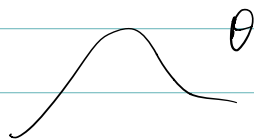


具体请看极大似然估计章节

单模型



$$\theta = \{\mu, \Sigma\}$$

$x \sim N(\mu, \Sigma)$

log似然函数

$$L(\theta | \bar{x}) = \log P(\bar{x} | \theta) = \sum \log P(x_i | \theta)$$

$$\bar{x} = \{x_1, \dots, x_n\}$$

$$x_i \stackrel{iid}{\sim} P(x | \theta)$$

MAP

$$P(\theta | x) \propto P(x | \theta) P(\theta)$$

\uparrow
后验

\uparrow
似然函数

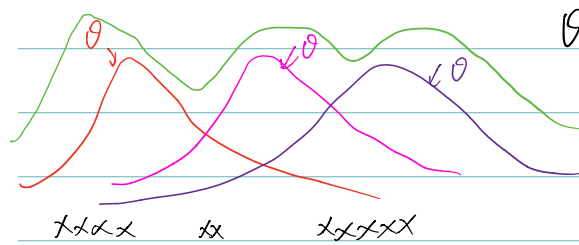
\uparrow
先验

$$\arg \max_{\theta} [\sum \log N(x_i | \mu, \Sigma)]$$

$$\mu_{MLE} = \frac{\partial \log(\mu, \Sigma | \bar{x})}{\partial \mu}$$

$$\Sigma_{MLE} = \frac{\partial \log(\mu, \Sigma | \bar{x})}{\partial \Sigma}$$

混合模型



$$\theta = \{\mu_1, \dots, \Sigma_1, \dots, \alpha_1, \dots, \alpha_{k-1}\}$$

$$P(x|\bar{\theta}) = \sum \alpha_k N(\mu_k, \Sigma_k) \quad \sum \alpha_k = 1.$$

$$\bar{\theta}_{MLE} = \underset{\theta}{\operatorname{argmax}} \left\{ \underbrace{\sum_{i=1}^N \log \left[\sum_{k=1}^K \alpha_k N(\mu_k, \Sigma_k) \right]}_{\ell(\theta|\bar{x})} \right\}$$

因为含有相加, 直接求解困难极大,
可以利用EM, 分开求解



