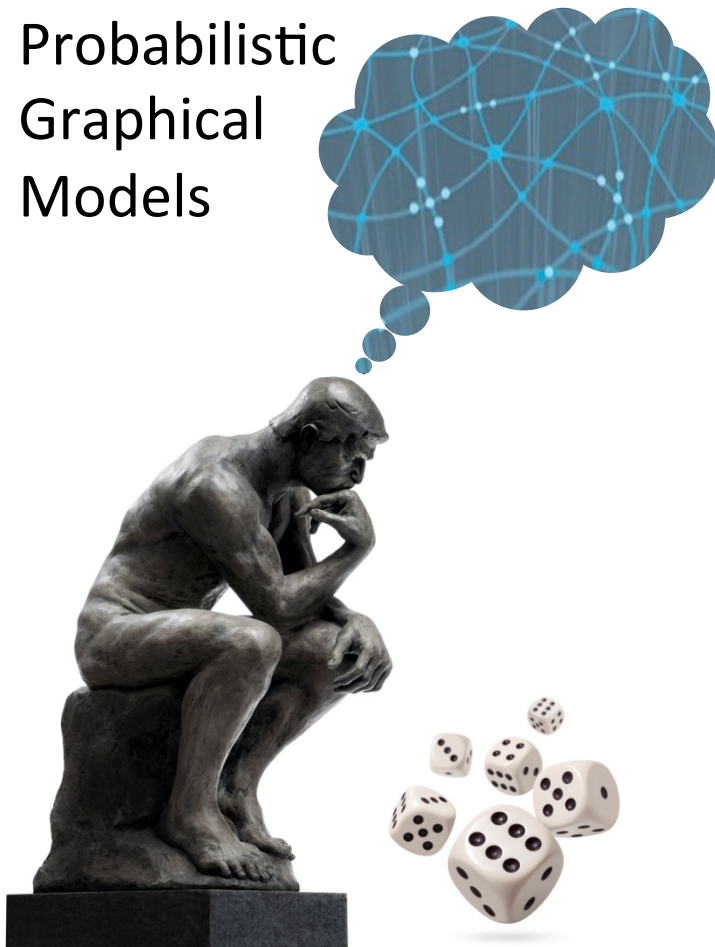


Probabilistic  
Graphical  
Models



Acting

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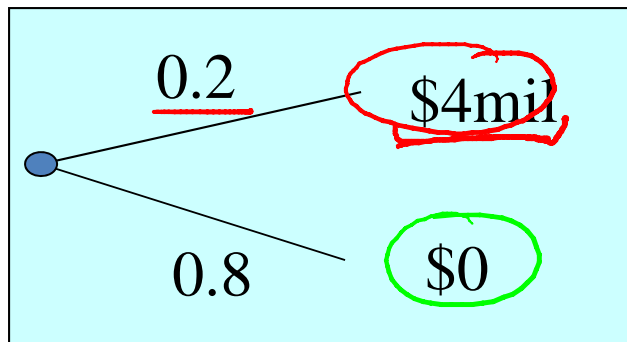
Decision Making

---

Utility  
Functions

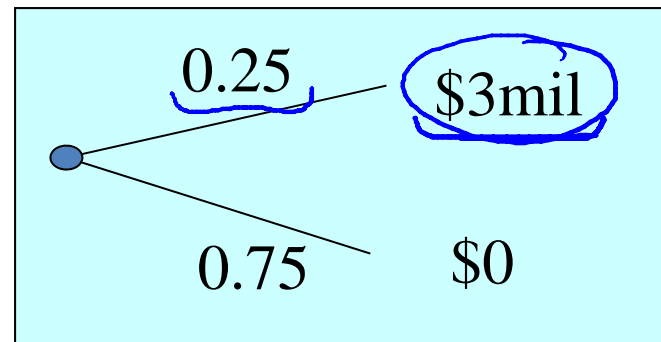
# Utilities and Preferences

lotteries



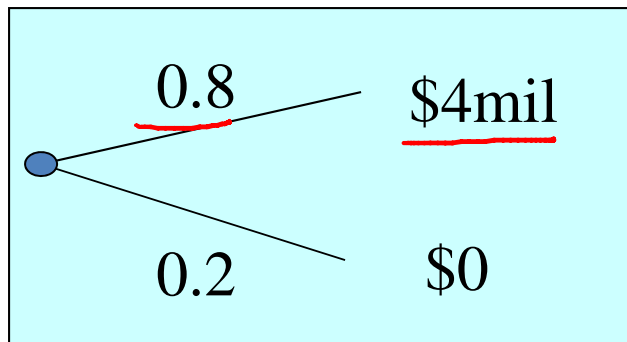
$$0.2 \times u(4) + 0.8 u(0)$$

$\sim$



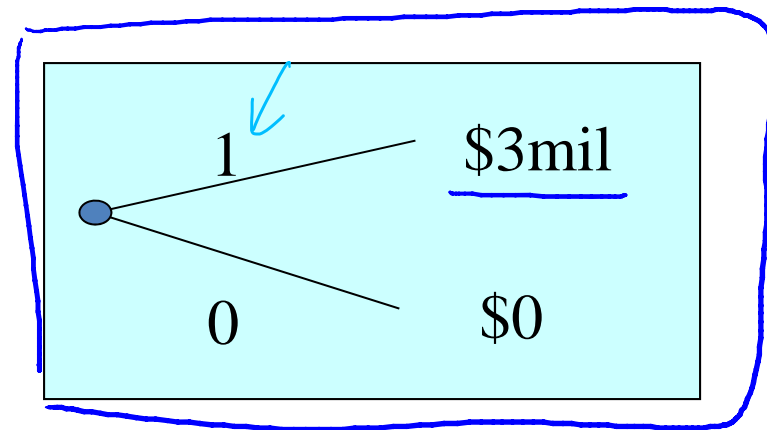
$$0.25 u(3) + 0.75 u(0)$$

# Utility = Payoff?



$$\begin{aligned} &\$4\text{mil} \times 0.8 = \\ &\quad \underline{\underline{\$3.2\text{mil}}} \end{aligned}$$

$\approx$



$\$3\text{mil}$

每次赢  $2^n$  元, 你愿意玩这个游戏一次

# St. Petersburg Paradox



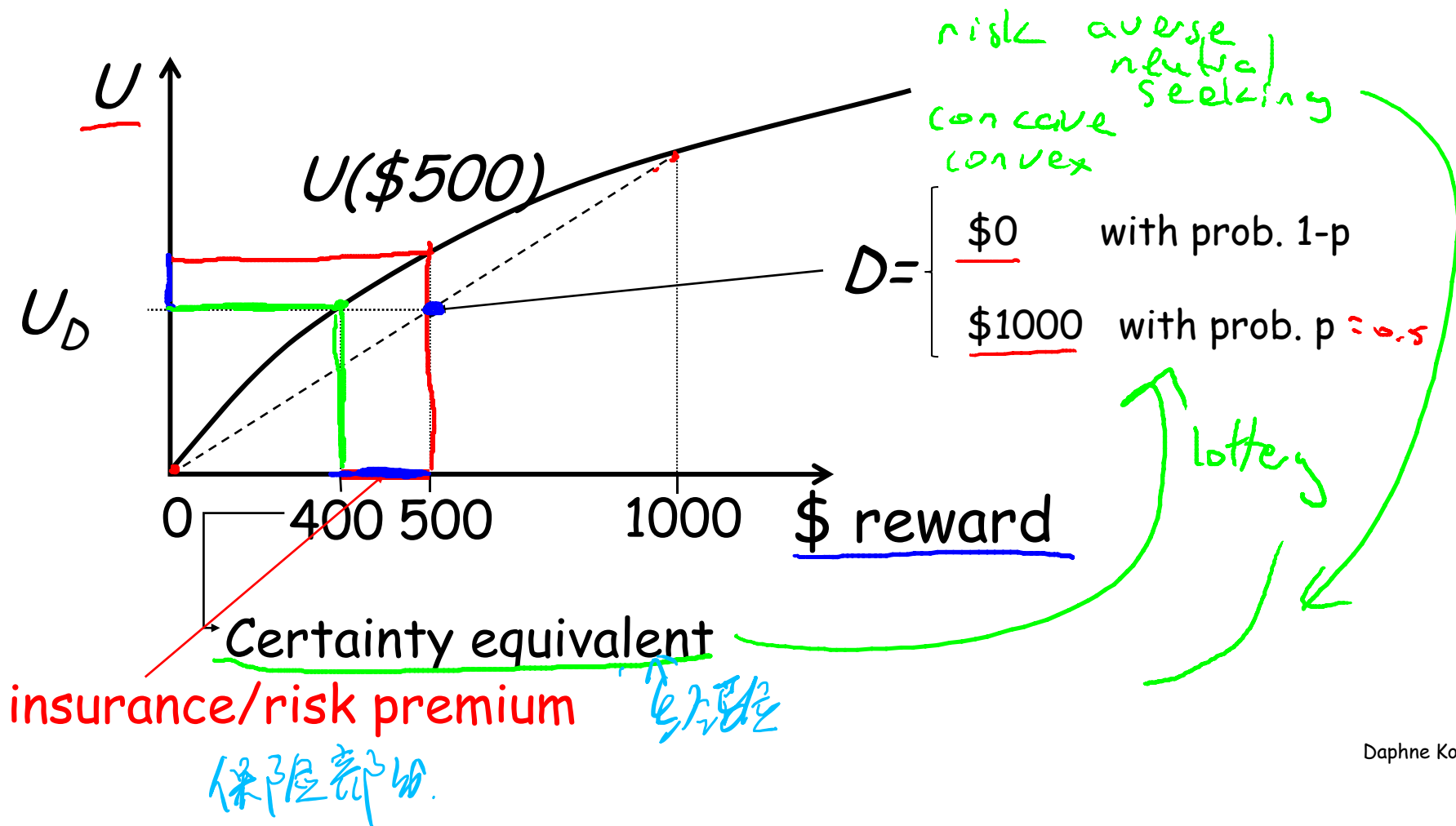
- Fair coin is tossed repeatedly until it comes up heads, say on the  $n^{\text{th}}$  toss
- Payoff =  $\$2^n$

$$\frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \dots = \infty$$

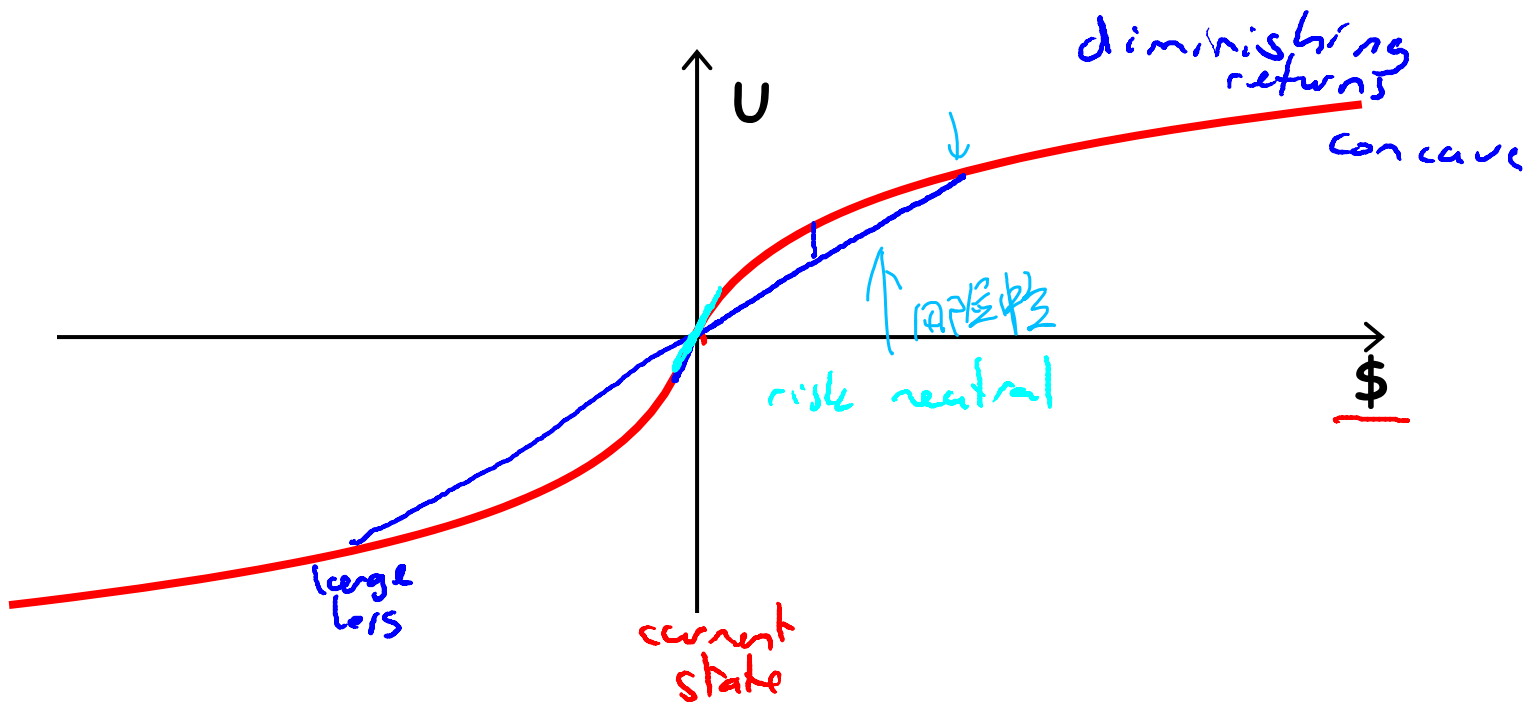
理论愿意

most people value  $\approx \$2$

实际愿意



# Typical Utility Curve



# Multi-Attribute Utilities

- All attributes affecting preferences must be integrated into one utility function

money, time, pleasure, ...

- Human life

- Micromorts

362天 24小时 2-1/4天  
 $1/1000000$  chance of death  $\approx \$20$  1980

- QALY (quality-adjusted life year)

质量与长度

生命

# Example: Prenatal diagnosis

$$\underline{U_1(T)} + \underline{U_2(K)} + \underline{U_3(D,L)} + \underline{U_4(L,F)} \quad \text{降低风险.}$$

Testing

Knowledge

Down's  
syndrome

Loss of  
fetus

Future  
pregnancy



# Summary

- Our utility function determines our preferences about decisions that involve uncertainty
- Utility generally depends on multiple factors
  - Money, time, chances of death, ...
- Relationship is usually nonlinear
  - Shape of utility curve determines attitude to risk
- Multi-attribute utilities can help decompose 分解 high-dimensional function into tractable pieces