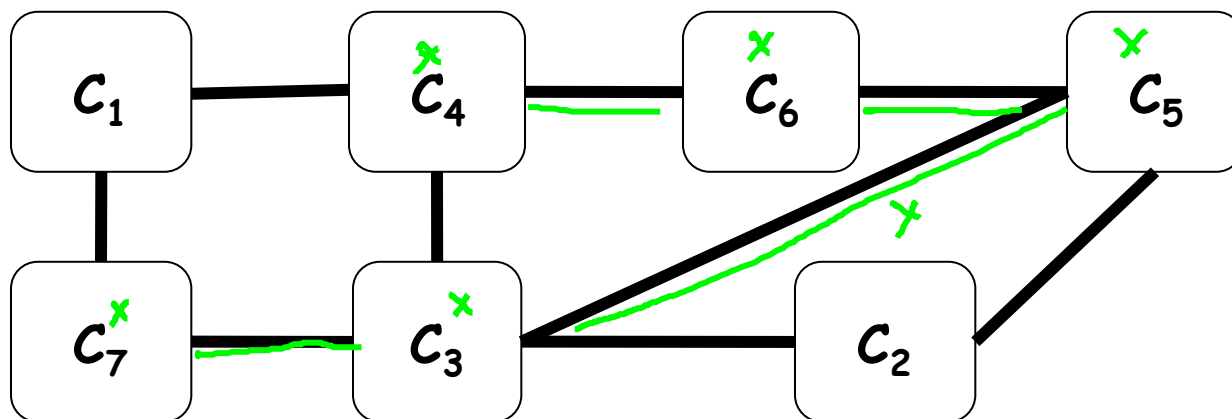
Running Intersection Property

- For each pair of clusters C_i, C_j and variable $X \in C_i \cap C_j$ there exists a unique path between C_i and C_j for which all clusters and sepsets contain X

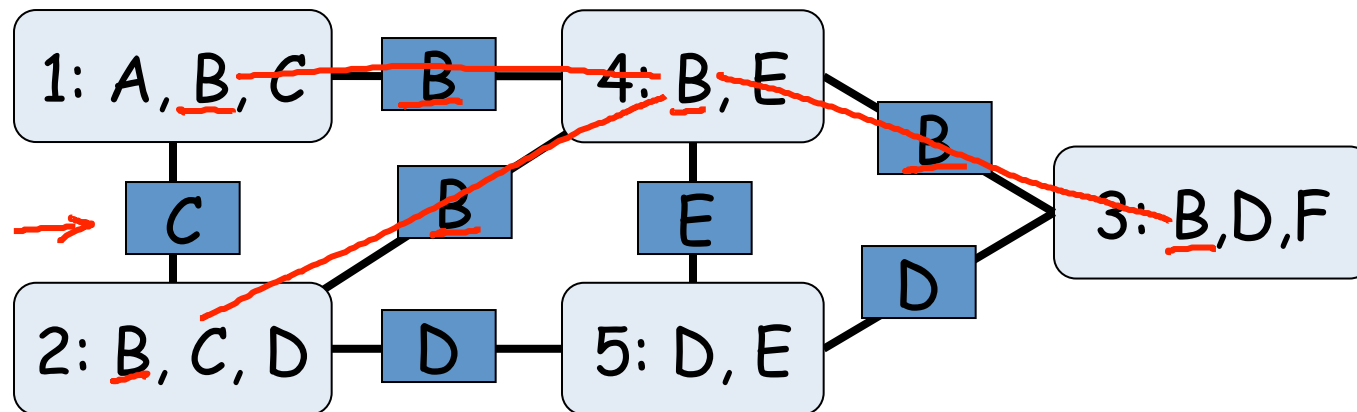


Running Intersection Property

- Equivalently: For any X , the set of clusters and sepsets containing X forms a tree



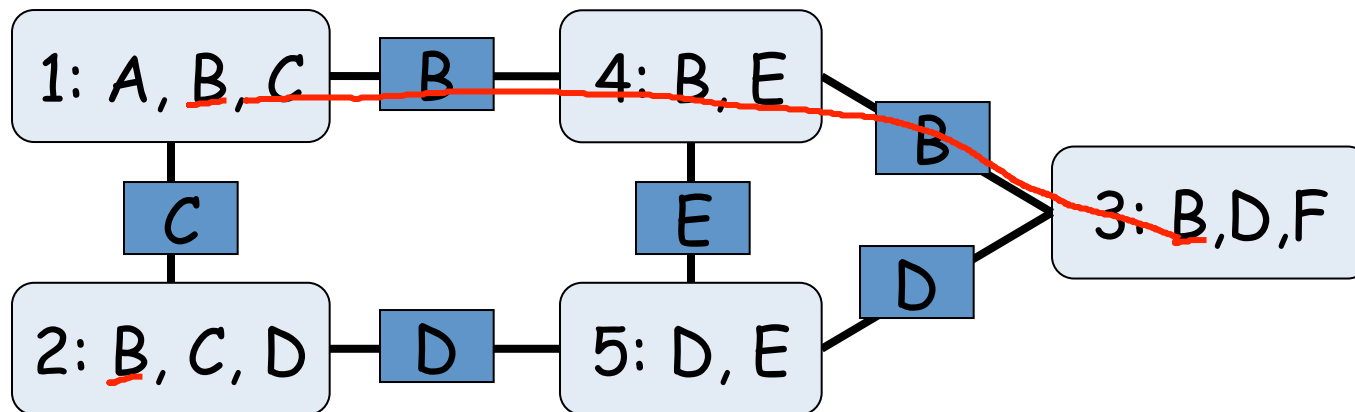
Example Cluster Graph



Illegal Cluster Graph I

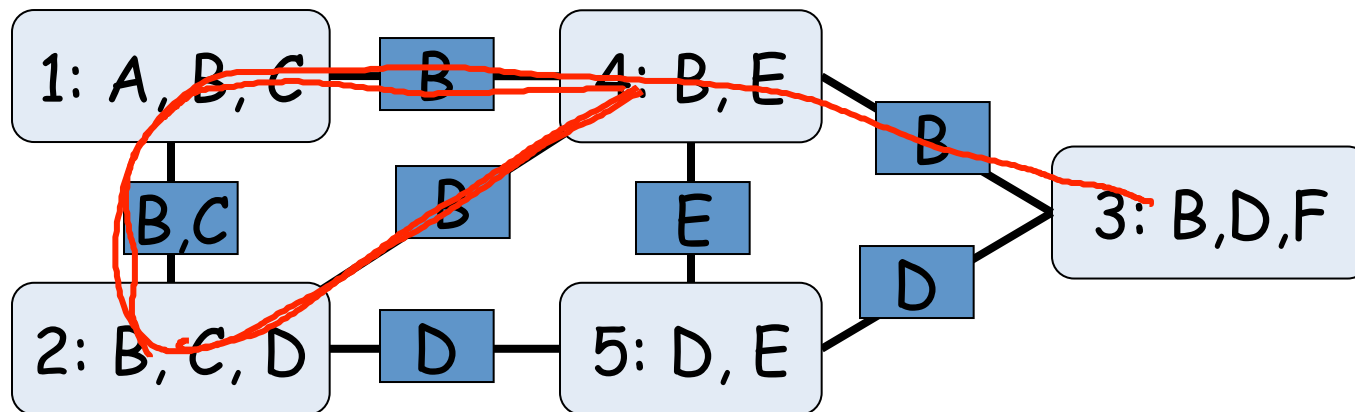
violates existence

（非）存在性，4. 2 都有 B，但不连通

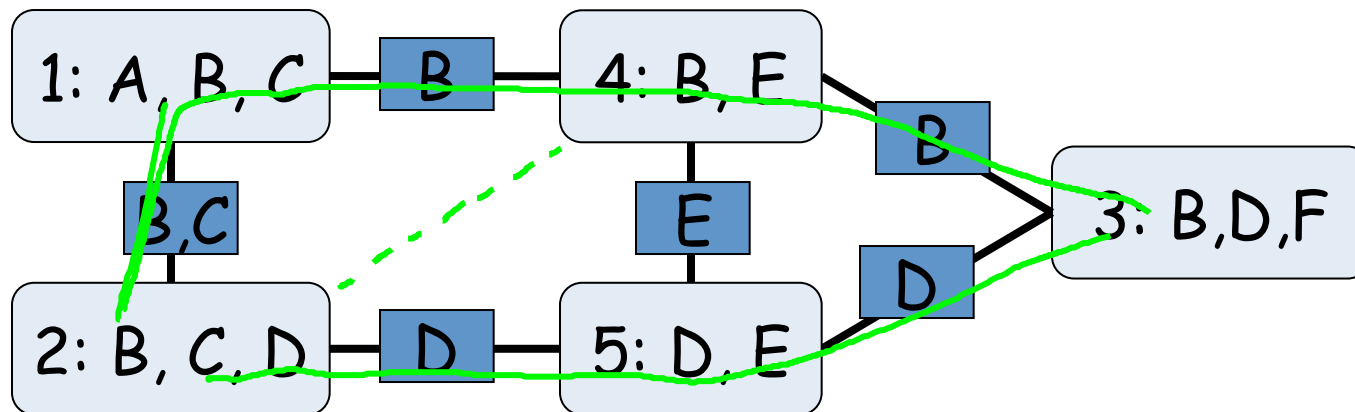


Illegal Cluster Graph II

violates uniqueness



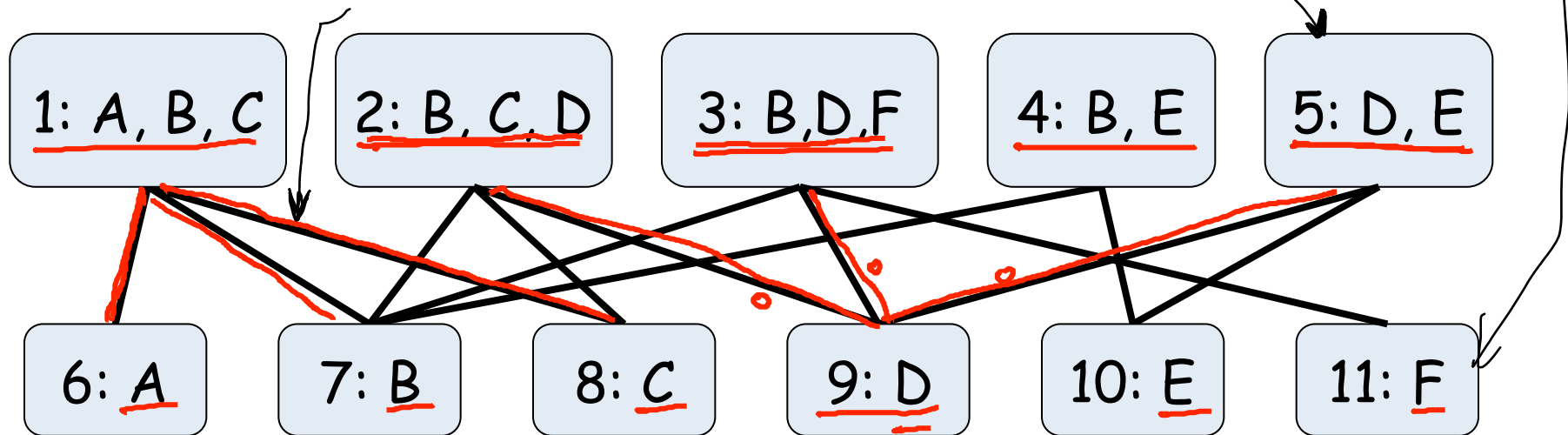
Alternative Legal Cluster Graph



Bethe Cluster Graph

big clusters = factor in Φ

- For each $\phi_k \in \Phi$, a factor cluster $C_k = \text{Scope}[\phi_k]$
- For each X_i a singleton cluster $\{X_i\}$
- Edge $C_k - X_i$ if $X_i \in C_k$



Summary

- Cluster graph must satisfy two properties
 - family preservation: allows Φ to be encoded
 - running intersection: connects all information about any variable, but without feedback loops 无环
- Bethe cluster graph is often first default
- Richer cluster graph structures can offer different tradeoffs wrt computational cost and preservation of dependencies