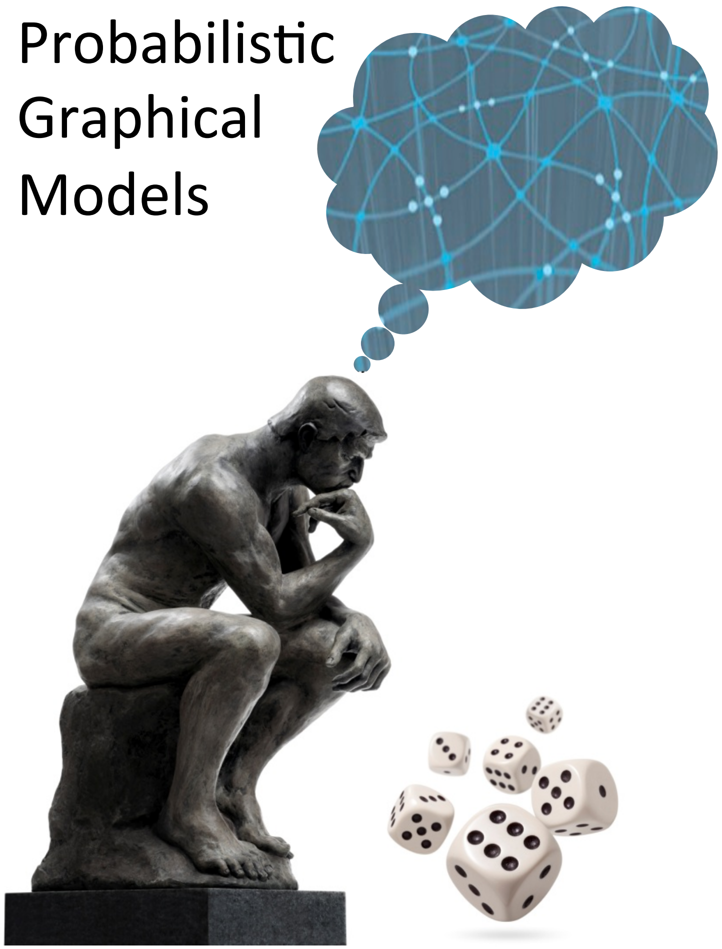
Probabilistic
Graphical
Models



Inference

Sampling Methods

Simple Sampling

Sampling-Based Estimation

independent, identically distributed

$\mathcal{D} = \{x[1], \ldots, x[M]\}$ sampled IID from P

If P(X=1) = p $= E_P[\mathbf{1}_{X=1}]$

fraction of 1's

Estimator for p: $T_{\mathcal{D}} = \frac{1}{M}\sum_{m=1}^{M} x[m]$

More generally, for any distribution P, function f:

indicator function

$$\boldsymbol{E}_P[f] \approx \frac{1}{M}\sum_{m=1}^{M} f(x[m])$$

f on samples

empirical expectation

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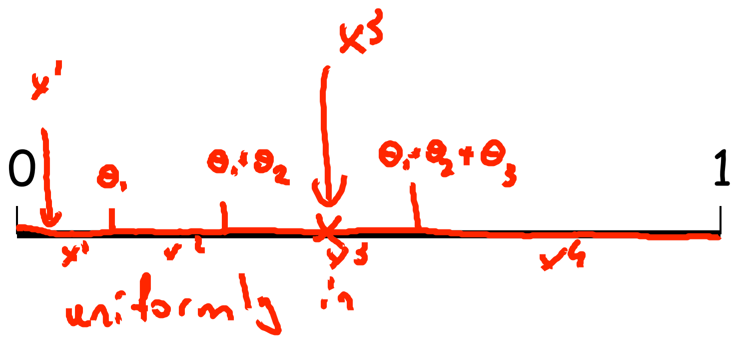
Sampling from Discrete Distribution

$Val(\underline{X}) = \{\underset{\theta^1}{x^1}, \ldots, \underset{\theta^k}{x^k}\}$ $\qquad$ $P(x^i) = \theta^i$



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Sampling-Based Estimation

$$T_{\mathcal{D}} = \frac{1}{M}\sum_{m=1}^{M} X[m]$$

Hoeffding Bound:

additive

\# samples

$$P_{\mathcal{D}}(T_{\mathcal{D}} \notin [p - \epsilon, p + \epsilon]) \leq 2e^{-2M\epsilon^2}$$

prob of a bad ~~data~~ sample set

estimator

is ε-away from p (

multiplicative

Chernoff Bound:

$$P_{\mathcal{D}}(T_{\mathcal{D}} \notin [p(1 - \epsilon), p(1 + \epsilon)]) \leq 2e^{-Mp\epsilon^2/3}$$

Sampling-Based Estimation

$$T_{\mathcal{D}} = \frac{1}{M}\sum_{m=1}^{M} X[m]$$

Hoeffding Bound:

$$P_{\mathcal{D}}(T_{\mathcal{D}} \notin [p-\epsilon, p+\epsilon]) \leq 2e^{-2M\epsilon^2} < \delta$$

For additive bound ε on error with probability > 1-δ:

$$\boxed{M \geq \frac{\ln(2/\delta)}{2\epsilon^2}}$$

Chernoff Bound:

$$P_{\mathcal{D}}(T_{\mathcal{D}} \notin [p(1-\epsilon), p(1+\epsilon)]) \leq 2e^{-Mp\epsilon^2/3}$$

For multiplicative bound ε on error with probability > 1-δ:

$$\boxed{M \geq 3\frac{\ln(2/\delta)}{p\epsilon^2}}$$

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Forward sampling from a BN

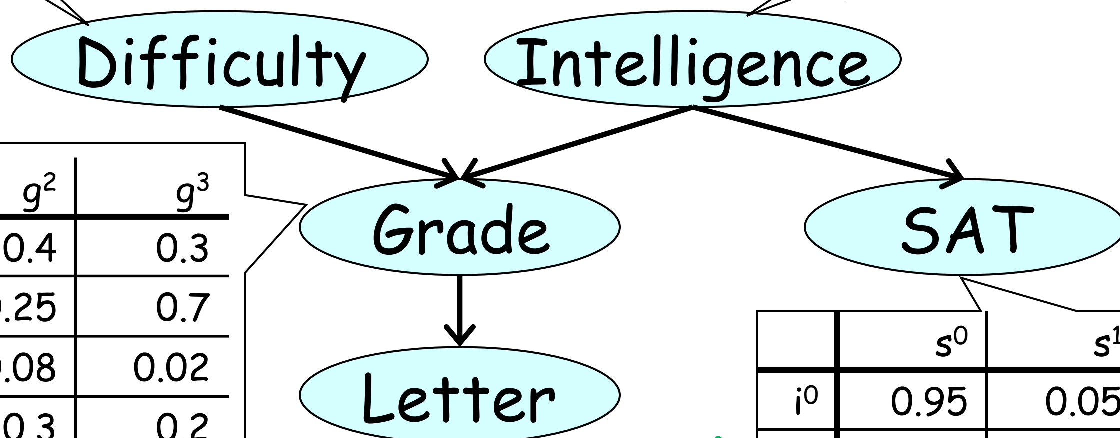
$P(D, I, G, S, L)$

d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

$d^0\ i^1\ g^1\ s^0\ l^1$

$d^1\ i^1\ g^2\ s^1\ l^0$



	g^1	g^2	g^3
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^1,d^0	0.9	0.08	0.02
i^1,d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

topological order
parents before children

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Forward Sampling for Querying

- Goal: Estimate P(Y=y)
  - Generate samples from BN
  - Compute fraction # samples where Y=y

For additive bound ε on error with probability > 1-δ: $M \geq \frac{\ln(2/\delta)}{2\epsilon^2}$

For multiplicative bound ε on error with probability > 1-δ: $M \geq 3\frac{\ln(2/\delta)}{P(y)\epsilon^2}$

Queries with Evidence

- Goal: Estimate P(Y=y | **E=e**)
- Rejection sampling algorithm
  - Generate samples from BN
  - Throw away all those where **E≠e**
  - Compute fraction where Y=y

remaining samples are sampled from P(| E=e)

Expected fraction of samples kept ~ P(**e**)

\# samples needed grows exponentially with # of observed variables

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Summary

- Generating samples from a BN is easy
- (ε,δ)-bounds exist, but usefulness is limited:
  - Additive bounds: useless for low probability events
  - Multiplicative bounds: # samples grows as $1/P(y)$
- With evidence, # of required samples grows exponentially with # of observed variables

- Forward sampling generally infeasible for MNs