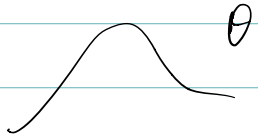
具体请看极大似然相关章节.

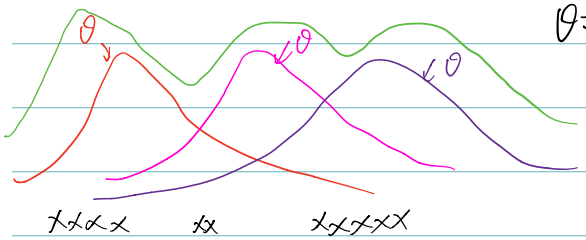
单峰高斯



$\theta = \{\mu, \Sigma\}$

- log似然函数
  - $L(\theta|\bar{x}) = \log p(\bar{x}|\theta) = \sum \log p(x_i|\theta)$.
  - $\bar{x} = \{x_1, \cdots, x_n\}$
  - $x_i \overset{iid}{\sim} p(x|\theta)$
- MAP
  - $p(\theta|x) \propto p(x|\theta)\, p(\theta)$
    - $p(\theta|x)$: 后验
    - $p(x|\theta)$: 似然函数
    - $p(\theta)$: 先验.
  - $\arg\max_\theta \left[\sum \log N(x_i|\mu, \Sigma)\right)$
    - $\mu_{MLE} = \frac{\partial L(\mu, \Sigma|\bar{x})}{\partial \mu}$
    - $\Sigma_{MLE} = \frac{\partial L(\mu, \Sigma|\bar{x})}{\partial \Sigma}$.

混合模型



$\theta = \{\mu_1, \cdots, \Sigma_1, \cdots, \alpha_1 \cdots, \alpha_{k-1}\}$

$$P(x|\theta) = \sum \alpha_k N(\mu_k, \Sigma_k) \qquad \sum \alpha_k = 1.$$

$$\hat{\theta}_{MLE} = \arg\max_{\theta} \underbrace{\left\{ \sum_{i=1}^{N} \log \left[\sum_{k=1}^{k} \alpha_k N(\mu_k, \Sigma_k) \right] \right\}}_{L(\theta|X)}$$

- 因为log有相加，直接求解困难较大，
  可以利用EM，分开求解