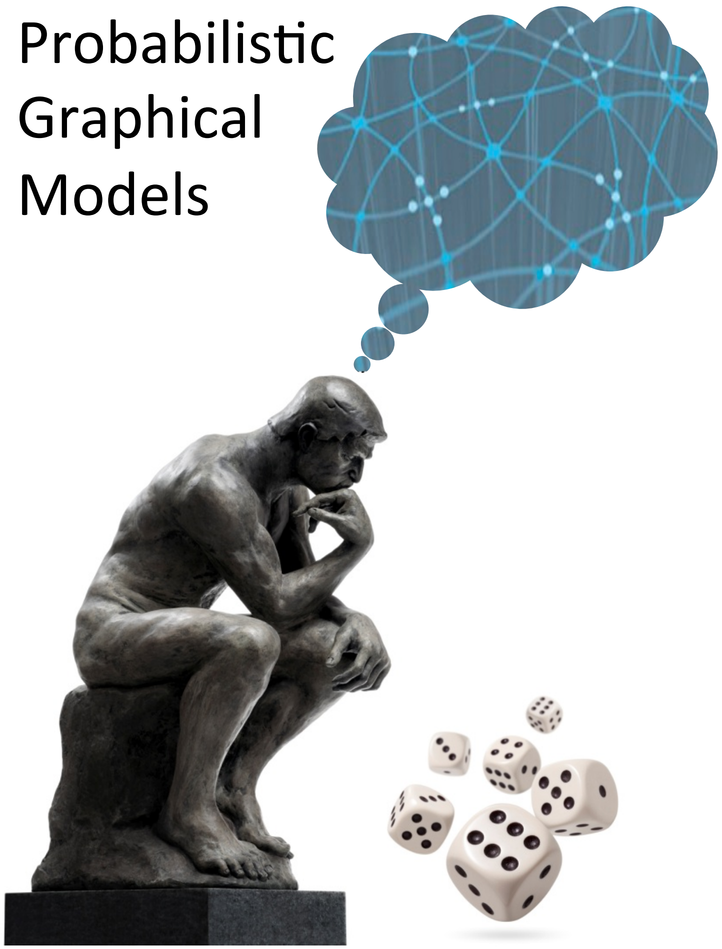
Probabilistic
Graphical
Models



Inference

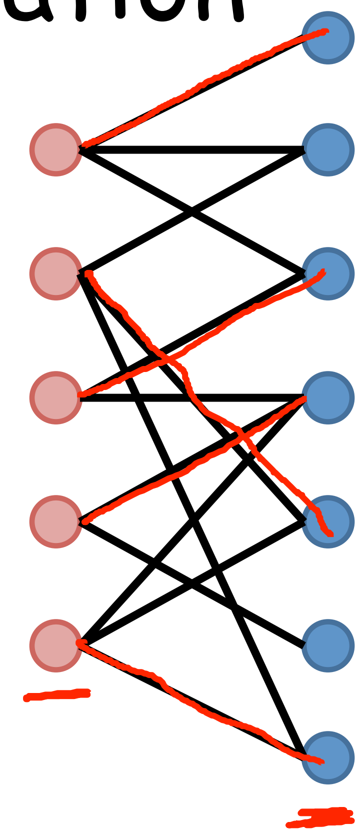
MAP

Tractable MAP Problems

Correspondence /data association

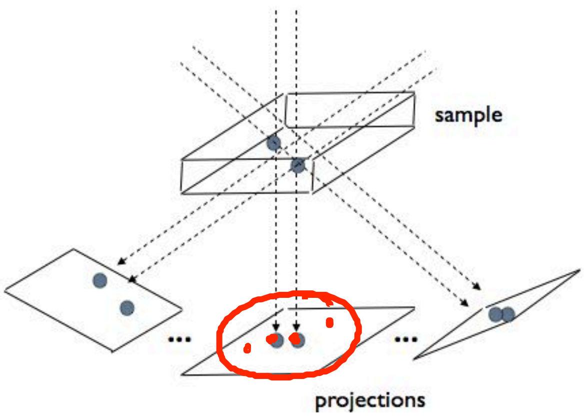
$X_{ij} = \begin{cases} 1 \text{ if } i \text{ matched to } j \\ 0 \text{ otherwise} \end{cases}$

θ_{ij} = quality of "match" between i and j

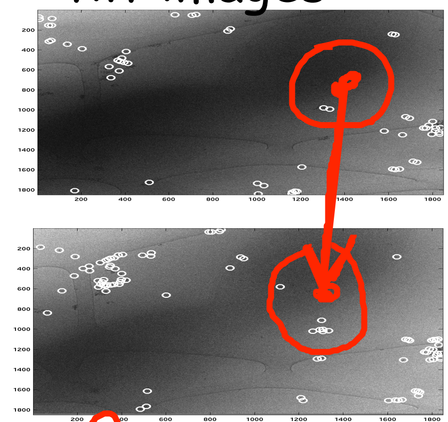


- Find highest scoring matching
  - maximize $\Sigma_{ij}\ \theta_{ij}\ X_{ij}$
  - subject to mutual exclusion constraint
- Easily solved using matching algorithms
- Many applications
  - matching sensor readings to objects
  - matching features in two related images
  - matching mentions in text to entities

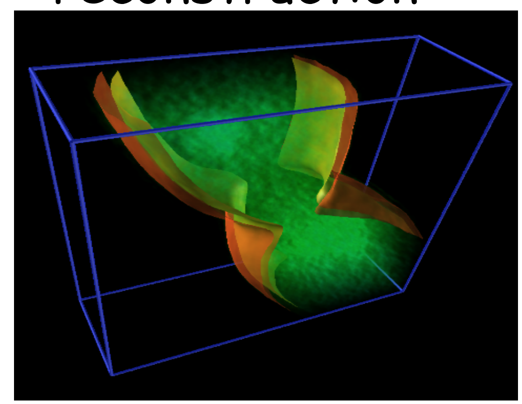
3D Cell Reconstruction



correspond
tilt images



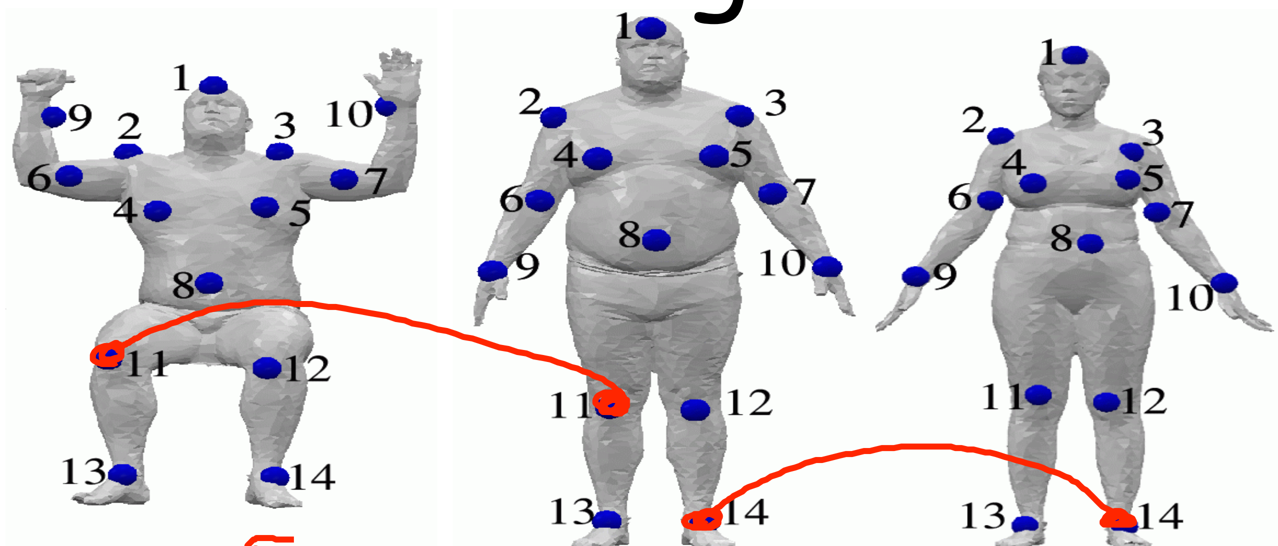
compute 3D
reconstruction



- Matching weights: similarity of location and local neighborhood appearance

Duchi, Tarlow, Elidan, and Koller, NIPS 2006. Amat, Moussavi, Comolli, Elidan, Downing, Horowitz, Journal of Strurctural Biology, 2006.

Mesh Registration



- Matching weights: similarity of location and local neighborhood appearance

pose θ_{ij} shape

[Anguelov, Koller, Srinivasan, Thrun, Pang, Davis, NIPS 2004]

Associative potentials

- Arbitrary network over binary variables using only singleton θ_i and supermodular pairwise potentials θ_{ij}
  - Exact solution using algorithms for finding minimum cuts in graphs
- Many related variants admit efficient exact or approximate solutions
  - Metric MRFs

vision

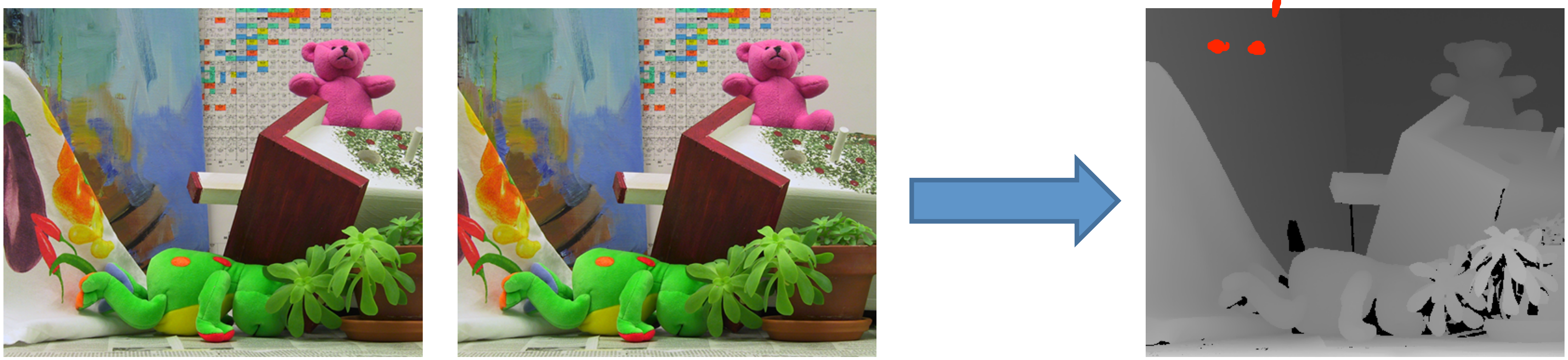
x_j

x_i	0	1
0	a	b
1	c	d

$a+d \geq b+c$

Example: Depth Reconstruction

depth (z)



view 1 view 2 depth reconstruction

denoising, infilling, FG/BG segmentation

Scharstein & Szeliski, "High-accuracy stereo depth maps using structured light" Proc. IEEE CVPR 2003

Cardinality Factors

- A factor over arbitrarily many binary variables $X_1, \ldots, X_k$
- $\text{Score}(X_1, \ldots, X_k) = f(\Sigma_i X_i)$
- Example applications:
  - soft parity constraints
  - prior on # pixels in a given category
  - prior on # of instances assigned to a given cluster

	A	B	C	D	score
0	0	0	0	0	
	0	0	0	1	
	0	0	1	0	
	0	0	1	1	
1	0	1	0	0	
	0	1	0	1	
	0	1	1	0	
	0	1	1	1	
1	1	0	0	0	
	1	0	0	1	
	1	0	1	0	
	1	0	1	1	
	1	1	0	0	
	1	1	0	1	
	1	1	1	0	
	1	1	1	1	

Daphne Koller

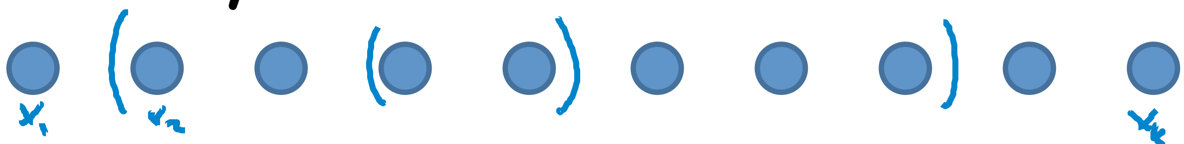
Sparse Pattern Factors

- A factor over variables $X_1,\ldots,X_k$
  - Score($X_1, \ldots, X_k$) specified for some small # of assignments $x_1,\ldots,x_k$
  - Constant for all other assignments
- Examples: give higher score to combinations that occur in real data
  - In spelling, letter combinations that occur in dictionary
  - 5×5 image patches that appear in natural images

A	B	C	D	score
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

Convexity Factors

- Ordered binary variables $X_1,\ldots,X_k$
- Convexity constraints



- Examples:
  - Convexity of "parts" in image segmentation
  - Contiguity of word labeling in text
  - Temporal contiguity of subactivities

Summary

- Many specialized models admit tractable MAP solution
  - Many do not have tractable algorithms for computing marginals
- These specialized models are useful
  - On their own
  - As a component in a larger model with other types of factors