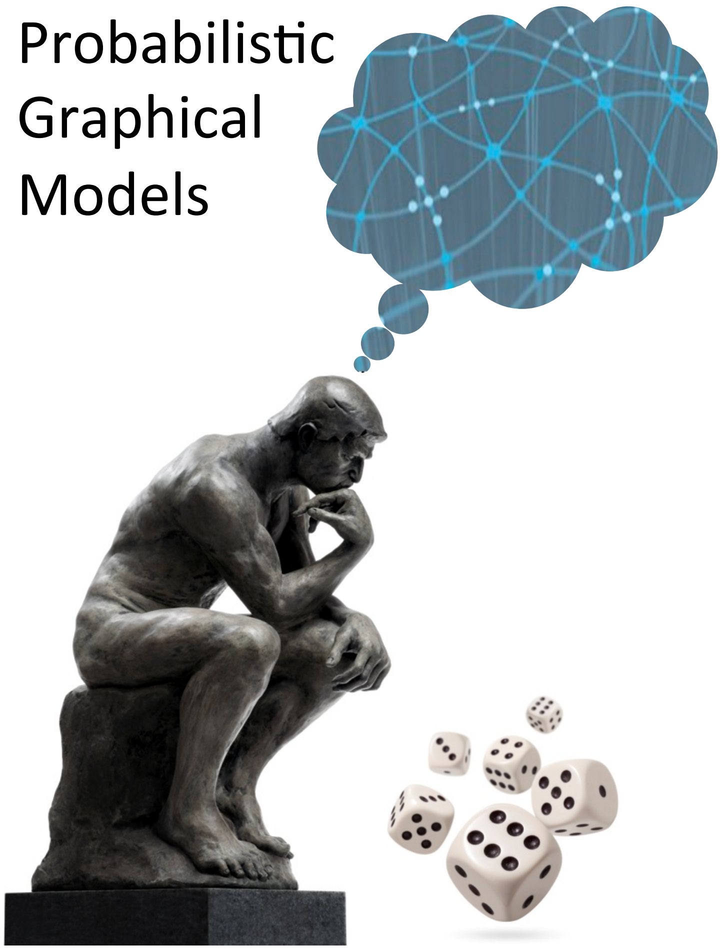
Probabilistic Graphical Models

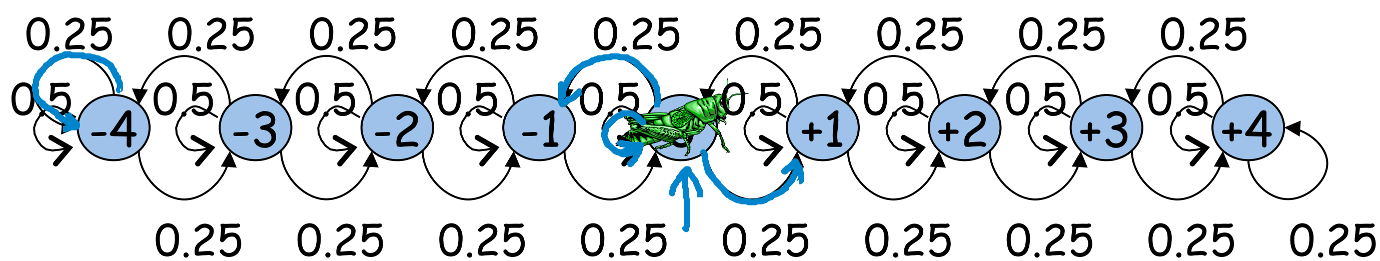


Inference

Sampling Methods

Markov Chain Monte Carlo

Markov Chain



- A Markov chain defines a probabilistic transition model $T(x \rightarrow x')$ over states x:
  – for all x: $\sum_{x'} T(x \rightarrow x') = 1$

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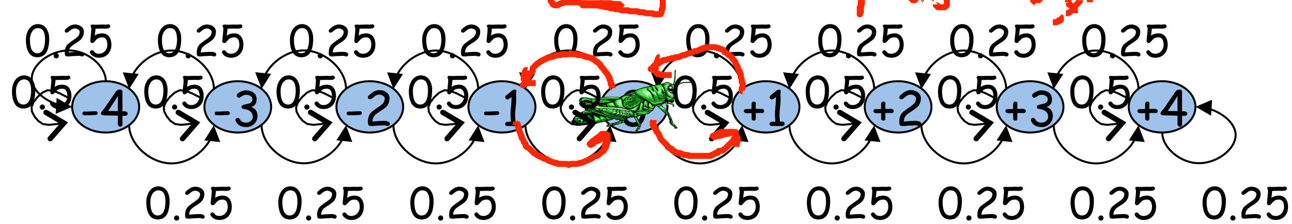
Temporal Dynamics

step   state   time t

$$P^{(t+1)}(X^{(t+1)} = x') = \underbrace{\sum_x}_{} P^{(t)}(X^{(t)} = x)T(x \to x')$$

pairs   x, x'



	-2	-1	0	+1	+2
P(0)	0	0	1	0	0
P(1)	0	.25	.5	.25	0
P(2)	$.25^2$= .0625	2×(.5×.25) = .25	$.5^2+2\times.25^2$ = .375	2×(.5×.25) = .25	$.25^2$= .0625

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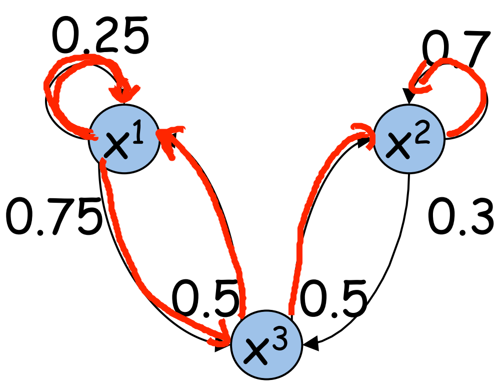
Stationary Distribution

$$P^{(t)}(x') \approx P^{(t+1)}(x') = \sum_x P^{(t)}(x)T(x \to x')$$

$$\pi(x') = \sum_x \pi(x)T(x \to x')$$



$$\begin{aligned}
\pi(x^1) &= 0.25\pi(x^1) + 0.5\pi(x^3) \\
\pi(x^2) &= 0.7\pi(x^2) + 0.5\pi(x^3) \\
\pi(x^3) &= 0.75\pi(x^1) + 0.3\pi(x^2)
\end{aligned}$$

$$\pi(x^1) + \pi(x^2) + \pi(x^3) = 1$$

$$\begin{aligned}
\pi(x^1) &= 0.2 \\
\pi(x^2) &= 0.5 \\
\pi(x^3) &= 0.3
\end{aligned}$$

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Regular Markov Chains

- A Markov chain is regular if there exists k such that, for every x, x′, the probability of getting from x to x′ in exactly k steps is > 0

- Theorem: A regular Markov chain converges to a unique stationary distribution regardless of start state

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Regular Markov Chains

- A Markov chain is regular if there exists k such that, for every x, x′, the probability of getting from x to x′ in exactly k steps is > 0

  k = distance between furthest x, x′

- Sufficient conditions for regularity:
  - Every two states x, x′ are connected with path of prob > 0
  - For every state, there is a self-transition