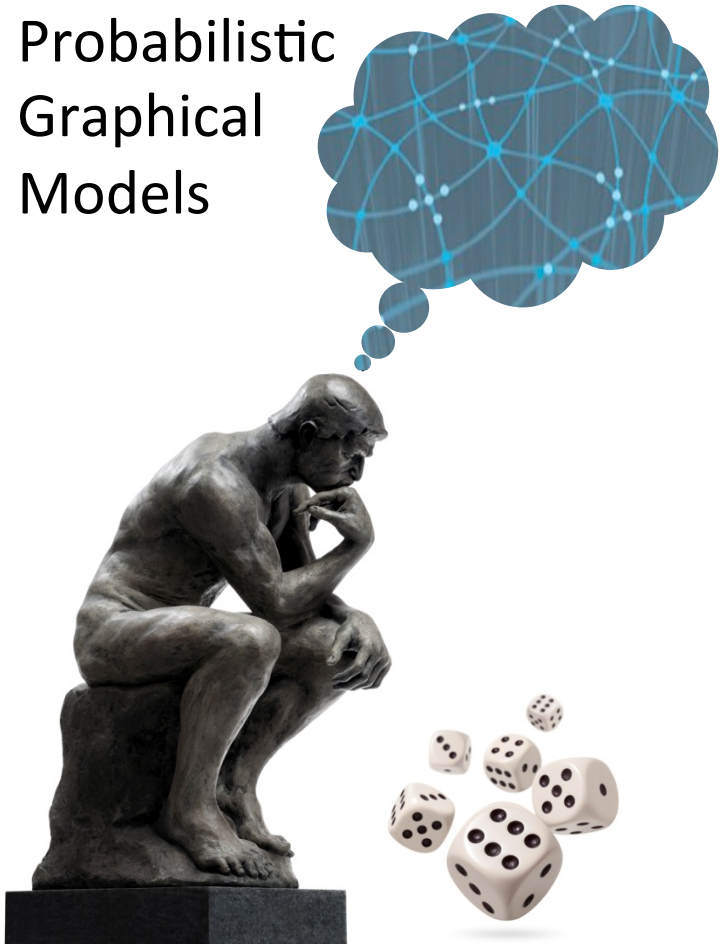
Probabilistic Graphical Models



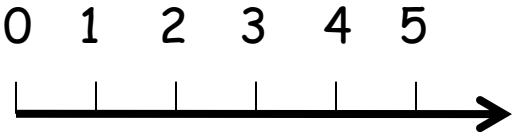
Representation

Template Models

Temporal Models

Distributions over Trajectories

discretize time



- Pick time granularity Δ ← sensor
- $X^{(t)}$ – variable X at time $t\Delta$
- $X^{(t:t')} = \{X^{(t)}, \ldots, X^{(t')}\}$ $(t \le t')$

- Want to represent $P(X^{(t:t')})$ for any t, t'

Markov Assumption

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(0:t)})$$

chain rule for probabilities

time flows forward

state at t+1

state at 0..t

$$(\boldsymbol{X}^{(t+1)} \perp \boldsymbol{X}^{(0:t-1)} \mid \boldsymbol{X}^{(t)})$$

forgetting

next step

past

present

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)})$$

Is this true?

X = Location of robot

probably not

velocity

$L^{t+1} \perp L^{t-1} \mid L^t$?

enrich state by adding v! and other variables
(adding dependencies back in time - semi-Markov)

Time Invariance

- Template probability model $P(\mathbf{X}' \mid \mathbf{X})$
- For all t:

$$P(\mathbf{X}^{(t+1)} \mid \mathbf{X}^{(t)}) = P(\mathbf{X}' \mid \mathbf{X})$$

next time

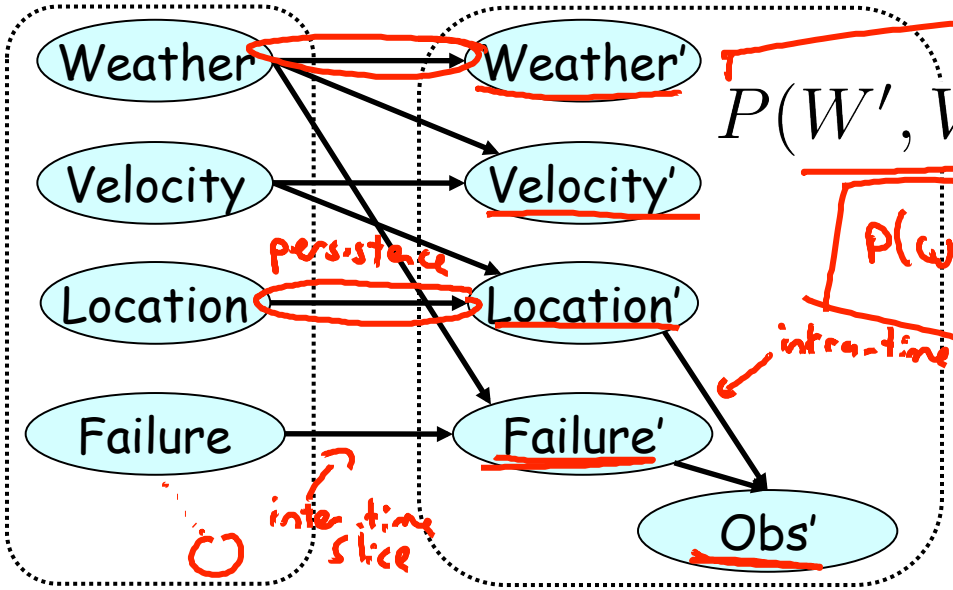
current time

traffic

time of day, day of week, football

enrich model by including

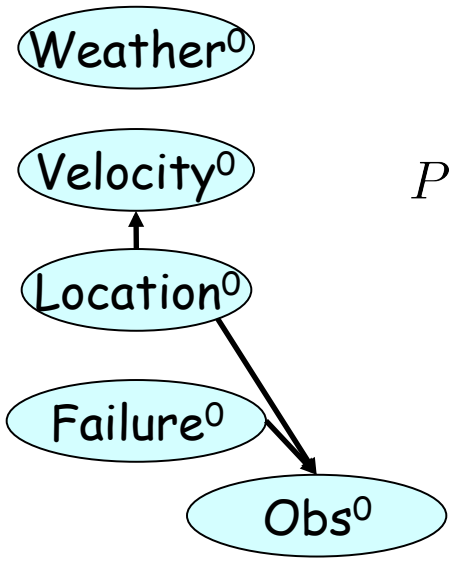
Template Transition Model



$$P(W', V', L', F', O' \mid W, V, L, F) =$$

$$P(W' \mid W) \cdot P(V' \mid W, V) \cdot P(L' \mid L, V) \cdot P(F' \mid F, W) \cdot P(O' \mid L', F')$$

Initial State Distribution



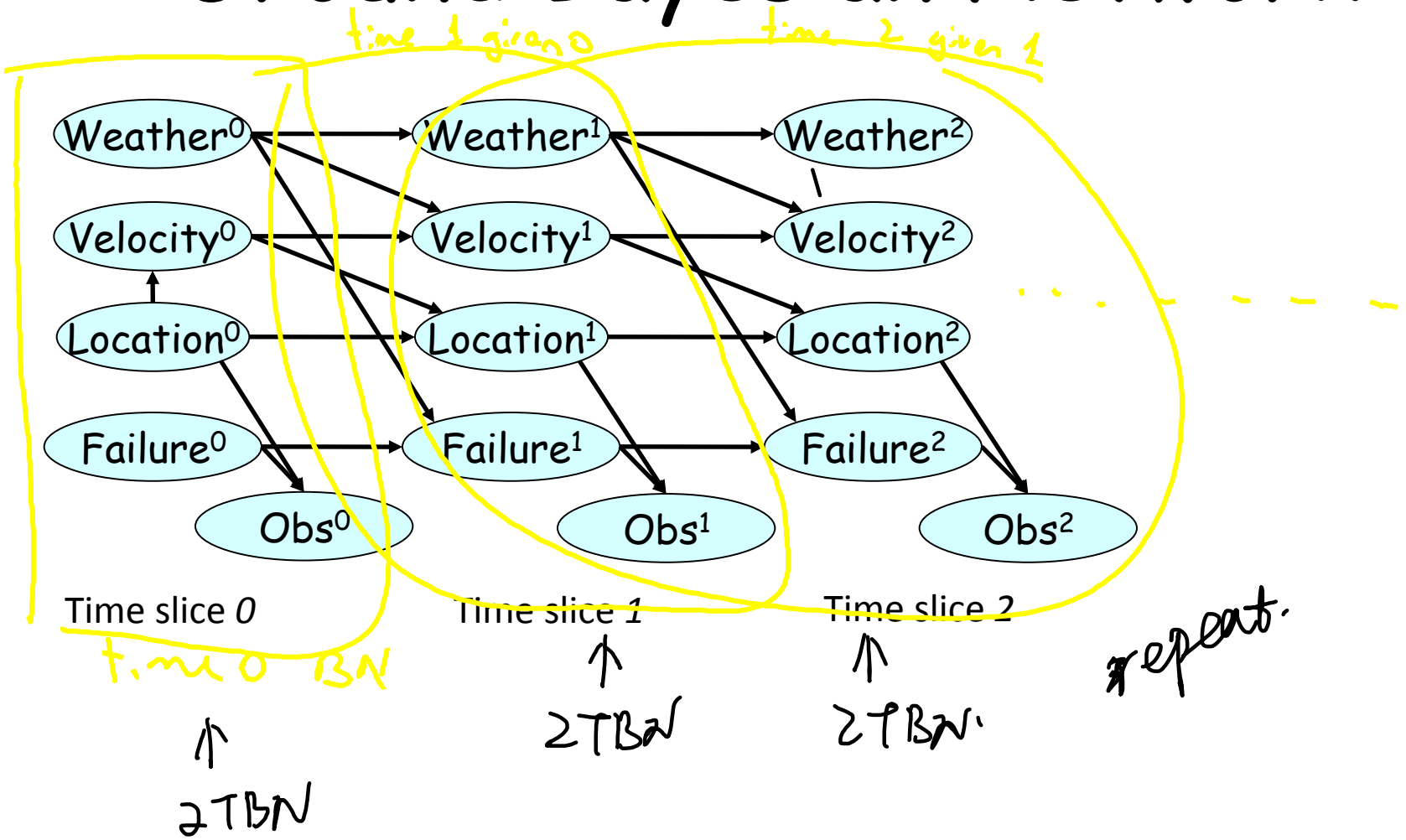
Time slice *0*

$$P(W^{(0)}, V^{(0)}, L^{(0)}, F^{(0)}, O^{(0)}) =$$

$$P(W^{(0)})P(V^{(0)} \mid L^{(0)})P(L^{(0)})P(F^{(0)})P(O^{(0)} \mid F^{(0)}, L^{(0)})$$

chain rule

Ground Bayesian Network



2-time-slice Bayesian Network

- A transition model (2TBN) over $X_1,\ldots,X_n$ is specified as a BN fragment such that:
  - The nodes include $X_1',\ldots,X_n'$ and a subset of $X_1,\ldots,X_n$
  - Only the nodes $X_1',\ldots,X_n'$ have parents and a CPD
- The 2TBN defines a conditional distribution

$$P(\boldsymbol{X}' \mid \boldsymbol{X}) = \prod_{i=1}^{n} P(X_i' \mid \mathbf{Pa}_{X_i'})$$

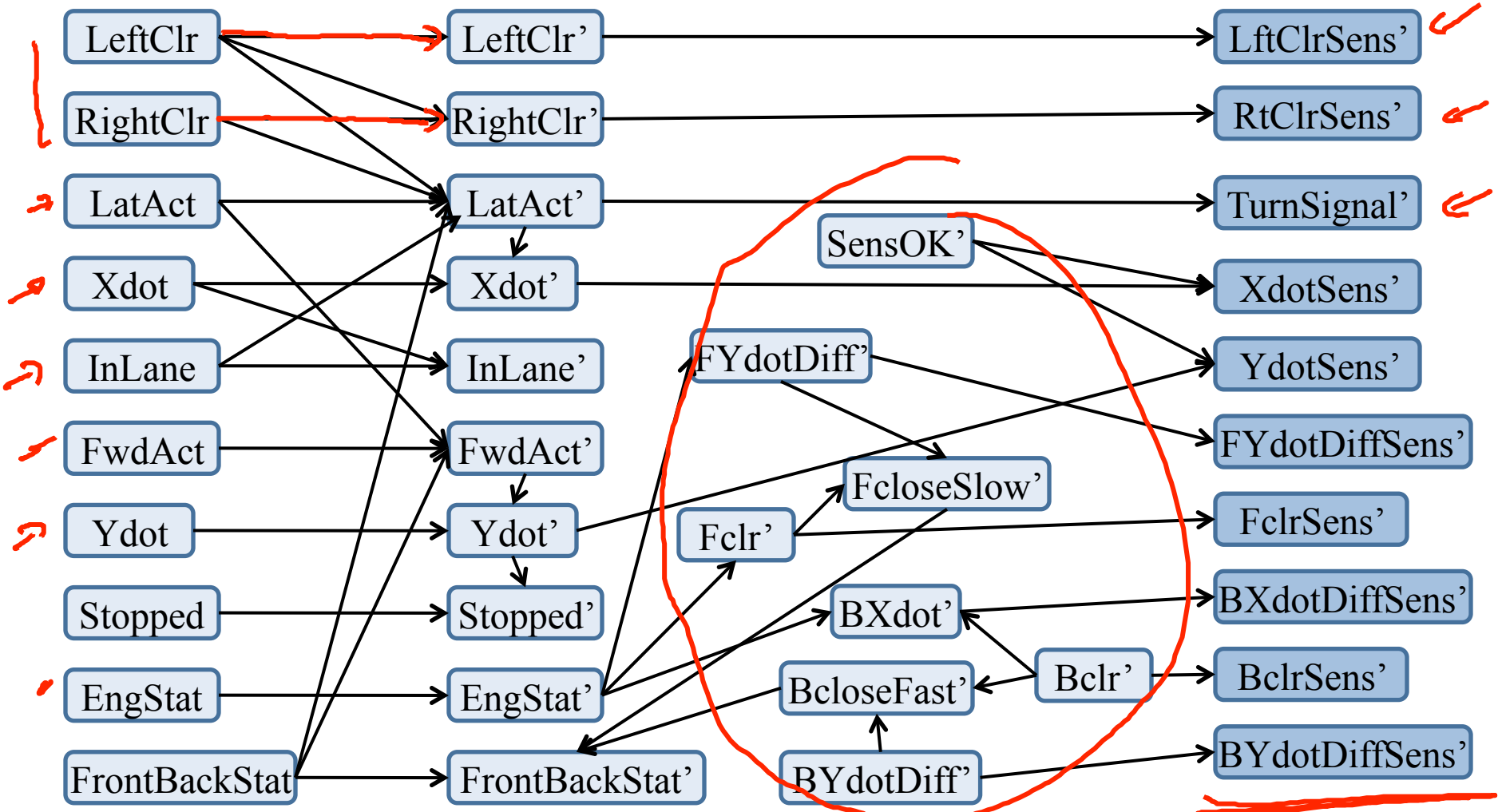
Dynamic Bayesian Network

- A dynamic Bayesian network (DBN) over $X_1,\ldots,X_n$ is defined by a
  - 2TBN $BN_{\rightarrow}$ over $X_1,\ldots,X_n$ dynamics
  - a Bayesian network $BN^{(0)}$ over $X_1^{(0)},\ldots,X_n^{(0)}$ time 0

Ground Network

- For a trajectory over 0,…,T we define a ground (unrolled network) such that
  - The dependency model for $X_1^{(0)}$,…,$X_n^{(0)}$ is copied from $BN^{(0)}$
  - The dependency model for $X_1^{(t)}$,…,$X_n^{(t)}$ for all t > 0 is copied from $BN_{\rightarrow}$

2-TBN no 重复.



Huang, Koller, Malik, Ogasawara, Rao, Russell, Weber, AAAI 94

2TBN repeat → 但意义、时间.

Summary

- DBNS are a compact representation for encoding structured distributions over arbitrarily long temporal trajectories
- They make assumptions that may require appropriate model (re)design:
  - Markov assumption
  - Time invariance

利用假设简化网络