$$\begin{split} P(x_{N+1}|X_N) &= \sum_{z_{N+1}} P(x_{N+1}, z_{N+1}|X_N) = \sum_{z_{N+1}} P(x_{N+1}|z_{N+1}) P(z_{N+1}|X_N) \\ P(x_{N+1}|X_N) &= \sum_{z_{N+1}} P(x_{N+1}|z_{N+1}) \sum_{z_N} P(z_{N+1}, z_N|X_N) = \sum_{z_{N+1}} P(x_{N+1}|z_{N+1}) \sum_{z_N} P(z_{N+1}|z_N) P(z_N|X_N) \\ P(x_{N+1}|X_N) &= \sum_{z_{N+1}} P(x_{N+1}|z_{N+1}) \sum_{z_N} P(z_{N+1}|z_N) \frac{P(z_N, X_N)}{P(X_N)} \\ P(x_{N+1}|X_N) &= \sum_{z_{N+1}} P(x_{N+1}|z_{N+1}) \sum_{z_N} P(z_{N+1}|z_N) \frac{\alpha(z_N)}{P(X_N)} \end{split}$$

由上可知,要预测观测值必须知道过去的隐藏状态值,从而确定转换概率 $P(z_{N+1}|z_N)$ 和发射概率 $P(x_{N+1}|z_{N+1})$ ,然后可以确定下一观测值的概率,如果不确定过去的隐藏状态值,可以先通过维特比算法确定最有可能的隐藏状态序列。

```
1
       # 已知当前序列预测未来(下一个)观测值的概率
2
       def predict(self, X, x_next, Z_seq=np.array([]), istrain=True):
           if self.trained == False or istrain == False: # 需要根据该序列重新训练
3
               self.train(X)
5
           X_{length} = len(X)
7
           if Z_seq.any():
               Z = np.zeros((X_length, self.n_state))
9
               for i in range(X_length):
                   Z[i][int(Z_seq[i])] = 1
10
11
           else:
12
               Z = np.ones((X_length, self.n_state))
           # 向前向后传递因子
13
14
           alpha, \_ = self.forward(X, Z) # P(x,z)
           prob_x_next = self.emit_prob(np.array([x_next]))*np.dot(alpha[X_length - 1], self.transmat_prob)
15
16
           return prob_x_next
```



