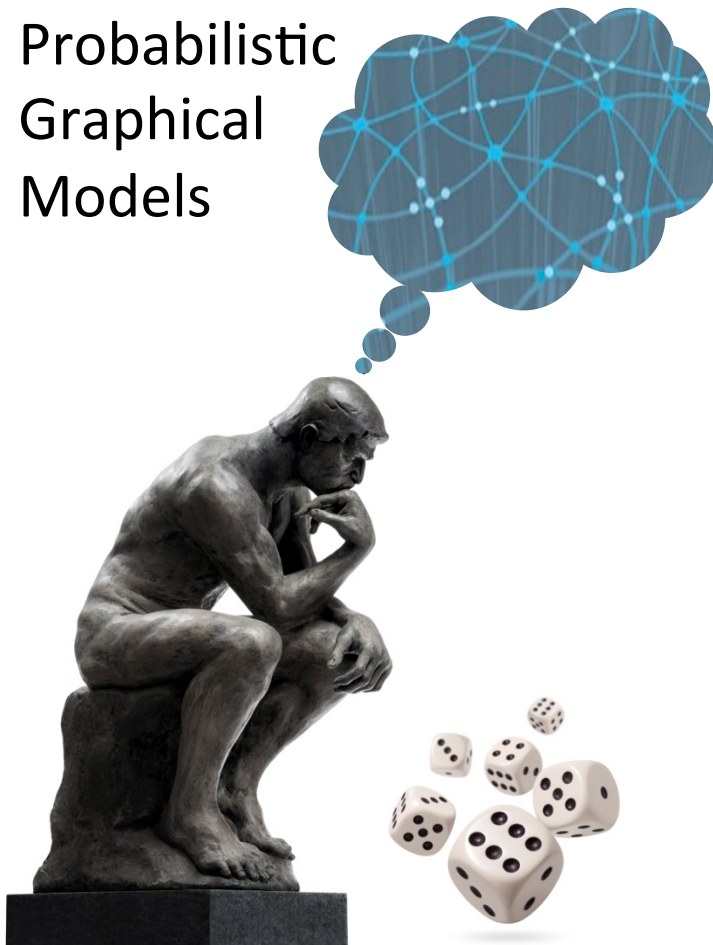


Probabilistic  
Graphical  
Models



Inference

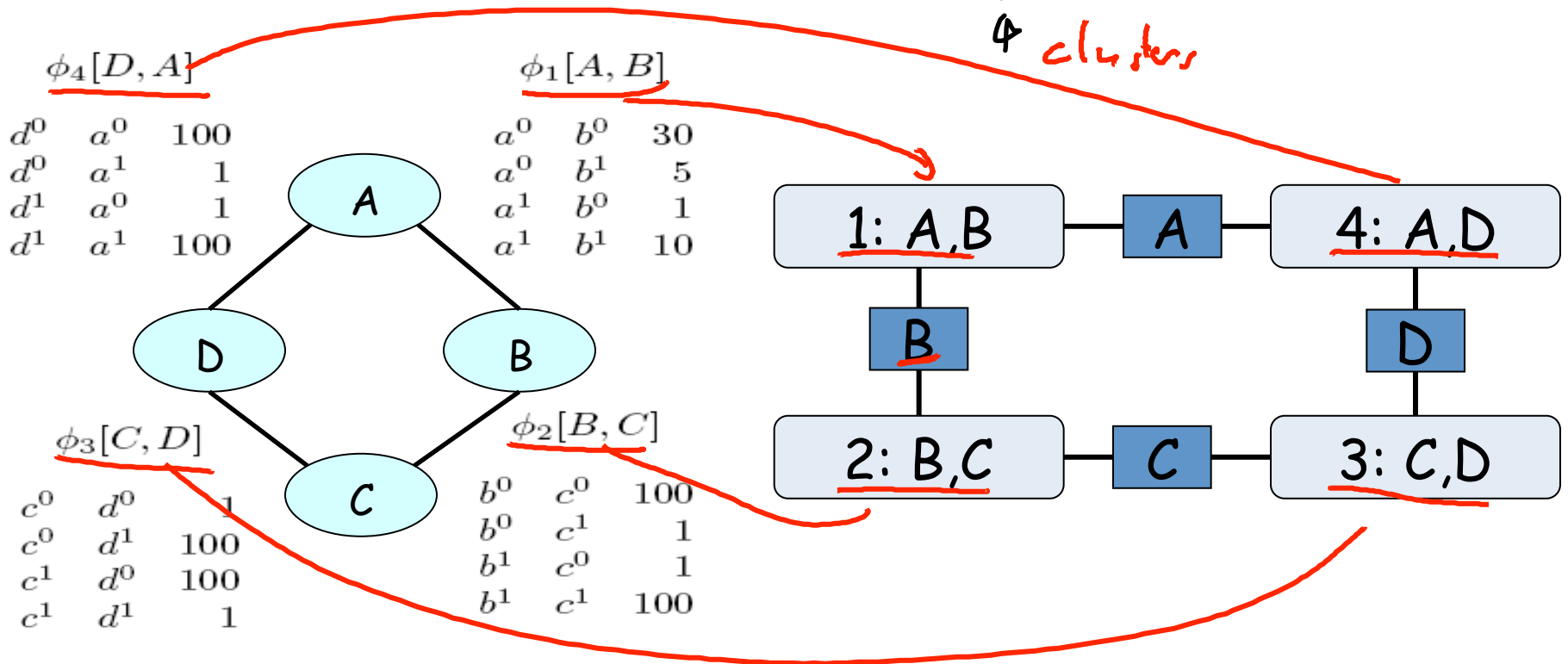
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Message Passing

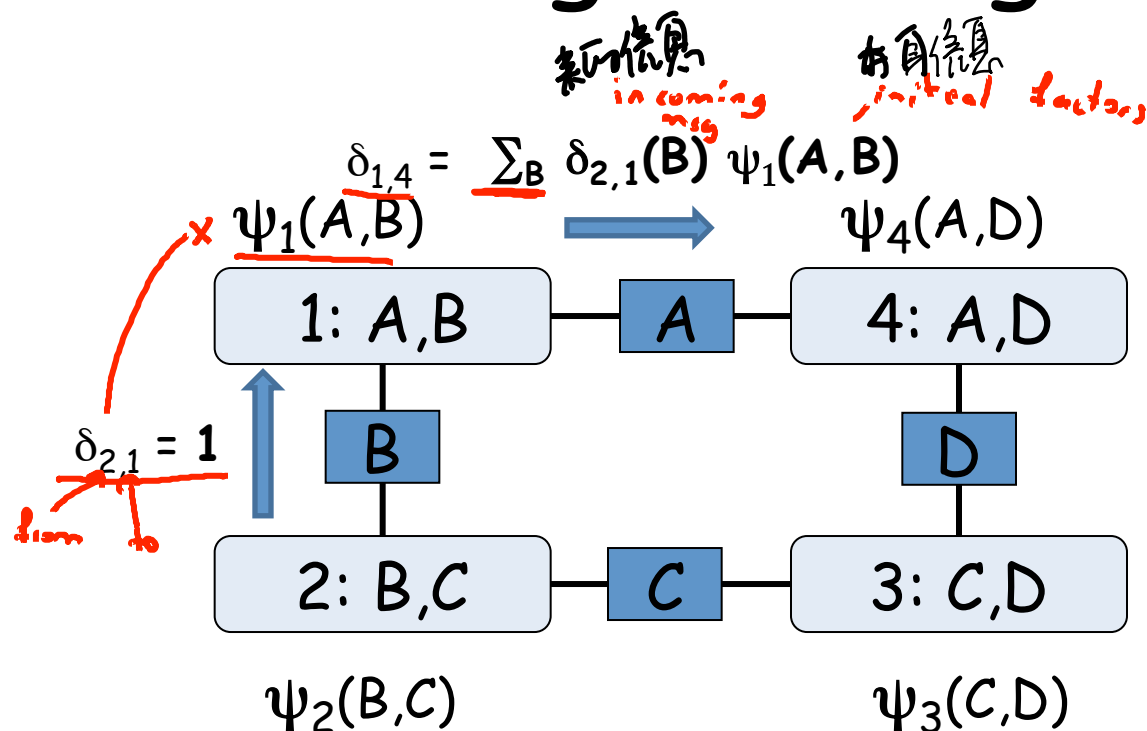
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Belief  
Propagation  
Algorithm

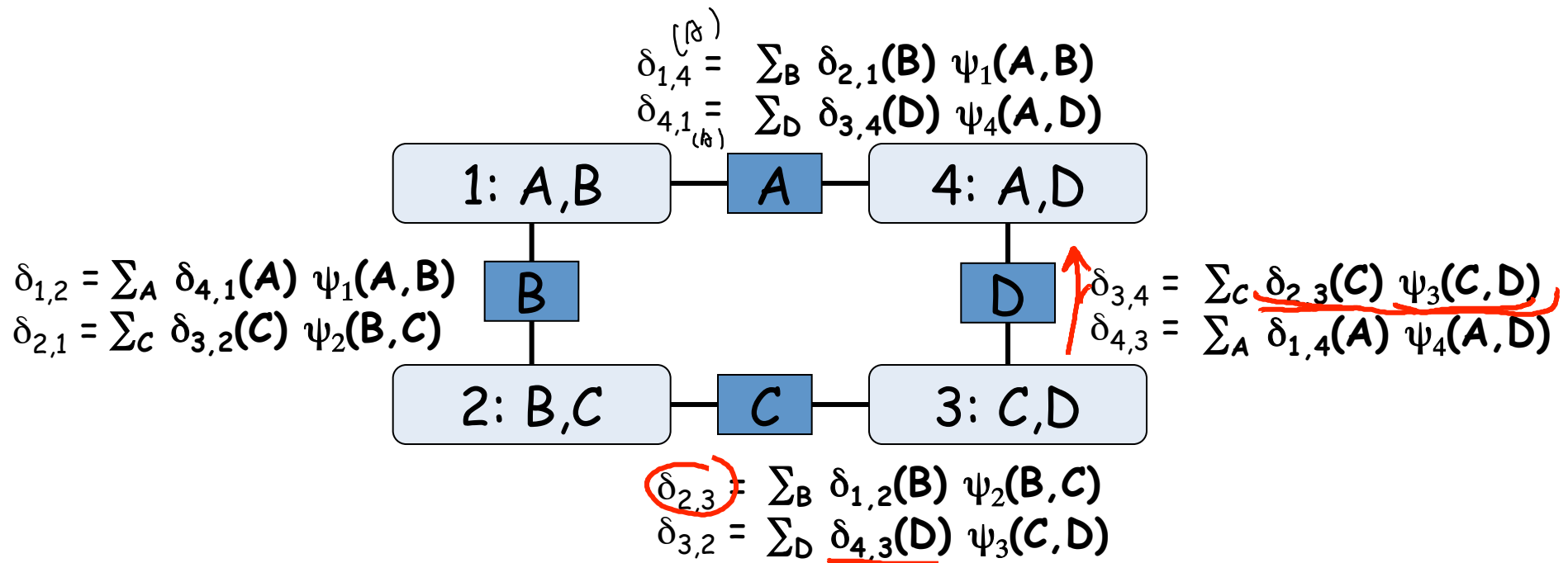
# Cluster Graph



# Passing Messages



# Passing Messages

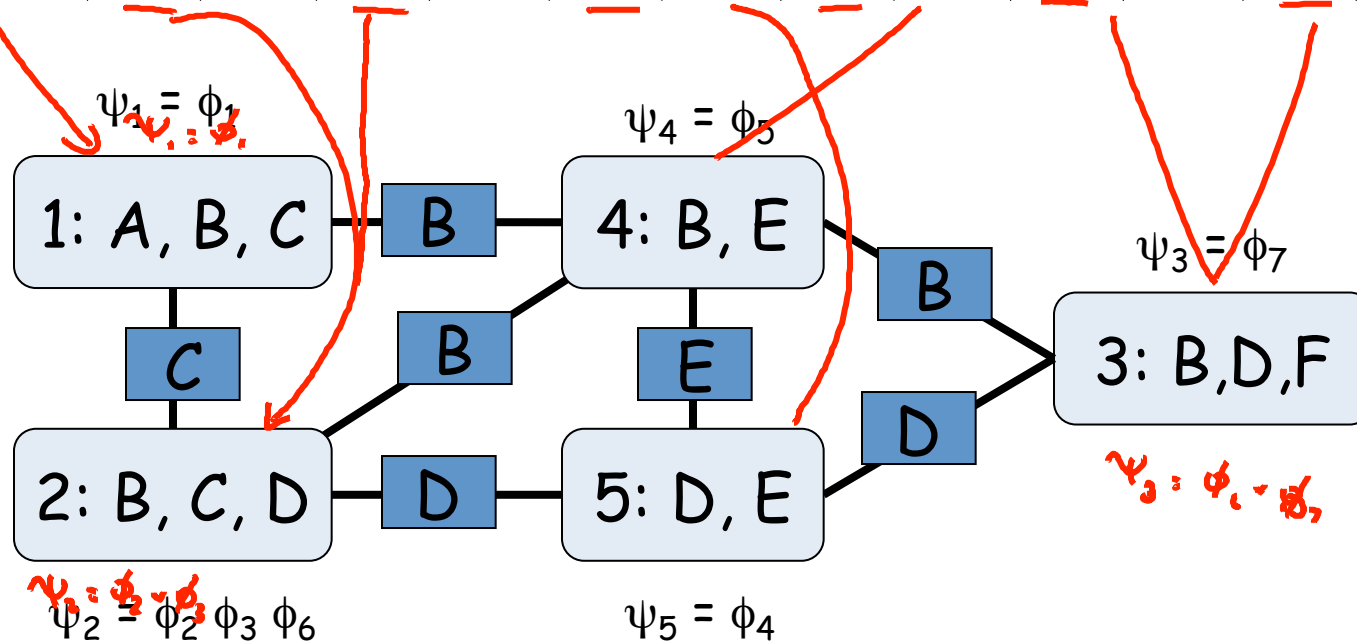


# Cluster Graphs

- Undirected graph such that:
  - nodes are clusters  $\mathcal{C}_i \subseteq \{X_1, \dots, X_n\}$  *Subjects of variables*
  - edge between  $\mathcal{C}_i$  and  $\mathcal{C}_j$  associated with *in graph* sepset  $S_{i,j} \subseteq \mathcal{C}_i \cap \mathcal{C}_j$  *Variables that they talk about*
- Given set of factors  $\Phi$ , we assign each  $\phi_k$  to a cluster  $\mathcal{C}_{\alpha(k)}$  s.t.  $\text{Scope}[\phi_k] \subseteq \mathcal{C}_{\alpha(k)}$
- Define  $\psi_i(\mathcal{C}_i)$  =  $\prod_{k: \alpha(k)=i} \phi_k$  *subset* *all factors assigned to it*

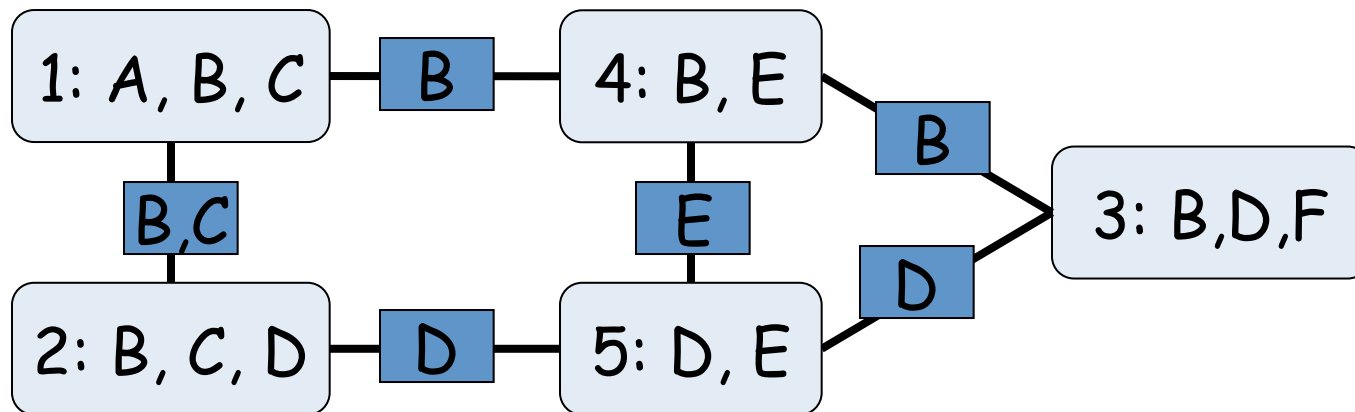
# Example Cluster Graph

$\phi_1(A, B, C)$ ,  $\phi_2(B, C)$ ,  $\phi_3(B, D)$ ,  $\phi_4(D, E)$ ,  $\phi_5(B, E)$ ,  $\phi_6(B, D)$ ,  $\phi_7(B, D, F)$



# Different Cluster Graph

$\phi_1(A, B, C), \phi_2(B, C), \phi_3(B, D), \phi_4(D, E), \phi_5(B, E), \phi_6(B, D), \phi_7(B, D, F)$



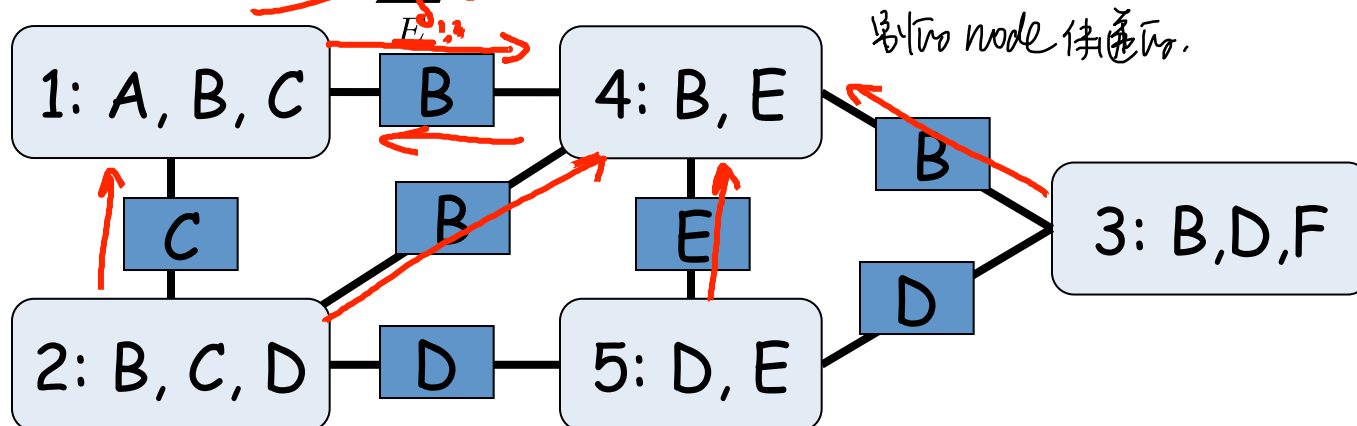
# Message Passing

$$\delta_{1 \rightarrow 4}(B) = \sum_{A, C} \psi_1(A, B, C) \delta_{2 \rightarrow 1}(C)$$

自有的
传递来的

$$\delta_{4 \rightarrow 1}(B) = \sum_{E} \psi_4(B, E) \times \delta_{2 \rightarrow 4}(B) \times \delta_{5 \rightarrow 4}(E) \times \delta_{3 \rightarrow 4}(B)$$

自己 node 固有的
别的 node 传递来的



$$\delta_{i \rightarrow j}(S_{i,j}) = \sum_{C_{i-S_{i,j}}} \psi_i \times \prod_{k \in (\mathcal{N}_i - \{j\})} \delta_{k \rightarrow i}$$

in coming msgs other than from j

$i$  node 所有节点减  
去连接的点

邻域中除  $j$



# Belief Propagation Algorithm

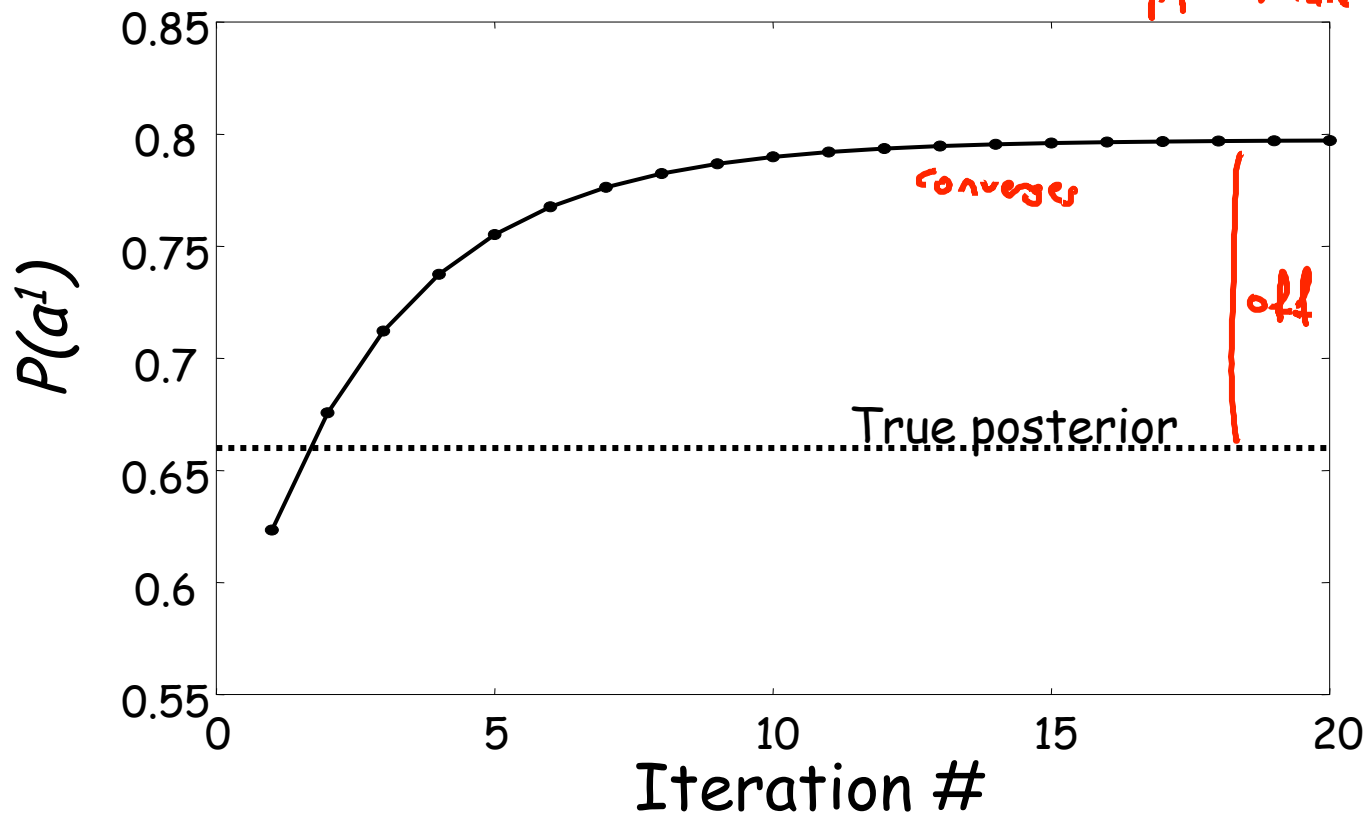
- Assign each factor  $\phi_k \in \Phi$  to a cluster  $\mathcal{C}_{\alpha(k)}$
- Construct initial potentials  $\psi_i(C_i) = \prod_{k:\alpha(k)=i} \phi_k$
- Initialize all messages to be 1
- Repeat *until when?*
  - Select edge (i,j) and pass message *round robin*

$$\delta_{i \rightarrow j}(S_{i,j}) = \sum_{C_i - S_{i,j} \in \mathcal{C}_i} \psi_i \times \prod_{k \in (\mathcal{N}_i - \{j\})} \delta_{k \rightarrow i}$$

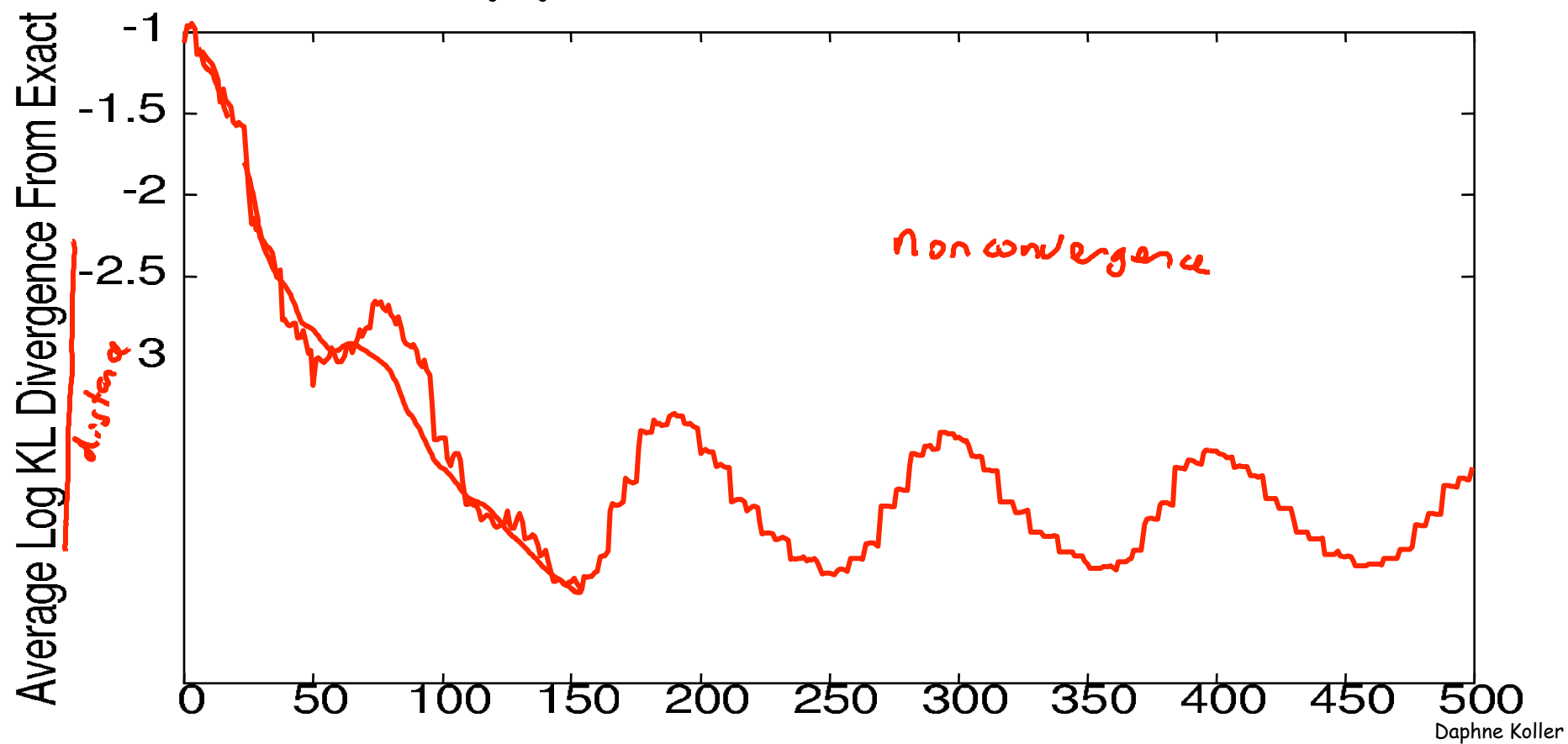
*Handwritten notes:  $\psi_i$  is a factor,  $\mathcal{N}_i$  is the set of neighbors of node i.*

- Compute  $\beta_i(C_i)$  *belief*  $= \psi_i \times \prod_{k \in \mathcal{N}_i} \delta_{k \rightarrow i}$  *all neighbors*

# Belief Propagation Run



# Different BP Run



# Summary

- Graph of clusters connected by sepsets
- Adjacent clusters pass information to each other about variables in sepset
  - Message from  $i$  to  $j$  summarizes everything  $i$  knows, except information obtained from  $j$  message,
- Algorithm may not converge not marginals of  $\tilde{P}_3$
- The resulting beliefs are pseudo-marginals
- Nevertheless, very useful in practice