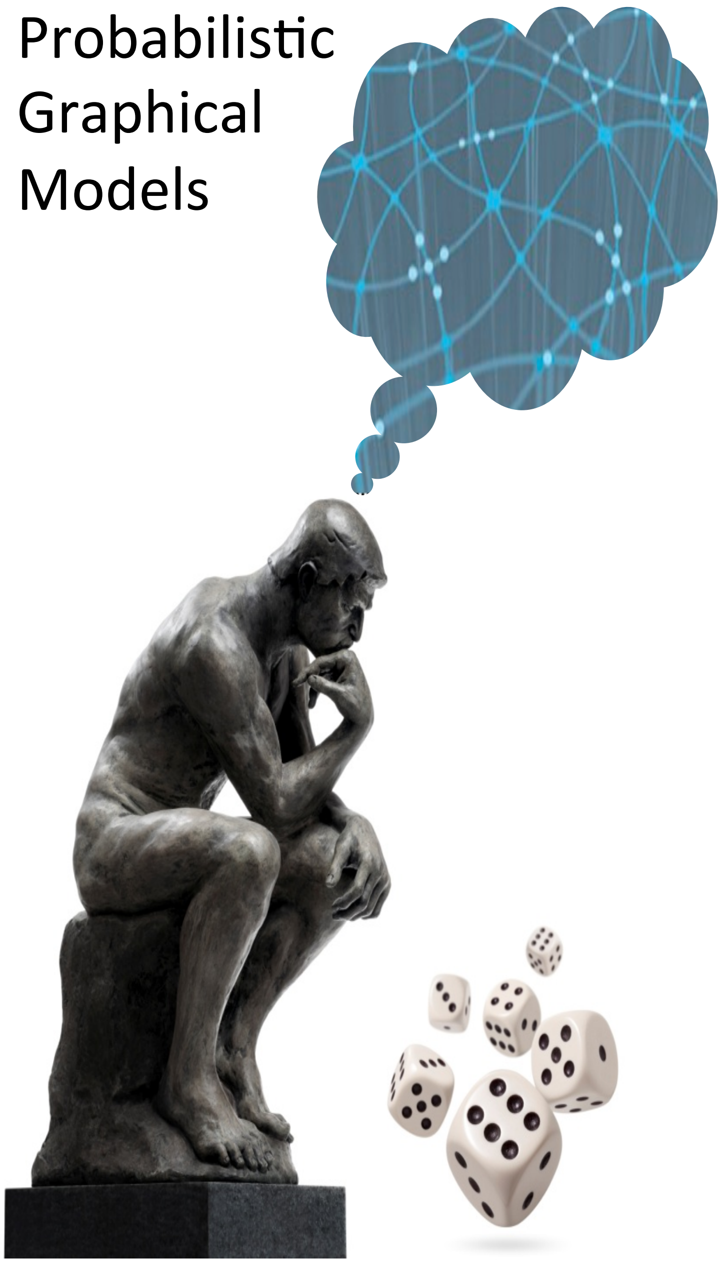
Probabilistic
Graphical
Models



Inference

MAP

Dual Decomposition

Problem Formulation

- Singleton factors $\theta_i(x_i)$
- Large factors $\theta_F(\boldsymbol{x}_F)$

θ

$$\text{MAP}(\boldsymbol{\theta}) = \max_{\boldsymbol{x}} \left(\sum_{i=1}^{n} \theta_i(x_i) + \sum_F \theta_F(\boldsymbol{x}_F) \right)$$

Divide and Conquer

$$\mathrm{MAP}(\boldsymbol{\theta}) = \max_{\boldsymbol{x}} \left(\sum_{i=1}^{n} \theta_i(x_i) + \sum_{F} \theta_F(\boldsymbol{x}_F) \right)$$

$$\sum_{i=1}^{n} \max_{x_i} \theta_i(x_i) + \sum_{F} \max_{\boldsymbol{x}_F} \theta_F(\boldsymbol{x}_F)$$

local decision making

Divide and Conquer

$$\mathrm{MAP}(\boldsymbol{\theta}) = \max_{\boldsymbol{x}} \left(\sum_{i=1}^{n} \theta_i(x_i) + \sum_{F} \theta_F(\boldsymbol{x}_F) \right)$$

i slave

i∈F

F slave

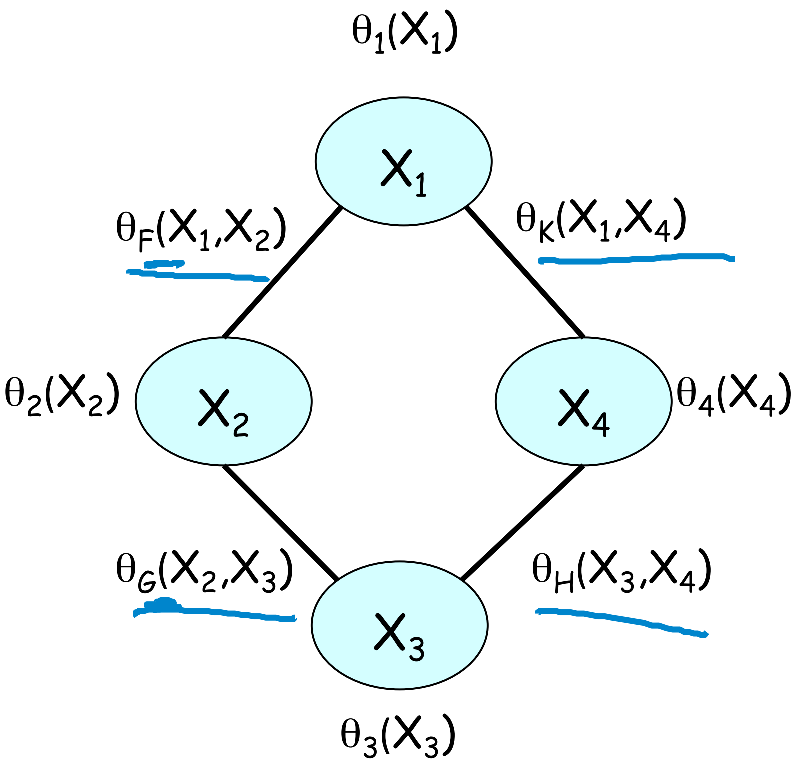
$$= \max_{\boldsymbol{x}} \left(\sum_{i=1}^{n} \left(\theta_i(x_i) + \sum_{F:i\in F} \lambda_{Fi}(x_i)\right) + \sum_{F} \left(\theta_F(\boldsymbol{x}_F) - \sum_{i\in F} \lambda_{Fi}(x_i)\right) \right)$$

agree with i slaves

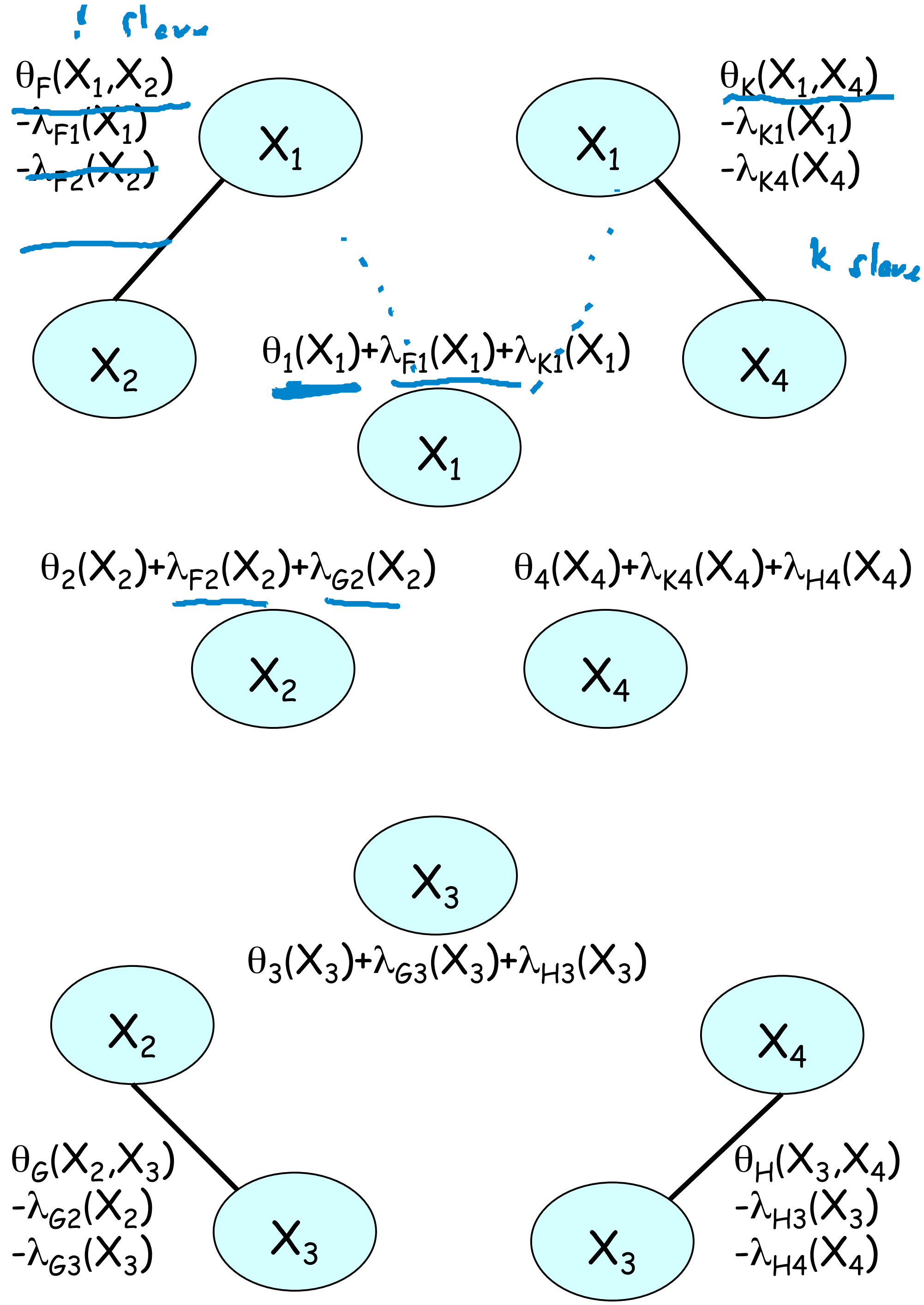
messages between F and i

$$L(\boldsymbol{\lambda}) = \sum_{i=1}^{n} \underbrace{\max_{x_i} \left(\theta_i(x_i) + \sum_{F:i\in F} \lambda_{Fi}(x_i) \right)}_{\bar{\theta}_i^{\lambda}} + \sum_{F} \underbrace{\max_{\boldsymbol{x}_F} \left(\theta_F(\boldsymbol{x}_F) - \sum_{i\in F} \lambda_{Fi}(x_i) \right)}_{\bar{\theta}_F^{\lambda}}$$

L(λ) is upper bound on MAP(θ) for any setting of λ's



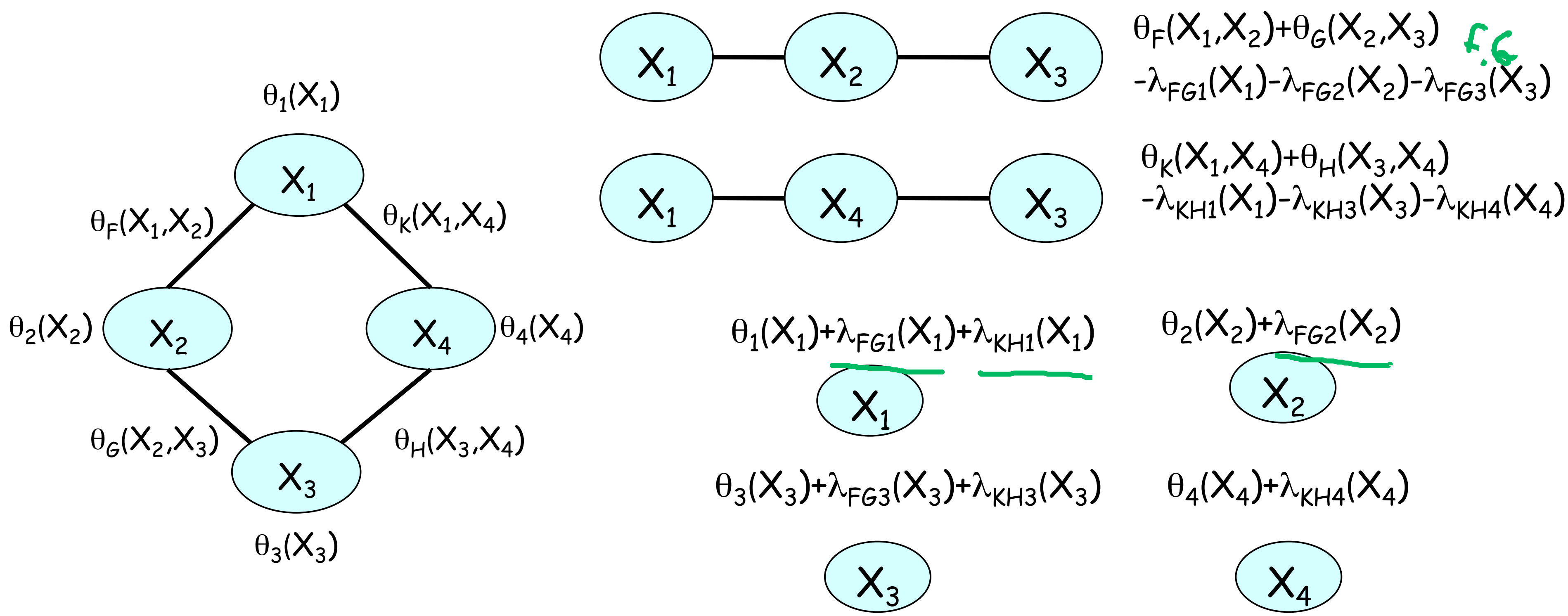
$\theta_1(X_1)$
X_1
$\theta_F(X_1,X_2)$
$\theta_K(X_1,X_4)$
$\theta_2(X_2)$
X_2
X_4
$\theta_4(X_4)$
$\theta_G(X_2,X_3)$
$\theta_H(X_3,X_4)$
X_3
$\theta_3(X_3)$



F slave
$\theta_F(X_1,X_2)$
$-\lambda_{F1}(X_1)$
$-\lambda_{F2}(X_2)$
X_1
X_2
X_1
$\theta_K(X_1,X_4)$
$-\lambda_{K1}(X_1)$
$-\lambda_{K4}(X_4)$
k slave
X_4
$\theta_1(X_1)+\lambda_{F1}(X_1)+\lambda_{K1}(X_1)$
X_1
$\theta_2(X_2)+\lambda_{F2}(X_2)+\lambda_{G2}(X_2)$
X_2
$\theta_4(X_4)+\lambda_{K4}(X_4)+\lambda_{H4}(X_4)$
X_4
X_3
$\theta_3(X_3)+\lambda_{G3}(X_3)+\lambda_{H3}(X_3)$
X_2
$\theta_G(X_2,X_3)$
$-\lambda_{G2}(X_2)$
$-\lambda_{G3}(X_3)$
X_3
X_4
$\theta_H(X_3,X_4)$
$-\lambda_{H3}(X_3)$
$-\lambda_{H4}(X_4)$
X_3

Divide and Conquer

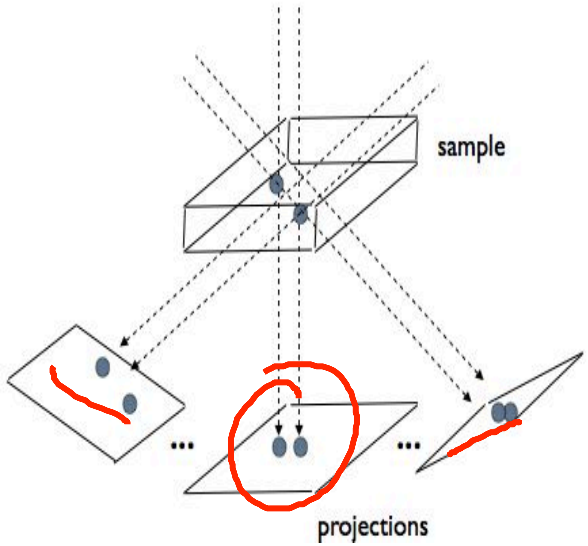
- Slaves don't have to be factors in original model
  - Subsets of factors that admit tractable solution to local maximization task



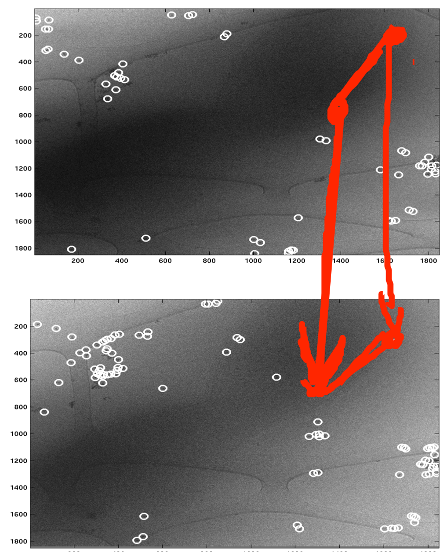
Divide and Conquer

- In pairwise networks, often divide factors into set of disjoint trees
  - Each edge factor assigned to exactly one tree
- Other tractable classes of factor sets
  - Matchings
  - Associative models
  - …

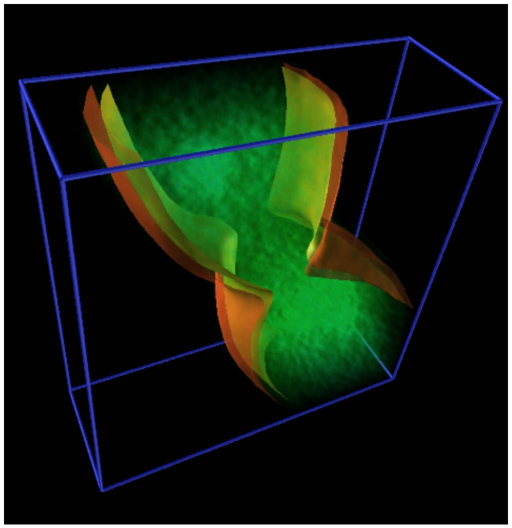
Example: 3D Cell Reconstruction



correspond tilt images



compute 3D reconstruction



matching

- Matching weights: similarity of location and local neighborhood appearance
- Pairwise potentials: approximate preservation of relative marker positions across images

Duchi, Tarlow, Elidan, and Koller, NIPS 2006. Amat, Moussavi, Comolli, Elidan, Downing, Horowitz, Journal of Strurctural Biology, 2006.