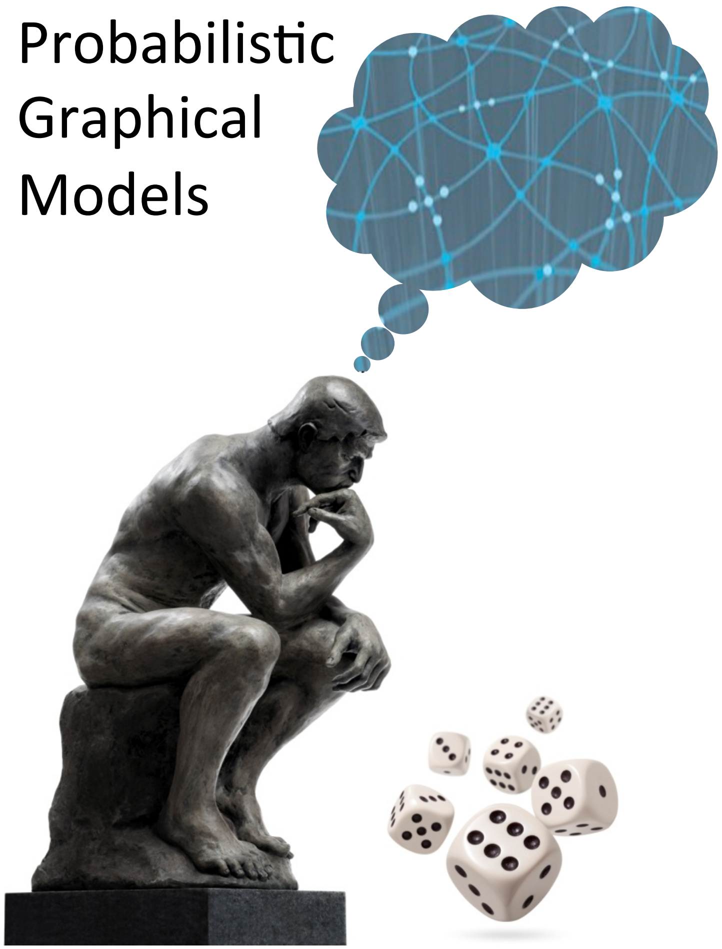
你不只是利用数据，算出结果，
而是需要做决策。

Probabilistic
Graphical
Models



Acting

Decision Making

Maximum Expected Utility

Simple Decision Making

A simple decision making situation $\mathcal{D}$:

- A set of possible actions $Val(A)=\{a^1,\ldots,a^K\}$ 动作
- A set of states $Val(\mathbf{X}) = \{\mathbf{x}^1,\ldots,\mathbf{x}^N\}$ 状态
- A distribution $P(\mathbf{X} \mid A)$
- A utility function $U(\mathbf{X}, A)$ prefer 反映选择

Expected Utility

选择结果的期望

$$\mathrm{EU}[\mathcal{D}[a]] = \sum_{\boldsymbol{x}} \underbrace{P(\boldsymbol{x} \mid a)}\underbrace{U(\boldsymbol{x}, a)}$$

- Want to choose action a that maximizes the expected utility

Max. expected utility

$$\boxed{a^* = \mathrm{argmax}_a \mathrm{EU}[\mathcal{D}[a]]}$$

最好的选择.

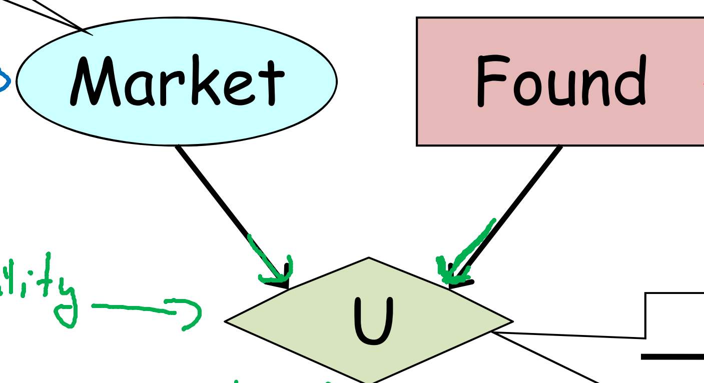
Simple Influence Diagram

poor moderate great

m^0	m^1	m^2
0.5	0.3	0.2

is an action, not a random variable, not have CPD.

random variables



Action

Utility

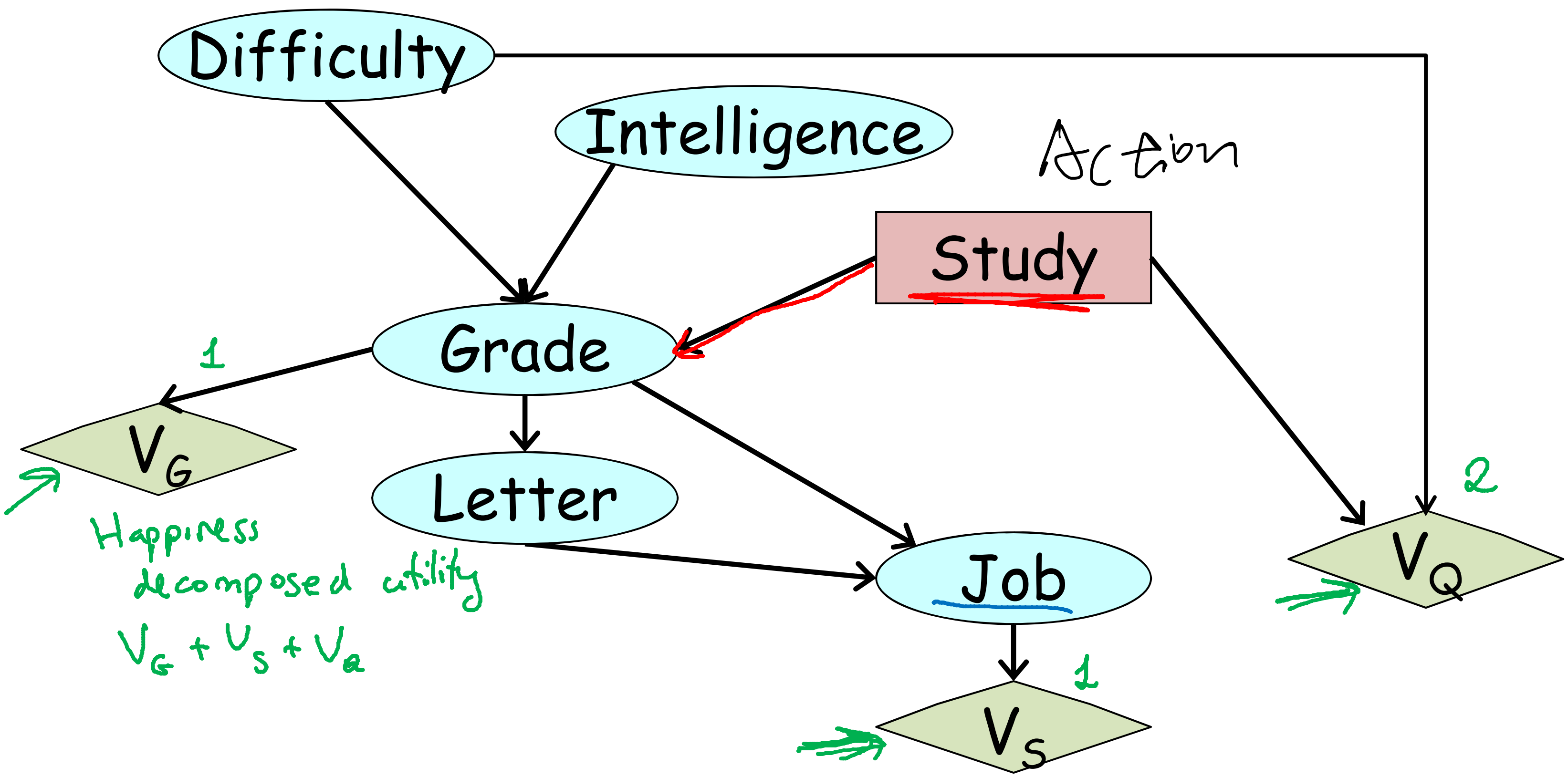
$U(F,M)$

$EU(f^0) = 0$

$\boxed{EU(f^1)} = 0.5 \times -7 + 0.3 \times 5 + 0.2 \times 20 = 2$

	f^0 (do not found)	f^1 (found)
m^0	0	-7
m^1	0	5
m^2	0	20

More Complex Influence Diagram



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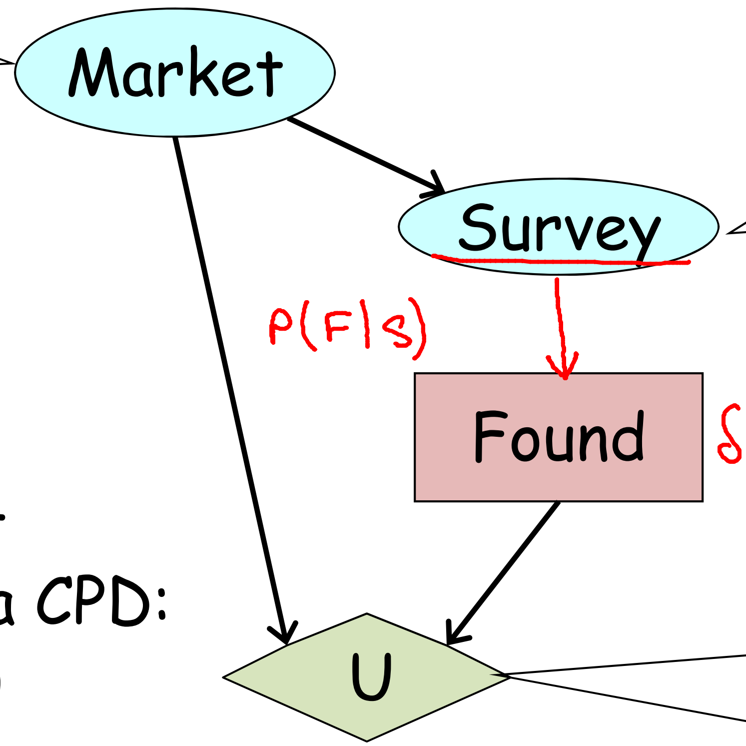
Information Edges

m^0	m^1	m^2
0.5	0.3	0.2



CPD P(S|M)

	s^0	s^1	s^2
m^0	0.6	0.3	0.1
m^1	0.3	0.4	0.3
m^2	0.1	0.4	0.5

	f^0	f^1
m^0	0	-7
m^1	0	5
m^2	0	20

Decision rule δ at action node A is a CPD: $P(A \mid \text{Parents}(A))$

$A \rightarrow$ 没有CPD 不是变量

$\delta_A \rightarrow$ 有CPD.

Expected Utility with Information

joint prob. dist over $X \cup \{A\}$

$$\mathrm{EU}[\mathcal{D}[\boxed{\delta_A}]] = \sum_{\boldsymbol{x},a} \underbrace{P_{\delta_A}(\boldsymbol{x},a)}_{\Delta} \underbrace{U(\boldsymbol{x},a)}$$

- Want to choose the decision rule $\boxed{\delta_A}$ that maximizes the expected utility

$$\mathrm{argmax}_{\delta_A} \mathrm{EU}[\mathcal{D}[\delta_A]]$$

$$\mathrm{MEU}(\mathcal{D}) = \max_{\delta_A} \mathrm{EU}[\mathcal{D}[\delta_A]]$$

Finding MEU Decision Rules

优化目标.

optimize

$$\mathrm{EU}[\mathcal{D}[\delta_A]] = \sum_{\boldsymbol{x},a} P_{\delta_A}(\boldsymbol{x},a)U(\boldsymbol{x},a)$$

表示整个组图.

factor

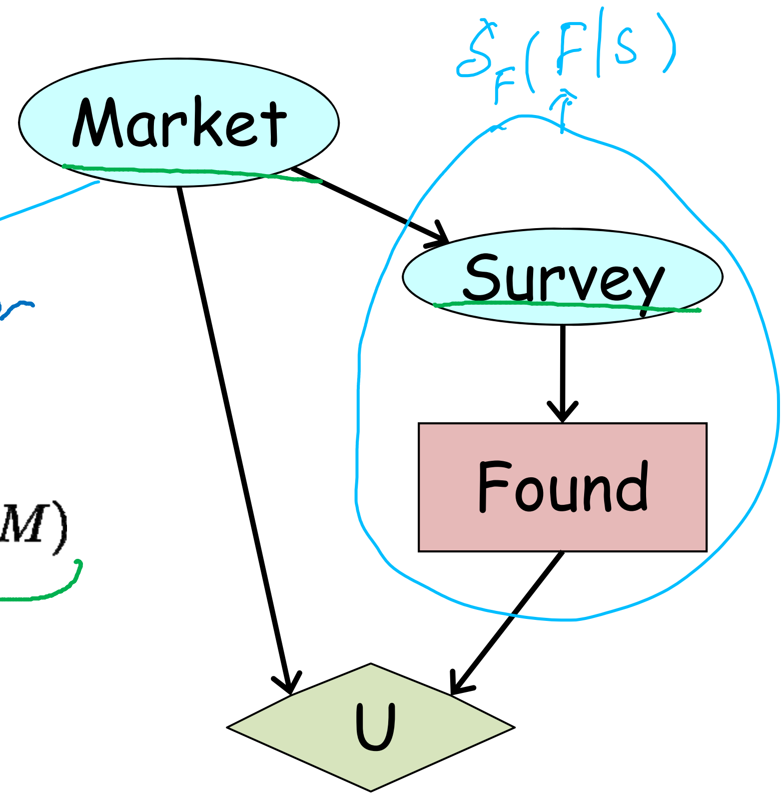
$$\sum_{M,S,F} P(M)P(S \mid M)\delta_F(F \mid S)U(F,M) =$$

$$= \sum_{S,F} \delta_F(F \mid S) \sum_M P(M)P(S \mid M)U(F,M)$$

消除M.

$$= \sum_{S,F} \delta_F(F \mid S)\mu(F,S)$$

优化目标中的变量不能被消除.



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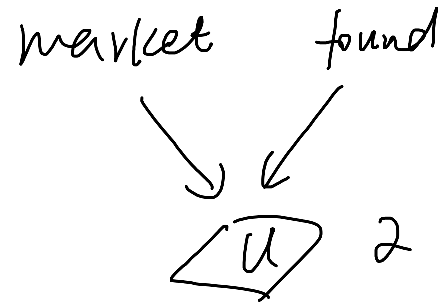


Finding MEU Decision Rules

$$\sum_{S,F} \delta_F(F \mid S) \boxed{\sum_M P(M) P(S \mid M) U(F,M)}$$

$$= \boxed{\sum_{S,F} \delta_F(F \mid S) \mu(F,S)}$$

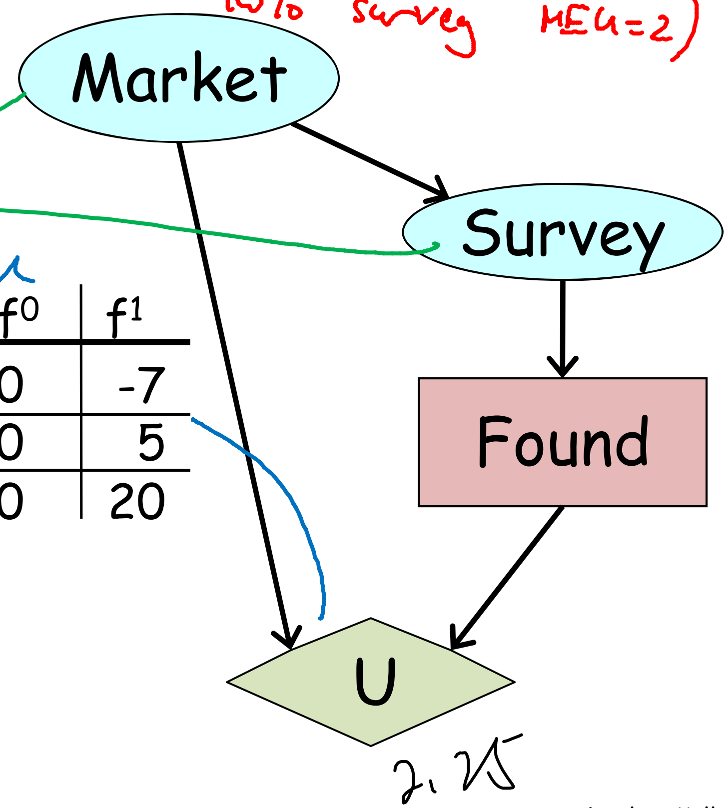
(w/o survey MEU=2)

m⁰	m¹	m²
0.5	0.3	0.2

	s^0	s^1	s^2
m^0	0.6	0.3	0.1
m^1	0.3	0.4	0.3
m^2	0.1	0.4	0.5

u

	f^0	f^1
m^0	0	-7
m^1	0	5
m^2	0	20



	f^0	f^1
s^0	0	-1.25
s^1	0	1.15
s^2	0	2.1

$s_0 \to f_0$

$s_1 \to f_1$

$s_2 \to f_1$

0
+ 1.15
+ 2.1
3.25

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More Generally

$$\mathrm{EU}[\mathcal{D}[\delta_A]] = \sum_{\boldsymbol{x},a} \overbrace{P_{\delta_A}}^{\text{joint dist.}}(\boldsymbol{x},a)U(\boldsymbol{x},a)$$

$\underline{\boldsymbol{Z}} = \mathbf{Pa}_A$ observations prior to A

$\boldsymbol{W} = \{X_1,\ldots,X_n\} - \boldsymbol{Z}$

$$= \sum_{X_1,\ldots,X_n,A} \left(\underbrace{\left(\prod_i P(X_i \mid \mathbf{Pa}_{X_i}) \right)}_{\text{变量}} \underbrace{U(\mathbf{Pa}_U)}_{\text{perferene}} \underbrace{\delta_A(A \mid \boldsymbol{Z})}_{\text{动作变量}} \right)$$

$$= \sum_{\boldsymbol{Z},A} \underline{\delta_A(A \mid \boldsymbol{Z})} \underbrace{\sum_{\boldsymbol{W}} \left(\left(\prod_i P(X_i \mid \mathbf{Pa}_{X_i}) \right) U(\mathbf{Pa}_U) \right)}_{\text{prob}}$$

CPD

$$= \sum_{\boldsymbol{Z},A} \delta_A(A \mid \boldsymbol{Z}) \underbrace{\mu(A,\boldsymbol{Z})}_{\text{只是动作变量和父变量关系.CPD}} \longrightarrow \delta^*_A(a \mid \boldsymbol{z}) = \begin{cases} 1 & a = \mathrm{argmax}_A \mu(A,\boldsymbol{z}) \\ 0 & \text{otherwise} \end{cases}$$

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MEU Algorithm Summary

- To compute MEU & optimize decision at *A*:
  - Treat *A* as random variable with arbitrary CPD
  - Introduce utility factor with scope Pa_U
  - VE Eliminate all variables except *A*, **Z** (*A*'s parents) to produce factor $\mu(A, \mathbf{Z})$
  - For each **z**, set:

$$\delta^*_A(a \mid z) = \begin{cases} 1 & a = \text{argmax}_A \mu(A, z) \\ 0 & \text{otherwise} \end{cases}$$

Decision Making under Uncertainty

- MEU principle provides rigorous foundation
- PGMs provide structured representation for probabilities, actions, and utilities
- PGM inference methods (VE) can be used for
  - Finding the optimal strategy
  - Determining overall value of the decision situation
- Efficient methods also exist for:
  - Multiple utility components
  - Multiple decisions